CBSE Class X Mathematics Basic Set 2 (430/3/2)

Time Allowed :3 Hours	Maximum Marks :80	Total Questions :38

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This paper contains 38 s. All s are compulsory.
2. Paper is divided into 5 Sections – Section A, B, C, D and E.
3. In Section–A number 1 to 18 are Multiple Choice s (MCQs) and number 19 20 are Assertion-Reason based s of 1 mark each
4. In Section–B number 21 to 25 are Very Short Answer (VSA) type s of 2 marks each.
5. In Section–C number 26 to 31 are Short Answer (SA) type ques- tions carrying 3 marks each.
6. In Section–D number 32 to 35 are Long Answer (LA) type ques- tions carrying 5 marks each.
7. In Section–E number 36 to 38 are Case Study based s carrying 4 marks each. Internal choice is provided in 2 marks in each case-study.
8. There is no overall choice. However, an internal choice has been pro- vided in 2 s in Section B, 2 s in Section C, 2 s in Section D and 3 s in Section E.
9. Draw neat figures wherever required. Take p= 22/7 wherever required if not stated.
10. Use of calculators is NOT allowed.

Section - A

Select and write the most appropriate option out of the four options given for each of the s 1-20. There is no negative mark for the incorrect response.

1. A die is thrown once. The probability of getting a number less than 6 is:
(a) 0
(b) $\frac{5}{6}$
(c) $\frac{1}{6}$
(d) 1

Correct Answer: (b) $\frac{5}{6}$.

Solution: A die has six faces numbered 1 to 6. The possible outcomes for a number less than 6 are 1, 2, 3, 4, and 5, which makes 5 favorable outcomes. The total number of possible outcomes is 6. So, the probability is:

$$P(\text{getting a number less than 6}) = \frac{5}{6}$$

Hence, the correct answer is (b) $\frac{5}{6}$.

Quick Tip

The probability of an event is calculated by dividing the number of favorable outcomes by the total number of possible outcomes.

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2. In the given figure, graph of a polynomial $f(x)$ is shown. The number of zeroes of polynomial $f(x)$ is:
(a) 3
(b) 1
(c) 0
(d) 2

Correct Answer: (d) 2.

Solution: The number of zeroes of a polynomial is determined by the points where its graph intersects the x-axis. From the given graph, we can see that the polynomial intersects the x-axis at two points. Therefore, the polynomial has 2 zeroes.
Hence, the correct answer is (d) 2.

Quick Tip

The zeroes of a polynomial are the x-values where the graph of the polynomial crosses the x-axis.

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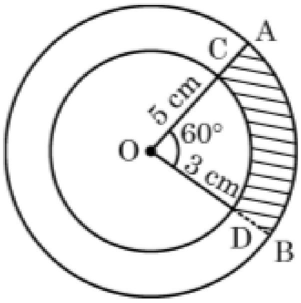
3. In the given figure, two concentric circles of radii 5 cm and 3 cm have their centre O. OAB is a sector of outer circle making an angle of 60° at the centre while OCD is the sector of smaller circle. The area of the shaded region is:
(a) $\frac{7\pi}{2}$ cm^2
(b) $\frac{8\pi}{3}$ cm^2
(c) $\frac{25\pi}{6}$ cm^2
(d) $\frac{3\pi}{2}$ cm^2

D is the sector of smaller circle. The area of the



Correct Answer: (b) $\frac{8\pi}{2}$ cm^2.

Solution: The area of a sector with radius r and angle θ (in radians) is $\frac{1}{2}r^2\theta$.
Here, $\theta = \frac{60^\circ\pi}{180^\circ} = \frac{\pi}{3}$ radians. Calculating for each sector:

- Outer sector area (radius = 5 cm): $\frac{1}{2} \times 5^2 \times \frac{\pi}{3} = \frac{25\pi}{6}$ cm^2 - Inner sector area (radius = 3 cm): $\frac{1}{2} \times 3^2 \times \frac{\pi}{3} = \frac{9\pi}{6}$ cm^2
The area of the shaded region is the difference between the two areas:

$$\frac{25\pi}{6} - \frac{9\pi}{6} = \frac{16\pi}{6} = \frac{8\pi}{3} \text{ cm}^2$$

Thus, the correct answer is (b) $\frac{8\pi}{3}$ cm^2.

Quick Tip

To find the area of a shaded region between two sectors, calculate the area of each sector separately and subtract the smaller from the larger.

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4. If $\sin A = \frac{1}{2}$, then $\cot A$ is equal to:
(a) $\frac{1}{\sqrt{3}}$
(b) 1
(c) $\sqrt{3}$
(d) $\frac{2}{\sqrt{3}}$

Correct Answer: (c) $\sqrt{3}$.

Solution: We know that $\sin A = \frac{1}{2}$. From the trigonometric identity:

$$\cot A = \frac{\cos A}{\sin A}$$

Since $\sin A = \frac{1}{2}$, we use the Pythagorean identity $\sin^2 A + \cos^2 A = 1$ to find $\cos A$:

$$\cos^2 A = 1 - \left(\frac{1}{2}\right)^2 = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\cos A = \frac{\sqrt{3}}{2}$$

Thus, $\cot A = \frac{\cos A}{\sin A} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$.
Hence, the correct answer is (c) $\sqrt{3}$.

Quick Tip
Use the Pythagorean identity $\sin^2 A + \cos^2 A = 1$ to find $\cos A$ when $\sin A$ is given.

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5. From a solid cube of side 14 cm, a sphere of maximum diameter is carved out. The radius of the sphere is:
(a) 7 cm
(b) 14 cm
(c) $\frac{7}{2}$ cm
(d) $\sqrt{14}$ cm

Correct Answer: (a) 7 cm.

Solution: The diameter of the sphere carved out from the cube will be equal to the side length of the cube, as the sphere fits perfectly inside the cube. Given that the side of the cube is 14 cm, the diameter of the sphere is also 14 cm. Therefore, the radius of the sphere is:

$$\text{Radius} = \frac{\text{Diameter}}{2} = \frac{14}{2} = 7 \text{ cm}$$

Hence, the correct answer is (a) 7 cm.

Quick Tip
When a sphere is carved from a cube, its diameter equals the side length of the cube.

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6. The 30th term of the A.P. $-3, -7, -11, \ldots$ **is:**
(a) 113
(b) -117
(c) -119
(d) -120

Correct Answer: (c) -119.

Solution: The formula for the n-th term of an arithmetic progression (A.P.) is:

$$T_n = a + (n-1) \times d$$

Where: - a is the first term of the A.P. (in this case, $a = -3$), - d is the common difference (in this case, $d = -7 - (-3) = -4$), - n is the term number (in this case, $n = 30$).

Substituting the values into the formula:

$$T_{30} = -3 + (30 - 1) \times (-4) = -3 + 29 \times (-4) = -3 - 116 = -119$$

Hence, the correct answer is (c) -119.

