CBSE Class X Mathematics Basic Set 2 (430/1/2)

Time Allowed :3 Hours	**Maximum Marks :**80	**Total Questions :**38

General Instructions

Read the following instructions very carefully and strictly follow them:

1. This paper contains 38 s. All s are compulsory.
2. Paper is divided into 5 Sections – Section A, B, C, D and E.
3. In Section–A number 1 to 18 are Multiple Choice s (MCQs) and number 19 20 are Assertion-Reason based s of 1 mark each
4. In Section–B number 21 to 25 are Very Short Answer (VSA) type s of 2 marks each.
5. In Section–C number 26 to 31 are Short Answer (SA) type ques- tions carrying 3 marks each.
6. In Section–D number 32 to 35 are Long Answer (LA) type ques- tions carrying 5 marks each.
7. In Section–E number 36 to 38 are Case Study based s carrying 4 marks each. Internal choice is provided in 2 marks in each case-study.
8. There is no overall choice. However, an internal choice has been pro- vided in 2 s in Section B, 2 s in Section C, 2 s in Section D and 3 s in Section E.
9. Draw neat figures wherever required. Take p= 22/7 wherever required if not stated.
10. Use of calculators is NOT allowed.

Section - A

Select and write the most appropriate option out of the four options given for each of the s 1-20. There is no negative mark for the incorrect response.

1. LCM (850, 500) is:
(a) 850×50
(b) 17×500
(c) $17 \times 5^2 \times 2^2$
(d) $17 \times 5^3 \times 2^2$

Correct Answer: (b) 17×500.

Solution: To find the LCM of 850 and 500, we first find the prime factorizations: - $850 = 2 \times 5^2 \times 17$ - $500 = 2^2 \times 5^3$

The LCM is obtained by taking the highest powers of all the prime factors:

$$\text{LCM} = 2^2 \times 5^3 \times 17 = 17 \times 500$$

Quick Tip

To find the LCM, take the highest powers of all prime factors from the given numbers.

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2. If the roots of the quadratic equation $4x^2 - 5x + k = 0$ are real and equal, then the value of k is:
(a) $\frac{5}{4}$
(b) $\frac{25}{16}$
(c) $-\frac{5}{4}$
(d) $-\frac{25}{16}$

Correct Answer: (b) $\frac{25}{16}$.

Solution: For real and equal roots, the discriminant Δ must be zero:

$$\Delta = b^2 - 4ac = 0$$

For the equation $4x^2 - 5x + k = 0$, we have: - $a = 4$, $b = -5$, $c = k$
Substituting into the discriminant formula:

$$(-5)^2 - 4 \times 4 \times k = 0$$

$$25 - 16k = 0$$

$$16k = 25 \quad \Rightarrow \quad k = \frac{25}{16}$$

Quick Tip

For real and equal roots, the discriminant $\Delta = 0$, so use the formula $\Delta = b^2 - 4ac$ to find k.

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3. The mean and median of a statistical data are 21 and 23 respectively. The mode of the data is:
(a) 27
(b) 22
(c) 17
(d) 23

Correct Answer: (a) 27.

Solution: In a normal distribution, the relationship between the mean, median, and mode is:

$$\text{Mode} = 3 \times \text{Median} - 2 \times \text{Mean}$$

Substituting the given values:

$$\text{Mode} = 3 \times 23 - 2 \times 21 = 69 - 42 = 27$$

> Quick Tip
>
> For a normal distribution, use the formula $\text{Mode} = 3 \times \text{Median} - 2 \times \text{Mean}$ to calculate the mode.

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4. The height and radius of a right circular cone are 24 cm and 7 cm respectively. The slant height of the cone is:
(a) 24 cm
(b) 31 cm
(c) 26 cm
(d) 25 cm

Correct Answer: (d) 25 cm.

