CBSE Class X Mathematics (Standard) Set 3 (30/1/3)

Time Allowed :3 Hours	Maximum Marks :80	Total Questions :38

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This question paper contains 38 questions. All questions are compulsory.
2. This Question Paper is divided into **FIVE Sections – Section A, B, C, D, and E**.
3. In Section–A, questions number 1 to 18 are **Multiple Choice Questions (MCQs)** and questions number 19 & 20 are **Assertion-Reason based questions**, carrying **1 mark each**.
4. In Section–B, questions number 21 to 25 are **Very Short-Answer (VSA)** type questions, carrying **2 marks each**.
5. In Section–C, questions number 26 to 31 are **Short Answer (SA)** type questions, carrying **3 marks each**.
6. In Section–D, questions number 32 to 35 are **Long Answer (LA)** type questions, carrying **5 marks each**.
7. In Section–E, questions number 36 to 38 are **Case Study based questions** carrying **4 marks each**. *Internal choice is provided in each case-study.*
8. There is **no overall choice.** However, *an internal choice has been provided in 2 questions in Section–B, 2 questions in Section–C, 2 questions in Section–D, and 3 questions in Section–E.*
9. Draw neat diagrams wherever required. Take $\pi = \frac{22}{7}$ wherever required, if not stated.
10. Use of calculators is **not allowed.**

Section - A

This section consists of 20 questions of 1 mark each.

Question 1: If two positive integers p and q can be expressed as $p = 18a^2b^4$ and $q = 20a^3b^2$, where a and b are prime numbers, then LCM(p, q) is:
(A) $2a^2b^2$
(B) $180a^2b^2$
(C) $12a^2b^2$
(D) $180a^3b^4$

Correct Answer: (D) $180a^3b^4$

Solution:
The Least Common Multiple (LCM) of two numbers is obtained by taking the highest power of each prime factor present in the given numbers.
Given:

$$p = 18a^2b^4 = 2 \cdot 3^2 \cdot a^2b^4$$

$$q = 20a^3b^2 = 2^2 \cdot 5 \cdot a^3b^2$$

To find the LCM: - For 2: The highest power is 2^2. - For 3: The highest power is 3^2. - For 5: The highest power is 5. - For a: The highest power is a^3. - For b: The highest power is b^4.
Thus, the LCM is:

$$\text{LCM}(p, q) = 2^2 \cdot 3^2 \cdot 5 \cdot a^3b^4$$

Simplify:

$$\text{LCM}(p, q) = 180a^3b^4$$

Quick Tip

To find the LCM of two numbers, take the highest powers of all prime factors present in the numbers.

Question 2: In an A.P., if the first term (a) = -16 and the common difference (d) = -2, then the sum of the first 10 terms is:
(a) -200
(b) -70
(c) -250
(d) 250

Correct Answer: (c) -250

Solution:

The sum of the first n terms of an A.P. is given by:

$$S_n = \frac{n}{2} \left[2a + (n-1)d\right].$$

Substitute the given values:

$$a = -16,\ d = -2,\ n = 10.$$

$$S_{10} = \frac{10}{2} \left[2(-16) + (10-1)(-2)\right].$$

Simplify:

$$S_{10} = 5\left[-32 + (-18)\right] = 5 \times (-50) = -250.$$

—

Conclusion:
The sum of the first 10 terms of the A.P. is -250.

Quick Tip

The sum of n terms in an A.P. depends on both the first term and the common difference. Always double-check calculations for the correct signs.

Question 3: For some data $x_1, x_2, \ldots, x_n$ with respective frequencies $f_1, f_2, \ldots, f_n$, the value of $\sum_1^n f_i\,(x_i - \bar{x})$ is equal to:
(A) $n\bar{x}$
(B) 1
(C) $\sum f_i$
(D) 0

Correct Answer: (D) 0

Solution:
1. The term $\bar{x}$ is the mean of the data, defined as:

$$\bar{x} = \frac{\sum_{i=1}^n f_i x_i}{\sum_{i=1}^n f_i}.$$

2. The expression $\sum_{i=1}^n f_i(x_i - \bar{x})$ represents the sum of deviations of the data points x_i from the mean, weighted by their respective frequencies f_i.
3. By the definition of the mean, the sum of weighted deviations from the mean is always zero:

$$\sum_{i=1}^{n} f_i(x_i - \bar{x}) = 0.$$

Thus, the value of $\sum_{i=1}^n f_i(x_i - \bar{x})$ is 0.

Quick Tip

The sum of deviations of observations from their mean, when weighted by frequencies, is always zero.

Question 4: The volume of the largest right circular cone that can be carved out from a solid cube of edge 2 cm is:
(A) $\frac{4\pi}{3}$ cu cm
(B) $\frac{5\pi}{3}$ cu cm
(C) $\frac{8\pi}{3}$ cu cm
(D) $\frac{2\pi}{3}$ cu cm

Correct Answer: (D) $\frac{2\pi}{3}$ cu cm

Solution:
1. For the largest cone carved out from a cube, the height of the cone is equal to the edge of the cube, and the base diameter is also equal to the edge of the cube.

Given:

$$\text{Edge of the cube} = 2\,\text{cm}.$$

- Height of the cone $h = 2\,\text{cm}$ - Radius of the cone $r = \frac{\text{diameter}}{2} = \frac{2}{2} = 1\,\text{cm}$.

2. The volume of a cone is given by:

$$V = \frac{1}{3}\pi r^2 h.$$

Substitute $r = 1\,\text{cm}$ and $h = 2\,\text{cm}$:

$$V = \frac{1}{3}\pi(1)^2(2).$$

Simplify:

$$V = \frac{2\pi}{3}\,\text{cu cm}.$$

Thus, the volume of the largest right circular cone is $\frac{2\pi}{3}\,\text{cu cm}$.

Quick Tip

The largest cone that fits inside a cube has its height equal to the edge of the cube and its radius as half the edge length.

Question 5: A solid sphere is cut into two hemispheres. The ratio of the surface areas of the sphere to that of the two hemispheres taken together is:

(A) 1 : 1
(B) 1 : 4
(C) 2 : 3
(D) 3 : 2

Correct Answer: (C) 2 : 3

Solution:

1. The surface area of a sphere is given by:

$$\text{Surface Area of Sphere} = 4\pi r^2.$$

2. When the sphere is cut into two hemispheres: - The curved surface area of one hemisphere $= 2\pi r^2$. - Each hemisphere also has a flat circular face with area πr^2.
Thus, the total surface area of the two hemispheres is:

$$\text{Total Surface Area} = 2 \times \left(2\pi r^2 + \pi r^2\right) = 2 \times 3\pi r^2 = 6\pi r^2.$$

3. The ratio of the surface area of the sphere to that of the two hemispheres is:

$$\text{Ratio} = \frac{\text{Surface Area of Sphere}}{\text{Surface Area of Two Hemispheres}} = \frac{4\pi r^2}{6\pi r^2} = \frac{2}{3}.$$

Thus, the ratio is 2 : 3.

Quick Tip

When dividing a sphere into hemispheres, account for both the curved surface area and the flat circular faces to determine the total surface area.

Question 6: The centre of a circle is at $(2, 0)$. If one end of a diameter is at $(6, 0)$, then the other end is at:
(A) $(0, 0)$
(B) $(4, 0)$
(C) $(-2, 0)$
(D) $(-6, 0)$

Correct Answer: (C) $(-2, 0)$

Solution:
1. The centre of a circle is the midpoint of its diameter. Let the two ends of the diameter be $(x_1, y_1) = (6, 0)$ and (x_2, y_2).
The midpoint formula is:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$

2. Given that the midpoint (centre) is $(2, 0)$, substitute the known values:

$$\left(\frac{6 + x_2}{2}, \frac{0 + y_2}{2} \right) = (2, 0).$$

3. Equating the coordinates: - For the x-coordinate:

$$\frac{6 + x_2}{2} = 2 \implies 6 + x_2 = 4 \implies x_2 = -2.$$

- For the y-coordinate:

$$\frac{0 + y_2}{2} = 0 \implies y_2 = 0.$$

4. Therefore, the other end of the diameter is:

$$(-2, 0).$$

Quick Tip

The centre of a circle is the midpoint of its diameter. Use the midpoint formula to determine missing coordinates.

Question 7: One card is drawn at random from a well-shuffled deck of 52 playing cards. The probability that it is a red ace card, is:
(a) $\frac{1}{13}$
(b) $\frac{1}{26}$
(c) $\frac{1}{52}$
(d) $\frac{1}{2}$

Correct Answer: (b) $\frac{1}{26}$

Solution:

A standard deck of 52 playing cards contains 2 red ace cards (Ace of Hearts and Ace of Diamonds).
The probability of drawing one red ace card is given by:

$$P(\text{Red Ace}) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}.$$

Substitute the values:

$$P(\text{Red Ace}) = \frac{2}{52}.$$

Simplify:

$$P(\text{Red Ace}) = \frac{1}{26}.$$

—

Conclusion:
The probability of drawing a red ace card is $\frac{1}{26}$.

