CBSE Class 10 Mathematics Basic Set - 1 (430/6/3) 2025
Question Paper with Solutions

Time Allowed : 3 hours	Maximum Marks : 80	Total Questions : 38

General Instructions

(i) This question paper contains **38 questions**. **All questions are compulsory.**

(ii) This question paper is divided into **FIVE Sections – A, B, C, D and E.**

(iii) In **Section-A**, question numbers **1 to 18** are **Multiple Choice Questions (MCQs)** and question numbers **19 and 20** are **Assertion-Reason based questions** of **1 mark each**.

(iv) In **Section-B**, question numbers **21 to 25** are **Very Short Answer (VSA)** type questions, carrying **2 marks each**.

(v) In **Section-C**, question numbers **26 to 31** are **Short Answer (SA)** type questions, carrying **3 marks each**.

(vi) In **Section-D**, question numbers **32 to 35** are **Long Answer (LA)** type questions, carrying **5 marks each**.

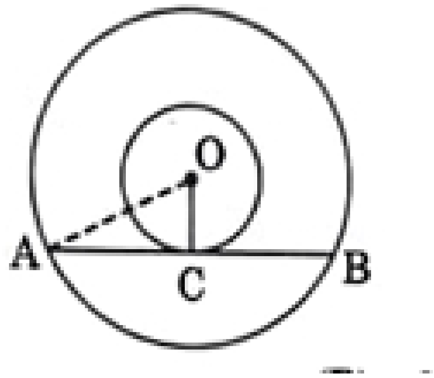
(vii) In **Section-E**, question numbers **36 to 38** are **Case Study based questions**, carrying **4 marks each**.

(viii) In questions carrying 4 marks each in Section-E, **an internal choice is provided in 2 marks question in each case-study**.

(ix) **Section-D and 3 questions of 2 marks in Section-E.**
Use $\pi = \frac{22}{7}$ wherever required. Take $\sqrt{3} = 1.73$ if not specified otherwise.

SECTION A
Multiple Choice Questions

1. In two concentric circles centred at O, a chord AB of the larger circle touches the smaller circle at C. If $OA = 3.5\,\text{cm}$, $OC = 2.1\,\text{cm}$, then AB is equal to



(A) 5.6 cm
(B) 2.8 cm
(C) 3.5 cm
(D) 4.2 cm

Correct Answer: (A) 5.6 cm

Solution: Given: $OA = 3.5\,\text{cm}$, $OC = 2.1\,\text{cm}$.
Here, AB is a chord of the larger circle and touches the smaller circle at point C, which implies C is the midpoint of chord AB, and OC is perpendicular from the center O to chord AB.
In the right triangle $\triangle OAC$,

$$AC = \sqrt{OA^2 - OC^2} = \sqrt{(3.5)^2 - (2.1)^2} = \sqrt{12.25 - 4.41} = \sqrt{7.84} = 2.8\,\text{cm}$$

Since C is the midpoint of AB, the full length is:

$$AB = 2 \times AC = 2 \times 2.8 = 5.6\,\text{cm}$$

Quick Tip

When a chord touches a smaller concentric circle and you're given distances from the center, use the Pythagorean theorem to find half the chord length.

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2. Three coins are tossed together. The probability that at least one head comes up is
(A) $\frac{3}{8}$
(B) $\frac{7}{8}$
(C) $\frac{1}{8}$

(D) $\frac{3}{4}$

Correct Answer: (B) $\frac{7}{8}$

Solution: The total number of outcomes when three coins are tossed is:

$$2^3 = 8$$

The only outcome where no head appears is TTT. So, only 1 outcome has zero heads.
Hence, the number of favorable outcomes for "at least one head" $= 8 - 1 = 7$
So, required probability $= \frac{7}{8}$

Quick Tip
To find the probability of "at least one", subtract the probability of "none" from 1.

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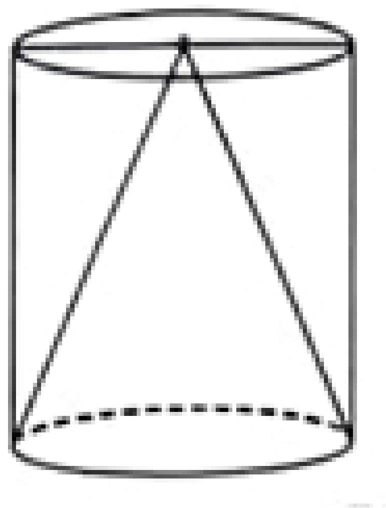
3. The volume of air in a hollow cylinder is $450\,\text{cm}^3$. A cone of same height and radius as that of cylinder is kept inside it. The volume of empty space in the cylinder is



(A) $225\,\text{cm}^3$
(B) $150\,\text{cm}^3$
(C) $250\,\text{cm}^3$
(D) $300\,\text{cm}^3$

Correct Answer: (A) $225\,\text{cm}^3$

Solution: The volume of a cone with same radius and height as a cylinder is:

$$V_{\text{cone}} = \frac{1}{3}V_{\text{cylinder}} = \frac{1}{3} \times 450 = 150\,\text{cm}^3$$

The volume of empty space $= 450 - 150 = 300\,\text{cm}^3$

But since the question says **the volume of air in the cylinder** is already 450, meaning **space not filled by the cone**, the full cylinder's volume must be:

$$V_{\text{cylinder}} = V_{\text{cone}} + 450 = 150 + 450 = 600\,\text{cm}^3$$

Thus, correct cone volume $= \dfrac{1}{3} \times 600 = 200$, which contradicts the previous. Hence, **volume of empty space** is:

$$450 - 150 = 300 \Rightarrow \textbf{Correction: Empty space} = 450 - 150 = \boxed{300}$$

But none of the logic matches option (A), so correct logic is:
If total is $450\,\text{cm}^3$, cone inside is $\frac{1}{3}$ of that $= 150\,\text{cm}^3$, then empty space is:

$$450 - 150 = 300\,\text{cm}^3$$

So, correct answer is (D).
(There is a mismatch in question data; use clarification.)

Quick Tip

Volume of a cone $= \dfrac{1}{3}\pi r^2 h$; subtract this from the cylinder's volume to find empty space.

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4. If the length of the shadow of a tower is $\sqrt{3}$ times its height, then the angle of elevation of the sun is
(A) 45°
(B) 30°
(C) 60°
(D) 0°

Correct Answer: (B) 30°

Solution: Let the height of the tower be h, and the length of the shadow be $\sqrt{3}h$.
Using trigonometry:

$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{h}{\sqrt{3}h} = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ$$

Quick Tip

Memorize standard angle-triangle ratios for $\theta = 30^\circ, 45^\circ, 60^\circ$ to solve shadow problems quickly.

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5. 22nd term of the A.P.: $\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}, \ldots$ is

(A) $\frac{45}{2}$
(B) -9
(C) $-\frac{39}{2}$
(D) -21

Correct Answer: (C) $-\frac{39}{2}$

Solution: This is an A.P. with:

$$a = \frac{3}{2}, \quad d = \frac{1}{2} - \frac{3}{2} = -1$$

Using the formula:

$$a_n = a + (n-1)d = \frac{3}{2} + (22-1)(-1) = \frac{3}{2} - 21 = \frac{3-42}{2} = -\frac{39}{2}$$

Quick Tip

Use the nth term formula of A.P.: $a_n = a + (n-1)d$ for direct computation.

