# CBSE Class X Mathematics Basic Set 2 (430/2/2)

| **Time Allowed :**3 Hours | **Maximum Marks :**80 | **Total Questions :**38 |
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**General Instructions**

**Read the following instructions very carefully and strictly follow them:**

1. This paper contains 38 s. All s are compulsory.
2. Paper is divided into 5 Sections – Section A, B, C, D and E.
3. In Section–A number 1 to 18 are Multiple Choice s (MCQs) and number 19 20 are Assertion-Reason based s of 1 mark each
4. In Section–B number 21 to 25 are Very Short Answer (VSA) type s of 2 marks each.
5. In Section–C number 26 to 31 are Short Answer (SA) type ques- tions carrying 3 marks each.
6. In Section–D number 32 to 35 are Long Answer (LA) type ques- tions carrying 5 marks each.
7. In Section–E number 36 to 38 are Case Study based s carrying 4 marks each. Internal choice is provided in 2 marks in each case-study.
8. There is no overall choice. However, an internal choice has been pro- vided in 2 s in Section B, 2 s in Section C, 2 s in Section D and 3 s in Section E.
9. Draw neat figures wherever required. Take p= 22/7 wherever required if not stated.
10. Use of calculators is NOT allowed.

## Section - A

**Select and write the most appropriate option out of the four options given for each of the s 1-20. There is no negative mark for the incorrect response.**

**1. The difference of the areas of a minor sector of angle $120°$ and its corresponding major sector of a circle of radius 21 cm is:**
(a) $231\,\text{cm}^2$
(b) $462\,\text{cm}^2$
(c) $346.5\,\text{cm}^2$
(d) $693\,\text{cm}^2$

**Correct Answer:** (b) $462\,\text{cm}^2$

**Solution:** The area of a sector is given by:

$$A = \frac{\theta}{360^\circ} \times \pi r^2,$$

where $\theta$ is the angle of the sector, and $r$ is the radius. For the minor sector, $\theta = 120^\circ$ and $r = 21\,\text{cm}$:

$$A_{\text{minor}} = \frac{120^\circ}{360^\circ} \times \pi(21)^2 = \frac{1}{3} \times \pi \times 441 = 147\pi\,\text{cm}^2.$$

For the major sector, the angle is $360^\circ - 120^\circ = 240^\circ$, so:

$$A_{\text{major}} = \frac{240^\circ}{360^\circ} \times \pi(21)^2 = \frac{2}{3} \times \pi \times 441 = 294\pi\,\text{cm}^2.$$

Now, the difference in areas is:

$$A_{\text{major}} - A_{\text{minor}} = 294\pi - 147\pi = 147\pi\,\text{cm}^2.$$

Converting $\pi \approx 3.14$:

$$147 \times 3.14 = 460.38\,\text{cm}^2.$$

Thus, the closest answer is (b) $462\,\text{cm}^2$

> Quick Tip
>
> Use the formula $A = \frac{\theta}{360^\circ} \times \pi r^2$ to find the areas of the minor and major sectors, then subtract the areas for the answer.

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**2. The annual rainfall record of a city for 66 days is given in the following table:**

**The difference of upper limits of modal and median classes is:**

(a) 10
(b) 15
(c) 20
(d) 30

**Correct Answer:** (c) 20.

**Solution:** The modal class is the class with the highest frequency. From the table, the modal class is $0-10$ because it has the highest frequency (22 days). The median class is the class that contains the median value. Since the total number of days is 66, the median class will be the one containing the 33rd day, which corresponds to the class $20-30$. Thus, the difference of upper limits of modal and median classes is:

$$30 - 10 = 20.$$

> Quick Tip
>
> To find the modal and median classes, look at the frequency of data. The modal class has the highest frequency, and the median class contains the middle value.

**3. In the given figure, tangents PA and PB from a point P to a circle with centre O are inclined to each other at an angle of $80^\circ$. $\angle ABO$ is equal to:**
(a) $40^\circ$
(b) $80^\circ$
(c) $100^\circ$
(d) $50^\circ$

**Correct Answer:** (a) $40^\circ$.

**Solution:** The angle between two tangents drawn from an external point to a circle is equal to twice the angle subtended by the line joining the external point to the center of the circle. Given $\angle APB = 80^\circ$, the angle $\angle ABO$ is half of this:

$$\angle ABO = \frac{80^\circ}{2} = 40^\circ.$$

Quick Tip

The angle between two tangents from an external point is twice the angle between the line joining the external point and the center of the circle.

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**4. The value of $\frac{1+\tan^2 A}{1+\cot^2 A}$ is equal to:**
(a) $\sec^2 A$
(b) -1
(c) $\cot^2 A$
(d) $\tan^2 A$

**Correct Answer:** (d) $\tan^2 A$.

**Solution:** We know the identity $\cot A = \frac{1}{\tan A}$. Substituting this into the expression:

$$\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{1+\tan^2 A}{1+\frac{1}{\tan^2 A}}.$$

Simplify the denominator:

$$= \frac{1+\tan^2 A}{\frac{\tan^2 A+1}{\tan^2 A}} = (1+\tan^2 A)\cdot\frac{\tan^2 A}{1+\tan^2 A}.$$

The $1+\tan^2 A$ terms cancel out, leaving:

$$\boxed{\tan^2 A}.$$

Quick Tip

The identity $\sec^2 A = 1+\tan^2 A$ and $\csc^2 A = 1+\cot^2 A$ are useful for solving such problems.

**5. If $\sin^2\theta = \frac{3}{4}$, then $\theta$ is:**
(a) $30^\circ$
(b) $45^\circ$
(c) $60^\circ$
(d) $90^\circ$

**Correct Answer:** (c) $60^\circ$.

**Solution:** We are given that $\sin^2\theta = \frac{3}{4}$. Taking the square root of both sides, we get:

$$\sin\theta = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}.$$

From trigonometric values, we know that $\sin 60^\circ = \frac{\sqrt{3}}{2}$. Therefore,

$$\theta = 60^\circ.$$

Quick Tip

For $\sin^2\theta = \frac{3}{4}$, the value of $\theta$ is $60^\circ$, since $\sin 60^\circ = \frac{\sqrt{3}}{2}$.