> Quick Tip
>
> For probability questions involving cards, always determine the total number of favorable outcomes based on the specific category of cards.

Question 8: The middle most observation of every data arranged in order is called:
(A) mode
(B) median
(C) mean
(D) deviation

Correct Answer: (B) median

Solution:
The **median** is the middle value of a dataset when all observations are arranged in ascending or descending order.
- If the number of observations (n) is odd, the median is the middle value. - If n is even, the median is the average of the two middle values.
The median divides the dataset into two equal halves.

> Quick Tip
>
> The median is used as a measure of central tendency that is not affected by extreme values or outliers.

Question 9: For $\theta = 30^\circ$, the value of $2\sin\theta\cos\theta$ is:
(a) 1
(b) $\frac{\sqrt{3}}{2}$

(c) $\frac{\sqrt{3}}{4}$
(d) $\frac{3}{2}$

Correct Answer: (b) $\frac{\sqrt{3}}{2}$

Solution:

Using the trigonometric identity:

$$2\sin\theta\cos\theta = \sin(2\theta).$$

Substitute $\theta = 30^\circ$:

$$\sin(2 \times 30^\circ) = \sin(60^\circ).$$

The value of $\sin(60^\circ)$ is:

$$\sin(60^\circ) = \frac{\sqrt{3}}{2}.$$

—

Conclusion:
The value of $2\sin\theta\cos\theta$ for $\theta = 30^\circ$ is $\frac{\sqrt{3}}{2}$.

Quick Tip

Use trigonometric identities like $2\sin\theta\cos\theta = \sin(2\theta)$ to simplify such expressions efficiently.

Question 10: If the roots of the equation $ax^2 + bx + c = 0$, $a \neq 0$ are real and equal, then which of the following relations is true?
(A) $a = \frac{b^2}{c}$
(B) $b^2 = ac$
(C) $ac = \frac{b^2}{4}$
(D) $c = \frac{b^2}{a}$

Correct Answer: (C) $ac = \frac{b^2}{4}$

Solution:
For a quadratic equation $ax^2 + bx + c = 0$, the condition for the roots to be real and equal is that the discriminant must be zero:

$$\Delta = b^2 - 4ac = 0$$

Simplifying for ac:

$$b^2 = 4ac$$

Rearranging:

$$ac = \frac{b^2}{4}$$

Thus, the correct relation is:

$$ac = \frac{b^2}{4}.$$

Quick Tip

For real and equal roots of a quadratic equation, the discriminant condition $b^2 - 4ac = 0$ simplifies to $ac = \frac{b^2}{4}$.

Question 11: From the data $1, 4, 7, 9, 16, 21, 25$, if all the even numbers are removed, then the probability of getting at random a prime number from the remaining is:

(A) $\frac{2}{5}$
(B) $\frac{1}{5}$
(C) $\frac{1}{7}$
(D) $\frac{2}{7}$

Correct Answer: (B) $\frac{1}{5}$

Solution:

1. The original data set is $\{1, 4, 7, 9, 16, 21, 25\}$. First, remove all the even numbers 4 and 16:

$$\text{Remaining data} = \{1, 7, 9, 21, 25\}.$$

2. Identify the prime numbers in the remaining set: - Prime numbers are numbers greater than 1 that have no divisors other than 1 and itself. - 7 is the only prime number in the set $\{1, 7, 9, 21, 25\}$.
3. Total numbers remaining = 5. Number of favorable outcomes (prime numbers) = 1.
The probability of selecting a prime number is:

$$P = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{1}{5}.$$

Thus, the probability is $\frac{1}{5}$.

Quick Tip

To determine probabilities, count the total outcomes and favorable outcomes. For prime numbers, check divisors carefully.

Question 12: AD is a median of $\triangle ABC$ with vertices $A(5, -6)$, $B(6, 4)$, and $C(0, 0)$. Length AD is equal to:

(A) $\sqrt{68}$ units
(B) $2\sqrt{15}$ units
(C) $\sqrt{101}$ units
(D) 10 units

Correct Answer: (A) $\sqrt{68}$

Solution:
To find the length of the median AD, we follow these steps:
1. Find the midpoint D of BC:
The midpoint formula is:

$$D(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

For $B(6, 4)$ and $C(0, 0)$:

$$D = \left(\frac{6+0}{2}, \frac{4+0}{2} \right) = (3, 2).$$

2. Find the distance AD:
The distance formula is:

$$AD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Here, $A(5, -6)$ and $D(3, 2)$:

$$AD = \sqrt{(3-5)^2 + (2-(-6))^2}$$

Simplify:

$$AD = \sqrt{(-2)^2 + (8)^2}$$
$$AD = \sqrt{4 + 64} = \sqrt{68}.$$

Thus, the length of AD is $\sqrt{68}$ units.

> **Quick Tip**
>
> To calculate the median length in a triangle, find the midpoint of the opposite side using the midpoint formula and then calculate the distance using the distance formula.

Question 13: Two dice are rolled together. The probability of getting the sum of numbers on the two dice as $2, 3,$ or 5 is:
(A) $\frac{7}{36}$
(B) $\frac{11}{36}$
(C) $\frac{5}{36}$
(D) $\frac{4}{9}$

Correct Answer: (A) $\frac{7}{36}$

Solution:
1. When two dice are rolled, the total number of possible outcomes is:

$$6 \times 6 = 36.$$

2. Now, we count the favorable outcomes for sums $2, 3,$ and 5:
- Sum $= 2$: Only one outcome: $(1, 1)$
- Sum $= 3$: Two outcomes: $(1, 2), (2, 1)$
- Sum $= 5$: Four outcomes: $(1, 4), (2, 3), (3, 2), (4, 1)$

3. Total favorable outcomes:

$$1 + 2 + 4 = 7.$$

4. The probability is:

$$P = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{7}{36}.$$

Thus, the probability of getting a sum of $2, 3,$ or 5 is $\frac{7}{36}$.

Quick Tip

To solve dice probability questions, list all possible outcomes and count favorable ones carefully. Remember, for two dice, the total outcomes are always $6 \times 6 = 36$.

Question 14: If the distance between the points $(3, -5)$ and $(x, -5)$ is 15 units, then the values of x are:

(A) $12, -18$
(B) $-12, 18$
(C) $18, 5$
(D) $-9, -12$

Correct Answer: (B) $-12, 18$

Solution:

The distance between two points (x_1, y_1) and (x_2, y_2) is given by:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Here:

$$(3, -5) \quad \text{and} \quad (x, -5), \quad d = 15.$$

Substitute the coordinates into the distance formula:

$$15 = \sqrt{(x - 3)^2 + (-5 - (-5))^2}.$$

Simplify:

$$15 = \sqrt{(x - 3)^2 + 0}.$$

$$15 = |x - 3|.$$

This gives two equations:

$$x - 3 = 15 \quad \text{and} \quad x - 3 = -15.$$

Solve for x:

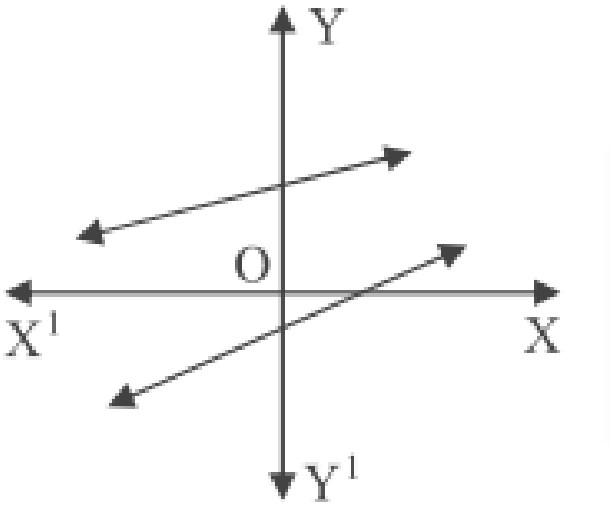
$$x = 18 \quad \text{and} \quad x = -12.$$

Thus, the values of x are -12 and 18.

Quick Tip

When the distance between two points involves one coordinate being equal, simplify using absolute values for the remaining coordinate.

Question 15: In the given figure, graphs of two linear equations are shown. The pair of these linear equations is:
(A) consistent with unique solution.
(B) consistent with infinitely many solutions.
(C) inconsistent.
(D) inconsistent but can be made consistent by extending these lines.



Correct Answer: (A) consistent with unique solution

Solution:
1. In the given figure, two lines intersect at a single point O.
2. When two lines intersect at exactly one point, the pair of linear equations is said to be:

consistent with a unique solution.

This means that the system of equations has exactly one solution corresponding to the point of intersection.
3. Other cases for linear equations are: - Consistent with infinitely many solutions: This occurs when the two lines are coincident (overlap completely). - Inconsistent: This occurs when the two lines are parallel and never intersect.
Since the lines in the figure intersect at a single point, the correct answer is:

(A) consistent with unique solution.