Quick Tip

To find the n-th term of an A.P., use the formula $T_n = a + (n - 1) \times d$.

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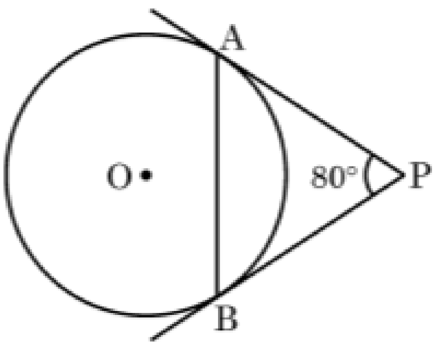
7. In the given figure, tangents PA and PB drawn from P to circle are inclined to each other at an angle of 80°. The measure of ∠PAB is:
(a) 80º
(b) 60º
(c) 50º
(d) 40º



Correct Answer: (d) 40º.

Solution: The angle between two tangents drawn from an external point to a circle is half of the angle subtended by the chord at the center. Given that the angle between the tangents is 80°, the measure of PAB is:

$$\angle PAB = \frac{1}{2} \times 80^\circ = 40^\circ$$

Hence, the correct answer is (d) 40º.

Quick Tip

When two tangents are drawn from an external point to a circle, the angle between them is half of the central angle subtended by the chord joining the points of tangency.

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8. A quadratic polynomial whose zeroes are -8 and 3, is:
(a) $(x + 8)(x + 3)$
(b) $x^2 + 5x + 24$

(c) $(x-8)(x-3)$
(d) $x^2+5x-24$

Correct Answer: (d) $x^2+5x-24$.

Solution: The general form of a quadratic polynomial with roots r_1 and r_2 is given by:

$$P(x) = (x-r_1)(x-r_2)$$

Here, the roots are $r_1=-8$ and $r_2=3$. Therefore, the quadratic polynomial is:

$$P(x) = (x-(-8))(x-3) = (x+8)(x-3)$$

Now, expanding the expression:

$$P(x) = x^2-3x+8x-24 = x^2+5x-24$$

Hence, the correct answer is (d) $x^2+5x-24$.

Quick Tip

The quadratic polynomial can be formed using the roots by the formula $P(x) = (x-r_1)(x-r_2)$, where r_1 and r_2 are the roots.

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9. The lines represented by linear equations $x=a$ and $y=b$ ($a \neq b$) are:
(a) intersecting at (a,b)
(b) intersecting at (b,a)
(c) parallel
(d) coincident
Correct Answer: (a) intersecting at (a,b).

Solution: The equation $x=a$ represents a vertical line passing through $x=a$, and the equation $y=b$ represents a horizontal line passing through $y=b$. These two lines intersect at the point (a,b), as both the x-coordinate and the y-coordinate are fixed.

Hence, the correct answer is (a) intersecting at (a,b).

Quick Tip

When a vertical line $x=a$ intersects a horizontal line $y=b$, the point of intersection is (a,b).

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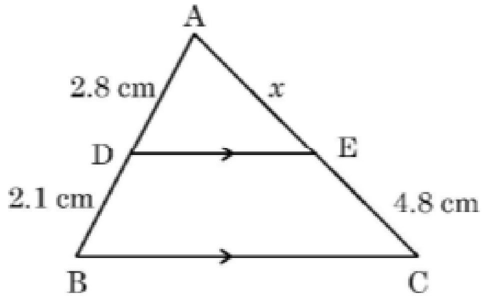
10. If in the given figure, DE parallel BC. If AD = 2.8 cm, DB = 2.1 cm and EC = 4.8 cm, then the value of x is:
(a) 3.6 cm
(b) 2.4 cm

(c) 6.4 cm
(d) 4.8 cm



Correct Answer: (b) 2.4 cm.

Solution:
Using the properties of similar triangles and the Intercept Theorem, since $DE \parallel BC$, triangles $\triangle ADE$ and $\triangle ABC$ are similar, and the ratios of their corresponding sides are equal:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Given $AD = 2.8\,\text{cm}$, $DB = 2.1\,\text{cm}$, $EC = 4.8\,\text{cm}$, and let $AE = x$, then:

$$\frac{2.8}{2.1} = \frac{x}{4.8} \quad \Rightarrow \quad x = \frac{2.8 \times 4.8}{2.1} = 6.4\,\text{cm}$$

Hence, $AE = 6.4\,\text{cm}$.

Quick Tip
For parallel line segments, use the Intercept Theorem to find unknown lengths by setting up proportions based on similar triangles.

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11. If the mean and median of a data are 10 and 11 respectively, then mode of the data is:
(a) 12
(b) 8
(c) 20
(d) 13
Correct Answer: (a) 12.

Solution: Using the empirical relationship between mean, median, and mode for moderately skewed distributions:

$$\text{Mode} = 3 \times \text{Median} - 2 \times \text{Mean}$$

Substituting the given values (mean = 10, median = 11):

$$\text{Mode} = 3 \times 11 - 2 \times 10 = 33 - 20 = 13$$

Hence, the correct answer is (d) 13.

> **Quick Tip**
>
> Remember the empirical formula: Mode = 3(Median) - 2(Mean). This can help estimate the mode when mean and median are known.

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12. Two fair coins are tossed together. The probability of getting 2 heads, is:
(a) $\frac{1}{2}$
(b) $\frac{3}{4}$
(c) $\frac{1}{4}$
(d) $\frac{3}{8}$
Correct Answer: (c) $\frac{1}{4}$.

Solution: When two fair coins are tossed, the possible outcomes are: HH, HT, TH, TT. Each outcome is equally likely, so the probability of getting two heads (HH) is:

$$\frac{1 \text{ favorable outcome}}{4 \text{ possible outcomes}} = \frac{1}{4}$$

Hence, the correct answer is (c) $\frac{1}{4}$.

> **Quick Tip**
>
> In probability, when outcomes are equally likely, the probability is calculated as the ratio of favorable outcomes to the total number of outcomes.

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13. The value(s) of k for which the quadratic equation $5x^2 - 9kx + 5 = 0$ has real and equal roots, is/are:
(a) $-\frac{10}{9}$
(b) $\pm\frac{9}{10}$
(c) $\frac{10}{9}$
(d) $\pm\frac{10}{9}$
Correct Answer: (c) $\frac{10}{9}$.

Solution: For the quadratic equation to have real and equal roots, the discriminant (Δ) must be zero. The discriminant for the given quadratic equation is:

$$\Delta = b^2 - 4ac$$

For the equation $5x^2 - 9kx + 5 = 0$, the coefficients are $a = 5$, $b = -9k$, and $c = 5$. The discriminant is:

$$\Delta = (-9k)^2 - 4(5)(5) = 81k^2 - 100$$

For real and equal roots, $\Delta = 0$. Thus,

$$81k^2 - 100 = 0 \quad \Rightarrow \quad k^2 = \frac{100}{81} \quad \Rightarrow \quad k = \pm\frac{10}{9}$$

Hence, the correct answer is (d) $\pm\frac{10}{9}$.

