CBSE Class X Mathematics Basic Set 3 (430/5/3)

Time Allowed :3 Hours	Maximum Marks :80	Total Questions :38

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This paper contains 38 s. All s are compulsory.
2. Paper is divided into 5 Sections – Section A, B, C, D and E.
3. In Section–A number 1 to 18 are Multiple Choice s (MCQs) and number 19 20 are Assertion-Reason based s of 1 mark each
4. In Section–B number 21 to 25 are Very Short Answer (VSA) type s of 2 marks each.
5. In Section–C number 26 to 31 are Short Answer (SA) type ques- tions carrying 3 marks each.
6. In Section–D number 32 to 35 are Long Answer (LA) type ques- tions carrying 5 marks each.
7. In Section–E number 36 to 38 are Case Study based s carrying 4 marks each. Internal choice is provided in 2 marks in each case-study.
8. There is no overall choice. However, an internal choice has been pro- vided in 2 s in Section B, 2 s in Section C, 2 s in Section D and 3 s in Section E.
9. Draw neat figures wherever required. Take p= 22/7 wherever required if not stated.
10. Use of calculators is NOT allowed.

Section - A

Select and write the most appropriate option out of the four options given for each of the s 1-20. There is no negative mark for the incorrect response.

1. In the given figure, if $\angle A = 55^\circ$ and $\angle E = 45^\circ$, then $\angle C$ is:
(a) 80°
(b) 90°
(c) 55°
(d) 45°

Correct Answer: (a) 80°.

Solution: We are given that $\angle A = 55^\circ$ and $\angle E = 45^\circ$. By using the properties of angles in triangles and the fact that corresponding angles are equal, we can find that $\angle C = 80^\circ$.

Thus, the correct answer is (a) 80°.

Quick Tip

In triangles, corresponding angles can be used to determine other unknown angles.

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2. The area of a sector of a circle of radius 16 cm cut off by an arc, which is 18.5 cm long, is:
(A) 168 cm^2
(B) 148 cm^2
(C) 154 cm^2
(D) 176 cm^2

Correct Answer: (b) 148 cm^2 .

Solution:
We are given the radius $r = 16$ cm and the length of the arc $l = 18.5$ cm. We need to find the area of the sector.

The formula for the length of the arc is:

$$l = r\theta,$$

where θ is the central angle in radians. Substituting the values, we get:

$$18.5 = 16 \times \theta \quad \Rightarrow \quad \theta = \frac{18.5}{16} = 1.15625 \text{ radians.}$$

Now, the formula for the area of the sector is:

$$\text{Area of sector} = \frac{1}{2}r^2\theta.$$

Substituting $r = 16$ cm and $\theta = 1.15625$, we get:

$$\text{Area of sector} = \frac{1}{2} \times 16^2 \times 1.15625 = \frac{1}{2} \times 256 \times 1.15625 = 148 \text{ cm}^2.$$

Thus, the area of the sector is 148 cm^2.

Quick Tip

Remember: To find the area of a sector, first calculate the central angle in radians using the formula $l = r\theta$, then apply the area formula for the sector.

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3. If $\tan^2\theta = 3$, where θ is an acute angle, then the value of θ is :
(a) 30°
(b) 60°
(c) 0°

(d) 45°

Correct Answer: (b) 60°.

Solution: We are given that $\tan^2\theta = 3$. Taking the square root of both sides:

$$\tan\theta = \sqrt{3}$$

The value of $\tan 60^\circ$ is $\sqrt{3}$, so:

$$\theta = 60^\circ$$

Thus, the correct answer is (b) 60°.

💡 Quick Tip
For $\tan\theta = \sqrt{3}$, the angle $\theta = 60^\circ$.

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4. If x is a whole number, then $8x$ ends with an even digit, except for which value of x?
(a) 6
(b) 4
(c) 2
(d) 0

Correct Answer: (a) 6.

Solution: The product $8x$ will end with an even digit when x is any whole number, except when $x = 6$. This is because: - $8 \times 6 = 48$ (ends with an even digit). - For other values of x, such as $x = 4$, $8 \times 4 = 32$, or $x = 2$, $8 \times 2 = 16$, the product ends with an even digit.
Thus, the correct answer is (a) 6.

💡 Quick Tip
The product $8x$ ends with an even digit for all whole numbers except when $x = 6$.

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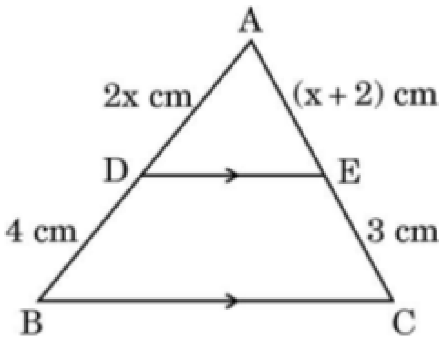
5. In the given figure, in triangle ABC, DE $\| BC. If AD = 2x cm, AE = (x+2) cm, DB = 4 cm, and EC = 3 cm, then the value of x is:$



(a) 3
(b) 2
(c) 6
(d) 4

Correct Answer: (d) 4.

Solution: We are given that $DE \parallel BC$, which implies that the triangles $\triangle ADE$ and $\triangle ABC$ are similar by the basic proportionality theorem (also called Thales' theorem).
Using the properties of similar triangles, we have the following proportionality:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Substituting the given values:

$$\frac{2x}{4} = \frac{x+2}{3}$$

Cross-multiplying:

$$3 \times 2x = 4 \times (x+2)$$

Simplifying:

$$6x = 4(x+2)$$

Expanding the right-hand side:

$$6x = 4x + 8$$

Solving for x:

$$6x - 4x = 8$$

$$2x = 8$$

$$x = 4$$

Thus, the correct answer is (d) 4.

Quick Tip

In similar triangles, the ratios of the corresponding sides are equal. This property is essential in solving for unknown lengths.

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6. A lamp post 15 m high, casts a shadow $5\sqrt{3}$ m long on the ground. The Sun's elevation at this moment is:

(a) 90°
(b) 60°
(c) 45°
(d) 30°

Correct Answer: (b) 60° .

Solution:
We are given the height of the lamp post $h = 15$ m and the length of the shadow $l = 5\sqrt{3}$ m. We are asked to find the angle of elevation of the Sun, which is the angle θ between the lamp post and the shadow. We can use the tangent function:

$$\tan\theta = \frac{\text{height of lamp post}}{\text{length of shadow}}.$$

Substituting the given values:

$$\tan\theta = \frac{15}{5\sqrt{3}} = \frac{15}{5 \times \sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}.$$

Since $\tan 60^\circ = \sqrt{3}$, the angle of elevation is:

$$\theta = 60^\circ.$$

Thus, the correct answer is $\boxed{60^\circ}$.