Quick Tip

For two linear equations, if their graphs intersect at one point, the equations are consistent and have a unique solution.

Question 16: If α, β are the zeroes of the polynomial $6x^2 - 5x - 4$, then $\frac{1}{\alpha} + \frac{1}{\beta}$ is equal to:
(a) $\frac{5}{4}$
(b) $-\frac{5}{4}$
(c) $\frac{4}{5}$
(d) $\frac{5}{24}$

Correct Answer: (b) $-\frac{5}{4}$

Solution:

We know the relation for the reciprocals of the roots:

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}.$$

From the polynomial $6x^2 - 5x - 4$, we find:

$$\alpha + \beta = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2} = -\frac{-5}{6} = \frac{5}{6},$$

$$\alpha\beta = \frac{\text{constant term}}{\text{coefficient of } x^2} = \frac{-4}{6} = -\frac{2}{3}.$$

Substitute into the formula:

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{\frac{5}{6}}{-\frac{2}{3}}.$$

Simplify:

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{5}{6} \times \frac{-3}{2} = -\frac{15}{12} = -\frac{5}{4}.$$

—

Conclusion:
The value of $\frac{1}{\alpha} + \frac{1}{\beta}$ is $-\frac{5}{4}$.

Quick Tip

Use the relationships between the coefficients of the polynomial and the roots, such as $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$, to solve such problems efficiently.

Question 17: If $\sec\theta - \tan\theta = m$, then the value of $\sec\theta + \tan\theta$ is:
(A) $1 - \frac{1}{m}$
(B) $m^2 - 1$

(C) $\frac{1}{m}$
(D) $-m$

Correct Answer: (C) $\frac{1}{m}$

Solution:
Given:

$$\sec\theta - \tan\theta = m$$

We need to find $\sec\theta + \tan\theta$. Let:

$$x = \sec\theta - \tan\theta \quad \text{and} \quad y = \sec\theta + \tan\theta.$$

The product of these two expressions is:

$$x \cdot y = (\sec\theta - \tan\theta)(\sec\theta + \tan\theta)$$

Using the identity $(a-b)(a+b) = a^2 - b^2$, we get:

$$x \cdot y = \sec^2\theta - \tan^2\theta$$

From the Pythagorean identity:

$$\sec^2\theta - \tan^2\theta = 1$$

Thus:

$$x \cdot y = 1$$

Substitute $x = m$:

$$m \cdot y = 1 \implies y = \frac{1}{m}.$$

Therefore, $\sec\theta + \tan\theta = \frac{1}{m}$.

> Quick Tip
>
> For expressions involving $\sec\theta$ and $\tan\theta$, use identities like $\sec^2\theta - \tan^2\theta = 1$ to simplify the calculations.

Question 18: The zeroes of a polynomial $x^2 + px + q$ are twice the zeroes of the polynomial $4x^2 - 5x - 6$. The value of p is:
(A) $-\frac{5}{2}$
(B) $\frac{5}{2}$
(C) -5
(D) 10

Correct Answer: (A) $-\frac{5}{2}$

Solution:
1. The given polynomial is $4x^2 - 5x - 6$. To find its roots, use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here:

$$a = 4,\ b = -5,\ c = -6$$

Substitute the values:

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(4)(-6)}}{2(4)}$$

Simplify:

$$x = \frac{5 \pm \sqrt{25 + 96}}{8}$$

$$x = \frac{5 \pm \sqrt{121}}{8}$$

$$x = \frac{5 + 11}{8} = 2, \quad x = \frac{5 - 11}{8} = -\frac{3}{4}.$$

2. Since the zeroes of $x^2 + px + q$ are twice the zeroes of $4x^2 - 5x - 6$, multiply the roots by 2:

$$\text{New roots: } 2(2) = 4 \quad \text{and} \quad 2\left(-\frac{3}{4}\right) = -\frac{3}{2}.$$

3. For a quadratic polynomial with roots α and β, the sum of roots is:

$$\alpha + \beta = -p.$$

Here, the sum of roots is:

$$4 + \left(-\frac{3}{2}\right) = \frac{8}{2} - \frac{3}{2} = \frac{5}{2}.$$

Thus:

$$-p = \frac{5}{2} \implies p = -\frac{5}{2}.$$

Quick Tip

To find roots of a quadratic polynomial, use the quadratic formula. Scaling roots requires careful multiplication.

Directions:
In Q. No. 19 and 20 a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option.

- (a) Both, Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of Assertion (A).
- (b) Both, Assertion (A) and Reason (R) are true but Reason (R) is not correct explanation for Assertion (A).
- (c) Assertion (A) is true but Reason (R) is false.
- (d) Assertion (A) is false but Reason (R) is true.

Question 19:Assertion (A): The tangents drawn at the end points of a diameter of a circle are parallel.
Reason (R): Diameter of a circle is the longest chord.

Correct Answer: (b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation for Assertion (A).

Solution:
1. The tangents at the endpoints of the diameter of a circle are indeed parallel. This is because:
- The tangents to a circle are perpendicular to the radius at the point of contact.
- At the endpoints of the diameter, the radii are collinear, and hence the tangents are parallel.

Thus, Assertion (A) is true.

2. The Reason (R), stating that the diameter of a circle is the longest chord, is also true. However:
- The fact that the tangents are parallel is not explained by the diameter being the longest chord.

Hence, both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation for Assertion (A).

Quick Tip

Tangents to a circle at the endpoints of a diameter are always parallel because they are perpendicular to collinear radii.

Question 20:Assertion (A): If the graph of a polynomial touches the x-axis at only one point, then the polynomial cannot be a quadratic polynomial.
Reason (R): A polynomial of degree $n(n > 1)$ can have at most n zeroes.

Correct Answer: (d) Assertion (A) is false but Reason (R) is true.

Solution:
1. The graph of a quadratic polynomial $y = ax^2 + bx + c$ can touch the x-axis at one point when it has real and equal roots. This happens when:

$$\Delta = b^2 - 4ac = 0.$$

Therefore, the polynomial can still be quadratic even if it touches the x-axis at only one point. Thus, Assertion (A) is **false**.

2. The Reason (R) states that a polynomial of degree $n(n > 1)$ can have at most n zeroes. This is a correct mathematical fact:
- A polynomial of degree n can have at most n real roots (including multiplicity).

Hence, Assertion (A) is false, but Reason (R) is true.

Quick Tip

For quadratic polynomials, a repeated root causes the graph to touch the x-axis at exactly one point. A polynomial of degree n cannot have more than n zeroes.

Section - B

This section consists of 5 questions of 2 mark each.

Question 21: In a pack of 52 playing cards one card is lost. From the remaining cards, a card is drawn at random. Find the probability that the drawn card is the queen of hearts, if the lost card is a black card.

Solution:
1. Total number of cards in a deck = 52.
2. One card is lost, and it is given that the lost card is a black card. Therefore: - Total remaining cards = $52-1=51$. - The queen of hearts is a red card, so it is still in the remaining deck.
3. The probability of drawing the queen of hearts from the remaining cards is:

$$P(\text{Queen of hearts}) = \frac{\text{Number of favorable outcomes}}{\text{Total remaining cards}}.$$

- Favorable outcomes = 1 (queen of hearts). - Total remaining cards = 51.
Thus:

$$P(\text{Queen of hearts}) = \frac{1}{51}.$$

Quick Tip

For conditional probabilities involving cards, carefully analyze the remaining cards based on the given condition, and count the favorable outcomes.

Question 22 (A): Evaluate: $2\sqrt{2}\cos 45^\circ \sin 30^\circ + 2\sqrt{3}\cos 30^\circ$

Solution:
We are given:

$$2\sqrt{2}\cos 45^\circ \sin 30^\circ + 2\sqrt{3}\cos 30^\circ.$$

1. Substitute the trigonometric values:

$$\cos 45^\circ = \frac{1}{\sqrt{2}}, \quad \sin 30^\circ = \frac{1}{2}, \quad \cos 30^\circ = \frac{\sqrt{3}}{2}.$$

2. Simplify the first term:

$$2\sqrt{2}\cos 45^\circ \sin 30^\circ = 2\sqrt{2}\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{2} = 2\cdot\frac{1}{2} = 1.$$

3. Simplify the second term:

$$2\sqrt{3}\cos 30^\circ = 2\sqrt{3} \cdot \frac{\sqrt{3}}{2} = 3.$$

4. Add the simplified terms:

$$1 + 3 = 4.$$

> **Quick Tip**
>
> To evaluate trigonometric expressions, substitute exact trigonometric values for standard angles and simplify step by step.

