CBSE Class X Mathematics (Standard) Set 3 (30/2/3)

Time Allowed :3 Hours	Maximum Marks :80	Total Questions :38

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This question paper contains 38 questions. All questions are compulsory.
2. This Question Paper is divided into **FIVE Sections – Section A, B, C, D, and E**.
3. In Section–A, questions number 1 to 18 are **Multiple Choice Questions (MCQs)** and questions number 19 & 20 are **Assertion-Reason based questions**, carrying **1 mark each**.
4. In Section–B, questions number 21 to 25 are **Very Short-Answer (VSA)** type questions, carrying **2 marks each**.
5. In Section–C, questions number 26 to 31 are **Short Answer (SA)** type questions, carrying **3 marks each**.
6. In Section–D, questions number 32 to 35 are **Long Answer (LA)** type questions, carrying **5 marks each**.
7. In Section–E, questions number 36 to 38 are **Case Study based questions** carrying **4 marks each**. *Internal choice is provided in each case-study.*
8. There is **no overall choice.** However, *an internal choice has been provided in 2 questions in Section–B, 2 questions in Section–C, 2 questions in Section–D, and 3 questions in Section–E.*
9. Draw neat diagrams wherever required. Take $\pi = \frac{22}{7}$ wherever required, if not stated.
10. Use of calculators is **not allowed.**

Section A

1. The distance between the points $(a\cos\theta, -a\sin\theta)$ and $(a\sin\theta, a\cos\theta)$ is:
(A) a
(B) $a\sqrt{2}$
(C) 0
(D) $2a$

Correct Answer: (B) $a\sqrt{2}$

Solution:
The formula for the distance between two points (x_1, y_1) and (x_2, y_2) is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Substitute the given points $(a\cos\theta, -a\sin\theta)$ and $(a\sin\theta, a\cos\theta)$:

$$d = \sqrt{(a\sin\theta - a\cos\theta)^2 + (a\cos\theta + a\sin\theta)^2}.$$

Simplify both terms:

$$(a\sin\theta - a\cos\theta)^2 = a^2(\sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta),$$
$$(a\cos\theta + a\sin\theta)^2 = a^2(\cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta).$$

Since $\sin^2\theta + \cos^2\theta = 1$:

$$d = \sqrt{a^2(1) + a^2(1)} = \sqrt{2a^2} = a\sqrt{2}.$$

Quick Tip

Use the distance formula and trigonometric identities like $\sin^2\theta + \cos^2\theta = 1$ to simplify expressions.

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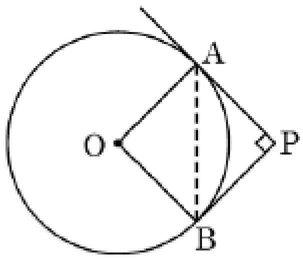
2. In the given figure, tangents PA and PB to the circle centered at O, from point P, are perpendicular to each other. If $PA = 5\,\text{cm}$, then the length of AB is equal to:



(A) $5\,\text{cm}$
(B) $5\sqrt{2}\,\text{cm}$
(C) $2\sqrt{5}\,\text{cm}$

(D) 10 cm

Correct Answer: (B) $5\sqrt{2}$ **cm**

Solution:
In the given figure: - PA and PB are tangents to the circle from an external point P, - $\triangle OAP$ and $\triangle OBP$ are right-angled triangles with OP as the hypotenuse.
By the Pythagoras theorem in $\triangle OAP$:

$$OP = \sqrt{OA^2 + PA^2},$$

where OA (radius) $= r$ and $PA = 5\,\text{cm}$.
Since PA and PB are perpendicular: - $\triangle APB$ is a right-angled isosceles triangle with $AP = BP = 5\,\text{cm}$. - The hypotenuse AB is given by:

$$AB = \sqrt{AP^2 + BP^2} = \sqrt{5^2 + 5^2} = \sqrt{25 + 25} = \sqrt{50} = 5\sqrt{2}\,\text{cm}.$$

Quick Tip
For right-angled isosceles triangles, the hypotenuse is given by side $\times \sqrt{2}$.

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3. Which term of the A.P. $-29, -26, -23, \ldots, 61$ **is 16?**
(A) 11^{th}
(B) 16^{th}
(C) 10^{th}
(D) 31^{st}

Correct Answer: (B) $16^{\textbf{th}}$

Solution:
1. The general term of an A.P. is given by:

$$a_n = a + (n-1)d,$$

where a is the first term, d is the common difference, and n is the term number.
2. Here, $a = -29$ and $d = -26 - (-29) = 3$.
3. Substitute $a_n = 16$:

$$16 = -29 + (n-1)(3).$$

4. Simplify:

$$16 + 29 = (n-1)(3) \implies 45 = 3(n-1).$$

5. Solve for n:

$$n - 1 = 15 \implies n = 16.$$

6. Therefore, the 16^{th} term of the A.P. is 16.

Quick Tip

To find the term number in an A.P., rearrange the formula for the nth term: $a_n = a + (n-1)d$.

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4. A box contains cards numbered 6 to 55. A card is drawn at random from the box. The probability that the drawn card has a number which is a perfect square, is:
(A) $\frac{7}{50}$
(B) $\frac{7}{55}$
(C) $\frac{1}{10}$
(D) $\frac{5}{49}$

Correct Answer: (C) $\frac{1}{10}$

Solution:
The cards are numbered from 6 to 55. Therefore, the total number of cards is:

$$55 - 6 + 1 = 50.$$

The perfect squares between 6 and 55 are:

$$9, 16, 25, 36, 49 \quad \text{(Perfect squares of } 3, 4, 5, 6, \text{ and } 7).$$

The total number of perfect squares is 5.
The probability of drawing a card with a perfect square is:

$$P = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{5}{50} = \frac{1}{10}.$$

Quick Tip

To find probabilities, count favorable outcomes and divide by total outcomes. List all perfect squares or multiples carefully.

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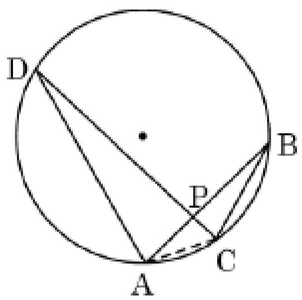
5. AB and CD are two chords of a circle intersecting at P. Choose the correct statement from the following:



(A) $\triangle ADP \sim \triangle CBA$
(B) $\triangle ADP \sim \triangle BPC$
(C) $\triangle ADP \sim \triangle BCP$
(D) $\triangle ADP \sim \triangle CBP$

Correct Answer: (D) $\triangle ADP \sim \triangle CBP$

Solution:
In the given figure: - Chords AB and CD intersect at P.
Using the property of intersecting chords, the corresponding angles are equal:

$$\angle ADP = \angle CBP \quad \text{(Vertically opposite angles)},$$

$$\angle APD = \angle BPC \quad \text{(Common angle)}.$$

By the AA (Angle-Angle) similarity criterion:

$$\triangle ADP \sim \triangle CBP.$$

Quick Tip

For intersecting chords, use vertical angles and common angles to prove similarity using the AA criterion.

