CBSE Class 10 Mathematics Basic 2026 Question Paper with Solutions

Time Allowed :3 Hours	Maximum Marks :80	Total Questions :38

General Instructions

Read the following instructions very carefully and strictly follow them:

1. *This question paper hass **38** questions. **All** questions are compulsory.*
2. *This question paper contains **five** sections - Section **A, B, C, D** and **E**.*
3. ***Section A** - Questions no. **1** to **18** are Multiple Choice Type Questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.*
4. ***Section B** - Questions no. **21** to **25** are Very-Short Answer (VSA) Type Questions, carrying 2 marks each.*
5. ***Section C** - Questions no. **26** to **31** are Short-Answer (SA) Type Questions, carrying 3 marks each.*
6. ***Section D** - Questions no. **32** to **35** are Long-Answer (LA) Type Question, carrying 5 marks each.*
7. ***Section E** - Questions no. **36** to **38** are case study based questions carrying 4 marks each. Internal choice is provided in 2 marks questions in each case study.*
8. *There is no overall choice given in the question paper. However, an internal choice has been provided in 2 questions in Sections B, 2 questions in Section C, 2 questions in Section D and 3 questions in Section E.*
9. *Use of calculator is **not** allowed.*
10. *Please write down the serial number of the question in the answer-book at the given place before attempting it.*
11. *15 minute time has been allotted to read this question paper. The question paper will be distributed at 10:15 a.m. From 10:15 a.m. to 10:30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period.*

1. The HCF of $2^2 \cdot 3^3$ and $3^2 \cdot 2^3$ is :
(a) 1
(b) 2.3
(c) $2^2 \cdot 3^2$
(d) $2^3 \cdot 3^3$

Correct Answer: (c) $2^2 \cdot 3^2$

Solution:

Step 1: Understanding the Concept:
The Highest Common Factor (HCF) of a set of numbers expressed in their prime factorization is the product of the lowest powers of all common prime factors.
Step 2: Key Formula or Approach:
Given two numbers:

$$N_1 = 2^2 \cdot 3^3$$

$$N_2 = 2^3 \cdot 3^2$$

Step 3: Detailed Explanation:
Identify the common prime factors in both expressions, which are 2 and 3.
Compare the exponents for prime factor 2: The powers are 2 and 3. The minimum is 2. So, we take 2^2.
Compare the exponents for prime factor 3: The powers are 3 and 2. The minimum is 2. So, we take 3^2.
The HCF is the product of these terms:

$$\text{HCF} = 2^2 \cdot 3^2$$

Step 4: Final Answer:
The HCF is $2^2 \cdot 3^2$.

> Quick Tip
>
> For HCF, think "Highest" common factor but use the "Lowest" powers. For LCM, think "Lowest" common multiple but use the "Highest" powers.

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2. A letter is selected from the letters of the word FEBRUARY. The probability that it is a vowel is :
(a) $\frac{1}{8}$
(b) $\frac{2}{8}$
(c) $\frac{3}{8}$
(d) $\frac{3}{7}$

Correct Answer: (c) $\frac{3}{8}$

Solution:
Step 1: Understanding the Concept:
Probability measures the likelihood of an event occurring and is calculated as the ratio of favorable outcomes to total possible outcomes.
Step 2: Key Formula or Approach:

$$P(\text{Event}) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

Step 3: Detailed Explanation:
The letters in the word "FEBRUARY" are F, E, B, R, U, A, R, Y.

Total number of letters = 8.
The vowels in the word are E, U, and A.
Number of vowels (favorable outcomes) = 3.
Applying the formula:

$$P(\text{vowel}) = \frac{3}{8}$$

Step 4: Final Answer:
The probability is $\frac{3}{8}$.

> 💡 Quick Tip
>
> Always double-check the total count of letters and ensure you identify all vowels (A, E, I, O, U) correctly.

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3. Which of the following numbers will not end with 0 for any natural number n?
(a) $4n$
(b) 4^n
(c) $3^n + 1$
(d) 10^{n+1}

Correct Answer: (b) 4^n

Solution:
Step 1: Understanding the Concept:
For a number to end with the digit 0, its prime factorization must include both 2 and 5 as factors.
Step 2: Detailed Explanation:
Evaluate each option:
(a) $4n$: If $n = 5$, $4 \times 5 = 20$. This ends in 0.
(b) 4^n: The prime factorization of 4 is 2×2. Therefore, $4^n = (2^2)^n = 2^{2n}$. It contains only the prime factor 2. Since 5 is missing, it can never end in 0.
(c) $3^n + 1$: If $n = 2$, $3^2 + 1 = 9 + 1 = 10$. This ends in 0.
(d) 10^{n+1}: Since 10 is the base, it will always end in 0 for any natural $n \geq 1$.
Thus, 4^n is the only number that will never end in 0.
Step 3: Final Answer:
The number 4^n will not end with 0 for any natural number n.

> 💡 Quick Tip
>
> A number a^n ends in zero only if the base a is divisible by 10 or has both 2 and 5 in its prime factorization.

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4. The system of linear equations $px + qy = r$ and $p_1x + q_1y = r_1$ has a unique solution, if:
(a) $pq \neq p_1q_1$
(b) $pp_1 \neq qq_1$
(c) $pq_1 \neq qp_1$
(d) $pqr \neq p_1q_1r_1$

Correct Answer: (c) $pq_1 \neq qp_1$

Solution:
Step 1: Understanding the Concept:
A pair of linear equations $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ represents two lines. A unique solution exists if the lines intersect at exactly one point.
Step 2: Key Formula or Approach:
The condition for a unique solution is:

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Step 3: Detailed Explanation:
In the given system:
$a_1 = p$, $b_1 = q$
$a_2 = p_1$, $b_2 = q_1$
The condition becomes:

$$\frac{p}{p_1} \neq \frac{q}{q_1}$$

Cross-multiplying the denominators, we get:

$$pq_1 \neq qp_1$$

Step 4: Final Answer:
The system has a unique solution if $pq_1 \neq qp_1$.

Quick Tip

Think of the condition as the cross-product of the coefficients of x and y not being equal.

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5. Which of the equations among the following is/are quadratic equation(s)?
$q_1 : x^2 + x = (x+1)^2$, $q_2 : x - 1 = x^2 - 1$, $q_3 : x^4 = x^2$, $q_4 : \sqrt{x} = x^2\sqrt{x} + 1$
(a) q_1 only
(b) q_1, q_2 and q_3 only
(c) q_2 only
(d) q_2 and q_4 only

Correct Answer: (c) q_2 only

Solution:
Step 1: Understanding the Concept:
A quadratic equation is a polynomial equation of degree 2, typically in the form $ax^2 + bx + c = 0$ where $a \neq 0$.
Step 2: Detailed Explanation:
Let's analyze each equation individually:
1. $q_1 : x^2 + x = (x+1)^2 \Rightarrow x^2 + x = x^2 + 2x + 1$.
After simplification: $x = 2x + 1 \Rightarrow x + 1 = 0$. This is a linear equation (degree 1).
2. $q_2 : x - 1 = x^2 - 1 \Rightarrow x^2 - x = 0$.
This is a polynomial of degree 2. Hence, it is a quadratic equation.
3. $q_3 : x^4 = x^2$. This is a polynomial of degree 4. It is not quadratic.
4. $q_4 : \sqrt{x} = x^2\sqrt{x} + 1$. This is not a polynomial equation because the exponent of x is not a non-negative integer ($\sqrt{x} = x^{1/2}$).
Therefore, only q_2 is a quadratic equation.
Step 3: Final Answer:
Only q_2 is a quadratic equation.

Quick Tip

Always simplify the equation completely before determining its degree. Terms with the highest power might cancel out.

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6. The discriminant of the quadratic equation $ax^2 + x + a = 0$ is :
(a) $\sqrt{1 - 4a^2}$
(b) $1 - 4a^2$
(c) $4a^2 - 1$
(d) $\sqrt{4a^2 - 1}$

Correct Answer: (b) $1 - 4a^2$

Solution:
Step 1: Understanding the Concept:
The discriminant of a quadratic equation $Ax^2 + Bx + C = 0$ determines the nature of the roots and is a part of the quadratic formula.
Step 2: Key Formula or Approach:
The discriminant (D) is given by:

$$D = B^2 - 4AC$$

Step 3: Detailed Explanation:
For the given equation $ax^2 + x + a = 0$:
$A = a$
$B = 1$
$C = a$
Substituting these into the discriminant formula:

$$D = (1)^2 - 4(a)(a)$$

$$D = 1 - 4a^2$$

Step 4: Final Answer:
The discriminant is $1 - 4a^2$.

Quick Tip

The discriminant is just the expression inside the square root of the quadratic formula. It does not include the square root sign itself.

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7. The distance between points (3, 0) and (0, -3) is :
(a) 3 units
(b) 6 units
(c) $\sqrt{6}$ units
(d) $\sqrt{18}$ units

Correct Answer: (d) $\sqrt{18}$ units

Solution:
Step 1: Understanding the Concept:
Distance between two points in a Cartesian plane can be found using the distance formula derived from the Pythagorean Theorem.
Step 2: Key Formula or Approach:
The distance d between points (x_1, y_1) and (x_2, y_2) is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Step 3: Detailed Explanation:
Given points are $(x_1, y_1) = (3, 0)$ and $(x_2, y_2) = (0, -3)$.
Substituting values:

$$d = \sqrt{(0-3)^2 + (-3-0)^2}$$

$$d = \sqrt{(-3)^2 + (-3)^2}$$

$$d = \sqrt{9+9}$$

$$d = \sqrt{18} \text{ units}$$

Step 4: Final Answer:
The distance is $\sqrt{18}$ units.