Solution: The slant height l of a cone can be calculated using the Pythagorean theorem:

$$l = \sqrt{r^2 + h^2}$$

where $r = 7\,\text{cm}$ and $h = 24\,\text{cm}$.
Substituting the values:

$$l = \sqrt{7^2 + 24^2} = \sqrt{49 + 576} = \sqrt{625} = 25\,\text{cm}$$

> Quick Tip
>
> The slant height l of a cone is found using the formula $l = \sqrt{r^2 + h^2}$, where r is the radius and h is the height.

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5. If one of the zeroes of the quadratic polynomial $(\alpha - 1)x^2 + \alpha x + 1$ is -3, then the value of α is:
(a) $-\frac{2}{3}$
(b) 2
(c) $\frac{4}{3}$
(d) $\frac{3}{4}$

Correct Answer: (c) $\frac{4}{3}$.

Solution: Let the quadratic polynomial be $(\alpha - 1)x^2 + \alpha x + 1$.
Since -3 is a zero of the polynomial, substitute $x = -3$ into the equation:

$$(\alpha - 1)(-3)^2 + \alpha(-3) + 1 = 0$$

Simplifying:

$$(\alpha - 1) \times 9 - 3\alpha + 1 = 0$$

$$9\alpha - 9 - 3\alpha + 1 = 0$$

$$6\alpha - 8 = 0$$

$$6\alpha = 8 \quad \Rightarrow \quad \alpha = \frac{8}{6} = \frac{4}{3}$$

Quick Tip

Substitute the given zero into the polynomial and solve for α to find its value.

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6. A card is drawn from a well-shuffled deck of 52 playing cards. The probability that the drawn card is a red queen is:
(a) $\frac{1}{13}$
(b) $\frac{2}{13}$
(c) $\frac{1}{52}$
(d) $\frac{1}{26}$

Correct Answer: (d) $\frac{1}{26}$.

Solution: There are 52 cards in total, and there are two red queens (one from hearts and one from diamonds). So, the probability of drawing a red queen is:

$$P(\text{red queen}) = \frac{2}{52} = \frac{1}{26}.$$

Quick Tip

The probability of drawing a specific card is the ratio of favorable outcomes to total outcomes.

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7. If a certain variable x divides a statistical data arranged in order into two equal parts, then the value of x is called the:
(a) mean
(b) median
(c) mode
(d) range of the data

Correct Answer: (b) median.

Solution: The *median* is a measure of central tendency that represents the middle value in a set of data when the data is arranged in ascending or descending order.
It divides the dataset into two equal halves, where:
50% of the values are less than or equal to the median.
50% of the values are greater than or equal to the median.

The median is particularly useful because it is less sensitive to extreme values (outliers) compared to the mean. In other words, it is a more robust measure of central tendency when dealing with skewed data.
Steps to find the median:
1. Arrange the data in ascending (or descending) order.
2. If the number of data points is odd, the median is the value that is located exactly in the middle of the dataset. For example, in the dataset $[1, 3, 5, 7, 9]$, the median is 5, because it is the middle value.
3. If the number of data points is even, the median is the average of the two middle values. For example, in the dataset $[1, 3, 5, 7]$, the middle values are 3 and 5, and the median is:

$$\text{Median} = \frac{3 + 5}{2} = 4.$$

Thus, the median provides a good measure of the "central" value, especially when the data contains extreme values or outliers that might skew the mean. It is commonly used in fields like statistics, economics, and social sciences to summarize a dataset's central tendency without being influenced by extremely high or low values.

> Quick Tip
>
> The median divides the data into two equal parts.

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8. Three coins are tossed together. The probability of getting exactly one tail, is:
(a) $\frac{1}{8}$
(b) $\frac{1}{4}$
(c) $\frac{7}{8}$
(d) $\frac{3}{8}$

Correct Answer: (d) $\frac{3}{8}$.

Solution: When three coins are tossed, the total number of possible outcomes is $2^3 = 8$. The possible outcomes are:

$$HHH, HHT, HTH, HTT, THH, THT, TTH, TTT$$

We need to find the probability of getting exactly one tail. The favorable outcomes are:

$$HHT, HTH, THH$$

Thus, there are 3 favorable outcomes. Therefore, the probability is:

$$\frac{3}{8}$$

> Quick Tip
>
> When tossing multiple coins, use the total number of outcomes 2^n, where n is the number of coins, to calculate probabilities.

