

CBSE Class 12 Physics (55/1/1) 2024 Question Paper with Solutions

Time Allowed :3 Hours	Maximum Marks :70	Total Questions :33
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General Instructions

Read the following instructions very carefully and strictly follow them:

1. This question paper contains 33 questions. All questions are compulsory.
2. This question paper is divided into five sections Sections A, B, C, D and E.
3. In Section A Questions no. 1 to 16 are Multiple Choice type questions. Each question carries 1 mark.
4. In Section B Questions no. 17 to 21 are Very Short Answer type questions. Each question carries 2 marks.
5. (v) In Section C Questions no. 22 to 28 are Short Answer type questions. Each question carries 3 marks.
6. In Section D Questions no. 29 and 30 are case study-based questions. Each question carries 4 marks.
7. In Section E Questions no. 31 to 33 are Long Answer type questions. Each question carries 5 marks.
8. There is no overall choice given in the question paper. However, an internal choice has been provided in few questions in all the Sections except Section A.
9. Kindly note that there is a separate question paper for Visually Impaired candidates.
10. Use of calculators is not allowed.

Section A

1. A thin plastic rod is bent into a circular ring of radius R . It is uniformly charged with charge density λ . The magnitude of the electric field at its centre is:

- (A) $\frac{\lambda}{2\epsilon_0 R}$
(B) Zero
(C) $\frac{\lambda}{4\pi\epsilon_0 R}$
(D) $\frac{\lambda}{4\epsilon_0 R}$

Correct Answer: (B) Zero

Solution:

The electric field at the center of a uniformly charged circular ring is zero. This is because the contributions to the electric field from all elements of the ring cancel out due to symmetry.

The electric field due to a small charge element dq on the ring at the center is:

$$dE = \frac{k dq}{R^2},$$

where $k = \frac{1}{4\pi\epsilon_0}$. However, the vector components of the electric field from opposite elements of the ring cancel out due to symmetry, resulting in a net electric field of:

$$\boxed{0}.$$

💡 Quick Tip

For symmetric charge distributions, evaluate the net electric field by summing the contributions from all elements. Symmetry often results in cancellation of components at specific points.

2. Ten capacitors, each of capacitance $1 \mu\text{F}$, are connected in parallel to a source of 100 V . The total energy stored in the system is equal to:

- (A) 10^{-2} J
- (B) 10^{-3} J
- (C) $0.5 \times 10^{-3} \text{ J}$
- (D) $5.0 \times 10^{-2} \text{ J}$

Correct Answer: (D) $5.0 \times 10^{-2} \text{ J}$

Solution:

The energy stored in a single capacitor is given by the formula:

$$E = \frac{1}{2}CV^2$$

Where: - E is the energy stored, - C is the capacitance, - V is the voltage across the capacitor.
For one capacitor, with $C = 1\mu\text{F} = 1 \times 10^{-6} \text{ F}$ and $V = 100 \text{ V}$:

$$E_1 = \frac{1}{2}(1 \times 10^{-6})(100)^2 = \frac{1}{2} \times 10^{-6} \times 10^4 = 5 \times 10^{-2} \text{ J}$$

Since the capacitors are connected in parallel, the total energy stored in the system is the sum of the energies of all the capacitors. There are 10 capacitors, so the total energy is:

$$E_{\text{total}} = 10 \times 5 \times 10^{-2} = 5.0 \times 10^{-2} \text{ J}$$

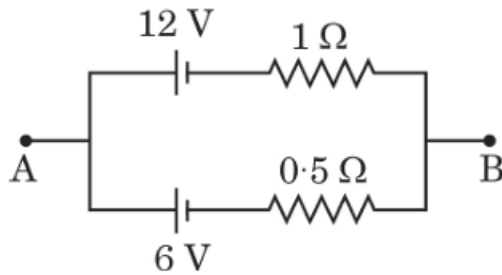
Thus, the correct answer is:

$$\text{(D) } 5.0 \times 10^{-2} \text{ J.}$$

💡 Quick Tip

For capacitors in parallel, the total energy stored is the sum of the energies stored in each individual capacitor.

3. Consider the circuit shown in the figure. The potential difference between points A and B is:



- (A) 6 V
- (B) 8 V
- (C) 9 V
- (D) 12 V

Correct Answer: (B) 8 V

Solution:

We will apply Kirchhoff's Voltage Law (KVL) to find the potential difference between points A and B. According to KVL, the sum of the potential differences around any closed loop must be zero.

The resistors and voltage sources are arranged as follows: - There is a 12 V source and a 1 Ω resistor in series. - A 0.5 Ω resistor is in series with a 6 V source.

First, calculate the current flowing through the circuit. The total resistance in the circuit is:

$$R_{\text{total}} = 1\ \Omega + 0.5\ \Omega = 1.5\ \Omega$$

Now, calculate the total voltage in the circuit:

$$V_{\text{total}} = 12\ \text{V} - 6\ \text{V} = 6\ \text{V}$$

Using Ohm's Law, the current I flowing through the circuit is:

$$I = \frac{V_{\text{total}}}{R_{\text{total}}} = \frac{6}{1.5} = 4\ \text{A}$$

Now, calculate the potential drop across the 0.5 Ω resistor:

$$V_{\text{drop}} = I \times 0.5 = 4 \times 0.5 = 2\ \text{V}$$

Thus, the potential difference between points A and B is:

$$\text{Potential difference} = 6\ \text{V} + 2\ \text{V} = 8\ \text{V}$$

Thus, the correct answer is:

(B) 8 V.

💡 Quick Tip

By using Kirchhoff's Voltage Law and Ohm's law, you can calculate the potential difference in a simple series circuit with resistors and voltage sources.

4. A loop carrying a current I clockwise is placed in $x - y$ plane, in a uniform magnetic field directed along z -axis. The tendency of the loop will be to:

- (A) move along x -axis
- (B) move along y -axis
- (C) shrink
- (D) expand

Correct Answer: (C) shrink

Solution:

The magnetic field exerts a force on each segment of the current-carrying loop. Due to the interaction between the magnetic field and the current, the magnetic forces tend to pull the loop inward, reducing its area. This tendency to shrink can be explained by the Lorentz force acting on the current elements, which generates a net torque compressing the loop.

Thus, the loop tends to:

shrink

.

💡 Quick Tip

For a current-carrying loop in a uniform magnetic field, consider the forces and torques acting on each segment. The tendency of a loop in such a field is to shrink due to the inward forces generated by the magnetic interaction.

5.

A 10 cm long wire lies along y -axis. It carries a current of 1.0 A in positive y -direction. A magnetic field $\vec{B} = (5 \text{ mT})\hat{j} - (8 \text{ mT})\hat{k}$ exists in the region. The force on the wire is:

- (A) $(0.8 \text{ mN})\hat{i}$
- (B) $-(0.8 \text{ mN})\hat{i}$
- (C) $(80 \text{ mN})\hat{i}$
- (D) $-(80 \text{ mN})\hat{i}$

Correct Answer: (B) $-(0.8 \text{ mN})\hat{i}$

Solution:

The magnetic force $\vec{F}$ on a current-carrying wire is given by:

$$\vec{F} = I(\vec{L} \times \vec{B}),$$

where:

- $I = 1.0 \text{ A}$ is the current,
- $\vec{L} = (10 \text{ cm}) \hat{j} = (0.1 \text{ m}) \hat{j}$ is the length vector of the wire,
- $\vec{B} = (5 \text{ mT}) \hat{j} - (8 \text{ mT}) \hat{k} = (5 \times 10^{-3}) \hat{j} - (8 \times 10^{-3}) \hat{k}$.

Step 1: Compute $\vec{L} \times \vec{B}$:

$$\vec{L} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0.1 & 0 \\ 0 & 5 \times 10^{-3} & -8 \times 10^{-3} \end{vmatrix}.$$

Expand the determinant:

$$\vec{L} \times \vec{B} = \hat{i} (0.1 \cdot (-8 \times 10^{-3}) - 0 \cdot (5 \times 10^{-3})) - \hat{j} (0 \cdot (-8 \times 10^{-3}) - 0 \cdot 0) + \hat{k} (0 \cdot (5 \times 10^{-3}) - 0.1 \cdot 0).$$

Simplify:

$$\vec{L} \times \vec{B} = \hat{i} (-0.8 \times 10^{-3}) + 0 + 0.$$

Thus:

$$\vec{L} \times \vec{B} = -(0.8 \times 10^{-3}) \hat{i}.$$

Step 2: Calculate $\vec{F}$:

$$\vec{F} = I(\vec{L} \times \vec{B}) = 1.0 \cdot -(0.8 \times 10^{-3}) \hat{i}.$$

Simplify:

$$\vec{F} = -0.8 \text{ mN} \hat{i}.$$

Thus, the force on the wire is:

$$\boxed{-0.8 \text{ mN} \hat{i}}.$$

💡 Quick Tip

To calculate the magnetic force on a wire, use $\vec{F} = I(\vec{L} \times \vec{B})$. Expand the cross product carefully, considering the directions of the vectors involved.

6. A galvanometer of resistance $G \Omega$ is converted into an ammeter of range 0 to 1 A. If the current through the galvanometer is 0.1% of 1 A, the resistance of the ammeter is:

- (A) $\frac{G}{999} \Omega$
- (B) $\frac{G}{1000} \Omega$
- (C) $\frac{G}{1001} \Omega$

(D) $\frac{G}{100.1} \Omega$

Correct Answer: (B) $\frac{G}{1000} \Omega$

Solution:

To convert a galvanometer into an ammeter, a shunt resistance S is connected in parallel with the galvanometer. The current through the galvanometer is given as $I_g = 0.001$ A (0.1% of 1 A). The total current through the ammeter is $I = 1$ A.

The current through the shunt is:

$$I_s = I - I_g = 1 - 0.001 = 0.999 \text{ A.}$$

The potential difference across the galvanometer and the shunt is the same:

$$I_g G = I_s S.$$

Substitute I_s and rearrange for S :

$$S = \frac{I_g G}{I_s} = \frac{(0.001)G}{0.999}.$$

Simplify:

$$S = \frac{G}{999}.$$

The total resistance of the ammeter is the parallel combination of G and S :

$$R_{\text{ammeter}} = \frac{GS}{G + S}.$$

Substitute $S = \frac{G}{999}$:

$$R_{\text{ammeter}} = \frac{G \cdot \frac{G}{999}}{G + \frac{G}{999}} = \frac{G^2}{999G + G} = \frac{G}{1000}.$$

Thus, the resistance of the ammeter is:

$$\boxed{\frac{G}{1000} \Omega}.$$

💡 Quick Tip

To convert a galvanometer into an ammeter, always calculate the shunt resistance S using $S = \frac{I_g G}{I_s}$. The total resistance of the ammeter is the effective parallel resistance of G and S .