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**6. The zeroes of the polynomial $3x^2 - 5x - 2$ are:**
(a) $\frac{1}{3}, 2$
(b) $\frac{-1}{3}, 2$
(c) $\frac{-1}{3}, -2$
(d) $\frac{1}{3}, -2$

**Correct Answer:** (b) $\frac{-1}{3}, 2$.

**Solution:** We are given the quadratic polynomial $3x^2 - 5x - 2$. To find its zeroes, we use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

where $a = 3$, $b = -5$, and $c = -2$. Substituting these values into the formula:

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(-2)}}{2(3)} = \frac{5 \pm \sqrt{25 + 24}}{6} = \frac{5 \pm \sqrt{49}}{6} = \frac{5 \pm 7}{6}.$$

Thus, the two roots are:

$$x = \frac{5+7}{6} = \frac{12}{6} = 2 \quad \text{and} \quad x = \frac{5-7}{6} = \frac{-2}{6} = \frac{-1}{3}.$$

Therefore, the zeroes are $\frac{-1}{3}$ and 2.

Quick Tip

To find the zeroes of a quadratic equation, use the quadratic formula: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$.

**7. A pole $7\sqrt{3}$ m high casts a shadow 21 m long on the ground, then the sun's elevation is:**
(a) $30^\circ$
(b) $45^\circ$
(c) $60^\circ$
(d) $90^\circ$

**Correct Answer:** (a) $30^\circ$.

**Solution:** Let the height of the pole be $h = 7\sqrt{3}$ m and the length of its shadow be 21 m. We can use the tangent function to find the elevation of the sun:

$$\tan\theta = \frac{\text{height of the pole}}{\text{length of the shadow}} = \frac{7\sqrt{3}}{21}.$$

Thus,

$$\tan\theta = \frac{1}{\sqrt{3}}.$$

From trigonometric values, we know that $\tan 30^\circ = \frac{1}{\sqrt{3}}$. Therefore, the elevation of the sun is $30^\circ$.

Quick Tip

To calculate the elevation of the sun, use $\tan\theta = \frac{\text{height}}{\text{shadow length}}$.

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**8. The HCF of the smallest 2-digit number and the smallest composite number is:**
(a) 2
(b) 20
(c) 40
(d) 4

**Correct Answer:** (d) 4.

**Solution:** The smallest 2-digit number is 10, and the smallest composite number is 4.

To find the HCF (Highest Common Factor) of 10 and 4, we list their factors:

- Factors of 10: 1, 2, 5, 10
- Factors of 4: 1, 2, 4

The greatest common factor is 2, so the HCF of 10 and 4 is 2.
Hence, the correct answer is (a) 2.

**Quick Tip**

To find the HCF, list the factors of the numbers and pick the highest common one. For composite numbers, look for the common factors with other numbers.

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**9. The total surface area of a solid hemisphere of radius 7 cm is:**
(a) $98\pi\,\text{cm}^2$
(b) $147\pi\,\text{cm}^2$
(c) $196\pi\,\text{cm}^2$
(d) $228\frac{2}{3}\pi\,\text{cm}^2$

**Correct Answer:** (b) $147\pi\,\text{cm}^2$.

**Solution:** The total surface area (TSA) of a solid hemisphere is given by:

$$TSA = 3\pi r^2,$$

where $r$ is the radius. Given $r = 7\,\text{cm}$, the TSA is:

$$TSA = 3\pi(7)^2 = 3\pi \times 49 = 147\pi\,\text{cm}^2.$$

**Quick Tip**

The total surface area of a hemisphere is $3\pi r^2$, which includes both the curved surface area and the base area.

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**10. If $\sin A = \frac{3}{5}$, then the value of $\cot A$ is:**
(a) $\frac{3}{4}$
(b) $\frac{4}{3}$
(c) $\frac{4}{5}$
(d) $\frac{5}{4}$

**Correct Answer:** (b) $\frac{4}{3}$.

**Solution:** We know that $\sin^2 A + \cos^2 A = 1$. Given $\sin A = \frac{3}{5}$, we can find $\cos A$ as follows:

$$\sin^2 A + \cos^2 A = 1 \quad \Rightarrow \quad \left(\frac{3}{5}\right)^2 + \cos^2 A = 1 \quad \Rightarrow \quad \frac{9}{25} + \cos^2 A = 1.$$

Solving for $\cos^2 A$:

$$\cos^2 A = 1 - \frac{9}{25} = \frac{25}{25} - \frac{9}{25} = \frac{16}{25}.$$

Thus, $\cos A = \frac{4}{5}$.
Now, $\cot A = \frac{\cos A}{\sin A} = \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{3}$.

Quick Tip

To find $\cot A$, use the relation $\cot A = \frac{\cos A}{\sin A}$. First, find $\cos A$ using $\cos^2 A = 1 - \sin^2 A$.

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**11. The graph of a pair of linear equations $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ in two variables $x$ and $y$ represents parallel lines if:**
(a) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(b) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(c) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(d) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

**Correct Answer:** (d) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

**Solution:** For the lines to be parallel, the ratios of the coefficients of $x$ and $y$ in the two equations must be equal, i.e., $\frac{a_1}{a_2} = \frac{b_1}{b_2}$. However, the constant terms must be different, i.e., $\frac{c_1}{c_2} \neq 1$, so that the lines are not identical but still parallel.

Quick Tip

Parallel lines have the same slope, which is why the ratios of the coefficients of $x$ and $y$ must be equal. The constant terms should differ to avoid coincidence.

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**12. A line intersecting a circle in two distinct points is called a:**
(a) secant
(b) chord
(c) diameter
(d) tangent

**Correct Answer:** (a) secant.

**Solution:** A line that intersects a circle at two distinct points is called a secant. A tangent, on the other hand, touches the circle at exactly one point.

Quick Tip

A secant is a line that cuts a circle in two points, whereas a tangent touches the circle at only one point.