Quick Tip

If you see $\sqrt{18}$ in the options, select it. Note that $\sqrt{18}$ is also equal to $3\sqrt{2}$.

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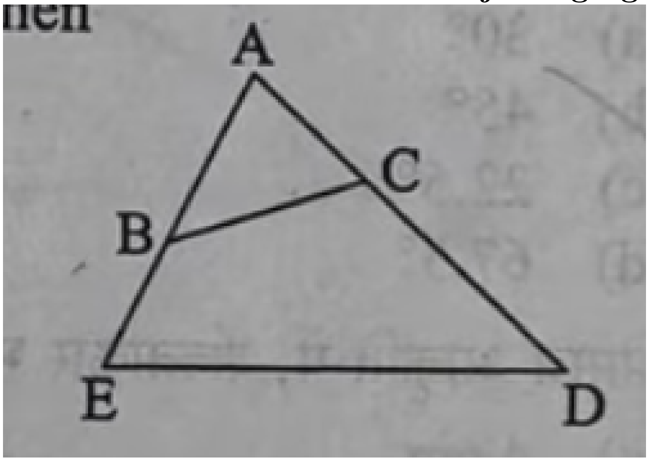
8. If $\triangle ABC \sim \triangle ADE$ in the adjoining figure, then which of the following is true?



(a) $\frac{AB}{BE} = \frac{AC}{CD}$
(b) $\frac{AB}{AD} = \frac{AC}{AE}$
(c) $\frac{AB}{BC} = \frac{AE}{DE}$
(d) $\frac{AC}{AD} = \frac{AB}{AE}$

Correct Answer: (b) $\frac{AB}{AD} = \frac{AC}{AE}$

Solution:
Step 1: Understanding the Concept:
Similar triangles have proportional corresponding sides. The order of vertices in the similarity statement determines the correspondence.
Step 2: Detailed Explanation:
Given $\triangle ABC \sim \triangle ADE$.
The correspondence is:
- Vertex A corresponds to Vertex A.
- Vertex B corresponds to Vertex D.
- Vertex C corresponds to Vertex E.
The ratio of corresponding sides is:

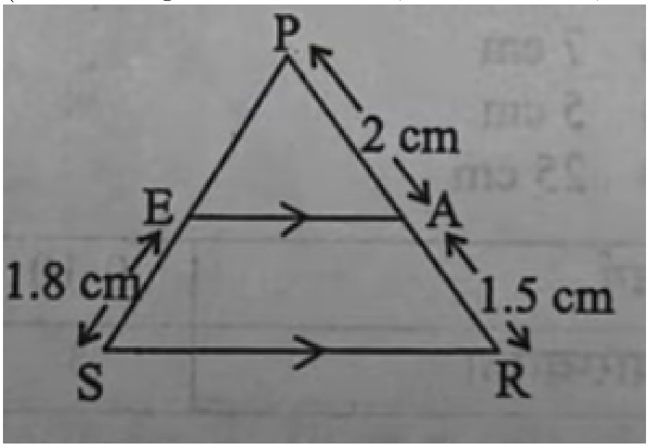
$$\frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$$

Looking at the options, $\frac{AB}{AD} = \frac{AC}{AE}$ matches this fundamental property.
Step 3: Final Answer:
The correct statement is $\frac{AB}{AD} = \frac{AC}{AE}$.

Quick Tip

Match the first two letters of the first triangle with the first two of the second, and so on: AB/AD, BC/DE, AC/AE.

9. In the adjoining figure, if $EA \parallel SR$ and $PE = x$ cm, then the value of $5x$ is :
(Given in figure: $PA = 2$ cm, $ES = 1.8$ cm, $AR = 1.5$ cm)



(a) 2.4 cm
(b) 12 cm
(c) 1.35 cm
(d) 6.75 cm

Correct Answer: (b) 12 cm

Solution:
Step 1: Understanding the Concept:
The Basic Proportionality Theorem (BPT) states that if a line is parallel to one side of a triangle, it divides the other two sides in the same ratio.
Step 2: Key Formula or Approach:
In $\triangle PSR$, since $EA \parallel SR$:

$$\frac{PE}{ES} = \frac{PA}{AR}$$

Step 3: Detailed Explanation:
From the figure, we have:
$PE = x$, $ES = 1.8$, $PA = 2$, $AR = 1.5$.
Apply the BPT:

$$\frac{x}{1.8} = \frac{2}{1.5}$$

Cross-multiply to solve for x:

$$1.5x = 2 \times 1.8$$

$$1.5x = 3.6$$

$$x = \frac{3.6}{1.5} = \frac{36}{15} = 2.4 \text{ cm}$$

The question asks for the value of $5x$:

$$5x = 5 \times 2.4 = 12 \text{ cm}$$

Step 4: Final Answer:
The value of $5x$ is 12 cm.

Quick Tip

Read the final requirement carefully. Many students calculate x and immediately pick option (a). Here, you need $5x$.

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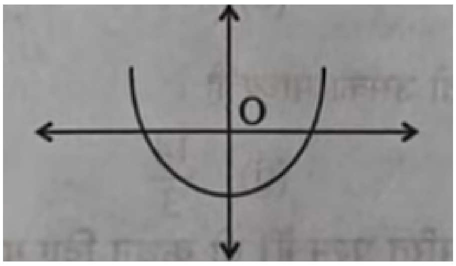
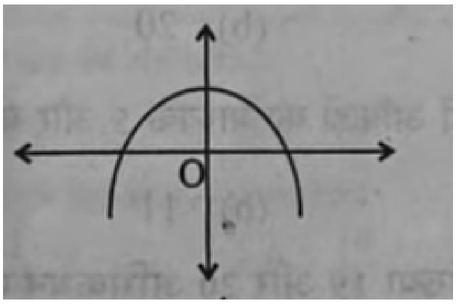
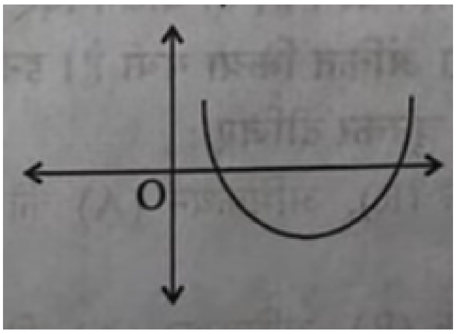
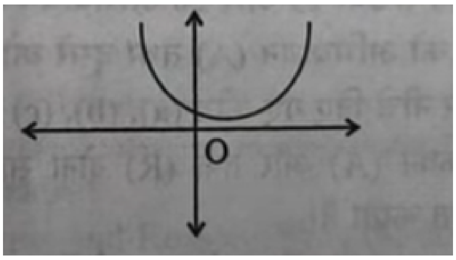
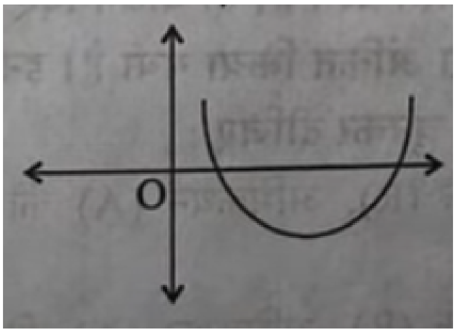
10. Which of the following graphs represents a polynomial with both zeroes being positive?

(a) 

(b) 

(c) 

(d) 

Correct Answer: (c) 

Solution:
Step 1: Understanding the Concept:
Zeroes of a polynomial are the x-coordinates of points where the graph intersects the x-axis. For zeroes to be positive, these intersections must occur to the right of the y-axis.
Step 2: Detailed Explanation:
Looking at the graphs:
- Graph (a) has one zero on the negative x-axis and one on the positive.
- Graph (b) also has one negative and one positive zero.
- Graph (c) is a parabola that intersects the x-axis at two distinct points, both of which are located to the right of the origin (positive x-values).
- Graph (d) does not touch the x-axis, indicating no real zeroes.
Therefore, Graph (c) represents both zeroes being positive.
Step 3: Final Answer:
Graph (c) represents both zeroes being positive.

> Quick Tip
>
> Check the intersection points relative to the origin (0,0). "Positive zeroes" mean the parabola must cross the x-axis in the region where $x > 0$.

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11. The system of equations $x = 2$ and $x = 3$ has :
(a) unique solution (2, 3)
(b) two solutions (2, 0) and (3, 0)
(c) no solution
(d) infinitely many solutions

Correct Answer: (c) no solution

Solution:
Step 1: Understanding the Concept:
A solution to a system of equations exists where the geometric representations (lines) of those equations intersect.
Step 2: Detailed Explanation:
- The equation $x = 2$ represents a vertical line passing through 2 on the x-axis.
- The equation $x = 3$ represents a vertical line passing through 3 on the x-axis.
- Since both are vertical lines, they are parallel to each other.

- Parallel lines do not intersect at any point.
- Because there is no intersection, there is no value of x that can be both 2 and 3 simultaneously.

Thus, the system has no solution.

Step 3: Final Answer:

The system has no solution.

> Quick Tip
>
> Linear equations like $x = a$ and $x = b$ where $a \neq b$ are always parallel. Parallel lines mean "no solution".

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12. The numbers $x, x + 4$ and $x + 8$ are in A.P. with common difference :

(a) x

(b) $4 + x$

(c) 4

(d) 0

Correct Answer: (c) 4

Solution:

Step 1: Understanding the Concept:

An Arithmetic Progression (A.P.) is a sequence where the difference between any two consecutive terms is constant. This constant is the common difference.