7. The reactance of a capacitor of capacitance C connected to an AC source of frequency ω is X . If the capacitance of the capacitor is doubled and the frequency of the source is tripled, the reactance will become:

- (A) $\frac{X}{6}$
(B) $6X$

- (C) $\frac{2}{3}X$
 (D) $\frac{3}{2}X$

Correct Answer: (A) $\frac{X}{6}$

Solution:

The capacitive reactance X_C is given by:

$$X_C = \frac{1}{\omega C},$$

where ω is the angular frequency, and C is the capacitance.

Initially, the reactance is:

$$X = \frac{1}{\omega C}.$$

When the capacitance is doubled ($C' = 2C$) and the frequency is tripled ($\omega' = 3\omega$), the new reactance becomes:

$$X'_C = \frac{1}{\omega' C'} = \frac{1}{(3\omega)(2C)} = \frac{1}{6\omega C}.$$

Compare with the initial reactance:

$$X'_C = \frac{X}{6}.$$

Thus, the new reactance is:

$$\boxed{\frac{X}{6}}.$$

💡 Quick Tip

For capacitive reactance, remember that $X_C = \frac{1}{\omega C}$. If the capacitance or frequency changes, calculate the new reactance proportionally.

8. In the four regions, I, II, III, and IV, the electric fields are described as:

Region I: $E_x = E_0 \sin(kz - \omega t)$

Region II: $E_x = E_0$

Region III: $E_x = E_0 \sin kz$

Region IV: $E_x = E_0 \cos kz$

The displacement current will exist in the region:

- (A) I
 (B) IV
 (C) II
 (D) III

Correct Answer: (A) I

Solution:

Displacement current exists in regions where there is a time-varying electric field. In Region I, the electric field is:

$$E_x = E_0 \sin(kz - \omega t).$$

This electric field varies with time due to the presence of the term $-\omega t$. The displacement current density J_d is given by:

$$J_d = \epsilon_0 \frac{\partial E_x}{\partial t}.$$

Differentiating E_x with respect to time:

$$\frac{\partial E_x}{\partial t} = -\omega E_0 \cos(kz - \omega t).$$

Thus, a displacement current exists in Region I. In the other regions (II, III, IV), the electric field does not vary with time, so there is no displacement current.

Therefore, the displacement current exists in:

Region I.

💡 Quick Tip

Displacement current arises from time-varying electric fields. Check for terms involving t in the electric field expressions to identify regions with displacement current.

9. The transition of electron that gives rise to the formation of the second spectral line of the Balmer series in the spectrum of hydrogen atom corresponds to:

- (A) $n_f = 2$ and $n_i = 3$
- (B) $n_f = 3$ and $n_i = 4$
- (C) $n_f = 2$ and $n_i = 4$
- (D) $n_f = 2$ and $n_i = \infty$

Correct Answer: (C) $n_f = 2$ and $n_i = 4$

Solution:

The Balmer series corresponds to transitions where the final energy level is $n_f = 2$. The second spectral line occurs when the transition happens from $n_i = 4$ to $n_f = 2$.

Using the formula for the wavelength of emitted radiation:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right),$$

where:

- R_H is the Rydberg constant,
- $n_f = 2$ and $n_i = 4$ for the second spectral line.

Substitute $n_f = 2$ and $n_i = 4$ into the equation:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{2^2} - \frac{1}{4^2} \right) = R_H \left(\frac{1}{4} - \frac{1}{16} \right).$$

Simplify:

$$\frac{1}{\lambda} = R_H \cdot \frac{4 - 1}{16} = R_H \cdot \frac{3}{16}.$$

Thus, the correct transition for the second line in the Balmer series is:

$$n_f = 2 \text{ and } n_i = 4.$$

 Quick Tip

For the Balmer series, transitions end at $n_f = 2$. The spectral lines are numbered based on the value of n_i (e.g., $n_i = 3$ for the first line, $n_i = 4$ for the second line).

10. Ge is doped with As. Due to doping:

- (A) the structure of Ge lattice is distorted.
- (B) the number of conduction electrons increases.
- (C) the number of holes increases.
- (D) the number of conduction electrons decreases.

Correct Answer: (B) the number of conduction electrons increases.

Solution:

When germanium (Ge), a group-IV element, is doped with arsenic (As), a group-V element, it creates an n -type semiconductor. Arsenic contributes extra electrons (conduction electrons) to the material because it has five valence electrons compared to four in germanium. These extra electrons become free to conduct electricity, thus increasing the number of conduction electrons.

Thus, the effect of doping Ge with As is:

the number of conduction electrons increases.

 Quick Tip

Doping a semiconductor with a group-V element increases the number of conduction electrons, resulting in an n -type semiconductor.

11. Two beams, A and B whose photon energies are 3.3 eV and 11.3 eV respectively, illuminate a metallic surface (work function 2.3 eV) successively. The ratio of maximum speed of electrons emitted due to beam A to that due to beam B is:

- (A) 3
- (B) 9
- (C) $\frac{1}{3}$
- (D) $\frac{1}{9}$

Correct Answer: (C) $\frac{1}{3}$

Solution:

The maximum kinetic energy of emitted electrons is given by Einstein's photoelectric equation:

$$K_{\max} = h\nu - \phi,$$

where:

- $h\nu$ is the photon energy,
- ϕ is the work function of the metal.

For beam A:

$$K_{\max,A} = 3.3 - 2.3 = 1.0 \text{ eV}.$$

For beam B:

$$K_{\max,B} = 11.3 - 2.3 = 9.0 \text{ eV}.$$

The maximum speed of the emitted electrons is related to the kinetic energy by:

$$K_{\max} = \frac{1}{2}mv_{\max}^2.$$

Thus:

$$v_{\max} = \sqrt{\frac{2K_{\max}}{m}}.$$

The ratio of speeds is:

$$\frac{v_{\max,A}}{v_{\max,B}} = \sqrt{\frac{K_{\max,A}}{K_{\max,B}}}.$$

Substitute the values:

$$\frac{v_{\max,A}}{v_{\max,B}} = \sqrt{\frac{1.0}{9.0}} = \frac{1}{3}.$$

Thus, the ratio of maximum speeds is:

$$\boxed{\frac{1}{3}}.$$

💡 Quick Tip

For photoelectric problems, use $K_{\max} = h\nu - \phi$ to calculate the kinetic energy of emitted electrons and relate it to their speed using $v_{\max} = \sqrt{\frac{2K_{\max}}{m}}$.

12. The waves associated with a moving electron and a moving proton have the same wavelength λ . It implies that they have the same:

- (A) momentum
- (B) angular momentum
- (C) speed
- (D) energy

Correct Answer: (A) momentum

Solution:

The de Broglie wavelength λ of a particle is given by:

$$\lambda = \frac{h}{p},$$

where:

- h is Planck's constant,
- p is the momentum of the particle.

If two particles have the same de Broglie wavelength, then their momenta must be the same, because λ is inversely proportional to p . However, their masses may differ, and thus their speeds and energies can be different.

For a moving electron and a moving proton, having the same wavelength implies:

$$p_{\text{electron}} = p_{\text{proton}}.$$

Thus, the correct answer is:

momentum.

💡 Quick Tip

The de Broglie wavelength λ depends only on the momentum p of the particle. If two particles have the same λ , they must have the same momentum, regardless of their masses or speeds.

Questions number 13 to 16 are Assertion (A) and Reason (R) type questions. Two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below. (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false and Reason (R) is also false.

13.

Assertion (A): In photoelectric effect, the kinetic energy of the emitted photoelectrons increases with increase in the intensity of the incident light.

Reason (R): Photoelectric current depends on the wavelength of the incident light.

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation

of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (D) Assertion (A) is false and Reason (R) is also false.

Solution:

In the photoelectric effect, the kinetic energy of the emitted photoelectrons depends on the frequency (or wavelength) of the incident light, not its intensity. Hence, Assertion (A) is false. The photoelectric current depends on the intensity of the incident light and not its wavelength, so Reason (R) is also false.

Thus, the correct answer is:

Assertion (A) is false and Reason (R) is also false.

💡 Quick Tip

Remember, in the photoelectric effect, the intensity of light affects the photoelectric current, while the frequency (or wavelength) of light determines the kinetic energy of the emitted electrons.

14.

Assertion (A): The mutual inductance between two coils is maximum when the coils are wound on each other.

Reason (R): The flux linkage between two coils is maximum when they are wound on each other.

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Solution:

Mutual inductance between two coils depends on the extent of magnetic flux linkage between them. When the coils are wound on each other, the magnetic flux linkage is maximum, and hence the mutual inductance is also maximum.

Therefore, both the Assertion (A) and the Reason (R) are true, and Reason (R) correctly explains Assertion (A).

Thus, the correct answer is:

Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion

 Quick Tip

Mutual inductance is maximum when the coils are wound on each other because the magnetic flux linkage is maximized in this configuration.

15. Assertion (A): Two long parallel wires, freely suspended and connected in series to a battery, move apart.

Reason (R): Two wires carrying current in opposite directions repel each other.

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

Solution:

The assertion states that two long parallel wires, freely suspended and connected to a battery in series, move apart. This behavior is due to the magnetic forces acting on the wires. When electric current flows through the wires, they generate magnetic fields around them.

The reason states that two wires carrying current in opposite directions repel each other. This is a result of the force between two parallel currents, described by Ampère's Law. When currents flow in opposite directions in parallel wires, the magnetic fields generated by the wires exert a repulsive force on each other, causing the wires to move apart.

Thus, both the assertion and the reason are true, and the reason correctly explains the assertion.

Thus, the correct answer is:

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion

 Quick Tip

The force between two current-carrying wires depends on the direction of the currents. When currents are in opposite directions, the wires repel each other, and when currents are in the same direction, the wires attract each other.

Question 16:

Assertion (A): Plane and convex mirrors cannot produce real images under any circumstance.

Reason (R): A virtual image cannot serve as an object to produce a real image.

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false and Reason (R) is also false.

Correct Answer: (D) Assertion (A) is false and reason (R) is also false.

Solution:

- The assertion (A) is false because while it is true that a plane mirror cannot produce a real image, a convex mirror can produce virtual images but does not produce real images under any circumstance. Thus, the assertion is incorrect.