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6. Two dice are rolled together. The probability of getting the sum of the two numbers to be more than 10, is:
(A) $\frac{1}{9}$
(B) $\frac{1}{6}$
(C) $\frac{7}{12}$
(D) $\frac{1}{12}$

Correct Answer: (D) $\frac{1}{12}$

Solution:
The total number of outcomes when two dice are rolled is:

$$6 \times 6 = 36.$$

The sums greater than 10 are 11 and 12: - For sum = 11: Possible pairs are $(5, 6), (6, 5)$ (2 outcomes), - For sum = 12: Possible pair is $(6, 6)$ (1 outcome).
Thus, the total favorable outcomes are:

$$2 + 1 = 3.$$

The probability of getting a sum greater than 10 is:

$$P = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{3}{36} = \frac{1}{12}.$$

Quick Tip

When rolling two dice, list all possible outcomes systematically to find sums or conditions.

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7. After an examination, a teacher wants to know the marks obtained by the maximum number of the students in her class. She requires to calculate ___ of marks.
(A) median
(B) mode
(C) mean
(D) range

Correct Answer: (B) mode

Solution:
- Mode is the value that appears most frequently in a data set. - Since the teacher wants to know the marks obtained by the maximum number of students, she must calculate the **mode**.

Quick Tip

Mode is the most frequently occurring value in a dataset. Median and mean provide different measures of central tendency.

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8. The roots of the quadratic equation $4x^2 - 5x + 4 = 0$ are:
(A) irrational
(B) rational and distinct
(C) not real
(D) rational and equal

Correct Answer: (C) not real

Solution:
The roots of a quadratic equation $ax^2 + bx + c = 0$ are determined using the discriminant:

$$D = b^2 - 4ac.$$

Substitute $a = 4$, $b = -5$, and $c = 4$:

$$D = (-5)^2 - 4(4)(4) = 25 - 64 = -39.$$

Since $D < 0$, the roots are not real.

Quick Tip

When $D > 0$, roots are real and distinct. If $D = 0$, roots are real and equal. When $D < 0$, roots are not real.

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9. The common difference of an A.P. in which $a_{20} - a_{15} = 20$, is:
(A) 4
(B) 5
(C) 4d
(D) 5d

Correct Answer: (A) 4

Solution:
In an A.P., the general term is given by:

$$a_n = a + (n-1)d,$$

where a is the first term, d is the common difference, and n is the term number.
1. The difference between a_{20} and a_{15} is:

$$a_{20} - a_{15} = [a + (20-1)d] - [a + (15-1)d].$$

2. Simplify:

$$a_{20} - a_{15} = (a + 19d) - (a + 14d) = 5d.$$

Given $a_{20} - a_{15} = 20$, we get:

$$5d = 20 \implies d = 4.$$

Quick Tip

The difference between two terms in an A.P. is proportional to the common difference: $a_m - a_n = (m-n)d$.

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10. If the value of each observation in a data is increased by 2, then the median of the new data:
(A) increases by 2
(B) increases by $2n$
(C) remains same
(D) decreases by 2

Correct Answer: (A) increases by 2

Solution:
- The **median** is the middle value of an ordered dataset. - If each observation in the dataset is increased by a constant value (e.g., 2), then the median will also increase by the same value.

Thus, the new median will increase by 2.

> **Quick Tip**
>
> When a constant is added to each observation, measures of central tendency (mean, median) increase by the same constant.

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11. The perimeters of two similar triangles ABC and PQR are 56 cm and 48 cm respectively. PQ/AB is equal to:
(A) $\frac{7}{8}$
(B) $\frac{6}{7}$
(C) $\frac{7}{6}$
(D) $\frac{8}{7}$

Correct Answer: (B) $\frac{6}{7}$

Solution:
For similar triangles, the ratio of corresponding sides is equal to the ratio of their perimeters.
Given: - Perimeter of $\triangle ABC = 56\,\text{cm}$, - Perimeter of $\triangle PQR = 48\,\text{cm}$.
The ratio of corresponding sides (e.g., PQ/AB) is:

$$\frac{PQ}{AB} = \frac{\text{Perimeter of PQR}}{\text{Perimeter of ABC}} = \frac{48}{56}.$$

Simplify:

$$\frac{48}{56} = \frac{6}{7}.$$

> **Quick Tip**
>
> For similar triangles, the ratio of perimeters is equal to the ratio of corresponding sides.

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12. If α and $\beta\,(\alpha > \beta)$ are the zeroes of the polynomial $-x^2 + 8x + 9$, then $(\alpha - \beta)$ is equal to:
(A) -10
(B) 10
(C) ± 10
(D) 8

Correct Answer: (B) 10

Solution:
The general form of a quadratic equation is:

$$ax^2 + bx + c = 0.$$

Here, $a = -1$, $b = 8$, and $c = 9$.
The difference between the roots α and β is given by:

$$\alpha - \beta = \frac{\sqrt{b^2 - 4ac}}{|a|}.$$

1. Calculate the discriminant:

$$b^2 - 4ac = 8^2 - 4(-1)(9) = 64 + 36 = 100.$$

2. Substitute into the formula:

$$\alpha - \beta = \frac{\sqrt{100}}{|-1|} = \frac{10}{1} = 10.$$

Quick Tip

Use the formula $\alpha - \beta = \frac{\sqrt{b^2-4ac}}{|a|}$ to find the difference between roots of a quadratic equation.

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13. The value of k for which the system of equations $3x - y + 8 = 0$ and $6x - ky + 16 = 0$ has infinitely many solutions, is:
(A) -2
(B) 2
(C) $\frac{1}{2}$
(D) $-\frac{1}{2}$

Correct Answer: (B) 2

Solution:
For two linear equations:

$$a_1x + b_1y + c_1 = 0 \quad \text{and} \quad a_2x + b_2y + c_2 = 0,$$

to have infinitely many solutions, the condition is:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}.$$

1. Compare the coefficients of the given equations:

$$3x - y + 8 = 0 \implies a_1 = 3,\ b_1 = -1,\ c_1 = 8,$$

$$6x - ky + 16 = 0 \implies a_2 = 6,\ b_2 = -k,\ c_2 = 16.$$

2. Use $\frac{a_1}{a_2} = \frac{b_1}{b_2}$:

$$\frac{3}{6} = \frac{-1}{-k}.$$

Simplify:

$$\frac{1}{2} = \frac{1}{k} \implies k = 2.$$

Quick Tip

For infinitely many solutions, check the condition: $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

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14. If $\sin\theta = \cos\theta\,(0^\circ < \theta < 90^\circ)$, then value of $(\sec\theta \cdot \sin\theta)$ is:
(A) $\frac{1}{\sqrt{2}}$
(B) $\sqrt{2}$
(C) 1
(D) 0

Correct Answer: (C) 1

Solution:
Given $\sin\theta = \cos\theta$, divide both sides by $\cos\theta$:

$$\tan\theta = 1.$$

The angle θ for which $\tan\theta = 1$ is 45°.
Now, calculate $\sec\theta \cdot \sin\theta$:

$$\sec\theta = \frac{1}{\cos\theta} \quad \text{and} \quad \sin\theta = \frac{1}{\sqrt{2}} \text{ at } \theta = 45^\circ.$$

$$\sec\theta \cdot \sin\theta = \frac{1}{\cos 45^\circ} \cdot \sin 45^\circ = \frac{1}{\frac{1}{\sqrt{2}}} \cdot \frac{1}{\sqrt{2}} = 1.$$

Quick Tip

When $\sin\theta = \cos\theta$, the angle θ is 45°, and trigonometric values simplify accordingly.