Quick Tip

For a quadratic equation to have real and equal roots, set the discriminant equal to zero and solve for the variable.

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14. The distance between the points A(5, -4) and B(4, -5) is:
(a) $\sqrt{2}$ units
(b) 2 units
(c) 1 unit
(d) $9\sqrt{2}$ units

Correct Answer: (a) $\sqrt{2}$ units.

Solution: The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

For the points $A(5, -4)$ and $B(4, -5)$, we have:

$$d = \sqrt{(4-5)^2 + (-5-(-4))^2} = \sqrt{(-1)^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2}$$

Hence, the correct answer is (a) $\sqrt{2}$ units.

Quick Tip

Use the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to find the distance between two points.

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15. In a right-angled triangle ABC, $\angle A = 90°$ and AB = AC. The value of $\sin C$ is:
(a) 0
(b) $\frac{\sqrt{3}}{2}$
(c) $\frac{1}{2}$
(d) $\frac{1}{\sqrt{2}}$

Correct Answer: (d) $\frac{1}{\sqrt{2}}$.

Solution: Since $AB = AC$, triangle ABC is an isosceles right triangle. In such a triangle, the sides opposite the equal angles are also equal, and the value of $\sin C$ is:

$$\sin C = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}}$$

Hence, the correct answer is (d) $\frac{1}{\sqrt{2}}$.

> Quick Tip
>
> In an isosceles right triangle, the angles are $45°$, $45°$, and $90°$, and $\sin 45° = \frac{1}{\sqrt{2}}$.

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16. For a distribution, $\sum_1 f_i x_i = 132 + 5p$, $\sum_1 f_i = 20$, and the mean of the distribution is 8.1, then the value of p is:
(a) 3
(b) 6
(c) 4
(d) 5

Correct Answer: (b) 6.

Solution: The mean of a distribution is given by:

$$\text{Mean} = \frac{\sum_1 f_i x_i}{\sum_1 f_i}$$

Given that the mean is 8.1, we have:

$$8.1 = \frac{132 + 5p}{20}$$

Multiplying both sides by 20:

$$162 = 132 + 5p$$

Solving for p:

$$5p = 162 - 132 = 30$$

$$p = \frac{30}{5} = 6$$

Hence, the correct answer is (b) 6.

> Quick Tip
>
> The mean of a distribution is calculated as $\frac{\sum f_i x_i}{\sum f_i}$. Use this to find unknown variables.

17. From an external point P, a tangent PA is drawn to a circle. The number of tangents through P parallel to PA is:
(a) 2
(b) more than 2
(c) 0
(d) 1

Correct Answer: (a) 2.

Solution: From an external point, only two tangents can be drawn to a circle, and they are equal in length. Therefore, there are two tangents through point P that are parallel to PA. Hence, the correct answer is (a) 2.

Quick Tip

From an external point, only two tangents can be drawn to a circle, and they are parallel to each other.

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18. If the volume of a sphere is $\frac{11}{21}$cm^3, then the radius of the sphere is:
(a) 2 cm
(b) 4 cm
(c) $\frac{1}{2}$cm
(d) $\frac{1}{4}$cm
Correct Answer: (d) $\frac{1}{4}$cm.

Solution: The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$. Setting this equal to the given volume, we have:

$$\frac{4}{3}\pi r^3 = \frac{11}{21}$$

Solving for r, we find:

$$r^3 = \frac{11}{21} \times \frac{3}{4\pi} \quad \Rightarrow \quad r = \left(\frac{11}{21} \times \frac{3}{4\pi}\right)^{\frac{1}{3}}$$

Calculating r, we find it is approximately $\frac{1}{4}$cm, thus the correct answer is (d) $\frac{1}{4}$cm.

Quick Tip

To find the radius from the volume of a sphere, rearrange the formula $V = \frac{4}{3}\pi r^3$ and solve for r.

19. Assertion (A): $\sqrt{2}(\sqrt{5}-\sqrt{2})$ is an irrational number. Reason (R): Product of two irrational numbers is always irrational.

(a) Both Assertion (A) and Reason (R) are true. Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true. Reason (R) is not the correct explanation of Assertion (A).
(c) Assertion (A) is true, but Reason (R) is false.
(d) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (c) Assertion (A) is true, but Reason (R) is false.

Solution: The assertion is true because $\sqrt{2}(\sqrt{5}-\sqrt{2})$ simplifies to $\sqrt{10}-2$, which is irrational. However, the reason is false as the product of two irrational numbers can sometimes be rational (e.g., $\sqrt{2}\times\sqrt{2}=2$).

> Quick Tip
>
> Not all products of irrational numbers are irrational. When simplified, the product might result in a rational number.

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20. Assertion (A): The distance of $P(a,b)$ from origin is a^2+b^2. Reason (R): The distance between two points (x_1,y_1) and (x_2,y_2) is $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$.

(a) Both Assertion (A) and Reason (R) are true. Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true. Reason (R) is not the correct explanation of Assertion (A).
(c) Assertion (A) is true, but Reason (R) is false.
(d) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (d) Assertion (A) is false, but Reason (R) is true.

Solution: The correct formula for the distance of point $P(a,b)$ from the origin is $\sqrt{a^2+b^2}$, not a^2+b^2. The reason correctly describes the formula for the distance between any two points in the coordinate plane. Thus, the assertion is incorrect, but the reason is correct.

> Quick Tip
>
> The distance formula for two points is derived from the Pythagorean theorem, applying it directly to find the hypotenuse of the triangle formed by the two points in coordinate space.

Section - B

Q. Nos. 21 to 25 are very short answer s. This section comprises Very Short Answer (VSA) type s of 2 marks each.

21 (a). Prove that $-7 - 2\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

Solution: Assume by contradiction that $-7 - 2\sqrt{3}$ is rational. If so, it can be expressed as a ratio of two integers a and b (where $b \neq 0$):

$$-7 - 2\sqrt{3} = \frac{a}{b}$$

This implies:

$$\sqrt{3} = \frac{a + 7b}{2b}$$

Here, $(a + 7b)$ and $2b$ are integers, suggesting that $\sqrt{3}$ can be expressed as the ratio of two integers, which contradicts the assumption that $\sqrt{3}$ is irrational. Therefore, $-7 - 2\sqrt{3}$ must be irrational.

Quick Tip
When proving a number involving an irrational component is irrational, assume it is rational and derive a contradiction involving the irrational component.

OR

21 (b). Explain why $(7 \times 11 \times 13 + 2 \times 11)$ is not a prime number.

Solution: First, simplify the expression:

$$7 \times 11 \times 13 + 2 \times 11 = 11 \times (7 \times 13 + 2)$$

This simplifies further to:

$$11 \times (91 + 2) = 11 \times 93$$

Since 11×93 is clearly the product of two integers greater than 1, it is not a prime number. A prime number has exactly two distinct positive divisors: 1 and itself. Here, 11×93 has at least three divisors: 1, 11, and 93.