- The reason (R) is also false. A virtual image formed by one mirror can indeed serve as an object for another mirror, and a real image can be formed by using this object. Thus, the statement that a virtual image cannot serve as an object for producing a real image is also incorrect.

Therefore, the correct answer is:

(D) Assertion (A) is false and reason (R) is also false.

 Quick Tip

A convex mirror always forms a virtual image, but a real image can be formed by concave mirrors. Also, a virtual image formed by one mirror can act as an object for another mirror.

Section B

17. Find the temperature at which the resistance of a wire made of silver will be twice its resistance at 20°C . Take 20°C as the reference temperature and the temperature coefficient of resistance of silver at 20°C as $4.0 \times 10^{-3} \text{ K}^{-1}$.

Correct Answer 270°C

Solution:

The resistance of a material varies with temperature according to the formula:

$$R_T = R_0(1 + \alpha\Delta T),$$

where:

- R_T is the resistance at temperature T ,
- R_0 is the resistance at the reference temperature T_0 ,
- α is the temperature coefficient of resistance,
- $\Delta T = T - T_0$ is the temperature difference.

Given that $R_T = 2R_0$, $\alpha = 4.0 \times 10^{-3} \text{ K}^{-1}$, and $T_0 = 20^\circ\text{C}$, substitute these values:

$$2R_0 = R_0(1 + \alpha\Delta T).$$

Cancel R_0 on both sides:

$$2 = 1 + \alpha\Delta T.$$

Rearrange to find ΔT :

$$\Delta T = \frac{2 - 1}{\alpha} = \frac{1}{4.0 \times 10^{-3}} = 250 \text{ K}.$$

Calculate the temperature T :

$$T = T_0 + \Delta T = 20 + 250 = 270^\circ\text{C}.$$

Thus, the temperature at which the resistance of the wire is twice its resistance at 20°C is:

$$\boxed{270^\circ\text{C}}.$$

Quick Tip

To calculate the temperature at which resistance changes, always use the formula $R_T = R_0(1 + \alpha\Delta T)$ and solve for ΔT .

18 (a) Monochromatic light of frequency $5.0 \times 10^{14} \text{ Hz}$ passes from air into a medium of refractive index 1.5. Find the wavelength of the light (i) reflected, and (ii) refracted at the interface of the two media.

Solution:

The wavelength λ of light is related to its speed and frequency by:

$$\lambda = \frac{v}{f}.$$

For light in air:

$$v_{\text{air}} = c = 3.0 \times 10^8 \text{ m/s}.$$

The wavelength in air is:

$$\lambda_{\text{air}} = \frac{v_{\text{air}}}{f} = \frac{3.0 \times 10^8}{5.0 \times 10^{14}} = 600 \text{ nm}.$$

(i) The wavelength of the reflected light remains the same as in air:

$$\lambda_{\text{reflected}} = 600 \text{ nm}.$$

(ii) For refracted light in the medium:

$$v_{\text{medium}} = \frac{v_{\text{air}}}{n} = \frac{3.0 \times 10^8}{1.5} = 2.0 \times 10^8 \text{ m/s}.$$

The wavelength in the medium is:

$$\lambda_{\text{refracted}} = \frac{v_{\text{medium}}}{f} = \frac{2.0 \times 10^8}{5.0 \times 10^{14}} = 400 \text{ nm}.$$

Thus:

$$\lambda_{\text{refracted}} = 400 \text{ nm}.$$

💡 Quick Tip

The wavelength of reflected light does not change as it remains in the same medium.
The wavelength of refracted light changes and is inversely proportional to the refractive index of the medium.

18 (b). A plano-convex lens of focal length 16 cm is made of a material of refractive index 1.4. Calculate the radius of the curved surface of the lens.

Solution:

The lens formula is given by:

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

For the given lens with a refractive index $\mu = 1.4$, and the relation becomes:

$$\frac{1}{16} = (1.4 - 1) \left(\frac{1}{R} - \frac{1}{\infty} \right)$$

Since $\frac{1}{\infty} = 0$, the equation simplifies to:

$$\frac{1}{16} = 0.4 \times \frac{1}{R}$$

Solving for R :

$$R = 16 \times 0.4 = 6.4 \text{ cm}$$

Thus, the radius of the curved surface is 6.4 cm.

💡 Quick Tip

For plano-convex lenses, use the lens maker's formula and remember that one radius of curvature is ∞ for the plane surface.

19. An object is placed 30 cm in front of a concave mirror of radius of curvature 40 cm. Find the (i) position of the image formed and (ii) magnification of the image.

Solution:

(i) The lens formula is given by:

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

Substituting the given values:

$$\frac{1}{v} + \frac{1}{-30} = \frac{1}{-20}$$

Now, solving for v :

$$\begin{aligned} \frac{1}{v} &= \frac{1}{-20} - \frac{1}{-30} \\ \frac{1}{v} &= \frac{-3 + 2}{60} = \frac{-1}{60} \end{aligned}$$

$$v = -60 \text{ cm}$$

Thus, the value of v is -60 cm.

(ii) The magnification (m) is given by:

$$m = -\frac{v}{u}$$

Substituting the values of $v = -60$ cm and $u = -30$ cm:

$$\begin{aligned} m &= -\left(\frac{-60}{-30}\right) \\ m &= -2 \end{aligned}$$

Thus, the magnification is $m = -2$.

💡 Quick Tip

For mirrors, use the mirror formula $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$ and apply the sign convention carefully for object and image distances.

20. Consider a neutron (mass m) of kinetic energy E and a photon of the same energy. Let λ_n and λ_p be the de Broglie wavelength of the neutron and the wavelength of the photon, respectively. Obtain an expression for $\frac{\lambda_n}{\lambda_p}$.

Solution:

We are given the equation for energy E :

$$E = \frac{hc}{\lambda_p} \Rightarrow \lambda_p = \frac{hc}{E}$$

Now, using the relation $\lambda_n = \frac{h}{p}$, where p is the momentum, we can substitute the expression for p :

$$\lambda_n = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

Now, substituting the expression for p :

$$\frac{\lambda_n}{\lambda_p} = \frac{\frac{h}{\sqrt{2mE}}}{\frac{h}{Ehc}} = \frac{E}{\sqrt{2mc^2}}$$

Thus, we get the expression:

$$\frac{\lambda_n}{\lambda_p} = \sqrt{\frac{E}{2mc^2}}$$

This gives us the desired expression for $\frac{\lambda_n}{\lambda_p}$.

💡 Quick Tip

For de Broglie wavelength problems, use $\lambda = \frac{h}{p}$ and carefully relate the momentum or energy for each particle or photon.

21. Plot a graph showing the variation of current with voltage for the material GaAs. On the graph, mark the region where:

- (a) resistance is negative, and
- (b) Ohm's law is obeyed.

Solution:

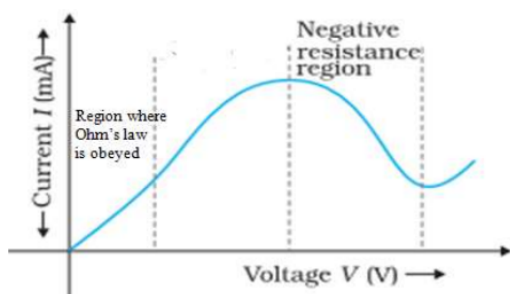
Gallium arsenide (GaAs) exhibits negative resistance in certain regions of its current-voltage (I - V) characteristic curve. The graph typically shows a linear region (Ohm's law), followed by

a region of negative slope (negative resistance), and then a saturation region.

The regions can be identified as follows:

- (a) The region where resistance is negative corresponds to the portion of the I - V curve with a negative slope (decreasing current with increasing voltage).
- (b) The region where Ohm's law is obeyed corresponds to the initial linear part of the I - V curve.

A qualitative sketch of the I - V graph is shown below:



The graph marks the linear region for Ohm's law and the negative resistance region.

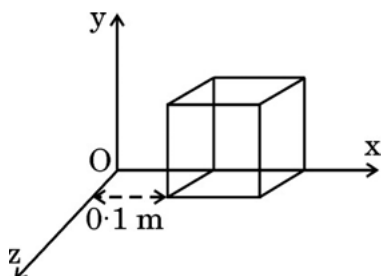
💡 Quick Tip

For semiconductors like GaAs, identify negative resistance regions by looking for sections of the I - V curve with a negative slope.

Section C

22. A cube of side 0.1 m is placed, as shown in the figure, in a region where electric field $\vec{E} = 500x \hat{i}$ exists. Here x is in meters and E in N/C. Calculate:

- the flux passing through the cube, and
- the charge within the cube.



Solution:

The electric flux Φ through a surface is given by:

$$\Phi = \int \vec{E} \cdot d\vec{A},$$

where:

- $\vec{E}$ is the electric field,
- $d\vec{A}$ is the area vector perpendicular to the surface.

(a) The flux through the cube:

The electric field varies as $\vec{E} = 500x\hat{i}$. Only the two faces of the cube perpendicular to the x -axis contribute to the flux. Let the cube extend from $x = 0$ to $x = 0.1$ m.

At $x = 0$: The electric field is:

$$\vec{E} = 500 \times 0 = 0 \text{ N/C}.$$

At $x = 0.1$ m: The electric field is:

$$\vec{E} = 500 \times 0.1 = 50 \text{ N/C}.$$

The area of each face of the cube is:

$$A = (0.1)^2 = 0.01 \text{ m}^2.$$

The flux through the face at $x = 0$ is:

$$\Phi_1 = \vec{E} \cdot A = 0 \cdot 0.01 = 0 \text{ Nm}^2/\text{C}.$$

The flux through the face at $x = 0.1$ m is:

$$\Phi_2 = \vec{E} \cdot A = 50 \cdot 0.01 = 0.5 \text{ Nm}^2/\text{C}.$$

The net flux through the cube is:

$$\Phi = \Phi_2 - \Phi_1 = 0.5 - 0 = 0.5 \text{ Nm}^2/\text{C}.$$

Thus, the flux passing through the cube is:

$$\boxed{\Phi = 0.5 \text{ Nm}^2/\text{C}}.$$

(b) The charge within the cube:

Using Gauss's law:

$$\Phi = \frac{q_{\text{enclosed}}}{\epsilon_0},$$

where $\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2/\text{Nm}^2$ is the permittivity of free space.