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15. Point P divides the line segment joining the points $A(4,-5)$ and $B(1,2)$ in the ratio $5:2$. Coordinates of point P are:
(A) $\left(\frac{5}{2}, -\frac{3}{2}\right)$
(B) $\left(\frac{11}{7}, 0\right)$
(C) $\left(\frac{13}{7}, 0\right)$
(D) $\left(0, \frac{13}{7}\right)$

Correct Answer: (C) $\left(\frac{13}{7}, 0\right)$

Solution:
To find the coordinates of P, use the section formula:

$$P = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right),$$

where $m_1 : m_2 = 5 : 2$, $A(4, -5)$, and $B(1, 2)$.
1. Calculate the x-coordinate:

$$x = \frac{(5)(1) + (2)(4)}{5+2} = \frac{5+8}{7} = \frac{13}{7}.$$

2. Calculate the y-coordinate:

$$y = \frac{(5)(2) + (2)(-5)}{5+2} = \frac{10-10}{7} = 0.$$

Thus, the coordinates of point P are:

$$P = \left(\frac{13}{7}, 0\right).$$

Quick Tip

Use the section formula $\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$ to find a point dividing a line segment in a given ratio.

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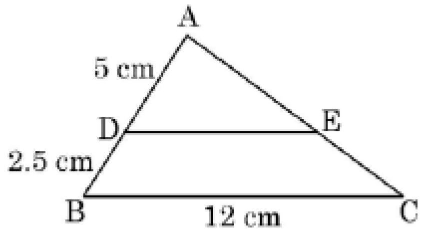
16. In the given figure $\triangle ABC$ is shown. DE is parallel to BC. If $AD = 5\,$cm, $DB = 2.5\,$cm, and $BC = 12\,$cm, then DE is equal to:



(A) $10\,$cm
(B) $6\,$cm
(C) $8\,$cm
(D) $7.5\,$cm

Correct Answer: (C) $8\,$**cm**

Solution:
Using the property of similar triangles, the ratio of corresponding sides is equal.
Given: - $AD : DB = 5 : 2.5 = 2 : 1$.
Thus, $DE : BC = AD : AB$:

$$\frac{DE}{BC} = \frac{AD}{AB}.$$

1. Find AB:

$$AB = AD + DB = 5 + 2.5 = 7.5\,\text{cm}.$$

2. Use the ratio to calculate DE:

$$\frac{DE}{12} = \frac{5}{7.5}.$$

Simplify:

$$DE = 12 \times \frac{2}{3} = 8\,\text{cm}.$$

Quick Tip

In similar triangles, the ratio of corresponding sides is proportional. Use this property to find missing side lengths.

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17. If the HCF of $(2520, 6600) = 40$ and LCM $(2520, 6600) = 252 \times k$, then the value of k is:

(A) 1650
(B) 1600
(C) 165
(D) 1625

Correct Answer: (A) 1650

Solution:
The relationship between HCF and LCM of two numbers is:

$$\text{HCF} \times \text{LCM} = \text{Product of the two numbers}.$$

Given: - $\text{HCF}(2520, 6600) = 40$, - $\text{LCM}(2520, 6600) = 252 \times k$.
1. Calculate the product of the two numbers:

$$2520 \times 6600.$$

2. Substitute into the equation:

$$40 \times (252 \times k) = 2520 \times 6600.$$

Simplify:

$$252 \times k = \frac{2520 \times 6600}{40}.$$

3. Simplify further:

$$252 \times k = 415800.$$

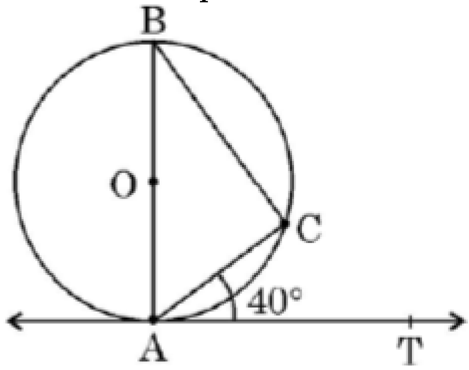
$$k = \frac{415800}{252} = 1650.$$

Thus, the value of k is 1650.

Quick Tip

The formula HCF $\times$ LCM $=$ Product of two numbers is key to solving HCF-LCM problems.

18. In the given figure, AT is tangent to a circle centered at O. If $\angle CAT = 40°$, then $\angle CBA$ is equal to:



(A) $70°$
(B) $50°$
(C) $65°$
(D) $40°$

Correct Answer: (D) $40°$

Solution:
When a tangent meets a circle, the angle between the tangent and the chord drawn from the point of contact is equal to the angle subtended by the chord in the alternate segment (Alternate Segment Theorem).
Given:

$$\angle CAT = 40°.$$

By the alternate segment theorem:

$$\angle CBA = \angle CAT = 40°.$$

Quick Tip

The angle between a tangent and a chord equals the angle subtended by the chord in the alternate segment (Alternate Segment Theorem).

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19. Assertion (A): If $\sin A = \frac{1}{3}$ $(0° < A < 90°)$, then the value of $\cos A$ is $\frac{2\sqrt{2}}{3}$.
Reason (R): For every angle θ, $\sin^2\theta + \cos^2\theta = 1$.
Options:
(A) Both Assertion (A) and Reason (R) are true. Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true. Reason (R) does not give correct explanation of Assertion (A).
(C) Assertion (A) is true but Reason (R) is not true.
(D) Assertion (A) is not true but Reason (R) is true.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true. Reason (R) is the correct explanation of Assertion (A).

Solution:
1. Given $\sin A = \frac{1}{3}$, use the identity:

$$\sin^2 A + \cos^2 A = 1.$$

2. Substitute $\sin A$:

$$\left(\frac{1}{3}\right)^2 + \cos^2 A = 1.$$

$$\frac{1}{9} + \cos^2 A = 1.$$

Simplify:

$$\cos^2 A = 1 - \frac{1}{9} = \frac{8}{9}.$$

$$\cos A = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}.$$

Both Assertion (A) and Reason (R) are true, and Reason (R) explains the Assertion correctly.

> Quick Tip
>
> To find $\cos A$ when $\sin A$ is given, use the Pythagorean identity: $\sin^2 A + \cos^2 A = 1$.

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20. Two cubes each of edge length 10 cm are joined together. The total surface area of the newly formed cuboid is $1200\,\text{cm}^2$.
Reason (R): Area of each surface of a cube of side 10 cm is $100\,\text{cm}^2$.
Options:
(A) Both Assertion (A) and Reason (R) are true. Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true. Reason (R) does not give correct explanation of Assertion (A).
(C) Assertion (A) is true but Reason (R) is not true.
(D) Assertion (A) is not true but Reason (R) is true.

Correct Answer: (D) Assertion (A) is not true but Reason (R) is true.

Solution:
1. The surface area of one cube of edge length 10 cm is:

$$6 \times \text{side}^2 = 6 \times 10^2 = 600\,\text{cm}^2.$$

2. When two cubes are joined to form a cuboid: - Total surface area is calculated considering the shared face:

$$\text{New surface area} = 2 \times 600 - 2 \times 100 = 1000\,\text{cm}^2 \quad (\text{not } 1200).$$

Thus, Assertion (A) is false.
3. However, Reason (R) is correct: - Each surface of a cube with side 10 cm has an area of:

$$\text{side}^2 = 100\,\text{cm}^2.$$

Quick Tip

When two solids are joined, subtract the shared surface area to calculate the new total surface area.

