CBSE Class X Mathematics (Standard) Set 2 (30/2/2)

Time Allowed :3 Hours	**Maximum Marks :**80	**Total Questions :**38

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This question paper contains 38 questions. All questions are compulsory.
2. This Question Paper is divided into **FIVE Sections – Section A, B, C, D, and E**.
3. In Section–A, questions number 1 to 18 are **Multiple Choice Questions (MCQs)** and questions number 19 & 20 are **Assertion-Reason based questions**, carrying **1 mark each**.
4. In Section–B, questions number 21 to 25 are **Very Short-Answer (VSA)** type questions, carrying **2 marks each**.
5. In Section–C, questions number 26 to 31 are **Short Answer (SA)** type questions, carrying **3 marks each**.
6. In Section–D, questions number 32 to 35 are **Long Answer (LA)** type questions, carrying **5 marks each**.
7. In Section–E, questions number 36 to 38 are **Case Study based questions** carrying **4 marks each**. *Internal choice is provided in each case-study.*
8. There is **no overall choice.** However, *an internal choice has been provided in 2 questions in Section–B, 2 questions in Section–C, 2 questions in Section–D, and 3 questions in Section–E.*
9. Draw neat diagrams wherever required. Take $\pi = \frac{22}{7}$ wherever required, if not stated.
10. Use of calculators is **not allowed.**

Section A

1. Which term of the A.P. -29, -26, -23, ..., 61 is 16?

(A) 11^{th}
(B) 16^{th}
(C) 10^{th}
(D) 31^{st}

Correct Answer: (B) 16^{th}

Solution:
1. The general term of an A.P. is given by:

$$a_n = a + (n-1)d,$$

where a is the first term, d is the common difference, and n is the term number.
2. Here, $a = -29$ and $d = -26 - (-29) = 3$.
3. Substitute $a_n = 16$:

$$16 = -29 + (n-1)(3).$$

4. Simplify:

$$16 + 29 = (n-1)(3) \implies 45 = 3(n-1).$$

5. Solve for n:

$$n - 1 = 15 \implies n = 16.$$

6. Therefore, the 16^{th} term of the A.P. is 16.

Quick Tip

To find the term number in an A.P., rearrange the formula for the nth term: $a_n = a + (n-1)d$.

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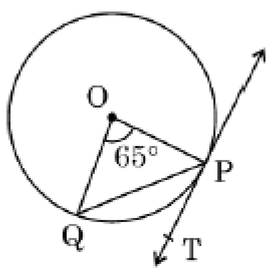
2. In the given figure, PT is tangent to a circle with center O. Chord PQ subtends an angle of 65° at the center. The measure of $\angle QPT$ is:



(A) 65°
(B) 57.5°
(C) 67.5°
(D) 32.5°

Correct Answer: (D) 32.5°

Solution:
In this problem: - PT is tangent to the circle, and the chord PQ subtends $\angle POQ = 65^\circ$ at the center. - By the circle theorem, the angle between a tangent and a chord drawn from the point of contact equals half the angle subtended by the chord at the center.
Thus:
$$\angle QPT = \frac{1}{2} \cdot \angle POQ$$
Substitute:
$$\angle QPT = \frac{1}{2} \cdot 65^\circ = 32.5^\circ$$

> Quick Tip
>
> The angle between a tangent and a chord at the point of contact equals half the angle subtended by the chord at the center.

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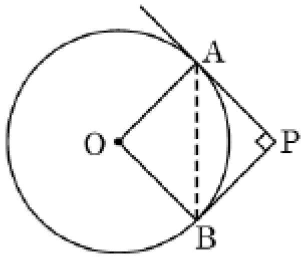
3. In the given figure, tangents PA and PB to the circle centered at O, from point P are perpendicular to each other. If PA = 5 cm, then the length of AB is equal to:



(A) 5 cm
(B) $5\sqrt{2}$ cm
(C) $2\sqrt{5}$ cm
(D) 10 cm

Correct Answer: (B) $5\sqrt{2}$ **cm**

Solution:
In the given problem: - PA and PB are tangents from point P to the circle, and they are perpendicular to each other. - The triangle OAP forms a right-angled triangle where:
$$OP = \text{radius of circle}, \quad OA = OB = \text{radius of circle}, \quad PA = 5\,\text{cm}.$$
The length of AB (chord subtending a 90° angle) can be determined using Pythagoras' theorem:
$$AB = \sqrt{PA^2 + PB^2} = \sqrt{5^2 + 5^2} = \sqrt{50} = 5\sqrt{2}.$$
Thus, the length of AB is $5\sqrt{2}$ cm.

Quick Tip

For tangents perpendicular to each other, use the Pythagorean theorem to calculate lengths involving right-angled triangles.

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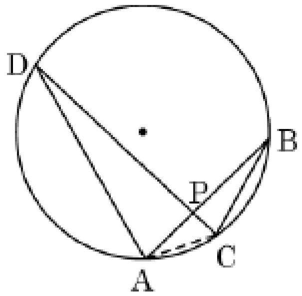
4. AB and CD are two chords of a circle intersecting at P. Choose the correct statement from the following:



(A) $\triangle ADP \sim \triangle ACB$
(B) $\triangle ADP \sim \triangle BPC$
(C) $\triangle ADP \sim \triangle BCP$
(D) $\triangle ADP \sim \triangle CBP$

Correct Answer: (D) $\triangle ADP \sim \triangle CBP$

Solution:
When two chords intersect inside a circle at a point, the triangles formed are similar by the AA (Angle-Angle) similarity criterion:
$\angle ADP = \angle CBP$ (vertically opposite angles),
$\angle DAP = \angle BCP$ (angles subtended by the same arc).
Thus, $\triangle ADP \sim \triangle CBP$.

Quick Tip

For intersecting chords, identify similar triangles using angle relationships and the AA similarity rule.

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5. If value of each observation in a data is increased by 2, then the median of the new data:
(A) increases by 2
(B) increases by $2n$
(C) remains same
(D) decreases by 2

Correct Answer: (A) increases by 2

Solution:
The median is a positional average. If each observation in a dataset is increased by 2, the position of the median remains the same, but its value increases by 2.
Thus, the new median is the original median + 2.

> Quick Tip
>
> Shifting data values (adding/subtracting a constant) shifts the median by the same amount.

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6. If α and β are zeroes of the polynomial $2x^2 - 9x + 5$, then the value of $\alpha^2 + \beta^2$ is:

(A) $\frac{1}{4}$
(B) $\frac{61}{4}$
(C) 1
(D) $\frac{71}{4}$

Correct Answer: (B) $\frac{61}{4}$

Solution:
For a quadratic polynomial $ax^2 + bx + c$, if α and β are the roots, we use the identities:

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}, \quad \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta.$$

Given: $a = 2$, $b = -9$, and $c = 5$:

$$\alpha + \beta = -\frac{-9}{2} = \frac{9}{2}, \quad \alpha\beta = \frac{5}{2}.$$

Substitute into the formula:

$$\alpha^2 + \beta^2 = \left(\frac{9}{2}\right)^2 - 2 \cdot \frac{5}{2}.$$

Simplify:

$$\alpha^2 + \beta^2 = \frac{81}{4} - \frac{10}{2} = \frac{81}{4} - \frac{20}{4} = \frac{61}{4}.$$

> Quick Tip
>
> For quadratic roots, use $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ to simplify calculations.

