



General Instructions

- (i) The test is of 1 hour duration.
- (ii) The question paper consists of 50 questions. The maximum marks are 250.
- (iii) 5 marks are awarded for every correct answer, and 1 mark is deducted for every wrong answer.

1. Let A be a non-singular 3×3 matrix satisfying $A^3 - 6A^2 + 11A - 6I = O$. If $B = A^2 - 5A + 7I$, then find $\det(B)$ given that $\det(A) = 6$.

- (A) 1
- (B) 8
- (C) 27
- (D) 64

2. If $\begin{vmatrix} x+a & y & z \\ x & y+b & z \\ x & y & z+c \end{vmatrix} = abc$, where $a, b, c \neq 0$, then find the value of $\frac{x}{a} + \frac{y}{b} + \frac{z}{c}$.

- (A) 0
- (B) 1
- (C) 2
- (D) 3

3. If $y = \left(\frac{x+1}{x-1}\right)^x$, then find $\frac{dy}{dx}$.

(A) $y \left[\ln \left(\frac{x+1}{x-1} \right) - \frac{2x}{x^2-1} \right]$

(B) $y \left[\ln \left(\frac{x+1}{x-1} \right) + \frac{2x}{x^2-1} \right]$

(C) $y [\ln(x+1) - \ln(x-1)]$

(D) $\frac{2y}{x^2-1}$

4. Evaluate: $\int \frac{x^2+1}{x^4+1} dx$

(A) $\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{x^2-1}{\sqrt{2}x} \right) + C$

(B) $\tan^{-1} x + C$

(C) $\frac{1}{2} \ln(x^2+1) + C$

(D) $\frac{x}{x^2+1} + C$

5. Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$, and $\vec{c} = \lambda\hat{i} + \mu\hat{j} + 3\hat{k}$. If $[\vec{a} \ \vec{b} \ \vec{c}] = 0$ and $\vec{c} \cdot (\vec{a} + \vec{b}) = 10$, then find the value of $\lambda + \mu$.

(A) 1

(B) 2

(C) 3

(D) 4

6. Find the equation of the plane which passes through the point $(1, -2, 3)$, contains the line of intersection of the planes $x + y + z = 1$ and $2x - y + 3z = 4$, and is perpendicular to the plane $x - 2y + 2z + 5 = 0$.

- (A) $5x - y + z - 10 = 0$
(B) $x + y + z - 1 = 0$
(C) $2x - y + 3z - 4 = 0$
(D) $3x + y - z + 2 = 0$
-

7. Let A and B be two 3×3 matrices such that $A^2 - 4A + 3I = O$ and $B = A^{-1} + 2A$. Find the determinant of B if $\det(A) = 3$.

- (A) 9
(B) 27
(C) 81
(D) 3
-

8. If $f(x) = \left(\frac{1+\sin x}{1-\sin x}\right)^{\tan x}$, then find $\lim_{x \rightarrow 0} \frac{\ln f(x)}{x^2}$.

- (A) 1
(B) 2
(C) 4
(D) 0
-

9. If $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = \hat{i} + 3\hat{j} - \hat{k}$, and vector $\vec{c} = \lambda\hat{i} + 2\hat{j} + \mu\hat{k}$ is perpendicular to both $\vec{a}$ and $\vec{b}$, then find the value of $\lambda + \mu$.

- (A) -4
(B) -2
(C) 2
(D) 4
-

10. If $y = (x^{\sin x})^{\tan x}$, then find $\frac{dy}{dx}$.

- (A) $y[\sec^2 x \sin x \ln x + \tan x \cos x \ln x + \frac{\tan x \sin x}{x}]$
(B) $y(\cos x + \sin x)$
(C) $y \tan x$
-

(D) $x^{\sin x}$