Rearrange to find q_{enclosed} :

$$q_{\text{enclosed}} = \Phi \cdot \epsilon_0.$$

Substitute the values:

$$q_{\text{enclosed}} = 0.5 \cdot 8.854 \times 10^{-12} = 4.427 \times 10^{-12} \text{ C}.$$

Thus, the charge within the cube is:

$$\boxed{q_{\text{enclosed}} = 4.43 \times 10^{-12} \text{ C}}.$$

Quick Tip

For calculating flux through a cube with a non-uniform field, consider the electric field at each relevant face and use Gauss's law to find the enclosed charge.

23 (a) Define ‘current density’. Is it a scalar or a vector? An electric field $\vec{E}$ is maintained in a metallic conductor. If n be the number of electrons (mass m , charge $-e$) per unit volume in the conductor and τ its relaxation time, show that the current density $\vec{j} = \alpha \vec{E}$, where $\alpha = \left(\frac{ne^2}{m}\right) \tau$.

Solution:

Definition of Current Density:

The current density $\vec{j}$ is defined as the current per unit cross-sectional area. Mathematically:

$$\vec{j} = \frac{\vec{I}}{A},$$

where $\vec{I}$ is the current and A is the cross-sectional area. Current density is a vector quantity because it has both magnitude and direction.

Derivation:

The current I through a conductor is given by:

$$I = nqAv_d,$$

where:

- n is the number of charge carriers per unit volume,
- q is the charge of each carrier,
- A is the cross-sectional area,
- v_d is the drift velocity of the charge carriers.

Substitute v_d using $v_d = \mu E$, where μ is the mobility of charge carriers and E is the electric field:

$$I = nqA(\mu E).$$

The current density $\vec{j}$ is:

$$\vec{j} = \frac{I}{A} = nq(\mu E).$$

Using $\mu = \frac{e\tau}{m}$, where τ is the relaxation time and m is the mass of the charge carriers, substitute μ :

$$\vec{j} = nq \left(\frac{e\tau}{m} \right) \vec{E}.$$

Simplify:

$$\vec{j} = \left(\frac{ne^2\tau}{m} \right) \vec{E}.$$

Thus, $\alpha = \frac{ne^2\tau}{m}$, and:

$$\boxed{\vec{j} = \alpha \vec{E}}.$$

💡 Quick Tip

Current density $\vec{j}$ is always a vector quantity. It is proportional to the electric field $\vec{E}$, with proportionality constant $\alpha = \frac{ne^2\tau}{m}$.

23 (b). What is a Wheatstone bridge? Obtain the necessary conditions under which the Wheatstone bridge is balanced.

Solution:

Definition of Wheatstone Bridge:

A Wheatstone bridge is a circuit used to measure unknown resistances. It consists of four resistors arranged in a diamond shape, with a galvanometer connected between two opposite nodes.

Condition for Balance:

Let the resistances of the four arms of the Wheatstone bridge be R_1 , R_2 , R_3 , and R_4 . The bridge is balanced when no current flows through the galvanometer. This occurs when:

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}.$$

Derivation:

At balance, the potentials at the two points connected to the galvanometer are equal. Using Ohm's law:

$$\frac{V_{AB}}{R_1} = \frac{V_{AD}}{R_2}, \quad \frac{V_{AB}}{R_3} = \frac{V_{AD}}{R_4}.$$

Simplify:

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}.$$

Thus, the condition for the Wheatstone bridge to be balanced is:

$$\boxed{\frac{R_1}{R_2} = \frac{R_3}{R_4}}.$$

 Quick Tip

For a balanced Wheatstone bridge, ensure $\frac{R_1}{R_2} = \frac{R_3}{R_4}$. This eliminates current through the galvanometer.

24. A proton with kinetic energy $1.3384 \times 10^{-14} \text{ J}$ moving horizontally from north to south, enters a uniform magnetic field $B = 2.0 \text{ mT}$ directed eastward. Calculate:

- (a) the speed of the proton,
- (b) the magnitude of acceleration of the proton,
- (c) the radius of the path traced by the proton.

[Take q/m for proton $= 1.0 \times 10^8 \text{ C/kg}$].

Solution:

- a) The velocity v is given by the equation:

$$v = \sqrt{\frac{2 \times \text{K.E.}}{m}}$$

Substituting the values:

$$v = 4 \times 10^6 \text{ m/s}$$

b) The acceleration a is given by the equation:

$$a = \frac{qvB}{m}$$

Substituting the values:

$$a = 8 \times 10^{11} \text{ m/s}^2$$

c) The radius r of the path is given by:

$$r = \frac{mv}{Bq}$$

Substituting the values:

$$r = 20 \text{ m}$$

💡 Quick Tip

For a charged particle in a magnetic field, use $F = qvB$ to calculate force, $a = F/m$ for acceleration, and $r = \frac{mv}{qB}$ for the radius of the path.

25. An inductor, a capacitor, and a resistor are connected in series with an AC source $v = v_m \sin \omega t$. Derive an expression for the average power dissipated in the circuit. Also, obtain the expression for the resonant frequency of the circuit.

Solution:

Deriving an expression for the average power dissipated in a series LCR circuit:

The voltage is given by:

$$v = v_m \sin(\omega t)$$

The current is:

$$i = i_m \sin(\omega t + \varphi)$$

The power P is given by the product of voltage and current:

$$P = v \times i = (v_m \sin(\omega t)) \times (i_m \sin(\omega t + \varphi))$$

This simplifies to:

$$P = \frac{v_m i_m}{2} [\cos \varphi - \cos(2\omega t + \varphi)] \quad (1)$$

The average power over a cycle is given by averaging the two terms on the right-hand side of equation (1). The second term is time-dependent, while the first term is constant. Therefore, the average power P is:

$$P = \frac{v_m i_m}{2} \cos \varphi$$

Obtaining the expression for the resonant frequency:

At resonance, the reactance of the capacitor X_C equals the reactance of the inductor X_L :

$$\frac{1}{\omega C} = \omega L$$

This gives the angular frequency ω as:

$$\omega = \frac{1}{\sqrt{LC}}$$

Thus, the resonant frequency f is:

$$f = \frac{1}{2\pi\sqrt{LC}}$$

💡 Quick Tip

The average power dissipated in an AC circuit depends only on the resistive part of the impedance, while the resonant frequency minimizes the overall impedance of the circuit.

26(a) "The wavelength of the electromagnetic wave is often correlated with the characteristic size of the system that radiates." Give two examples to justify this statement.

Solution: 1. **Radio Waves:** Large antenna arrays are used for radio wave transmission because the wavelength of radio waves is in meters, requiring similarly sized systems to efficiently radiate or receive these waves.

2. **Microwaves:** Microwaves have a wavelength on the order of centimeters, which matches the size of cavities or waveguides used for their propagation in microwave ovens or radar systems.

💡 Quick Tip

For efficient radiation or absorption, the system's size should match the wavelength of the electromagnetic wave.

26(b).

(i) Why do long-distance radio broadcasts use short-wave bands?

(ii) Why are optical and radio telescopes built on the ground, but X-ray astronomy is possible only from satellites orbiting the Earth?

Solution: (i) Long-distance radio broadcasts use short-wave bands because these bands undergo ionospheric reflection, allowing them to travel large distances beyond the line of sight.

(ii) Optical and radio telescopes can be built on the ground because these waves can penetrate the Earth's atmosphere. X-rays, however, are absorbed by the atmosphere, making it necessary to place X-ray telescopes on satellites.

💡 Quick Tip

The Earth's atmosphere selectively allows certain frequencies (like optical and radio waves) while blocking others (like X-rays and gamma rays).

27. Write the drawbacks of Rutherford's atomic model. How did Bohr remove them? Show that different orbits in Bohr's atom are not equally spaced.

Solution:

Drawbacks of Rutherford's atomic model:

(i) According to classical electromagnetic theory, an accelerating charged particle emits radiation in the form of electromagnetic waves. The energy of an accelerating electron should therefore continuously decrease, causing the electron to spiral inward and eventually fall into the nucleus. As a result, such an atom cannot be stable.

(ii) As the electrons spiral inward, their angular velocities and hence their frequencies would change continuously. This would lead to the emission of a continuous spectrum, which contradicts the line spectrum that is actually observed.

Bohr's explanation:

Bohr postulated stable orbits in which electrons do not radiate energy. The postulates are as follows:

- (i) An electron in an atom can revolve in certain stable orbits without the emission of radiant energy.
- (ii) The electron revolves around the nucleus in only those orbits for which the angular momentum is an integral multiple of $\frac{h}{2\pi}$.

- **(iii)** An electron might make a transition from one of its specified non-radiating orbits to another of lower energy. When this happens, a photon is emitted with energy equal to the difference in energy between the initial and final states.

The radius of the n^{th} orbit is given by:

$$r_n = \frac{n^2 h^2}{4\pi^2 m e^2} \quad \text{or} \quad r_n \propto n^2$$

Alternatively:

The difference in the radius of consecutive orbits is:

$$r_{n+1} - r_n = k [(n+1)^2 - n^2]$$

This simplifies to:

$$r_{n+1} - r_n = k(2n+1)$$

which depends on n , meaning it is not a constant.

💡 Quick Tip

Bohr's model successfully explained atomic stability and spectral lines by introducing quantized energy levels.

28.

- State any two properties of a nucleus.**
- Why is the density of a nucleus much more than that of an atom?**
- Show that the density of nuclear matter is the same for all nuclei.**

Solution:

(a) Properties of a Nucleus:

- The nucleus contains protons and neutrons, collectively called nucleons.
- The nucleus has a positive charge due to the presence of protons.

(b) Nuclear Density: - The nucleus is much denser than the atom because most of the mass of an atom is concentrated in its tiny nucleus, whereas the atom's size includes the much larger electron cloud.

(c) Density of Nuclear Matter: - The volume of a nucleus is proportional to A , where A is the mass number. The radius R of a nucleus is given by:

$$R = R_0 A^{1/3}$$

- The density of the nucleus is:

$$\rho = \frac{\text{Mass of Nucleus}}{\text{Volume of Nucleus}} = \frac{m_N A}{\frac{4}{3}\pi R^3} = \frac{m_N}{\frac{4}{3}\pi R_0^3}$$

- Since R_0 and m_N are constants, the density is the same for all nuclei.

💡 Quick Tip

The density of nuclear matter is constant because the nucleus's radius increases with $A^{1/3}$, while its mass is proportional to A .

Questions number 29 and 30 are case study-based questions. Read the following paragraphs and answer the questions that follow .