Step 2: Key Formula or Approach:

Common difference d is calculated as:

$$d = a_2 - a_1 = a_3 - a_2$$

Step 3: Detailed Explanation:

Let $a_1 = x$, $a_2 = x + 4$, and $a_3 = x + 8$.

Calculate the difference between the second and first term:

$$d = (x + 4) - x = 4$$

Calculate the difference between the third and second term:

$$d = (x + 8) - (x + 4) = x + 8 - x - 4 = 4$$

Since the difference is constant and equal to 4, the common difference is 4.

Step 4: Final Answer:

The common difference is 4.

> Quick Tip
>
> The common difference is independent of the variable x in this case because the variable part is the same in every term.

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13. Which of the following statements is not true?
(a) $\sin 0^\circ = \cos 0^\circ$
(b) $\tan 30^\circ = \cot 60^\circ$
(c) $\sin 30^\circ = \cos 60^\circ$
(d) $\sin 45^\circ = \frac{1}{\sec 45^\circ}$

Correct Answer: (a) $\sin 0^\circ = \cos 0^\circ$

Solution:
Step 1: Understanding the Concept:
This question tests the knowledge of trigonometric values at standard angles and basic identities.
Step 2: Detailed Explanation:
Evaluate each statement:
(a) $\sin 0^\circ = 0$ and $\cos 0^\circ = 1$. Since $0 \neq 1$, this statement is **not true**.
(b) $\tan 30^\circ = \frac{1}{\sqrt{3}}$ and $\cot 60^\circ = \frac{1}{\sqrt{3}}$. This is true.
(c) $\sin 30^\circ = \frac{1}{2}$ and $\cos 60^\circ = \frac{1}{2}$. This is true.
(d) $\frac{1}{\sec 45^\circ} = \cos 45^\circ = \frac{1}{\sqrt{2}}$ and $\sin 45^\circ = \frac{1}{\sqrt{2}}$. This is true.
Step 3: Final Answer:
The statement $\sin 0^\circ = \cos 0^\circ$ is not true.

Quick Tip

Remember complementary angle properties: $\sin\theta = \cos(90^\circ - \theta)$. This helps verify (b) and (c) without calculating values.

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14. If $\sqrt{3}\sin A = \cos A$, then the measure of A is :
(A) 90°
(B) 60°
(C) 45°
(D) 30°

Correct Answer: (D) 30°

Solution:
Step 1: Understanding the Concept:
The question involves solving a basic trigonometric equation relating the sine and cosine of an angle A.
To find the value of the angle, we can transform the equation to involve a single trigonometric ratio, such as tangent ($\tan A$), which is defined as the ratio of sine to cosine.
Step 2: Key Formula or Approach:
We use the fundamental trigonometric relationship:

$$\tan A = \frac{\sin A}{\cos A}$$

Once the equation is in terms of $\tan A$, we compare the result with standard trigonometric values to find the angle A.

Step 3: Detailed Explanation:

Given the equation:

$$\sqrt{3}\sin A = \cos A$$

Divide both sides of the equation by $\cos A$ (assuming $\cos A \neq 0$):

$$\frac{\sqrt{3}\sin A}{\cos A} = \frac{\cos A}{\cos A}$$

Substituting $\frac{\sin A}{\cos A}$ with $\tan A$, we get:

$$\sqrt{3}\tan A = 1$$

Now, divide both sides by $\sqrt{3}$ to isolate $\tan A$:

$$\tan A = \frac{1}{\sqrt{3}}$$

We know from the standard trigonometric table that for an acute angle:

$$\tan 30^\circ = \frac{1}{\sqrt{3}}$$

By comparing $\tan A = \tan 30^\circ$, we find the value of angle A:

$$A = 30^\circ$$

Step 4: Final Answer:

The measure of angle A is 30°.

> Quick Tip
>
> When you see an equation with both sin and cos, always try to convert it to tan by dividing.
> A useful mnemonic for the tan values is: $\tan 30^\circ = \frac{1}{\sqrt{3}}$ (the "smaller" value for the smaller angle) and $\tan 60^\circ = \sqrt{3}$ (the "larger" value for the larger angle).

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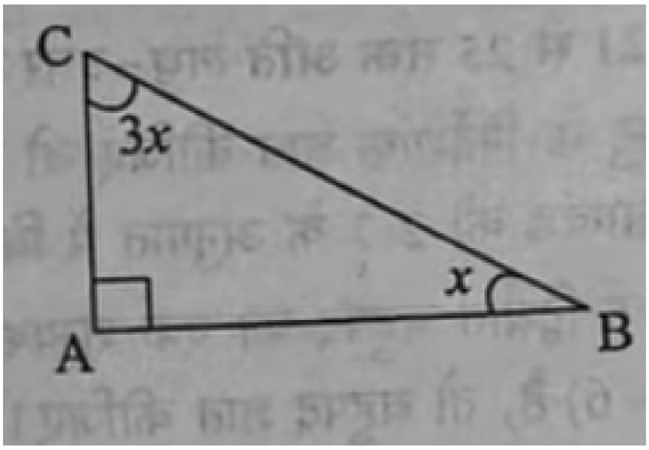
15. In the adjoining figure, the angle of elevation of the point C from the point B, is :



(A) 30°
(B) 45°
(C) 22.5°
(D) 67.5°

Correct Answer: (C) 22.5°

Solution:
Step 1: Understanding the Concept:
The angle of elevation of a point is the angle formed between the horizontal line of sight and the line of sight from the observer's eye to the point above the horizontal.
In the given right-angled triangle $\triangle ABC$, $\angle A = 90°$.
The angle of elevation of point C from point B is the angle $\angle ABC$, which is given as x.
Step 2: Key Formula or Approach:
We use the Angle Sum Property of a triangle, which states that the sum of all interior angles of a triangle is 180°.

$$\angle A + \angle B + \angle C = 180°$$

Step 3: Detailed Explanation:
From the figure, we have the following angle measures:
$\angle A = 90°$ (as indicated by the square symbol at vertex A)
$\angle B = x$
$\angle C = 3x$
Applying the angle sum property:

$$90° + x + 3x = 180°$$

$$90° + 4x = 180°$$

Subtracting 90° from both sides:

$$4x = 180° - 90°$$

$$4x = 90°$$

Dividing by 4:

$$x = \frac{90^\circ}{4}$$

$$x = 22.5^\circ$$

The angle of elevation of point C from point B is $\angle ABC = x = 22.5^\circ$.
Step 4: Final Answer:
The angle of elevation is 22.5°.

💡 Quick Tip

In a right-angled triangle, the sum of the two acute angles is always 90°.
Here, $x + 3x = 90^\circ \Rightarrow 4x = 90^\circ \Rightarrow x = 22.5^\circ$.
This saves a step by not involving the 180° total.

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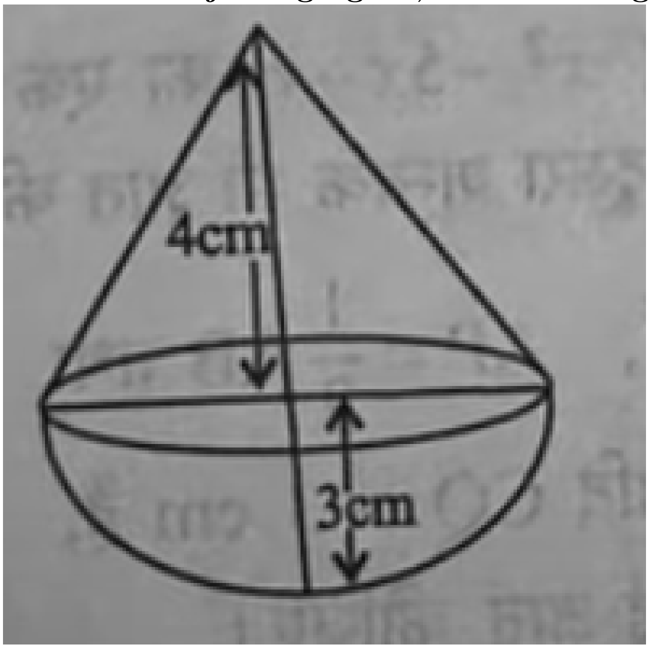
16. In the adjoining figure, the slant height of the conical part is :



(A) 4 cm
(B) 7 cm
(C) 5 cm
(D) 25 cm

Correct Answer: (C) 5 cm

Solution:
Step 1: Understanding the Concept:
The figure represents a composite solid consisting of a cone mounted on a hemisphere.
To find the slant height of the conical part, we need to identify the vertical height (h) and the base radius (r) of the cone.
Step 2: Key Formula or Approach:

The slant height (l) of a cone is related to its vertical height (h) and radius (r) by the Pythagorean theorem, as they form a right-angled triangle.

$$l = \sqrt{r^2 + h^2}$$

Step 3: Detailed Explanation:
From the given figure:
Vertical height of the cone (h) = 4 cm
Radius of the hemisphere (which is also the radius of the cone's base, r) = 3 cm
Now, substituting these values into the slant height formula:

$$l = \sqrt{3^2 + 4^2}$$

$$l = \sqrt{9 + 16}$$

$$l = \sqrt{25}$$

$$l = 5 \text{ cm}$$

Step 4: Final Answer:
The slant height of the conical part is 5 cm.

Quick Tip

Remember the standard Pythagorean triplet (3, 4, 5).
If the legs of a right triangle are 3 and 4, the hypotenuse (slant height in this case) must be 5.

17.