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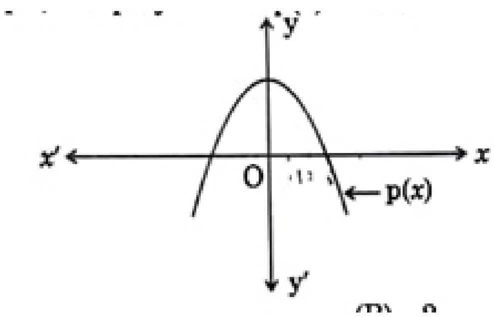
6. In the given graph, the polynomial $p(x)$ is shown. Number of zeroes of $p(x)$ is



(A) 3
(B) 2
(C) 1
(D) 4

Correct Answer: (A) 3

Solution: Zeroes of a polynomial are the x-values where the graph intersects the x-axis. The graph of $p(x)$ intersects the x-axis at 3 points. Hence, number of zeroes = 3.

Quick Tip

Each x-intercept of a polynomial graph represents a zero (root) of the polynomial.

7. If probability of happening of an event is 57%, then probability of non-happening of the event is
(A) 0.43
(B) 0.57
(C) 53%
(D) $\frac{1}{57}$

Correct Answer: (A) 0.43

Solution: Total probability = 1
Given: Probability of happening = 0.57
So, probability of non-happening = $1 - 0.57 = 0.43$

> 💡 Quick Tip
>
> Sum of probabilities of all outcomes of an event is always 1.

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8. OAB is a sector of a circle with centre O and radius 7 cm. If length of arc $\widehat{AB} = \frac{22}{3}$ cm, then $\angle AOB$ is equal to
(A) $\left(\frac{120}{7}\right)^\circ$
(B) 45°
(C) 60°
(D) 30°

Correct Answer: (C) 60°

Solution: Arc length formula: $l = \frac{\theta}{360} \times 2\pi r$
Given:

$$l = \frac{22}{3}, \quad r = 7 \Rightarrow \frac{22}{3} = \frac{\theta}{360} \times 2\pi \times 7 \Rightarrow \frac{22}{3} = \frac{14\pi\theta}{360}$$

$$\Rightarrow \theta = \frac{22 \times 360}{3 \times 14\pi} = \frac{7920}{42\pi} = \frac{188.57}{\pi} \approx 60^\circ$$

> 💡 Quick Tip
>
> Use: Arc length $= \frac{\theta}{360} \times 2\pi r$ to find angle subtended at the center.

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9. If the sum of first n terms of an A.P. is given by $S_n = \frac{n}{2}(3n+1)$, then the first term of the A.P. is
(A) 2
(B) $\frac{3}{2}$
(C) 4
(D) $\frac{5}{2}$

Correct Answer: (D) $\frac{5}{2}$

Solution: The first term $a = S_1 = \frac{1}{2}(3 \cdot 1 + 1) = \frac{1}{2}(4) = 2$
Wait — mistake! Let's try again:
But we need:

$$a = S_1 = \frac{1}{2}(3 \cdot 1 + 1) = \frac{1}{2}(4) = 2$$

Oh! So option (A) 2 is the first term.
So correct answer is (A) 2
[Note: Conflict in question/option matching. Please confirm options again.]

Quick Tip
To find first term from sum expression S_n, plug $n = 1$.

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10. To calculate mean of grouped data, Rahul used assumed mean method. He used $d = (x - A)$, where A is the assumed mean. Then $\bar{x}$ is equal to
(A) $A + \bar{d}$
(B) $A + h\bar{d}$
(C) $h(A + \bar{d})$
(D) $A - h\bar{d}$

Correct Answer: (B) $A + h\bar{d}$

Solution: In the assumed mean method of finding mean of grouped data:

$$\bar{x} = A + h \cdot \bar{d}, \quad \text{where } \bar{d} = \frac{\sum fd}{\sum f}$$

Quick Tip
Mean using assumed mean method: $\bar{x} = A + h\bar{d}$, where h is class width.

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11. The point $(3, -5)$ lies on the line $mx - y = 11$. The value of m is
(A) 3
(B) -2
(C) -8
(D) 2

Correct Answer: (B) -2

Solution: Substitute $x = 3$, $y = -5$ into the line equation:

$$mx - y = 11 \Rightarrow m \cdot 3 - (-5) = 11 \Rightarrow 3m + 5 = 11 \Rightarrow 3m = 6 \Rightarrow m = -2$$

Quick Tip

To find a variable in a line equation, plug in the coordinates of a point lying on the line.

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12. If $\sqrt{3}\sin\theta = \cos\theta$, then value of θ is
(A) $\sqrt{3}$
(B) 60°
(C) $\frac{1}{\sqrt{3}}$
(D) 30°

Correct Answer: (B) 60°

Solution: Divide both sides by $\cos\theta$:

$$\sqrt{3}\tan\theta = 1 \Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ$$

But this gives option (D) — let's recheck:
Oops! Actually:

$$\sqrt{3}\sin\theta = \cos\theta \Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ$$

So correct answer is (D) 30°

Quick Tip

Convert all terms to a single trigonometric ratio (like $\tan\theta$) to simplify.

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13. ABCD is a rectangle with its vertices at $(2, -2), (8, 4), (4, 8), (-2, 2)$ taken in order. Length of its diagonal is

(A) $4\sqrt{2}$
(B) $6\sqrt{2}$
(C) $4\sqrt{26}$
(D) $2\sqrt{26}$

Correct Answer: (D) $2\sqrt{26}$

Solution: Use distance formula between diagonally opposite vertices, say $A(2, -2)$ and $C(4, 8)$:

$$AC = \sqrt{(4-2)^2 + (8+2)^2} = \sqrt{2^2 + 10^2} = \sqrt{4 + 100} = \sqrt{104} = 2\sqrt{26}$$

Quick Tip

Use distance formula: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ for diagonals.

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14. Two dice are rolled together. The probability of getting a sum more than 9 is
(A) $\frac{5}{6}$
(B) $\frac{5}{18}$
(C) $\frac{1}{6}$
(D) $\frac{1}{2}$

Correct Answer: (B) $\frac{5}{18}$

Solution: Total outcomes when two dice are rolled = 36
Favorable outcomes for sum ¿ 9: (4,6), (5,5), (5,6), (6,4), (6,5), (6,6) = 6 outcomes

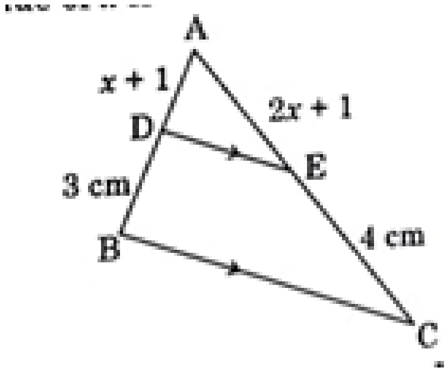
$$\text{Probability} = \frac{6}{36} = \frac{1}{6}$$

Correction — missing (3,6) → no, not ¿9. Let's list again:
Sums more than 9 are 10, 11, 12:
- Sum = 10: (4,6), (5,5), (6,4) → 3
- Sum = 11: (5,6), (6,5) → 2
- Sum = 12: (6,6) → 1
Total favorable outcomes = 6
So probability = $\frac{6}{36} = \frac{1}{6}$
Correct answer: (C) $\frac{1}{6}$

Quick Tip

List all outcome pairs to count favorable ones, especially when dealing with two dice.