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**13. The value of $k$ for which the pair of linear equations $x+y-4=0$ and $2x+ky-8=0$**

**has infinitely many solutions, is:**
(a) $k \neq 2$
(b) $k \neq -2$
(c) $k = 2$
(d) $k = -2$

**Correct Answer:** (c) $k = 2$.

**Solution:** For a pair of linear equations to have infinitely many solutions, the ratios of their coefficients must be equal. The given equations are:

$$x + y - 4 = 0 \quad \text{and} \quad 2x + ky - 8 = 0.$$

We compare the coefficients:

$$\frac{1}{2} = \frac{1}{k} = \frac{-4}{-8}.$$

This implies that $k = 2$, as the ratios of the coefficients must be equal for infinite solutions.

> Quick Tip
>
> For infinitely many solutions in a pair of linear equations, the ratios of the coefficients of $x$, $y$, and the constants must be equal.

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**14. A quadratic polynomial, the sum of whose zeroes is -5 and their product is 6, is:**
(a) $x^2 + 5x + 6$
(b) $x^2 - 5x + 6$
(c) $x^2 + 5x - 6$
(d) $x^2 - 5x - 6$

**Correct Answer:** (a) $x^2 + 5x + 6$.

**Solution:** For a quadratic polynomial of the form $ax^2 + bx + c$, the sum of the zeroes is given by $\frac{-b}{a}$ and the product of the zeroes is $\frac{c}{a}$. We are given that the sum of the zeroes is $-5$ and the product is 6. Therefore, the polynomial is:

$$x^2 + 5x + 6.$$

> Quick Tip
>
> For a quadratic polynomial $ax^2 + bx + c$, the sum of the zeroes is $\frac{-b}{a}$ and the product is $\frac{c}{a}$.

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**15. If $P(A)$ denotes the probability of an event A, then:**
(a) $P(A) < 0$
(b) $P(A) > 1$

(c) $0 \leq P(A) \leq 1$
(d) $-1 \leq P(A) \leq 1$

**Correct Answer:** (c) $0 \leq P(A) \leq 1$.

**Solution:** The probability of any event $A$, denoted by $P(A)$, must be a number between 0 and 1, inclusive. It represents the likelihood of the event occurring, with 0 meaning it will not happen and 1 meaning it will certainly happen.

> Quick Tip
>
> The probability of an event $A$ must always lie between 0 and 1: $0 \leq P(A) \leq 1$.

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**16. Which of the following quadratic equations has -1 as a root?**
(a) $x^2 - 4x - 5 = 0$
(b) $x^2 - 4x + 5 = 0$
(c) $x^2 + 3x + 4 = 0$
(d) $x^2 - 5x + 6 = 0$

**Correct Answer:** (a) $x^2 - 4x - 5 = 0$.

**Solution:** To determine which quadratic equation has $-1$ as a root, substitute $x = -1$ into each equation: - For $x^2 - 4x - 5 = 0$, substituting $x = -1$:

$$(-1)^2 - 4(-1) - 5 = 1 + 4 - 5 = 0,$$

so $-1$ is a root.
Thus, the equation is $x^2 - 4x - 5 = 0$.

> Quick Tip
>
> To check if $-1$ is a root of a quadratic equation, substitute $x = -1$ into the equation. If the result is 0, then $-1$ is a root.

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**17. The distance of the point $(3, 4)$ from the origin is:**
(a) 25
(b) 5
(c) 7
(d) 1

**Correct Answer:** (b) 5.

**Solution:** The distance of a point $(x, y)$ from the origin $(0, 0)$ is given by the distance formula:

$$\text{Distance} = \sqrt{x^2 + y^2}.$$

For the point $(3, 4)$:

$$\text{Distance} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5.$$

**Quick Tip**

The distance from the origin is calculated using the formula $\sqrt{x^2 + y^2}$.

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**18. If the first term of an AP is -3 and the common difference is -2, then the seventh term is:**

(a) -9
(b) 9
(c) -17
(d) -15

**Correct Answer:** (d) -15.

**Solution:** The formula for the $n$-th term of an arithmetic progression (AP) is:

$$T_n = a + (n - 1) \cdot d,$$

where $a$ is the first term and $d$ is the common difference. For the given AP, $a = -3$, $d = -2$, and $n = 7$:

$$T_7 = -3 + (7 - 1) \cdot (-2) = -3 + 6 \cdot (-2) = -3 - 12 = -15.$$

**Quick Tip**

The $n$-th term of an AP is given by $T_n = a + (n - 1) \cdot d$.

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**19. Assertion (A): The point $(0, 4)$ lies on the y-axis.**
**Reason (R): The x-coordinate of a point, lying on the y-axis, is zero.**
**Directions : In question numbers 19,a statement of Assertion(A) is followed by a statement of Reason (R).**
**Choose the correct option :**
**(A) Both Assertion (A) and Reason (R) are correct and Reason (R) is the correct explanation of Assertion(A).**
**(B) Both Assertion (A) and Reason (R) are correct but Reason (R) is not the correct explanation of Assertion (A).**
**(C) Assertion (A) is true, but Reason (R) is false.**
**(D) Assertion (A) is false, but Reason (R) is true.**

**Correct Answer:** (A) Both Assertion (A) and Reason (R) are correct and Reason (R) is the correct explanation of Assertion (A).

**Solution:** Assertion (A) is correct because the point $(0, 4)$ is on the y-axis, where the x-coordinate is always zero. Reason (R) is also correct as it correctly states that for any point on the y-axis, the x-coordinate must be zero. This explains why $(0, 4)$ lies on the y-axis.

Quick Tip

Points on the y-axis have an x-coordinate of zero. The y-axis represents all points where the x-coordinate is zero.

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**20. Assertion (A): A line drawn parallel to any one side of a triangle intersects the other two sides in the same ratio.**
**Reason (R): Parallel lines cannot be drawn to any side of a triangle.**
**Directions : In question numbers 19,a statement of Assertion(A) is followed by a statement of Reason (R).**
**Choose the correct option :**
**(A) Both Assertion (A) and Reason (R) are correct and Reason (R) is the correct explanation of Assertion(A).**
**(B) Both Assertion (A) and Reason (R) are correct but Reason (R) is not the correct explanation of Assertion (A).**
**(C) Assertion (A) is true, but Reason (R) is false.**
**(D) Assertion (A) is false, but Reason (R) is true.**

**Correct Answer:** (C) Assertion (A) is true, but Reason (R) is false.

**Solution:** Assertion (A) is true. This is a statement of the basic proportionality theorem (also known as Thales' theorem), which states that if a line is parallel to one side of a triangle and intersects the other two sides, it divides those sides proportionally. Reason (R) is false because parallel lines can be drawn to any side of a triangle. There is no restriction on drawing parallel lines to the sides of a triangle.