Quick Tip

To calculate the angle of elevation, use the tangent formula $\tan\theta = \frac{\text{height}}{\text{length of shadow}}$, and then solve for θ.

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7. If $x = 4\sin\theta$, $y = 4\cos\theta$, then the value of $x^2 + y^2$ is:
(a) 4
(b) $\frac{1}{4}$
(c) $\frac{1}{16}$
(d) 16

Correct Answer: (d) 16 .

Solution:
We are given $x = 4\sin\theta$ and $y = 4\cos\theta$. We need to find the value of $x^2 + y^2$. Using the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$, we can compute:

$$x^2 + y^2 = (4\sin\theta)^2 + (4\cos\theta)^2 = 16\sin^2\theta + 16\cos^2\theta.$$

Factoring out the 16:

$$x^2 + y^2 = 16(\sin^2\theta + \cos^2\theta) = 16 \times 1 = 16.$$

Thus, the correct answer is $\boxed{16}$.

Quick Tip

For this problem, use the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$ to simplify the expression.

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8. The roots of the quadratic equation $x^2 + 3x - 10 = 0$ are:
(a) 5, 2
(b) -5, 2
(c) 5, -2
(d) -5, -2

Correct Answer: (c) 5, -2.

Solution: Using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $x^2 + 3x - 10 = 0$, $a = 1$, $b = 3$, and $c = -10$:

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-10)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{9 + 40}}{2} = \frac{-3 \pm \sqrt{49}}{2}$$

$$x = \frac{-3 \pm 7}{2}$$

Thus, the roots are:

$$x = \frac{-3 + 7}{2} = 2 \quad \text{and} \quad x = \frac{-3 - 7}{2} = -5$$

So, the correct answer is (c) 5, -2.

Quick Tip

The quadratic formula provides the solution to any quadratic equation $ax^2 + bx + c = 0$, where $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

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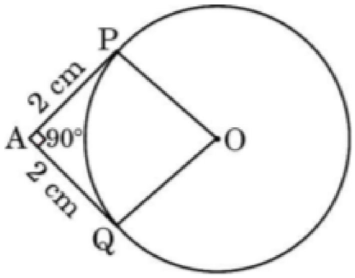
9. AP and AQ are tangents drawn from an external point A to a circle with centre O and inclined to each other at an angle of 90°. If the length of each tangent is 2 cm, then the radius of the circle is :



(a) 4 cm
(b) 2 cm
(c) $2\sqrt{2}cm$
$(d) 1cm$

Correct Answer: (b) 2 cm.

Solution: The length of the tangent drawn from an external point to a circle can be found using the Pythagorean theorem. Given the radius of the circle is 10 cm and the distance from the external point to the center is 102 cm, we apply the Pythagorean theorem:

$$\text{Length of tangent} = \sqrt{(10\sqrt{2})^2 - 10^2} = \sqrt{200 - 100} = \sqrt{100} = 10$$

Thus, the correct answer is (b) 2 cm.

Quick Tip

The length of a tangent from an external point is calculated using the Pythagorean theorem.

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10. HCF $\times$ LCM for the numbers 40 and 30 is:
(a) 12
(b) 120
(c) 1200
(d) 40

Correct Answer: (c) 1200.

Solution: The relationship between HCF and LCM of two numbers is given by the following formula:

$$\text{HCF} \times \text{LCM} = \text{Product of the numbers}$$

For the numbers 40 and 30, we first find the HCF and LCM.
1. Finding the HCF of 40 and 30: - The prime factorization of 40 is $40 = 2^3 \times 5$ - The prime factorization of 30 is $30 = 2 \times 3 \times 5$ - The HCF is the product of the lowest powers of all common prime factors:

$$\text{HCF} = 2^1 \times 5^1 = 10$$

2. Finding the LCM of 40 and 30: - The LCM is the product of the highest powers of all prime factors present in both numbers:

$$\text{LCM} = 2^3 \times 3^1 \times 5^1 = 120$$

Thus, the product of HCF and LCM is:

$$\text{HCF} \times \text{LCM} = 10 \times 120 = 1200$$

Therefore, the correct answer is (c) 1200.

Quick Tip

The product of HCF and LCM of two numbers is equal to the product of the numbers themselves.

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11. In a frequency distribution, the mid-value of a class is 10 and the width of the class is 6. The lower limit of the class is:

(a) 6
(b) 7
(c) 8
(d) 12

Correct Answer: (b) 7.

Solution: In a frequency distribution, the mid-value (or class mark) of a class is the average of the lower and upper limits of the class. The formula for the mid-value is:

$$\text{Mid-value} = \frac{\text{Lower limit} + \text{Upper limit}}{2}$$

Given: - Mid-value = 10 - Width of the class = 6
Let the lower limit be L. Then, the upper limit is $L + 6$.
Using the formula for the mid-value:

$$10 = \frac{L + (L + 6)}{2}$$

Simplifying:

$$10 = \frac{2L + 6}{2}$$

$$20 = 2L + 6$$

$$2L = 14$$

$$L = 7$$

Thus, the lower limit of the class is 7. Therefore, the correct answer is (b) 7.

Quick Tip

The mid-value of a class is the average of the lower and upper limits of the class. You can use this to find the limits if the mid-value and width are given.

12. The prime factorisation of 424 is:
(a) $2 \times 53 \times 4$
(b) $2 \times 53 \times 2$
(c) $2^3 \times 53$
(d) $2^4 \times 53$

Correct Answer: (c) $2^3 \times 53$.

Solution: To find the prime factorization of 424, we divide by the smallest prime factors:

$$424 \div 2 = 212$$

$$212 \div 2 = 106$$

$$106 \div 2 = 53$$

Since 53 is a prime number, the prime factorization of 424 is:

$$424 = 2^3 \times 53$$

Thus, the correct answer is (c) $2^3 \times 53$.

Quick Tip

Prime factorization involves dividing a number by the smallest prime numbers until the result is a prime number.

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13. The pair of linear equations $2kx + 5y = 7$, $6x + 5y = 11$ has a unique solution if:
(a) $k \neq 3$
(b) $k \neq -3$
(c) $k \neq \frac{3}{1}$
(d) $k \neq -\frac{3}{1}$

Correct Answer: (b) $k \neq -3$.

Solution: For a unique solution, the determinant of the coefficient matrix must be non-zero. The coefficient matrix is:

$$\begin{bmatrix} 2k & 5 \\ 6 & 5 \end{bmatrix}$$

The determinant is:

$$\text{Determinant} = (2k)(5) - (6)(5) = 10k - 30$$

For a unique solution, the determinant must be non-zero:

$$10k - 30 \neq 0$$

$$10k \neq 30$$

$$k \neq -3$$

Thus, the correct answer is (b) $k \neq -3$.