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7. After an examination, a teacher wants to know the marks obtained by the maximum number of students in her class. She requires to calculate:

(A) median
(B) mode
(C) mean

(D) range

Correct Answer: (B) mode

Solution:
The mode of a dataset is the value that occurs most frequently. To find the marks obtained by the maximum number of students, the teacher needs to calculate the mode.

Quick Tip
Mode is the most frequently occurring value in a dataset. It represents the "most common" outcome.

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8. The value of k for which the system of equations $3x - y + 8 = 0$ and $6x - ky + 16 = 0$ has infinitely many solutions, is:
(A) -2
(B) 2
(C) $\frac{1}{2}$
(D) $-\frac{1}{2}$

Correct Answer: (B) 2

Solution:
For two linear equations to have infinitely many solutions, their ratios of coefficients must be equal:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}.$$

Given equations:

$$3x - y + 8 = 0 \quad \text{and} \quad 6x - ky + 16 = 0,$$

the ratios are:

$$\frac{3}{6} = \frac{-1}{-k} = \frac{8}{16}.$$

Simplify:

$$\frac{3}{6} = \frac{1}{2} \quad \text{and} \quad \frac{-1}{-k} = \frac{1}{2}.$$

Thus:

$$k = 2.$$

Quick Tip
For infinitely many solutions, equate the ratios of coefficients: $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

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9. Point P divides the line segment joining the points A(4, -5) and B(1, 2) in the ratio 5:2. Coordinates of point P are:
(A) $\left(5, -\frac{3}{2}\right)$
(B) $\left(\frac{11}{7}, 0\right)$
(C) $\left(\frac{13}{7}, 0\right)$
(D) $\left(0, \frac{13}{7}\right)$

Correct Answer: (C) $\left(\frac{13}{7}, 0\right)$

Solution:
The coordinates of a point dividing a line segment internally in the ratio $m : n$ are:

$$P = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right).$$

Here:

$$A(4, -5),\ B(1, 2),\ m = 5,\ n = 2.$$

Substitute into the formula:

$$P_x = \frac{5(1) + 2(4)}{5+2} = \frac{5+8}{7} = \frac{13}{7}, \quad P_y = \frac{5(2) + 2(-5)}{5+2} = \frac{10-10}{7} = 0.$$

Thus, $P = \left(\frac{13}{7}, 0\right)$.

Quick Tip

Use the section formula: $P = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$ for internal division.

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10. A box contains cards numbered 6 to 55. A card is drawn at random from the box. The probability that the drawn card has a number which is a perfect square, is:
(A) $\frac{7}{50}$
(B) $\frac{7}{55}$
(C) $\frac{1}{10}$
(D) $\frac{5}{49}$

Correct Answer: (C) $\frac{1}{10}$

Solution:
The numbers from 6 to 55 are inclusive. The total number of cards is:

$$55 - 6 + 1 = 50.$$

The perfect squares between 6 and 55 are: $9, 16, 25, 36, 49$ (5 numbers). Therefore, the probability is:

$$P = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{5}{50} = \frac{1}{10}.$$

Quick Tip

Count carefully when determining perfect squares in a range. Use the formula for probability: $P = \frac{\text{Favorable outcomes}}{\text{Total outcomes}}$.

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11. If $\sin\theta = \cos\theta$, $(0^\circ < \theta < 90^\circ)$, then value of $(\sec\theta \cdot \sin\theta)$ is:
(A) $\frac{1}{\sqrt{2}}$
(B) $\sqrt{2}$
(C) 1
(D) 0

Correct Answer: (C) 1

Solution:
If $\sin\theta = \cos\theta$, then:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = 1 \implies \theta = 45^\circ.$$

At $\theta = 45^\circ$:

$$\sin\theta = \cos\theta = \frac{1}{\sqrt{2}}, \quad \sec\theta = \frac{1}{\cos\theta} = \sqrt{2}.$$

Now calculate:

$$\sec\theta \cdot \sin\theta = \sqrt{2} \cdot \frac{1}{\sqrt{2}} = 1.$$

Quick Tip

When $\sin\theta = \cos\theta$, θ is always 45°. Use standard trigonometric values for simplification.

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12. Two dice are rolled together. The probability of getting the sum of the two numbers to be more than 10, is:
(A) $\frac{1}{9}$
(B) $\frac{1}{6}$
(C) $\frac{7}{12}$
(D) $\frac{1}{12}$

Correct Answer: (D) $\frac{1}{12}$

Solution:
The total outcomes when rolling two dice is:

$$6 \times 6 = 36.$$

The favorable outcomes for sums greater than 10 are: - $(5, 6), (6, 5), (6, 6)$, giving a total of 3 outcomes.

Thus, the probability is:

$$P = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{3}{36} = \frac{1}{12}.$$

Quick Tip

For sums with two dice, list all possible outcomes systematically to avoid errors in counting.

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13. Value of k for which $x = 2$ is a solution of the equation $5x^2 - 4x + (2 + k) = 0$, is:

(A) 10
(B) -10
(C) 14
(D) -14

Correct Answer: (D) -14

Solution:
Since $x = 2$ is a solution, substitute $x = 2$ into the equation:

$$5x^2 - 4x + (2 + k) = 0.$$

Substitute $x = 2$:

$$5(2)^2 - 4(2) + (2 + k) = 0.$$

Simplify:

$$5(4) - 8 + (2 + k) = 0 \implies 20 - 8 + 2 + k = 0.$$

$$14 + k = 0 \implies k = -14.$$

Quick Tip

To check if a value satisfies an equation, substitute it into the equation and simplify to solve for unknown parameters.

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14. The perimeters of two similar triangles ABC and PQR are 56 cm and 48 cm respectively. PQ/AB is equal to:

(A) $\frac{7}{8}$
(B) $\frac{6}{7}$
(C) $\frac{7}{6}$
(D) $\frac{8}{7}$

Correct Answer: (B) $\frac{6}{7}$

Solution:
For similar triangles, the ratio of their perimeters equals the ratio of their corresponding sides. Therefore:

$$\frac{\text{Perimeter of ABC}}{\text{Perimeter of PQR}} = \frac{\text{AB}}{\text{PQ}}.$$

Substitute the values:

$$\frac{56}{48} = \frac{\text{AB}}{\text{PQ}} = \frac{7}{6}.$$

Thus, $\frac{\text{PQ}}{\text{AB}} = \frac{6}{7}$.