29. A lens is a transparent medium bounded by two surfaces, with one or both surfaces being spherical. The focal length of a lens is determined by the radii of curvature of its two surfaces and the refractive index of its medium with respect to that of the surrounding medium. The power of a lens is reciprocal of its focal length. If a number of lenses are kept in contact, the power of the combination is the algebraic sum of the powers of the individual lenses.

(i). A double-convex lens, with each face having the same radius of curvature R , is made of glass of refractive index n . Its power is:

(A) $\frac{2(n-1)}{R}$

(B) $\frac{(2n-1)}{R}$

(C) $\frac{(n-1)}{2R}$)

(D) $\frac{(2n-1)}{2R}$

Correct Answer: (A) $\frac{2(n-1)}{R}$

Solution: The lens maker's formula is given by:

$$\frac{1}{f} = (n - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

For a double-convex lens, both surfaces have the same radius of curvature R , and:

$$\frac{1}{f} = (n - 1) \left(\frac{1}{R} - \frac{-1}{R} \right)$$

$$\frac{1}{f} = (n - 1) \cdot \frac{2}{R}$$

The power P of the lens is:

$$P = \frac{1}{f} = \frac{2(n - 1)}{R}$$

💡 Quick Tip

For a double-convex lens, use the lens maker's formula with the radii of curvature appropriately signed. The total power is proportional to the refractive index and inversely proportional to the radius.

(ii). A double-convex lens of power P , with each face having the same radius of curvature, is cut into two equal parts perpendicular to its principal axis. The power of one part of the lens will be:

- (A) $2P$
- (B) P
- (C) $4P$
- (D) $P/2$

Correct Answer: (D) $P/2$

Solution:

When a lens is cut into two equal parts perpendicular to its principal axis, the curvature of each part remains the same as the original lens. However, the effective thickness of each part is halved. The power of a lens is inversely proportional to its focal length, and the focal length is proportional to the thickness of the lens.

Since the thickness is halved, the focal length of one part will also be halved. This results in the power of one part being twice that of the original lens. Therefore, the power of one part of the lens will be:

$$\frac{P}{2}$$

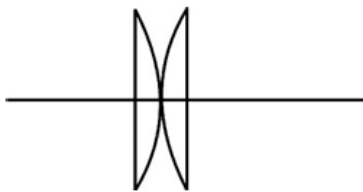
Thus, the correct answer is:

$$\boxed{\frac{P}{2}}.$$

💡 Quick Tip

Cutting a lens perpendicular to its principal axis does not change its focal length or power.

(iii). The above two parts are kept in contact with each other as shown in the figure. The power of the combination will be:



(A) $P_{\frac{2}{3}}$

(B) P

(C) $2P$

(D) $P_{\frac{4}{3}}$

Correct Answer: (C) $2P$

Solution: When two lenses are kept in contact, the total power of the combination is the algebraic sum of their individual powers. Since both parts have power P , the total power is:

$$P_{\text{total}} = P + P = 2P$$

💡 Quick Tip

For lenses in contact, the combined power is the sum of their individual powers.

(iv)(a). A double-convex lens of power P , with each face having the same radius of curvature, is cut along its principal axis. The two parts are arranged as shown in the figure. The power of the combination will be:



(A) Zero

(B) P

(C) $2P$

(D) $\frac{P}{2}$

Correct Answer: (C) $2P$

Solution:

When a double-convex lens is cut along its principal axis, the resulting two parts will each behave as a plano-convex lens. The focal length of each part will be halved, and the power is inversely proportional to the focal length. Since the focal length of each part is halved, the power of each part will be doubled.

When these two parts are combined, their powers will add up. Therefore, the total power of the combination will be:

$$P_{\text{total}} = P + P = 2P$$

Thus, the power of the combination is $2P$.

Thus, the correct answer is:

$$\boxed{2P}.$$

💡 Quick Tip

- When a convex lens is cut along its principal axis, each half acts as a plano-convex lens.
- The focal length of each part is halved, and since power is inversely proportional to focal length, the power of each part doubles.
- The total power of the combination is the sum of individual powers, making it $2P$.
- This concept is useful in understanding how lens cutting affects optical power in practical applications.

(iv)(b). Two convex lenses of focal lengths 60 cm and 20 cm are held coaxially in contact with each other. The power of the combination is:

- (A) 6.6 D
- (B) 15 D
- (C) $\frac{1}{15}$ D
- (D) $\frac{1}{80}$ D

Correct Answer: (A) 6.6 D

Solution:

The power P of a lens is given by the equation:

$$P = \frac{1}{f}$$

Where f is the focal length of the lens in meters. The power of two lenses in contact is the sum of their individual powers:

$$P_{\text{total}} = P_1 + P_2$$

For the first lens with a focal length of 60 cm (or 0.6 m):

$$P_1 = \frac{1}{0.6} = 1.67 \text{ D}$$

For the second lens with a focal length of 20 cm (or 0.2 m):

$$P_2 = \frac{1}{0.2} = 5 \text{ D}$$

The total power of the combination is:

$$P_{\text{total}} = 1.67 \text{ D} + 5 \text{ D} = 6.67 \text{ D}$$

Thus, the correct answer is approximately 6.6 D.

6.6 D

💡 Quick Tip

The combined power of coaxial lenses in contact is the algebraic sum of their individual powers. Always use focal lengths in the same unit and ensure consistency in calculations.

Question 30:

Junction Diode as a Rectifier : The process of conversion of an ac voltage into a dc voltage is called rectification and the device which performs this conversion is called a rectifier. The characteristics of a p-n junction diode reveal that when a p-n junction diode is forward biased, it offers a low resistance and when it is reverse biased, it offers a high resistance. Hence, a p-n junction diode conducts only when it is forward biased. This property of a p-n junction diode makes it suitable for its use as a rectifier.

Thus, when an ac voltage is applied across a p-n junction, it conducts only during those alternate half cycles for which it is forward biased. A rectifier which rectifies only half cycle of an ac voltage is called a half-wave rectifier and one that rectifies both the half cycles is known as a full-wave rectifier.

(i).

The root mean square (RMS) value of an alternating voltage applied to a full-wave rectifier is $\frac{V_0}{\sqrt{2}}$. Then the root mean square value of the rectified output voltage is:

- (A) $V_0 \sqrt{2}$
- (B) $V_0^2 \sqrt{2}$
- (C) $2V_0 \sqrt{2}$
- (D) $V_0 \sqrt{2\sqrt{2}}$

Correct Answer: (A) $\frac{V_0}{\sqrt{2}}$

Solution: For a full-wave rectifier, the rectified output voltage waveform consists of the absolute values of the input AC voltage waveform. The RMS value of the output voltage is calculated as:

$$V_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T V_{\text{rectified}}^2 dt}$$

Since the full-wave rectifier converts both halves of the alternating current waveform into positive polarity, the RMS value remains the same as that of the input AC voltage waveform:

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$$

Thus, the RMS value of the rectified output voltage is $\frac{V_0}{\sqrt{2}}$.

💡 Quick Tip

In a full-wave rectifier, the RMS value of the output voltage is the same as the RMS value of the input AC voltage, as it retains the waveform's overall power distribution.

(ii). In a full-wave rectifier, the current in each of the diodes flows for:

- (A) Complete cycle of the input signal
- (B) Half cycle of the input signal
- (C) Less than half cycle of the input signal
- (D) Only for the positive half cycle of the input signal

Correct Answer: (B) Half cycle of the input signal

Solution: In a full-wave rectifier circuit, there are two diodes that conduct current alternately during the positive and negative half-cycles of the AC input signal. Each diode conducts for only half of the input signal cycle. The conduction process works as follows:

- During the positive half-cycle, one diode is forward biased, allowing current to flow.
- During the negative half-cycle, the other diode becomes forward biased and conducts current.

Thus, each diode conducts current for half the cycle of the input signal.

💡 Quick Tip

In a full-wave rectifier, each diode operates alternately, conducting current for half the input signal's cycle.

(iii). In a full-wave rectifier:

- (A) Both diodes are forward biased at the same time.
- (B) Both diodes are reverse biased at the same time.
- (C) One is forward biased and the other is reverse biased at the same time.

(D) Both are forward biased in the first half of the cycle and reverse biased in the second half.

Correct Answer: (C) One is forward biased and the other is reverse biased at the same time.

Solution: In a full-wave rectifier, the two diodes are connected such that:

- During the positive half-cycle of the AC input, one diode is forward biased (allowing current to flow), while the other is reverse biased (blocking current).
- During the negative half-cycle, the roles are reversed. The previously reverse-biased diode becomes forward biased and conducts current, while the other diode becomes reverse biased and blocks current.

At any given time, one diode is conducting (forward biased), and the other diode is blocking (reverse biased).

💡 Quick Tip

In a full-wave rectifier, one diode is forward biased, and the other is reverse biased during each half-cycle of the input signal.

(iv)(a). An alternating voltage of frequency 50 Hz is applied to a half-wave rectifier. Then the ripple frequency of the output will be:

- (A) 100 Hz
- (B) 50 Hz
- (C) 25 Hz
- (D) 150 Hz

Correct Answer: (B) 50 Hz

Solution:

In a half-wave rectifier, only one half of the alternating current (either positive or negative half-cycle) is used for rectification. The output waveform consists of pulses that correspond to the frequency of the input AC signal.

Therefore, the frequency of the ripple in the output is the same as the input frequency:

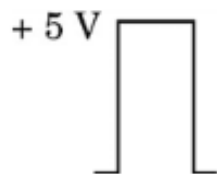
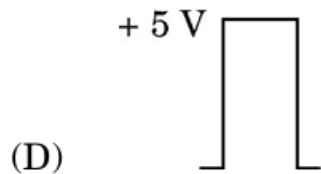
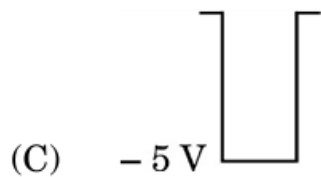
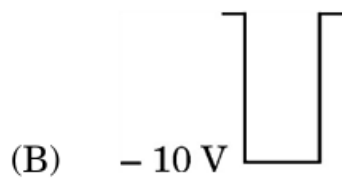
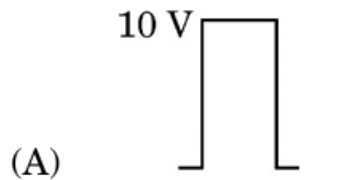
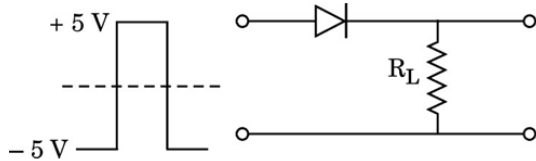
$$f_{\text{ripple}} = f_{\text{input}} = 50 \text{ Hz}$$

Thus, the ripple frequency of the output is 50 Hz.