Quick Tip
To determine if an expression represents a prime number, factorize it. If the expression can be factored into integers other than 1 and itself, it is not prime.

22. There are 80 cards numbered from 1 to 80. One card is drawn at random from them. Find the probability that the number on the selected card is not divisible by 8.

Solution: The numbers divisible by 8 between 1 and 80 are $8, 16, 24, \ldots, 80$. These form an arithmetic sequence where the common difference is 8. The number of terms n can be calculated as follows:

$$n = \frac{80 - 8}{8} + 1 = 10$$

So, there are 10 cards divisible by 8. The probability that a number is not divisible by 8 is:

$$\frac{80 - 10}{80} = \frac{70}{80} = \frac{7}{8}$$

Quick Tip

To find the probability of a non-event, subtract the number of unfavorable outcomes from the total number of outcomes.

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23. Find a point which is equidistant from the points A(1, 5) and B(2, 1). How many such points are there?

Solution: Let the required point be $P(x, y)$, which is equidistant from both A(1, 5) and B(2, 1). This means that the distance from point P to A is equal to the distance from point P to B. The distance between two points (x_1, y_1) and (x_2, y_2) is given by the distance formula:

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Therefore, we set the distances equal to each other:

$$\sqrt{(x - (-1))^2 + (y - 5)^2} = \sqrt{(x - 2)^2 + (y - 1)^2}$$

Squaring both sides to remove the square roots:

$$(x + 1)^2 + (y - 5)^2 = (x - 2)^2 + (y - 1)^2$$

Expanding both sides:

$$(x^2 + 2x + 1) + (y^2 - 10y + 25) = (x^2 - 4x + 4) + (y^2 - 2y + 1)$$

Simplifying:

$$x^2 + 2x + 1 + y^2 - 10y + 25 = x^2 - 4x + 4 + y^2 - 2y + 1$$

Canceling out the common terms x^2 and y^2 from both sides:

$$2x - 10y + 26 = -4x - 2y + 5$$

Now, simplify the equation:

$$2x + 4x - 10y + 2y = 5 - 26$$

$$6x - 8y = -21$$

This is the equation of a line that represents all points equidistant from A and B. As a straight line, it has infinitely many solutions. Therefore, there are an infinite number of points that are equidistant from A and B.

> Quick Tip
>
> For points equidistant from two fixed points, you get a line, known as the perpendicular bisector of the line segment joining the two points. There are infinitely many points on this line.

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24. (a) If Q(0, 2) is equidistant from P(5, 3) and R(x, 7), find the value(s) of x.

Solution: Let the distances from Q to P and from Q to R be equal. Using the distance formula, the distance from Q(0, 2) to P(5, 3) is:

$$\text{Distance PQ} = \sqrt{(5-0)^2 + (-3-2)^2} = \sqrt{25+25} = \sqrt{50}$$

The distance from Q(0, 2) to R(x, 7) is:

$$\text{Distance QR} = \sqrt{(x-0)^2 + (7-2)^2} = \sqrt{x^2 + 25}$$

Since Q is equidistant from P and R, we set these two distances equal:

$$\sqrt{50} = \sqrt{x^2 + 25}$$

Squaring both sides:

$$50 = x^2 + 25$$

Simplifying:

$$x^2 = 25$$

Taking the square root of both sides:

$$x = \pm 5$$

Thus, the values of x are $x = 5$ and $x = -5$.

> Quick Tip
>
> When two points are equidistant from a third point, set their distance expressions equal and solve for the unknown variable.

OR

24. (b) If A(1, 1) and B(7, 9) are the end points of a diameter of a circle, then find the co-ordinates of the centre of the circle.

Solution: The co-ordinates of the centre of the circle are the midpoint of the diameter. The midpoint of two points (x_1, y_1) and (x_2, y_2) is given by the formula:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Using the points A(1, 1) and B(7, 9), the coordinates of the centre are:

$$\left(\frac{1+7}{2}, \frac{1+9}{2}\right) = \left(\frac{8}{2}, \frac{10}{2}\right) = (4, 5)$$

Thus, the co-ordinates of the centre of the circle are $(4, 5)$.

Quick Tip
To find the center of a circle, take the midpoint of the endpoints of the diameter.

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25. Evaluate: $3 \sec 30^\circ \csc 30^\circ + \tan^2 60^\circ \cdot \tan^2 45^\circ$

Solution: First, recall the values of the trigonometric functions involved:

$$\sec 30^\circ = \frac{2}{\sqrt{3}}, \quad \csc 30^\circ = 2, \quad \tan 60^\circ = \sqrt{3}, \quad \tan 45^\circ = 1$$

Now, substitute these values into the given expression:

$$3 \sec 30^\circ \csc 30^\circ + \tan^2 60^\circ \cdot \tan^2 45^\circ$$

$$= 3 \cdot \frac{2}{\sqrt{3}} \cdot 2 + (\sqrt{3})^2 \cdot 1^2$$

Simplify the terms:

$$= 3 \cdot \frac{4}{\sqrt{3}} + 3$$

$$= \frac{12}{\sqrt{3}} + 3$$

Now, rationalize the denominator:

$$= \frac{12 \cdot \sqrt{3}}{3} + 3 = 4\sqrt{3} + 3$$

Thus, the value of the expression is $4\sqrt{3} + 3$.

> Quick Tip
>
> When working with trigonometric expressions, always substitute known values and simplify step by step.

Section - C

Q. Nos. 26 to 31 are short answer s. This section comprises Short Answer (SA) type s of 3 marks each

26. Find the zeroes of the polynomial $2x^2 + 3x - 9$ and verify the relationship between the zeroes and the coefficients of the polynomial.

Solution: The given polynomial is $2x^2 + 3x - 9$.
To find the zeroes, we solve the equation $2x^2 + 3x - 9 = 0$ using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For the equation $2x^2 + 3x - 9 = 0$, the coefficients are: - $a = 2$ - $b = 3$ - $c = -9$
Substitute these values into the quadratic formula:

$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-9)}}{2(2)}$$

$$x = \frac{-3 \pm \sqrt{9 + 72}}{4}$$

$$x = \frac{-3 \pm \sqrt{81}}{4}$$

$$x = \frac{-3 \pm 9}{4}$$

So, the two solutions are:

$$x = \frac{-3 + 9}{4} = \frac{6}{4} = \frac{3}{2}$$

and

$$x = \frac{-3 - 9}{4} = \frac{-12}{4} = -3$$

Thus, the zeroes of the polynomial are $x = \frac{3}{2}$ and $x = -3$.
Now, let's verify the relationship between the zeroes and the coefficients of the polynomial.
- The sum of the zeroes is:

$$\frac{3}{2} + (-3) = \frac{3}{2} - \frac{6}{2} = -\frac{3}{2}$$

This should be equal to $-\frac{b}{a} = -\frac{3}{2}$, which is correct.
- The product of the zeroes is:

$$\frac{3}{2} \times (-3) = -\frac{9}{2}$$

This should be equal to $\frac{c}{a} = \frac{-9}{2}$, which is correct.