Class	0-10	10-20	20-30	30-40	40-50
Frequency	3	5	7	9	11

The upper limit of the median class of the above data is :
(A) 10
(B) 20
(C) 30
(D) 40

Correct Answer: (D) 40

Solution:
Step 1: Understanding the Concept:
The median class is the class interval whose cumulative frequency is greater than or equal to $\frac{N}{2}$, where N is the total frequency.
Step 2: Key Formula or Approach:
1. Calculate the total frequency (N).
2. Calculate $\frac{N}{2}$.

3. Find the cumulative frequencies (cf) for each class.
4. Identify the class where $cf \geq \frac{N}{2}$.

Step 3: Detailed Explanation:

First, let's construct a cumulative frequency table:

Class Interval	Frequency (f)	Cumulative Frequency (cf)
0-10	3	3
10-20	5	$3 + 5 = 8$
20-30	7	$8 + 7 = 15$
30-40	9	$15 + 9 = 24$
40-50	11	$24 + 11 = 35$

Total frequency (N) = 35.

$$\frac{N}{2} = \frac{35}{2} = 17.5$$

We look for the class whose cumulative frequency is just greater than or equal to 17.5.
- For class 20-30, $cf = 15$ (which is < 17.5).
- For class 30-40, $cf = 24$ (which is ≥ 17.5).

Therefore, the median class is 30-40.
The question asks for the upper limit of the median class.
In the interval 30-40, the lower limit is 30 and the upper limit is 40.

Step 4: Final Answer:

The upper limit of the median class is 40.

> Quick Tip
>
> Always double-check your cumulative frequency additions.
> The last cumulative frequency must equal the sum of all individual frequencies.
> In this case, $3 + 5 + 7 + 9 + 11 = 35$, which matches.

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18. If for a data, median is 5 and mode is 4, then mean is equal to :

(a) 7
(b) 11
(c) $\frac{11}{2}$
(d) $\frac{14}{3}$

Correct Answer: (c) $\frac{11}{2}$

Solution:

Step 1: Understanding the Concept:

The three measures of central tendency (Mean, Median, and Mode) for a frequency distribution are related by an empirical formula.

Step 2: Key Formula or Approach:

The empirical relationship is:

$$\text{Mode} = 3 \times \text{Median} - 2 \times \text{Mean}$$

Step 3: Detailed Explanation:
Given:
Median = 5
Mode = 4
Let Mean be $\bar{x}$.
Substituting the given values into the formula:

$$4 = 3(5) - 2(\bar{x})$$

$$4 = 15 - 2\bar{x}$$

Transpose $2\bar{x}$ to the left side and 4 to the right side:

$$2\bar{x} = 15 - 4$$

$$2\bar{x} = 11$$

$$\bar{x} = \frac{11}{2}$$

Step 4: Final Answer:
The mean of the data is $\frac{11}{2}$.

> Quick Tip
>
> Remember the formula using the "3-2-1" rule: 3 Median - 2 Mean = 1 Mode. Also, note that "Median" has more letters than "Mean", and it is multiplied by the larger number (3).

---

19. Assertion (A) : From a bag containing 5 red balls, 2 white balls and 3 green balls, the probability of drawing a non-white ball is $\frac{4}{5}$.
Reason (R) : For any event E, $P(E) + P(\textbf{not E}) = 1$.
(a) Both, Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(b) Both, Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(c) Assertion (A) is true, but Reason (R) is false.
(d) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (a) Both, Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Solution:
Step 1: Understanding the Concept:
The probability of a non-occurrence of an event is the complement of the probability of its occurrence.

Step 2: Detailed Explanation:
For Assertion (A):
Total number of balls $= 5(\text{red}) + 2(\text{white}) + 3(\text{green}) = 10$.
Number of white balls $= 2$.
Probability of drawing a white ball, $P(W) = \frac{2}{10} = \frac{1}{5}$.
Probability of drawing a non-white ball $= 1 - P(W) = 1 - \frac{1}{5} = \frac{4}{5}$.
Alternatively, non-white balls = Red + Green $= 5 + 3 = 8$.
$P(\text{non-white}) = \frac{8}{10} = \frac{4}{5}$.
Thus, Assertion (A) is true.
For Reason (R):
The sum of probability of an event and its complement is always 1. This is a fundamental property.
Thus, Reason (R) is true and correctly explains the logic used in Assertion (A).
Step 3: Final Answer:
Both (A) and (R) are true and (R) is the correct explanation of (A).

Quick Tip

In probability, the phrase "non-white" is exactly the complement of "white". Always check if it is faster to calculate the event directly or via its complement.

---

20. Assertion (A) : $7 \times 2 + 3$ **is a composite number.**
Reason (R) : A composite number has more than two factors.
(a) Both, Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(b) Both, Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(c) Assertion (A) is true, but Reason (R) is false.
(d) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (d) Assertion (A) is false, but Reason (R) is true.

Solution:
Step 1: Understanding the Concept:
A composite number is a positive integer greater than 1 that has at least one divisor other than 1 and itself. A prime number has exactly two factors.
Step 2: Detailed Explanation:
For Assertion (A):
Evaluate the expression: $7 \times 2 + 3 = 14 + 3 = 17$.
17 is a prime number because its only factors are 1 and 17.
Therefore, it is NOT a composite number. Assertion (A) is false.
For Reason (R):
By definition, a composite number is an integer greater than 1 that has more than two factors.
Therefore, Reason (R) is true.
Step 3: Final Answer:
Assertion (A) is false, but Reason (R) is true.

Quick Tip

Don't confuse composite numbers with odd numbers. While 17 is odd, it is prime. While 9 is odd, it is composite. Always evaluate the numerical expression before judging.

---

21. Find the coordinates of the point which divides the line segment joining the points $A(-6, 10)$ and $B(3, -8)$ in the ratio $2 : 7$.

Correct Answer: $(-4, 6)$

Solution:
Step 1: Understanding the Concept:
The coordinates of a point dividing a line segment in a given ratio can be determined using the internal section formula.
Step 2: Key Formula or Approach:
Section Formula: If a point $P(x, y)$ divides the segment joining (x_1, y_1) and (x_2, y_2) in ratio $m : n$, then:

$$x = \frac{mx_2 + nx_1}{m + n}, \quad y = \frac{my_2 + ny_1}{m + n}$$

Step 3: Detailed Explanation:
Given: $A(-6, 10) \implies x_1 = -6, y_1 = 10$.
$B(3, -8) \implies x_2 = 3, y_2 = -8$.
Ratio $m : n = 2 : 7$.
Applying the formula for x:

$$x = \frac{2(3) + 7(-6)}{2 + 7} = \frac{6 - 42}{9} = \frac{-36}{9} = -4$$

Applying the formula for y:

$$y = \frac{2(-8) + 7(10)}{2 + 7} = \frac{-16 + 70}{9} = \frac{54}{9} = 6$$

The coordinates of the point are $(-4, 6)$.
Step 4: Final Answer:
The required point is $(-4, 6)$.

Quick Tip

Be very careful with negative signs during calculation. In the numerator, multiply the ratio 'm' with the 'far' coordinate x_2, and 'n' with the 'near' coordinate x_1.

22(A). One zero of a quadratic polynomial is twice the other. If the sum of zeroes is (-6), find the polynomial.

Correct Answer: $k(x^2 + 6x + 8)$

Solution:
Step 1: Understanding the Concept:
A quadratic polynomial with zeroes α and β is given by $p(x) = k[x^2 - (\alpha + \beta)x + \alpha\beta]$.
Step 2: Detailed Explanation:
Let the zeroes be α and 2α.
Given: Sum of zeroes $= -6$.

$$\alpha + 2\alpha = -6$$

$$3\alpha = -6 \implies \alpha = -2$$

So, the zeroes are -2 and $2(-2) = -4$.
Product of zeroes $= (-2) \times (-4) = 8$.
The quadratic polynomial is:

$$p(x) = k[x^2 - (\text{sum of zeroes})x + (\text{product of zeroes})]$$

$$p(x) = k[x^2 - (-6)x + 8]$$

$$p(x) = k(x^2 + 6x + 8)$$

Step 3: Final Answer:
The polynomial is $x^2 + 6x + 8$ (taking $k = 1$).

Quick Tip

If you know the sum and product of zeroes, the polynomial is always x^2-Sx+P. Don't forget the negative sign before the sum term.

22(B). If one zero of the polynomial $x^2 - 5x - c$ is (-1), find the value of c. Also, find the other zero.

Correct Answer: $c = 6$, other zero $= 6$

Solution:
Step 1: Understanding the Concept:
If α is a zero of $p(x)$, then $p(\alpha) = 0$. The relationship between zeroes and coefficients is $\alpha + \beta = -b/a$.
Step 2: Detailed Explanation:
Given $p(x) = x^2 - 5x - c$.
Since -1 is a zero:

$$(-1)^2 - 5(-1) - c = 0$$

$$1 + 5 - c = 0 \implies 6 - c = 0 \implies c = 6$$

Now, the polynomial is $x^2 - 5x - 6 = 0$.
Let the zeroes be $\alpha = -1$ and β.
Sum of zeroes $= -b/a = -(-5)/1 = 5$.

$$-1 + \beta = 5 \implies \beta = 6$$

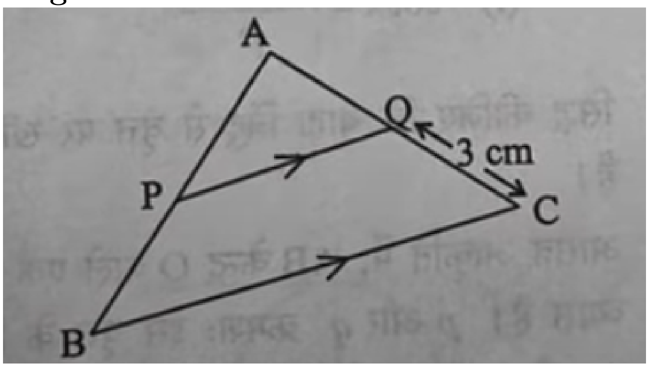
Step 3: Final Answer:
The value of c is 6 and the other zero is 6.