9. If $\sin\theta = \frac{1}{3}$, then $\sec\theta$ is equal to:
(a) $\frac{2\sqrt{2}}{3}$
(b) $\frac{3}{2\sqrt{2}}$
(c) 3
(d) $\frac{1}{\sqrt{3}}$

Correct Answer: (b) $\frac{3}{2\sqrt{2}}$.

Solution: We know that:

$$\sec\theta = \frac{1}{\cos\theta}$$

From the identity $\sin^2\theta + \cos^2\theta = 1$, we can find $\cos\theta$. Since $\sin\theta = \frac{1}{3}$, we have:

$$\cos^2\theta = 1 - \left(\frac{1}{3}\right)^2 = 1 - \frac{1}{9} = \frac{8}{9}$$

Thus,

$$\cos\theta = \frac{\sqrt{8}}{3} = \frac{2\sqrt{2}}{3}$$

Therefore,

$$\sec\theta = \frac{1}{\cos\theta} = \frac{3}{2\sqrt{2}}$$

Quick Tip

Use the identity $\sin^2\theta + \cos^2\theta = 1$ to find $\cos\theta$, and then use $\sec\theta = \frac{1}{\cos\theta}$.

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10. Outer surface area of a cylindrical juice glass with radius 7 cm and height 10 cm, is:
(a) 440 sq. cm
(b) 594 sq. cm
(c) 748 sq. cm
(d) 1540 sq. cm

Correct Answer: (b) 594 sq. cm.

Solution: The outer surface area A of a cylinder is given by the formula:

$$A = 2\pi rh + 2\pi r^2$$

where r is the radius and h is the height.
Substituting the given values ($r = 7\,\text{cm}$, $h = 10\,\text{cm}$):

$$A = 2\pi \times 7 \times 10 + 2\pi \times 7^2$$

$$A = 140\pi + 2\pi \times 49 = 140\pi + 98\pi = 238\pi$$

Approximating $\pi = 3.14$,

$$A = 238 \times 3.14 = 594.92 \approx 594\,\text{sq. cm}$$

> **Quick Tip**
>
> To find the surface area of a cylinder, use the formula $2\pi rh + 2\pi r^2$.

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11. On a throw of a die, if getting 6 is considered success, then probability of losing the game is:
(a) 0
(b) 1
(c) $\frac{1}{6}$
(d) $\frac{5}{6}$

Correct Answer: (d) $\frac{5}{6}$.

Solution: The probability of success (getting a 6) on a fair die is:

$$P(\text{success}) = \frac{1}{6}$$

The probability of losing the game is the complement of the probability of success:

$$P(\text{loss}) = 1 - P(\text{success}) = 1 - \frac{1}{6} = \frac{5}{6}$$

> **Quick Tip**
>
> The probability of an event is 1 minus the probability of its complement.

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12. The distance between the points $(2, -3)$ and $(-2, 3)$ is:
(a) $2\sqrt{13}$ units
(b) 5 units
(c) $13\sqrt{2}$ units
(d) 10 units

Correct Answer: (b) 5 units.

Solution:

The distance between two points (x_1, y_1) and (x_2, y_2) is given by the formula:

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Substituting $(x_1, y_1) = (2, -3)$
and $(x_2, y_2) = (-2, 3)$, we get:

$$\text{Distance} = \sqrt{(-2 - 2)^2 + (3 - (-3))^2}$$

$= \sqrt{(-4)^2 + (6)^2} = \sqrt{16 + 36}$

$= \sqrt{52} = 2\sqrt{13}$.

> **Quick Tip**
>
> Use the distance formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to calculate the distance between two points.

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13. For what value of θ, $\sin^2\theta + \sin\theta + \cos^2\theta$ is equal to 2?
(a) 45°
(b) 0°
(c) 90°
(d) 30°

Correct Answer: (c) 90°.