Section B

21(a). In what ratio is the line segment joining the points $(3, -5)$ and $(-1, 6)$ divided by the line $y = x$?

Solution:
To find the required ratio $k : 1$, assume the line divides the segment at point P with coordinates:

$$P = \left(\frac{kx_2 + x_1}{k+1}, \frac{ky_2 + y_1}{k+1} \right).$$

Given: - $A(3, -5)$ and $B(-1, 6)$, - The line $y = x$.
1. Equating the x- and y-coordinates at point P:

$$\frac{k(-1) + 3}{k+1} = \frac{k(6) + (-5)}{k+1}.$$

2. Simplify:

$$\frac{-k + 3}{k+1} = \frac{6k - 5}{k+1}.$$

Eliminate the denominator:

$$-k + 3 = 6k - 5.$$

Simplify for k:

$$3 + 5 = 6k + k \implies 8 = 7k \implies k = \frac{8}{7}.$$

Thus, the required ratio is:

$$8 : 7.$$

Quick Tip

To find the ratio in which a line divides a segment, equate the coordinates using the section formula and simplify.

21(b). $A(0, 6)$, $B(4, 4)$, and $C(1, 1)$ are vertices of a triangle ABC. Find the length of its median BE.

Solution:
To find the length of median BE, first determine the midpoint E of side AC.

1. The midpoint formula is:

$$E = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Substitute $A(0, 6)$ and $C(1, 1)$:

$$E = \left(\frac{0+1}{2}, \frac{6+1}{2}\right) = \left(\frac{1}{2}, \frac{7}{2}\right).$$

2. Now, calculate the length of BE using the distance formula:

$$BE = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Substitute $B(4, 4)$ and $E\left(\frac{1}{2}, \frac{7}{2}\right)$:

$$BE = \sqrt{\left(4 - \frac{1}{2}\right)^2 + \left(4 - \frac{7}{2}\right)^2}.$$

Simplify:

$$BE = \sqrt{\left(\frac{8-1}{2}\right)^2 + \left(\frac{8-7}{2}\right)^2} = \sqrt{\left(\frac{7}{2}\right)^2 + \left(\frac{1}{2}\right)^2}.$$

$$BE = \sqrt{\frac{49}{4} + \frac{1}{4}} = \sqrt{\frac{50}{4}} = \frac{\sqrt{50}}{2} = \frac{5\sqrt{2}}{2}.$$

Quick Tip

The length of a median can be found by calculating the midpoint of the opposite side and applying the distance formula.

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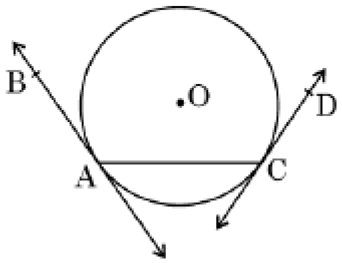
22. In the given figure, AB and CD are tangents to a circle centered at O. Is $\angle BAC = \angle DCA$? Justify your answer.



Solution:
To prove $\angle BAC = \angle DCA$, use the following steps:
1. Join OA and OC:

$$OA = OC \quad \text{(Radii of the same circle)}.$$

2. Observe that $\triangle OAC$ is isosceles:

$$\angle OAC = \angle OCA.$$

3. Tangents drawn from an external point to a circle are equal. Therefore:

$$AB = AC \quad \text{and} \quad CD = CA.$$

4. Since $\angle BAC$ and $\angle DCA$ are subtended by equal tangents and involve the same chord, they are equal:

$$\angle BAC = \angle DCA.$$

> 💡 Quick Tip
>
> For tangents to a circle, angles subtended by the tangents from the same external point are equal.

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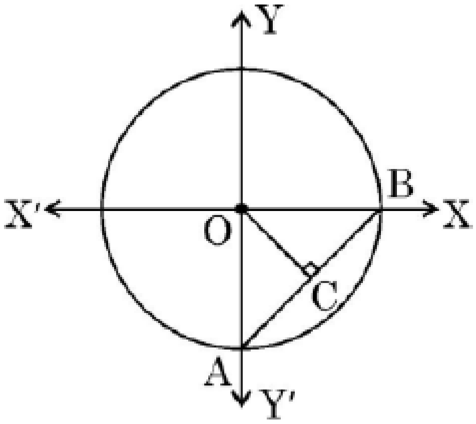
23. In the given figure, a circle centered at origin O has radius 7 cm. OC is a median of $\triangle OAB$. Find the length of median OC.



Solution:
Given: - Radius of the circle $OA = OB = 7\,\text{cm}$, - OC is a median of $\triangle OAB$.
$\angle AOB = 90^\circ$

$$\text{AB}^2 = 7^2 + 7^2 \implies AB = 7\sqrt{2}\,\text{cm}$$

$$\text{AC} = \frac{7\sqrt{2}}{2}\,\text{cm}$$

Now in $\triangle AOC$:

$$\text{OC}^2 = 7^2 - \left(\frac{7\sqrt{2}}{2}\right)^2$$

Simplify:

$$\text{OC} = \frac{7\sqrt{2}}{2}\,\text{cm}$$

Quick Tip

In an equilateral triangle, the length of the median is given by $\frac{\sqrt{3}}{2} \times$ side.

24(a). Evaluate: $2\sin^2 30^\circ \sec^2 60^\circ + \tan^2 60^\circ$.

Solution:
Substitute the trigonometric values: - $\sin 30^\circ = \frac{1}{2}$, - $\sec 60^\circ = 2$, - $\tan 60^\circ = \sqrt{3}$.
1. Evaluate each term:

$$2\sin^2 30^\circ = 2\left(\frac{1}{2}\right)^2 = 2 \times \frac{1}{4} = \frac{1}{2}.$$

$$\sec^2 60^\circ = 2^2 = 4.$$

$$\tan^2 60^\circ = (\sqrt{3})^2 = 3.$$

2. Combine:

$$2\sin^2 30^\circ \sec^2 60^\circ + \tan^2 60^\circ = \frac{1}{2} \times 4 + 3 = 2 + 3 = 4.$$

Quick Tip

Substitute known trigonometric values and simplify step-by-step to avoid errors.

24(b). If $2\sin A + \sqrt{3} = \cos A$ and $\cos(A - B) = 1$, find the measures of angles A and B.

Solution:
1. Given:

$$2\sin A + \sqrt{3} = \cos A.$$

Divide through by $\cos A$:

$$\frac{2\sin A}{\cos A} + \frac{\sqrt{3}}{\cos A} = 1.$$

Simplify:

$$2\tan A + \sqrt{3} = 1 \quad \Longrightarrow \tan A = \frac{1 - \sqrt{3}}{2}.$$

2. Given $\cos(A - B) = 1$, we know:

$$A - B = 0^\circ \quad \Longrightarrow A = B = 30^\circ.$$

Thus, $A = B = 30^\circ$.

Quick Tip

When $\cos(A - B) = 1$, the angles A and B are equal. Solve for A using trigonometric identities.