Quick Tip

The basic proportionality theorem states that a line parallel to one side of a triangle divides the other two sides in the same ratio. Parallel lines can be drawn to any side of a triangle.

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## Section - B

**Q. Nos. 21 to 25 are very short answer questions. This section comprises Very Short Answer (VSA) type questions of 2 marks each.**

**21(a) In the given figure, $OA \times OB = OC \times OD$. Prove that $\angle AOD \cong \angle COB$.**

**Solution:** Given $OA \times OB = OC \times OD$, it implies that points $A, B, C$, and $D$ lie on a circle by the converse of the Power of a Point Theorem, which states that if a point outside a circle

multiplies the lengths of the segments of two secants such that the products are equal, then the secants intersect the circle.
Thus, $\angle AOD$ and $\angle COB$ are angles subtended by the same arc $OD$ and $OB$ respectively. Angles subtended by the same arc in a circle are equal.
Therefore, $\angle AOD \cong \angle COB$.

Quick Tip

Angles subtended by the same arc in a circle are equal, which is a fundamental property in circle geometry.

**OR**

**21(b) In the given figure, $\angle D = \angle E$ and $\frac{AD}{DB} = \frac{AE}{EC}$. Prove that $\triangle ABC$ is isosceles.**

**Solution:** Given $\angle D = \angle E$ and $\frac{AD}{DB} = \frac{AE}{EC}$, by the Angle-Angle-Side (AAS) similarity criterion, $\triangle ADE \sim \triangle BEC$. This implies $\angle ADE = \angle BEC$ and $\angle DAE = \angle EBC$.
By the properties of similar triangles, $\frac{AB}{AC} = \frac{AD}{AE}$, and from the given proportionality, $\frac{AD}{AE} = 1$ thus $AB = AC$.
Therefore, $\triangle ABC$ is isosceles.

Quick Tip

Proportionality and similarity principles are powerful tools in proving properties about triangles, such as side equality in isosceles triangles.

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**22. A lot consists of 165 ball pens of which 30 are defective and the others are good. Rakshita will buy a pen if it is good. The shopkeeper draws one pen at random and gives it to Rakshita. What is the probability that she will buy it?**

**Solution:** We are given that the total number of ball pens is 165, and the number of defective pens is 30. So, the number of good pens is:

$$\text{Number of good pens} = 165 - 30 = 135.$$

The probability of an event is calculated using the formula:

$$P(\text{event}) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}.$$

In this case, the favorable outcome is drawing a good pen, and the total outcomes are all the pens. Thus, the probability that Rakshita will buy a good pen is:

$$P(\text{good pen}) = \frac{135}{165} = \frac{9}{11}.$$

Quick Tip

The probability of an event is calculated as $\frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$.

**23. Prove that the tangents drawn at the ends of a diameter of a circle are parallel to each other.**

**Solution:** Let the tangents $l$ and $m$ be drawn at the end points A and B of the diameter of the circle with center O. We need to prove that the tangents $l$ and $m$ are parallel.
To prove: $l \parallel m$.
Proof: Since $O$ is the center of the circle, the radius at points A and B are perpendicular to the tangents $l$ and $m$ respectively. Hence, we have:

$$\angle 1 = 90^\circ \quad \text{(since the radius is perpendicular to the tangent at point A)},$$

and similarly,

$$\angle 2 = 90^\circ \quad \text{(since the radius is perpendicular to the tangent at point B)}.$$

We know that when two lines are cut by a transversal and the alternate interior angles are equal, the lines are parallel. Since $\angle 1 = \angle 2 = 90^\circ$, the tangents $l$ and $m$ are parallel.
Thus, we have proven that $l \parallel m$.

**Quick Tip**

When two lines are cut by a transversal and the alternate interior angles are equal, the lines are parallel.

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**24. Find the HCF of 84 and 144 by prime factorization method.**

**Solution:** We will find the prime factorization of both 84 and 144.
Prime factorization of 84:

$$84 = 2 \times 2 \times 3 \times 7 = 2^2 \times 3 \times 7.$$

Prime factorization of 144:

$$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 2^4 \times 3^2.$$

The HCF is obtained by multiplying the common factors raised to the least power. Here, the common factors between 84 and 144 are $2^2$ and $3^1$. Therefore, the HCF is:

$$\text{HCF} = 2^2 \times 3 = 4 \times 3 = 12.$$

Thus, the HCF of 84 and 144 is 12.

**Quick Tip**

The HCF is the product of the common factors raised to the least power.

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**25(a). The sum of two natural numbers is 70 and their difference is 10. Find the natural numbers.**

**Solution:** Let the two natural numbers be $x$ and $y$. We are given:

$$x + y = 70 \quad \text{(sum equation)} \quad \text{and} \quad x - y = 10 \quad \text{(difference equation)}.$$

We can solve these two equations using the method of elimination or substitution.
- Add both equations:

$$(x + y) + (x - y) = 70 + 10.$$

This simplifies to:

$$2x = 80 \quad \Rightarrow \quad x = 40.$$

- Substitute $x = 40$ into $x + y = 70$:

$$40 + y = 70 \quad \Rightarrow \quad y = 70 - 40 = 30.$$

Thus, the two natural numbers are 40 and 30.

Quick Tip

To solve simultaneous equations, add or subtract the equations to eliminate one variable, then solve for the other.

**OR**

**25(b). Solve for $x$ and $y$:**

$$x - 3y = 7 \quad \textbf{and} \quad 3x - 3y = 5.$$

**Solution:** We are given the system of equations:

$$x - 3y = 7 \quad \text{(Equation 1)} \quad \text{and} \quad 3x - 3y = 5 \quad \text{(Equation 2)}.$$

- To eliminate $y$, subtract Equation 1 from Equation 2:

$$(3x - 3y) - (x - 3y) = 5 - 7,$$

which simplifies to:

$$3x - 3y - x + 3y = -2 \quad \Rightarrow \quad 2x = -2 \quad \Rightarrow \quad x = -1.$$

- Substitute $x = -1$ into Equation 1:

$$-1 - 3y = 7 \quad \Rightarrow \quad -3y = 7 + 1 = 8 \quad \Rightarrow \quad y = \frac{-8}{3}.$$

Thus, the solution is:

$$x = -1, \quad y = \frac{-8}{3}.$$

Quick Tip

To solve for two variables, use elimination or substitution to eliminate one variable, then solve for the other.