Quick Tip

You can also find the other zero using the product of zeroes: $\alpha\beta = c/a = -6/1 = -6$. Since $\alpha = -1$, $\beta = -6/(-1) = 6$.

---

23. In the adjoining figure, $AP = \frac{1}{2}AB$ and $PQ \parallel BC$. If $CQ = 3$ cm, then find the length of AC.



Correct Answer: 6 cm

Solution:
Step 1: Understanding the Concept:
The Basic Proportionality Theorem (BPT) states that if a line is parallel to one side of a triangle, it divides the other two sides in the same ratio.
Step 2: Detailed Explanation:
In $\triangle ABC$, $PQ \parallel BC$.
By BPT:

$$\frac{AP}{PB} = \frac{AQ}{QC}$$

Given $AP = \frac{1}{2}AB$.
This means P is the midpoint of AB, so $AP = PB$.

$$\frac{AP}{AP} = \frac{AQ}{QC} \implies 1 = \frac{AQ}{QC} \implies AQ = QC$$

Since $QC = 3$ cm, then $AQ = 3$ cm.

The total length of $AC = AQ + QC = 3 + 3 = 6$ cm.
Step 3: Final Answer:
The length of AC is 6 cm.

> Quick Tip
>
> If a line is parallel to the base and starts at the midpoint of one side, it will hit the midpoint of the other side. This is the Converse of the Midpoint Theorem.

24(A). Evaluate : $\sin^2 30° - \cos^2 45° + \cot^2 60°$

Correct Answer: $\frac{1}{12}$

Solution:
Step 1: Understanding the Concept:
Trigonometric values for specific standard angles $(30°, 45°, 60°)$ are used to evaluate algebraic expressions involving them.
Step 2: Detailed Explanation:
The standard values are:
$\sin 30° = \frac{1}{2}$
$\cos 45° = \frac{1}{\sqrt{2}}$
$\cot 60° = \frac{1}{\sqrt{3}}$
Substituting these values into the expression:

$$\left(\frac{1}{2}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2$$

$$= \frac{1}{4} - \frac{1}{2} + \frac{1}{3}$$

Taking the LCM of 4, 2, and 3, which is 12:

$$= \frac{3 - 6 + 4}{12} = \frac{1}{12}$$

Step 3: Final Answer:
The value of the expression is $\frac{1}{12}$.

> Quick Tip
>
> Always memorize the trigonometric table for $0°, 30°, 45°, 60°, 90°$. It saves time and ensures accuracy in basic calculation questions.

24(B). If $\sin(A + 2B) = 2\cos 60°$ **and** $A = 3B$**, find the measures of A and B.**

Correct Answer: $A = 54°$, $B = 18°$

Solution:
Step 1: Understanding the Concept:
Trigonometric ratios can be equated to find angles if the value corresponds to a standard angle value.
Step 2: Detailed Explanation:
Given: $\sin(A + 2B) = 2\cos 60^\circ$.
We know $\cos 60^\circ = \frac{1}{2}$.

$$\sin(A + 2B) = 2 \times \frac{1}{2} = 1$$

Since $\sin 90^\circ = 1$, we can equate the angles:

$$A + 2B = 90^\circ \quad \ldots (1)$$

Also given: $A = 3B$.
Substitute $A = 3B$ into equation (1):

$$3B + 2B = 90^\circ$$

$$5B = 90^\circ \implies B = 18^\circ$$

Now find A:

$$A = 3(18^\circ) = 54^\circ$$

Step 3: Final Answer:
The angles are $A = 54^\circ$ and $B = 18^\circ$.

Quick Tip

When $\sin\theta = 1$, $\theta = 90^\circ$. When $\cos\theta = 1$, $\theta = 0^\circ$. Always solve the numerical part on the RHS first.

---

25. A box consists of 60 wall clocks, out of which 40 are good, 15 have minor defects and the remaining are broken. A trader will reject the box, if the clock taken out from the box is broken. The trader randomly takes out one clock from the box. What is the probability that :
(i) the box will be rejected ?
(ii) the clock taken out of the box has minor defect ?

Correct Answer: (i) $\frac{1}{12}$, (ii) $\frac{1}{4}$

Solution:
Step 1: Understanding the Concept:
Probability = (Favorable outcomes) / (Total outcomes).
Step 2: Detailed Explanation:
Total wall clocks = 60.

Good clocks = 40.
Clocks with minor defects = 15.
Broken clocks = $60 - (40 + 15) = 60 - 55 = 5$.
(i) The box will be rejected:
The box is rejected if the selected clock is broken.
Favorable outcomes (Broken) = 5.

$$P(\text{Rejected}) = \frac{5}{60} = \frac{1}{12}$$

(ii) The clock has minor defect:
Favorable outcomes (Minor defect) = 15.

$$P(\text{Minor defect}) = \frac{15}{60} = \frac{1}{4}$$

Step 3: Final Answer:
(i) 1/12, (ii) 1/4.

> Quick Tip
>
> Always simplify your fractions to the lowest terms. Probability values must always be between 0 and 1 inclusive.

---

26. Given that $\sqrt{5}$ is an irrational number, prove that $3 + 2\sqrt{5}$ is also an irrational number.

Correct Answer: Proof by contradiction.

Solution:
Step 1: Understanding the Concept:
The sum and product of rational and irrational numbers follow specific properties. We usually prove irrationality using the method of contradiction.
Step 2: Detailed Explanation:
Let us assume that $3 + 2\sqrt{5}$ is a rational number.
Then, it can be expressed as $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

$$3 + 2\sqrt{5} = \frac{a}{b}$$

Subtract 3 from both sides:

$$2\sqrt{5} = \frac{a}{b} - 3 = \frac{a - 3b}{b}$$

Divide by 2:

$$\sqrt{5} = \frac{a - 3b}{2b}$$

Since a and b are integers, $a - 3b$ and $2b$ are also integers.
This means $\frac{a-3b}{2b}$ is a rational number.

This implies $\sqrt{5}$ is a rational number.
But it is given that $\sqrt{5}$ is irrational.
This is a contradiction to our assumption.
Therefore, $3 + 2\sqrt{5}$ must be an irrational number.
Step 3: Final Answer:
$3 + 2\sqrt{5}$ is irrational.

> Quick Tip
>
> In such proofs, isolate the irrational term ($\sqrt{5}$) on one side and show that the other side is a rational expression (combination of integers).

27(A). Solve the following system of equations graphically :
$x + 3y = 6$ **and** $2x - 3y = 12$
Also, find the area of the triangle formed by the lines $x + 3y = 6, x = 0$ **and** $y = 0$**.**

Correct Answer: Solution: $(6, 0)$, Area: 6 sq. units

Solution:
Step 1: Understanding the Concept:
Solving graphically involves plotting lines and finding their intersection. The area of a triangle with vertices on axes is calculated using $\frac{1}{2} \times \text{Base} \times \text{Height}$.
Step 2: Detailed Explanation:
For $x + 3y = 6$:
- If $x = 0, y = 2$. Point: $(0, 2)$.
- If $y = 0, x = 6$. Point: $(6, 0)$.
For $2x - 3y = 12$:
- If $x = 0, y = -4$. Point: $(0, -4)$.
- If $y = 0, x = 6$. Point: $(6, 0)$.
Plotting these points, the two lines intersect at $(6, 0)$.
So, the solution is $x = 6, y = 0$.
The second part asks for the area of the triangle formed by $x + 3y = 6$, $x = 0$ (y-axis) and $y = 0$ (x-axis).
The vertices of this triangle are $(0, 0), (6, 0)$, and $(0, 2)$.
Base = 6 units (on x-axis), Height = 2 units (on y-axis).

$$\text{Area} = \frac{1}{2} \times 6 \times 2 = 6 \text{ sq. units}$$

Step 3: Final Answer:
Intersection point is $(6, 0)$ and the area is 6 sq. units.

> Quick Tip
>
> For area with the axes, simply find the x-intercept and y-intercept of the line. The product of these intercepts divided by 2 gives the area.

27(B). One of the supplementary angles exceeds the other by $120°$. Express the given information as a system of linear equations in two variables. Hence, find the measure of both the angles.

Correct Answer: $150°$ and $30°$

Solution:
Step 1: Understanding the Concept:
Supplementary angles are two angles whose sum is $180°$.
Step 2: Detailed Explanation:
Let the two angles be x and y.
From the definition of supplementary angles:

$$x + y = 180° \quad \ldots(1)$$

Given: One angle exceeds the other by $120°$.

$$x - y = 120° \quad \ldots(2)$$

Adding equations (1) and (2):

$$(x + y) + (x - y) = 180° + 120°$$

$$2x = 300° \implies x = 150°$$

Substituting $x = 150°$ in equation (1):

$$150° + y = 180° \implies y = 30°$$

Step 3: Final Answer:
The angles are $150°$ and $30°$.

> Quick Tip
>
> Remember: Supplementary $= 180°$, Complementary $= 90°$. A quick check: $150 + 30 = 180$ and $150 - 30 = 120$.

