CBSE Class X Mathematics (Standard) Set 1 (30/5/1)

Time Allowed :3 Hours	**Maximum Marks :**80	**Total Questions :**38

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This question paper contains 38 questions. All questions are compulsory.
2. This Question Paper is divided into **FIVE Sections – Section A, B, C, D, and E**.
3. In Section–A, questions number 1 to 18 are **Multiple Choice Questions (MCQs)** and questions number 19 & 20 are **Assertion-Reason based questions**, carrying **1 mark each**.
4. In Section–B, questions number 21 to 25 are **Very Short-Answer (VSA)** type questions, carrying **2 marks each**.
5. In Section–C, questions number 26 to 31 are **Short Answer (SA)** type questions, carrying **3 marks each**.
6. In Section–D, questions number 32 to 35 are **Long Answer (LA)** type questions, carrying **5 marks each**.
7. In Section–E, questions number 36 to 38 are **Case Study based questions** carrying **4 marks each**. *Internal choice is provided in each case-study.*
8. There is **no overall choice.** However, *an internal choice has been provided in 2 questions in Section–B, 2 questions in Section–C, 2 questions in Section–D, and 3 questions in Section–E.*
9. Draw neat diagrams wherever required. Take $\pi = \frac{22}{7}$ wherever required, if not stated.
10. Use of calculators is **not allowed.**

Section A

1. The next (4^{th}) term of the A.P. $\sqrt{18}$, $\sqrt{50}$, $\sqrt{98}$, ... is:
(A) $\sqrt{128}$
(B) $\sqrt{140}$
(C) $\sqrt{162}$
(D) $\sqrt{200}$
Correct Answer: (C) $\sqrt{162}$

Solution: The given sequence is an arithmetic progression (A.P.) of square roots of numbers. The difference between consecutive terms is:

$$\sqrt{50} - \sqrt{18} = 2$$

and similarly:

$$\sqrt{98} - \sqrt{50} = 2$$

Thus, the common difference is 2. For the 4^{th} term:

$$\sqrt{98} + 2 = \sqrt{162}$$

Therefore, the next term is $\sqrt{162}$.

Quick Tip

In A.P., the common difference remains constant. Use this property to quickly find missing terms.

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2. If $\frac{x}{3} = 2\sin A$, $\frac{y}{3} = 2\cos A$, then the value of $x^2 + y^2$ is:
(A) 36
(B) 9
(C) 6
(D) 18
Correct Answer: (A) 36

Solution: From the given equations:

$$x = 6\sin A, \quad y = 6\cos A$$

We use the identity $\sin^2 A + \cos^2 A = 1$:

$$x^2 + y^2 = (6\sin A)^2 + (6\cos A)^2 = 36(\sin^2 A + \cos^2 A) = 36$$

Thus, $x^2 + y^2 = 36$.

Quick Tip

Remember the Pythagorean identity: $\sin^2 A + \cos^2 A = 1$. It's crucial for simplifying trigonometric equations.

3. If $4\sec\theta - 5 = 0$, then the value of $\cot\theta$ is:
(A) $\frac{3}{4}$
(B) $\frac{4}{5}$
(C) $\frac{5}{3}$
(D) $\frac{4}{3}$
Correct Answer: (D) $\frac{4}{3}$

Solution: Solving $4\sec\theta - 5 = 0$:

$$\sec\theta = \frac{5}{4} \quad \text{so,} \quad \cos\theta = \frac{4}{5}$$

Using $\sin^2\theta + \cos^2\theta = 1$:

$$\sin^2\theta = 1 - \cos^2\theta = 1 - \left(\frac{4}{5}\right)^2 = \frac{9}{25}$$

Thus:

$$\sin\theta = \frac{3}{5}$$

The value of $\cot\theta$ is:

$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{3}$$

💡 Quick Tip

When given $\sec\theta$ or $\csc\theta$, always start by finding $\cos\theta$ or $\sin\theta$ to simplify calculations.

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4. Which out of the following type of straight lines will be represented by the system of equations $3x + 4y = 5$ and $6x + 8y = 7$?
(A) Parallel
(B) Intersecting
(C) Coincident
(D) Perpendicular to each other
Correct Answer: (A) Parallel

Solution: The general form of a straight line equation is $ax + by = c$. The given equations are:

$$3x + 4y = 5 \quad \text{and} \quad 6x + 8y = 7$$

We compare the ratios of coefficients:

$$\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{4}{8} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{5}{7}$$

Here, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, which implies that the lines are parallel.

💡 Quick Tip

For two lines to be parallel, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$. Memorize this condition for quick identification.

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5. The ratio of the sum and product of the roots of the quadratic equation $5x^2 - 6x + 21 = 0$ is:
(A) $5:21$
(B) $2:7$
(C) $21:5$
(D) $7:2$
Correct Answer: (B) $2:7$

Solution: The standard quadratic equation is $ax^2 + bx + c = 0$. For the given equation $5x^2 - 6x + 21 = 0$:

$$\text{Sum of roots} = -\frac{b}{a} = -\frac{-6}{5} = \frac{6}{5}, \quad \text{Product of roots} = \frac{c}{a} = \frac{21}{5}$$

The ratio of the sum to the product is:

$$\frac{\text{Sum}}{\text{Product}} = \frac{\frac{6}{5}}{\frac{21}{5}} = \frac{6}{21} = \frac{2}{7}$$

Thus, the ratio is $2:7$.

Quick Tip
To find the ratio of sum and product of roots, use $-\frac{b}{a}$ for the sum and $\frac{c}{a}$ for the product. Simplify their ratio directly.

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6. For the data $2, 9, x+6, 2x+3, 5, 10, 5$, if the mean is 7, then the value of x is:
(A) 9
(B) 6
(C) 5
(D) 3
Correct Answer: (D) 3

Solution: The mean of the data is given as 7. The mean is calculated as:

$$\text{Mean} = \frac{\text{Sum of all terms}}{\text{Number of terms}}$$

The sum of the terms is:

$$2 + 9 + (x+6) + (2x+3) + 5 + 10 + 5 = 40 + 3x$$

Since there are 7 terms:

$$\frac{40+3x}{7} = 7 \quad \Rightarrow \quad 40 + 3x = 49 \quad \Rightarrow \quad 3x = 9 \quad \Rightarrow \quad x = 3$$

Quick Tip
To find the mean, multiply the given mean by the number of terms to calculate the total sum.

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7. One ticket is drawn at random from a bag containing tickets numbered 1 to 40. The probability that the selected ticket has a number which is a multiple of 7 is:
(A) $\frac{1}{7}$
(B) $\frac{1}{8}$
(C) $\frac{1}{5}$
(D) $\frac{7}{40}$
Correct Answer: (B) $\frac{1}{8}$

Solution: The numbers between 1 and 40 that are multiples of 7 are: $7, 14, 21, 28, 35$. There are 5 such numbers. The total number of tickets is 40. The probability is:

$$P = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{5}{40} = \frac{1}{8}$$

Quick Tip

To calculate probability, always count the favorable outcomes and divide by the total outcomes.