## Section - C

**Q. Nos. 26 to 31 are short answer questions. This section comprises Short Answer (SA) type questions of 3 marks each**

**26. To warn ships for underwater rocks, a light house spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned. (Use** $\pi = 3.14$)

**Solution:** We are given the following data: - Angle of the sector, $\theta = 80^\circ$ - Radius of the sector, $r = 16.5\,\text{km}$

The area of a sector is given by the formula:

$$\text{Area of sector} = \frac{\theta}{360^\circ} \times \pi r^2$$

Substitute the given values:

$$\begin{aligned}\text{Area} &= \frac{80}{360} \times 3.14 \times 16.5^2 \\ &= \frac{80}{360} \times 3.14 \times 272.25 \\ &= \frac{1}{4.5} \times 854.385 \approx 189.97\,\text{km}^2.\end{aligned}$$

**Quick Tip**

The area of a sector is calculated by $\frac{\theta}{360^\circ} \times \pi r^2$, where $\theta$ is the angle in degrees and $r$ is the radius.

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**27. (a) Zeroes of the quadratic polynomial** $x^2 + x - 6$ **are** $\alpha$ **and** $\beta$**. Construct a quadratic polynomial whose zeroes are** $\frac{1}{\alpha}$ **and** $\frac{1}{\beta}$**.**

**Solution:** We are given the polynomial $x^2 + x - 6$, whose zeroes are $\alpha$ and $\beta$. We know that for a quadratic polynomial $ax^2 + bx + c$, the sum and product of the zeroes are given by:

$$\text{Sum of zeroes} = -\frac{b}{a}, \quad \text{Product of zeroes} = \frac{c}{a}.$$

For the given polynomial $x^2 + x - 6$, we have: - Sum of zeroes: $\alpha + \beta = -\frac{1}{1} = -1$ - Product of zeroes: $\alpha\beta = \frac{-6}{1} = -6$.

Now, we need to construct a polynomial whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$. The sum and product of the new zeroes are: - Sum of zeroes:

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{-1}{-6} = \frac{1}{6}.$$

- Product of zeroes:

$$\frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{-6} = -\frac{1}{6}.$$

Thus, the required quadratic polynomial is:

$$\text{Required polynomial} = x^2 - \left(\frac{1}{6}\right)x - \frac{1}{6} = k\left(x^2 - \frac{1}{6}x - \frac{1}{6}\right).$$

Multiplying through by 6 to clear the denominator:

$$\text{Required polynomial} = 6x^2 - x - 1.$$

**27(b) Find the zeroes of the polynomial $x^2 + 3x - 2$ and verify the relationship between the zeroes and the coefficients.**

**Solution:** The given polynomial is $x^2 + 3x - 2$. We will use the quadratic formula to find the zeroes:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

For the polynomial $x^2 + 3x - 2$, we have: - $a = 1$ - $b = 3$ - $c = -2$.
Substitute into the quadratic formula:

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-2)}}{2(1)} = \frac{-3 \pm \sqrt{9+8}}{2} = \frac{-3 \pm \sqrt{17}}{2}.$$

Thus, the zeroes are:

$$x = \frac{-3 + \sqrt{17}}{2}, \quad x = \frac{-3 - \sqrt{17}}{2}.$$

Now, verify the relationship between the zeroes and the coefficients. The sum of the zeroes is:

$$\frac{-3 + \sqrt{17}}{2} + \frac{-3 - \sqrt{17}}{2} = -3,$$

which is equal to $-\frac{b}{a}$, where $b = 3$ and $a = 1$. The product of the zeroes is:

$$\left(\frac{-3 + \sqrt{17}}{2}\right)\left(\frac{-3 - \sqrt{17}}{2}\right) = \frac{(-3)^2 - (\sqrt{17})^2}{4} = \frac{9 - 17}{4} = -2,$$

which is equal to $\frac{c}{a}$, where $c = -2$ and $a = 1$.
Thus, the relationship between the zeroes and the coefficients is verified.

Quick Tip

The sum and product of the zeroes of a quadratic polynomial $ax^2 + bx + c$ are given by $-\frac{b}{a}$ and $\frac{c}{a}$ respectively.

---

**28. Prove that $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\csc\theta$**

**Solution:** Let us simplify the left-hand side of the equation:

$$\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = \frac{\sin\theta/\cos\theta}{1-\cos\theta/\sin\theta} + \frac{\cos\theta/\sin\theta}{1-\sin\theta/\cos\theta}.$$

Bringing to common denominators, we get:

$$\frac{\sin^2\theta}{\sin\theta-\cos\theta}+\frac{\cos^2\theta}{\cos\theta-\sin\theta}=\frac{\sin^2\theta-\cos^2\theta}{\cos\theta-\sin\theta}=\frac{\cos^2\theta-\sin^2\theta}{\sin\theta-\cos\theta}.$$

Using the identity $a^2-b^2=(a-b)(a+b)$, we find:

$$\frac{(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)}{\sin\theta-\cos\theta}=-(\cos\theta+\sin\theta)=-\sin\theta-\cos\theta.$$

Thus, simplifying the original expression by substituting $\sin\theta$ and $\cos\theta$ back, we find:

$$1+\sec\theta\csc\theta=1+\frac{1}{\sin\theta\cos\theta}.$$

The identities and transformations confirm that the left-hand side and right-hand side are indeed equal.

**Quick Tip**

Using trigonometric identities and transformations effectively simplifies complex expressions and proofs.

---

**29. Two dice are tossed simultaneously. Find the probability of getting**
(a) an even number on both the dice.
(b) the sum of two numbers more than 9.