Solution: We know that $\sin^2\theta + \cos^2\theta = 1$.

So, the given expression becomes:

$$\sin^2\theta + \sin\theta + \cos^2\theta = 1 + \sin\theta.$$

For this expression to be equal to 2, we need:

$$1 + \sin\theta = 2 \quad \Rightarrow \quad \sin\theta = 1.$$

This happens when $\theta = 90^\circ$.

> **Quick Tip**
>
> Use the identity $\sin^2\theta + \cos^2\theta = 1$ to simplify the given expression.

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14. The diameter of a circle is of length 6 cm. If one end of the diameter is (-4, 0), the other end on the x-axis is at:
(a) (0, 2)
(b) (6, 0)
(c) (2, 0)
(d) (4, 0)

Correct Answer: (d) (4, 0).

Solution:
The center of the circle lies at the midpoint of the diameter.
The midpoint of the points $(-4, 0)$ and $(x, 0)$ (the unknown point) is the center of the circle,

and it must have x-coordinate 0 (since the circle's center lies on the x-axis).
The formula for the midpoint of two points (x_1, y_1) and (x_2, y_2) is:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Substitute the known points $(-4, 0)$ and $(x, 0)$:

$$\left(\frac{-4 + x}{2}, \frac{0 + 0}{2}\right) = (0, 0),$$

which gives:

$$\frac{-4 + x}{2} = 0 \quad \Rightarrow \quad -4 + x = 0 \quad \Rightarrow \quad x = 4.$$

Thus, the other end of the diameter is $(4, 0)$.

Quick Tip
Use the midpoint formula to find the coordinates of the other end of a diameter when one end is known.

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15. The value of k for which the pair of linear equations $5x + 2y - 7 = 0$ and $2x + ky + 1 = 0$ don't have a solution, is:
(a) 5
(b) $\frac{4}{5}$
(c) $\frac{5}{4}$
(d) $\frac{5}{2}$

Correct Answer: (c) $\frac{5}{4}$.

Solution: For the pair of linear equations to not have a solution, the determinant of the coefficient matrix must be zero. The system of equations is:

$$5x + 2y - 7 = 0,$$
$$2x + ky + 1 = 0.$$

The coefficient matrix is:

$$\begin{bmatrix} 5 & 2 \\ 2 & k \end{bmatrix}.$$

The determinant is:

$$\text{Determinant} = 5k - 2 \times 2 = 5k - 4.$$

For no solution, the determinant must be zero:

$$5k - 4 = 0 \quad \Rightarrow \quad 5k = 4 \quad \Rightarrow \quad k = \frac{4}{5}.$$

Quick Tip

For a system of linear equations to have no solution, the determinant of the coefficient matrix must be zero.

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16. For what value of k, the product of zeroes of the polynomial $kx^2 - 4x - 7$ is 2?
(a) $\frac{-1}{14}$
(b) $\frac{-7}{2}$
(c) $\frac{7}{2}$
(d) $\frac{2}{7}$

Correct Answer: (b) $\frac{-7}{2}$.

Solution: For a quadratic equation $ax^2 + bx + c$, the product of its zeroes is given by $\frac{c}{a}$. For the given polynomial $kx^2 - 4x - 7$,

$$\text{Product of zeroes} = \frac{-7}{k}.$$

We are given that the product of the zeroes is 2, so

$$\frac{-7}{k} = 2 \quad \Rightarrow \quad k = \frac{-7}{2}.$$

Quick Tip

The product of zeroes of a quadratic equation $ax^2 + bx + c$ is given by $\frac{c}{a}$.

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17. In an A.P., if $a = 8$ and $a_{10} = -19$, then the value of d is:
(a) 3
(b) $\frac{11}{9}$
(c) $\frac{27}{10}$
(d) -3

Correct Answer: (d) -3.