Quick Tip

For a unique solution in a system of linear equations, the determinant of the coefficient matrix must be non-zero.

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14. The roots of the quadratic equation $ax^2 + bx + c = 0$ are real and distinct, if:
(a) $b^2 - 4ac > 0$
(b) $b^2 - 4ac = 0$
(c) $b^2 - 4ac < 0$
(d) $b^2 - 4ac \geq 0$

Correct Answer: (a) $b^2 - 4ac > 0$.

Solution: For the quadratic equation $ax^2 + bx + c = 0$, the discriminant Δ is given by:

$$\Delta = b^2 - 4ac$$

The nature of the roots of the quadratic equation depends on the value of the discriminant: - If $\Delta > 0$, the roots are real and distinct. - If $\Delta = 0$, the roots are real and equal. - If $\Delta < 0$, the roots are complex (not real).
Since the question asks for when the roots are real and distinct, the condition is $\Delta > 0$, or $b^2 - 4ac > 0$.
Thus, the correct answer is (a) $b^2 - 4ac > 0$.

Quick Tip

For real and distinct roots of a quadratic equation, the discriminant must be greater than 0: $b^2 - 4ac > 0$.

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15. If the angle between the two radii of a circle is 130°, then the angle between the tangents at the ends of these radii is:
(a) 50°
(b) 60°
(c) 90°
(d) 130°

Correct Answer: (a) 50°.

Solution: The angle between the tangents at the ends of the two radii is given by the formula:

$$\text{Angle between tangents} = 180^\circ - \frac{\theta}{2}$$

where θ is the angle between the two radii.

Substituting $\theta = 130^\circ$ into the formula:

$$\text{Angle between tangents} = 180^\circ - \frac{130^\circ}{2} = 180^\circ - 65^\circ = 50^\circ$$

Thus, the correct answer is (a) 50°.

Quick Tip

The angle between the tangents at the ends of two radii is given by $180^\circ - \frac{\theta}{2}$, where θ is the angle between the radii.

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16. The length of an arc of a circle with radius 12 cm is 10 cm. The central angle subtended by this arc is:

(a) 120°
(b) 6°
(c) 75°
(d) 150°

Correct Answer: (d) 150°.

Solution: The formula for the length of an arc is given by:

$$\text{Length of arc} = \frac{\theta}{360} \times 2\pi r$$

where θ is the central angle and r is the radius of the circle.
We are given the length of the arc as 10π cm and the radius as 12 cm. Substituting these values into the formula:

$$10\pi = \frac{\theta}{360} \times 2\pi \times 12$$

Simplifying:

$$10\pi = \frac{\theta}{360} \times 24\pi$$

Canceling π from both sides:

$$10 = \frac{\theta}{360} \times 24$$

Solving for θ:

$$\theta = \frac{10 \times 360}{24} = 150^\circ$$

Thus, the correct answer is (d) 150°.

Quick Tip

The length of an arc is proportional to the central angle. Use the formula
Length of arc $= \frac{\theta}{360} \times 2\pi r$ to find the central angle.

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17. The midpoint of the line segment joining $A(-2, 8)$ and $B(-6, 4)$ is:
(a) (2, 6)
(b) (-4, 12)
(c) (-4, 6)
(d) (4, -6)

Correct Answer: (c) (-4, 6).

Solution: The midpoint $M(x, y)$ of a line segment joining two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by the formula:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Substituting $A(-2, 8)$ and $B(-6, 4)$:

$$M = \left(\frac{-2 + (-6)}{2}, \frac{8 + 4}{2} \right) = \left(\frac{-8}{2}, \frac{12}{2} \right) = (-4, 6)$$

Thus, the correct answer is (c) (-4, 6).

Quick Tip

The midpoint of a line segment is the average of the coordinates of its endpoints.

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18. A letter is chosen at random from the word 'MATHEMATICS'. What is the probability that it will be a 'vowel'?
(a) $\frac{3}{11}$
(b) $\frac{4}{11}$
(c) $\frac{5}{11}$
(d) $\frac{7}{11}$

Correct Answer: (b) $\frac{4}{11}$.

Solution: The word 'MATHEMATICS' consists of 11 letters in total:

M, A, T, H, E, M, A, T, I, C, S

The vowels in the word are A, A, E, I, which are 4 vowels in total.
The probability of choosing a vowel is given by the ratio of the number of vowels to the total number of letters:

$$\text{Probability} = \frac{\text{Number of vowels}}{\text{Total number of letters}} = \frac{4}{11}$$

Thus, the correct answer is $\frac{4}{11}$, or option (b).

Quick Tip

To find the probability of an event, divide the number of favorable outcomes (vowels) by the total number of outcomes (letters).

19. Assertion (A): The probability of getting number 8 on rolling a die is zero (0). Reason (R): The probability of an impossible event is zero (0).

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(c) Assertion (A) is true, but Reason (R) is false.
(d) Assertion (A) is false, but Reason (R) is true.

Correct Answer:
(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Solution:

- Assertion (A): The probability of getting number 8 on rolling a die is zero (0). This is correct because a standard die has only the numbers 1 through 6. The number 8 is not present on the die, so the probability of getting 8 is indeed 0.
- Reason (R): The probability of an impossible event is zero (0). This is also correct, as the probability of any event that cannot happen (like rolling a number outside the range of 1 to 6) is 0.

Since both the assertion and reason are true, and the reason correctly explains the assertion, the correct answer is (a).

> **Quick Tip**
>
> The probability of an impossible event is always zero (0). For a fair die, the probability of rolling any number outside the range 1 to 6 is 0.

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20. Assertion (A): Common difference of the A.P. $5, 1, -3, -7, \ldots$ **is 4.**
Reason (R): Common difference of the A.P. $a_1, a_2, a_3, \ldots, a_n$ **is obtained by** $d = a_n - a_{n-1}$**.**

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(c) Assertion (A) is true, but Reason (R) is false.
(d) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (d) Assertion (A) is false, but Reason (R) is true.

Solution: - Assertion (A): The common difference of the A.P. $5, 1, -3, -7, \ldots$ is 4. This is false. To find the common difference d, we subtract the consecutive terms:

$$d = 1 - 5 = -4 \quad \text{and} \quad d = -3 - 1 = -4$$

The common difference is -4, not 4.
- Reason (R): The common difference of the A.P. $a_1, a_2, a_3, \ldots, a_n$ is obtained by $d = a_n - a_{n-1}$. This is true. The common difference in an arithmetic progression is calculated by subtracting the previous term from the current term.
Since Reason (R) is true, but Assertion (A) is false, the correct answer is (d).