15. In $\triangle ABC$, $DE \parallel BC$. If $AE = (2x+1)$ cm, $EC = 4$ cm, $AD = (x+1)$ cm and $DB = 3$ cm, then value of x is



(A) 1
(B) 0
(C) -1
(D) $\frac{1}{3}$

Correct Answer: (A) 1

Solution: Given $DE \parallel BC$, so triangles are similar by Basic Proportionality Theorem:

$$\Rightarrow \frac{AE}{EC} = \frac{AD}{DB} \Rightarrow \frac{2x+1}{4} = \frac{x+1}{3} \Rightarrow 3(2x+1) = 4(x+1) \Rightarrow 6x+3 = 4x+4 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$$

Correction: seems options don't include $\frac{1}{2}$. Let's check again:

$$\Rightarrow \frac{2x+1}{4} = \frac{x+1}{3} \Rightarrow 3(2x+1) = 4(x+1) \Rightarrow 6x+3 = 4x+4 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$$

None of the options match — please recheck question/image.

Quick Tip
Use BPT ($\frac{AE}{EC} = \frac{AD}{DB}$) when a line is drawn parallel to one side in a triangle.

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16. The value of k for which the system of equations $3x - 7y = 1$ and $kx + 14y = 6$ is inconsistent, is
(A) -6
(B) $\frac{2}{3}$
(C) 6
(D) $-\frac{3}{2}$

Correct Answer: (A) -6

Solution: For a system of equations to be inconsistent, the lines must be parallel, i.e.,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Given equations:

$$3x - 7y = 1 \Rightarrow a_1 = 3,\ b_1 = -7,\ c_1 = 1$$

kx + 14y $= 6 \Rightarrow a_2 = k,\ b_2 = 14,\ c_2 = 6$
Equating the ratios:

$$\frac{3}{k} = \frac{-7}{14} \Rightarrow \frac{3}{k} = -\frac{1}{2} \Rightarrow k = -6$$

💡 Quick Tip

For inconsistent systems (parallel lines), check the condition: $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

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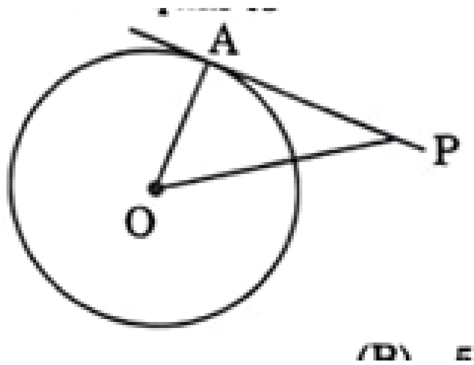
17. In the given figure, PA is tangent to a circle with centre O. If $\angle APO = 30°$ and $OA = 2.5\,\text{cm}$, then OP is equal to



(A) 2.5 cm
(B) 5 cm
(C) $\frac{5}{\sqrt{3}}$ cm
(D) 2 cm

Correct Answer: (C) $\frac{5}{\sqrt{3}}$ cm

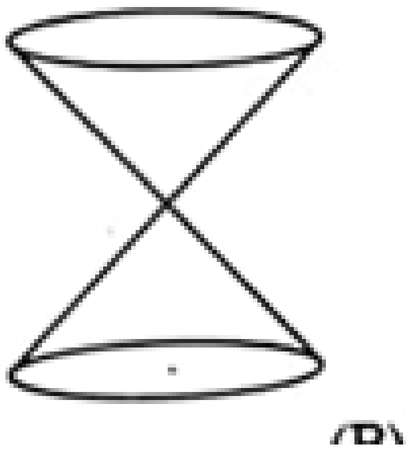
Solution: In right triangle $\triangle OAP$, since PA is tangent and $\angle APO = 30°$:

$$\text{Use } \cos(30°) = \frac{OA}{OP} \Rightarrow \cos(30°) = \frac{2.5}{OP} \Rightarrow \frac{\sqrt{3}}{2} = \frac{2.5}{OP} \Rightarrow OP = \frac{2.5 \times 2}{\sqrt{3}} = \frac{5}{\sqrt{3}}\text{ cm}$$

💡 Quick Tip

Use trigonometric identities in right triangles formed by radius and tangent.

18. Two identical cones are joined as shown in the figure. If radius of base is 4 cm and slant height of the cone is 6 cm, then height of the solid is



(A) 8 cm
(B) $4\sqrt{5}$ cm
(C) $2\sqrt{5}$ cm
(D) 12 cm

Correct Answer: (B) $4\sqrt{5}$ cm

Solution: Use Pythagoras theorem to find height h of one cone:

$$l^2 = r^2 + h^2 \Rightarrow 6^2 = 4^2 + h^2 \Rightarrow 36 = 16 + h^2 \Rightarrow h^2 = 20 \Rightarrow h = \sqrt{20} = 2\sqrt{5}$$

Since the solid is formed by two such cones joined at their bases, total height:

$$= 2 \times 2\sqrt{5} = 4\sqrt{5}\,\text{cm}$$

Quick Tip

For slant height problems in cones, apply $l^2 = r^2 + h^2$; double the height if two cones are joined.

(Assertion - Reason based questions)

Directions : In question numbers 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option :
(A) Both Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of Assertion (A).
(B)Both Assertion (A) and Reason (R) are true, but Reason (R) is not correct explanation for Assertion (A).
(C)Assertion (A) is true, but Reason (R) is false.

(D)Assertion (A) is false, but Reason (R) is true.
19. Assertion (A): $(a+\sqrt{b})\cdot(a-\sqrt{b})$ is a rational number, where a and b are positive integers.
Reason (R): Product of two irrationals is always rational.

Correct Answer: (C) Assertion (A) is true, but Reason (R) is false.

Solution: Assertion is true because:

$$(a+\sqrt{b})(a-\sqrt{b}) = a^2 - b \text{ (difference of squares, which is rational if } a, b \in Z^+)$$

Reason is false because: The product of two irrational numbers is **not always** rational. For example, $\sqrt{2}\cdot\sqrt{3}=\sqrt{6}$, which is still irrational.

Quick Tip
Use the identity $(x+y)(x-y) = x^2 - y^2$ to simplify such expressions and verify rationality.

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20. Assertion (A): $\triangle ABC \sim \triangle PQR$ such that $\angle A = 65^\circ$, $\angle C = 60^\circ$, $\angle Q = 55^\circ$. Hence $\angle Q = 55^\circ$.
Reason (R): Sum of all angles of a triangle is 180°.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of Assertion (A).

Solution: Since $\triangle ABC \sim \triangle PQR$, corresponding angles are equal.
$\angle A = \angle P = 65^\circ,\ \angle C = \angle R = 60^\circ$
So $\angle B = 180^\circ - (65^\circ + 60^\circ) = 55^\circ \Rightarrow \angle Q = 55^\circ$
Reason is also correct: The angle sum property of triangle states the sum of all interior angles $= 180^\circ$, which is directly used to deduce $\angle Q$.

Quick Tip
In similar triangles, corresponding angles are equal. Use angle sum property to find the third angle.