25. Can the number 8^n, n being a natural number, end with the digit 0? Give reasons.

Solution:
For a number to end in the digit 0, it must be divisible by 10. This means it must have 2 and 5 as its prime factors.
1. The number 8^n can be expressed as:

$$8^n = (2^3)^n = 2^{3n}.$$

2. Since 8^n only contains the prime factor 2 and no factor of 5, it cannot be divisible by 10. Thus, 8^n cannot end with the digit 0.

Quick Tip

A number ends in 0 if and only if it is divisible by both 2 and 5 (i.e., it contains 10 as a factor).

Section C

26.Prove that

$$\frac{\csc^2\theta - \sec^2\theta}{\csc^2\theta + \sec^2\theta} = \frac{3}{4}, \text{ if } \tan\theta = \frac{1}{\sqrt{7}}.$$

Solution:

$$\tan\theta = \frac{1}{\sqrt{7}} \implies \sec^2\theta = \frac{8}{7} \text{ and } \csc^2\theta = 8.$$

Substitute into the left-hand side (LHS):

$$\text{LHS} = \frac{\csc^2\theta - \sec^2\theta}{\csc^2\theta + \sec^2\theta} = \frac{8-\frac{8}{7}}{8+\frac{8}{7}}.$$

Simplify the numerator and denominator:

$$\text{Numerator} = 8 - \frac{8}{7} = \frac{56}{7} - \frac{8}{7} = \frac{48}{7},$$

$$\text{Denominator} = 8 + \frac{8}{7} = \frac{56}{7} + \frac{8}{7} = \frac{64}{7}.$$

Thus:

$$\text{LHS} = \frac{\frac{48}{7}}{\frac{64}{7}} = \frac{48}{64} = \frac{3}{4}.$$

Since LHS = RHS, the equation is verified.

Quick Tip

Use trigonometric identities like $\sec^2\theta = 1+\tan^2\theta$ and $\cos^2\theta = \frac{1}{\sec^2\theta}$ for simplifications.

27(a). If the sum of the first m terms of an A.P. is the same as the sum of its first n terms $(m \neq n)$, then show that the sum of the first $(m+n)$ terms is zero.

Solution:

$$S_m = S_n$$
$$\implies \frac{m}{2}\left[2a + (m-1)d\right] = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$\implies 2a(m-n) = d\left(n^2 - m^2\right) - d(n-m)$$
$$\implies 2a = -d(m+n-1)$$

or

$$2a + (m+n-1)d = 0$$

Thus,

$$S_{m+n} = \frac{m+n}{2}\left[2a + (m+n-1)d\right] = 0.$$

Quick Tip

Use the formula for the sum of an A.P. and simplify step-by-step to prove equal sums.

27(b). In an A.P., the sum of three consecutive terms is 24 and the sum of their squares is 194. Find the numbers.

Solution:
Let the three consecutive terms of the A.P. be:

$$a-d,\ a,\ a+d.$$

1. Given the sum of the terms:

$$(a-d) + a + (a+d) = 24.$$

Simplify:

$$3a = 24 \implies a = 8.$$

2. Given the sum of their squares:

$$(a-d)^2 + a^2 + (a+d)^2 = 194.$$

Substitute $a = 8$:

$$(8-d)^2 + 8^2 + (8+d)^2 = 194.$$

Simplify:

$$(64 - 16d + d^2) + 64 + (64 + 16d + d^2) = 194.$$

Combine terms:

$$2d^2 + 192 = 194.$$

Solve for d^2:

$$2d^2 = 2 \implies d^2 = 1 \implies d = \pm 1.$$

3. The three terms are: - For $d = 1$: $7, 8, 9$, - For $d = -1$: $9, 8, 7$.

Quick Tip

In an A.P., use algebraic expressions for consecutive terms and solve using given conditions.

---

28. Prove that $\sqrt{5}$ is an irrational number.

Solution:
Assume $\sqrt{5}$ is rational. Then it can be expressed as:

$$\sqrt{5} = \frac{p}{q},$$

where p and q are co-prime integers, and $q \neq 0$.
1. Square both sides:

$$5 = \frac{p^2}{q^2} \implies p^2 = 5q^2.$$

2. This implies p^2 is divisible by 5, so p must also be divisible by 5. Let:

$$p = 5k \text{ for some integer } k.$$

3. Substitute into the equation:

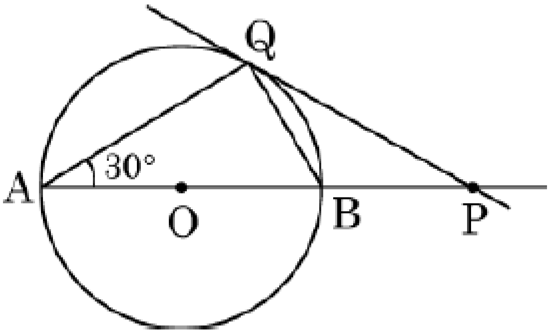
$$(5k)^2 = 5q^2 \implies 25k^2 = 5q^2 \implies q^2 = 5k^2.$$

This shows q^2 is divisible by 5, so q must also be divisible by 5.
4. Thus, p and q have a common factor of 5, contradicting the assumption that they are co-prime.
Therefore, $\sqrt{5}$ is irrational.

Quick Tip

To prove a square root is irrational, assume it is rational and show a contradiction using divisibility.

29(a). In the given figure, PQ is tangent to a circle centered at O and $\angle BAQ = 30^\circ$. Show that $BP = BQ$.



Solution:
To prove $BP = BQ$, follow these steps:
1. Join OQ and OA.
2. Since PQ is a tangent at Q, the radius OQ is perpendicular to PQ:

$$\angle OQP = 90^\circ.$$

3. Given $\angle BAQ = 30^\circ$, note that $\triangle OQA$ is isosceles because $OA = OQ$ (radii of the circle):

$$\angle OAQ = \angle OQA = 30^\circ.$$

4. In $\triangle OQB$, the following angles hold true:

$$\angle OQB = 60^\circ \quad \text{(sum of angles in a triangle)}.$$

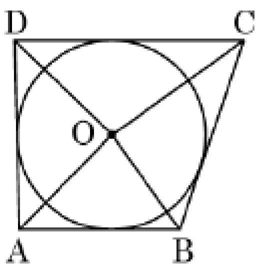
5. Since OQ bisects $\angle BQA$ and tangents drawn from an external point are equal, we conclude:

$$BP = BQ.$$

Quick Tip

Tangents drawn from the same external point to a circle are always equal in length.

29(b). In the given figure, AB, BC, CD, and DA are tangents to the circle with center O, forming a quadrilateral $ABCD$. Show that $\angle AOB + \angle COD = 180^\circ$.



Solution:
1. Join the center O to the vertices of the quadrilateral:

$$\text{Join } OP, OQ, OR, \text{ and } OS.$$

2. Note that each angle at the center subtended by the tangents forms a pair of supplementary angles. For example:

$$\angle AOB + \angle COD = 180^\circ.$$

3. Use the property of a cyclic quadrilateral, where the sum of opposite angles is 180°:

$$\angle AOB = \angle 1 + \angle 2 \quad \text{and} \quad \angle COD = \angle 3 + \angle 4.$$

4. Combine the angles:

$$\angle AOB + \angle COD = (\angle 1 + \angle 2) + (\angle 3 + \angle 4) = 360^\circ - \text{sum of other angles}.$$

5. By symmetry and the properties of tangents, the sum simplifies to:

$$\angle AOB + \angle COD = 180^\circ.$$

Quick Tip

For tangents forming a cyclic quadrilateral, the sum of angles subtended at the center is always 180°.