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8. The perimeter of the sector of a circle of radius 21 cm which subtends an angle of $60°$ **at the center of the circle, is:**
(A) 22 cm
(B) 43 cm
(C) 64 cm
(D) 462 cm
Correct Answer: (C) 64 **cm**

Solution: The perimeter of a sector is the sum of the arc length and twice the radius:

$$\text{Perimeter} = \text{Arc length} + 2 \times \text{Radius}$$

The arc length is given by:

$$\text{Arc length} = \frac{\theta}{360} \times 2\pi r = \frac{60}{360} \times 2\pi \times 21 = \frac{1}{6} \times 2 \times 22 \times 21 = 44\,\text{cm}$$

Thus, the perimeter is:

$$\text{Perimeter} = 44 + 2 \times 21 = 64\,\text{cm}$$

Quick Tip

The arc length is proportional to the angle subtended at the center: $\frac{\theta}{360} \times 2\pi r$.

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9. The length of an arc of a circle with radius 12 cm is 10π **cm. The angle subtended by the arc at the center of the circle, is:**
(A) $120°$
(B) $6°$
(C) $75°$
(D) $150°$
Correct Answer: (D) $150°$

Solution: The arc length is related to the angle as:

$$\text{Arc length} = \frac{\theta}{360} \times 2\pi r$$

Given Arc length $= 10\pi$ and $r = 12$:

$$10\pi = \frac{\theta}{360} \times 2\pi \times 12 \quad \Rightarrow \quad 10 = \frac{\theta}{360} \times 24 \quad \Rightarrow \quad \theta = \frac{10 \times 360}{24} = 150^\circ$$

Quick Tip

To calculate the angle subtended, rearrange the formula Arc length $= \frac{\theta}{360} \times 2\pi r$.

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10. The greatest number which divides 281 and 1249, leaving remainder 5 and 7 respectively, is:
(A) 23
(B) 276
(C) 138
(D) 69
Correct Answer: (C) 138

Solution: Subtract the remainders from the numbers:

$$281 - 5 = 276, \quad 1249 - 7 = 1242$$

The required number is the greatest common divisor (GCD) of 276 and 1242. Using the Euclidean algorithm:

$$1242 \div 276 = 4\,(\text{remainder } 138), \quad 276 \div 138 = 2\,(\text{remainder } 0)$$

Thus, the GCD is 138.

Quick Tip

When given remainders, subtract them first, then find the GCD of the results using the Euclidean algorithm.

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11. The number of terms in the A.P. $3, 6, 9, 12, \ldots, 111$ **is:**
(A) 36
(B) 40
(C) 37
(D) 30
Correct Answer: (C) 37

Solution: The formula for the n^{th} term of an A.P. is:

$$a_n = a + (n-1)d$$

Here, $a = 3$, $d = 3$, and $a_n = 111$. Substituting these values:

$$111 = 3 + (n-1)3 \quad \Rightarrow \quad 111 - 3 = 3(n-1) \quad \Rightarrow \quad 108 = 3(n-1)$$

$$n - 1 = 36 \quad \Rightarrow \quad n = 37$$

Thus, the number of terms is 37.

Quick Tip

Use the formula for the n^{th} term of an A.P.: $a_n = a + (n-1)d$, and solve for n.

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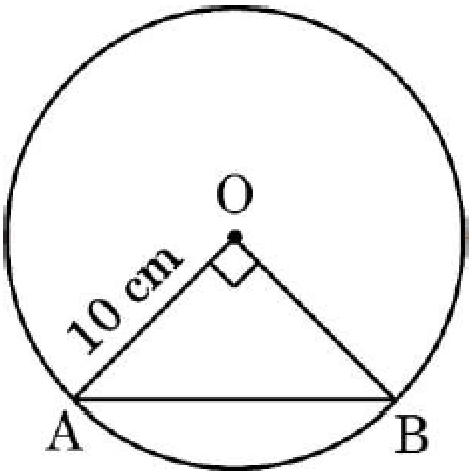
12. A chord of a circle of radius 10 cm subtends a right angle at its center. The length of the chord (in cm) is:



(A) $5\sqrt{2}$
(B) $10\sqrt{2}$
(C) $\frac{5}{\sqrt{2}}$
(D) 5
Correct Answer: (B) $10\sqrt{2}$

Solution: The chord length in a circle is related to the angle it subtends at the center. Since the angle is 90°, the chord forms an isosceles right triangle with the radii. Using the Pythagoras theorem:

$$AB^2 = AO^2 + BO^2 \quad \text{where } AO = BO = 10\,\text{cm}$$

$$AB^2 = 10^2 + 10^2 = 200 \quad \Rightarrow \quad AB = \sqrt{200} = 10\sqrt{2}\,\text{cm}$$

Thus, the length of the chord is $10\sqrt{2}$ cm.

Quick Tip

For chords subtending a 90° angle, apply the Pythagoras theorem to calculate the length directly.

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13. The LCM of three numbers $28, 44, 132$ is:
(A) 258
(B) 231
(C) 462
(D) 924
Correct Answer: (D) 924

Solution: To find the LCM of 28, 44, and 132, first find their prime factorizations:

$$28 = 2^2 \times 7, \quad 44 = 2^2 \times 11, \quad 132 = 2^2 \times 3 \times 11$$

The LCM is obtained by taking the highest powers of all prime factors:

$$\text{LCM} = 2^2 \times 3 \times 7 \times 11 = 924$$

Thus, the LCM is 924.

Quick Tip

To find the LCM, take the highest powers of all prime factors from the given numbers.

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14. If the product of two co-prime numbers is 553, then their HCF is:
(A) 1
(B) 553
(C) 7
(D) 79
Correct Answer: (A) 1

Solution: Co-prime numbers are numbers that have no common factors other than 1. The definition implies that their HCF is always 1. Therefore, the HCF of two co-prime numbers whose product is 553 is:

$$\text{HCF} = 1$$

Quick Tip

For co-prime numbers, the HCF is always 1, regardless of their product.

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15. If α and β are the zeroes of the polynomial $p(x) = kx^2 - 30x + 45k$ and $\alpha + \beta = \alpha\beta$, then the value of k is:
(A) $-\frac{2}{3}$
(B) $-\frac{3}{2}$
(C) $\frac{3}{2}$
(D) $\frac{2}{3}$
Correct Answer: (D) $\frac{2}{3}$

Solution: For a polynomial $p(x) = ax^2 + bx + c$, the sum and product of zeroes are given by:

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}$$

Here, $a = k$, $b = -30$, and $c = 45k$. Substituting these values:

$$\alpha + \beta = -\frac{-30}{k} = \frac{30}{k}, \quad \alpha\beta = \frac{45k}{k} = 45$$

The condition $\alpha + \beta = \alpha\beta$ gives:

$$\frac{30}{k} = 45 \quad \Rightarrow \quad 30 = 45k \quad \Rightarrow \quad k = \frac{2}{3}$$

Thus, the value of k is $\frac{2}{3}$.