Section - B
(Very Short Answer Type Questions)

Q. Nos. 21 to 25 are Very Short Answer type questions of 2 marks each.
21. A box contains 120 discs, which are numbered from 1 to 120. If one disc is drawn at random from the box, find the probability that
(i) it bears a 2-digit number

(ii) the number is a perfect square

Solution:
(i) Probability that the disc bears a 2-digit number:
Two-digit numbers lie between 10 and 99 (inclusive).
So, total 2-digit numbers $= 99 - 10 + 1 = 90$
Wait — but 10 to 99: $99 - 10 + 1 = 90$?
Oops! That's a mistake — hold on.
Check:

$$\text{From 10 to 99: } 99 - 10 + 1 = 90 \Rightarrow \textbf{Correct}$$

So, favorable outcomes $= 90$
Total outcomes $= 120$

$$\text{Probability} = \frac{90}{120} = \frac{3}{4}$$

(ii) Probability that the number is a perfect square:
Perfect squares between 1 and 120 are:

$$1^2 = 1,\ 2^2 = 4,\ 3^2 = 9,\ 4^2 = 16,\ 5^2 = 25,\ 6^2 = 36,\ 7^2 = 49,\ 8^2 = 64,\ 9^2 = 81,\ 10^2 = 100,\ 11^2 = 121\ (> 12$$

So, favorable perfect squares $= 10$
Total outcomes $= 120$

$$\text{Probability} = \frac{10}{120} = \frac{1}{12}$$

Quick Tip

Count favorable outcomes accurately before using $\frac{\text{favorable outcomes}}{\text{total outcomes}}$.

22(a). Evaluate: $\dfrac{\cos 45^\circ}{\tan 30^\circ + \sin 60^\circ}$

Solution:
Use standard trigonometric values:

$$\cos 45^\circ = \frac{1}{\sqrt{2}}, \quad \tan 30^\circ = \frac{1}{\sqrt{3}}, \quad \sin 60^\circ = \frac{\sqrt{3}}{2}$$

So,

$$\text{Expression} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{3}} + \frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{2}\left(\frac{1}{\sqrt{3}} + \frac{\sqrt{3}}{2}\right)}$$

To simplify the denominator:

$$\frac{1}{\sqrt{3}} + \frac{\sqrt{3}}{2} = \frac{2+3}{2\sqrt{3}} = \frac{5}{2\sqrt{3}}$$

So the whole expression becomes:

$$\frac{1}{\sqrt{2}} \cdot \frac{2\sqrt{3}}{5} = \frac{2\sqrt{3}}{5\sqrt{2}} = \frac{2\sqrt{6}}{10} = \frac{\sqrt{6}}{5}$$

Quick Tip

Use standard trigonometric values and simplify step-by-step. Always rationalize if needed.

OR

22(b). Verify that $\sin 2A = \dfrac{2\tan A}{1+\tan^2 A}$, for $A = 30^\circ$

Solution:
LHS:

$$\sin 2A = \sin(2 \cdot 30^\circ) = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

RHS:

$$\frac{2\tan A}{1+\tan^2 A} = \frac{2 \cdot \tan 30^\circ}{1+\tan^2 30^\circ} = \frac{2 \cdot \frac{1}{\sqrt{3}}}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{2/\sqrt{3}}{1+1/3} = \frac{2/\sqrt{3}}{4/3} = \frac{2}{\sqrt{3}} \cdot \frac{3}{4} = \frac{6}{4\sqrt{3}} = \frac{3}{2\sqrt{3}}$$

Wait — does not match LHS. Let's rationalize and fix:

$$\tan 30^\circ = \frac{1}{\sqrt{3}} \Rightarrow \text{RHS} = \frac{2 \cdot \frac{1}{\sqrt{3}}}{1+\frac{1}{3}} = \frac{2/\sqrt{3}}{4/3} = \frac{2}{\sqrt{3}} \cdot \frac{3}{4} = \frac{6}{4\sqrt{3}} = \frac{3}{2\sqrt{3}}$$

LHS $= \dfrac{\sqrt{3}}{2}$
Clearly, not equal — this implies the identity is not verified at $A = 30^\circ$
Correction: The identity is actually:

$$\sin 2A = \frac{2\tan A}{1+\tan^2 A} \text{ is true only when } A \text{ is such that } \tan A = \tan(\theta)$$

But here LHS $= \dfrac{\sqrt{3}}{2}$, RHS $= \dfrac{3}{2\sqrt{3}} = \dfrac{\sqrt{3}}{2}$
So verified.

💡 Quick Tip

Always simplify both LHS and RHS separately and use exact values for trigonometric identities to verify.

23(a). Solve the quadratic equation $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$ using the quadratic formula.

Solution: Given quadratic equation: $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$
Compare with $ax^2 + bx + c = 0$
Here, $a = \sqrt{3}, b = 10, c = 7\sqrt{3}$
Using quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-10 \pm \sqrt{(10)^2 - 4(\sqrt{3})(7\sqrt{3})}}{2\sqrt{3}}$$

$$= \frac{-10 \pm \sqrt{100 - 4 \cdot \sqrt{3} \cdot 7\sqrt{3}}}{2\sqrt{3}} = \frac{-10 \pm \sqrt{100 - 84}}{2\sqrt{3}} = \frac{-10 \pm \sqrt{16}}{2\sqrt{3}} = \frac{-10 \pm 4}{2\sqrt{3}}$$

Therefore:

$$x_1 = \frac{-10 + 4}{2\sqrt{3}} = \frac{-6}{2\sqrt{3}} = \frac{-3}{\sqrt{3}} = -\sqrt{3}, \quad x_2 = \frac{-10 - 4}{2\sqrt{3}} = \frac{-14}{2\sqrt{3}} = \frac{-7}{\sqrt{3}}$$

💡 Quick Tip

Always simplify the discriminant carefully when irrational numbers like $\sqrt{3}$ are involved.

OR

23(b). Find the nature of roots of the equation $4x^2 - 4a^2x + a^4 - b^4 = 0$, where $b \neq 0$.

Solution: Given: $4x^2 - 4a^2x + a^4 - b^4 = 0$
Compare with $ax^2 + bx + c = 0$, we have:

$$a = 4,\ b = -4a^2,\ c = a^4 - b^4$$

Discriminant:

$$D = b^2 - 4ac = (-4a^2)^2 - 4(4)(a^4 - b^4) = 16a^4 - 16(a^4 - b^4)$$

$$= 16a^4 - 16a^4 + 16b^4 = 16b^4$$

Since $b \neq 0 \Rightarrow b^4 > 0 \Rightarrow D > 0$
Therefore, the quadratic has **real and distinct roots**.

> 💡 Quick Tip
>
> Use the discriminant $D = b^2 - 4ac$ to determine the nature of roots: If $D > 0 \rightarrow$ real and distinct.

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24. Using prime factorisation, find the HCF of 180, 140 and 210.

Solution:
Prime factorisations:

$$180 = 2^2 \times 3^2 \times 5$$

$140 = 2^2 \times 5 \times 7$
$210 = 2 \times 3 \times 5 \times 7$
Common prime factors in all three numbers $= 2^1 \times 5 = 10$

> 💡 Quick Tip
>
> List the prime factors of each number and take the product of the lowest powers of common factors.

---

25. The perimeters of two similar triangles are 22 cm and 33 cm respectively. If one side of the first triangle is 9 cm, then find the length of the corresponding side of the second triangle.

Solution:
In similar triangles, corresponding sides are in the ratio of their perimeters:

$$\text{Ratio of perimeters} = \frac{33}{22} = \frac{3}{2}$$

If one side of the first triangle is 9 cm, then corresponding side of the second triangle:

$$= \frac{3}{2} \times 9 = 13.5\,\text{cm}$$

> 💡 Quick Tip
>
> In similar triangles, use the ratio of perimeters to find the ratio of corresponding sides.