---

30. A dealer sells an article for Rs.75 and gains as much percent as the cost price of the article. Find the cost price of the article.

Solution:
Let the cost price of the article be Rs.x.
1. The gain percentage is given as equal to the cost price. Therefore:

$$\text{Gain} = x \,\text{and}\, \text{Selling Price} = \text{Cost Price} + \text{Gain}.$$

2. Substituting values:

$$75 = x + x\left(\frac{x}{100}\right).$$

Simplify the equation:

$$75 = x + \frac{x^2}{100}.$$

Multiply through by 100 to eliminate the denominator:

$$7500 = 100x + x^2.$$

Rearrange:

$$x^2 + 100x - 7500 = 0.$$

3. Solve the quadratic equation using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

where $a = 1$, $b = 100$, $c = -7500$.

$$x = \frac{-100 \pm \sqrt{100^2 - 4(1)(-7500)}}{2(1)}.$$

Simplify:

$$x = \frac{-100 \pm \sqrt{10000 + 30000}}{2}.$$

$$x = \frac{-100 \pm \sqrt{40000}}{2}.$$

$$x = \frac{-100 \pm 200}{2}.$$

4. Solve for x:

$$x = \frac{100}{2} = 50 \quad \text{(Ignoring the negative value)}.$$

Thus, the cost price of the article is Rs.50.

Quick Tip

When the gain percentage equals the cost price, use the relationship:
Selling Price = Cost Price + Gain, and form a quadratic equation.

---

31. In a test, the marks obtained by 100 students (out of 50) are given below:

Marks obtained	Number of students
0 – 10	12
10 – 20	23
20 – 30	34
30 – 40	25
40 – 50	6

Find the mean marks of the students.

Solution:
1. To find the mean, use the formula:

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i},$$

where x_i is the mid-point of each class, and f_i is the frequency.

Marks Obtained	Number of students (f_i)	x_i	$f_i x_i$
0 – 10	12	5	60
10 – 20	23	15	345
20 – 30	34	25	850
30 – 40	25	35	875
40 – 50	6	45	270
Total	100		2400

2. Calculate the mean:

$$\text{Mean} = \frac{2400}{100} = 24.$$

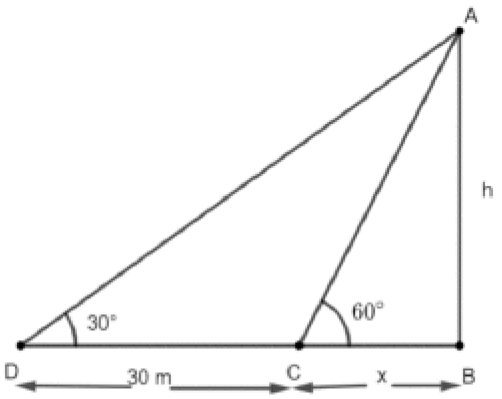
💡 Quick Tip

To find the mean in grouped data, use the mid-points of intervals as x_i and multiply by their corresponding frequencies.

Section D

32. A person standing on the bank of a river observes that the angle of elevation of the top of a tower on the opposite bank is $60°$. When he moves $10\,\text{m}$ away from the bank, the angle of elevation reduces to $30°$. Find the height of the tower and width of the river.

Solution:



Let the height of the tower BA be $h\,\text{m}$ and the width of the river BC be $x\,\text{m}$.

$$\tan 60° = \sqrt{3} = \frac{h}{x}$$

$$\implies h = \sqrt{3}x \quad \text{— (i)}$$

$$\tan 30° = \frac{1}{\sqrt{3}} = \frac{h}{30 + x}$$

$$\implies h\sqrt{3} = 30 + x \quad \text{— (ii)}$$

Substitute $h = \sqrt{3}x$ from (i) into (ii):

$$\sqrt{3}(\sqrt{3}x) = 30 + x$$

$$\implies 3x = 30 + x$$

$$\implies 2x = 30 \qquad \implies x = 15$$

Using (i):

$$h = \sqrt{3}x = 15 \times 1.732 = 25.98\,\text{m}$$

Thus, the height of the tower is 25.98 m and the width of the river is 15 m.

> 💡 Quick Tip
>
> For angle of elevation problems, use $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ and solve step-by-step for the unknowns.

---

33. The perimeter of a certain sector of a circle of radius 8.4 m is 30.8 m. Find the area of the sector.

Solution:
The perimeter of the sector is given by:

$$2r + \frac{2\pi r\theta}{360} = 20$$

Substitute $r = 5.6\,\text{m}$:

$$2(5.6) + \frac{2 \times \frac{22}{7} \times 5.6 \times \theta}{360} = 20$$

$$\implies 11.2 + \frac{22}{7} \times 5.6 \times \frac{\theta}{360} = 20$$

Solve for θ:

$$\frac{22}{7} \times 5.6 \times \frac{\theta}{360} = 20 - 11.2$$

$$\implies \frac{22}{7} \times 5.6 \times \frac{\theta}{360} = 8.8$$

$$\implies \theta = 90^\circ$$

The area of the sector is given by:

$$\text{Area} = \frac{\pi r^2\theta}{360}$$

$$\implies \text{Area} = \frac{22}{7} \times 5.6 \times 5.6 \times \frac{90}{360}$$

$$\implies \text{Area} = 24.64\,\text{m}^2$$

Thus, the area of the sector is $24.64\,\text{m}^2$.

> 💡 Quick Tip
>
> For sector problems, use the formulas:
>
> $$\text{Perimeter} = 2r + l \quad \text{and} \quad \text{Area} = \frac{1}{2} \times r \times l.$$

34(a). If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, prove that the other two sides are divided in the same ratio.

Solution:

To prove the Basic Proportionality Theorem (Thales' theorem):
1. Consider $\triangle ABC$, where $DE \parallel BC$, and D and E are points on AB and AC, respectively.
2. By construction, draw perpendiculars from B and C to line DE: - Let the perpendicular distances be h_1 and h_2.
3. Triangles $\triangle ADE$ and $\triangle ABC$ are similar because: - Corresponding angles are equal ($\angle ADE = \angle ABC$, $\angle AED = \angle ACB$).
4. From similarity, the sides are proportional:

$$\frac{AD}{DB} = \frac{AE}{EC}.$$

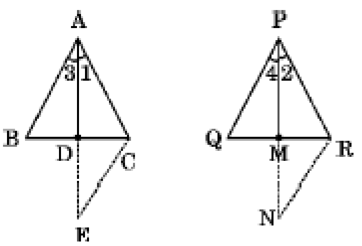
Thus, the theorem is proved: A line parallel to one side of a triangle divides the other two sides in the same ratio.

Quick Tip

In similar triangles, corresponding sides are proportional. Use this property to prove the Basic Proportionality Theorem.

34(b). Sides AB and AC and median AD to $\triangle ABC$ are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\triangle ABC \sim \triangle PQR$.