💡 Quick Tip

For a half-wave rectifier, the ripple frequency is the same as the input AC frequency. In contrast, for a full-wave rectifier, the ripple frequency is twice the input frequency.

(iv)(b). A signal, as shown in the figure, is applied to a p-n junction diode. Identify the output across the resistance R_L :



Correct Answer: (D)

Solution:

The circuit consists of a p-n junction diode in series with a load resistor R_L . The input signal alternates between $+5\text{ V}$ and -5 V . The behavior of the diode depends on the polarity of the input signal:

1. **Positive Half-Cycle ($+5\text{ V}$):** The diode is forward biased during the positive half-cycle. This allows current to flow through the load resistor R_L , and the output voltage across R_L is equal to the input voltage of $+5\text{ V}$.

2. **Negative Half-Cycle (-5 V):** The diode is reverse biased during the negative half-cycle. In this condition, the diode does not conduct, and no current flows through R_L . The output voltage across R_L is zero during this phase.

The output waveform across R_L consists only of the positive half-cycles of the input signal, and its amplitude is $+5\text{ V}$.

💡 Quick Tip

In a rectifier circuit, a forward-biased diode allows current flow, while a reverse-biased diode blocks current. The output waveform corresponds to the rectified portion of the input signal.

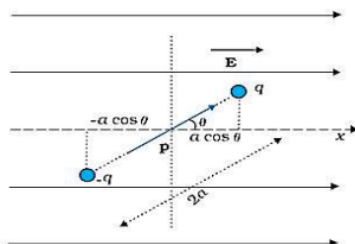
Section E

31.

(a)(i) Derive an expression for potential energy of an electric dipole $\vec{p}$ in an external uniform electric field $\vec{E}$. When is the potential energy of the dipole (1) maximum, and (2) minimum ?

Correct Answer: Maximum potential energy: $U = +pE$ when $\theta = 180^\circ$. Minimum potential energy: $U = -pE$ when $\theta = 0^\circ$.

Solution:



(i) The amount of work done in rotating the dipole from $\theta = \theta_0$ to $\theta = \theta_1$ by the external torque is:

$$W = \int_{\theta_0}^{\theta_1} \tau_{\text{ext}} d\theta$$

Since the torque $\tau_{\text{ext}} = pE \sin \theta$, we can write:

$$W = \int_{\theta_0}^{\theta_1} pE \sin \theta d\theta$$

The work done is then:

$$W = pE(\cos \theta_0 - \cos \theta_1)$$

For $\theta_0 = \frac{\pi}{2}$ and $\theta_1 = \theta$, we get:

$$W = pE \left(\cos \frac{\pi}{2} - \cos \theta \right) = -pE (\cos \theta)$$

The potential energy is given by:

$$U(\theta) = -pE \cos \theta = -\vec{p} \cdot \vec{E}$$

(1) The potential energy is maximum when:

$\vec{p}$ is antiparallel to $\vec{E}$

Alternatively:

$$\theta = 180^\circ \quad \text{or} \quad \pi \text{ radians}$$

(2) The potential energy is minimum when:

$\vec{p}$ is parallel to $\vec{E}$

Alternatively:

$$\theta = 0^\circ$$

💡 Quick Tip

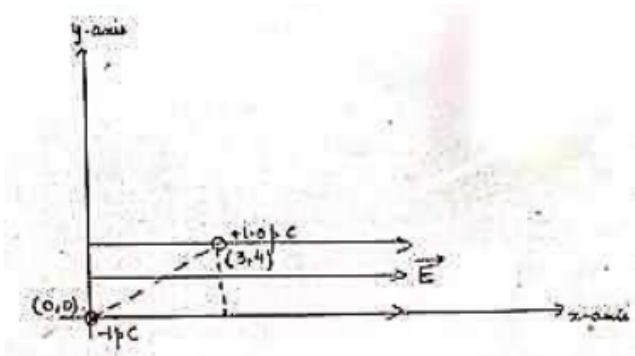
- The potential energy of an electric dipole in a uniform electric field is given by $U = -\vec{p} \cdot \vec{E} = -pE \cos \theta$.
- The potential energy is maximum ($U = +pE$) when the dipole is antiparallel to the field ($\theta = 180^\circ$).
- The potential energy is minimum ($U = -pE$) when the dipole is parallel to the field ($\theta = 0^\circ$).
- Understanding this concept helps in analyzing dipole behavior in external fields and potential energy changes due to rotations.

31.

(a)(ii) An electric dipole consists of point charges -1.0 pC and $+1.0 \text{ pC}$ located at $(0, 0)$ and $(3 \text{ mm}, 4 \text{ mm})$ respectively in the x - y plane. An electric field $\vec{E} = (1000 \text{ V/m}) \hat{i}$ is switched on in the region. Find the torque $\vec{\tau}$ acting on the dipole.

Correct Answer: Torque acting on the dipole: $\vec{\tau} = 12 \times 10^{-12} \text{ N} \cdot \text{m}$.

Solution: The torque τ is given by:



$$\tau = pE \sin \theta$$

This can be written as:

$$\tau = (2aq)E \sin \theta$$

Substituting the known values:

$$\tau = (5 \times 10^{-3} \times 1 \times 10^{-12} \times 10^3) \times \frac{4}{5}$$

Simplifying:

$$\tau = 4 \times 10^{-12} \text{ Nm}$$

The direction of the torque is along the negative Z-direction.

💡 Quick Tip

To solve torque problems for dipoles, always determine the displacement vector, dipole moment, and relative orientation with the electric field.

31 (b)(i):

An electric dipole (dipole moment $\vec{p} = p\hat{i}$), consisting of charges $-q$ and q , separated by distance $2a$, is placed along the x -axis, with its center at the origin. Show that the potential V , due to this dipole, at a point x ($x \gg a$) is equal to:

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{\vec{p} \cdot \hat{i}}{x^2}.$$

Correct Answer: Potential at x :

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{\vec{p} \cdot \hat{i}}{x^2}.$$

Solution:

Potential Due to an Electric Dipole

(i) The potential V due to two point charges $+q$ and $-q$ separated by a distance $2a$ can be calculated using:

$$V = \frac{1}{4\pi\epsilon_0} \left[\frac{q}{x-a} - \frac{q}{x+a} \right]$$

Simplifying this, we get:

$$V = \frac{q}{4\pi\epsilon_0} \left[\frac{x+a-x+a}{x^2-a^2} \right]$$

Thus, the potential becomes:

$$V = \frac{q}{4\pi\epsilon_0} \cdot \frac{2a}{x^2-a^2} = \frac{p}{4\pi\epsilon_0} \cdot \frac{1}{x^2-a^2}$$

where $p = 2aq$ is the dipole moment.

Since p is along the x-axis, the potential can be written as:

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{x^2-a^2}$$

If $x \gg a$, the potential simplifies to:

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{x^2}$$

Alternatively, we can use the geometry of the situation to derive the following expression:

$$V = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{r_1} - \frac{q}{r_2} \right)$$

where r_1 and r_2 are the distances from the charges to the point where the potential is being calculated.

By geometry:

$$r_1^2 = r^2 + a^2 - 2ar \cos \theta$$

$$r_2^2 = r^2 + a^2 + 2ar \cos \theta$$

Simplifying these equations:

$$r_1 = r \left(1 - \frac{2a \cos \theta}{r} \right)$$

$$r_2 = r \left(1 + \frac{2a \cos \theta}{r} \right)$$

Now using the binomial expansion and retaining terms up to the first order in $\frac{a}{r}$, we obtain:

$$\frac{1}{r_1} = \frac{1}{r} \left(1 - \frac{2a \cos \theta}{r} \right)^{-\frac{1}{2}} \approx \frac{1}{r} \left(1 + \frac{a \cos \theta}{r} \right)$$

$$\frac{1}{r_2} = \frac{1}{r} \left(1 + \frac{2a \cos \theta}{r} \right)^{-\frac{1}{2}} \approx \frac{1}{r} \left(1 - \frac{a \cos \theta}{r} \right)$$

Finally, the potential V can be expressed as:

$$V = \frac{q}{4\pi\epsilon_0} \cdot \frac{2a \cos \theta}{r^2}$$

Since $\vec{p} \cdot \hat{r} = p \cos \theta$, we can write:

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{r^2}$$

This is the rephrased solution using LaTeX code, providing a clear and structured approach to calculating the potential due to two charges arranged as a dipole.

31 (b)(ii): Two isolated metallic spheres S_1 and S_2 of radii 1 cm and 3 cm respectively are charged such that both have the same charge density

$$\sigma = \frac{2}{\pi} \times 10^{-9} \text{ C/m}^2.$$

They are placed far away from each other and connected by a thin wire. Calculate the new charge on sphere S_1 .

Correct Answer: New charge on S_1 : $Q_1 = 8.38 \text{ nC}$.

Solution:

Charge Redistribution Between Spheres

Charge on sphere S_1 :

$$Q_1 = \sigma \times \text{Surface Area} = \left(\frac{2 \times 10^{-9}}{\pi} \right) \times 4\pi(1 \times 10^{-2})^2 = 8 \times 10^{-13} \text{ C}$$

Charge on sphere S_2 :

$$Q_2 = \sigma \times \text{Surface Area} = \left(\frac{2 \times 10^{-9}}{\pi} \right) \times 4\pi(3 \times 10^{-2})^2 = 72 \times 10^{-13} \text{ C}$$

When the two spheres are connected by a thin wire, they acquire a common potential V , and the charge remains conserved. Therefore:

$$\begin{aligned} Q_1 + Q_2 &= Q'_1 + Q'_2 \\ &= C_1 V + C_2 V \end{aligned}$$

Thus,

$$Q_1 + Q_2 = (C_1 + C_2)V$$

The capacitances C_1 and C_2 are given by:

$$C_1 = 4\pi\epsilon_0 r_1 = \frac{1}{9} \times 10^{-11} \text{ F}$$

$$C_2 = 4\pi\epsilon_0 r_2 = \frac{1}{3} \times 10^{-11} \text{ F}$$

Now, we calculate the common potential V :

$$V = \frac{80 \times 10^{-13}}{\frac{1}{9} \times 10^{-11} + \frac{1}{3} \times 10^{-11}} = 1.8 \text{ V}$$

Substituting into $Q'_1 = C_1 V$, we get:

$$Q'_1 = C_1 \times V = \frac{1}{9} \times 10^{-11} \times 1.8 = 2 \times 10^{-12} \text{ C}$$

Alternatively:

Charge on sphere S_1 :

$$Q_1 = \sigma \times \text{Surface Area} = \left(\frac{2 \times 10^{-9}}{\pi} \right) \times 4\pi(1 \times 10^{-2})^2 = 8 \times 10^{-13} \text{ C}$$

Charge on sphere S_2 :

$$Q_2 = \sigma \times \text{Surface Area} = \left(\frac{2 \times 10^{-9}}{\pi} \right) \times 4\pi(3 \times 10^{-2})^2 = 72 \times 10^{-13} \text{ C}$$

When the spheres are connected by a thin wire, they acquire a common potential V , and the charge remains conserved. Therefore:

$$Q_1 + Q_2 = Q'_1 + Q'_2$$

Additionally, we have the relation for the charge distribution:

$$\frac{Q'_2}{Q'_1} = \frac{r_2}{r_1}$$

Solving this, we find:

$$Q'_1 = 2 \times 10^{-12} \text{ C}$$

💡 Quick Tip

When spheres are connected, charges redistribute such that their potentials are equal. Use the formula for potential and charge conservation to find the final charges.