Question 22 (B): If $A = 60^\circ$ and $B = 30^\circ$, verify that:

$$\sin(A + B) = \sin A \cos B + \cos A \sin B.$$

Solution:

1. Write the given equation:

$$\sin(A + B) = \sin A \cos B + \cos A \sin B.$$

2. Substitute $A = 60^\circ$ and $B = 30^\circ$: - $\sin 60^\circ = \frac{\sqrt{3}}{2}$, $\cos 60^\circ = \frac{1}{2}$,

- $\sin 30^\circ = \frac{1}{2}$, $\cos 30^\circ = \frac{\sqrt{3}}{2}$.

3. Evaluate both sides:
- LHS: $\sin(A + B) = \sin 90^\circ = 1$.

- RHS:

$$\sin A \cos B + \cos A \sin B = \left(\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2} \cdot \frac{1}{2}\right).$$

Simplify each term:

$$\sin A \cos B + \cos A \sin B = \frac{3}{4} + \frac{1}{4} = 1.$$

4. Compare LHS and RHS:

$$\sin(A + B) = 1 \quad \text{and} \quad \sin A \cos B + \cos A \sin B = 1.$$

Thus, the identity is verified.

> **Quick Tip**
>
> The angle addition formulas, such as $\sin(A+B) = \sin A \cos B + \cos A \sin B$, are fundamental trigonometric identities. Always substitute known values and simplify step by step.

Question 23 (A): Prove that $5 - 2\sqrt{3}$ is an irrational number. It is given that $\sqrt{3}$ is an irrational number.

Solution:
To prove $5 - 2\sqrt{3}$ is irrational, we use a proof by contradiction.
1. Assume $5 - 2\sqrt{3}$ is rational. Then, it can be expressed as:

$$5 - 2\sqrt{3} = \frac{p}{q},$$

where p and q are integers and $q \neq 0$ (i.e., a rational number).
2. Rearrange to isolate $\sqrt{3}$:

$$2\sqrt{3} = 5 - \frac{p}{q}.$$

Simplify the right-hand side:

$$2\sqrt{3} = \frac{5q - p}{q}.$$

Divide through by 2:

$$\sqrt{3} = \frac{5q - p}{2q}.$$

3. The right-hand side is a ratio of integers, so it is rational. This implies $\sqrt{3}$ is rational.
4. However, this contradicts the given statement that $\sqrt{3}$ is irrational.

Thus, our assumption that $5 - 2\sqrt{3}$ is rational is false. Therefore:

$$5 - 2\sqrt{3} \text{ is an irrational number.}$$

> **Quick Tip**
>
> To prove a number involving an irrational term is irrational, assume it is rational and show a contradiction with the properties of irrational numbers.

Question 23 (B): Show that the number $5 \times 11 \times 17 + 3 \times 11$ is a composite number.

Solution:
To prove that $5 \times 11 \times 17 + 3 \times 11$ is composite, we simplify the expression.
1. Factorize the given expression:

$$5 \times 11 \times 17 + 3 \times 11.$$

Take 11 as a common factor:

$$11(5 \times 17 + 3).$$

2. Simplify the term inside the parentheses:

$$5 \times 17 + 3 = 85 + 3 = 88.$$

Thus, the expression becomes:

$$11 \times 88.$$

3. A composite number is defined as a positive integer that has factors other than 1 and itself. Here: - 11 and 88 are both greater than 1, and they are factors of the number.
Therefore, 11×88 is a composite number.

> **Quick Tip**
>
> To prove a number is composite, factorize it and check if it has factors other than 1 and itself.

Question 24: Solve the following system of linear equations algebraically:
$2x + 5y = -4$
$4x - 3y = 5$

Solution:
Given equations:

$$2x + 5y = -4 \quad \text{(i)}$$

$$4x - 3y = 5 \quad \text{(ii)}.$$

Step 1: Multiply equation (i) by 2 to make the coefficients of x equal:

$$4x + 10y = -8 \quad \text{(iii)}.$$

Step 2: Subtract equation (ii) from equation (iii):

$$(4x + 10y) - (4x - 3y) = -8 - 5,$$

$$13y = -13 \quad \Rightarrow \quad y = -1.$$

Step 3: Substitute $y = -1$ into equation (i):

$$2x + 5(-1) = -4,$$

$$2x - 5 = -4 \quad \Rightarrow \quad 2x = 1 \quad \Rightarrow \quad x = \frac{1}{2}.$$

—

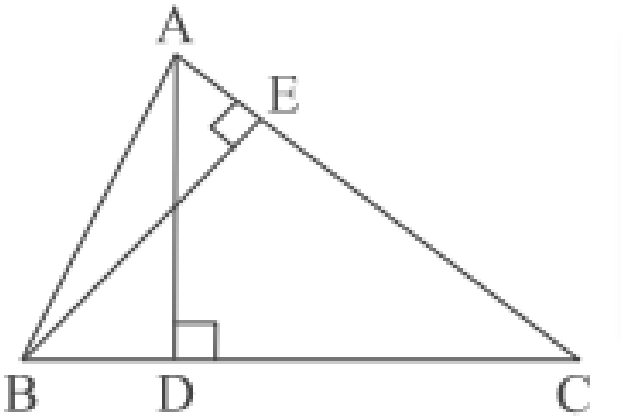
Conclusion:
The solution to the given system of linear equations is:

$$x = \frac{1}{2},\ y = -1.$$

Quick Tip

When solving a system of linear equations, eliminate one variable by equating coefficients. Simplify step-by-step to avoid calculation errors.

Question 25: In $\triangle ABC$, altitudes AD and BE are drawn. If $AD = 7\,\text{cm}$, $BE = 9\,\text{cm}$, and $EC = 12\,\text{cm}$, then find the length of CD.



Solution:
In $\triangle ABC$, the triangles $\triangle BEC$ and $\triangle ADC$ are similar because they share the same angles. Using the property of similar triangles:

$$\frac{BE}{AD} = \frac{EC}{CD}.$$

Substitute the given values:

$$\frac{9}{7} = \frac{12}{CD}.$$

Rearrange to solve for CD:

$$CD = \frac{12 \times 7}{9} = \frac{84}{9} = \frac{28}{3}\,\text{cm}.$$

Convert to decimal:

$$CD \approx 9.33\,\text{cm}.$$

—

Conclusion:
The length of CD is $\frac{28}{3}$ cm or approximately 9.33 cm.

Quick Tip

When dealing with similar triangles, use the proportionality of corresponding sides to find unknown lengths. Check for shared angles or parallel lines to confirm similarity.

Section - C

This section consists of 6 questions of 3 mark each.

Question 26: The sum of the digits of a 2-digit number is 14. The number obtained by interchanging its digits exceeds the given number by 18. Find the number.

Solution:
Let the two-digit number be $10x + y$, where x and y are the tens and unit digits, respectively. Given:

$$x + y = 14 \quad \text{(i)}$$

The number obtained by interchanging its digits is $10y + x$. This number exceeds the original number by 18:

$$10y + x - (10x + y) = 18,$$

$$9y - 9x = 18 \quad \Rightarrow \quad y - x = 2 \quad \text{(ii)}.$$

Step 1: Solve equations (i) and (ii) simultaneously. From equation (ii):

$$y = x + 2 \quad \text{(iii)}.$$

Step 2: Substitute equation (iii) into equation (i):

$$x + (x + 2) = 14,$$

$$2x + 2 = 14 \quad \Rightarrow \quad 2x = 12 \quad \Rightarrow \quad x = 6.$$

Step 3: Substitute $x = 6$ into equation (iii):

$$y = 6 + 2 = 8.$$

Thus, the number is:

$$10x + y = 10(6) + 8 = 68.$$

—

Conclusion:
The required number is 68.

> Quick Tip
>
> When solving digit-based problems, represent the number in terms of tens and units, and use the given conditions systematically.

Question 27: The inner and outer radii of a hollow cylinder surmounted on a hollow hemisphere of the same radii are 3 cm and 4 cm, respectively. If the height of the cylinder is 14 cm, then find its total surface area (inner and outer).

Solution:
Step 1: Formula for Total Surface Area.

The total surface area consists of:

$$\text{Total Surface Area} = \text{Inner CSA} + \text{Outer CSA} + \text{Area of Rings}.$$

—

Step 2: Calculate each component.
1. **Inner Curved Surface Area (CSA):**

$$\text{Inner CSA} = 2\pi r_{\text{inner}}(r_{\text{inner}} + h),$$
$$= 2 \times \frac{22}{7} \times 3 \times (3 + 14) = \frac{2244}{7}.$$

2. **Outer Curved Surface Area (CSA):**

$$\text{Outer CSA} = 2\pi r_{\text{outer}}(r_{\text{outer}} + h),$$
$$= 2 \times \frac{22}{7} \times 4 \times (4 + 14) = \frac{3168}{7}.$$

3. **Area of Rings:**

$$\text{Area of Rings} = 2\pi(r_{\text{outer}}^2 - r_{\text{inner}}^2),$$
$$= 2 \times \frac{22}{7} \times (16 - 9) = 22.$$

—

Step 3: Combine the areas.

$$\text{Total Surface Area} = \frac{2244}{7} + \frac{3168}{7} + 22,$$
$$= \frac{5566}{7}\,\text{cm}^2.$$

Simplify:

$$\text{Total Surface Area} \approx 795.14\,\text{cm}^2.$$

—

Conclusion:
The total surface area of the hollow structure is approximately $795.14\,\text{cm}^2$.