> **Quick Tip**
>
> For similar triangles, the ratio of perimeters is the same as the ratio of their corresponding sides.

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15. The sum of the first 200 natural numbers is:
(A) 2010
(B) 2000
(C) 20100
(D) 21000

Correct Answer: (C) 20100

Solution:
The formula for the sum of the first n natural numbers is:

$$S_n = \frac{n(n+1)}{2}.$$

Substitute $n = 200$:

$$S_{200} = \frac{200 \cdot (200+1)}{2} = \frac{200 \cdot 201}{2} = 20100.$$

> **Quick Tip**
>
> Use the formula $S_n = \frac{n(n+1)}{2}$ to calculate the sum of the first n natural numbers.

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16. If the HCF (2520, 6600) = 40 and LCM (2520, 6600) = 252 × k, then the value of k is:
(A) 1650
(B) 165
(C) 160
(D) 1625

Correct Answer: (A) 1650

Solution:
The relationship between HCF, LCM, and the product of two numbers is:

$$\text{HCF} \times \text{LCM} = \text{Product of the numbers.}$$

Substitute:

$$40 \times (252 \cdot k) = 2520 \cdot 6600.$$

Simplify:

$$40 \cdot 252 \cdot k = 2520 \cdot 6600 \quad \Rightarrow \quad k = \frac{2520 \cdot 6600}{40 \cdot 252}.$$

Simplify further:

$$k = 1650.$$

Quick Tip

For HCF and LCM problems, remember: HCF $\times$ LCM = Product of the two numbers.

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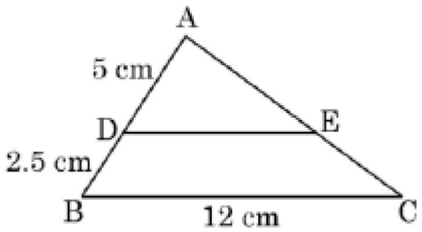
17. In the given figure, $\triangle ABC$ is shown. DE is parallel to BC. If AD = 5 cm, DB = 2.5 cm, and BC = 12 cm, then DE is equal to:



(A) 10 cm
(B) 6 cm
(C) 8 cm
(D) 7.5 cm

Correct Answer: (C) 8 cm

Solution:
Since DE $\parallel$ BC, by the Basic Proportionality Theorem (BPT), the ratio of corresponding sides is equal:

$$\frac{\text{AD}}{\text{AB}} = \frac{\text{DE}}{\text{BC}}.$$

Given: $AD = 5\,\text{cm}$, $DB = 2.5\,\text{cm}$, $BC = 12\,\text{cm}$. First, calculate AB:

$$AB = AD + DB = 5 + 2.5 = 7.5\,\text{cm}.$$

Now substitute into the ratio:

$$\frac{5}{7.5} = \frac{\text{DE}}{12}.$$

Simplify:

$$\text{DE} = \frac{5 \cdot 12}{7.5} = 8\,\text{cm}.$$

Quick Tip

When two sides are parallel in a triangle, apply the Basic Proportionality Theorem to find unknown lengths.

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18. XY0Z is a rectangle with vertices X(-3, 0), O(0, 0), Y(0, 4), and Z(x, y). The length of its each diagonal is:

(A) 5 units
(B) $\sqrt{5}$ units
(C) $x^2 + y^2$ units
(D) 4 units

Correct Answer: (A) 5 units

Solution:
To calculate the length of a diagonal in a rectangle, use the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Given vertices $X(-3, 0)$ and $Y(0, 4)$, the diagonal XY:

$$XY = \sqrt{(0 - (-3))^2 + (4 - 0)^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = 5\,\text{units}.$$

Similarly, both diagonals in a rectangle are equal, so each diagonal is 5 units.

Quick Tip

In rectangles, the diagonals are equal, and their length can be calculated using the distance formula.

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19. Assertion (A): Two cubes each of edge length 10 cm are joined together. The total surface area of newly formed cuboid is 1200 cm^2.
Reason (R): Area of each surface of a cube of side 10 cm is 100 cm^2.
Options:

(A) Both Assertion (A) and Reason (R) are true. Reason (R) is the correct explanation of Assertion (A).

(B) Both Assertion (A) and Reason (R) are true. Reason (R) does not give correct explanation of Assertion (A).

(C) Assertion (A) is true but Reason (R) is not true.

(D) Assertion (A) is not true but Reason (R) is true.

Correct Answer: (D) Assertion (A) is not true but Reason (R) is true.

Solution:
- The surface area of a cube with edge length 10 cm is:

$$\text{Area of one face} = 10 \times 10 = 100\,\text{cm}^2, \quad \text{Total surface area} = 6 \times 100 = 600\,\text{cm}^2.$$

- When two cubes are joined side by side, two faces overlap, reducing the total surface area:

$$\text{New surface area} = 600 + 600 - 2(100) = 1100\,\text{cm}^2.$$

Hence, Assertion (A) is false, but Reason (R) is true.

Quick Tip

When combining shapes, subtract the overlapping area to calculate the total surface area.

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20. Assertion (A): If $\sin A = \frac{1}{3}$ $(0° < A < 90°)$, then the value of $\cos A$ is $\frac{2\sqrt{2}}{3}$.
Reason (R): For every angle θ, $\sin^2\theta + \cos^2\theta = 1$.
Options:

(A) Both Assertion (A) and Reason (R) are true. Reason (R) is the correct explanation of Assertion (A).

(B) Both Assertion (A) and Reason (R) are true. Reason (R) does not give correct explanation of Assertion (A).

(C) Assertion (A) is true but Reason (R) is not true.

(D) Assertion (A) is not true but Reason (R) is true.

Correct Answer: (A) Both Assertion (A) and (R) are true. Reason (R) is the correct explanation of Assertion (A).

Solution:
Given $\sin A = \frac{1}{3}$, we use the Pythagorean identity:

$$\sin^2 A + \cos^2 A = 1.$$

Substitute $\sin A$:

$$\left(\frac{1}{3}\right)^2 + \cos^2 A = 1 \implies \frac{1}{9} + \cos^2 A = 1.$$

Simplify:

$$\cos^2 A = 1 - \frac{1}{9} = \frac{8}{9}.$$

Taking the square root:

$$\cos A = \sqrt{\frac{8}{9}} = \frac{\sqrt{8}}{3} = \frac{2\sqrt{2}}{3}.$$

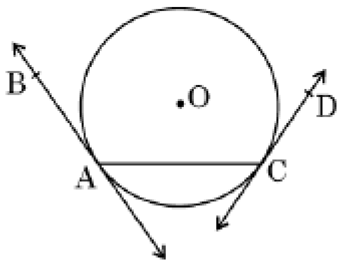
Thus, both Assertion (A) and Reason (R) are true, and Reason (R) explains Assertion (A).