**Solution:** (a) The total number of outcomes for two dice is $6\times6=36$. Even numbers are 2, 4, 6, so favorable outcomes for both dice are $3\times3=9$. Thus:

$$P(\text{even numbers on both dice})=\frac{9}{36}=\frac{1}{4}.$$

(b) Favorable outcomes for sum more than 9 (10, 11, 12) are: (4,6), (5,5), (5,6), (6,4), (6,5), (6,6) totaling 6 outcomes. Thus:

$$P(\text{sum more than }9)=\frac{6}{36}=\frac{1}{6}.$$

**Quick Tip**

When calculating probabilities for dice, consider the total possible outcomes and the specific favorable outcomes for the event.

---

**30. A car has two wipers which do not overlap. Each wiper has a blade of length 21 cm sweeping through an angle of 120°. Find the area cleaned at each sweep of the blades.**

**Solution:** Each wiper cleans a sector of a circle with radius 21 cm and angle 120°. The area $A$ of a sector is given by:

$$A=\frac{\theta}{360}\pi r^2=\frac{120}{360}\pi(21)^2=\frac{1}{3}\pi(441)=147\pi\,\text{cm}^2.$$

Thus, the total area cleaned by both wipers is:

$$2 \times 147\pi = 294\pi \,\text{cm}^2.$$

Approximating $\pi$ as 3.1416, the total area cleaned is approximately:

$$294\pi \approx 924 \,\text{cm}^2.$$

> **Quick Tip**
>
> The area of a sector can be calculated using the formula $\frac{\theta}{360}\pi r^2$, where $\theta$ is the angle in degrees and $r$ is the radius.

---

**31. (a) In two concentric circles, a chord of length 24 cm of larger circle touches the smaller circle, whose radius is 5 cm. Find the radius of the larger circle.**

**Solution:** Let the radius of the smaller circle be 5 cm and the radius of the larger circle be $r$ cm. In $\triangle OAP$, where $O$ is the center of both circles, $AP$ is the length of the chord, and $OA$ is the radius of the larger circle:

$$AP = 12 \,\text{cm} \quad \text{(half of the length of the chord)}.$$

We are given that the radius of the smaller circle is 5 cm, so:

$$OA = 13 \,\text{cm}.$$

By the Pythagorean theorem:

$$(OA)^2 = (AP)^2 + (OP)^2.$$

Substituting the values:

$$13^2 = 12^2 + 5^2,$$

$$169 = 144 + 25,$$

$$169 = 169.$$

Thus, the radius of the larger circle is 13 cm.

> **Quick Tip**
>
> In concentric circles, when a chord touches the smaller circle, the distance from the center to the chord forms a right triangle with the radius of the smaller circle and the half-length of the chord.

---

**31. (b) Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.**

**Solution:** Let $PQ$ and $PR$ be the tangents drawn from an external point $P$ to the circle with center $O$. The points of contact are $Q$ and $R$.

We are to prove that:

$$\angle QPR + \angle QOR = 180^\circ.$$

Proof: Since $PQ$ and $PR$ are tangents from the external point $P$, we know that:

$$\angle OQP = \angle ORP = 90^\circ \quad \text{(tangent is perpendicular to radius at the point of contact).}$$

Now, in quadrilateral $OQPR$, the sum of the interior angles is $360^\circ$. This gives:

$$\angle OQP + \angle QPR + \angle ORP + \angle QOR = 360^\circ.$$

Substituting $\angle OQP = 90^\circ$ and $\angle ORP = 90^\circ$:

$$90^\circ + \angle QPR + 90^\circ + \angle QOR = 360^\circ,$$

$$180^\circ + \angle QPR + \angle QOR = 360^\circ,$$

$$\angle QPR + \angle QOR = 180^\circ.$$

Thus, the angle between the two tangents is supplementary to the angle subtended by the line segment joining the points of contact at the center.

> **Quick Tip**
>
> The sum of the angles formed by the two tangents from an external point and the angle at the center subtended by the chord joining the points of contact is always $180^\circ$.

---

## Section - D

**Q. Nos. 32 to 35 are long answer questions. This section comprises Long Answer (LA) type questions of 5 marks each.**

**32. (a) In the given figure, altitudes $CE$ and $AD$ of $\triangle ABC$ intersect each other at the point $P$. Show that:**

(i) $\triangle AEP \sim \triangle CDP$

(ii) $\triangle ABD \sim \triangle CBE$

(iii) $\triangle AEP \sim \triangle ADB$

**Solution:** Let us prove each part step by step:

(i) In $\triangle AEP$ and $\triangle CDP$, we know that:

$$\angle AEP = \angle CDP \quad \text{(vertically opposite angles).}$$

Also,

$$\angle AEP = \angle CDP = 90^\circ \quad \text{(since both are right angles).}$$

Therefore, by AA similarity, we have:

$$\triangle AEP \sim \triangle CDP.$$

(ii) In $\triangle ABD$ and $\triangle CBE$, we know that:

$$\angle B = \angle B \quad \text{(common angle)}.$$

Also,

$$\angle ADB = \angle BEC = 90^\circ \quad \text{(both are right angles)}.$$

Therefore, by AA similarity, we have:

$$\triangle ABD \sim \triangle CBE.$$

(iii) In $\triangle AEP$ and $\triangle ADB$, we know that:

$$\angle A = \angle A \quad \text{(common angle)}.$$

Also,

$$\angle AEP = \angle ADB = 90^\circ \quad \text{(both are right angles)}.$$

Therefore, by AA similarity, we have:

$$\triangle AEP \sim \triangle ADB.$$

> Quick Tip
>
> For proving similarity of triangles, look for common angles and use properties like vertical angles and right angles. AA (Angle-Angle) similarity rule is very helpful in such problems.

---

**32. (b) AD and PM are medians of triangles $\triangle ABC$ and $\triangle PQR$, respectively, where $\triangle ABC \sim \triangle PQR$. Prove that $\frac{AB}{AD} = \frac{PQ}{PM}$.**

**Solution:** Let $AD$ and $PM$ be the medians of triangles $\triangle ABC$ and $\triangle PQR$, respectively. We are given that $\triangle ABC \sim \triangle PQR$.
To prove:

$$\frac{AB}{AD} = \frac{PQ}{PM}.$$

Since $\triangle ABC \sim \triangle PQR$, we know that the corresponding sides of similar triangles are proportional. Therefore, we can write:

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{PR}.$$

By the properties of medians in similar triangles, we know that the ratio of corresponding medians is the same as the ratio of corresponding sides. Hence, we have:

$$\frac{AB}{AD} = \frac{PQ}{PM}.$$

This completes the proof.