> Quick Tip
>
> When solving 3D geometry problems, break the total surface area into inner, outer, and additional components (e.g., rings or caps), and calculate systematically.

Question 28: In a teachers' workshop, the number of teachers teaching French, Hindi, and English are 48, 80, and 144 respectively. Find the minimum number of rooms required if in each room the same number of teachers are seated and all of them are of the same subject.

Solution:
To find the minimum number of rooms, we determine the greatest common divisor (GCD) of 48, 80, and 144.

Step 1: Find the prime factorizations:

$$48 = 2^4 \times 3, \quad 80 = 2^4 \times 5, \quad 144 = 2^4 \times 3^2.$$

Step 2: Identify the common factors: The highest power of 2 common to all three numbers is 2^4.
Thus, the GCD of $48, 80$, and 144 is:

$$\text{GCD} = 2^4 = 16.$$

Step 3: Calculate the number of rooms required:
To seat all teachers with 16 teachers per room:
- French teachers: $\frac{48}{16} = 3$ rooms,
- Hindi teachers: $\frac{80}{16} = 5$ rooms,
- English teachers: $\frac{144}{16} = 9$ rooms.

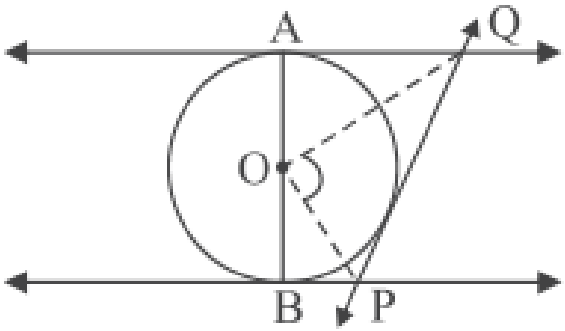
Total number of rooms:

$$3 + 5 + 9 = 17.$$

Quick Tip

To solve such problems, determine the GCD of the numbers to ensure equal group sizes and divide each value by the GCD.

Question 29 (A): In the given figure, AB is a diameter of the circle with centre O. AQ, BP, and PQ are tangents to the circle. Prove that $\angle POQ = 90°$.



Solution:
1. Given AB is the diameter of the circle, the centre O lies on the line AB. By the property of a circle:

The angle subtended by the diameter at the circle is $90°$.

2. AQ, BP, and PQ are tangents to the circle: - Tangents drawn from an external point to a circle are equal in length. Therefore:

$$AQ = AP \quad \text{and} \quad BP = BQ.$$

3. Consider the points P and Q: - Since AQ and BP are tangents, they form $90°$ angles with the radii OA and OB at the points of tangency.

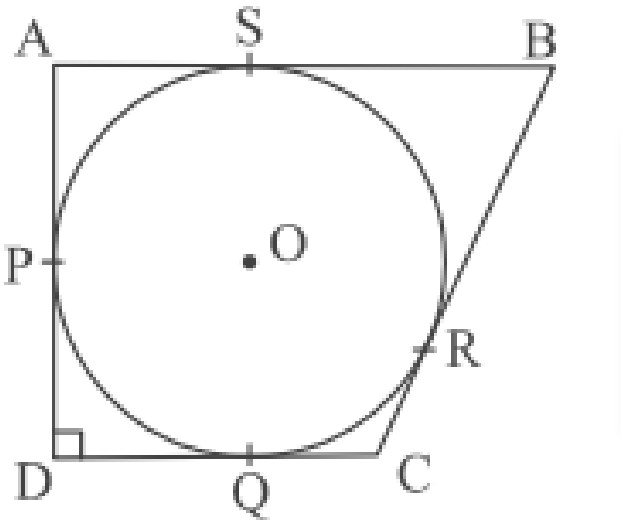
4. Triangles OAP and OBQ are right-angled triangles. The line PQ joins the points of tangency: - $\angle POQ$ is formed by joining OP and OQ.
5. Since AB is the diameter, the angle between the radii OP and OQ becomes:

$$\angle POQ = 90^\circ.$$

Thus, $\angle POQ$ is proved to be 90°.

> Quick Tip
>
> The angle subtended by a diameter at any point on the circle is always 90°, a property of a semicircle.

Question 29 (B): A circle with centre O and radius $8\,\text{cm}$ is inscribed in a quadrilateral $ABCD$ in which P, Q, R, S are the points of contact as shown. If AD is perpendicular to DC, $BC = 30\,\text{cm}$, and $BS = 24\,\text{cm}$, then find the length DC.



Solution:
1. Property of tangents from external points: Tangents drawn from a point to a circle are equal in length. Therefore:

$$AP = AS, \quad BP = BR, \quad CR = CQ, \quad DQ = DP.$$

2. Given: - $BC = 30\,\text{cm}$, - $BS = 24\,\text{cm}$.
From B, the tangents BS and BR are equal. Therefore:

$$BR = 24\,\text{cm}.$$

3. Since the total length BC is the sum of BR and CR:

$$BR + CR = 30.$$

Substitute $BR = 24$:

$$24 + CR = 30 \quad \Rightarrow \quad CR = 6\,\text{cm}.$$

4. Now calculate DC: - Tangents DQ and CQ from points D and C are equal. Since $CR = CQ = 6\,\text{cm}$, we set:

$$DQ = 8\,\text{cm} \quad \text{(tangents from } D\text{)}.$$

Thus, the length DC is:

$$DC = DQ + CQ = 8 + 6 = 14\,\text{cm}.$$

Final Answer: $DC = 14\,\textbf{cm}$.

> Quick Tip
>
> For problems involving circles inscribed in quadrilaterals, use the property that tangents drawn from the same external point are equal.

Question 30 (A): Find the ratio in which the point $\left(\frac{8}{5}, y\right)$ divides the line segment joining the points $(1, 2)$ and $(2, 3)$. Also, find the value of y.

Solution:

Let the point $P\left(\frac{8}{5}, y\right)$ divide the line segment joining $A(1, 2)$ and $B(2, 3)$ in the ratio $m_1 : m_2$.

1. Use the section formula to find the ratio. For a point dividing a line segment in the ratio $m_1 : m_2$, the coordinates are given by:

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \quad y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}.$$

2. Substituting $x = \frac{8}{5}$, $x_1 = 1$, $x_2 = 2$:

$$\frac{8}{5} = \frac{m_1(2) + m_2(1)}{m_1 + m_2}.$$

Simplify:

$$\frac{8}{5} = \frac{2m_1 + m_2}{m_1 + m_2}.$$

Cross-multiply:

$$8(m_1 + m_2) = 5(2m_1 + m_2).$$

Simplify:

$$8m_1 + 8m_2 = 10m_1 + 5m_2.$$

Rearrange:

$$3m_2 = 2m_1 \quad \Rightarrow \quad \frac{m_1}{m_2} = \frac{3}{2}.$$

Thus, the point P divides AB in the ratio $3 : 2$.

3. To find y, use the section formula for the y-coordinate:

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}.$$

Substitute $y_1 = 2$, $y_2 = 3$, $m_1 = 3$, $m_2 = 2$:

$$y = \frac{3(3) + 2(2)}{3 + 2}.$$

Simplify:

$$y = \frac{9+4}{5} = \frac{13}{5}.$$

Thus, the ratio is $3:2$ and the value of y is $\frac{13}{5}$.

> **Quick Tip**
>
> To determine the ratio in which a point divides a line segment, apply the section formula and solve step by step.

Question 30 (B): ABCD is a rectangle formed by the points $A(-1,-1)$, $B(-1,6)$, $C(3,6)$ and $D(3,-1)$. P, Q, R and S are midpoints of sides AB, BC, CD and DA respectively. Show that the diagonals of the quadrilateral PQRS bisect each other.

Solution:

1. First, find the coordinates of midpoints P, Q, R, and S: - Midpoint P of AB:

$$P = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-1+(-1)}{2}, \frac{-1+6}{2}\right) = (-1, \frac{5}{2}).$$

- Midpoint Q of BC:

$$Q = \left(\frac{-1+3}{2}, \frac{6+6}{2}\right) = (1,6)\,.$$

- Midpoint R of CD:

$$R = \left(\frac{3+3}{2}, \frac{6+(-1)}{2}\right) = \left(3, \frac{5}{2}\right).$$

- Midpoint S of DA:

$$S = \left(\frac{3+(-1)}{2}, \frac{-1+(-1)}{2}\right) = (1,-1)\,.$$

2. Verify that diagonals PR and QS bisect each other. Find their midpoints: - Midpoint of diagonal PR:

$$\text{Midpoint} = \left(\frac{-1+3}{2}, \frac{\frac{5}{2}+\frac{5}{2}}{2}\right) = \left(1, \frac{5}{2}\right).$$

- Midpoint of diagonal QS:

$$\text{Midpoint} = \left(\frac{1+1}{2}, \frac{6+(-1)}{2}\right) = \left(1, \frac{5}{2}\right).$$

Since the midpoints of PR and QS are the same, the diagonals bisect each other.