Quick Tip

Use $\sin^2\theta + \cos^2\theta = 1$ to find $\cos\theta$ when $\sin\theta$ is known.

Section B

21. In the given figure, AB and CD are tangents to a circle centered at O. Is $\angle BAC = \angle DCA$? Justify your answer.



Solution:
To prove $\angle BAC = \angle DCA$, follow the construction: - Join OA and OC, where O is the center of the circle.
Using the properties of tangents:
1. $OA \perp AB$ and $OC \perp CD$ (radius is perpendicular to tangents at the point of contact).
2. $\triangle OAB \cong \triangle OCD$ (by RHS congruence: right angle, hypotenuse, and corresponding sides are equal).
Therefore:

$$\angle BAC = \angle DCA \quad \text{(corresponding angles in congruent triangles).}$$

Quick Tip

For tangents drawn from external points, the angles subtended by tangents at the center are equal.

22(a). In what ratio is the line segment joining the points $(3, -5)$ and $(-1, 6)$ divided by the line $y = x$?

Solution:

1. Let the required ratio be $K : 1$.
2. The coordinates of point P dividing the line segment in the ratio $K : 1$ are:

$$\left(\frac{-K+3}{K+1}, \frac{6K-5}{K+1}\right).$$

3. Since point P lies on the line $y = x$, equate the coordinates:

$$\frac{-K+3}{K+1} = \frac{6K-5}{K+1}.$$

4. Simplify:

$$-K+3 = 6K-5.$$

$$7K = 8 \implies K = \frac{8}{7}.$$

5. The required ratio is:

$$8 : 7.$$

> **Quick Tip**
>
> When dividing a line segment, use the section formula and substitute the given condition to find the ratio.

22(b). $A(3,0)$, $B(6,4)$, and $C(-1,3)$ are vertices of a triangle ABC. Find the length of its median BE.

Solution:
1. The midpoint of AC is $E\left(\frac{3-1}{2}, \frac{0+3}{2}\right) = \left(1, \frac{3}{2}\right)$.
2. The length of the median BE is:

$$BE = \sqrt{(6-1)^2 + \left(4 - \frac{3}{2}\right)^2}.$$

3. Simplify:

$$BE = \sqrt{(6-1)^2 + \left(\frac{8}{2} - \frac{3}{2}\right)^2} = \sqrt{(5)^2 + \left(\frac{5}{2}\right)^2}.$$

4. Further simplify:

$$BE = \sqrt{25 + \frac{25}{4}} = \sqrt{\frac{100}{4} + \frac{25}{4}} = \sqrt{\frac{125}{4}} = \frac{\sqrt{125}}{2} = \frac{5\sqrt{5}}{2}.$$

> **Quick Tip**
>
> For finding the length of a median, calculate the midpoint of the opposite side and then use the distance formula.

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23. Explain why $7 \times 11 \times 13$ and $7 \times 3 \times 13$ are composite numbers.

Solution:
A composite number has more than two factors.
1. For $7 \times 11 \times 13$:

$$7 \times 11 \times 13 = 1001 \quad \text{(factors: } 1, 7, 11, 13, 77, 91, 143, 1001\text{)}.$$

It has more than two factors, so it is composite.
2. For $7 \times 3 \times 13$:

$$7 \times 3 \times 13 = 273 \quad \text{(factors: } 1, 3, 7, 13, 21, 39, 91, 273\text{)}.$$

It also has more than two factors, so it is composite.

> 💡 Quick Tip
>
> A number is composite if it has factors other than 1 and itself.

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24. The vertices of a triangle are $A(2,3)$, $B(4,1)$, and $C(1,-1)$. Find the length of the median BD.

Solution:
1. The midpoint of AC is:

$$D = \left(\frac{-2+1}{2}, \frac{4+(-6)}{2}\right) = \left(-\frac{1}{2}, -1\right).$$

2. The length of the median BD is:

$$BD = \sqrt{\left(4 - \left(-\frac{1}{2}\right)\right)^2 + (3-(-1))^2}.$$

3. Simplify:

$$BD = \sqrt{\left(4 + \frac{1}{2}\right)^2 + (3+1)^2}.$$

4. Further simplify:

$$BD = \sqrt{\left(\frac{8}{2} + \frac{1}{2}\right)^2 + 4^2} = \sqrt{\left(\frac{9}{2}\right)^2 + 16}.$$

5. Combine terms:

$$BD = \sqrt{\frac{81}{4} + \frac{64}{4}} = \sqrt{\frac{145}{4}} = \frac{\sqrt{145}}{2}.$$

> 💡 Quick Tip
>
> To calculate the length of a median, find the midpoint of the opposite side and then apply the distance formula.

25(a). Evaluate $2\sin^2 30^\circ \sec 60^\circ + \tan^2 60^\circ$.

Solution:
First, substitute the trigonometric values:

$$\sin 30^\circ = \frac{1}{2}, \quad \sec 60^\circ = 2, \quad \tan 60^\circ = \sqrt{3}.$$

Simplify step-by-step:

$$2\sin^2 30^\circ \sec 60^\circ + \tan^2 60^\circ = 2\left(\frac{1}{2}\right)^2 \cdot 2 + (\sqrt{3})^2.$$

$$= 2 \cdot \frac{1}{4} \cdot 2 + 3 = \frac{4}{4} + 3 = 1 + 3 = 4.$$

Quick Tip

Use standard trigonometric values to simplify step-by-step.

25(b). If $\sin(A+B) = \sqrt{3}/2$ and $\cos(A-B) = 1$, find the measures of angles A and B.

Solution:
1. From $\cos(A-B) = 1$, we know:

$$A - B = 0^\circ \implies A = B.$$

2. From $\sin(A+B) = \frac{\sqrt{3}}{2}$:

$$\sin(A+A) = \frac{\sqrt{3}}{2} \implies \sin(2A) = \frac{\sqrt{3}}{2}.$$

The angle for which $\sin\theta = \frac{\sqrt{3}}{2}$ is $\theta = 60^\circ$. Thus:

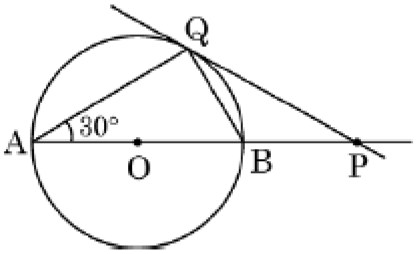
$$2A = 60^\circ \implies A = 30^\circ \quad \text{and} \quad B = 30^\circ.$$

Quick Tip

For trigonometric equations, identify standard values and simplify using known identities.

Section C

26(a). In the given figure, PQ is tangent to a circle centered at O, and $\angle BAQ = 30^\circ$. Show that $BP = BQ$.