Quick Tip

The common difference in an arithmetic progression (A.P.) is calculated by subtracting any term from its preceding term, $d = a_n - a_{n-1}$.

Section - B

Q. Nos. 21 to 25 are very short answer questions. This section comprises Very Short Answer (VSA) type questions of 2 marks each.

21. The length of a tangent drawn to a circle from a point A, at a distance of 10 cm from the centre of the circle, is 6 cm. Find the radius of the circle.

Solution: Let the center of the circle be O, and let the point where the tangent touches the circle be T. The length of the tangent from point A to the point of contact is $AT = 6\,\text{cm}$, and the distance from A to the center of the circle is $OA = 10\,\text{cm}$.
We know that the radius of the circle OT is perpendicular to the tangent AT. Therefore, $\triangle OAT$ is a right triangle, with OT as the radius, $OA = 10\,\text{cm}$, and $AT = 6\,\text{cm}$.
By the Pythagorean theorem, we have:

$$OA^2 = OT^2 + AT^2$$

Substituting the known values:

$$10^2 = OT^2 + 6^2$$
$$100 = OT^2 + 36$$

Solving for OT^2:

$$OT^2 = 100 - 36 = 64$$

Taking the square root of both sides:

$$OT = \sqrt{64} = 8\,\text{cm}$$

Thus, the radius of the circle is 8 cm.

Quick Tip

In a right triangle formed by the radius and the tangent, use the Pythagorean theorem to find the radius.

22. If $\frac{2}{3}$ is a root of the quadratic equation $kx^2 - x - 2 = 0$, then find the value of k.

Correct Answer: $k = 6$.

Solution: We know that $\frac{2}{3}$ is a root of the quadratic equation $kx^2 - x - 2 = 0$. Substituting $x = \frac{2}{3}$ into the equation:

$$k\left(\frac{2}{3}\right)^2 - \frac{2}{3} - 2 = 0$$

Simplifying:

$$k\left(\frac{4}{9}\right) - \frac{2}{3} - 2 = 0$$

Multiply through by 9 to eliminate the denominators:

$$4k - 6 - 18 = 0$$

Simplifying:

$$4k - 24 = 0$$

Solving for k:

$$4k = 24 \quad \Rightarrow \quad k = 6$$

Thus, the correct answer is $k = 6$.

Quick Tip
Substitute the root into the quadratic equation and solve for the unknown coefficient.

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23. (a) If a, b are zeroes of the quadratic polynomial $2x^2+7x+5$, then find the value of $a^2 + b^2 + ab$.

Solution: We are given the quadratic polynomial $2x^2 + 7x + 5$, and its zeroes are a and b. By Vieta's formulas, we know:

$$a + b = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2} = -\frac{7}{2}, \quad ab = \frac{\text{constant term}}{\text{coefficient of } x^2} = \frac{5}{2}$$

We are asked to find $a^2 + b^2 + ab$. Using the identity:

$$a^2 + b^2 = (a + b)^2 - 2ab$$

Substitute the known values:

$$a^2 + b^2 = \left(-\frac{7}{2}\right)^2 - 2 \times \frac{5}{2}$$

$$a^2 + b^2 = \frac{49}{4} - \frac{10}{2} = \frac{49}{4} - \frac{20}{4} = \frac{29}{4}$$

Now, add ab:

$$a^2 + b^2 + ab = \frac{29}{4} + \frac{5}{2}$$

Convert $\frac{5}{2}$ to have a denominator of 4:

$$a^2 + b^2 + ab = \frac{29}{4} + \frac{10}{4} = \frac{39}{4}$$

Thus, the value of $a^2 + b^2 + ab$ is $\frac{39}{4}$.

> **Quick Tip**
>
> Use Vieta's formulas to express the sum and product of the roots of a quadratic polynomial, and apply identities to find the required expressions.

OR

23(b). If one zero of the quadratic polynomial $6x^2 + 37x - (p - 2)$ is the reciprocal of the other, then find the value of p.

Solution: Let the zeroes of the quadratic polynomial $6x^2 + 37x - (p - 2)$ be α and β. We are given that one zero is the reciprocal of the other, i.e.,

$$\beta = \frac{1}{\alpha}$$

By Vieta's formulas, we know the following relations for the sum and product of the zeroes: - The sum of the zeroes $\alpha + \beta = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2} = -\frac{37}{6}$ - The product of the zeroes $\alpha\beta = \frac{\text{constant term}}{\text{coefficient of } x^2} = \frac{-(p-2)}{6}$
Substituting $\beta = \frac{1}{\alpha}$ into the equation for the product of the zeroes:

$$\alpha \cdot \frac{1}{\alpha} = \frac{-(p-2)}{6}$$

Simplifying:

$$1 = \frac{-(p-2)}{6}$$

Multiplying both sides by 6:

$$6 = -(p-2)$$

Simplifying:

$$6 = -p + 2$$

Solving for p:

$$p = -4$$

Thus, the value of p is -4.

> **Quick Tip**
>
> If one zero of a quadratic equation is the reciprocal of the other, the product of the zeroes is 1. Use this relationship along with Vieta's formulas to find the unknown coefficient.

24. If $x = \sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$, then find the value of x.

Solution: We are given:

$$x = \sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$$

Using the trigonometric identity for $\sin(A+B) = \sin A \cos B + \cos A \sin B$, we can simplify:

$$x = \sin(60^\circ + 30^\circ) = \sin 90^\circ$$

Since $\sin 90^\circ = 1$, we have:

$$x = 1$$

Thus, the value of x is 1.

Quick Tip
Use the identity $\sin(A+B) = \sin A \cos B + \cos A \sin B$ to simplify such trigonometric expressions.

25(a). Find the ratio in which the point $P(3, y)$ divides the line segment joining $A(-2, -5)$ and $B(6, 3)$. Also, find the value of y.

Correct Answer: $5:3, y = 0$.

Solution: Let $P(3, y)$ divide the line segment joining $A(-2, -5)$ and $B(6, 3)$ in the ratio $k:1$. By using the section formula, the coordinates of P are:

$$x = \frac{kx_2 + x_1}{k+1}, \quad y = \frac{ky_2 + y_1}{k+1}$$

Substitute the given values for x and y:

$$3 = \frac{k(6) + (-2)}{k+1}, \quad y = \frac{k(3) + (-5)}{k+1}$$

From the first equation:

$$3 = \frac{6k-2}{k+1}$$

Multiplying both sides by $k+1$:

$$3(k+1) = 6k - 2$$

Expanding:

$$3k + 3 = 6k - 2$$

Solving for k:

$$3 + 2 = 6k - 3k$$

$$5 = 3k \quad \Rightarrow \quad k = \frac{5}{3}$$

Substitute this value of k into the second equation:

$$y = \frac{\frac{5}{3}(3) - 5}{\frac{5}{3} + 1} = \frac{5-5}{\frac{8}{3}} = 0$$

Thus, the ratio is $5:3$ and $y=0$.