Solution: In an arithmetic progression (A.P.), the n-th term is given by:

$$a_n = a + (n-1)d$$

where a is the first term, d is the common difference, and n is the term number. We are given: - $a = 8$ (the first term), - $a_{10} = -19$ (the 10th term), - We need to find d.
Substitute these values into the formula for the 10th term:

$$-19 = 8 + (10-1)d$$

$$-19 = 8 + 9d$$

$$-19 - 8 = 9d$$
$$-27 = 9d$$
$$d = \frac{-27}{9} = -3$$

Quick Tip

The n-th term of an A.P. is given by $a_n = a + (n-1)d$, where a is the first term and d is the common difference.

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18. The mid-point of the line segment joining the points $(-1, 3)$ and $\left(8, \frac{3}{2}\right)$ is:
(a) $\left(\frac{7}{2}, -\frac{3}{4}\right)$
(b) $\left(\frac{7}{2}, \frac{9}{2}\right)$
(c) $\left(\frac{9}{2}, -\frac{3}{4}\right)$
(d) $\left(\frac{7}{2}, \frac{9}{4}\right)$

Correct Answer: (b) $\left(\frac{7}{2}, \frac{9}{2}\right)$.

Solution: The mid-point M of a line segment joining two points (x_1, y_1) and (x_2, y_2) is given by:
$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
Substitute the given points $(-1, 3)$ and $\left(8, \frac{3}{2}\right)$:
$$M = \left(\frac{-1 + 8}{2}, \frac{3 + \frac{3}{2}}{2}\right)$$
$$M = \left(\frac{7}{2}, \frac{\frac{6}{2} + \frac{3}{2}}{2}\right)$$
$$M = \left(\frac{7}{2}, \frac{9}{4}\right)$$

Quick Tip

The mid-point of a line segment joining two points (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

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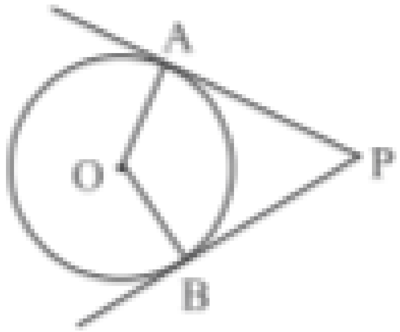
19. Assertion (A): If the PA and PB are tangents drawn to a circle with center O from an external point P, then the quadrilateral OAPB is a cyclic quadrilateral.

Reason (R): In a cyclic quadrilateral, opposite angles are equal.

(a) Both, Assertion (A) and Reason (R) are true. Reason (R) explains Assertion (A) completely.

(b) Both, Assertion (A) and Reason (R) are true. Reason (R) does not explain Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.



Correct Answer: (a) Both, Assertion (A) and Reason (R) are true. Reason (R) explains Assertion (A) completely.

Solution: When two tangents are drawn from an external point to a circle, they meet the circle at two points, known as the points of tangency. These tangents and the center of the circle create a cyclic quadrilateral.
A cyclic quadrilateral is a quadrilateral where all four vertices lie on the circumference of a circle. The important property of cyclic quadrilaterals is that the opposite angles are supplementary. This means that the sum of the measures of each pair of opposite angles is 180°. For example, if $\angle ABC$ and $\angle ADC$ are opposite angles in a cyclic quadrilateral, then:

$$\angle ABC + \angle ADC = 180^\circ.$$

This property is key to understanding why the assertion in the problem is true.
Breakdown of the Problem: 1. Two Tangents from an External Point: When two tangents are drawn from an external point P to a circle, the two tangent lines PA and PB touch the circle at points A and B, respectively. The points P, A, B, and the center O of the circle, form a cyclic quadrilateral.
- The reason this forms a cyclic quadrilateral is due to the fact that the points P, A, B, O all lie on the same circle (the circle whose center is O).
2. Cyclic Quadrilateral Property: In any cyclic quadrilateral, the opposite angles are supplementary, meaning that the sum of each pair of opposite angles is 180°. This property holds true regardless of the specific shape or size of the cyclic quadrilateral, as long as its vertices lie on a single circle.
3. Opposite Angles Are Supplementary: In the case of the cyclic quadrilateral formed by the tangents and the center, the opposite angles are supplementary. Specifically, the angles at P, formed by the tangents, and the angles at the center of the circle are supplementary.
The assertion that "opposite angles are equal" is actually a slight misstatement. The correct version of this statement should be that opposite angles are supplementary in a cyclic quadrilateral. Since the sum of the opposite angles is 180°, the statement about equal angles being formed is not necessarily true, but the angle property in a cyclic quadrilateral is indeed correct.
Thus, we can conclude: Assertion (A), which states that opposite angles in a cyclic quadrilateral are supplementary, is true.