Solution:
To prove $BP = BQ$, follow these steps:
1. Join OQ and use the tangent-secant theorem: - $OQ = OA$ (radii of the circle). 2. Use the angle properties:

$$\angle BAQ = 30^\circ \implies \angle OAQ = 90^\circ - 30^\circ = 60^\circ.$$

- Since OQ and OA are equal, $\triangle OAQ$ is isosceles.

$$\angle OQA = \angle OAQ = 60^\circ.$$

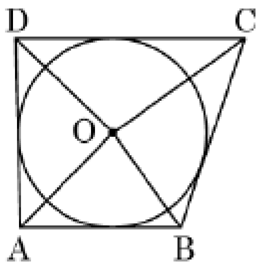
3. Now consider the line BP: - $\angle BPQ = 90^\circ - \angle OQA = 30^\circ$. - Since BP and BQ subtend equal angles, the lengths BP and BQ are equal.
Thus:

$$BP = BQ \quad \text{(Proved)}.$$

Quick Tip

For tangents drawn from a common point, the lengths of the tangents are always equal.

26(b). In the given figure, AB, BC, CD, and DA are tangents to the circle with center O, forming a quadrilateral $ABCD$. Show that $\angle AOB + \angle COD = 180^\circ$.



Solution:
To prove $\angle AOB + \angle COD = 180^\circ$, follow these steps:
1. Join OP, OQ, OR, and OS: - Here, AP and BP are tangents from A and B, respectively.
- Similarly, CR and DR are tangents from C and D.

2. Each pair of tangents forms a 90° angle at the point of contact with the circle. Therefore:

$$\angle AOB + \angle BOC + \angle COD + \angle DOA = 360^\circ.$$

3. Since $ABCD$ is a quadrilateral, opposite angles satisfy:

$$\angle AOB + \angle COD = 180^\circ.$$

Thus, the result is proven:

$$\angle AOB + \angle COD = 180^\circ.$$

Quick Tip

For tangents forming a quadrilateral, the sum of opposite angles at the center equals 180°.

27. In a test, the marks obtained by 100 students (out of 50) are given below:

Marks obtained	Number of students
0 – 10	12
10 – 20	23
20 – 30	34
30 – 40	25
40 – 50	6

Find the mean marks of the students.

Solution:
1. To find the mean, use the formula:

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i},$$

where x_i is the mid-point of each class, and f_i is the frequency.

Marks Obtained	Number of students (f_i)	x_i	$f_i x_i$
0 – 10	12	5	60
10 – 20	23	15	345
20 – 30	34	25	850
30 – 40	25	35	875
40 – 50	6	45	270
Total	100		2400

2. Calculate the mean:

$$\text{Mean} = \frac{2400}{100} = 24.$$

Quick Tip

To find the mean in grouped data, use the mid-points of intervals as x_i and multiply by their corresponding frequencies.

28. Three consecutive integers are such that the sum of the square of second and product of other two is 161. Find the three integers.

Solution:
Let the three consecutive integers be $x-1$, x, and $x+1$.
The condition is:

$$x^2 + (x-1)(x+1) = 161.$$

1. Expand $(x-1)(x+1)$:

$$x^2 + (x^2 - 1) = 161.$$

2. Simplify:

$$2x^2 - 1 = 161 \implies 2x^2 = 162 \implies x^2 = 81 \implies x = 9 \,\text{or}\, x = -9.$$

3. Find the integers: - For $x = 9$: The integers are $8, 9, 10$. - For $x = -9$: The integers are $-10, -9, -8$.

> Quick Tip
>
> For consecutive integers, represent them as $x-1$, x, and $x+1$, and carefully simplify the algebraic conditions.

29(a). If the sum of the first m terms of an A.P. is the same as the sum of its first n terms $(m \neq n)$, then show that the sum of its first $(m+n)$ terms is zero.

Solution:
1. The sum of the first m terms of an A.P. is:

$$S_m = \frac{m}{2}[2a + (m-1)d].$$

2. The sum of the first n terms of an A.P. is:

$$S_n = \frac{n}{2}[2a + (n-1)d].$$

3. Given $S_m = S_n$, equate the two:

$$\frac{m}{2}[2a + (m-1)d] = \frac{n}{2}[2a + (n-1)d].$$

4. Simplify:

$$m[2a + (m-1)d] = n[2a + (n-1)d].$$

5. Expand:

$$2am + m(m-1)d = 2an + n(n-1)d.$$

6. Rearrange:

$$2a(m-n) = d[n^2 - m^2 - n + m].$$

$$2a(m-n) = d(m+n)(n-m).$$

7. Simplify further:

$$2a = -d(m+n).$$

8. Substitute into the formula for the sum of the first $(m+n)$ terms:

$$S_{m+n} = \frac{m+n}{2}[2a + (m+n-1)d].$$

9. Replace $2a$:

$$S_{m+n} = \frac{m+n}{2}[-d(m+n) + (m+n-1)d].$$

10. Simplify:

$$S_{m+n} = \frac{m+n}{2}[d(-m-n+m+n-1)] = 0.$$

Quick Tip
Use the A.P. sum formula and substitute known conditions systematically to prove such relationships.

29(b). In an A.P., the sum of three consecutive terms is 24, and the sum of their squares is 194. Find the numbers.

Solution:
Let the three consecutive terms of the A.P. be $a-d$, a, and $a+d$.
1. Given:

$$(a-d) + a + (a+d) = 24.$$

Simplify:

$$3a = 24 \implies a = 8.$$

2. Given sum of squares:

$$(a-d)^2 + a^2 + (a+d)^2 = 194.$$

Substitute $a = 8$:

$$(8-d)^2 + 8^2 + (8+d)^2 = 194.$$

Simplify:

$$(64 - 16d + d^2) + 64 + (64 + 16d + d^2) = 194.$$

Combine like terms:

$$2d^2 + 192 = 194 \implies 2d^2 = 2 \implies d^2 = 1 \implies d = \pm 1.$$

3. Find the terms: - For $d = 1$: The terms are $7, 8, 9$. - For $d = -1$: The terms are $9, 8, 7$.

Quick Tip
For three consecutive terms in an A.P., represent them as $a-d$, a, and $a+d$, and use algebraic equations to solve.

30. Prove that $\sqrt{5}$ is an irrational number.

Solution:
To prove that $\sqrt{5}$ is irrational, assume the contrary.
1. Let $\sqrt{5}$ be rational. Then it can be expressed as:

$$\sqrt{5} = \frac{p}{q}, \quad \text{where } p \text{ and } q \text{ are co-prime integers (no common factors except 1).}$$

2. Squaring both sides:

$$5 = \frac{p^2}{q^2} \implies p^2 = 5q^2.$$

Thus, p^2 is divisible by 5. Therefore, p must also be divisible by 5 (property of primes).
3. Let $p = 5k$, where k is some integer. Substitute into $p^2 = 5q^2$:

$$(5k)^2 = 5q^2 \implies 25k^2 = 5q^2 \implies q^2 = 5k^2.$$

Thus, q^2 is divisible by 5, and therefore q must also be divisible by 5.
4. Since both p and q are divisible by 5, this contradicts the assumption that p and q are co-prime.
Therefore, $\sqrt{5}$ is irrational.