32 (a)(i): A resistor and a capacitor are connected in series to an AC source $v = v_m \sin \omega t$. Derive an expression for the impedance of the circuit.

Correct Answer: Impedance of the circuit:

$$Z = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$

Solution:

Derivation of Impedance

The total voltage in a series R-C circuit is given by:

$$v = v_R + v_C,$$

where:

- $v_R = iR$ is the voltage across the resistor,
- $v_C = i\frac{1}{j\omega C}$ is the voltage across the capacitor.

The current in the circuit is the same through all components, and the impedance Z of the circuit is given by:

$$Z = \frac{v}{i}.$$

The impedance of the resistor is purely real, $Z_R = R$, and the impedance of the capacitor is purely imaginary, $Z_C = -j\frac{1}{\omega C}$. The total impedance is:

$$Z = Z_R + Z_C = R - j\frac{1}{\omega C}.$$

The magnitude of the impedance is:

$$|Z| = \sqrt{\text{Re}(Z)^2 + \text{Im}(Z)^2}.$$

Substituting the real and imaginary parts:

$$|Z| = \sqrt{R^2 + \left(-\frac{1}{\omega C}\right)^2} = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}.$$

Thus, the impedance of the circuit is:

$$Z = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}.$$

💡 Quick Tip

- The total impedance Z in a series R-C circuit is given by:

$$Z = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$

- The resistor's impedance is purely real ($Z_R = R$), while the capacitor's impedance is imaginary ($Z_C = -j\frac{1}{\omega C}$). - The magnitude of impedance is determined by combining the real and imaginary parts using:

$$|Z| = \sqrt{\text{Re}(Z)^2 + \text{Im}(Z)^2}$$

- The total impedance determines the phase difference and voltage behavior in AC circuits involving resistors and capacitors.

32 (a)(ii). When does an inductor act as a conductor in a circuit? Give reason for it.

Correct Answer:

An inductor acts as a conductor in a DC circuit when the current is steady (constant). This is because the induced emf is zero in the absence of a changing magnetic flux.

Solution:

Inductor Acting as a Conductor

An inductor acts as a conductor in a DC circuit when the current is steady (i.e., not changing with time). This happens because:

- The induced emf in the inductor is given by Faraday's Law:

$$\mathcal{E} = -L \frac{dI}{dt}.$$

- When the current I is steady, $\frac{dI}{dt} = 0$, and hence the induced emf is zero.

With no opposing emf, the inductor behaves as a simple conductor with negligible resistance, allowing the current to flow freely.

💡 Quick Tip

In AC circuits, consider both the real and imaginary components of impedance. In DC circuits, inductors and capacitors behave differently depending on the steadiness of the current or voltage.

32 (a)(iii).

An electric lamp is designed to operate at 110 V DC and 11 A current. If the lamp is operated on 220 V, 50 Hz AC source with a coil in series, then find the inductance of the coil.

Correct Answer:

The inductance of the coil is $L = 0.064$ H.

Solution:

The lamp is designed for DC operation with the following specifications:

$$V_{\text{DC}} = 110 \text{ V}, \quad I = 11 \text{ A}.$$

The power consumed by the lamp is:

$$P = V_{\text{DC}} \cdot I = 110 \cdot 11 = 1210 \text{ W}.$$

When the lamp is connected to an AC source of 220 V and 50 Hz with a coil in series, the total impedance Z of the circuit is given by:

$$Z = \frac{V_{\text{AC}}}{I} = \frac{220}{11} = 20 \Omega.$$

The impedance of the circuit is the combination of the resistance of the lamp and the inductive reactance of the coil:

$$Z = \sqrt{R^2 + X_L^2}.$$

The resistance of the lamp is:

$$R = \frac{V_{\text{DC}}}{I} = \frac{110}{11} = 10 \Omega.$$

The inductive reactance X_L is given by:

$$X_L = \sqrt{Z^2 - R^2}.$$

Substituting the values:

$$X_L = \sqrt{20^2 - 10^2} = \sqrt{400 - 100} = \sqrt{300} = 10\sqrt{3} \Omega.$$

The inductive reactance is related to the inductance L by:

$$X_L = \omega L,$$

where $\omega = 2\pi f$ is the angular frequency of the AC source. For $f = 50 \text{ Hz}$:

$$\omega = 2\pi \cdot 50 = 100\pi \text{ rad/s}.$$

Substituting for L :

$$L = \frac{X_L}{\omega} = \frac{10\sqrt{3}}{100\pi}.$$

Simplifying:

$$L = \frac{\sqrt{3}}{10\pi} \text{ H}.$$

Approximating:

$$L \approx 0.064 \text{ H}.$$

💡 Quick Tip

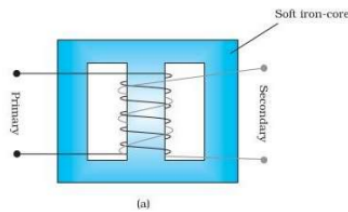
To calculate the inductance of a coil in an AC circuit, determine the total impedance, separate the resistive and reactive components, and use the relationship $X_L = \omega L$.

32 (b)(i):

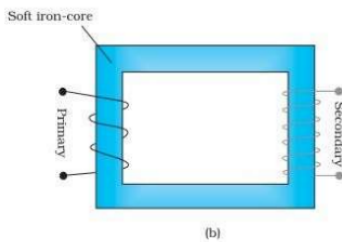
Draw a labelled diagram of a step-up transformer and describe its working principle. Explain any three causes for energy losses in a real transformer.

Correct Answer: The transformer does not violate the principle of conservation of energy.

Solution:



OR



Solution: Step-Up Transformer

A step-up transformer increases the voltage by having more turns in the secondary coil compared to the primary coil.

Working Principle:

A step-up transformer works on the principle of electromagnetic induction. When an alternating current flows through the primary coil, it generates a varying magnetic flux in the core. This varying flux induces an electromotive force (EMF) in the secondary coil according to Faraday's Law:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}.$$

The voltage ratio between the primary and secondary coils is given by the turns ratio:

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}.$$

Energy Losses in Transformers:

1. **Eddy Current Losses:** Circulating currents in the core produce heat, reducing efficiency. These are minimized by using laminated cores.
2. **Hysteresis Losses:** Energy is lost during repeated magnetization and demagnetization of the core. Soft magnetic materials help reduce this loss.
3. **Copper Losses:** Heat is generated due to the resistance of the windings. This can be reduced by using low-resistance materials for the coils.

💡 Quick Tip

- A step-up transformer- It operates on Faraday's Law of Electromagnetic Induction, where a varying magnetic flux induces an EMF in the secondary coil:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

- The voltage ratio follows the turns ratio formula:

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

- Energy Losses in Transformers: - Eddy Current Losses: Minimized using laminated cores. - Hysteresis Losses: Reduced using soft magnetic materials. - Copper Losses: Reduced by using low-resistance wire.

32 (b)(ii).

(ii) A step-up transformer converts a low voltage into high voltage. Does it violate the principle of conservation of energy? Explain.

Solution:

Conservation of Energy in Transformers

A step-up transformer increases the voltage but decreases the current proportionally, ensuring that the power remains constant (ignoring losses). The relationship is given by:

$$P_s = P_p \implies V_s I_s = V_p I_p.$$

Since the power output is equal to the power input (in an ideal transformer), the principle of conservation of energy is not violated. Any apparent violation is due to energy losses in the transformer, such as eddy current, hysteresis, and copper losses.

💡 Quick Tip

- A step-up transformer increases voltage while decreasing current proportionally, ensuring that power remains conserved (ignoring losses).

- The power relation is:

$$P_s = P_p \Rightarrow V_s I_s = V_p I_p$$

- The principle of conservation of energy is not violated because the total input and output power remain equal in an ideal transformer.

- Any apparent energy loss is due to eddy currents, hysteresis, and copper losses in practical transformers.

32 (b)(iii).

A step-up transformer has 200 and 3000 turns in its primary and secondary coils

respectively. The input voltage given to the primary coil is 90 V. Calculate:

- (1) The output voltage across the secondary coil
- (2) The current in the primary coil if the current in the secondary coil is 2.0 A.

Correct Answer: (1) The output voltage is 1350 V.

(2) The current in the primary coil is 30 A.

Solution: Calculations in a Step-Up Transformer

(1) Output Voltage:

The output voltage is determined by the turns ratio:

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}.$$

Substituting the given values:

$$\frac{V_s}{90} = \frac{3000}{200}.$$

Solving for V_s :

$$V_s = 90 \cdot \frac{3000}{200} = 1350 \text{ V}.$$

(2) Current in the Primary Coil:

The power in the primary and secondary coils is equal (neglecting losses):

$$P_s = P_p \implies V_s I_s = V_p I_p.$$

Rearranging for I_p :

$$I_p = \frac{V_s I_s}{V_p}.$$

Substituting the values:

$$I_p = \frac{1350 \cdot 2.0}{90} = 30 \text{ A}.$$

💡 Quick Tip

To solve transformer problems, always use the turns ratio for voltage and current calculations, and apply power conservation for ideal transformers.