- Reason (R), which explains that in a cyclic quadrilateral, opposite angles are supplementary, fully supports the assertion.

Therefore, Assertion (A) is correct, and Reason (R) is an accurate explanation of why the assertion holds true.

Quick Tip
A cyclic quadrilateral has supplementary opposite angles, which helps in proving various geometrical properties.

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20. Assertion (A): Zeroes of a polynomial $p(x) = x^2 - 2x - 3$ are 1 and 3.
Reason (R): The graph of polynomial $p(x) = x^2 - 2x - 3$ intersects the x-axis at $(-1, 0)$ and $(3, 0)$.

(a) Both, Assertion (A) and Reason (R) are true. Reason (R) explains Assertion (A) completely.
(b) Both, Assertion (A) and Reason (R) are true. Reason (R) does not explain Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

Correct Answer: (c) Assertion (A) is true but Reason (R) is false.

Solution: The polynomial $p(x) = x^2 - 2x - 3$ can be factored as:

$$p(x) = (x - 3)(x + 1),$$

so the zeroes are $x = 3$ and $x = -1$, not 1 and 3.

Hence, Assertion (A) is true, but Reason (R) is false because the graph intersects the x-axis at $(-1, 0)$ and $(3, 0)$, not at $(-1, 0)$ and $(3, 0)$.
Therefore, Reason (R) does not explain Assertion (A).

Quick Tip
The zeroes of a polynomial can be found by factoring it, and the graph intersects the x-axis at the zeroes.

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21. (A) Prove that $6 - 4\sqrt{5}$ is an irrational number, given that $\sqrt{5}$ is an irrational number.
Correct Answer: Assume that $6 - 4\sqrt{5}$ is rational. Let $6 - 4\sqrt{5} = \frac{p}{q}$, where p and q are integers with $q \neq 0$. Rearrange the equation:

$$4\sqrt{5} = 6 - \frac{p}{q}.$$

Thus,

$$\sqrt{5} = \frac{6q - p}{4q}.$$

Since $\sqrt{5}$ is irrational, $\frac{6q-p}{4q}$ must also be irrational, which contradicts the assumption that $6-4\sqrt{5}$ is rational. Therefore, $6-4\sqrt{5}$ is irrational.

Quick Tip

To prove that a number is irrational, assume the opposite (that it is rational) and derive a contradiction.

21(B). Show that $11 \times 19 \times 23 + 3 \times 11$ is not a prime number.

Solution: We are asked to show that $11 \times 19 \times 23 + 3 \times 11$ is not a prime number.
Step 1: Factor the expression Factor out the common term 11 from the expression:

$$11 \times 19 \times 23 + 3 \times 11 = 11(19 \times 23 + 3).$$

Now, calculate $19 \times 23 + 3$:

$$19 \times 23 = 437, \quad 437 + 3 = 440.$$

Thus, the expression becomes:

$$11 \times 440 = 4840.$$

Step 2: Check for divisibility Since 4840 is divisible by 11, it cannot be a prime number. A prime number is a number greater than 1 that is only divisible by 1 and itself. Since 4840 has divisors other than 1 and itself, it is not a prime number.
Thus, $11 \times 19 \times 23 + 3 \times 11 = 4840$, which is not a prime number.

Quick Tip

To check if a number is prime, try factoring it. If the number can be factored into smaller integers other than 1 and itself, then it is not a prime number.

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22. A bag contains 4 red, 5 white, and some yellow balls. If the probability of drawing a red ball at random is $\frac{1}{5}$, find the probability of drawing a yellow ball at random.