Quick Tip

To prove a square root is irrational, assume it is rational, derive a contradiction using divisibility, and conclude the proof.

31. Prove that $\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta$.

Solution:
To prove $\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta$, follow these steps:
1. Start with the Left-Hand Side (LHS):

$$\text{LHS} = \sin^4\theta + \cos^4\theta.$$

Factorize using the identity $a^2 + b^2 = (a+b)^2 - 2ab$:

$$\sin^4\theta + \cos^4\theta = (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta.$$

2. Use the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$:

$$\sin^4\theta + \cos^4\theta = (1)^2 - 2\sin^2\theta\cos^2\theta.$$

3. Simplify:

$$\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta.$$

Thus, the Right-Hand Side (RHS) is equal to the Left-Hand Side.

Quick Tip

Use the identity $a^2 + b^2 = (a+b)^2 - 2ab$ and apply $\sin^2\theta + \cos^2\theta = 1$ to simplify higher powers of sine and cosine.

Section D

32. The angle of elevation of an aircraft from a point A on the ground is $60°$. After a flight of 30 seconds, the angle of elevation changes to $30°$. The aircraft is flying at a constant height of $3500\sqrt{3}$ m at a uniform speed. Find the speed of the aircraft.

Solution:

1. Let P and Q be the positions of the aircraft at two different times. Given: - Height of the aircraft $= 3500\sqrt{3}$ m, - Angles of elevation: $\theta_1 = 60°$ and $\theta_2 = 30°$, - Time between observations = 30 seconds.

2. Use the tangent function for angles of elevation:

$$\tan 60° = \frac{\text{Height}}{\text{Base AC}} \quad \text{and} \quad \tan 30° = \frac{\text{Height}}{\text{Base AB}}.$$

Substitute $\tan 60° = \sqrt{3}$ and $\tan 30° = \frac{1}{\sqrt{3}}$:

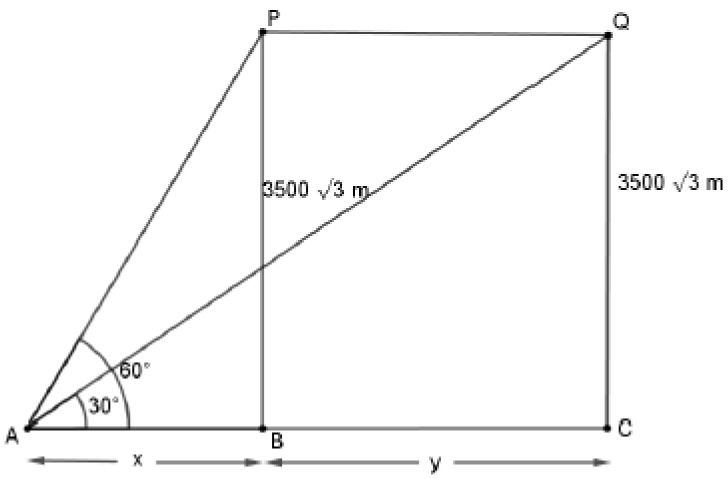
$$\text{Base AC} = \frac{3500\sqrt{3}}{\sqrt{3}} = 3500\,\text{m}, \quad \text{Base AB} = \frac{3500\sqrt{3}}{\frac{1}{\sqrt{3}}} = 3500 \times 3 = 10500\,\text{m}.$$

3. Distance traveled by the aircraft:

$$PQ = AB - AC = 10500 - 3500 = 7000\,\text{m}.$$

4. Speed of the aircraft:

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{7000}{30} = 233.3\,\text{m/s (approx)}.$$



Quick Tip

For angle of elevation problems, use $\tan\theta = \frac{\text{Height}}{\text{Base}}$ and apply basic trigonometry.

---

33. If the length of a rectangle is reduced by 5 cm and its breadth is increased by 2 cm, then the area of the rectangle is reduced by 80 cm^2. However, if we increase the length by 10 cm and decrease the breadth by 5 cm, its area is increased by 50 cm^2. Find the length and breadth of the rectangle.

Solution:
Let the length of the rectangle be x cm and the breadth be y cm.
1. First condition: Reducing the length by 5 cm and increasing the breadth by 2 cm reduces the area by 80 cm^2:

$$(x-5)(y+2) = xy - 80.$$

Simplify:

$$xy - 5y + 2x - 10 = xy - 80.$$

Cancel xy and rearrange:

$$-5y + 2x = -70. \tag{1}$$

2. Second condition: Increasing the length by 10 cm and decreasing the breadth by 5 cm increases the area by 50 cm^2:

$$(x+10)(y-5) = xy + 50.$$

Simplify:

$$xy - 5x + 10y - 50 = xy + 50.$$

Cancel xy and rearrange:

$$-5x + 10y = 100. \tag{2}$$

3. Solve equations (1) and (2): From equation (1):

$$2x - 5y = -70 \implies x = \frac{5y-70}{2}.$$

Substitute into equation (2):

$$-5\left(\frac{5y-70}{2}\right) + 10y = 100.$$

Simplify:

$$-\frac{25y-350}{2} + 10y = 100.$$

Multiply through by 2:

$$-25y + 350 + 20y = 200.$$

Simplify:

$$-5y = -150 \implies y = 30.$$

4. Find x: Substitute $y = 30$ into $2x - 5y = -70$:

$$2x - 5(30) = -70 \implies 2x - 150 = -70 \implies 2x = 80 \implies x = 40.$$

Thus, the length of the rectangle is 40 cm and the breadth is 30 cm.

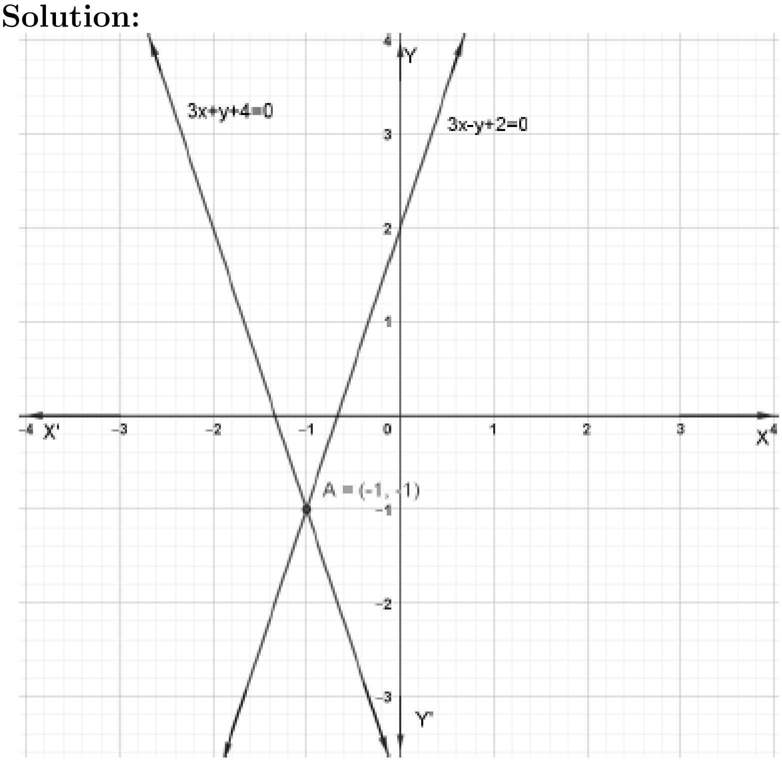
Quick Tip

Translate word problems into algebraic equations, solve them systematically, and verify the solutions.