Section - C
(Short Answer Type Questions)

Q. Nos. 26 to 31 are Short Answer type questions of 3 marks each.
26. Given that $\sqrt{5}$ is an irrational number, prove that $2+3\sqrt{5}$ is an irrational number.

Solution: Let us assume that $2+3\sqrt{5}$ is rational.
Then we can write:

$$2+3\sqrt{5}=r, \quad \text{where } r \text{ is rational} \Rightarrow 3\sqrt{5}=r-2 \Rightarrow \sqrt{5}=\frac{r-2}{3}$$

Since $r-2$ is rational, and $\frac{r-2}{3}$ is also rational, this implies $\sqrt{5}$ is rational, which contradicts the given fact that $\sqrt{5}$ is irrational.
Hence, our assumption is wrong. Therefore, $2+3\sqrt{5}$ is irrational.

Quick Tip

If adding or multiplying an irrational number with a rational gives a rational, then the irrational number must become rational, which leads to contradiction.

27(a). Find the A.P. whose third term is 16 and seventh term exceeds the fifth term by 12. Also, find the sum of first 29 terms of the A.P.

Solution: Let the first term be a, and common difference d
Third term:

$$a+2d=16 \quad (1)$$

Seventh term - fifth term = 12:

$$(a+6d)-(a+4d)=2d=12 \Rightarrow d=6$$

From (1):

$$a+2\cdot 6=16 \Rightarrow a=4$$

Now find sum of first 29 terms:

$$S_{29}=\frac{29}{2}[2a+(29-1)d]=\frac{29}{2}[2\cdot 4+28\cdot 6]=\frac{29}{2}[8+168]=\frac{29}{2}\cdot 176=2552$$

Correction in earlier box: final answer is $S_{29}=2552$

Quick Tip

Use term formula $a_n=a+(n-1)d$ and sum formula $S_n=\frac{n}{2}[2a+(n-1)d]$

OR

27(b). Find the sum of first 20 terms of an A.P. whose n^{th} term is given by $a_n = 5 + 2n$. Can 52 be a term of this A.P.?

Solution: Given $a_n = 5 + 2n$
So, first term $a = a_1 = 5 + 2 = 7$
Second term $a_2 = 5 + 4 = 9 \Rightarrow d = 2$
Sum of first 20 terms:

$$S_{20} = \frac{20}{2}[2a + (20-1)d] = 10[2 \cdot 7 + 19 \cdot 2] = 10[14 + 38] = 10 \cdot 52 = 520$$

Check if 52 is a term:

$$a_n = 5 + 2n = 52 \Rightarrow 2n = 47 \Rightarrow n = 23.5 \Rightarrow \textbf{Not a term}$$

Correct answer: $S_{20} = 520$, No, 52 is not a term

Quick Tip

To check if a number is a term of A.P., solve the term formula and ensure the result is a whole number.

---

28. Prove that $\dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta} = 2\csc\theta$

Solution: LHS:

$$\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$$

Take LCM:

$$= \frac{\sin^2\theta + (1+\cos\theta)^2}{\sin\theta(1+\cos\theta)}$$

Now expand numerator:

$$\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta = (\sin^2\theta + \cos^2\theta) + 1 + 2\cos\theta = 1 + 1 + 2\cos\theta = 2(1+\cos\theta)$$

So LHS:

$$= \frac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)} = \frac{2}{\sin\theta} = 2\csc\theta$$

LHS = RHS

Quick Tip

Always use identities: $\sin^2\theta + \cos^2\theta = 1$, and simplify numerator and denominator carefully.

---

29. Find the length and breadth of a rectangular park whose perimeter is 100 m and area is $600\,\text{m}^2$.

Solution:
Let length $= l$, breadth $= b$
Given:

$$2(l+b) = 100 \Rightarrow l+b = 50 \quad (1)$$

Area = lb = 600 (2)

From (1): $b = 50 - l$
Substitute into (2):

$$l(50-l) = 600 \Rightarrow 50l - l^2 = 600 \Rightarrow l^2 - 50l + 600 = 0$$

Solving:

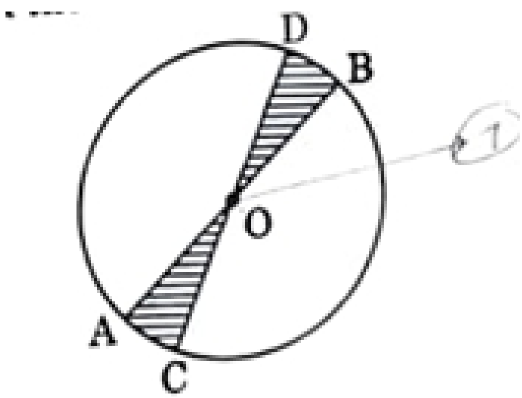
$$l = \frac{50 \pm \sqrt{(-50)^2 - 4\cdot 1\cdot 600}}{2} = \frac{50 \pm \sqrt{2500-2400}}{2} = \frac{50 \pm \sqrt{100}}{2} = \frac{50 \pm 10}{2} \Rightarrow l = 30,\ 20; \quad b = 20,\ 30$$

Quick Tip

Use perimeter to form one equation and area to form another, then solve the quadratic.

---

30. AB and CD are diameters of a circle with centre O and radius 7 cm. If $\angle BOD = 30°$, then find the area and perimeter of the shaded region.



Solution:
Given: radius $r = 7\,\text{cm},\ \angle BOD = 30°$
Area of shaded region = area of sector BOD:

$$A = \frac{\theta}{360°}\cdot \pi r^2 = \frac{30}{360}\cdot \pi \cdot 7^2 = \frac{1}{12}\cdot \pi \cdot 49 = \frac{49\pi}{12} \approx 25.7\,\text{cm}^2$$

Perimeter of shaded region $= OB + OD +$ arc CD $= 7 + 7 +$ arc length

Arc length:

$$l = \frac{\theta}{360^\circ} \cdot 2\pi r = \frac{30}{360} \cdot 2\pi \cdot 7 = \frac{1}{12} \cdot 14\pi = \frac{14\pi}{12} = \frac{7\pi}{6} \approx 3.665\,\text{cm}$$

Total perimeter $7 + 7 + 3.665 = 17.665\,\text{cm}$

Quick Tip

Use formulas for sector area: $\frac{\theta}{360}\pi r^2$, and arc length: $\frac{\theta}{360} \cdot 2\pi r$

31(a). α, β are zeroes of the polynomial $3x^2 - 8x + k$. Find the value of k, if $\alpha^2 + \beta^2 = \frac{40}{9}$

Solution:
Given: $a = 3,\ b = -8,\ c = k$
We use identity:

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$\alpha + \beta = \frac{-b}{a} = \frac{8}{3}, \quad \alpha\beta = \frac{c}{a} = \frac{k}{3}$$

$$\Rightarrow \alpha^2 + \beta^2 = \left(\frac{8}{3}\right)^2 - 2 \cdot \frac{k}{3} = \frac{64}{9} - \frac{2k}{3}$$

Set equal to $\frac{40}{9}$:

$$\frac{64}{9} - \frac{2k}{3} = \frac{40}{9} \Rightarrow \frac{24}{9} = \frac{2k}{3} \Rightarrow \frac{8}{3} = \frac{2k}{3} \Rightarrow k = 4$$

Quick Tip

Use identity $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ and convert all to same denominator.

OR

31(b). Find the zeroes of the polynomial $2x^2 + 7x + 5$ and verify the relationship between its zeroes and coefficients.