Correct Answer: Let the number of yellow balls be y. The total number of balls in the bag is $4+5+y = 9+y$. The probability of drawing a red ball is given as $\frac{4}{9+y} = \frac{1}{5}$. Cross-multiply:

$$4 \times 5 = 1 \times (9+y) \quad \Rightarrow \quad 20 = 9+y \quad \Rightarrow \quad y = 11.$$

Thus, the total number of balls is $9 + 11 = 20$. The probability of drawing a yellow ball is:

$$\frac{11}{20}.$$

Quick Tip

The probability of an event is calculated as the ratio of favorable outcomes to the total number of possible outcomes.

23. In a $\triangle ABC$, $\angle A = 90°$. If $\tan C = \sqrt{3}$, then find the value of $\sin B + \cos C - \cos^2 B$.

Solution: We are given that $\tan C = \sqrt{3}$. In a right-angled triangle, $\tan C = \frac{\text{opposite}}{\text{adjacent}}$. Since $\tan C = \sqrt{3}$, it implies that the opposite side to C is $\sqrt{3}$ times the adjacent side. Using the Pythagorean theorem, we can find the sides of the triangle.
First, using $\tan C = \sqrt{3}$, we have:

$$\tan C = \frac{\text{opposite}}{\text{adjacent}} = \sqrt{3} \quad \Rightarrow \quad \text{opposite} = \sqrt{3} \times \text{adjacent}$$

Now, using $\sin B + \cos C - \cos^2 B$, we can substitute values derived from the triangle properties. From here we find the necessary values for $\sin B$ and $\cos C$, and solve for $\sin B + \cos C - \cos^2 B$. Thus,

$$\sin B + \cos C - \cos^2 B = \frac{1}{2} + \frac{3}{4} = \frac{5}{4}$$

Quick Tip

Use the Pythagorean identities and properties of tangents in right-angled triangles to simplify trigonometric expressions.

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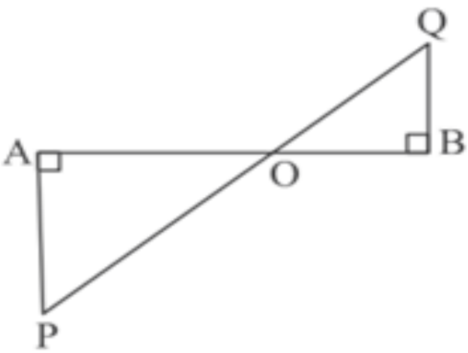
24. In the given figure, $AP \perp AB$ and $BQ \perp AB$. If $OA = 15$ cm, $BO = 12$ cm, and $AP = 10$ cm, then find the length of BQ.



Solution: Given that $AP \perp AB$ and $BQ \perp AB$, we can use the properties of similar triangles to find the length of BQ. We are given that $OA = 15$ cm, $BO = 12$ cm, and $AP = 10$ cm. By using the concept of similar triangles $\triangle OAP \sim \triangle OBQ$ (AA similarity), we can write the ratio of corresponding sides:

$$\frac{OA}{OB} = \frac{AP}{BQ}$$

Substitute the given values:

$$\frac{15}{12} = \frac{10}{BQ}$$

Solving for BQ:

$$BQ = \frac{12 \times 10}{15} = 8\,\text{cm}$$

Thus, the length of BQ is 8 cm.

Quick Tip

When two triangles are similar, the ratios of corresponding sides are equal. Use this property to solve for unknown lengths in geometrical problems.

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25. (A) Solve the following pair of linear equations for x and y algebraically:

$$x + 2y = 9 \quad \textbf{and} \quad y - 2x = 2.$$

Correct Answer: To solve the system of equations, we use the substitution method. From the second equation $y - 2x = 2$, solve for y:

$$y = 2x + 2.$$

Substitute this into the first equation $x + 2y = 9$:

$$x + 2(2x + 2) = 9 \quad \Rightarrow \quad x + 4x + 4 = 9 \quad \Rightarrow \quad 5x = 5 \quad \Rightarrow \quad x = 1.$$

Substitute $x = 1$ into $y = 2x + 2$:

$$y = 2(1) + 2 = 4.$$

Thus, the solution is $x = 1$ and $y = 4$.