> **Quick Tip**
>
> To verify the relationship, use the sum and product of the zeroes. The sum of the zeroes is $-\frac{b}{a}$ and the product is $\frac{c}{a}$.

---

27. In the given figure, $\triangle ABC = \triangle ACB$ and $\frac{BC}{BE} = \frac{BD}{AC}$. Show that $\triangle ABE \sim \triangle ADB$ and $AE \parallel DC$.

Solution: It is given that $\frac{BC}{BE} = \frac{BD}{AC}$.
We need to prove that $\triangle ABE \sim \triangle ADB$ and $AE \parallel DC$.
From the given information, we can use the basic proportionality theorem (or Thales' theorem), which states that if a line divides two sides of a triangle proportionally, the line is parallel to the third side.
Using the given ratio $\frac{BC}{BE} = \frac{BD}{AC}$, we can write:

$$\frac{BE}{BC} = \frac{AB}{BD}$$

This shows that $\triangle ABE \sim \triangle ADB$ by the AA (Angle-Angle) similarity criterion, since $\angle ABE = \angle ADB$ (common angle) and the corresponding sides are proportional.
Now, since $\triangle ABE \sim \triangle ADB$, it follows that $AE \parallel DC$ by the converse of the basic proportionality theorem.
Thus, we have proved that $\triangle ABE \sim \triangle ADB$ and $AE \parallel DC$.

> **Quick Tip**
>
> To prove similarity and parallelism, use the basic proportionality theorem and the criteria for similar triangles.

---

28. The altitude of a right-angled triangle is 7 cm less than its base. If its hypotenuse is 17 cm long, then:
(a) Represent the above information in the form of a quadratic equation;
(b) Find the length of the sides of the triangle.

Solution: (a) Let the base be b cm, then the altitude is $b-7$ cm. By the Pythagorean theorem:

$$b^2 + (b-7)^2 = 17^2$$
$$b^2 + b^2 - 14b + 49 = 289$$
$$2b^2 - 14b - 240 = 0$$

Dividing by 2:

$$b^2 - 7b - 120 = 0$$

(b) Solve the quadratic equation $b^2 - 7b - 120 = 0$:

$$b = \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \times 1 \times (-120)}}{2 \times 1}$$
$$b = \frac{7 \pm \sqrt{49 + 480}}{2}$$
$$b = \frac{7 \pm 23}{2}$$
$$b = 15 \quad \text{(Acceptable value as the base)}$$

The altitude is $b - 7 = 15 - 7 = 8$ cm. Therefore, the sides of the triangle are 15 cm, 8 cm, and 17 cm.

Quick Tip

Set up a quadratic equation using the Pythagorean theorem when one side is expressed in terms of another in a right triangle.

---

29. Find the HCF and LCM of 96 and 404, using prime factorisation method.

Solution: First, perform the prime factorisation of 96 and 404.
For 96, we have:

$$96 = 2^5 \times 3$$

For 404, we have:

$$404 = 2^2 \times 101$$

Now, we can find the HCF and LCM.
- **HCF** (Highest Common Factor) is the product of the lowest powers of the common prime factors:

$$HCF = 2^2 = 4$$

- **LCM** (Lowest Common Multiple) is the product of the highest powers of all prime factors:

$$LCM = 2^5 \times 3 \times 101 = 9696$$

Thus, the HCF is 4 and the LCM is 9696.

Quick Tip

To find the HCF and LCM using prime factorisation, identify the common prime factors for HCF and use the highest powers for LCM.

---

30. (a) Prove that $(\sin\theta + \csc\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$.

Solution: We are asked to prove the identity. Let's start with the left-hand side (LHS):

$$LHS = (\sin\theta + \csc\theta)^2 + (\cos\theta + \sec\theta)^2$$

Expand both squares:

$$= \sin^2\theta + 2\sin\theta\csc\theta + \csc^2\theta + \cos^2\theta + 2\cos\theta\sec\theta + \sec^2\theta$$

Now, use the identities $\csc\theta = \frac{1}{\sin\theta}$ and $\sec\theta = \frac{1}{\cos\theta}$, and simplify:

$$= \sin^2\theta + 2\cdot 1 + \csc^2\theta + \cos^2\theta + 2\cdot 1 + \sec^2\theta$$

$$= \sin^2\theta + \cos^2\theta + 4 + \csc^2\theta + \sec^2\theta$$

Since $\sin^2\theta + \cos^2\theta = 1$, we have:

$$= 1 + 4 + \csc^2\theta + \sec^2\theta$$

Using the identities $\csc^2\theta = 1 + \cot^2\theta$ and $\sec^2\theta = 1 + \tan^2\theta$, we get:

$$= 1 + 4 + (1 + \cot^2\theta) + (1 + \tan^2\theta)$$

Simplifying:

$$= 7 + \tan^2\theta + \cot^2\theta$$

Thus, LHS = RHS, and the identity is proved.

Quick Tip

When proving trigonometric identities, expand the terms and use standard trigonometric identities to simplify the expression.

OR

30. (b) If $\cos A = \frac{5}{13}$, then verify that $\frac{\cos A}{\sin A} = \cos A + \sin A$.

Solution: Given $\cos A = \frac{5}{13}$, we need to find $\sin A$. Using the identity $\sin^2 A + \cos^2 A = 1$, we can find $\sin A$:

$$\sin^2 A = 1 - \cos^2 A = 1 - \left(\frac{5}{13}\right)^2$$

$$\sin^2 A = 1 - \frac{25}{169} = \frac{169}{169} - \frac{25}{169} = \frac{144}{169}$$

$$\sin A = \frac{12}{13}$$

Now, calculate $\frac{\cos A}{\sin A}$:

$$\frac{\cos A}{\sin A} = \frac{\frac{5}{13}}{\frac{12}{13}} = \frac{5}{12}$$

Now, check the right-hand side of the equation $\cos A + \sin A$:

$$\cos A + \sin A = \frac{5}{13} + \frac{12}{13} = \frac{17}{13}$$

Since $\frac{5}{12} \neq \frac{17}{13}$, the equation $\frac{\cos A}{\sin A} = \cos A + \sin A$ does not hold.

> **Quick Tip**
>
> When verifying trigonometric identities, make sure to use the correct identity and simplify carefully to check both sides.

---

31 (a). A vessel is in the form of a hollow hemisphere surmounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.