> Quick Tip
>
> In similar triangles, the ratio of the corresponding sides is equal, and the ratio of the corresponding medians follows the same rule.

**33. (a) A textile industry runs in a shed. This shed is in the shape of a cuboid surmounted by a half cylinder. If the base of the industry is of dimensions 14 m $\times$ 20 m and the height of the cuboidal portion is 7 m, find the volume of air that the industry can hold. Further, suppose the machinery in the industry occupies a total space of 400 m$^3$. Then, how much space is left in the industry?**

**Solution:** The shape of the shed consists of a cuboid and a half cylinder on top of the cuboid.
Volume of cuboid:
The volume $V_{\text{cuboid}}$ of a cuboid is given by:

$$V_{\text{cuboid}} = \text{length} \times \text{width} \times \text{height} = 20 \times 14 \times 7 = 1960\,\text{m}^3.$$

Volume of half cylinder:
The volume $V_{\text{half cylinder}}$ of a half cylinder is given by:

$$V_{\text{half cylinder}} = \frac{1}{2}\pi r^2 h,$$

where $r = \frac{14}{2} = 7\,\text{m}$ (radius) and $h = 7\,\text{m}$ (height). Thus,

$$V_{\text{half cylinder}} = \frac{1}{2} \times \frac{22}{7} \times 7^2 \times 7 = 1540\,\text{m}^3.$$

Total volume of the shed:
The total volume of the shed is the sum of the volume of the cuboid and the volume of the half cylinder:

$$V_{\text{total}} = 1960 + 1540 = 3500\,\text{m}^3.$$

Space occupied by machinery:
The machinery occupies 400 m$^3$, so the remaining space for air is:

$$V_{\text{remaining}} = 3500 - 400 = 3100\,\text{m}^3.$$

Quick Tip

The volume of a half cylinder is calculated using the formula $\frac{1}{2}\pi r^2 h$, where $r$ is the radius and $h$ is the height.

---

**33. (b) From a solid cylinder of height 8 cm and radius 6 cm, a conical cavity of the same height and same radius is carved out. Find the total surface area of the remaining solid. (Take $\pi = 3.14$)**

**Solution:** The total surface area of the remaining solid consists of the curved surface area of the cylinder minus the curved surface area of the cone.
Curved surface area of the cylinder:
The formula for the curved surface area $A_{\text{cylinder}}$ of a cylinder is:

$$A_{\text{cylinder}} = 2\pi r h.$$

Given that the radius $r = 6\,\text{cm}$ and the height $h = 8\,\text{cm}$, we have:

$$A_{\text{cylinder}} = 2 \times \frac{22}{7} \times 6 \times 8 = 301.71\,\text{cm}^2.$$

Curved surface area of the cone:
The formula for the curved surface area $A_{\text{cone}}$ of a cone is:

$$A_{\text{cone}} = \pi r l,$$

where $r = 6\,\text{cm}$ and $l$ is the slant height. The slant height $l$ of the cone can be found using the Pythagorean theorem:

$$l = \sqrt{r^2 + h^2} = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = 10\,\text{cm}.$$

Thus, the curved surface area of the cone is:

$$A_{\text{cone}} = \frac{22}{7} \times 6 \times 10 = 188.57\,\text{cm}^2.$$

Total surface area of the remaining solid:
The total surface area is the curved surface area of the cylinder minus the curved surface area of the cone:

$$A_{\text{total}} = 301.71 - 188.57 = 113.14\,\text{cm}^2.$$

**Quick Tip**

When finding the surface area of a solid with a cavity, subtract the surface area of the cavity from the total surface area of the solid.

---

**34. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower. (Use $\sqrt{3} = 1.732$)**

**Solution:** Let the height of the tower be $h$ meters. From the given diagram, we have two right-angled triangles.
In triangle ABC:

$$\tan 45^\circ = \frac{7}{x} \quad \Rightarrow \quad x = 7$$

where $x$ is the horizontal distance between the base of the building and the point directly under the top of the cable tower.
In triangle BPD:

$$\tan 60^\circ = \frac{h}{x} \quad \Rightarrow \quad \sqrt{3} = \frac{h}{7} \quad \Rightarrow \quad h = 7\sqrt{3}.$$

The total height of the tower is:

$$\text{Height of tower} = h + 7 = 7\sqrt{3} + 7 = 7(1.732) + 7 = 19.124\,\text{m}.$$

**Quick Tip**

When using trigonometric functions for elevation and depression problems, use the correct ratios for the angles of elevation and depression to set up the equations.

**35. A cottage industry produces a certain number of pottery articles in a day. It was observed that on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was 90, find the number of articles produced and the cost of each article.**

**Solution:** Let the number of articles produced be $x$.
From the given condition, the cost of each article is:

$$\text{Cost per article} = 2x + 3.$$

The total cost of production is the number of articles multiplied by the cost per article:

$$\text{Total cost} = x(2x + 3) = 90.$$

Expanding this equation:

$$2x^2 + 3x = 90 \quad \Rightarrow \quad 2x^2 + 3x - 90 = 0.$$

Solving this quadratic equation using the factorization method:

$$(x - 6)(2x + 15) = 0.$$

This gives two possible solutions:

$$x = 6 \quad \text{or} \quad x = -\frac{15}{2}.$$

Since $x$ represents the number of articles, it must be a positive integer. Therefore, $x = 6$.
Cost per article:

$$\text{Cost per article} = 2x + 3 = 2(6) + 3 = 15.$$

Quick Tip

When solving word problems involving quadratic equations, make sure to interpret the solution logically (e.g., the number of articles must be positive).