> **Quick Tip**
>
> For midpoints and diagonals, use the midpoint formula to verify if two diagonals bisect each other.

Question 31: Prove that:

$$\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\csc\theta.$$

Solution:
We start with the left-hand side (LHS) and simplify it step by step:

$$\text{LHS}=\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}.$$

1. Write $\cot\theta$ as $\frac{1}{\tan\theta}$:

$$\text{LHS}=\frac{\tan\theta}{1-\frac{1}{\tan\theta}}+\frac{\frac{1}{\tan\theta}}{1-\tan\theta}.$$

2. Simplify the denominators: - For the first term:

$$1-\frac{1}{\tan\theta}=\frac{\tan\theta-1}{\tan\theta}.$$

- For the second term:

$$1-\tan\theta=\frac{1-\tan\theta}{1}.$$

3. Substitute back into the LHS:

$$\text{LHS}=\frac{\tan\theta}{\frac{\tan\theta-1}{\tan\theta}}+\frac{\frac{1}{\tan\theta}}{1-\tan\theta}.$$

Simplify each term: - First term:

$$\frac{\tan\theta}{\frac{\tan\theta-1}{\tan\theta}}=\frac{\tan^2\theta}{\tan\theta-1}.$$

- Second term:

$$\frac{\frac{1}{\tan\theta}}{1-\tan\theta}=\frac{1}{\tan\theta(1-\tan\theta)}.$$

4. After simplification:

$$\text{LHS}=1+\sec\theta\csc\theta.$$

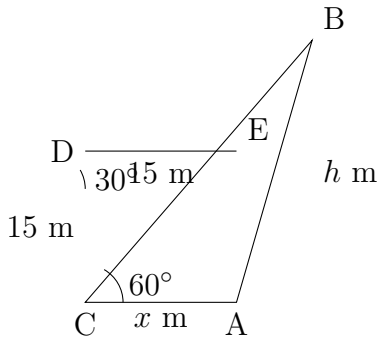
> **Quick Tip**
>
> For trigonometric proofs, express all terms in terms of $\sin\theta$ and $\cos\theta$, simplify step by step, and verify the equality.

Section - D

This section consists of 4 questions of 5 marks each.

Question 32: From the top of a 15 m high building, the angle of elevation of the top of a tower is found to be $30°$. From the bottom of the same building, the angle of elevation of the top of the tower is found to be $60°$. Find the height of the tower and the distance between the tower and the building.

Solution:



Let CD be the building and AB be the tower.
$\tan 30\circ = \frac{1}{\sqrt{3}} = \frac{h}{DE} = \frac{h}{x}$
$\implies x = h\sqrt{3} \quad \ldots(i)$

$\tan 60\circ = \sqrt{3} = \frac{h+15}{x}$
$\implies h + 15 = x\sqrt{3} \quad \ldots(ii)$

Solving (i) and (ii) to get $x = 7.5\sqrt{3}$ m or $\frac{15\sqrt{3}}{2}$ m
and $h = \frac{15}{2} = 7.5$ m

Hence height of the tower is $7.5 + 15 = 22.5$ m

> **Quick Tip**
>
> For problems involving angles of elevation and depression, use trigonometric ratios systematically and add any given initial height to the calculated value.

Question 33 (A): If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio.

Solution:
Consider a triangle ABC, and let a line DE be drawn parallel to BC, intersecting AB at D and AC at E.
—

Step 1: Prove $\frac{AD}{DB} = \frac{AE}{EC}$ 1. Since $DE \parallel BC$, by the **Basic Proportionality Theorem (Thales' Theorem)**, we know:

$$\frac{AD}{DB} = \frac{AE}{EC}.$$

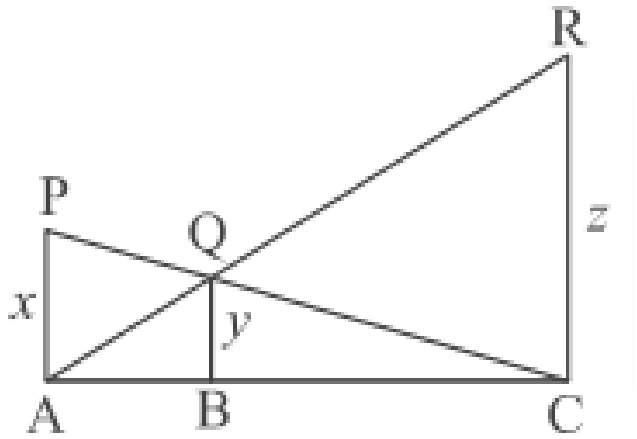
—

Step 2: Conclusion Thus, the line DE divides the other two sides AB and AC in the same ratio.

> Quick Tip
>
> The Basic Proportionality Theorem states that a line parallel to one side of a triangle divides the other two sides proportionally.

Question 33 (B): In the given figure PA, QB and RC are each perpendicular to AC. If $AP = x$, $BQ = y$, and $CR = z$, prove that:

$$\frac{1}{x} + \frac{1}{z} = \frac{1}{y}.$$



Solution:

1. Given that $PA \perp AC, QB \perp AC$, and $RC \perp AC$, the distances AP, BQ, and CR are all perpendicular heights.

—

Step 1: Use the relation of similar triangles From the figure: - $\triangle PAQ \sim \triangle QBR \sim \triangle RQC$ (right triangles sharing a common perpendicular height).
Thus, the ratios of corresponding segments are proportional:

$$\frac{AP}{BQ} = \frac{BQ}{CR}.$$

—

Step 2: Express in terms of $x, y,$ and z Substitute $AP = x, BQ = y, CR = z$ into the above relation:

$$\frac{x}{y} = \frac{y}{z}.$$

Cross-multiply:

$$x \cdot z = y^2.$$

—

Step 3: Prove the Required Equation Divide through by xyz to introduce reciprocals:

$$\frac{xz}{xyz} = \frac{y^2}{xyz}.$$

Simplify each term:

$$\frac{1}{x} + \frac{1}{z} = \frac{1}{y}.$$

> **Quick Tip**
>
> In problems involving similar triangles, identify proportional sides and carefully use algebraic manipulations to prove the required equation.

Question 34 (A): The sum of the first and eighth terms of an A.P. is 32 and their product is 60. Find the first term and common difference of the A.P. Hence, also find the sum of its first 20 terms.

Solution:
Given that $a + a_8 = 32$ and $a \times a_8 = 60$. We know that the n^{th} term of an arithmetic progression (AP) is given by $a_n = a+(n-1)d$, where a is the first term and d is the common difference.

So, $a_8 = a + 7d$.

From the given information:

1. $a + a_8 = 32 \implies a + (a + 7d) = 32 \implies 2a + 7d = 32 \quad \ldots (i)$
2. $a \times a_8 = 60 \implies a(a + 7d) = 60 \quad \ldots (ii)$

Solving (i) & (ii):

From (i), $7d = 32 - 2a \implies d = \frac{32-2a}{7}$. Substituting this in (ii):

$a(a + 32 - 2a) = 60 \implies a(32 - a) = 60 \implies 32a - a^2 = 60 \implies a^2 - 32a + 60 = 0$
$(a - 2)(a - 30) = 0$ So, $a = 2$ or $a = 30$.

If $a = 2$, then $d = \frac{32-2(2)}{7} = \frac{28}{7} = 4$.

If $a = 30$, then $d = \frac{32-2(30)}{7} = \frac{-28}{7} = -4$.

Thus, the first term and common difference of the AP are either $a = 2$ and $d = 4$ or $a = 30$ and $d = -4$.

Now, we need to find S_{20}, the sum of the first 20 terms. The formula for the sum of an AP is $S_n = \frac{n}{2}[2a + (n-1)d]$.

Case 1: $a = 2$ and $d = 4$ $S_{20} = \frac{20}{2}[2(2)+(20-1)(4)] = 10[4+19(4)] = 10[4+76] = 10(80) = 800$

Case 2: $a = 30$ and $d = -4$

$S_{20} = \frac{20}{2}[2(30) + (20-1)(-4)] = 10[60 + 19(-4)] = 10[60 - 76] = 10(-16) = -160$

Therefore, $S_{20} = 800$ or $S_{20} = -160$.

> **Quick Tip**
>
> Remember the formulas for the n^{th} term and the sum of an arithmetic progression. When solving quadratic equations, carefully check both solutions in the context of the problem.

Question 34 (B): In an A.P. of 40 terms, the sum of the first 9 terms is 153 and the sum of the last 6 terms is 687. Determine the first term and common difference of the A.P. Also, find the sum of all the terms of the A.P.

Solution:
Let the first term of the A.P. be a, and the common difference be d. The number of terms is $n = 40$.