34(a). Using graphical method, solve the following system of equations:

$$3x + y + 4 = 0 \quad \text{and} \quad 3x - y + 2 = 0.$$

Solution:



1. Rewrite both equations in slope-intercept form ($y = mx + c$):

$$3x + y + 4 = 0 \implies y = -3x - 4, \quad 3x - y + 2 = 0 \implies y = 3x + 2.$$

2. Plot the two lines on the graph: - The first line ($y = -3x - 4$) has a slope of -3 and a y-intercept of -4. - The second line ($y = 3x + 2$) has a slope of 3 and a y-intercept of 2.
3. Intersection of the two lines: From the graph, the lines intersect at $x = 2$ and $y = -2$.
Thus, the solution to the system of equations is:

$$x = 2, \, y = -2.$$

Quick Tip

For solving equations graphically, convert them to slope-intercept form ($y = mx + c$) and plot the lines to find their intersection point.

34(b). Tara scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks deducted for each wrong answer, Tara would have scored 50 marks. Assuming that Tara attempted all questions, find the total number of questions in the test.

Solution:
Let the number of correct answers be x and the number of incorrect answers be y.
1. From the given conditions:

$$3x - 1y = 40 \quad \text{and} \quad 4x - 2y = 50.$$

2. Simplify the second equation:

$$4x - 2y = 50 \implies 2x - y = 25. \tag{1}$$

3. Solve equations $3x - y = 40$ and $2x - y = 25$: Subtract equation (1) from the first equation:

$$(3x - y) - (2x - y) = 40 - 25.$$

Simplify:

$$x = 15.$$

Substitute $x = 15$ into $2x - y = 25$:

$$2(15) - y = 25 \implies 30 - y = 25 \implies y = 5.$$

4. Total number of questions:

$$x + y = 15 + 5 = 20.$$

Thus, the total number of questions is 20.

Quick Tip

Translate word problems into equations and solve them systematically using elimination or substitution methods.

35(a). If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, prove that the other two sides are divided in the same ratio.

Solution:
To prove the Basic Proportionality Theorem (Thales' theorem):
1. Consider $\triangle ABC$, where $DE \parallel BC$, and D and E are points on AB and AC, respectively.
2. By construction, draw perpendiculars from B and C to line DE: - Let the perpendicular distances be h_1 and h_2.
3. Triangles $\triangle ADE$ and $\triangle ABC$ are similar because: - Corresponding angles are equal ($\angle ADE = \angle ABC$, $\angle AED = \angle ACB$).

4. From similarity, the sides are proportional:

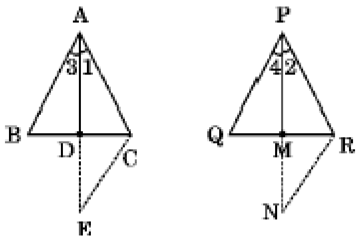
$$\frac{AD}{DB} = \frac{AE}{EC}.$$

Thus, the theorem is proved: A line parallel to one side of a triangle divides the other two sides in the same ratio.

> **Quick Tip**
>
> In similar triangles, corresponding sides are proportional. Use this property to prove the Basic Proportionality Theorem.

35(b). Sides AB and AC and median AD to $\triangle ABC$ are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\triangle ABC \sim \triangle PQR$.

Solution:
To prove $\triangle ABC \sim \triangle PQR$, follow these steps:



1. Extend median AD and PM: - Produce AD to E such that $AD = DE$ and join EC. - Produce PM to N such that $PM = MN$ and join NR.
2. Show triangles are congruent: - In $\triangle ABD$ and $\triangle EMC$: - $AD = DE$ (construction), - $AB = EC$ (given proportional sides), - $\angle BAD = \angle EDC$ (corresponding angles in congruent parts).
Thus:

$$\triangle ABD \cong \triangle EMC.$$

3. Prove proportionality in sides: - Similarly, $PQ = NR$ and $PM = MN$ (construction). - Since $AB : PQ = AC : PR$, we have:

$$\triangle ABC \sim \triangle PQR \quad \text{(by SSS similarity criterion).}$$

4. Angles are equal: - From the similarity condition:

$$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R.$$

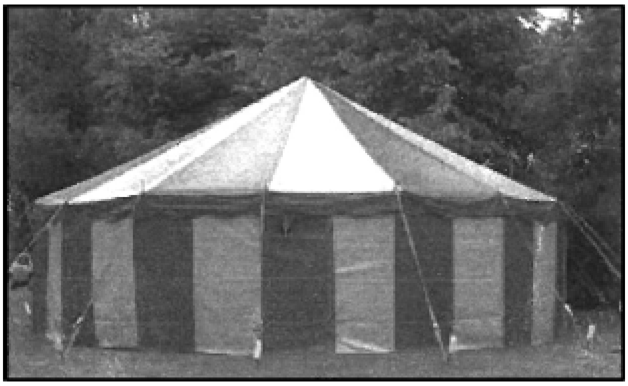
Thus, the triangles $\triangle ABC$ and $\triangle PQR$ are similar.

> **Quick Tip**
>
> To prove two triangles are similar, check proportionality of corresponding sides and equality of included angles (SSS or SAS similarity criteria).

Section E

36. The word 'circus' has the same root as 'circle.' In a closed circular area, various entertainment acts including human skill and animal training are presented before the crowd. A circus tent is cylindrical up to a height of $8\,\text{m}$ and conical above it. The diameter of the base is $28\,\text{m}$ and the total height of the tent is $18.5\,\text{m}$.
Based on the above, answer the following questions:

1. Find the slant height of the conical part.
2. Determine the floor area of the tent.
3. (a) Find the area of the cloth used for making the tent.
   (b) Find the total volume of air inside an empty tent.