Quick Tip

Use the relationships $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$ to solve for unknowns in quadratic equations.

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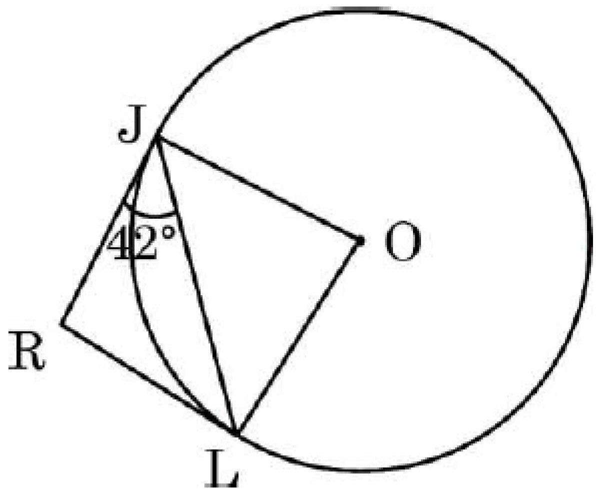
16. In the given figure, RJ and RL are two tangents to the circle. If $\angle RJL = 42°$, then the measure of $\angle JOL$ is:



(A) 42°
(B) 84°
(C) 96°
(D) 138°
Correct Answer: (B) 84°

Solution: In a circle, the angle subtended by two tangents at the center is twice the angle subtended at the point of tangency. Given $\angle RJL = 42°$, we calculate:

$$\angle JOL = 2 \times \angle RJL = 2 \times 42° = 84°$$

Thus, $\angle JOL = 84°$.

Quick Tip

For tangents to a circle, the angle subtended at the center is twice the angle between the tangents.

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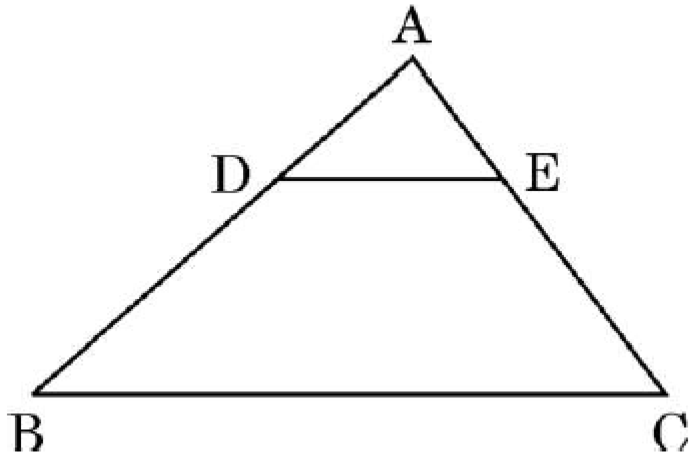
17. In the given figure, in $\triangle ABC$, $DE \parallel BC$. If $AD = 2.4$ cm, $DB = 4$ cm, and $AE = 2$ cm, then the length of AC is:



(A) $\frac{10}{3}$ cm
(B) $\frac{3}{10}$ cm
(C) $\frac{16}{3}$ cm
(D) 1.2 cm
Correct Answer: (C) $\frac{16}{3}$ cm

Solution: Using the Basic Proportionality Theorem (Thales' theorem), we know:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Substituting $AD = 2.4$, $DB = 4$, and $AE = 2$:

$$\frac{2.4}{4} = \frac{2}{EC} \quad \Rightarrow \quad EC = \frac{4 \times 2}{2.4} = \frac{10}{3}\text{ cm}$$

Thus, $AC = AD + DB = 2.4 + \frac{10}{3} = \frac{16}{3}$ cm.

> 💡 Quick Tip
>
> For parallel lines in a triangle, use the Basic Proportionality Theorem: $\frac{AD}{DB} = \frac{AE}{EC}$.

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18. If a vertical pole of length 7.5 m casts a shadow 5 m long on the ground and at the same time, a tower casts a shadow 24 m long, then the height of the tower is:
(A) 20 m
(B) 40 m
(C) 60 m
(D) 36 m
Correct Answer: (D) 36 m

Solution: Using the concept of similar triangles, the ratio of the height of the object to the length of its shadow is constant. Let the height of the tower be h. Then:

$$\frac{\text{Height of pole}}{\text{Length of pole's shadow}} = \frac{\text{Height of tower}}{\text{Length of tower's shadow}}$$

Substituting the values:

$$\frac{7.5}{5} = \frac{h}{24} \quad \Rightarrow \quad h = \frac{7.5 \times 24}{5} = 36\,\text{m}$$

Thus, the height of the tower is 36 m.

Quick Tip

For problems involving shadows, use the ratio of height to shadow length for similar triangles: $\frac{h_1}{s_1} = \frac{h_2}{s_2}$.

19.Assertion (A): ABCD is a trapezium with $DC \parallel AB$. E and F are points on AD and BC, respectively, such that $EF \parallel AB$. Then:

$$\frac{AE}{ED} = \frac{BF}{FC}.$$

Reason (R): Any line parallel to parallel sides of a trapezium divides the non-parallel sides proportionally.

Options:

(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (A)

Solution: The assertion states a property of trapeziums when a line parallel to the parallel sides divides the non-parallel sides proportionally. The reason explicitly explains this property. Therefore, both the assertion and the reason are true, and the reason correctly explains the assertion.

Quick Tip

For parallel lines in trapeziums, use the Basic Proportionality Theorem to establish proportional relationships.

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20. Assertion (A): Degree of a zero polynomial is not defined.

Reason (R): Degree of a non-zero constant polynomial is 0.

Options:

(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (B)

Solution: The degree of a zero polynomial is undefined because it does not have any terms with a non-zero coefficient. The reason correctly states that the degree of a non-zero constant polynomial is 0. However, the reason does not directly explain why the degree of a zero polynomial is undefined. Therefore, both the assertion and the reason are true, but the reason is not the correct explanation of the assertion.

Quick Tip

A zero polynomial has no degree, while the degree of a non-zero constant polynomial is always 0.

Section B

21(a). If two tangents inclined at an angle of 60° are drawn to a circle of radius 3 cm, then find the length of each tangent.

Solution: Let the tangents meet at point P outside the circle, and let A and B be the points of tangency. The center of the circle is O, and the radius is 3 cm. The angle subtended at P by the tangents is 60°. The line OP bisects $\angle APB$ into two equal angles of 30°. In $\triangle APO$:

$$\angle APO = 30^\circ$$

Using the trigonometric relation $\tan\theta = \frac{\text{perpendicular}}{\text{base}}$:

$$\tan 30^\circ = \frac{OA}{AP} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{3}{AP}$$

Solving for AP:

$$AP = 3\sqrt{3}\,\text{cm}$$

Thus, the length of each tangent is $3\sqrt{3}$ cm.