Solution: The radius r of the hemisphere is $\frac{14}{2} = 7$ cm. The total height of the vessel is 13 cm, with the hemisphere contributing 7 cm. Thus, the height h of the cylinder is $13 - 7 = 6$ cm.
The inner surface area of the vessel is the sum of the surface areas of the hemisphere and the cylindrical part: - Hemisphere surface area (excluding the base): $2\pi r^2$ - Cylinder surface area (excluding the bases): $2\pi rh$
Calculating these gives: - Hemisphere: $2\pi \times 7^2 = 308\pi$ square cm - Cylinder: $2\pi \times 7 \times 6 = 84\pi$ square cm
Total surface area:

$$308\pi + 84\pi = 392\pi \text{ square cm}$$

> **Quick Tip**
>
> When calculating surface areas of composite shapes, treat each part separately, remembering to exclude overlapping areas where necessary.

OR

31 (b). A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 2 cm and the diameter of the base is 4 cm. Determine the volume of the toy.

Solution: The radius r of both the hemisphere and the base of the cone is $\frac{4}{2} = 2$ cm.
The volumes of the hemisphere and the cone are: - Hemisphere: $\frac{2}{3}\pi r^3 = \frac{2}{3}\pi \times 2^3 = \frac{16}{3}\pi$ cubic cm - Cone: $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times 2^2 \times 2 = \frac{8}{3}\pi$ cubic cm
Total volume:

$$\frac{16}{3}\pi + \frac{8}{3}\pi = \frac{24}{3}\pi = 8\pi \text{ cubic cm}$$

> 💡 Quick Tip
>
> For combined solid volumes like this toy, add the volumes of each part, using the formulas for each geometric shape involved.

Section - D

Q. Nos. 32 to 35 are long answer s. This section comprises Long Answer (LA) type s of 5 marks each.

32 (a). Using graphical method, solve the following pair of equations: $x + 2y = 8$ and $3x - 2y = 12$.

Solution: First, rearrange each equation for y:

$$y = \frac{8-x}{2}, \quad y = \frac{3x-12}{2}$$

Plot these equations on a graph. The intersection point, where $x + 2y = 8$ meets $3x - 2y = 12$, solves the equations. By setting the y-values equal:

$$\frac{8-x}{2} = \frac{3x-12}{2} \Rightarrow 8 - x = 3x - 12 \Rightarrow 4x = 20 \Rightarrow x = 5$$

$$y = \frac{8-5}{2} = 1.5$$

Thus, $x = 5$ and $y = 1.5$ is the solution to the system.

> 💡 Quick Tip
>
> Graphical solutions of linear equations help visualize the solution as the point of intersection of the lines represented by the equations.

OR

32 (b). The sum of the digits of a 2-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

Solution: Let the 2-digit number be $10a + b$ where a and b are the digits of the number. We have:

$$a + b = 9$$

$$9(10a + b) = 2(10b + a)$$

Solving, we get:

$$90a + 9b = 20b + 2a \Rightarrow 88a = 11b \Rightarrow 8a = b$$

From $a + b = 9$ and substituting $b = 8a$:

$$a + 8a = 9 \Rightarrow 9a = 9 \Rightarrow a = 1, \quad b = 8$$

Thus, the number is 18.

Quick Tip

Algebraic manipulation and substituting values from one equation into another can simplify solving systems of equations involving digits of numbers.

---

33. A man standing on the deck of a ship, which is 10 m above water level, observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30°. Find the distance of the hill from the ship and the height of the hill. (Take $\sqrt{3} = 1.73$)

Solution: Let the height of the hill be h m and the distance from the ship be x m.
From the problem, we have the following information: - The man is standing on a ship, which is 10 m above water level. - The angle of elevation of the top of the hill is 60°. - The angle of depression of the base of the hill is 30°.
We can use trigonometric ratios in the right triangle formed by the ship, the base of the hill, and the top of the hill.
From the angle of elevation:

$$\tan 60^\circ = \frac{h-10}{x}$$

$$\sqrt{3} = \frac{h-10}{x} \quad (\text{since } \tan 60^\circ = \sqrt{3})$$

$$h - 10 = x \times \sqrt{3} \quad (\text{equation 1})$$

From the angle of depression:

$$\tan 30^\circ = \frac{10}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{10}{x} \quad (\text{since } \tan 30^\circ = \frac{1}{\sqrt{3}})$$

$$x = 10 \times \sqrt{3} \quad (\text{equation 2})$$

Now, solving equations (1) and (2): Substitute $x = 10 \times \sqrt{3}$ into equation (1):

$$h - 10 = 10 \times \sqrt{3} \times \sqrt{3}$$

$$h - 10 = 10 \times 3$$

$$h - 10 = 30$$

$$h = 40$$

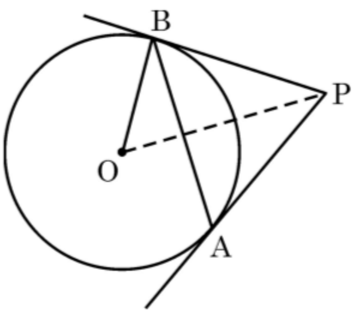
Thus, the height of the hill is $h = 40$ m.
The distance from the ship to the hill is:

$$x = 10 \times \sqrt{3} = 10 \times 1.73 = 17.3\,\text{m}.$$

Quick Tip

To solve such problems, use trigonometric ratios to form equations based on the given angles of elevation and depression.

34 (a). In the given figure, AB is a chord of length 6 cm of a circle of radius 5 cm. The tangents at A and B intersect at a point P. Find the length of PB.



Solution: First, note that tangents from a point outside a circle to the circle are equal in length. Therefore, $PA = PB$.

To find PB, use the properties of a right triangle formed by the radius and the tangent line at the point of tangency. The distance from the center of the circle O to P can be calculated using the Pythagorean theorem in triangle OAP, where $OA = 5$ cm (radius) and $AB = 6$ cm is the chord.

The perpendicular from O to AB divides AB into two equal parts of 3 cm each. Let the distance from O to the midpoint M of AB be OM. Then:

$$OM = \sqrt{OA^2 - AM^2} = \sqrt{5^2 - 3^2} = \sqrt{16} = 4\,\text{cm}.$$

Using triangle OMP (where MP is tangent and OM is perpendicular to AB):

$$OP = \sqrt{OM^2 + MP^2} = \sqrt{4^2 + 5^2} = \sqrt{41}\,\text{cm}.$$

Since $PA = PB$ and OP is the hypotenuse in the right triangle OMP, PB is also $\sqrt{41}$ cm.

Quick Tip
When dealing with tangents from a point to a circle, the segments of the tangents are equal, and the tangent segment forms a right angle with the radius at the point of tangency.

OR

34 (b). Prove that the parallelogram circumscribing a circle is a rhombus. Also, find the area of the rhombus, if the radius of the circle is 3 cm and the length of one side of the rhombus is 10 cm.

Solution: A parallelogram circumscribing a circle implies that each side of the parallelogram touches the circle. In such a configuration, opposite sides of the parallelogram are equal in length, and each pair of tangents from a single external point to a circle are equal.