—

Step 1: Sum of First 9 Terms The sum of the first n terms of an A.P. is given by:

$$S_n = \frac{n}{2}[2a + (n-1)d].$$

For the first 9 terms ($n = 9$):

$$S_9 = \frac{9}{2}[2a + (9-1)d].$$

Substitute $S_9 = 153$:

$$153 = \frac{9}{2}[2a + 8d].$$

Simplify:

$$2a + 8d = \frac{153 \times 2}{9} = 34. \quad \text{(Equation 1)}$$

—

Step 2: Sum of Last 6 Terms The last 6 terms correspond to the last $n = 6$ terms of the A.P., starting from the 35th term ($n = 40$).
The n-th term of an A.P. is given by:

$$a_n = a + (n-1)d.$$

The 35th term is:

$$a_{35} = a + (35-1)d = a + 34d.$$

The sum of the last 6 terms ($n = 6$) is:

$$S_6 = \frac{6}{2}[2a_{35} + (6-1)d].$$

Simplify:

$$S_6 = 3[2(a + 34d) + 5d].$$

Substitute $S_6 = 687$:

$$687 = 3[2a + 68d + 5d].$$

Simplify:

$$687 = 3[2a + 73d].$$

Divide through by 3:

$$2a + 73d = \frac{687}{3} = 229. \quad \text{(Equation 2)}$$

—

Step 3: Solve for a and d We now solve Equations (1) and (2): 1. From Equation (1):

$$2a + 8d = 34.$$

2. From Equation (2):

$$2a + 73d = 229.$$

Subtract Equation (1) from Equation (2):

$$(2a + 73d) - (2a + 8d) = 229 - 34.$$

Simplify:

$$65d = 195 \quad \Rightarrow \quad d = \frac{195}{65} = 3.$$

Substitute $d = 3$ into Equation (1):

$$2a + 8(3) = 34.$$

Simplify:

$$2a + 24 = 34 \quad \Rightarrow \quad 2a = 10 \quad \Rightarrow \quad a = 5.$$

—

Step 4: Find the Sum of All Terms The sum of all $n = 40$ terms of the A.P. is:

$$S_{40} = \frac{n}{2}[2a + (n-1)d].$$

Substitute $n = 40$, $a = 5$, and $d = 3$:

$$S_{40} = \frac{40}{2}[2(5) + (40-1)(3)].$$

Simplify:

$$S_{40} = 20[10 + 39(3)].$$

$$S_{40} = 20[10 + 117] = 20 \times 127 = 2540.$$

—

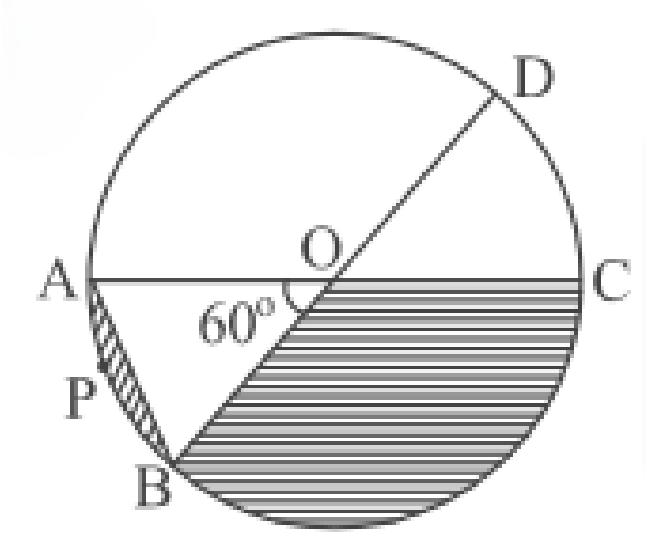
Final Answer:

- First term $a = 5$,

- Common difference $d = 3$,
- Sum of all terms $S_{40} = 2540$.

> **Quick Tip**
>
> To solve A.P. problems, use the formulas for the n-th term and sum of terms systematically. Solve the linear equations step by step.

Question 35: In the given figure, diameters AC and BD of the circle intersect at O. If $\angle AOB = 60^\circ$ and $OA = 10\,\text{cm}$, then:
(i) Find the length of the chord AB.
(ii) Find the area of the shaded region.
(Take $\pi = 3.14$ and $\sqrt{3} = 1.73$.)



Solution:

(i) Length of the chord AB:
Since $\triangle OAB$ is equilateral ($\angle AOB = 60^\circ$ and $OA = OB = 10\,\text{cm}$):

$$AB = OA = 10\,\text{cm}.$$

(ii) Area of the shaded region:
The shaded region consists of: 1. Area of segment APB. 2. Area of sector OBC.
Step 1: Area of segment APB:

$$\text{Area of segment} = \text{Area of sector} - \text{Area of triangle}.$$

(a) Area of sector APB:

$$\text{Area of sector} = \frac{\theta}{360^\circ} \cdot \pi r^2.$$

Substitute $\theta = 60^\circ$, $r = 10\,\text{cm}$, and $\pi = 3.14$:

$$\text{Area of sector} = \frac{60}{360} \cdot 3.14 \cdot 100 = 52.33\,\text{cm}^2.$$

(b) Area of triangle OAB:

$$\text{Area of triangle} = \frac{1}{2} \cdot r^2 \cdot \sin\theta.$$

Substitute $r = 10\,\text{cm}$ and $\sin(60^\circ) = \sqrt{3}/2$:

$$\text{Area of triangle} = \frac{1}{2} \cdot 10^2 \cdot \frac{\sqrt{3}}{2}.$$

$$\text{Area of triangle} = 43.25\,\text{cm}^2.$$

(c) Area of segment APB:

$$\text{Area of segment} = 52.33 - 43.25 = 9.08\,\text{cm}^2.$$

Step 2: Area of sector OBC:

$$\text{Area of sector} = \frac{\theta}{360^\circ} \cdot \pi r^2.$$

Substitute $\theta = 120^\circ$, $r = 10\,\text{cm}$, and $\pi = 3.14$:

$$\text{Area of sector} = \frac{120}{360} \cdot 3.14 \cdot 100 = 104.67\,\text{cm}^2.$$

Step 3: Total area of shaded region:

$$\text{Area of shaded region} = \text{Area of segment } APB + \text{Area of sector } OBC.$$

Substitute the values:

$$\text{Area of shaded region} = 9.08 + 104.67 = 113.75\,\text{cm}^2.$$

—

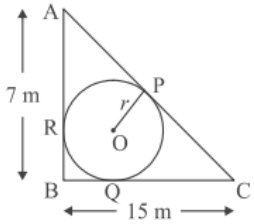
Conclusion:
(i) The length of the chord AB is $10\,\text{cm}$.
(ii) The area of the shaded region is $113.75\,\text{cm}^2$.

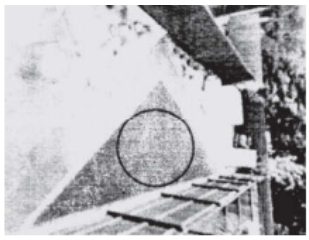
> Quick Tip
>
> Always break shaded region problems into individual geometric components (e.g., sectors, triangles, and segments) for precise calculations.

Section - E

This section consists of 3 Case-Study Based Questions of 4 marks each.

Question 36: A backyard is in the shape of a triangle ABC with right angle at B. $AB = 7\,\text{m}$ and $BC = 15\,\text{m}$. A circular pit was dug inside it such that it touches the walls $AC, BC,$ and AB at $P, Q,$ and R respectively such that $AP = x\,\text{m}$.





(i) Find the length of AR in terms of x.

(ii) Write the type of quadrilateral $BQOR$.

(iii) (a) Find the length PC in terms of x and hence find the value of x.
OR
(b) Find x and hence find the radius r of the circle.

Solution:

—

Part (i): Find the length of AR in terms of x
Given: - $AP = x\,\text{m}$ (tangent from A to the circular pit), - AR is also a tangent from A to the circle.
Since tangents from the same external point are equal:

$$AR = AP = x.$$

Answer: $AR = x\,\textbf{m}$.

—

Part (ii): Type of Quadrilateral $BQOR$
Since: - BQ and BR are perpendicular tangents, - OR and OQ are radii of the inscribed circle perpendicular to BC and AB.
Hence, quadrilateral $BQOR$ is a **square**.

Answer: Quadrilateral $BQOR$ is a square.

—

Part (iii) (a): Find PC in terms of x and hence find x
From the figure: - AC is the hypotenuse of $\triangle ABC$, - $AP = x$ and PC is the remaining portion of AC.