33 (a)(i):

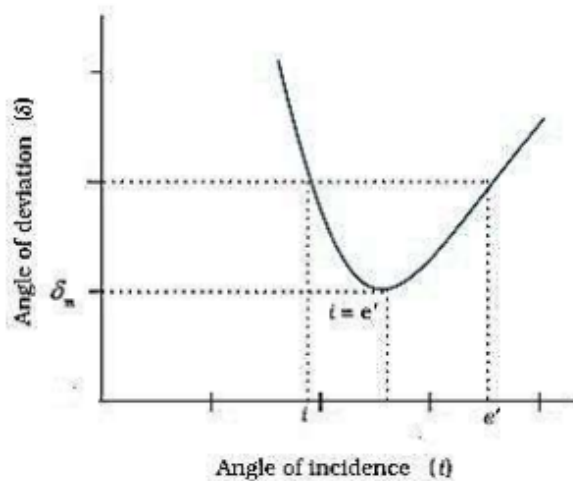
(i) A ray of light passes through a triangular prism. Show graphically how the angle of deviation varies with the angle of incidence. Hence, define the angle of minimum deviation.

Correct Answer:

(i) The graph of the angle of deviation vs. the angle of incidence has a minimum point, which corresponds to the angle of minimum deviation.

Solution:

a (i): Graphical Representation and Definition of Angle of Minimum Deviation



When a ray of light passes through a triangular prism, the angle of deviation (δ) varies with the angle of incidence (i). A graphical representation of δ vs. i shows a characteristic curve:

The graph is symmetric about the point of minimum deviation. At this point, the angle of incidence and the angle of emergence are equal. The angle of minimum deviation, denoted as δ_m , occurs when the light ray inside the prism is parallel to its base.

Definition: The angle of minimum deviation (δ_m) is the smallest angle of deviation of a light ray as it passes through a prism. It occurs when the light ray travels symmetrically through the prism.

💡 Quick Tip

- The angle of deviation (δ) of a light ray passing through a prism varies with the angle of incidence (i).
- The graph of δ vs. i is symmetric and shows a minimum point, corresponding to the angle of minimum deviation (δ_m).
- The angle of minimum deviation occurs when the light ray inside the prism is parallel to its base, and the angle of incidence equals the angle of emergence.
- This principle helps in determining the refractive index of the prism material using the relation:

$$n = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

where A is the prism angle.

33 (a)(ii). A ray of light is incident normally on a refracting face of a prism of prism angle A and suffers a deviation of angle δ . Prove that the refractive index n of the material of the prism is given by:

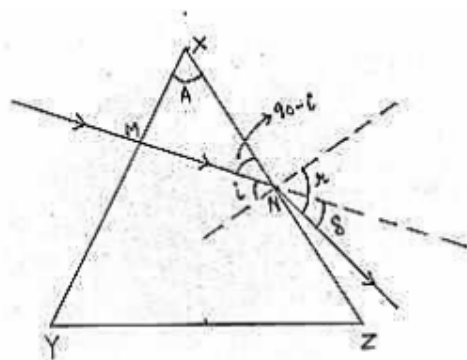
$$n = \frac{\sin(A + \delta)}{\sin A}.$$

Correct Answer The refractive index n is given by:

$$n = \frac{\sin(A + \delta)}{\sin A}.$$

Solution:

Derivation of Refractive Index of the Prism



When a ray of light is incident normally on a refracting face of a prism, the angle of incidence $i = 0$, and the ray enters the prism without deviation. Inside the prism, the ray undergoes refraction at the second face, resulting in a total deviation angle δ .

Using the geometry of the prism:

$$\delta = i_e - i_r,$$

where:

- i_e is the angle of emergence,
- i_r is the angle of refraction inside the prism.

From Snell's law at the second face:

$$n = \frac{\sin i_e}{\sin i_r}.$$

At the point of minimum deviation ($\delta = \delta_m$), the angles of incidence and emergence are equal:

$$i_e = A + \frac{\delta}{2}.$$

Substituting:

$$n = \frac{\sin(A + \delta)}{\sin A}.$$

Thus, the refractive index of the prism is:

$$n = \frac{\sin(A + \delta)}{\sin A}.$$

💡 Quick Tip

For triangular prisms, the angle of minimum deviation provides a simple way to calculate the refractive index using the geometry of the prism and Snell's law.

33 (a)(iii)

The refractive index of the material of a prism is $\sqrt{2}$. If the refracting angle of the prism is 60° , find:

1. The angle of minimum deviation, and
2. The angle of incidence.

Correct Answer:

- (1) Angle of minimum deviation: $\delta_m = 30^\circ$.
- (2) Angle of incidence: $i = 45^\circ$.

Solution:

(1): Angle of Minimum Deviation

The refractive index n of the material of the prism is related to the angle of the prism A and the angle of minimum deviation δ_m by the formula:

$$n = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}.$$

Substituting the given values:

- $n = \sqrt{2}$,
- $A = 60^\circ$.

Rearranging the formula:

$$\sin\left(\frac{A + \delta_m}{2}\right) = n \cdot \sin\left(\frac{A}{2}\right).$$

Calculating $\sin\left(\frac{A}{2}\right)$:

$$\frac{A}{2} = \frac{60^\circ}{2} = 30^\circ, \quad \sin 30^\circ = \frac{1}{2}.$$

Substituting:

$$\sin\left(\frac{A + \delta_m}{2}\right) = \sqrt{2} \cdot \frac{1}{2} = \frac{\sqrt{2}}{2}.$$

The angle whose sine is $\frac{\sqrt{2}}{2}$ is 45° :

$$\frac{A + \delta_m}{2} = 45^\circ.$$

Solving for δ_m :

$$A + \delta_m = 90^\circ \implies \delta_m = 90^\circ - 60^\circ = 30^\circ.$$

Thus, the angle of minimum deviation is:

$$\boxed{\delta_m = 30^\circ}.$$

(2): Angle of Incidence

At the angle of minimum deviation, the angle of incidence i is equal to the angle of emergence. Using the geometry of the prism, the relation between the angle of incidence, the angle of refraction r , and the prism angle A is:

$$r = \frac{A}{2}.$$

Substituting $A = 60^\circ$:

$$r = \frac{60^\circ}{2} = 30^\circ.$$

Using Snell's law at the first face of the prism:

$$n = \frac{\sin i}{\sin r}.$$

Rearranging for i :

$$\sin i = n \cdot \sin r.$$

Substituting:

$$\sin i = \sqrt{2} \cdot \sin 30^\circ = \sqrt{2} \cdot \frac{1}{2} = \frac{\sqrt{2}}{2}.$$

The angle whose sine is $\frac{\sqrt{2}}{2}$ is 45° :

$$i = 45^\circ.$$

Thus, the angle of incidence is:

$$i = 45^\circ.$$

💡 Quick Tip

For minimum deviation in a prism, the ray travels symmetrically, and Snell's law at the interface simplifies calculations for refractive index and angles.

33 (b):

(i) State Huygens' principle. A plane wave is incident at an angle i on a reflecting surface. Construct the corresponding reflected wavefront. Using this diagram, prove that the angle of reflection is equal to the angle of incidence.

Correct Answer:

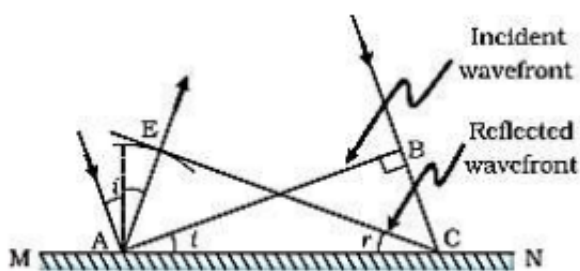
The angle of reflection is equal to the angle of incidence: $\theta_r = \theta_i$.

Solution:

Huygens' Principle and Angle of Reflection

Huygens' principle states that every point on a wavefront acts as a source of secondary wavelets, which spread out in all directions with the same speed as the wave. The new wavefront is the envelope of these secondary wavelets.

Diagram:



In the diagram, consider:

- The incident wavefront AB approaching the reflecting surface at an angle θ_i ,
- The reflected wavefront CD leaving the surface at an angle θ_r .

From the geometry of the wavefronts:

$$\theta_r = \theta_i.$$

Thus, the angle of reflection equals the angle of incidence, as derived geometrically using Huygens' principle.

💡 Quick Tip

- Huygens' Principle states that every point on a wavefront acts as a source of secondary wavelets that spread out in all directions.
- The new wavefront is formed as the envelope of these secondary wavelets.
- For reflection, the incident and reflected wavefronts follow the relation:

$$\theta_r = \theta_i$$

where θ_i (angle of incidence) = θ_r (angle of reflection).

- This principle provides a geometrical explanation of the Law of Reflection.

33 (b):

(ii) What are the coherent sources of light? Can two independent sodium lamps act like coherent sources? Explain.

Correct Answer: (ii) Coherent sources have a constant phase difference. Two independent sodium lamps cannot act as coherent sources.

Solution:

Coherent Sources of Light

Coherent sources of light are sources that emit waves with:

- A constant phase difference,
- The same frequency.

Explanation: Two independent sodium lamps cannot act as coherent sources because their emissions are random and do not maintain a constant phase difference. Only light derived from a single source (e.g., using a beam splitter) can produce coherent sources.

💡 Quick Tip

- Coherent sources emit waves with: - A constant phase difference. - The same frequency. - Two independent sodium lamps cannot be coherent sources because their emissions are random and do not maintain a constant phase relationship. - Coherent light can be obtained from a single source, such as using a beam splitter to create two coherent beams.

33 (b).

(iii) A beam of light consisting of a known wavelength 520 nm and an unknown wavelength λ , used in Young's double slit experiment, produces two interference patterns such that the fourth bright fringe of unknown wavelength coincides with the fifth bright fringe of known wavelength. Find the value of λ .

Solution

The unknown wavelength λ is 416 nm.

Finding the Unknown Wavelength λ

In Young's double slit experiment, the position of the bright fringes is determined by:

$$x = n \cdot \frac{\lambda D}{d},$$

where:

- n is the fringe order,
- λ is the wavelength of light,
- D is the distance between the slits and the screen,
- d is the distance between the slits.

For the fourth bright fringe of the unknown wavelength to coincide with the fifth bright fringe of the known wavelength:

$$4\lambda = 5 \cdot 520 \text{ nm.}$$

Solving for λ :

$$\lambda = \frac{5 \cdot 520}{4} = 650 \text{ nm.}$$

Quick Tip

In interference problems, use the relationship between fringe order and wavelength to solve for unknowns. Coincidence of fringes gives a proportional relationship.