Since the parallelogram has all four sides tangential to the circle, each side must be of equal length to balance the tangential points, thus making it a rhombus.

To find the area of the rhombus: The formula for the area A of a rhombus given the side s and the radius r of the inscribed circle is:

$$A = 2 \times s \times r.$$

Given $s = 10$ cm and $r = 3$ cm:

$$A = 2 \times 10 \times 3 = 60\,\text{cm}^2.$$

Quick Tip

In geometry, a parallelogram that circumscribes a circle is always a rhombus because the circle imposes equal lengths on all tangents drawn from points outside the circle to the circle.

---

35. Find the mean and median of the following distribution:

Class	Frequency
$0 - 20$	5
$20 - 40$	8
$40 - 60$	10
$60 - 80$	12
$80 - 100$	7
$100 - 120$	8

Solution: We are given the frequency distribution in class intervals. To find the mean and median, we follow these steps:
Step 1: Finding the Mean
First, calculate the class marks (x) for each class interval. The class mark is the midpoint of the class interval, given by:

$$x = \frac{\text{Lower limit} + \text{Upper limit}}{2}$$

So, the class marks for each class are:

Class Interval (C.I.)	x
$0 - 20$	10
$20 - 40$	30
$40 - 60$	50
$60 - 80$	70
$80 - 100$	90
$100 - 120$	110

Next, calculate the frequency times class mark $(f \times x)$ and the cumulative frequency (cf):

Class Interval (C.I.) Cumulative Frequency (cf)	x	f	$f \times x$
$0 - 20$ 5	10	5	50
$20 - 40$ 13	30	8	240
$40 - 60$ 23	50	10	500
$60 - 80$ 35	70	12	840
$80 - 100$ 42	90	7	630
$100 - 120$ 50	110	8	880

Now, calculate the mean using the formula:

$$\text{Mean} = \frac{\sum(f \times x)}{\sum f}$$

$$\text{Mean} = \frac{50 + 240 + 500 + 840 + 630 + 880}{50} = \frac{3140}{50} = 62.8$$

Thus, the mean is 62.8.
Step 2: Finding the Median
To find the median, we use the following formula:

$$\text{Median} = L + \left(\frac{\frac{N}{2} - C}{f} \right) \times h$$

Where: - L is the lower limit of the median class. - N is the total frequency. - C is the cumulative frequency of the class before the median class. - f is the frequency of the median class. - h is the class width.
From the cumulative frequency table, the total frequency $N = 50$, so $\frac{N}{2} = 25$.
The median class is the class where the cumulative frequency just exceeds 25. This is the class $60 - 80$, where the cumulative frequency is 23 and the next cumulative frequency is 35.
- $L = 60$ (lower limit of the median class), - $C = 23$ (cumulative frequency of the class before the median class), - $f = 12$ (frequency of the median class), - $h = 20$ (class width).
Now, substitute these values into the median formula:

$$\text{Median} = 60 + \left(\frac{25 - 23}{12} \right) \times 20 = 60 + \left(\frac{2}{12} \right) \times 20 = 60 + \frac{40}{12} = 60 + 3.33 = 63.33$$

Thus, the median is approximately 63.3.

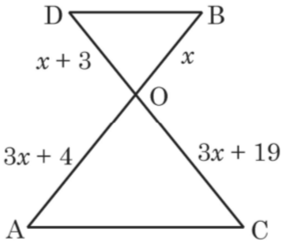
Quick Tip

To calculate the mean, multiply each class mark by its frequency, sum the products, and divide by the total frequency. For the median, find the median class and use the formula to compute the value.

Section - E

This section comprises 3 case study based s of 4 marks each.

36 In the figure given below, a folding table is shown:



The legs of the table are represented by line segments AB and CD intersecting at O. Join AC and BD.
Considering the table top is parallel to the ground, and $OB = x$, $OD = x + 3$, $OC = 3x + 19$, and $OA = 3x + 4$, answer the following s:
(i) Prove that $\triangle OAC$ is similar to $\triangle OBD$.
(ii) Prove that $\frac{OA}{OB} = \frac{AC}{BD}$.
(iii) (a) Observe the figure and find the value of x. Hence, find the length of OC.
OR
(iii) (b) Observe the figure and find $\frac{BD}{AC}$.

Solution:
(i) To prove that $\triangle OAC \sim \triangle OBD$: - In $\triangle OAC$ and $\triangle OBD$, $\angle AOC = \angle BOD$ because vertically opposite angles are equal. - Since the table top is parallel to the ground, $AC \parallel BD$, and hence, $\angle OCA = \angle OBD$ (corresponding angles). Thus, $\triangle OAC \sim \triangle OBD$ by AA similarity criterion.

(ii) To prove that $\frac{OA}{AC} = \frac{OB}{BD}$:
From the similarity of $\triangle OAC$ and $\triangle OBD$, the sides of the similar triangles are proportional. Therefore:

$$\frac{OA}{AC} = \frac{OB}{BD}.$$

Let us verify using the given values: - $OA = 3x+4$, $AC = OC - OA = (3x+19) - (3x+4) = 15$.
- $OB = x$, $BD = OD - OB = (x+3) - x = 3$.
Substitute these values:

$$\frac{OA}{AC} = \frac{3x+4}{15}, \quad \frac{OB}{BD} = \frac{x}{3}.$$

Using $x = 2$, substitute into both expressions:

$$\frac{OA}{AC} = \frac{3(2)+4}{15} = \frac{10}{15} = \frac{2}{3}, \quad \frac{OB}{BD} = \frac{2}{3}.$$

Since $\frac{OA}{AC} = \frac{OB}{BD}$, the relationship is verified.
(iii) (a) To find x and the length of OC: From the similarity of $\triangle OAC$ and $\triangle OBD$, we know:

$$\frac{OA}{OB} = \frac{OC}{OD}.$$

Substituting the given values $OA = 3x + 4$, $OB = x$, $OC = 3x + 19$, and $OD = x + 3$:

$$\frac{3x + 4}{x} = \frac{3x + 19}{x + 3}.$$

Cross-multiply:

$$(3x + 4)(x + 3) = x(3x + 19).$$

Expand:

$$3x^2 + 9x + 4x + 12 = 3x^2 + 19x.$$

Simplify:

$$13x + 12 = 19x \quad \Rightarrow \quad 6x = 12 \quad \Rightarrow \quad x = 2.$$

Substitute $x = 2$ into $OC = 3x + 19$:

$$OC = 3(2) + 19 = 25\,\text{cm}.$$

OR

(iii) (b) To find $\frac{BD}{AC}$: Using the similarity of triangles $\triangle OAC$ and $\triangle OBD$, and substituting $OA = 3x + 4 = 10$ (for $x = 2$), $OB = x = 2$, $OC = 25$, and $OD = 5$:

$$\frac{BD}{AC} = \frac{OD}{OA} = \frac{5}{10} = \frac{1}{2}.$$

Quick Tip

For proving similarity in geometry, use angle-angle (AA) similarity, and apply proportions to find unknown lengths. Always substitute values carefully for verification.