Quick Tip

For tangents inclined at an angle, divide the angle into two right triangles and apply basic trigonometry to calculate the tangent length.

21(b). Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Solution: Let P and Q be the points of tangency at the ends of the diameter AB, and let XY and PQ be the tangents at A and B, respectively. The radius at the points of tangency is perpendicular to the tangent. Thus:

$$\angle OAY = \angle OBP = 90^\circ$$

Since $\angle OAY$ and $\angle OBP$ form alternate interior angles and the lines XY and PQ are transversals, the tangents PQ and XY are parallel:

$$PQ \parallel XY$$

Quick Tip

For tangents at the ends of a diameter, use the property that the radius is perpendicular to the tangent.

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22. Evaluate:

$$2\tan 30^\circ \cdot \sec 60^\circ \cdot \tan 45^\circ \div \left(1 - \sin^2 60^\circ\right)$$

Solution: Substituting the values:

$$\tan 30^\circ = \frac{1}{\sqrt{3}}, \quad \sec 60^\circ = 2, \quad \tan 45^\circ = 1, \quad \sin^2 60^\circ = \frac{3}{4}$$

$$= \frac{2 \times \frac{1}{\sqrt{3}} \times 2 \times 1}{1 - \frac{3}{4}} = \frac{\frac{4}{\sqrt{3}}}{\frac{1}{4}} = \frac{16}{\sqrt{3}} \quad \text{or} \quad \frac{16\sqrt{3}}{3}$$

Thus, the answer is $\frac{16\sqrt{3}}{3}$.

Quick Tip

Remember trigonometric values for standard angles (30°, 45°, 60°) to simplify such problems quickly.

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23. If α, β are zeroes of the polynomial $p(x) = 5x^2 - 6x + 1$, then find the value of $\alpha + \beta + \alpha\beta$.

Solution: For a quadratic polynomial $ax^2 + bx + c$, the sum and product of zeroes are:

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}$$

Substituting $a = 5$, $b = -6$, $c = 1$:

$$\alpha + \beta = -\frac{-6}{5} = \frac{6}{5}, \quad \alpha\beta = \frac{1}{5}$$

The required value is:

$$\alpha + \beta + \alpha\beta = \frac{6}{5} + \frac{1}{5} = \frac{7}{5}$$

Thus, the answer is $\frac{7}{5}$.

Quick Tip

For quadratic polynomials, use $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$ to quickly compute related values.

24(a). Find the ratio in which the point $P(-4, 6)$ divides the line segment joining the points $A(-6, 10)$ and $B(3, -8)$.

Solution: Let the ratio be $k : 1$. Using the section formula for the x-coordinate:

$$x_P = \frac{kx_B + x_A}{k+1}$$

Substitute $x_P = -4$, $x_A = -6$, and $x_B = 3$:

$$-4 = \frac{3k - 6}{k+1}$$

Solving for k:

$$-4(k+1) = 3k - 6 \quad \Rightarrow \quad -4k - 4 = 3k - 6 \quad \Rightarrow \quad -7k = -2 \quad \Rightarrow \quad k = \frac{2}{7}$$

Thus, the required ratio is $2 : 7$.

Quick Tip

For division of a line segment, use the section formula: $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$.

24(b). Prove that the points $(3, 0)$, $(6, 4)$, and $(-1, 3)$ are the vertices of an isosceles triangle.

Solution: Let $A(3, 0)$, $B(6, 4)$, and $C(-1, 3)$. Calculate the lengths of the sides using the distance formula:

$$AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2} = \sqrt{(6-3)^2 + (4-0)^2} = \sqrt{3^2 + 4^2} = \sqrt{9+16} = 5$$

$$BC = \sqrt{(x_C - x_B)^2 + (y_C - y_B)^2} = \sqrt{(-1-6)^2 + (3-4)^2} = \sqrt{(-7)^2 + (-1)^2} = \sqrt{49+1} = \sqrt{50}$$

$$CA = \sqrt{(x_C - x_A)^2 + (y_C - y_A)^2} = \sqrt{(-1-3)^2 + (3-0)^2} = \sqrt{(-4)^2 + 3^2} = \sqrt{16+9} = 5$$

Since $AB = CA$, the triangle $\triangle ABC$ is isosceles.

Quick Tip

For proving a triangle is isosceles, calculate the lengths of all sides using the distance formula: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

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25. A carton consists of 60 shirts of which 48 are good, 8 have major defects, and 4 have minor defects. Nigam, a trader, will accept the shirts which are good, but Anmol, another trader, will only reject the shirts which have major defects. One shirt is drawn at random from the carton. Find the probability that it is acceptable to Anmol.

Solution: The total number of shirts is 60. Anmol will accept all shirts except those with major defects. The number of shirts without major defects is:

$$48\,(\text{good}) + 4\,(\text{minor defects}) = 52$$

The probability that the shirt is acceptable to Anmol is:

$$P(\text{Anmol will accept the shirt}) = \frac{\text{Number of acceptable shirts}}{\text{Total number of shirts}} = \frac{52}{60} = \frac{13}{15}$$

Thus, the probability is $\frac{13}{15}$.

> **Quick Tip**
>
> To calculate probability, divide the favorable outcomes by the total outcomes.

Section C

26(a). Prove that $\sqrt{3}$ is an irrational number.

Solution: Assume $\sqrt{3}$ is a rational number. Then it can be expressed as:

$$\sqrt{3} = \frac{p}{q}, \quad \text{where } p, q \in Z, q \neq 0, \text{ and } p, q \text{ are coprime.}$$

Squaring both sides:

$$3q^2 = p^2 \quad \Rightarrow \quad p^2 \text{ is divisible by 3.}$$

If p^2 is divisible by 3, then p is also divisible by 3 (property of prime numbers). Let $p = 3a$, where $a \in Z$. Substituting $p = 3a$ into $3q^2 = p^2$:

$$3q^2 = (3a)^2 \quad \Rightarrow \quad 3q^2 = 9a^2 \quad \Rightarrow \quad q^2 = 3a^2.$$

Thus, q^2 is divisible by 3, which implies that q is also divisible by 3.
Since both p and q are divisible by 3, this contradicts the assumption that p and q are coprime. Therefore, $\sqrt{3}$ cannot be a rational number. Hence, $\sqrt{3}$ is irrational.

> **Quick Tip**
>
> To prove a number is irrational, assume it is rational, derive a contradiction using properties of integers, and conclude the assumption is false.

26(b). Prove that $\left(\sqrt{2} + \sqrt{3}\right)^2$ is an irrational number, given that $\sqrt{6}$ is an irrational number.