---

28. If the point $P(x, y)$ is equidistant from the points $(3, 6)$ and $(-3, 4)$, obtain the relation between x and y. Hence, find the coordinates of point P if it lies on x-axis.

Correct Answer: $3x + y = 5$; Point $(5/3, 0)$

Solution:
Step 1: Understanding the Concept:
If P is equidistant from A and B, then the distance $PA = PB$. We use the distance formula.

Step 2: Detailed Explanation:
Let $A = (3, 6)$ and $B = (-3, 4)$.

$$PA = PB \implies PA^2 = PB^2$$

Using distance formula $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$:

$$(x-3)^2 + (y-6)^2 = (x+3)^2 + (y-4)^2$$

$$x^2 - 6x + 9 + y^2 - 12y + 36 = x^2 + 6x + 9 + y^2 - 8y + 16$$

Cancel $x^2, y^2, 9$ from both sides:

$$-6x - 12y + 36 = 6x - 8y + 16$$

$$-12x - 4y = 16 - 36$$

$$-12x - 4y = -20$$

Divide by -4:

$$3x + y = 5 \quad \ldots \text{(The relation)}$$

If P lies on the x-axis, then $y = 0$.

$$3x + 0 = 5 \implies x = \frac{5}{3}$$

Step 3: Final Answer:
The relation is $3x + y = 5$ and the point is $(5/3, 0)$.

> Quick Tip
>
> Points on the x-axis always have $y = 0$. This simplification usually leads directly to the answer once the relation is found.

29(A). Prove that : $\frac{\sin A - \tan A}{\sin A + \tan A} = \frac{1 - \sec A}{1 + \sec A}$

Correct Answer: Proof

Solution:
Step 1: Understanding the Concept:
To prove trigonometric identities, convert terms into their basic components (sin and cos) or use reciprocal identities.
Step 2: Detailed Explanation:
LHS $= \frac{\sin A - \tan A}{\sin A + \tan A}$
Substitute $\tan A = \frac{\sin A}{\cos A}$:

$$= \frac{\sin A - \frac{\sin A}{\cos A}}{\sin A + \frac{\sin A}{\cos A}}$$

Factor out $\sin A$ from numerator and denominator:

$$= \frac{\sin A \left(1 - \frac{1}{\cos A}\right)}{\sin A \left(1 + \frac{1}{\cos A}\right)}$$

Cancel $\sin A$:

$$= \frac{1 - \frac{1}{\cos A}}{1 + \frac{1}{\cos A}}$$

We know $\frac{1}{\cos A} = \sec A$:

$$= \frac{1 - \sec A}{1 + \sec A} = \text{RHS}$$

Hence Proved.

Step 3: Final Answer:

LHS = RHS.

Quick Tip

Often, "taking common" is the hidden step in simplifying trigonometric fractions. Look for terms like $\sin A$ or $\cos A$ that appear in both parts of a numerator/denominator.

29(B). If $\sin x = p$, then prove that :

(i) $\cot x = \frac{\sqrt{1-p^2}}{p}$

(ii) $\frac{1+\tan^2 x}{1+\cot^2 x} = \frac{p^2}{1-p^2}$

Correct Answer: Proof

Solution:

Step 1: Understanding the Concept:

Use the fundamental identity $\sin^2 x + \cos^2 x = 1$ to find related trigonometric ratios.

Step 2: Detailed Explanation:

Given: $\sin x = p$.

Then $\cos x = \sqrt{1 - \sin^2 x} = \sqrt{1 - p^2}$.

(i) For $\cot x$:

$$\cot x = \frac{\cos x}{\sin x} = \frac{\sqrt{1 - p^2}}{p}$$

Hence Proved.

(ii) For the expression:

LHS $= \frac{1+\tan^2 x}{1+\cot^2 x}$

Using identities $1+\tan^2 x = \sec^2 x$ and $1+\cot^2 x = \csc^2 x$:

$$= \frac{\sec^2 x}{\csc^2 x} = \frac{1/\cos^2 x}{1/\sin^2 x} = \frac{\sin^2 x}{\cos^2 x} = \tan^2 x$$

Substitute values of $\sin x$ and $\cos x$:

$$= \frac{(p)^2}{(\sqrt{1-p^2})^2} = \frac{p^2}{1-p^2} = \text{RHS}$$

Hence Proved.
Step 3: Final Answer:
LHS = RHS for both parts.

> Quick Tip
>
> The identity $\frac{1+\tan^2 A}{1+\cot^2 A} = \tan^2 A$ is very useful and appears frequently in competitive exams.

---

30. Prove that the lengths of tangents drawn from an external point to a circle are equal.

Correct Answer: Proof based question.

Solution:
Step 1: Understanding the Concept:
This theorem is a fundamental property of circles stating that two tangents drawn from a single point outside the circle to the points of tangency have identical lengths.
Step 2: Key Formula or Approach:
The proof relies on establishing the congruence of two right-angled triangles formed by the radii, the tangents, and the line connecting the external point to the center.
Step 3: Detailed Explanation:
Given: A circle with centre O, an external point P, and two tangents PA and PB meeting the circle at A and B respectively.
To Prove: $PA = PB$.
Construction: Join OA, OB, and OP.
Proof:
In $\triangle OAP$ and $\triangle OBP$:
1. $\angle OAP = \angle OBP = 90°$
(A radius is always perpendicular to the tangent at the point of contact).
2. $OA = OB$
(Radii of the same circle are equal).
3. $OP = OP$
(Common hypotenuse for both triangles).
Therefore, $\triangle OAP \cong \triangle OBP$ by the RHS (Right angle-Hypotenuse-Side) congruence criterion.
By CPCT (Corresponding Parts of Congruent Triangles):

$$PA = PB$$

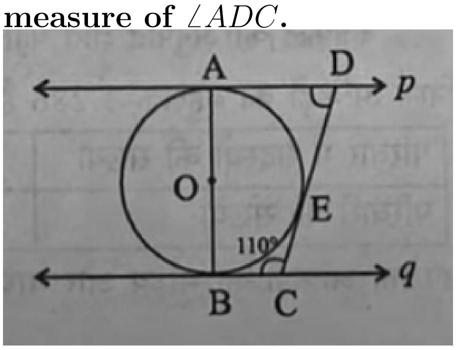
Step 4: Final Answer:
Hence, the lengths of tangents drawn from an external point to a circle are proved to be equal.

Quick Tip

Remember that $\triangle OAP \cong \triangle OBP$ also implies $\angle APO = \angle BPO$, meaning the line joining the external point to the center bisects the angle between the tangents.

---

31. In the adjoining figure, AB is the diameter of the circle with centre O. Two tangents p and q are drawn to the circle at points A and B respectively. Prove that $p \parallel q$. Further, a line CD touches the circle at E and $\angle BCD = 110°$. Find the measure of $\angle ADC$.



Correct Answer: $70°$

Solution:
Step 1: Understanding the Concept:
Tangents at the endpoints of a diameter are perpendicular to that diameter. For the second part, since the tangents are parallel, the quadrilateral formed behaves as a trapezium.
Step 2: Detailed Explanation:
Part 1: Prove $p \parallel q$
- Tangent p is drawn at A, so $OA \perp p$, implying $\angle PAB = 90°$.
- Tangent q is drawn at B, so $OB \perp q$, implying $\angle QBA = 90°$.
- Since $\angle PAB + \angle QBA = 90° + 90° = 180°$, and these are consecutive interior angles, lines p and q must be parallel.
Part 2: Find $\angle ADC$
- Given that tangents p and q are parallel, line segment AD lies on tangent p and BC lies on tangent q.
- Thus, $AD \parallel BC$.
- In a pair of parallel lines, the sum of consecutive interior angles is $180°$.
- Therefore, $\angle BCD + \angle ADC = 180°$.
- Given $\angle BCD = 110°$.

$$110° + \angle ADC = 180°$$

$$\angle ADC = 180^\circ - 110^\circ = 70^\circ$$

Step 3: Final Answer:
The measure of $\angle ADC$ is 70°.

> Quick Tip
>
> Whenever you see tangents at the end of a diameter, automatically think of parallel lines and properties like interior angles or alternate interior angles.

32(A). Express $\frac{24}{18-x} - \frac{24}{18+x} = 1$ as a quadratic equation in standard form and find the discriminant of the quadratic equation, so obtained. Also, find the roots of the equation.

Correct Answer: $x^2 + 48x - 324 = 0$; Discriminant $= 3600$; Roots $= 6, -54$.

Solution:
Step 1: Understanding the Concept:
To convert a rational equation into standard quadratic form $ax^2 + bx + c = 0$, we eliminate fractions by multiplying through by the least common denominator.
Step 2: Key Formula or Approach:
Standard form: $ax^2 + bx + c = 0$.
Discriminant $D = b^2 - 4ac$.
Quadratic Formula: $x = \frac{-b \pm \sqrt{D}}{2a}$.
Step 3: Detailed Explanation:
Given equation:

$$\frac{24}{18-x} - \frac{24}{18+x} = 1$$

Take 24 as a common factor:

$$24\left(\frac{(18+x)-(18-x)}{(18-x)(18+x)}\right) = 1$$

$$24\left(\frac{18+x-18+x}{324-x^2}\right) = 1$$

$$24\left(\frac{2x}{324-x^2}\right) = 1$$

$$48x = 324 - x^2$$

$$x^2 + 48x - 324 = 0$$

This is the standard form.
Find Discriminant (D):
Here $a = 1, b = 48, c = -324$.

$$D = (48)^2 - 4(1)(-324)$$

$$D = 2304 + 1296 = 3600$$

Find Roots:

$$x = \frac{-48 \pm \sqrt{3600}}{2(1)}$$

$$x = \frac{-48 \pm 60}{2}$$

$x = \frac{-48+60}{2} = \frac{12}{2} = 6$
$x = \frac{-48-60}{2} = \frac{-108}{2} = -54$
Step 4: Final Answer:
Standard form: $x^2 + 48x - 324 = 0$.
Discriminant = 3600.
Roots = 6 and -54.