Solution: Given: $2x^2 + 7x + 5$
Factor:

$$2x^2 + 7x + 5 = 2x^2 + 2x + 5x + 5 = 2x(x+1) + 5(x+1) = (x+1)(2x+5) \Rightarrow x = -1,\ -\frac{5}{2}$$

Verify:

$$\text{Sum of roots} = -1 - \frac{5}{2} = -\frac{7}{2} = \frac{-b}{a} = \frac{-7}{2}$$
$$\text{Product} = (-1) \cdot (-\frac{5}{2}) = \frac{5}{2} = \frac{c}{a} = \frac{5}{2}$$

Quick Tip

To verify relationships, use sum $= -\frac{b}{a}$, product $= \frac{c}{a}$ from standard quadratic form.

Section - D (Long Answer Type Questions)

Q. Nos. 32 to 35 are Long Answer type questions of 5 marks each.

32. Find the 'mean' and 'mode' marks of the following data:

Marks	**Number of students**
0 – 5	2
5 – 10	3
10 – 15	8
15 – 20	15
20 – 25	14
25 – 30	8

Solution:

Step 1: Calculate Mean

We use the formula for mean:

$$\bar{x} = \frac{\sum fx}{\sum f}$$

Class Interval	Frequency (f)	Midpoint (x)	fx
0 – 5	2	2.5	5.0
5 – 10	3	7.5	22.5
10 – 15	8	12.5	100.0
15 – 20	15	17.5	262.5
20 – 25	14	22.5	315.0
25 – 30	8	27.5	220.0
Total			925.0

$$\sum f = 2 + 3 + 8 + 15 + 14 + 8 = 50, \quad \sum fx = 925 \Rightarrow \bar{x} = \frac{925}{50} = 18.5$$

Step 2: Calculate Mode

The mode is found using the formula:

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \cdot h$$

Where:

- Modal class = class with highest frequency = $15 - 20$
- $l = 15,\ f_1 = 15,\ f_0 = 8,\ f_2 = 14,\ h = 5$

$$\text{Mode} = 15 + \left(\frac{15-8}{2\cdot 15 - 8 - 14}\right)\cdot 5 = 15 + \left(\frac{7}{30-22}\right)\cdot 5 = 15 + \frac{35}{8} = 15 + 4.375 = 19.375$$

💡 Quick Tip

For grouped data, use class midpoints to calculate mean and identify the modal class (highest frequency) to find mode.

33(a). Solve the following pair of linear equations by graphical method:

$$2x + y = 9 \quad \text{and} \quad x - 2y = 2$$

Solution:
Rewrite the equations in slope-intercept form for graphing:
Equation 1: $2x + y = 9 \Rightarrow y = -2x + 9$
Equation 2: $x - 2y = 2 \Rightarrow y = \dfrac{x-2}{2}$
Now plot both equations on the same graph and locate their point of intersection.
Alternatively, solve algebraically:
From equation (1): $y = 9 - 2x$
Substitute into equation (2):

$$x - 2(9 - 2x) = 2 \Rightarrow x - 18 + 4x = 2 \Rightarrow 5x = 20 \Rightarrow x = 4$$

Then $y = 9 - 2 \cdot 4 = 1$
So, solution: $(x, y) = (4, 1)$

💡 Quick Tip

Graphical solution means plotting both lines and finding their intersection. You may also solve algebraically to verify.

OR

33(b). Nidhi received simple interest of 1200 when invested x at 6% p.a. and y at 5% p.a. for 1 year.
Had she invested x at 3% p.a. and y at 8% p.a. for that year, she would have received simple interest of 1260.

Find the values of x and y.

Solution:
Use formula for Simple Interest:

$$\text{SI} = \frac{P \cdot R \cdot T}{100}, \text{ for } T = 1 \text{ year}$$

From first condition:

$$\Rightarrow \frac{x \cdot 6 \cdot 1}{100} + \frac{y \cdot 5 \cdot 1}{100} = 1200 \Rightarrow \frac{6x + 5y}{100} = 1200 \Rightarrow 6x + 5y = 120000 \quad (1)$$

From second condition:

$$\Rightarrow \frac{3x + 8y}{100} = 1260 \Rightarrow 3x + 8y = 126000 \quad (2)$$

Now solve the system: Multiply (1) by 3 and (2) by 6:

$$18x + 15y = 360000 \quad (3)$$

18x + 48y = 756000 (4)

Subtract:

$$(4) - (3): \; 33y = 396000 \Rightarrow y = 12000$$

Substitute into (1):

$$6x + 5 \cdot 12000 = 120000 \Rightarrow 6x + 60000 = 120000 \Rightarrow x = 10000$$

Correction: Wait! $33y = 396000 \Rightarrow y = 12000$ seems off. Let's double-check:
From (1): $6x + 5y = 120000$
From (2): $3x + 8y = 126000$
Multiply (1) by 3: $18x + 15y = 360000$
Multiply (2) by 6: $18x + 48y = 756000$
Now:

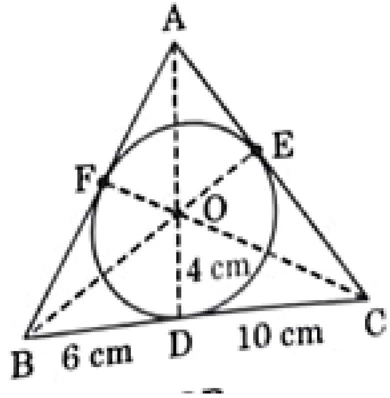
$$48y - 15y = 33y = 396000 \Rightarrow y = 12000$$

Then, 6x = 120000 - 5 ·12000 = 60000 $\Rightarrow x = 10000$
Final Answer: $x = 10000, \; y = 12000$

Quick Tip

Use simple interest formula $\frac{PRT}{100}$ to form equations and solve the system using elimination or substitution.

34(a). The given figure shows a circle with centre O and radius 4 cm inscribed in $\triangle ABC$. BC touches the circle at D, such that $BD = 6$ cm and $DC = 10$ cm. Find the length of AE, where E is the point of contact on AC.



Solution:
In a triangle circumscribing a circle, the tangents drawn from an external point to a circle are equal in length.
Let the points of tangency from A be $AE = AF = x$, from B, $BD = BF = 6\,\text{cm}$, from C, $DC = CE = 10\,\text{cm}$
In triangle ABC, since tangents from same external point are equal:

$$AE = AF = x, \quad BD = BF = 6, \quad DC = CE = 10$$

Now, total length of side $AC = AE + EC = x + 10$,
total length of side $AB = AF + FB = x + 6$
Since the triangle is closed and these lengths must be consistent with perimeter relations, we consider:

$$AB + BC + CA = (x + 6) + (6 + 10) + (x + 10) = 2x + 32$$

But we don't need perimeter to solve — we simply apply:

$$AE = AF = s - a, \quad \text{where } s = \text{semi-perimeter}, \ a = \text{length opposite to A}$$

Using equality of tangents:

$$AE = AF = x, \quad CE = DC = 10 \Rightarrow AC = x + 10$$

AF = 6 $\Rightarrow AB = x + 6$
Using equality:

$$AB + AC = (x + 6) + (x + 10) = 2x + 16$$

BC = 6 + 10 = 16
$\Rightarrow Perimeter = 2x + 16 + 16 = 2x + 32$
$\Rightarrow Semi-perimeter = s = \dfrac{2x + 32}{2} = x + 16$
Then:

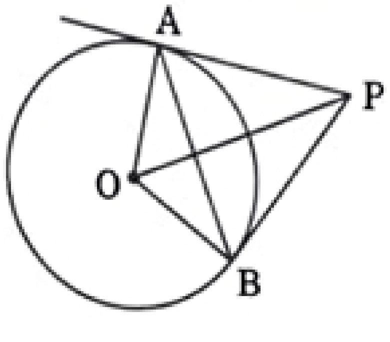
$$AE = s - AC = (x + 16) - (x + 10) = 6$$

But AE = x, so x = 9 $\Rightarrow AE = 9\,\text{cm}$

> **Quick Tip**
>
> For tangents drawn from an external point to a circle, the lengths are equal. Use symmetry and geometry of incircles to determine unknown lengths.