Solution: Expanding $\left(\sqrt{2} + \sqrt{3}\right)^2$:

$$\left(\sqrt{2} + \sqrt{3}\right)^2 = 2 + 3 + 2\sqrt{6} = 5 + 2\sqrt{6}.$$

Assume, to the contrary, that $5 + 2\sqrt{6}$ is rational. Then:

$$5 + 2\sqrt{6} = \frac{a}{b}, \quad \text{where } a, b \in Z,\ b \neq 0.$$

Rearranging for $\sqrt{6}$:

$$\sqrt{6} = \frac{a - 5b}{2b}.$$

Here, $\sqrt{6}$ is expressed as a ratio of integers, implying it is rational. This contradicts the fact that $\sqrt{6}$ is irrational. Therefore, our assumption is wrong.
Thus, $5 + 2\sqrt{6}$, and hence $\left(\sqrt{2} + \sqrt{3}\right)^2$, is an irrational number.

> **Quick Tip**
>
> For proofs of irrationality, assume the given number is rational and derive a contradiction using known properties of irrational numbers.

27(a). If the sum of the first 14 terms of an A.P. is 1050 and the first term is 10, then find the 20$^{\text{th}}$ term and the n^{th} term.

Solution: The sum of the first n terms of an A.P. is given by:

$$S_n = \frac{n}{2}\big(2a + (n-1)d\big).$$

Substituting $S_{14} = 1050$, $a = 10$, and $n = 14$:

$$\frac{14}{2}\big(20 + 13d\big) = 1050 \quad \Rightarrow \quad 7\big(20 + 13d\big) = 1050 \quad \Rightarrow \quad 20 + 13d = 150 \quad \Rightarrow \quad d = 10.$$

The 20$^{\text{th}}$ term is:

$$a_{20} = a + 19d = 10 + 19 \times 10 = 200.$$

The n^{th} term is:

$$a_n = a + (n-1)d = 10 + (n-1) \times 10 = 10n.$$

Thus, $a_{20} = 200$ and $a_n = 10n$.

> **Quick Tip**
>
> For the n^{th} term of an A.P., use $a_n = a + (n-1)d$. For the sum of n terms, use $S_n = \frac{n}{2}\big(2a + (n-1)d\big)$.

27(b). The first term of an A.P. is 5, the last term is 45, and the sum of all the terms is 400. Find the number of terms and the common difference of the A.P.

Solution: The sum of n terms of an A.P. is given by:

$$S_n = \frac{n}{2}\big(a + l\big),$$

where a is the first term, l is the last term, and n is the number of terms. Substituting $S_n = 400$, $a = 5$, and $l = 45$:

$$\frac{n}{2}(5+45) = 400 \quad \Rightarrow \quad \frac{n}{2} \times 50 = 400 \quad \Rightarrow \quad n = 16.$$

The common difference d is:

$$a_n = a + (n-1)d \quad \Rightarrow \quad 45 = 5 + 15d \quad \Rightarrow \quad 15d = 40 \quad \Rightarrow \quad d = \frac{8}{3}.$$

Thus, $n = 16$ and $d = \frac{8}{3}$.

Quick Tip

For finding n and d, use $S_n = \frac{n}{2}(a+l)$ for the sum and $a_n = a + (n-1)d$ for the last term.

---

28. Prove that the parallelogram circumscribing a circle is a rhombus.

Solution: Let $ABCD$ be a parallelogram circumscribing a circle, and let the points of tangency be P, Q, R, and S on sides AB, BC, CD, and DA, respectively.
For a circle inscribed in a quadrilateral, the tangents drawn from an external point to the circle are equal in length. Thus:

$$AP = AS, \quad BP = BQ, \quad CR = CQ, \quad DR = DS.$$

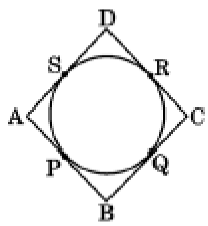
Adding these:

$$(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ),$$
$$AB + CD = AD + BC.$$

Since $AB = CD$ and $AD = BC$ (opposite sides of a parallelogram are equal), we get:

$$2AB = 2BC \quad \Rightarrow \quad AB = BC.$$

Thus, all sides of the parallelogram are equal, and $ABCD$ is a rhombus.



Quick Tip

For proving that a parallelogram is a rhombus, show that all sides are equal using the properties of tangents and the parallelogram.

---

29. Prove that:

$$\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}=1+\sec A\csc A.$$

Solution: The left-hand side (LHS) is:

$$\text{LHS}=\frac{\sin A}{\cos A(1-\cos A/\sin A)}+\frac{\cos A}{\sin A(1-\sin A/\cos A)}.$$

Simplify the denominators:

$$\text{LHS}=\frac{\sin^2 A}{\sin A-\cos A}+\frac{\cos^2 A}{\cos A-\sin A}.$$

Combine into a single fraction:

$$\text{LHS}=\frac{\sin^3 A-\cos^3 A}{(\sin A-\cos A)\sin A\cos A}.$$

Factorize $\sin^3 A-\cos^3 A$:

$$\sin^3 A-\cos^3 A=(\sin A-\cos A)(\sin^2 A+\sin A\cos A+\cos^2 A).$$

Substitute $\sin^2 A+\cos^2 A=1$:

$$\text{LHS}=\frac{(\sin A-\cos A)(1+\sin A\cos A)}{(\sin A-\cos A)\sin A\cos A}.$$

Cancel $\sin A-\cos A$:

$$\text{LHS}=\frac{1+\sin A\cos A}{\sin A\cos A}.$$

Split the fraction:

$$\text{LHS}=\frac{1}{\sin A\cos A}+1=\sec A\csc A+1.$$

Thus:

$$\text{LHS}=\text{RHS}.$$

Quick Tip

For trigonometric proofs, use algebraic simplifications and trigonometric identities like $\sin^2 A+\cos^2 A=1$ and factorization techniques.

---

30. Three unbiased coins are tossed simultaneously. Find the probability of getting:

(i) At least one head.
(ii) Exactly one tail.
(iii) Two heads and one tail.

Solution: The total number of possible outcomes when three coins are tossed is $2^3=8$. The sample space is:

$$\{HHH,HHT,HTH,HTT,THH,THT,TTH,TTT\}.$$

(i) **Probability of at least one head:**
The complement is getting no heads (TTT). Thus:

$$P(\text{at least one head}) = 1 - P(\text{no heads}) = 1 - \frac{1}{8} = \frac{7}{8}.$$

(ii) **Probability of exactly one tail:**
The favorable outcomes are $\{HHT, HTH, THH\}$. The number of favorable outcomes is 3. Thus:

$$P(\text{exactly one tail}) = \frac{3}{8}.$$

(iii) **Probability of two heads and one tail:**
The favorable outcomes are the same as in part (ii): $\{HHT, HTH, THH\}$. Thus:

$$P(\text{two heads and one tail}) = \frac{3}{8}.$$

Quick Tip

For coin toss probabilities, calculate the total outcomes as 2^n (where n is the number of coins) and count favorable outcomes to find probabilities.

---

31. An arc of a circle of radius 10 cm subtends a right angle at the center of the circle. Find the area of the corresponding major sector. (Use $\pi = 3.14$).