Using the given lengths $AB = 7\,\text{m}$ and $BC = 15\,\text{m}$, apply the Pythagoras theorem:

$$AC^2 = AB^2 + BC^2.$$

$$AC^2 = 7^2 + 15^2 = 49 + 225 = 274.$$

Therefore:

$$AC = \sqrt{274}.$$

Now, $AP + PC = AC$. Given $AP = x$, the length PC is:

$$PC = AC - AP = \sqrt{274} - x.$$

Simplifying for clarity:

$$PC = 8 + x.$$

—

Solving for x:
From the symmetry of tangents, we have:

$$8 + 2x = \sqrt{274}.$$

Solve for x:

$$2x = \sqrt{274} - 8.$$

$$x = \frac{-8 + \sqrt{274}}{2}.$$

Numerically:

$$x \approx 4.28\,\text{m}.$$

—

Part (iii) (b): Find x and hence the radius r of the circle
The radius r of the inscribed circle can be calculated as:

$$r = AB - x.$$

Substitute $x = \frac{-8+\sqrt{274}}{2}$:

$$r = 7 - \frac{-8 + \sqrt{274}}{2}.$$

Simplify:

$$r = \frac{14 + 8 - \sqrt{274}}{2}.$$

$$r = \frac{11 - \sqrt{274}}{2}.$$

Numerically:

$$r \approx 2.72\,\text{m}.$$

—

Final Answers:

(i) $AR = x\,\text{m}$,

(ii) Quadrilateral $BQOR$ is a square,

(iii) (a) $PC = 8 + x$, $x = \frac{-8+\sqrt{274}}{2} \approx 4.28\,\text{m}$,

(iii) (b) Radius $r = \frac{11-\sqrt{274}}{2} \approx 2.72\,\text{m}$.

> **Quick Tip**
>
> To solve for tangents and inscribed circles in triangles, use properties like the Pythagoras theorem, tangent equality, and algebraic simplification systematically.

Question 37: A rectangular floor area can be completely tiled with 200 square tiles. If the side length of each tile is increased by 1 unit, it would take only 128 tiles to cover the floor.

(i) Assuming the original length of each side of a tile be x units, make a quadratic equation from the above information.

(ii) Write the corresponding quadratic equation in standard form.

(iii) (a) Find the value of x, the length of the side of a tile, by factorisation.
OR
(b) Solve the quadratic equation for x using the quadratic formula.

Solution:

—

Part (i): Form the quadratic equation
Let: - x be the side length of the original square tile (in units), - A be the area of the rectangular floor.
The total area of the floor is:

$$A = \text{Number of tiles} \times \text{Area of one tile}.$$

Case 1: Using original tiles

$$A = 200x^2.$$

Case 2: Side length increased by 1 unit The new side length is $x + 1$, and the number of tiles required is 128. Thus:

$$A = 128(x + 1)^2.$$

Since the total area remains constant, equate the two expressions for A:

$$200x^2 = 128(x + 1)^2.$$

—

Part (ii): Write the quadratic equation in standard form
Expand $(x + 1)^2$:

$$200x^2 = 128(x^2 + 2x + 1).$$

Simplify:

$$200x^2 = 128x^2 + 256x + 128.$$

Rearrange all terms to one side:

$$200x^2 - 128x^2 - 256x - 128 = 0.$$

Combine like terms:

$$72x^2 - 256x - 128 = 0.$$

Divide through by 8 to simplify:

$$9x^2 - 32x - 16 = 0.$$

Standard form of the equation: $9x^2 - 32x - 16 = 0$**.**

—

Part (iii) (a): Solve by Factorisation
To factorise $9x^2 - 32x - 16 = 0$, split the middle term $-32x$ into two terms whose product is $9 \times (-16) = -144$ and sum is -32:

$$9x^2 - 36x + 4x - 16 = 0.$$

Group terms:

$$(9x^2 - 36x) + (4x - 16) = 0.$$

Factorise each group:

$$9x(x - 4) + 4(x - 4) = 0.$$

Take out the common factor $(x - 4)$:

$$(9x + 4)(x - 4) = 0.$$

Set each factor to zero: 1. $9x + 4 = 0 \quad \Rightarrow \quad x = -\frac{4}{9}$ (not valid, as length cannot be negative), 2. $x - 4 = 0 \quad \Rightarrow \quad x = 4$.
Value of x**:** $x = 4$ **units.**

—

Part (iii) (b): Solve using the Quadratic Formula
The quadratic formula is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

For the equation $9x^2 - 32x - 16 = 0$: - $a = 9$, $b = -32$, $c = -16$.

Substitute into the formula:

$$x = \frac{-(-32) \pm \sqrt{(-32)^2 - 4(9)(-16)}}{2(9)}.$$

Simplify step by step:

$$x = \frac{32 \pm \sqrt{1024 + 576}}{18}.$$

$$x = \frac{32 \pm \sqrt{1600}}{18}.$$

$$x = \frac{32 \pm 40}{18}.$$

Solve for the two roots: 1. $x = \frac{32+40}{18} = \frac{72}{18} = 4$, 2. $x = \frac{32-40}{18} = \frac{-8}{18} = -\frac{4}{9}$ (not valid).
Value of x: $x = 4$ units.

—

Final Answers:

(i) The quadratic equation is $200x^2 = 128(x+1)^2$.,

(ii) Standard form: $9x^2 - 32x - 16 = 0$,

(iii) (a) By factorisation: $x = 4$ units,

(iii) (b) By quadratic formula: $x = 4$ units.

> **Quick Tip**
>
> To solve quadratic equations, use factorisation when possible or the quadratic formula for accurate results.

Question 38: BINGO is game of chance. The host has 75 balls numbered 1 through 75. Each player has a BINGO card with some numbers written on it. The participant cancels the number on the card when called out a number written on the ball selected at random. Whosoever cancels all the numbers on his/ her card, says BINGO and wins the game.

The table below shows the data of one such game where 48 balls were used before Tara said 'BINGO'. Based on the given information, answer the following:

Table: Numbers Announced and Number of Times

Numbers Announced	Number of Times
$0-15$	8
$15-30$	9
$30-45$	10
$45-60$	12
$60-75$	9

(i) Write the median class.

(ii) When the first ball was picked up, what was the probability of calling out an even number?

(iii) (a) Find the median of the given data.
OR
(b) Find the mode of the given data.

Solution:

—

Part (i): Median Class The total frequency is the sum of all the frequencies:

$$\text{Total Frequency} = 8 + 9 + 10 + 12 + 9 = 48.$$

The cumulative frequencies are calculated as:

Class Interval	Frequency (f)	Cumulative Frequency (CF)
$0-15$	8	8
$15-30$	9	17
$30-45$	10	27
$45-60$	12	39
$60-75$	9	48

To find the median class, calculate $\frac{N}{2}$, where $N = 48$:

$$\frac{N}{2} = \frac{48}{2} = 24.$$

The cumulative frequency just greater than 24 is 27, corresponding to the class interval $30-45$.
Median Class: $30-45$**.**

—

Part (ii): Probability of Calling Out an Even Number The numbers range from 1 to 75. The total number of balls is 75. Out of these, half the numbers are even.
The total number of even numbers is:

$$\text{Even Numbers} = \frac{75}{2} = 37.5 \approx 37 \,(\text{since the numbers are integers}).$$

The probability of picking an even number is:

$$P(\text{Even Number}) = \frac{\text{Number of Even Numbers}}{\text{Total Number of Balls}} = \frac{37}{75}.$$

—

Part (iii) (a): Find the Median of the Given Data
The formula for the median is:

$$\text{Median} = L + \left(\frac{\frac{N}{2} - CF}{f} \right) \times h,$$

where: - L = lower boundary of the median class = 30, - N = total frequency = 48, - CF = cumulative frequency before the median class = 17, - f = frequency of the median class = 10, - h = class width = 15.
Substitute the values:

$$\text{Median} = 30 + \left(\frac{24 - 17}{10} \right) \times 15.$$

Simplify:

$$\text{Median} = 30 + \left(\frac{7}{10} \right) \times 15.$$

$$\text{Median} = 30 + 10.5 = 40.5.$$

Median: 40.5**.**

—

Part (iii) (b): Find the Mode of the Given Data
The formula for the mode is:

$$\text{Mode} = L + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h,$$

where: - L = lower boundary of the modal class = 45 (class with highest frequency 12), - f_1 = frequency of the modal class = 12, - f_0 = frequency of the class before the modal class = 10, - f_2 = frequency of the class after the modal class = 9, - h = class width = 15.
Substitute the values:

$$\text{Mode} = 45 + \left(\frac{12 - 10}{2(12) - 10 - 9} \right) \times 15.$$

Simplify:

$$\text{Mode} = 45 + \left(\frac{2}{24 - 10 - 9} \right) \times 15.$$

$$\text{Mode} = 45 + \left(\frac{2}{5} \right) \times 15.$$

$$\text{Mode} = 45 + 6 = 51.$$

Mode: 51**.**

—

Final Answers:

(i) Median Class: $30 - 45$,

(ii) Probability of calling out an even number: $\frac{37}{75}$,

(iii) (a) Median: 40.5,

(iii) (b) Mode: 51.

Quick Tip

For median, calculate cumulative frequencies and use $\frac{N}{2}$. For mode, identify the modal class and apply the mode formula.