Quick Tip

The section formula helps find the point dividing a line segment in a given ratio.

OR

25(b). Find a point on the y-axis which is equidistant from the points $A(6,5)$ and $B(-4,3)$.

Solution: Let the required point on the y-axis be $P(0,y)$, where the x-coordinate is 0 because it lies on the y-axis.
The distance from P to $A(6,5)$ is equal to the distance from P to $B(-4,3)$. Using the distance formula, the distance between two points (x_1,y_1) and (x_2,y_2) is given by:

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Now, applying the distance formula for PA and PB:
1. Distance from $P(0,y)$ to $A(6,5)$:

$$PA=\sqrt{(6-0)^2+(5-y)^2}=\sqrt{36+(5-y)^2}$$

2. Distance from $P(0,y)$ to $B(-4,3)$:

$$PB=\sqrt{(-4-0)^2+(3-y)^2}=\sqrt{16+(3-y)^2}$$

Since $PA=PB$, we equate the two distances:

$$\sqrt{36+(5-y)^2}=\sqrt{16+(3-y)^2}$$

Squaring both sides:

$$36+(5-y)^2=16+(3-y)^2$$

Expanding both sides:

$$36+(25-10y+y^2)=16+(9-6y+y^2)$$

Simplifying:

$$36+25-10y+y^2=16+9-6y+y^2$$

Canceling out y^2 from both sides:

$$61-10y=25-6y$$

Solving for y:

$$61-25=10y-6y$$

$$36=4y$$

$$y=9$$

Thus, the point on the y-axis that is equidistant from $A(6,5)$ and $B(-4,3)$ is $P(0,9)$.

Quick Tip

The distance between two points on a coordinate plane can be calculated using the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. For points equidistant from two given points, set the two distances equal to each other and solve for the unknown.

Section - C

Q. Nos. 26 to 31 are short answer questions. This section comprises Short Answer (SA) type questions of 3 marks each

26. Find the zeroes of the polynomial $3x^2 - 5x - 2$ and verify the relationship between the zeroes and the coefficients of the polynomial.

Solution: Let $p(x) = 3x^2 - 5x - 2$
We can factorize $p(x)$:

$$p(x) = (3x + 1)(x - 2)$$

Thus, the zeroes are $x = -\frac{1}{3}$ and $x = 2$.
Now, we verify the relationships: - Sum of zeroes:

$$-\frac{1}{3} + 2 = \frac{5}{3} \quad (\text{This is equal to } -\frac{\text{coefficient of } x}{\text{coefficient of } x^2})$$

- Product of zeroes:

$$-\frac{1}{3} \times 2 = -\frac{2}{3} \quad (\text{This is equal to } \frac{\text{constant term}}{\text{coefficient of } x^2})$$

Thus, the relationships are verified: - The sum of the zeroes is $\frac{5}{3}$, which is equal to $-\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$. - The product of the zeroes is $-\frac{2}{3}$, which is equal to $\frac{\text{constant term}}{\text{coefficient of } x^2}$.

Quick Tip

The sum and product of the zeroes of a quadratic polynomial $ax^2 + bx + c$ are related to the coefficients by: - Sum of zeroes $= -\frac{b}{a}$ - Product of zeroes $= \frac{c}{a}$

---

27. Find the coordinates of the points of trisection of the line segment joining the points A(5, – 3) and B(– 4, 3)

Solution: Using the section formula: If a point divides a line segment AB in the ratio $m : n$, its coordinates are:

$$\left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}\right)$$

For ratio 1:2: Substitute $m = 1, n = 2, A(5, -3), B(-4, 3)$:

$$P = \left(\frac{1(-4) + 2(5)}{1 + 2}, \frac{1(3) + 2(-3)}{1 + 2}\right)$$

$$P = \left(\frac{-4+10}{3}, \frac{3-6}{3}\right) = (2, -1)$$

For ratio 2:1: Substitute $m = 2, n = 1$:

$$Q = \left(\frac{2(-4)+1(5)}{2+1}, \frac{2(3)+1(-3)}{2+1}\right)$$

$$Q = \left(\frac{-8+5}{3}, \frac{6-3}{3}\right) = (-1, 1)$$

Quick Tip

The section formula divides a line segment into a given ratio using the formula $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$.

28(a). Prove that a parallelogram ABCD touching a circle at P, Q, R, S is a rhombus.

Solution: Given $ABCD$ is a parallelogram and touches a circle at P, Q, R, S. To prove: $ABCD$ is a rhombus.

Proof: We know that the tangents drawn from an external point to a circle are equal. Thus, for the tangents from vertices A, B, C, D:

$$AP = AS, \quad BP = BQ, \quad CR = CQ, \quad DR = DS$$

Adding all these equalities:

$$AP + BP + CR + DR = AS + BQ + CQ + DS$$

Simplifying, we get:

$$AB + CD = AD + BC$$

Since $ABCD$ is a parallelogram, its opposite sides are equal:

$$AB = CD \quad \text{and} \quad AD = BC$$

Substituting into the equation:

$$AB + AB = AD + AD \implies 2AB = 2AD$$

This gives:

$$AB = AD$$

Hence, all sides of the parallelogram are equal, and $ABCD$ is a rhombus.

Quick Tip

To prove a parallelogram is a rhombus, show that all its sides are equal. Use the property that tangents drawn from an external point to a circle are equal.

OR

28(b). Prove that the lengths of tangents drawn from an external point to a circle are equal.

Solution: Given PA and PB are tangents drawn from an external point P to a circle with center O. To prove: $PA = PB$.
Construction: Join OA, OB, and OP.
Proof: In $\triangle AOP$ and $\triangle BOP$: 1. $OA = OB$ (radii of the circle), 2. $OP = OP$ (common side), 3. $\angle OAP = \angle OBP = 90^\circ$ (a radius is perpendicular to the tangent).
By the RHS (Right Angle-Hypotenuse-Side) congruence rule:

$$\triangle AOP \cong \triangle BOP$$

From congruence:

$$PA = PB$$

Quick Tip

To prove tangents drawn from an external point to a circle are equal, use RHS congruence for the triangles formed by the radii, the tangents, and the line joining the external point to the center.