Solution:



To prove $\triangle ABC \sim \triangle PQR$, follow these steps:
1. Extend median AD and PM: - Produce AD to E such that $AD = DE$ and join EC. - Produce PM to N such that $PM = MN$ and join NR.
2. Show triangles are congruent: - In $\triangle ABD$ and $\triangle EMC$: - $AD = DE$ (construction), - $AB = EC$ (given proportional sides), - $\angle BAD = \angle EDC$ (corresponding angles in congruent parts).
Thus:

$$\triangle ABD \cong \triangle EMC.$$

3. Prove proportionality in sides: - Similarly, $PQ = NR$ and $PM = MN$ (construction). - Since $AB : PQ = AC : PR$, we have:

$$\triangle ABC \sim \triangle PQR \quad \text{(by SSS similarity criterion)}.$$

4. Angles are equal: - From the similarity condition:

$$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R.$$

Thus, the triangles $\triangle ABC$ and $\triangle PQR$ are similar.

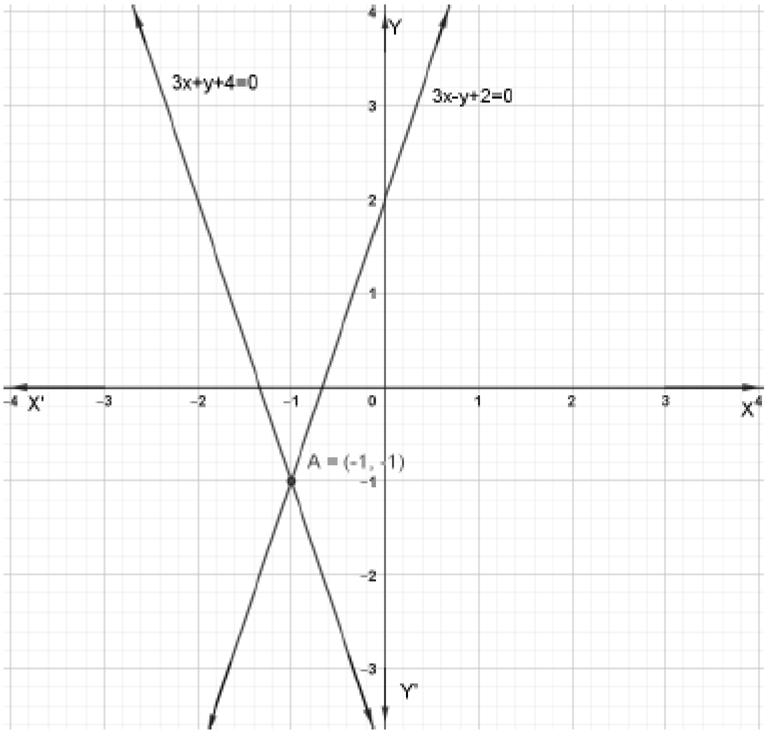
Quick Tip

To prove two triangles are similar, check proportionality of corresponding sides and equality of included angles (SSS or SAS similarity criteria).

35(a). Using graphical method, solve the following system of equations:

$$3x + y + 4 = 0 \quad \text{and} \quad 3x - y + 2 = 0.$$

Solution:



1. Rewrite both equations in slope-intercept form ($y = mx + c$):

$$3x + y + 4 = 0 \implies y = -3x - 4, \quad 3x - y + 2 = 0 \implies y = 3x + 2.$$

2. Plot the two lines on the graph: - The first line ($y = -3x - 4$) has a slope of -3 and a y-intercept of -4. - The second line ($y = 3x + 2$) has a slope of 3 and a y-intercept of 2.
3. Intersection of the two lines: From the graph, the lines intersect at $x = -1$ and $y = -1$. Thus, the solution to the system of equations is:

$$x = -1, \, y = -1.$$

Quick Tip

For solving equations graphically, convert them to slope-intercept form ($y = mx + c$) and plot the lines to find their intersection point.

35(b). Tara scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks deducted for each wrong answer, Tara would have scored 50 marks. Assuming that Tara attempted all questions, find the total number of questions in the test.

Solution:
Let the number of correct answers be x and the number of incorrect answers be y.
1. From the given conditions:

$$3x - 1y = 40 \quad \text{and} \quad 4x - 2y = 50.$$

2. Simplify the second equation:

$$4x - 2y = 50 \implies 2x - y = 25. \tag{1}$$

3. Solve equations $3x - y = 40$ and $2x - y = 25$: Subtract equation (1) from the first equation:

$$(3x - y) - (2x - y) = 40 - 25.$$

Simplify:

$$x = 15.$$

Substitute $x = 15$ into $2x - y = 25$:

$$2(15) - y = 25 \implies 30 - y = 25 \implies y = 5.$$

4. Total number of questions:

$$x + y = 15 + 5 = 20.$$

Thus, the total number of questions is 20.

Quick Tip

Translate word problems into equations and solve them systematically using elimination or substitution methods.

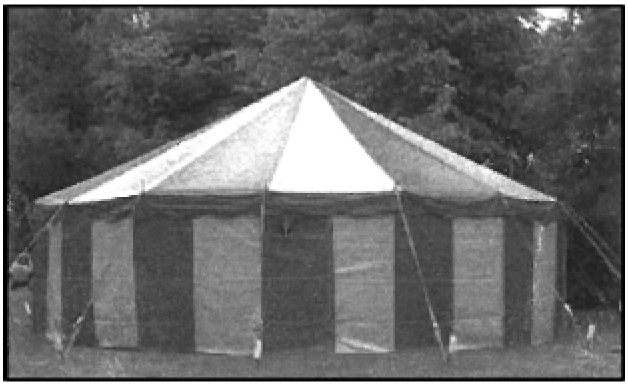
Section E

36. The word 'circus' has the same root as 'circle.' In a closed circular area, various entertainment acts including human skill and animal training are presented before the crowd. A circus tent is cylindrical up to a height of $8\,\text{m}$ and conical above it. The diameter of the base is $28\,\text{m}$ and the total height of the tent is $18.5\,\text{m}$.
Based on the above, answer the following questions:

1. Find the slant height of the conical part.

2. Determine the floor area of the tent.

3. (a) Find the area of the cloth used for making the tent.
   (b) Find the total volume of air inside an empty tent.



Solution:
(i) Height of the conical part:

$$\text{Height of conical part} = \text{Total height} - \text{Cylindrical height} = 18.5 - 8 = 10.5\,\text{m}.$$

Radius of the conical part:

$$\text{Radius} = \frac{\text{Diameter}}{2} = \frac{28}{2} = 14\,\text{m}.$$

Slant height:

$$l = \sqrt{(10.5)^2 + (14)^2} = \sqrt{110.25 + 196} = \sqrt{306.25} = 17.5\,\text{m}.$$

(ii) Floor area of the tent:

$$\text{Floor area} = \pi r^2 = \frac{22}{7} \times 14 \times 14 = 616\,\text{m}^2.$$

(iii)(a) Area of cloth used for making the tent:

- Curved surface area of the cylindrical part:

$$\text{CSA (cylinder)} = 2\pi rh = 2 \times \frac{22}{7} \times 14 \times 8 = 704\,\text{m}^2.$$

- Curved surface area of the conical part:

$$\text{CSA (cone)} = \pi rl = \frac{22}{7} \times 14 \times 17.5 = 770\,\text{m}^2.$$