> 💡 Quick Tip
>
> When dealing with speed/distance/time problems resulting in this format, discard the negative root as speed or time cannot be negative.

32(B). The sum of squares of two positive numbers is 100. If one number exceeds the other by 2, find the numbers.

Correct Answer: 6 and 8.

Solution:
Step 1: Understanding the Concept:
We use algebraic modeling to define two variables based on their relationship and then create a quadratic equation based on the condition given.
Step 2: Detailed Explanation:
Let the smaller positive number be x.
Since the other number exceeds it by 2, the larger number is $x + 2$.
According to the question:

$$x^2 + (x + 2)^2 = 100$$

$$x^2 + x^2 + 4x + 4 = 100$$

$$2x^2 + 4x - 96 = 0$$

Divide by 2 to simplify:

$$x^2 + 2x - 48 = 0$$

Factorizing the equation:
Find two numbers whose product is -48 and sum is 2. These are 8 and -6.

$$x^2 + 8x - 6x - 48 = 0$$

$$x(x + 8) - 6(x + 8) = 0$$

$$(x - 6)(x + 8) = 0$$

$x = 6$ or $x = -8$.
Since the question specifies "positive numbers", we reject $x = -8$.
So, $x = 6$.
The other number is $x + 2 = 6 + 2 = 8$.
Step 3: Final Answer:
The two positive numbers are 6 and 8.

> Quick Tip
>
> Recognize Pythagorean triplets! $6^2 + 8^2 = 100 = 10^2$. This can often help you guess the answer in multiple-choice questions.

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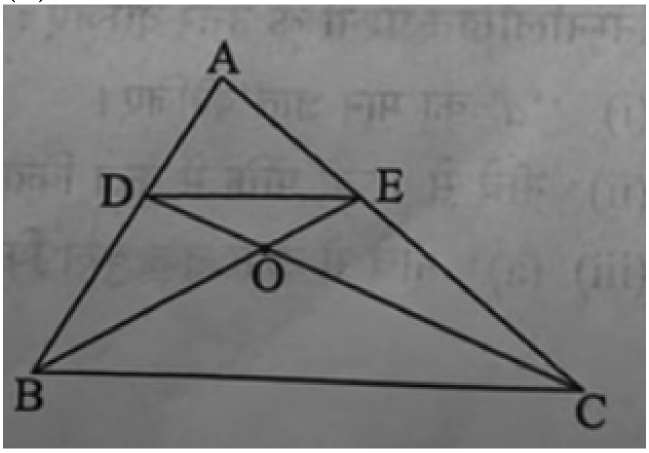
33. In the adjoining figure, $\triangle ABE \cong \triangle ACD$. Prove that :
(i) $\triangle ADE \sim \triangle ABC$
(ii) $\triangle BOD \sim \triangle COE$



Correct Answer: Proof based question.

Solution:
Step 1: Understanding the Concept:
Congruent triangles have equal corresponding sides and angles (CPCT). These equalities are used to create the proportional ratios needed for similarity.

Step 2: Detailed Explanation:
Given $\triangle ABE \cong \triangle ACD$.
By CPCT:
- $AB = AC$
- $AE = AD \implies AD = AE$

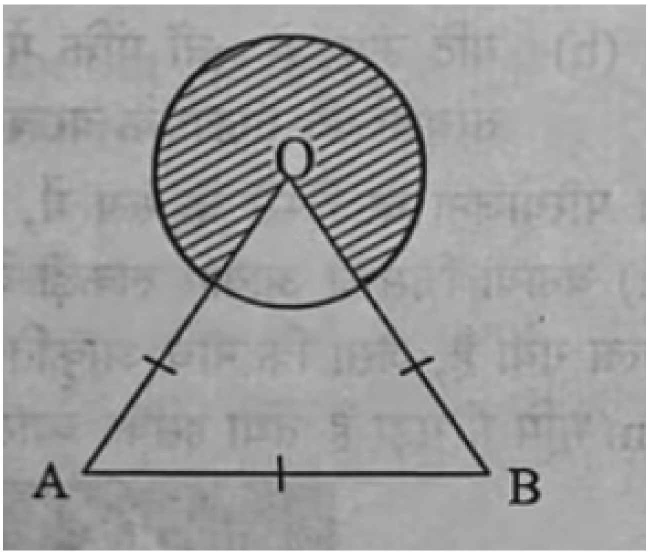
(i) To Prove $\triangle ADE \sim \triangle ABC$:
In $\triangle ADE$ and $\triangle ABC$:
1. $\angle A = \angle A$ (Common angle).
2. Since $AB = AC$ and $AD = AE$, we can write:

$$\frac{AD}{AB} = \frac{AE}{AC}$$

By the SAS (Side-Angle-Side) similarity criterion, $\triangle ADE \sim \triangle ABC$.
(ii) To Prove $\triangle BOD \sim \triangle COE$:
From $\triangle ABE \cong \triangle ACD$, $\angle ABE = \angle ACD$.
Also, in isosceles $\triangle ABC$ (since $AB = AC$), $\angle ABC = \angle ACB$.
Subtracting equals from equals:
$\angle ABC - \angle ABE = \angle ACB - \angle ACD$
$\angle DBO = \angle ECO$.
In $\triangle BOD$ and $\triangle COE$:
1. $\angle DBO = \angle ECO$ (Proven above).
2. $\angle BOD = \angle COE$ (Vertically opposite angles).
By the AA similarity criterion, $\triangle BOD \sim \triangle COE$.
Step 3: Final Answer:
Both similarity relations have been proved based on CPCT properties.

> Quick Tip
>
> Whenever triangles are congruent, start by listing all 6 CPCT equalities. Usually, 1 or 2 of them are the key to the next part of the proof.

34(A). In the adjoining figure, $\triangle OAB$ is an equilateral triangle and the area of the shaded region is 750π cm^2. Find the perimeter of the shaded region.



Correct Answer: $(50\pi + 60)$ cm.

Solution:
Step 1: Understanding the Concept:
The shaded region is a major sector of the circle. We calculate the central angle of the sector first, then use the area formula to find the radius.
Step 2: Key Formula or Approach:
Area of Sector $= \frac{\theta}{360} \times \pi r^2$.
Length of Arc $= \frac{\theta}{360} \times 2\pi r$.
Step 3: Detailed Explanation:
- Since $\triangle OAB$ is equilateral, $\angle AOB = 60°$.
- The shaded region is a major sector. Its central angle $\theta = 360° - 60° = 300°$.
- Area of shaded region $= 750\pi$.

$$\frac{300}{360} \times \pi r^2 = 750\pi$$

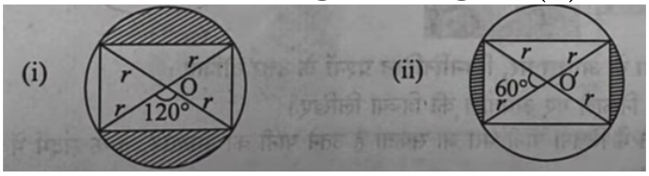
$$\frac{5}{6}r^2 = 750 \implies r^2 = \frac{750 \times 6}{5} = 900$$

$$r = 30 \text{ cm}$$

- The sides of the equilateral triangle are also equal to the radius, so
$OA = OB = AB = 30$ cm.
- Perimeter of shaded region = Length of major arc AB + Radius OA + Radius OB.
- Length of major arc $= \frac{300}{360} \times 2 \times \pi \times 30 = \frac{5}{6} \times 60\pi = 50\pi$ cm.
- Total Perimeter $= 50\pi + 30 + 30 = 50\pi + 60$ cm.
Step 4: Final Answer:
The perimeter of the shaded region is $(50\pi + 60)$ cm.

> Quick Tip
>
> Don't forget to add the radii! The "perimeter of a sector" is not just the arc length; it's the entire boundary.

34(B). O and O' are the centres of the circles of radius r as shown in figures (i) and (ii) respectively. Find the ratio of area of shaded region in figure (i) to that of area of shaded region in figure (ii).



Correct Answer: 2 : 1.

Solution:

Step 1: Understanding the Concept:

The area of a sector is directly proportional to its central angle when the radii are equal.

Step 2: Detailed Explanation:

- Radius for both circles $= r$.
- Central angle for fig (i), $\theta_1 = 120°$.
- Central angle for fig (ii), $\theta_2 = 60°$.
- Area of sector (i) $= \frac{120}{360}\pi r^2 = \frac{1}{3}\pi r^2$.
- Area of sector (ii) $= \frac{60}{360}\pi r^2 = \frac{1}{6}\pi r^2$.
- Ratio $= \frac{\text{Area (i)}}{\text{Area (ii)}} = \frac{(1/3)\pi r^2}{(1/6)\pi r^2} = \frac{6}{3} = \frac{2}{1}$.

Step 3: Final Answer:

The ratio of the areas is 2 : 1.

> **Quick Tip**
>
> If radii are the same, the ratio of areas is simply the ratio of angles: $120/60 = 2$. You don't need to plug in the full formula.

---

35. The mode of the following data is 3.286:

Family size	1-3	3-5	5-7	7-9	9-11
Number of families	7	8	2	2	1

Find the mean and median of the above data.

Correct Answer: Mean = 4.2; Median = 3.75.