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37. While preparing for a competitive examination, Akbar came across a match-stick pattern-based . The pattern is given below:

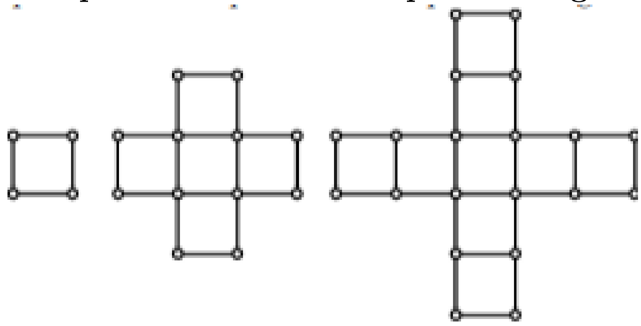


Fig. (1) **Fig. (2)** **Fig. (3)**

Based on the above information, answer the following s:
(i) Write the first term and common difference of the A.P. formed by the number of squares in each figure.
(ii) Write the first term and common difference of the A.P. formed by the number of sticks used in each figure.
(iii) (a) How many squares are there in Fig. (10)? Also, write the number of sticks used in Fig. (10).

OR

(iii) (b) If 88 sticks are used to make the n-th figure (Fig. n), find the value of n. How many squares are formed in this figure?

Solution:

(i) For the number of squares in each figure, observe the pattern:

$$\text{First term } (a) = 1, \quad \text{Common difference } (d) = 1.$$

(ii) For the number of sticks used in each figure:

$$\text{First term } (a) = 4, \quad \text{Common difference } (d) = 3.$$

(iii) (a) Using the formula for the n-th term of an A.P., $a_n = a + (n-1)d$:

$$a_{10} = 1 + (10-1)(1) = 10 \,(\text{squares}).$$

For the sticks:

$$S_{10} = 4 + (10-1)(3) = 31 \,(\text{sticks}).$$

OR

(iii) (b) If 88 sticks are used:

$$S_n = 4 + (n-1)(3), \quad 88 = 4 + (n-1)(3).$$

$$88 - 4 = 3(n-1), \quad 84 = 3(n-1), \quad n = \frac{84}{3} + 1 = 29.$$

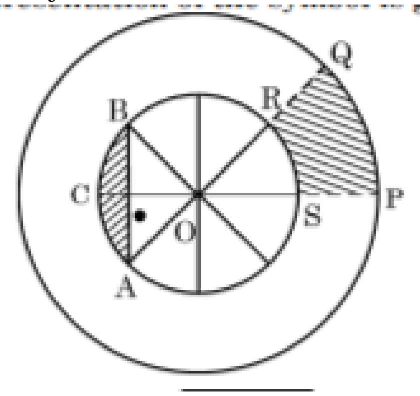
Thus, $n = 29$. For squares:

$$a_{29} = 1 + (29-1)(1) = 29 \,(\text{squares}).$$

Quick Tip
The n-th term of an A.P. is given by $a_n = a+(n-1)d$, and the total count for repetitive patterns like this can be calculated using basic A.P. formulas.

---

38. NSS (National Service Scheme) aims to connect the students to the community and to involve them in problem-solving processes. NSS symbol is based on the 'Rath' wheel of the Konark Sun Temple situated in Odisha. The wheel signifies the progress cycle of life.



Observe the figure given above. The diameters of the inner circle are equally placed. Given that $OP = 21\,\text{cm}$ and $OS = 10\,\text{cm}$.
Based on the above information, answer the following s:
(i) Find $\angle ROS$.
(ii) Find the perimeter of sector OPQ.
(iii) (a) Find the area of the shaded region $PQRS$.
OR
(iii) (b) Find the area of the shaded region ACB, i.e., the segment ACB.

Solution:
(i) The angle $\angle ROS$ is part of a circle divided into equal sectors. Since the diameters are equally placed, the circle is divided into six equal parts. Therefore:

$$\angle ROS = \frac{360^\circ}{6} = 60^\circ.$$

(ii) The perimeter of sector OPQ is given by:

$$\text{Perimeter of sector } = \text{Arc length } + 2 \times \text{Radius}.$$

The arc length is:

$$\text{Arc length } = \frac{\theta}{360^\circ} \times 2\pi r,$$

where $\theta = 60^\circ$ and $r = 21\,\text{cm}$. Substituting values:

$$\text{Arc length } = \frac{60}{360} \times 2\pi \times 21 = \frac{1}{6} \times 2\pi \times 21 = 22\pi\,\text{cm}.$$

Adding the radius:

$$\text{Perimeter of sector } = 22\pi + 2 \times 21 = 22\pi + 42\,\text{cm}.$$

(iii) (a) The area of the shaded region $PQRS$ is the difference between the area of the larger sector and the smaller sector. The areas are given by:

$$\text{Area of sector OPQ} = \frac{\theta}{360^\circ} \times \pi r^2,$$

where $r = 21\,\text{cm}$, and:

$$\text{Area of sector OSQ} = \frac{\theta}{360^\circ} \times \pi r^2,$$

where $r = 10\,\text{cm}$. Substituting:

$$\text{Area of sector OPQ} = \frac{60}{360} \times \pi \times 21^2 = \frac{1}{6} \times \pi \times 441 = 73.5\pi\,\text{cm}^2.$$

$$\text{Area of sector OSQ} = \frac{60}{360} \times \pi \times 10^2 = \frac{1}{6} \times \pi \times 100 = 16.67\pi\,\text{cm}^2.$$

$$\text{Area of shaded region PQRS} = 73.5\pi - 16.67\pi = 56.83\pi\,\text{cm}^2.$$

OR

(iii) (b) The area of the shaded region ACB (the segment ACB) is the area of the larger sector $OACB$ minus the area of the corresponding triangle OAB. First, calculate the sector area:

$$\text{Area of sector OACB} = \frac{\theta}{360^\circ} \times \pi r^2 = \frac{60}{360} \times \pi \times 21^2 = 73.5\pi\,\text{cm}^2.$$

Next, calculate the area of triangle OAB using:

$$\text{Area of } \triangle OAB = \frac{1}{2} \times r^2 \times \sin\theta = \frac{1}{2} \times 21^2 \times \sin 60^\circ.$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2}, \quad \text{Area of } \triangle OAB = \frac{1}{2} \times 441 \times \frac{\sqrt{3}}{2} = 110.25\sqrt{3}\,\text{cm}^2.$$

Finally:

$$\text{Area of shaded region ACB} = 73.5\pi - 110.25\sqrt{3}\,\text{cm}^2.$$

Quick Tip

For sector-related problems, use the formulas for arc length and sector area. Subtract the triangle area for segments.