Total area of the cloth:

$$\text{Total cloth area} = 704 + 770 = 1474\,\text{m}^2.$$

(iii)(b) Volume of air inside the tent:

- Volume of the cylindrical part:

$$\text{Volume (cylinder)} = \pi r^2 h = \frac{22}{7} \times 14 \times 14 \times 8 = 4928\,\text{m}^3.$$

- Volume of the conical part:

$$\text{Volume (cone)} = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 14 \times 14 \times 10.5 = 2156\,\text{m}^3.$$

Total volume:

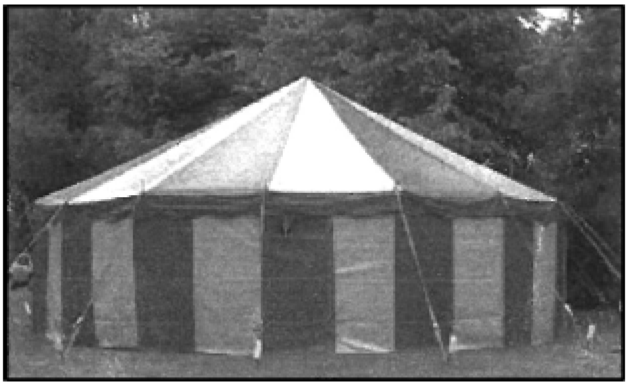
$$\text{Total volume} = 4928 + 2156 = 7084\,\text{m}^3.$$

Quick Tip

For composite shapes, calculate areas and volumes of individual parts separately and then sum them up.

---

37. A circus tent is in cylindrical shape up to a height of 8 m and conical above it. The diameter of the base is 14 m, and the height of the conical part is 10.5 m.



Based on the above, answer the following questions:
(i) Find the slant height of the conical part.
(ii) Determine the floor area of the tent.
(iii) Find the area of cloth used to make the tent.
(iv) Find the volume of air inside an empty tent.

Solution:
(i) Slant height of conical part:
The slant height (l) of the cone is calculated using the Pythagoras theorem:

$$l = \sqrt{(r)^2 + (h)^2},$$

where $r = 7\,\text{m}$ (radius of the base) and $h = 10.5\,\text{m}$ (height of the cone):

$$l = \sqrt{(7)^2 + (10.5)^2} = \sqrt{49 + 110.25} = \sqrt{159.25} \approx 12.6\,\text{m}.$$

(ii) Floor area of the tent:
The floor area is the area of the circular base:

$$\text{Floor area} = \pi r^2.$$

Substitute $r = 7\,\text{m}$:

$$\text{Floor area} = \pi(7)^2 = 22/7 \times 49 = 154\,\text{m}^2.$$

(iii) Area of cloth used:
The area of cloth includes the curved surface area of the cylinder and the cone:

$$\text{Total area} = \text{CSA of cylinder} + \text{CSA of cone}.$$

- CSA of cylinder:

$$\text{CSA}_{\text{cylinder}} = 2\pi r h,$$

where $h = 8\,\text{m}$:

$$\text{CSA}_{\text{cylinder}} = 2 \times \frac{22}{7} \times 7 \times 8 = 352\,\text{m}^2.$$

- CSA of cone:

$$\text{CSA}_{\text{cone}} = \pi r l.$$

Substitute $r = 7$ and $l = 12.6$:

$$\text{CSA}_{\text{cone}} = \frac{22}{7} \times 7 \times 12.6 = 277.2\,\text{m}^2.$$

- Total area of cloth:

$$\text{Total area} = 352 + 277.2 = 629.2\,\text{m}^2.$$

(iv) Volume of air inside the tent:
The total volume includes the volume of the cylinder and cone:

$$\text{Total volume} = \text{Volume of cylinder} + \text{Volume of cone}.$$

- Volume of cylinder:

$$\text{Volume}_{\text{cylinder}} = \pi r^2 h.$$

Substitute $r = 7$ and $h = 8$:

$$\text{Volume}_{\text{cylinder}} = \frac{22}{7} \times (7)^2 \times 8 = 1232\,\text{m}^3.$$

- Volume of cone:

$$\text{Volume}_{\text{cone}} = \frac{1}{3}\pi r^2 h.$$

Substitute $r = 7$ and $h = 10.5$:

$$\text{Volume}_{\text{cone}} = \frac{1}{3} \times \frac{22}{7} \times (7)^2 \times 10.5 = 539\,\text{m}^3.$$

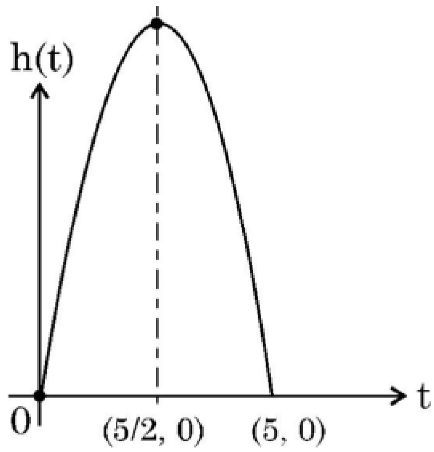
- Total volume:

$$\text{Total volume} = 1232 + 539 = 1771\,\text{m}^3.$$

Quick Tip

For combined shapes, calculate individual areas or volumes (cylinder and cone here) and add them to get the total.

38. A ball is thrown in the air so that t seconds after it is thrown, its height h metre above its starting point is given by the polynomial $h = 25t - 5t^2$.



Observe the graph of the polynomial and answer the following questions:

(i) Write the zeroes of the given polynomial.

(ii) Find the maximum height achieved by the ball.

(iii) (a) After throwing upward, how much time did the ball take to reach the height of 30 m?
(b) Find the two different values of t when the height of the ball was 20 m.

Solution:
(i) The polynomial $h = 25t - 5t^2$ can be written as:

$$h = 5t(5 - t).$$

The zeroes of the polynomial are $t = 0$ and $t = 5$.
(ii) The maximum height of the ball occurs at the vertex of the parabola.
The formula for the time to reach the maximum height is:

$$t = -\frac{b}{2a},$$

where $a = -5$ and $b = 25$.
Substitute:

$$t = -\frac{25}{2(-5)} = \frac{25}{10} = 2.5\,\text{seconds}.$$

Maximum height:

$$h = 25t - 5t^2 = 25(2.5) - 5(2.5)^2.$$

Simplify:

$$h = 62.5 - 31.25 = 31.25\,\text{m}.$$

(iii)(a) To find the time when the height is 30 m, substitute $h = 30$:

$$30 = 25t - 5t^2.$$

Rearrange:

$$-5t^2 + 25t - 30 = 0.$$

Divide by -5:

$$t^2 - 5t + 6 = 0.$$

Factorize:

$$(t - 3)(t - 2) = 0.$$

Thus, $t = 2$ seconds and $t = 3$ seconds.

(iii)(b) To find the time when the height is 20 m, substitute $h = 20$:

$$20 = 25t - 5t^2.$$

Rearrange:

$$-5t^2 + 25t - 20 = 0.$$

Divide by -5:

$$t^2 - 5t + 4 = 0.$$

Factorize:

$$(t - 4)(t - 1) = 0.$$

Thus, $t = 1$ second and $t = 4$ seconds.

Quick Tip

For motion problems, use the quadratic formula or factoring to solve for time when the height is known. The vertex formula gives the maximum or minimum value.