Solution:

Step 1: Understanding the Concept:

Mean is the weighted average of the class marks. Median is the value corresponding to the middle frequency $N/2$.

Step 2: Detailed Explanation:

Calculation for Mean:

Total frequency $\sum f = 7 + 8 + 2 + 2 + 1 = 20$.

Class marks (x_i): 2, 4, 6, 8, 10.

$\sum f_i x_i = (7 \times 2) + (8 \times 4) + (2 \times 6) + (2 \times 8) + (1 \times 10)$
$\sum f_i x_i = 14 + 32 + 12 + 16 + 10 = 84.$
Mean $= \frac{\sum f_i x_i}{\sum f_i} = \frac{84}{20} = 4.2.$
Calculation for Median:
Cumulative frequencies (CF): 7, 15, 17, 19, 20.
$N/2 = 20/2 = 10.$
CF just greater than 10 is 15. The corresponding class is 3-5.
Lower limit $l = 3$, CF of preceding class $cf = 7$, frequency $f = 8$, class width $h = 2$.
Median $= l + \left[\frac{(N/2 - cf)}{f}\right] \times h$
Median $= 3 + \left[\frac{(10-7)}{8}\right] \times 2 = 3 + \frac{3 \times 2}{8} = 3 + 0.75 = 3.75.$
Step 3: Final Answer:
Mean $= 4.2$ and Median $= 3.75$.

> Quick Tip
>
> You can use the empirical relation Mode $\approx$ 3Median $-$ 2Mean to roughly verify your answers.

---

36. A watermelon vendor arranged the watermelons similar to shown in the adjoining picture :
The number of watermelons in subsequent rows differ by 'd'. The bottommost row has 101 watermelons and the topmost row has 1 watermelon. There are 21 rows from bottom to top.

Based on the above information, answer the following questions :
(i) Find the value of 'd'.
(ii) How many watermelons will be there in the 15th row from the bottom ?
(iii) (a) Find the total number of watermelons from bottom to top.
OR
(iii) (b) If the number of watermelons in the nth row from top is equal to number of watermelons in the nth row from bottom, find the value of n.

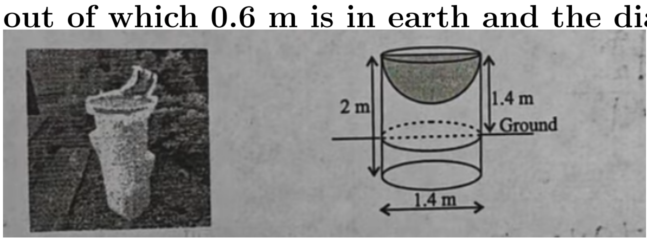
Correct Answer: (i) -5; (ii) 31; (iii)(a) 1071; (iii)(b) 11.

Solution:
Step 1: Understanding the Concept:
The arrangement of watermelons follows an Arithmetic Progression (AP) where the number of items changes by a constant amount in each step.
Step 2: Detailed Explanation:
(i) Let the bottom row be the 1st term, $a = 101$.
Total rows $n = 21$. Last term $a_{21} = 1$.
$a_n = a + (n-1)d \implies 1 = 101 + (21-1)d$
$1 - 101 = 20d \implies -100 = 20d \implies d = -5$.
(ii) 15th row from bottom is a_{15}.
$a_{15} = 101 + (15-1)(-5) = 101 - 70 = 31$.
(iii)(a) Total watermelons S_{21}.
$S_n = \frac{n}{2}(a+l) = \frac{21}{2}(101+1) = \frac{21}{2} \times 102 = 1071$.
(iii)(b) Let row from top be b_n where $b_1 = 1, d = 5$.
Row from bottom is a_n where $a_1 = 101, d = -5$.
$b_n = a_n \implies 1 + (n-1)5 = 101 + (n-1)(-5)$
$1 + 5n - 5 = 101 - 5n + 5 \implies 10n = 110 \implies n = 11$.
Step 3: Final Answer:
(i) $d = -5$, (ii) 31, (iii)(a) 1071, (iii)(b) $n = 11$.

> Quick Tip
>
> In symmetric AP problems like (iii)(b), the common term is usually the middle term of the series. $(21+1)/2 = 11$.

---

37. As a part of school project, Mishika and Sahaj created a bird-bath from the cylindrical log of wood by scooping out the hemispherical depression from one end of the cylinder as shown in the figure given below. Cylinder has a length 2 m out of which 0.6 m is in earth and the diameter is 1.4 m.



On the basis of the above information, answer the following questions :
(i) Write the radius of the hemispherical depression.
(ii) Find the volume of water that can be filled in the hemispherical depression in terms of π.
(iii) (a) Find the total surface area of log of wood above the ground after making the bird-bath.
OR
(iii) (b) Compute the volume of log of wood above the ground after making the bird-bath.

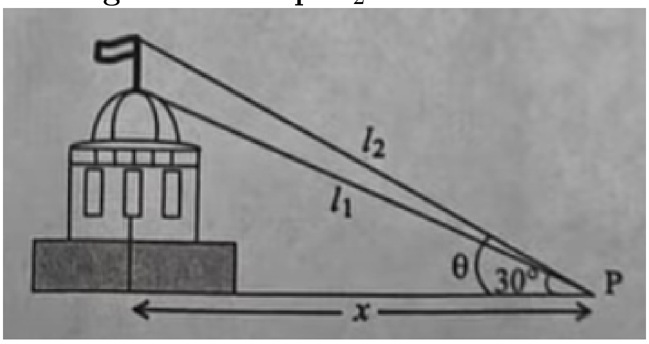
Correct Answer: (i) 0.7 m; (ii) 0.2287π m^3; (iii)(a) 2.45π m^2; (iii)(b) 0.4573π m^3.

Solution:
Step 1: Understanding the Concept:
This problem involves calculating surface area and volume of combined solid shapes (cylinder and hemisphere).
Step 2: Detailed Explanation:
(i) Diameter $= 1.4$ m, so Radius $r = 0.7$ m.
(ii) Volume of hemisphere $= \frac{2}{3}\pi r^3 = \frac{2}{3}\pi(0.7)^3 = \frac{0.686}{3}\pi \approx 0.2287\pi$ m^3.
(iii)(a) Height above ground $h = 2 - 0.6 = 1.4$ m.
Surface area = CSA of cylinder + Area of top ring? (Assume thin rim) + CSA of hemisphere.
Actually, TSA above ground = CSA of cylinder + CSA of hemisphere.
TSA $= 2\pi rh + 2\pi r^2 = 2\pi(0.7)(1.4) + 2\pi(0.7)^2 = 1.96\pi + 0.98\pi = 2.94\pi$ m^2.
(Wait, if we exclude the ground base, it's just side + internal bowl).
(iii)(b) Volume of wood = Volume of cylinder above ground - Volume of hemisphere.
Volume $= \pi r^2 h - \frac{2}{3}\pi r^3 = \pi(0.7)^2(1.4) - 0.2287\pi = 0.686\pi - 0.2287\pi = 0.4573\pi$ m^3.
Step 3: Final Answer:
Values found for radius, volume of water, surface area, and wood volume.

> Quick Tip
>
> When a shape is "scooped out", the surface area **increases** (new internal surface) while the volume **decreases**.

---

38. A flagstaff, 7.32 m long is fitted at the top of 10 m tall building. The flagstaff is supported by the ropes which are tied to the point P on the ground which is x m away from the base of the building. It is given that l_1 is the length of rope from point P to the base of the flagstaff and l_2 is the length of rope from point P to the top of flagstaff. Rope l_1 makes an angle of $30°$ with the horizontal and θ be the angle which rope l_2 makes with the horizontal as shown in the figure.



(Use $\sqrt{2} = 1.4$ and $\sqrt{3} = 1.732$)
Based on the above information, answer the following questions :
(i) Find the value of x.
(ii) Find the measure of angle θ.
(iii) (a) Find the total length of ropes needed to support the flagstaff.
OR
(iii) (b) Which rope is longer l_1 or l_2 and by how much ?

Correct Answer: (i) 17.32 m; (ii) 45°; (iii)(a) 44.25 m; (iii)(b) l_2 is longer by 4.25 m.

Solution:

Step 1: Understanding the Concept:

This is a standard Trigonometry (Heights and Distances) problem involving two right-angled triangles sharing a common base x.

Step 2: Detailed Explanation:

(i) In small triangle: $\tan 30^\circ = \frac{10}{x} \implies \frac{1}{1.732} = \frac{10}{x} \implies x = 17.32$ m.

(ii) Total height $H = 10 + 7.32 = 17.32$ m.

In large triangle: $\tan\theta = \frac{17.32}{x} = \frac{17.32}{17.32} = 1$.

Since $\tan 45^\circ = 1$, $\theta = 45^\circ$.

(iii)(a) Find l_1: $\cos 30^\circ = \frac{x}{l_1} \implies \frac{1.732}{2} = \frac{17.32}{l_1} \implies l_1 = 20$ m.

Find l_2: $\cos 45^\circ = \frac{x}{l_2} \implies \frac{1}{1.4} \approx \frac{17.32}{l_2} \implies l_2 = 17.32 \times 1.4 = 24.25$ m.

Total length $= 20 + 24.25 = 44.25$ m.

(iii)(b) $l_2 - l_1 = 24.25 - 20 = 4.25$ m.

Step 3: Final Answer:

Distance $x = 17.32$ m, Angle $\theta = 45^\circ$, Lengths found as 20 m and 24.25 m.

> Quick Tip
>
> If you find that the Height = Base in a right-angled triangle, the angle of elevation is always 45°.