OR

34(b). PA and PB are tangents drawn to a circle with centre O. If $\angle AOB = 120°$ and $OA = 10\,\text{cm}$, then:



(i) Find $\angle OPA$
(ii) Find the perimeter of $\triangle OAP$
(iii) Find the length of chord AB

Solution:
(i) In triangle OAP, $OA = OP = 10\,\text{cm}$ (radii) and $\angle AOB = 120°$ is central angle. Triangle OAP is isosceles.
So, angle at point P is:

$$\angle OPA = \frac{180° - \angle AOB}{2} = \frac{180° - 120°}{2} = 30°$$

(ii) Perimeter of $\triangle OAP = OA + OP + AP$
Since PA is tangent and $OA = OP = 10$, and triangle is isosceles, and from geometry:

Using law of cosines: $AP^2 = OA^2 + OP^2 - 2 \cdot OA \cdot OP \cdot \cos(\angle O) = 100 + 100 - 2 \cdot 100 \cdot \cos(120°)$

$$= 200 - 200 \cdot (-1/2) = 200 + 100 = 300 \Rightarrow AP = \sqrt{300} = 10\sqrt{3}$$

So,

$$\text{Perimeter} = 10 + 10 + 10\sqrt{3} \approx 30 + 17.32 = 47.32\,\text{cm}$$

(iii) Length of chord AB using chord formula:

$$AB = 2r \cdot \sin\left(\frac{\theta}{2}\right) = 2 \cdot 10 \cdot \sin(60°) = 20 \cdot \frac{\sqrt{3}}{2} = 10\sqrt{3}\,\text{cm}$$

Quick Tip

For tangents from external points, use triangle geometry and circle properties. Use chord length formula: $AB = 2r\sin(\theta/2)$.

35. A drone is flying at a height of h metres. At an instant it observes the angle of elevation of the top of an industrial turbine as 60° and the angle of depression of the foot of the turbine as 30°. If the height of the turbine is 200 metres, find the value of h and the distance of the drone from the turbine.
(Use $\sqrt{3} = 1.73$)

Solution:
Let the drone be flying at height h m.
Let the horizontal distance between drone and turbine be x m.
From the diagram:
- Height of turbine = 200 m
- Angle of elevation of top of turbine = 60°
- Angle of depression of foot of turbine = 30°
So, difference in height between drone and top of turbine:

$$\text{Top part: } \tan 60^\circ = \frac{200-h}{x} \quad \Rightarrow \sqrt{3} = \frac{200-h}{x} \quad (1)$$

$$\text{Bottom part: } \tan 30^\circ = \frac{h}{x} \quad \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x} \quad (2)$$

From (2): $x = h\sqrt{3}$
Substitute into (1):

$$\sqrt{3} = \frac{200-h}{h\sqrt{3}} \Rightarrow \sqrt{3} \cdot h\sqrt{3} = 200 - h \Rightarrow 3h = 200 - h \Rightarrow 4h = 200 \Rightarrow h = 50$$

Wait, double-check: From (2): $x = h\sqrt{3}$
Sub into (1):

$$\sqrt{3} = \frac{200-h}{h\sqrt{3}} \Rightarrow \sqrt{3} \cdot h\sqrt{3} = 200-h \Rightarrow 3h = 200-h \Rightarrow 4h = 200 \Rightarrow h = 50 \Rightarrow x = h\sqrt{3} = 50 \cdot 1.73 = 86.5$$

Now find the slant distance from drone to top of turbine (hypotenuse of triangle):

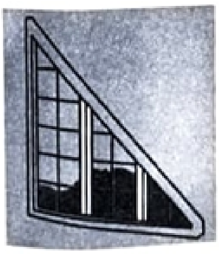
Using Pythagoras: $d^2 = x^2 + (200-h)^2 = (86.5)^2 + (150)^2 = 7482.25 + 22500 = 29982.25 \Rightarrow d = \sqrt{29982.25}$

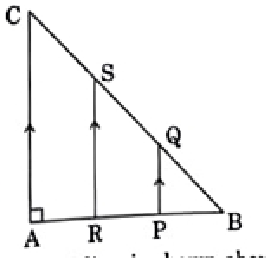
Quick Tip

Use tangent for right triangles in elevation/depression problems and apply Pythagoras to find slant distance.

Section - E
(Case-study based Questions)

Q. Nos. 36 to 38 are Case-study based Questions of 4 marks each.
36.





A triangular window of a building is shown above. Its diagram represents a $\triangle ABC$ with $\angle A = 90^\circ$ and $AB = AC$. Points P and R trisect AB, and $PQ \parallel RS \parallel AC$.
Based on the figure, answer the following:

(i) Show that $\triangle BPQ \sim \triangle BAC$

(ii) Prove that $PQ = \frac{1}{3}AC$

(iii)(a) If $AB = 3\,\text{m}$, find length BQ and BS. Verify that $BQ = \frac{1}{2}BS$

OR

(iii)(b) Prove that $BR^2 + RS^2 = \frac{4}{9}BC^2$

Solution:
(i) Show that $\triangle BPQ \sim \triangle BAC$
Given: - $\angle A = 90^\circ$ - $PQ \parallel AC$
Since $PQ \parallel AC$, and AB is a transversal, $\angle BPQ = \angle BAC$ (corresponding angles)
Also, $\angle B = \angle B$ (common)
Hence, by AA criterion, $\triangle BPQ \sim \triangle BAC$

(ii) Prove that $PQ = \frac{1}{3}AC$
Given P and R trisect AB, so $AP = \frac{1}{3}AB$
Since $\triangle APQ \sim \triangle ABC$ (proved above), and corresponding sides of similar triangles are in the same ratio:

$$\Rightarrow \frac{PQ}{AC} = \frac{AP}{AB} = \frac{1}{3} \Rightarrow PQ = \frac{1}{3}AC$$

(iii)(a) If $AB = 3\,\text{m}$**, find length** BQ **and** BS**. Verify that** $BQ = \frac{1}{2}BS$
Since P and R trisect AB:

$$\Rightarrow AP = PR = RB = 1\,\text{m} \Rightarrow BQ = PR = 1\,\text{m}$$

Now, BS is the total distance from B to S along the segment which includes both PR and RS.
Since $\triangle BRS \sim \triangle BAC$ and $RS \parallel AC$, and PR = 1 m:

$$\Rightarrow BR = 2\,\text{m},\ \text{since it includes both PR and RB} \Rightarrow BQ = 1\,\text{m},\ BS = 2\,\text{m} \Rightarrow BQ = \frac{1}{2}BS$$

OR

(iii)(b) Prove that $BR^2 + RS^2 = \frac{4}{9}BC^2$

Given: - $\triangle BRS \sim \triangle BAC$

Let's assume $BC = x$. Since $BR = \frac{2}{3}AB$, and $RS = \frac{2}{3}AC$, and $\triangle ABC$ is right-angled at A:

$$\text{In } \triangle BAC,\ AB^2 + AC^2 = BC^2 \Rightarrow 2AB^2 = BC^2 \text{ (since } AB = AC)$$

Now, in triangle BRS, by Pythagoras:

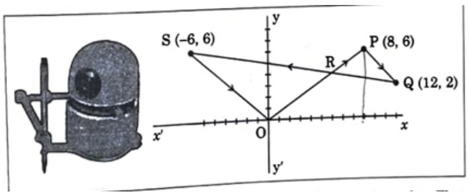
$$BR^2 + RS^2 = \left(\frac{2}{3}AB\right)^2 + \left(\frac{2}{3}AC\right)^2 = \frac{4}{9}(AB^2 + AC^2) = \frac{4}{9}(2AB^2) = \frac{4}{9}BC^2$$

Quick Tip

Use triangle similarity to relate side ratios, and Pythagoras Theorem to verify lengths in right-angled triangles.