Solution: The total area of the circle is:

$$\text{Area of circle} = \pi r^2 = 3.14 \times 10 \times 10 = 314\,\text{cm}^2.$$

The area of the minor sector (subtending 90°) is:

$$\text{Area of minor sector} = \frac{\theta}{360} \times \pi r^2 = \frac{90}{360} \times 3.14 \times 10^2 = \frac{314}{4} = 78.5\,\text{cm}^2.$$

The area of the major sector is:

$$\text{Area of major sector} = \text{Total area of circle} - \text{Area of minor sector} = 314 - 78.5 = 235.5\,\text{cm}^2.$$

Thus, the area of the major sector is $235.5\,\text{cm}^2$.

Quick Tip

For sectors of a circle, use Area of sector $= \frac{\theta}{360} \times \pi r^2$, where θ is the angle subtended at the center.

Section D

32(a). Find the value of k for which the quadratic equation $(k+1)x^2 - 6(k+1)x + 3(k+9) = 0$, $k \neq -1$, has real and equal roots.

Solution: For real and equal roots, the discriminant D must be zero:

$$D = b^2 - 4ac = 0.$$

Substitute $a = (k+1)$, $b = -6(k+1)$, and $c = 3(k+9)$:

$$\big[-6(k+1)\big]^2 - 4(k+1) \cdot 3(k+9) = 0.$$

Simplify:

$$36(k+1)^2 - 12(k+1)(k+9) = 0.$$

Factorize:

$$(k+1)\big[36(k+1) - 12(k+9)\big] = 0.$$

Simplify further:

$$36(k+1) - 12(k+9) = 36k + 36 - 12k - 108 = 24k - 72.$$

Thus:

$$(k+1)(24k - 72) = 0.$$

Solve for k:

$$k + 1 = 0 \quad \text{or} \quad 24k - 72 = 0.$$

Since $k \neq -1$, solve $24k - 72 = 0$:

$$k = \frac{72}{24} = 3.$$

Thus, $k = 3$.

> **Quick Tip**
>
> For quadratic equations, use $D = b^2 - 4ac$ to check the nature of roots (real, equal, or imaginary).

32(b). The age of a man is twice the square of the age of his son. Eight years hence, the age of the man will be 4 years more than three times the age of his son. Find their present ages.

Solution: Let the present age of the son be x years. Then the present age of the man is $2x^2$ years.
Eight years hence, the age of the son and the man will be $(x+8)$ years and $(2x^2+8)$ years, respectively. According to the problem:

$$2x^2 + 8 = 3(x+8) + 4.$$

Simplify:

$$2x^2 + 8 = 3x + 24 + 4 \quad \Rightarrow \quad 2x^2 - 3x - 20 = 0.$$

Factorize the quadratic equation:

$$(2x+5)(x-4) = 0.$$

Thus:

$$x = -\frac{5}{2} \quad \text{(not possible as age cannot be negative)} \quad \text{or} \quad x = 4.$$

Therefore, the present age of the son is 4 years, and the present age of the man is:

$$2x^2 = 2(4)^2 = 32\,\text{years}.$$

Quick Tip

For age problems, create equations based on relationships and solve systematically, ensuring solutions are realistic.

---

33. From a point on a bridge across the river, the angles of depression of the banks on opposite sides of the river are 30° and 60° respectively. If the bridge is at a height of $4\,\text{m}$ from the banks, find the width of the river.

Solution: Let AB be the width of the river, and let P be the point on the bridge. The height of the bridge is $PQ = 4\,\text{m}$, and the width of the river is $AB = AQ + QB$.
In $\triangle PAQ$, using $\tan 30^\circ = \frac{\text{Opposite}}{\text{Adjacent}}$:

$$\tan 30^\circ = \frac{PQ}{AQ} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{4}{x}.$$

Solve for x:

$$x = 4\sqrt{3}.$$

In $\triangle PBQ$, using $\tan 60^\circ = \frac{\text{Opposite}}{\text{Adjacent}}$:

$$\tan 60^\circ = \frac{PQ}{BQ} \quad \Rightarrow \quad \sqrt{3} = \frac{4}{y}.$$

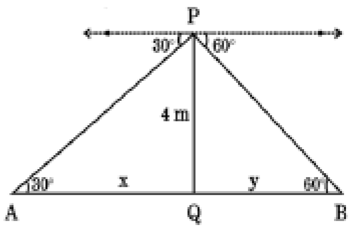
Solve for y:

$$y = \frac{4}{\sqrt{3}}.$$

The total width of the river is:

$$AB = AQ + QB = x + y = 4\sqrt{3} + \frac{4}{\sqrt{3}} = \frac{12}{3} + \frac{4}{\sqrt{3}} = \frac{16}{3}\sqrt{3}\,\text{m}.$$

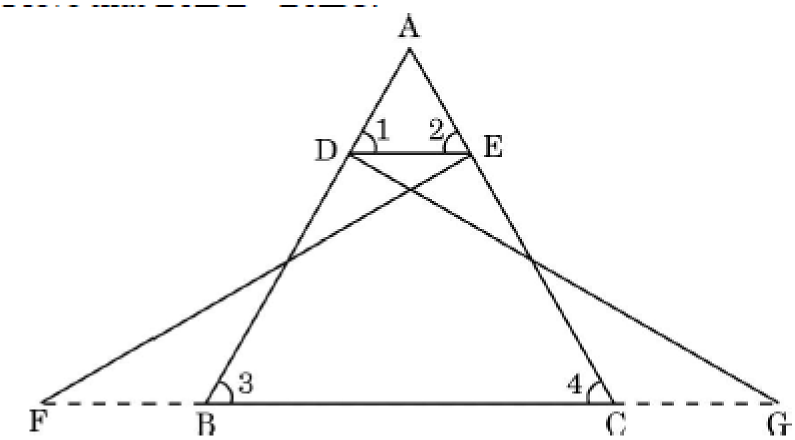
Thus, the width of the river is $\frac{16}{3}\sqrt{3}\,\text{m}$.



Quick Tip

For angle of depression problems, use trigonometric ratios like $\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$ to solve for distances.

34(a). In the given figure, $\triangle FEC \cong \triangle GDB$ and $\angle 1 = \angle 2$. Prove that $\triangle ADE \sim \triangle ABC$.