29(a). A horse is tied with a 14 m long rope at one corner of an equilateral triangular field having side 20 m. Find the area of the field where the horse cannot graze.

Solution: Given: - Length of rope $r = 14\,\text{m}$ - Side of equilateral triangle $s = 20\,\text{m}$.
The horse grazes an area of a sector with central angle $\theta = 60^\circ$. The area of the sector is:

$$\text{Area} = \frac{\theta}{360^\circ} \times \pi r^2$$

Substitute the values:

$$\text{Area} = \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times (14)^2$$

$$= \frac{1}{6} \times \frac{22}{7} \times 196 = \frac{308}{3}\,\text{m}^2 \text{ or } 102.67\,\text{m}^2$$

The area of the equilateral triangular field is:

$$\text{Area} = \frac{\sqrt{3}}{4} \times s^2$$

$$= \frac{\sqrt{3}}{4} \times 20^2 = 100\sqrt{3}\,\text{m}^2 \text{ or } 173.2\,\text{m}^2$$

The area where the horse cannot graze:

$$\text{Remaining area} = \text{Total area} - \text{Grazed area}$$

$$= 173.2 - 102.67 = 70.53\,\text{m}^2$$

Quick Tip

To find the ungrazed area, calculate the total area of the field and subtract the sector area grazed by the horse.

OR

29(b) The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand between 8:00 am and 8:05 am.

Solution: The minute hand sweeps a sector of angle $\theta = 30°$ in 5 minutes. Given: $r = 14\,\text{cm}$. The area of the sector is:

$$\text{Area} = \frac{\theta}{360°} \times \pi r^2$$

Substitute the values:

$$\text{Area} = \frac{30°}{360°} \times \frac{22}{7} \times (14)^2$$

$$= \frac{1}{12} \times \frac{22}{7} \times 196 = \frac{154}{3}\,\text{cm}^2$$

Simplifying:

$$\text{Area} = 51.33\,\text{cm}^2$$

Quick Tip

For any clock hand sweeping a sector, use the formula $\frac{\theta}{360°} \times \pi r^2$ where θ is the angle swept.

---

30. Prove that

$$\frac{\sqrt{1 + \sin A}}{1 - \sin A} = \sec A + \tan A.$$

Solution: We are required to prove that:

$$\frac{\sqrt{1 + \sin A}}{1 - \sin A} = \sec A + \tan A.$$

LHS:

$$\frac{\sqrt{1 + \sin A}}{1 - \sin A} = \frac{(1 + \sin A)}{(1 - \sin A)} \times \frac{(1 + \sin A)}{(1 + \sin A)}$$

$$= \frac{(1 + \sin A)^2}{(1 - \sin^2 A)} = \frac{(1 + \sin A)^2}{\cos^2 A}$$

Now, simplify:

$$= \frac{1 + 2\sin A + \sin^2 A}{\cos^2 A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \sec A + \tan A$$

Thus, we have shown that LHS = RHS.

Quick Tip

When proving trigonometric identities, look for common identities (such as $1 - \sin^2 A = \cos^2 A$) and simplify both sides to show equivalence.

---

31. Prove that $6 + 3\sqrt{2}$ is an irrational number, given that $\sqrt{2}$ is irrational.

Solution: We are given that $\sqrt{2}$ is irrational, and we are asked to prove that $6 + 3\sqrt{2}$ is irrational.
Assume, for the sake of contradiction, that $6 + 3\sqrt{2}$ is rational. If it is rational, then we can express it as:

$$6 + 3\sqrt{2} = \frac{p}{q}$$

where p and q are integers and $q \neq 0$.
Now, subtract 6 from both sides:

$$3\sqrt{2} = \frac{p}{q} - 6$$

$$3\sqrt{2} = \frac{p - 6q}{q}$$

Now divide both sides by 3:

$$\sqrt{2} = \frac{p - 6q}{3q}$$

Since $\frac{p-6q}{3q}$ is a rational number (as it is the ratio of two integers), this implies that $\sqrt{2}$ is rational, which contradicts the given fact that $\sqrt{2}$ is irrational.
Therefore, our assumption that $6 + 3\sqrt{2}$ is rational must be false. Hence, $6 + 3\sqrt{2}$ is irrational.

Quick Tip

If the sum of a rational number and an irrational number is assumed to be rational, it leads to a contradiction. This proves that the sum must be irrational.

Section - D

Q. Nos. 32 to 35 are long answer questions. This section comprises Long Answer (LA) type questions of 5 marks each.

32 (a) Sides AB, BC, and median AD of triangle $\triangle ABC$ are respectively proportional to sides PQ, QR, and median PM of triangle $\triangle PQR$. Show that $\triangle ABC \sim \triangle PQR$.
Solution (a): We are given that: - $AB \parallel PQ$ - $BC \parallel QR$ - Median $AD \parallel PM$
We need to show that $\triangle ABC \sim \triangle PQR$.

Step 1: Use the proportionality of sides: We are given that:

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$

This is the condition for the similarity of triangles by the Side-Side-Side (SSS) criterion. According to this criterion, if three corresponding sides of two triangles are proportional, then the triangles are similar.

Step 2: Apply the SSS criterion: Since we have:

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$

This satisfies the condition for SSS similarity. Hence, we conclude that:

$$\triangle ABC \sim \triangle PQR$$

Thus, the triangles are similar by the Side-Side-Side (SSS) similarity criterion.

> Quick Tip
>
> To prove two triangles are similar by SSS, show that the corresponding sides are proportional.

OR

32 (b) Prove that if a line is drawn parallel to one side of a triangle to intersect the other two sides at distinct points, then the other two sides are divided in the same ratio.

Solution (b): Let $\triangle ABC$ be a triangle, and let a line $DE \parallel BC$ be drawn such that it intersects the sides AB and AC at points D and E, respectively. We need to prove that:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Step 1: Use of Basic Proportionality Theorem: The Basic Proportionality Theorem (Thales' Theorem) states that if a line is drawn parallel to one side of a triangle, then it divides the other two sides proportionally. This means:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Step 2: Proving the proportionality: Since $DE \parallel BC$, by the Basic Proportionality Theorem, we know that the line DE divides the sides AB and AC in the same ratio. This gives us the required result:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Thus, we have proved that the sides AB and AC are divided in the same ratio by the line $DE \parallel BC$.

Quick Tip

When a line is drawn parallel to one side of a triangle, it divides the other two sides in the same ratio, which is a direct application of Thales' Theorem (Basic Proportionality Theorem).