Solution:
(i) Height of the conical part:

$$\text{Height of conical part} = \text{Total height} - \text{Cylindrical height} = 18.5 - 8 = 10.5\,\text{m}.$$

Radius of the conical part:

$$\text{Radius} = \frac{\text{Diameter}}{2} = \frac{28}{2} = 14\,\text{m}.$$

Slant height:

$$l = \sqrt{(10.5)^2 + (14)^2} = \sqrt{110.25 + 196} = \sqrt{306.25} = 17.5\,\text{m}.$$

(ii) Floor area of the tent:

$$\text{Floor area} = \pi r^2 = \frac{22}{7} \times 14 \times 14 = 616\,\text{m}^2.$$

(iii)(a) Area of cloth used for making the tent:

- Curved surface area of the cylindrical part:

$$\text{CSA (cylinder)} = 2\pi r h = 2 \times \frac{22}{7} \times 14 \times 8 = 704\,\text{m}^2.$$

- Curved surface area of the conical part:

$$\text{CSA (cone)} = \pi r l = \frac{22}{7} \times 14 \times 17.5 = 770\,\text{m}^2.$$

Total area of the cloth:

$$\text{Total cloth area} = 704 + 770 = 1474\,\text{m}^2.$$

(iii)(b) Volume of air inside the tent:

- Volume of the cylindrical part:

$$\text{Volume (cylinder)} = \pi r^2 h = \frac{22}{7} \times 14 \times 14 \times 8 = 4928\,\text{m}^3.$$

- Volume of the conical part:

$$\text{Volume (cone)} = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 14 \times 14 \times 10.5 = 2156\,\text{m}^3.$$

Total volume:

$$\text{Total volume} = 4928 + 2156 = 7084\,\text{m}^3.$$

Quick Tip

For composite shapes, calculate areas and volumes of individual parts separately and then sum them up.

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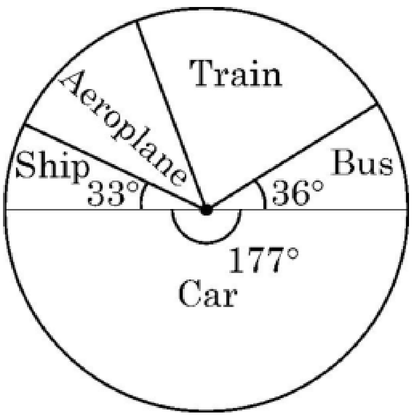
37. In a survey on holidays, 120 people were asked to state which type of transport they used on their last holiday. The following pie chart shows the results of the survey:



Observe the pie chart and answer the following questions:

(i) If one person is selected at random, find the probability that he/she travelled by bus or ship.

(ii) Which is the most favourite mode of transport, and how many people used it?

(iii) (a) A person is selected at random. If the probability that he did not use train is $\frac{4}{5}$, find the number of people who used train.
(b) The probability that a randomly selected person used aeroplane is $\frac{7}{60}$. Find the revenue collected by the air company at the rate of Rs. 5,000 per person.

Solution:
(i) Probability of travelling by bus or ship:

$$P(\text{bus or ship}) = \frac{\text{Angle for bus + Angle for ship}}{\text{Total angle}} = \frac{36^\circ + 33^\circ}{360^\circ} = \frac{69}{360} = \frac{23}{120}.$$

(ii) Car is the most favourite mode of transport.
Number of people who used cars:

$$\text{Number of people} = \frac{\text{Angle for car}}{\text{Total angle}} \times \text{Total people surveyed.}$$

Substitute:

$$\text{Number of people} = \frac{177}{360} \times 120 = 59.$$

(iii)(a) Probability of not using a train is:

$$P(\text{not using train}) = \frac{4}{5}.$$

Thus, probability of using a train:

$$P(\text{using train}) = 1 - \frac{4}{5} = \frac{1}{5}.$$

Number of people who used a train:

$$\text{Number of people} = \frac{1}{5} \times 120 = 24.$$

(iii)(b) Probability of using an aeroplane is:

$$P(\text{using aeroplane}) = \frac{7}{60}.$$

Number of people who used aeroplanes:

$$\text{Number of people} = \frac{7}{60} \times 120 = 14.$$

Revenue collected by the air company:

$$\text{Revenue} = 14 \times 5000 = Rs.70,000.$$

Quick Tip

To calculate the number of people based on probabilities or angles, use the formula:
Number of people = Fraction or Probability × Total surveyed people.

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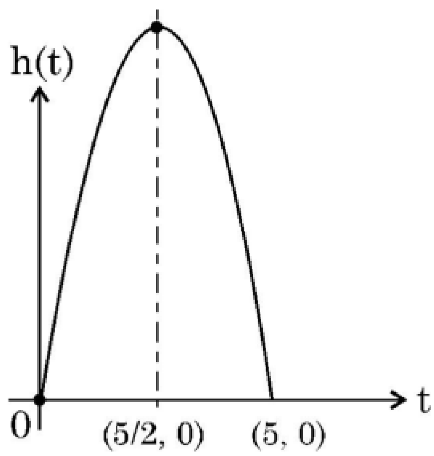
38. A ball is thrown in the air so that t seconds after it is thrown, its height h metre above its starting point is given by the polynomial $h = 25t - 5t^2$.



Observe the graph of the polynomial and answer the following questions:

(i) Write the zeroes of the given polynomial.

(ii) Find the maximum height achieved by the ball.

(iii) (a) After throwing upward, how much time did the ball take to reach the height of 30 m?
(b) Find the two different values of t when the height of the ball was 20 m.

Solution:
(i) The polynomial $h = 25t - 5t^2$ can be written as:

$$h = 5t(5 - t).$$

The zeroes of the polynomial are $t = 0$ and $t = 5$.
(ii) The maximum height of the ball occurs at the vertex of the parabola.
The formula for the time to reach the maximum height is:

$$t = -\frac{b}{2a},$$

where $a = -5$ and $b = 25$.
Substitute:

$$t = -\frac{25}{2(-5)} = \frac{25}{10} = 2.5\,\text{seconds}.$$

Maximum height:

$$h = 25t - 5t^2 = 25(2.5) - 5(2.5)^2.$$

Simplify:

$$h = 62.5 - 31.25 = 31.25\,\text{m}.$$

(iii)(a) To find the time when the height is 30 m, substitute $h = 30$:

$$30 = 25t - 5t^2.$$

Rearrange:

$$-5t^2 + 25t - 30 = 0.$$

Divide by -5:

$$t^2 - 5t + 6 = 0.$$

Factorize:

$$(t-3)(t-2)=0.$$

Thus, $t = 2$ seconds and $t = 3$ seconds.

(iii)(b) To find the time when the height is 20 m, substitute $h = 20$:

$$20 = 25t - 5t^2.$$

Rearrange:

$$-5t^2 + 25t - 20 = 0.$$

Divide by -5:

$$t^2 - 5t + 4 = 0.$$

Factorize:

$$(t-4)(t-1)=0.$$

Thus, $t = 1$ second and $t = 4$ seconds.

Quick Tip

For motion problems, use the quadratic formula or factoring to solve for time when the height is known. The vertex formula gives the maximum or minimum value.