Solution: Given $\triangle FEC \cong \triangle GDB$, we have:

$$\angle 3 = \angle 4.$$

In $\triangle ABC$, since $\angle 3 = \angle 4$:

$$AB = AC \quad \text{(sides opposite equal angles in } \triangle ABC\text{)}. \quad \ldots(i)$$

In $\triangle ADE$, since $\angle 1 = \angle 2$:

$$AD = AE \quad \text{(sides opposite equal angles in } \triangle ADE\text{)}. \quad \ldots(ii)$$

Dividing equation (ii) by equation (i):

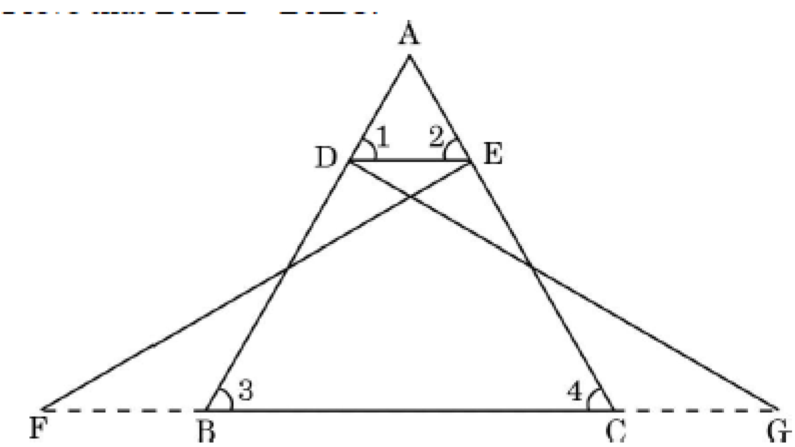
$$\frac{AD}{AB} = \frac{AE}{AC}.$$

Also, since $DE \parallel BC$:

$$\angle 1 = \angle 3 \quad \text{and} \quad \angle 2 = \angle 4 \quad \text{(corresponding angles)}.$$

Therefore, by the AA similarity criterion:

$$\triangle ADE \sim \triangle ABC.$$



💡 Quick Tip

To prove triangles similar, use the AA similarity criterion, which states that two triangles are similar if two corresponding angles are equal.

34(b). Sides AB and AC and median AD of $\triangle ABC$ are respectively proportional to sides PQ and PR and median PM of $\triangle PQR$. Show that $\triangle ABC \sim \triangle PQR$.

Solution: Produce AD to E such that $AD = DE$, and join EC. Similarly, produce PM to L such that $PM = ML$, and join LR.
In $\triangle ABD$ and $\triangle ECD$, we have:

$$AD = DE \quad \text{(construction)},$$

$$\angle ABD = \angle ECD \quad \text{(vertically opposite angles)},$$

$$AB = EC \quad \text{(proportionality given)}.$$

Thus:

$$\triangle ABD \cong \triangle ECD \quad \text{(by SAS criterion)}.$$

Similarly, in $\triangle PLR$:

$$PM = ML, \quad PQ = LR, \quad \angle P = \angle L.$$

Thus:

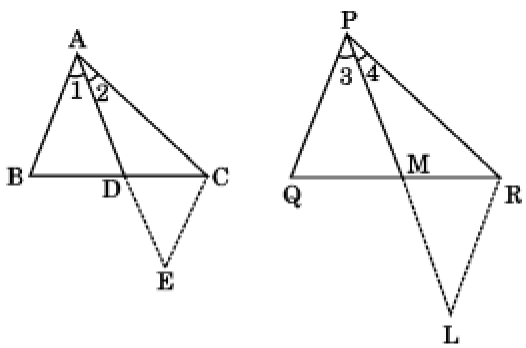
$$\triangle PMR \sim \triangle PLR \quad \text{(by SAS similarity)}.$$

From proportionality:

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}.$$

Using this proportionality, we conclude:

$$\triangle ABC \sim \triangle PQR \quad \text{(by SSS similarity criterion)}.$$



💡 Quick Tip

To prove triangles similar, check proportionality between corresponding sides and include equal angles when possible.

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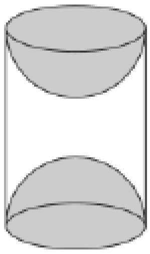
35. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in the figure. If the height of the cylinder is $5.8\,\text{cm}$ and its base is of radius $2.1\,\text{cm}$, find the total surface area of the article.



Solution:
The total surface area of the article is the sum of: 1. The curved surface area (CSA) of the cylinder. 2. The curved surface areas of the two hemispheres.

1. CSA of the cylinder:

$$\text{CSA of cylinder} = 2\pi rh.$$

Substitute $r = 2.1\,\text{cm}$ and $h = 5.8\,\text{cm}$:

$$\text{CSA of cylinder} = 2 \times \frac{22}{7} \times 2.1 \times 5.8 = 76.56\,\text{cm}^2.$$

2. CSA of two hemispheres: The CSA of one hemisphere is given by $2\pi r^2$. Therefore, the CSA of two hemispheres is:

$$\text{CSA of two hemispheres} = 4\pi r^2.$$

Substitute $r = 2.1\,\text{cm}$:

$$\text{CSA of two hemispheres} = 4 \times \frac{22}{7} \times 2.1 \times 2.1 = 55.44\,\text{cm}^2.$$

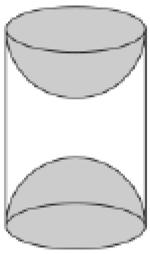
Total Surface Area:

$$\text{Total Surface Area} = \text{CSA of cylinder} + \text{CSA of two hemispheres}.$$

Substitute the values:

$$\text{Total Surface Area} = 76.56 + 55.44 = 132\,\text{cm}^2.$$

Thus, the total surface area of the article is $132\,\text{cm}^2$.



Quick Tip

For such composite shapes, calculate the curved surface areas of individual components and sum them up to find the total surface area.

Section E

36.Essel World is one of India's largest amusement parks that offers a diverse range of thrilling rides, water attractions, and entertainment options for visitors of all ages. The park is known for its iconic "Water Kingdom" section, making it a popular destination for family outings and fun-filled adventure. The ticket charges for the park are 150 per child and 250 per adult.
On a day, the cashier of the park found that 300 tickets were sold and an amount of 55,000 was collected.

Based on the above, answer the following questions:

(i) If the number of children visited be x and the number of adults visited be y, then write the given situation algebraically.

(ii) (a) How many children visited the amusement park that day?
OR
(b) How many adults visited the amusement park that day?

(iii) How much amount will be collected if 250 children and 100 adults visit the amusement park?

Solution:

(i) Given:

$$x + y = 300 \quad \ldots(1)$$

$$150x + 250y = 55000 \quad \ldots(2)$$

(ii) Solving equations (1) and (2): From equation (1):

$$y = 300 - x.$$

Substitute y into equation (2):

$$150x + 250(300 - x) = 55000,$$

$$150x + 75000 - 250x = 55000 \quad \Rightarrow \quad -100x + 75000 = 55000 \quad \Rightarrow \quad -100x = -20000$$

$\Rightarrow \quad x = 200$. Thus, the number of children is $x = 200$, and the number of adults is:

$$y = 300 - 200 = 100.$$

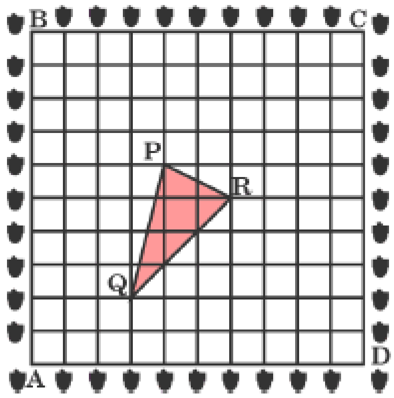
(iii) If 250 children and 100 adults visit:

$$\text{Amount collected} = 150(250) + 250(100) = 37500 + 25000 = 62500.$$

💡 Quick Tip

To solve word problems involving tickets and money, convert the given information into equations and solve systematically using substitution or elimination.