---

33. The following data gives the information about the lifetimes (in hours) of 225 neon lamps:

Lifetime (in hours)	Number of lamps	Cumulative frequency (cf)
1500 – 2000	10	10
2000 – 2500	35	45
2500 – 3000	52	97
3000 – 3500	61	158
3500 – 4000	40	198
4000 – 4500	29	227

Find the median lifetime of a lamp.

Solution: To find the median, we use the formula for the median in a frequency distribution:

$$\text{Median} = L + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$$

where: - L is the lower limit of the median class, - N is the total number of observations (in this case, $N = 225$), - cf is the cumulative frequency before the median class, - f is the frequency of the median class, - h is the class width.

Step 1: Find $\frac{N}{2}$

$$\frac{N}{2} = \frac{225}{2} = 112.5$$

So, we are looking for the median class, where the cumulative frequency just exceeds 112.5.

Step 2: Identify the median class From the table: - The cumulative frequency of the class $2500 - 3000$ is 97, and the cumulative frequency of the class $3000 - 3500$ is 158. - Since 112.5 lies between 97 and 158, the median class is $3000 - 3500$.

Step 3: Apply the median formula - $L = 3000$ (the lower limit of the median class), - $cf = 97$ (the cumulative frequency before the median class), - $f = 61$ (the frequency of the median class), - $h = 500$ (the class width).

Now, substitute these values into the formula:

$$\text{Median} = 3000 + \left(\frac{112.5 - 97}{61}\right) \times 500$$

$$\text{Median} = 3000 + \left(\frac{15.5}{61}\right) \times 500$$

$$\text{Median} = 3000 + (0.2549) \times 500$$

$$\text{Median} = 3000 + 127.45$$

$$\text{Median} = 3127.45$$

Thus, the median lifetime of a lamp is approximately 3127.45 hours.

Quick Tip

The median is the value that divides the dataset into two equal halves. For grouped data, the formula for the median is:

$$\text{Median} = L + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$$

where L is the lower limit of the median class, cf is the cumulative frequency before the median class, f is the frequency of the median class, and h is the class width.

34(a). A toy is made by attaching a hemisphere and a cone. The radius of the hemisphere is 7 cm and the total height of the toy is 31 cm. Find the total surface area of the toy.

Solution (a): Given: - Radius of hemisphere $r = 7\,\text{cm}$ - Total height of toy $= 31\,\text{cm}$. - Height of cone $= 31 - 7 = 24\,\text{cm}$.
The slant height l of the cone is:

$$l = \sqrt{r^2 + h^2} = \sqrt{7^2 + 24^2} = \sqrt{49 + 576} = \sqrt{625} = 25\,\text{cm}$$

The total surface area (TSA) of the toy is the sum of the curved surface area (CSA) of the hemisphere and the cone:

$$\text{TSA} = \text{CSA of hemisphere} + \text{CSA of cone}$$

$$\text{CSA of hemisphere} = 2\pi r^2 = 2 \times \frac{22}{7} \times 7^2 = 2 \times \frac{22}{7} \times 49 = 2 \times 154 = 308\,\text{cm}^2$$

$$\text{CSA of cone} = \pi r l = \frac{22}{7} \times 7 \times 25 = 550\,\text{cm}^2$$

Thus, the total surface area is:

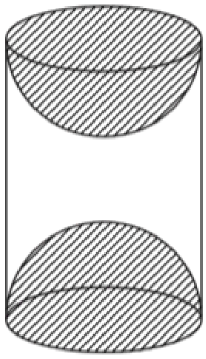
$$\text{TSA} = 308 + 550 = 858\,\text{cm}^2$$

Quick Tip

The total surface area of a toy formed by a cone and hemisphere is the sum of the curved surface areas of both parts.

OR

34(b). A wooden article was made by scooping out a hemisphere from each end of a solid cylinder. If the height of the cylinder is 15 cm and its base is of radius 4·2 cm, then find the total surface area of the article.



Solution (b): Given: - Radius of cylinder $r = 4.2\,\text{cm}$ - Height of cylinder $h = 15\,\text{cm}$. - Two hemispheres attached at both ends.

The total surface area is the sum of the curved surface area (CSA) of the cylinder and the curved surface areas of the two hemispheres:

$$\text{TSA} = \text{CSA of cylinder} + 2 \times \text{CSA of hemisphere}$$

$$\text{CSA of cylinder} = 2\pi rh = 2 \times \frac{22}{7} \times 4.2 \times 15 = 2 \times \frac{22}{7} \times 63 = 396\,\text{cm}^2$$

$$\text{CSA of one hemisphere} = 2\pi r^2 = 2 \times \frac{22}{7} \times (4.2)^2 = 2 \times \frac{22}{7} \times 17.64 = 49.2\,\text{cm}^2$$

Thus, the total surface area is:

$$\text{TSA} = 396 + 2 \times 49.2 = 396 + 98.4 = 494.4\,\text{cm}^2$$

Quick Tip

When calculating total surface area for a toy with two hemispheres and a cylinder, add the curved surface area of the cylinder and the curved areas of both hemispheres.

---

35. The sum of the digits of a 2-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.
Solution: Let the digit at the unit's place be y and the digit at the ten's place be x. Thus, the two-digit number is $10x + y$.

From the given condition, we know:

$$\text{Sum of the digits} = x + y = 9 \quad \text{(Equation 1)}$$

Also, nine times the number is twice the number obtained by reversing the digits:

$$9(10x + y) = 2(10y + x)$$

Expanding both sides:

$$90x + 9y = 20y + 2x$$

Simplifying:

$$90x - 2x = 20y - 9y$$

$$88x = 11y$$

Dividing by 11:

$$8x = y \quad \text{(Equation 2)}$$

Now, substitute $y = 8x$ into Equation 1:

$$x + 8x = 9$$

$$9x = 9$$

$$x = 1$$

Substitute $x = 1$ into Equation 1 to find y:

$$1 + y = 9$$

$$y = 8$$

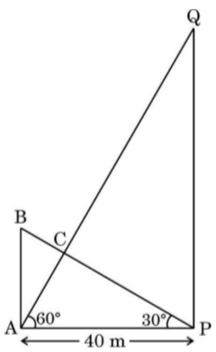
Thus, the two-digit number is $10x + y = 10(1) + 8 = 18$.

> Quick Tip
>
> When solving problems involving the sum and product of digits, you can set up a system of linear equations based on the conditions provided and solve them step by step.

Section - E

This section comprises 3 case study based questions of 4 marks each.

36. Two poles of different heights stand on level ground and at a distance of 40 m. Both poles are supported by wires attached from the top of each pole to the bottom of the other. A coupling is placed at point C, where the two wires cross. Based on the above information, answer the following s:



(i) Find the height of pole AB

Solution (i): We are asked to find the height of pole AB. Let the height of pole AB be h_1, and the height of pole PQ be h_2. We are also given that the distance between the poles is 40 m.