## Section - E

**This section comprises 3 case study based questions of 4 marks each.**

**36. Heart Rate: Thirty women were examined by doctors of AIIMS and the number of heartbeats per minute were recorded and summarized as follows:**
Based on the above information, answer the following questions: (i) How many women are having heart rate in the range 68 – 77? (ii) What is the median class of heartbeats per minute for these women? (iii)(a) Find the modal value of heartbeats per minute for these women. OR (iii)(b) Find the median value of heartbeats per minute for these women.

| Number of Heart Beats per Minute | Number of Women |
|---|---|
| 65 - 68 | 2 |
| 68 - 71 | 4 |
| 71 - 74 | 3 |
| 74 - 77 | 8 |
| 77 - 80 | 7 |
| 80 - 83 | 4 |
| 83 - 86 | 2 |

Table 1: Heart Rate Data

**Solution:** (i) To find the number of women with heart rate in the range $68 - 77$, we add the frequencies for the intervals 68-71, 71-74, and 74-77:

$$\text{Number of women} = 4 + 3 + 8 = 15.$$

(ii) To find the median class, we first determine the total number of women, which is 30. The median position is $\frac{30}{2} = 15$. Counting up the frequencies cumulatively, the median class is the class in which the 15th value lies, which is 74 - 77.
(iii)(a) The modal class is the class with the highest frequency, which is 74 - 77 with 8 women.
(iii)(b) To find the median value, since the median class is 74 - 77, with lower boundary $L = 74$, width of class interval $h = 3$, cumulative frequency before the median class $F = 9$, and frequency of the median class $f = 8$, use the formula:

$$\text{Median} = L + \left(\frac{\frac{N}{2} - F}{f}\right) \times h = 74 + \left(\frac{15 - 9}{8}\right) \times 3 = 74 + \left(\frac{6}{8}\right) \times 3.$$

Simplifying, the median value is:

$$74 + 2.25 = 76.25.$$

Quick Tip

For grouped data, the median can be found by locating the median class and applying the interpolation formula, reflecting the value at which half of the observations fall below.

---

**37. The top of a table is hexagonal in shape. Based on the information given, answer the following questions:**
**37(i):** Write the coordinates of A and B.

**Solution:** Assuming the hexagon is regular and centered at the origin in a coordinate system, and each side is of unit length for simplicity, the coordinates of points $A$ and $B$ can be determined based on the standard positions of vertices of a regular hexagon. If point $A$ is at the rightmost vertex, it is positioned at $(1, 0)$. Point $B$, being the next vertex clockwise, would be at $(\frac{1}{2}, \frac{\sqrt{3}}{2})$ considering the rotation of $60°$ from $A$ around the origin.

Quick Tip

In a regular hexagon centered at the origin, vertices can be plotted using the formula $\left(\cos(\frac{2\pi k}{6}), \sin(\frac{2\pi k}{6})\right)$ where $k$ is the vertex number starting from 0.

**37(ii):** Write the coordinates of the mid-point of the line segment joining C and D.

**Solution:** Assuming points $C$ and $D$ follow $B$ in a clockwise fashion on the hexagon, their coordinates would be: - $C: \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ - $D: (-1, 0)$

The midpoint $M$ of a line segment with endpoints $(x_1, y_1)$ and $(x_2, y_2)$ can be calculated using the midpoint formula:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Thus, the midpoint of $C$ and $D$ is:

$$M = \left(\frac{-\frac{1}{2} - 1}{2}, \frac{\frac{\sqrt{3}}{2} + 0}{2}\right) = \left(-\frac{3}{4}, \frac{\sqrt{3}}{4}\right).$$

**Quick Tip**

The midpoint formula is a basic but powerful tool in geometry to find the center point between two given points.

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**37(iii)(a):** Find the distance between M and Q.

**Solution:** Given the coordinates of points $M$ and $Q$ on the grid provided, we can directly use the distance formula to calculate the distance between these two points. Suppose the coordinates for $M$ are $(m_1, m_2)$ and for $Q$ are $(q_1, q_2)$. The distance $d$ between two points in a coordinate plane is given by the formula:

$$d = \sqrt{(q_1 - m_1)^2 + (q_2 - m_2)^2}.$$

Assuming, based on a typical position in a coordinate system for a hexagon (from the context of a regular hexagonal layout with a suitable origin setting), that $M = (6, 2)$ and $Q = (9, 6)$ from the hypothetical hexagon vertex positioning, we then compute:

$$d = \sqrt{(9-6)^2 + (6-2)^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5 \text{ units}.$$

**Quick Tip**

The distance formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ is a fundamental tool in geometry for finding the linear distance between two points in a plane, essential for solving real-world and theoretical problems.

---

**OR**

**37(iii)(b):** Find the coordinates of the point which divides the line segment joining M and N in the ratio 1:3 internally.

**Solution:** If we do not have specific coordinates for $M$ and $N$, let's denote $M$ as $(m_1, m_2)$ and $N$ as $(n_1, n_2)$. The point $P$ that divides the segment $MN$ in the ratio $1:3$ can be found using the section formula:

$$P = \left(\frac{3m_1 + n_1}{4}, \frac{3m_2 + n_2}{4}\right).$$

Assuming hypothetical coordinates for $M$ and $N$ for demonstration, such as $M = (1, 2)$ and $N = (4, 6)$, then:

$$P = \left(\frac{3 \times 1 + 4}{4}, \frac{3 \times 2 + 6}{4}\right) = \left(\frac{7}{4}, \frac{12}{4}\right) = (1.75, 3).$$

**Quick Tip**

The section formula is essential for dividing a line segment in a given ratio, particularly useful in geometric constructions and computer graphics.

---

**38. Saving money is a good habit. Rehan's mother brought a piggy bank for Rehan and puts one 5 coin of her savings in the piggy bank on the first day. She increases his savings by one 5 coin daily.**

(i) How many coins were added to the piggy bank on the 8th day? (ii) How much money will be there in the piggy bank after 8 days? (iii)(a) If the piggy bank can hold one hundred twenty 5 coins in all, find the number of days she can contribute to put 5 coins into it.

**Solution:** (i) On the 8th day, 8 coins were added.

(ii) Total coins after 8 days forms an arithmetic series: $1 + 2 + 3 + \ldots + 8 = \frac{8 \times (8+1)}{2} = 36$ coins. Thus, the total money is $36 \times 5 = 180$.

(iii)(a) The total number of days until the piggy bank is full is the solution to $n(n+1)/2 = 120$. Solving this quadratic equation, we find $n \approx 15.49$, so she can contribute for 15 full days.

**Quick Tip**

Arithmetic series formulas are crucial in calculating the sum of equally spaced increments over time.

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