37.A garden is in the shape of a square. The gardener grew saplings of Ashoka trees on the boundary of the garden at a distance of 1 m from each other. He wants to decorate the garden with rose plants. He chose a triangular region inside the garden to grow rose plants. In the above situation, the gardener took help from the students of class 10. They made a chart for it which looks like the given figure.



Based on the above, answer the following questions:

(i) If A is taken as the origin, what are the coordinates of the vertices of $\triangle PQR$?

(ii) (a) Find distances PQ and QR.
OR
(b) Find the coordinates of the point which divides the line segment joining points P and R in the ratio 2 : 1 internally.

(iii) Find out if $\triangle PQR$ is an isosceles triangle.

Solution:

(i) The coordinates of P, Q, and R are:

$$P(4,6), \quad Q(3,2), \quad R(6,5).$$

(ii) (a) **Finding distances PQ and QR:** Using the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$:

$$PQ = \sqrt{(4-3)^2 + (6-2)^2} = \sqrt{1+16} = \sqrt{17}.$$

$$QR = \sqrt{(3-6)^2 + (2-5)^2} = \sqrt{9+9} = \sqrt{18}.$$

(b) **Finding the coordinates of the point dividing PR in the ratio $2:1$ internally:** Using the section formula:

$$x = \frac{mx_2 + nx_1}{m+n}, \quad y = \frac{my_2 + ny_1}{m+n}.$$

Substituting $P(4,6)$, $R(6,5)$, $m=2$, $n=1$:

$$x = \frac{2\cdot 6 + 1\cdot 4}{2+1} = \frac{12+4}{3} = \frac{16}{3},$$

$$y = \frac{2\cdot 5 + 1\cdot 6}{2+1} = \frac{10+6}{3} = \frac{16}{3}.$$

The point is $\left(\frac{16}{3}, \frac{16}{3}\right)$.

(iii) **Checking if $\triangle PQR$ is isosceles:** Find PR using the distance formula:

$$PR = \sqrt{(4-6)^2 + (6-5)^2} = \sqrt{4+1} = \sqrt{5}.$$

Comparing the sides:

$$PQ = \sqrt{17}, \quad QR = \sqrt{18}, \quad PR = \sqrt{5}.$$

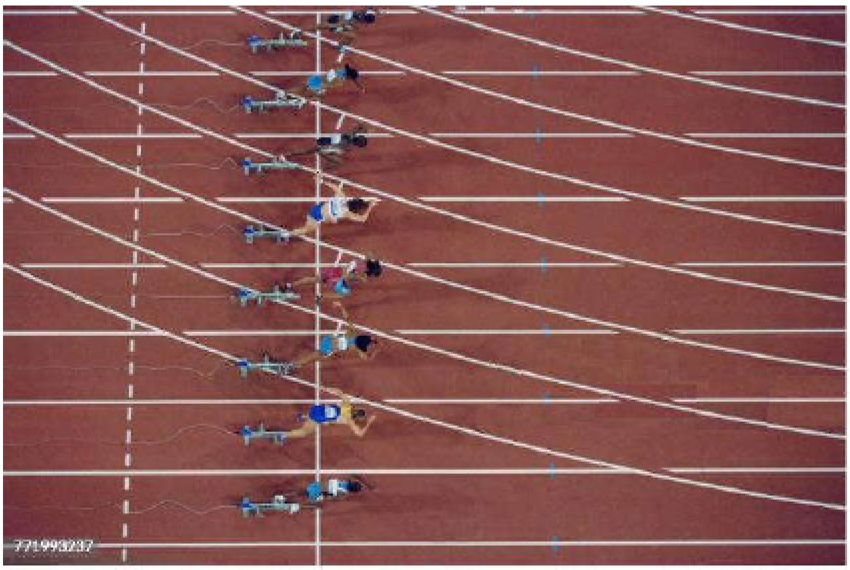
Since $PQ \neq QR \neq PR$, $\triangle PQR$ is not isosceles.

Quick Tip

For coordinate geometry problems, use the distance formula and section formula systematically to find lengths and points.

---

38. Activities like running or cycling reduce stress and the risk of mental disorders like depression. Running helps build endurance. Children develop stronger bones and muscles and are less prone to gain weight. The physical education teacher of a school has decided to conduct an inter-school running tournament in his school premises. The time taken by a group of students to run 100 m was noted as follows:



Time (in seconds) vs Number of Students:

Time (in seconds)	Number of Students
$0-20$	8
$20-40$	10
$40-60$	13
$60-80$	6
$80-100$	3

Based on the above, answer the following questions:

(i) What is the median class of the above-given data?

(ii) (a) Find the mean time taken by the students to finish the race.
OR

(b) Find the mode of the above-given data.

(iii) How many students took time less than 60 seconds?

Solution:

(i) **Finding the median class:**
The cumulative frequency table is:

Time (in seconds)	Number of Students (f)	Cumulative Frequency (cf)
$0-20$	8	8
$20-40$	10	18
$40-60$	13	31
$60-80$	6	37
$80-100$	3	40

The total frequency $N = 40$. The median class is the class containing the $N/2 = 20^{\text{th}}$ observation. Thus, the median class is $40 - 60$.

(ii) (a) **Finding the mean time:**
Using the formula:

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i}.$$

Compute $f_i x_i$ for each class:

Time (in seconds)	Midpoint (x_i)	f_i	$f_i x_i$
$0-20$	10	8	80
$20-40$	30	10	300
$40-60$	50	13	650
$60-80$	70	6	420
$80-100$	90	3	270
Total	–	40	1720

Substituting:

$$\text{Mean} = \frac{1720}{40} = 43\,\text{seconds}.$$

(b) **Finding the mode:**
Using the formula:

$$\text{Mode} = l + \left(\frac{f_m - f_1}{2f_m - f_1 - f_2}\right) \cdot h,$$

where:

$$l = 40,\ h = 20,\ f_m = 13,\ f_1 = 10,\ f_2 = 6.$$

Substituting:

$$\text{Mode} = 40 + \left(\frac{13 - 10}{2 \cdot 13 - 10 - 6}\right) \cdot 20 = 40 + \left(\frac{3}{10}\right) \cdot 20 = 40 + 6 = 46\,\text{seconds}.$$

(iii) **Finding students who took less than 60 seconds:**
From the cumulative frequency table, the number of students who took less than 60 seconds is:

$$\text{cf at 60 seconds} = 31.$$

Quick Tip

To find the median, cumulative frequency helps identify the median class. For the mean, use $\frac{\sum f_i x_i}{\sum f_i}$. For the mode, apply the mode formula based on the frequency distribution.