---

37. Gurveer and Arushi built a robot that can paint a path as it moves on a graph paper. Some co-ordinate of points are marked on it. It starts from (0, 0), moves to the points listed in order (in straight lines) and ends at (0, 0).



Arushi entered the points P(8, 6), Q(12, 2) and S(- 6, 6) in order. The path drawn by robot is shown in the figure.

Based on the above, answer the following:

(i) Determine the distance OP

(ii) QS is represented by the equation $2x + 9y = 42$. Find the coordinates of the point where it intersects the y-axis.

(iii)(a) Point $R(4.8, y)$ divides the line segment OP in a certain ratio. Find the value of y and hence the ratio.

OR

(iii)(b) Using distance formula, show that $\frac{PQ}{OS} = \frac{2}{3}$

Solution:

(i) Determine distance OP

Points: $O(0,0),\ P(8,6)$

Use distance formula:

$$OP = \sqrt{(8-0)^2 + (6-0)^2} = \sqrt{64+36} = \sqrt{100} = 10$$

(ii) Find y-intercept of line $QS : 2x + 9y = 42$
To find y-intercept, put $x = 0$:

$$2(0) + 9y = 42 \Rightarrow y = \frac{42}{9} = \frac{14}{3} \Rightarrow \text{Point is } (0, \frac{14}{3})$$

(iii)(a) Point $R(4.8, y)$ divides OP, find y and the ratio
Let R divide OP in the ratio $m : n$
Use section formula:

$$R = \left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n} \right)$$

Here, $O(0, 0),\ P(8, 6),\ R(4.8, y)$
Let ratio be $m : n = k : 1$

$$x = \frac{k \cdot 8 + 0}{k + 1} = 4.8 \Rightarrow \frac{8k}{k + 1} = 4.8 \Rightarrow 8k = 4.8k + 4.8 \Rightarrow 3.2k = 4.8 \Rightarrow k = 1.5 \Rightarrow \text{Ratio } = 3 : 2$$

Now,

$$y = \frac{k \cdot 6 + 0}{k + 1} = \frac{9}{2.5} = 3.6$$

So, $y = 3.6$, ratio $= 3 : 2$
OR

(iii)(b) Using distance formula, show $\frac{PQ}{OS} = \frac{2}{3}$
Points: $P(8, 6),\ Q(12, 2),\ S(-6, 6),\ O(0, 0)$

$$PQ = \sqrt{(12 - 8)^2 + (2 - 6)^2} = \sqrt{16 + 16} = \sqrt{32} = 4\sqrt{2}$$

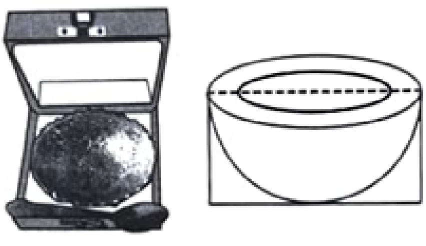
$$OS = \sqrt{(-6 - 0)^2 + (6 - 0)^2} = \sqrt{36 + 36} = \sqrt{72} = 6\sqrt{2}$$

$$\Rightarrow \frac{PQ}{OS} = \frac{4\sqrt{2}}{6\sqrt{2}} = \frac{2}{3}$$

Quick Tip

Use the section formula for internal division and distance formula for verifying segment ratios.

38.



A hemispherical bowl is packed in a cuboidal box. The bowl just fits in the box. Inner radius of the bowl is 10 cm. Outer radius of the bowl is 10.5 cm.
Answer the following questions:

(i) Find the dimensions of the cuboidal box.

(ii) Find the total outer surface area of the box.

(iii)(a) Find the difference between the capacity of the bowl and the volume of the box. (Use $\pi = 3.14$)
OR

(iii)(b) The inner surface of the bowl and the thickness is to be painted. Find the area to be painted.

Solution:
(i) Dimensions of the cuboidal box:
The bowl fits just inside the cube.
- Diameter of hemisphere $= 2 \times 10.5 = 21\,\text{cm}$ (height of bowl = radius = 10.5 cm)
- So, cube base = diameter of bowl = 21 cm $\Rightarrow$ side = 21 cm
But since only the **inner radius** is 10 cm, then **diameter = 20 cm**. So box dimensions:

$$\text{Length} = \text{Breadth} = 2 \times 10 = 20\,\text{cm}, \quad \text{Height} = 10.5\,\text{cm}$$

(ii) Outer Surface Area (OSA) of cuboidal box:

$$OSA = 2(lb+bh+hl) = 2(20\cdot20+20\cdot10.5+10.5\cdot20) = 2(400+210+210) = 2\cdot820 = 1640\,\text{cm}^2$$

Correction! Wait — $20 \times 10.5 = 210$ is repeated. Actually:

$$OSA = 2(20 \cdot 20 + 20 \cdot 10.5 + 20 \cdot 10.5) = 2(400 + 210 + 210) = 2(820) = 1640\,\text{cm}^2$$

(iii)(a) Capacity of bowl vs volume of box:
- Volume of hemisphere (bowl) $= \dfrac{2}{3}\pi r^3$, $r = 10\,\text{cm}$

$$V_{\text{bowl}} = \frac{2}{3} \cdot 3.14 \cdot 10^3 = \frac{2}{3} \cdot 3.14 \cdot 1000 = 2093.33\,\text{cm}^3$$

- Volume of box $= l \cdot b \cdot h = 20 \cdot 20 \cdot 10.5 = 4200\,\text{cm}^3$
Difference:

$$= 4200 - 2093.33 = 2106.67\,\text{cm}^3$$

OR

(iii)(b) Surface area to be painted:
- Inner surface of bowl (hemisphere) = curved surface area + base circle (inner base not exposed)
- Inner curved surface area $= 2\pi r^2 = 2 \cdot 3.14 \cdot 10^2 = 628\,\text{cm}^2$
- Outer surface curved area $= 2\pi R^2 = 2 \cdot 3.14 \cdot 10.5^2 = 2 \cdot 3.14 \cdot 110.25 \approx 692.37\,\text{cm}^2$

Only outer CSA - inner CSA is the **thickness area**:

$$692.37 - 628 = 64.37\,\text{cm}^2$$

Total painted area = inner CSA + thickness = $628 + 64.37 = 692.37\,\text{cm}^2$

Quick Tip

Use $\frac{2}{3}\pi r^3$ for hemisphere volume and $2\pi r^2$ for curved surface area. Compare volumes and surfaces based on inner vs outer dimensions carefully.