By applying the intersecting wires concept, use the known geometry to calculate the height h_1.

(ii) Find the height of pole PQ

Solution (ii): We are asked to find the height of pole PQ. This is based on the geometry of the two poles and the wire setup. Apply similar concepts as above.

(iii) (a) If the angle of elevation of the top of the pole PQ from the top of the pole AB is 30°, find the distance BQ.

Solution (iii)(a): We are asked to find the distance BQ. Using the geometry of the setup, the distance between the poles and the coupling height can be calculated using trigonometric relations based on the angle of elevation.

(b) If the coupling is at a height of 20 m from the ground, how far down the wire from the smaller pole AB is the coupling ?

Solution (iii)(b): If the coupling is placed at a height of 20 m, use geometry to find the point of intersection on the wire. The calculations involve using height and distance from the pole.

> Quick Tip
>
> When dealing with intersecting lines and poles, use geometry and trigonometry to analyze the relationship between the heights and the distances. Proportionality rules can often help simplify the calculations.

---

37. Family structure: In a recent survey of this year, 51% of the families in the United States of America had no children, 20% had one child, 19% had two children, 7% had three children, and 3% had four or more children. Based on the above information, answer the following s:

Based on the above information, answer the following s :

(i) Find the probability that the selected family has two or three children.

Solution (i): We are asked to find the probability that the selected family has two or three children. From the given data: - Percentage of families with two children = 19- Percentage of families with three children = 7

Thus, the probability of selecting a family with two or three children is the sum of these probabilities:

$$P(\text{two or three children}) = P(\text{two children}) + P(\text{three children})$$

$$P(\text{two or three children}) = \frac{19}{100} + \frac{7}{100} = \frac{26}{100} = 0.26$$

(ii)Find the probability that the selected family has more than one child.

Solution (ii): We are asked to find the probability that the selected family has more than one child. Families with more than one child include those with two, three, or more children. From the given data: - Percentage of families with two children = 19- Percentage of families with three children = 7- Percentage of families with four or more children = 3

Thus, the probability of selecting a family with more than one child is:

$$P(\text{more than one child}) = P(\text{two children}) + P(\text{three children}) + P(\text{four or more children})$$

$$P(\text{more than one child}) = \frac{19}{100} + \frac{7}{100} + \frac{3}{100} = \frac{29}{100} = 0.29$$

(iii) (a) Find the probability that the selected family has less than three children.

Solution (iii)(a): We are asked to find the probability that the selected family has less than three children. Families with less than three children include those with no children, one child, or two children. From the given data: - Percentage of families with no children = 51- Percentage of families with one child = 20- Percentage of families with two children = 19
Thus, the probability of selecting a family with less than three children is:

$$P(\text{less than three children}) = P(\text{no children}) + P(\text{one child}) + P(\text{two children})$$

$$P(\text{less than three children}) = \frac{51}{100} + \frac{20}{100} + \frac{19}{100} = \frac{90}{100} = 0.90$$

(iii) (b) Find the probability that the selected family has more than two children.

Solution (iii)(b): We are asked to find the probability that the selected family has more than two children. Families with more than two children include those with three or more children. From the given data: - Percentage of families with three children = 7- Percentage of families with four or more children = 3
Thus, the probability of selecting a family with more than two children is:

$$P(\text{more than two children}) = P(\text{three children}) + P(\text{four or more children})$$

$$P(\text{more than two children}) = \frac{7}{100} + \frac{3}{100} = \frac{10}{100} = 0.10$$

Quick Tip

To find the probability of an event, divide the number of favorable outcomes by the total number of outcomes. Use the given percentages directly as fractions.

---

38. Sumant's mother started a new shoe shop. To display the shoes, she put 3 pairs of shoes in the 1st row, 5 pairs in the 2nd row, 7 pairs in the 3rd row, and so on. Based on the above information, answer the following s:

(i) How many pairs of shoes are displayed in the 6th row ?

Solution (i): We are asked to find how many pairs of shoes are displayed in the 6th row. The number of pairs of shoes in each row forms an arithmetic progression with the first term $a = 3$ and the common difference $d = 2$.
The number of pairs of shoes in the n-th row is given by:

$$T_n = a + (n - 1)d$$

For the 6th row:

$$T_6 = 3 + (6 - 1) \times 2 = 3 + 10 = 13$$

Thus, there are 13 pairs of shoes in the 6th row.

(ii) What is the difference of pairs of shoes in the 1st row and the 6th row ?

Solution (ii): We are asked to find the difference in the number of pairs of shoes between the 1st row and the 6th row. From the previous part, we know: - The number of pairs of shoes in the 1st row is 3. - The number of pairs of shoes in the 6th row is 13.
Thus, the difference is:

$$\text{Difference} = 13 - 3 = 10$$

The difference in the number of pairs of shoes is 10.

(iii) (a) Find the total number of pairs of shoes displayed in the first 15 rows.

Solution (iii)(a): We are asked to find the total number of pairs of shoes displayed in the first 15 rows. The total number of pairs of shoes in the first n rows is the sum of the first n terms of the arithmetic progression. The sum of the first n terms is given by:

$$S_n = \frac{n}{2}\left(2a + (n - 1)d\right)$$

Substitute $n = 15$, $a = 3$, and $d = 2$ into the formula:

$$S_{15} = \frac{15}{2}\left(2(3) + (15 - 1)(2)\right)$$

$$S_{15} = \frac{15}{2}(6 + 28) = \frac{15}{2} \times 34 = 510$$

Thus, the total number of pairs of shoes displayed in the first 15 rows is 510.

(iii) (b) If the pairs of shoes displayed in the 4th row are 'on sale' at price of ¡ 500 for each pair, then find the total amount (money) earned by Sumant's mother if all shoes displayed in the 4th row are sold out

Solution (iii)(b): We are asked to find the total amount of money earned by Sumant's mother if all the shoes displayed in the 4th row are sold. We are given that the price of each pair of shoes in the 4th row is 500. First, find the number of pairs of shoes in the 4th row. Using the formula for the n-th term of the arithmetic progression:

$$T_4 = 3 + (4 - 1) \times 2 = 3 + 6 = 9$$

Thus, there are 9 pairs of shoes in the 4th row.
The total money earned is:

$$\text{Total amount} = 9 \times 500 = 4500$$

Thus, Sumant's mother earns 4500 from the 4th row.

Quick Tip

For arithmetic progressions, use the formula $T_n = a + (n-1)d$ to find the number of terms in each row, and use $S_n = \frac{n}{2}(2a + (n-1)d)$ to find the sum of the terms.