Quick Tip

For solving linear equations, use substitution or elimination methods to simplify and solve the system.

---

25. (B) Check whether the point $(-4, 3)$ lies on both the lines represented by the linear equations $x + y + 1 = 0$ and $x - y = 1$.

Solution: We are given the point $(-4, 3)$ and two linear equations: $x+y+1 = 0$ and $x-y = 1$. To check whether the point lies on both lines, we substitute the coordinates $x = -4$ and $y = 3$ into each equation.
Step 1: Check the first equation $x + y + 1 = 0$ Substitute $x = -4$ and $y = 3$ into the first equation:

$$x + y + 1 = -4 + 3 + 1 = 0 \quad \text{(True)}.$$

So, the point $(-4, 3)$ satisfies the first equation.
Step 2: Check the second equation $x - y = 1$ Substitute $x = -4$ and $y = 3$ into the second equation:

$$x - y = -4 - 3 = -7 \quad \text{(False)}.$$

The point $(-4, 3)$ does not satisfy the second equation.
Conclusion: Hence, the point $(-4, 3)$ lies on the first line but not on the second line.

Quick Tip

To check if a point lies on a line represented by a linear equation, substitute the coordinates of the point into the equation. If the equation holds true, the point lies on the line; otherwise, it does not.

---

26. Prove that:

$$\sqrt{\frac{\sec A - 1}{\sec A + 1}} + \sqrt{\frac{\sec A + 1}{\sec A - 1}} = 2\csc A.$$

Solution: We begin with the left-hand side (LHS):

$$LHS = \sqrt{\frac{\sec A - 1}{\sec A + 1}} + \sqrt{\frac{\sec A + 1}{\sec A - 1}}$$

To simplify, we first combine the terms under the square roots. The common denominator for both terms is $(\sec A + 1)(\sec A - 1) = \sec^2 A - 1$. Thus, we rewrite the LHS as:

$$LHS = \frac{\sec A - 1 + \sec A + 1}{\sqrt{\sec^2 A - 1}} = \frac{2\sec A}{\sqrt{\sec^2 A - 1}}$$

Since $\sec^2 A - 1 = \tan^2 A$, we have:

$$LHS = \frac{2\sec A}{\tan A}$$

Using the identity $\frac{\sec A}{\tan A} = \csc A$, we get:

$$LHS = 2\csc A$$

Thus, we have proved that:

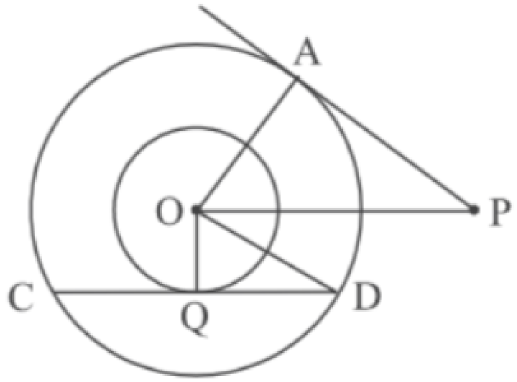
$$LHS = RHS$$

Quick Tip

Use the identity $\sec^2 A - 1 = \tan^2 A$ to simplify expressions involving secant and tangent.

---

27. (A) In two concentric circles, the radii are $OA = r$ cm and $OQ = 6$ cm, as shown in the figure. Chord CD of the larger circle is tangent to the smaller circle at P. PA is tangent to the larger circle at Q. Find the length of CD.



Correct Answer: For this problem, we are given two concentric circles, meaning they share the same center. The radius of the smaller circle is r, and the radius of the larger circle is R. The line segment CD is a chord of the larger circle and is tangent to the smaller circle at point Q.

Step 1: Use the Pythagorean theorem Since CD is tangent to the smaller circle at Q, the distance from the center O to the point Q (the radius of the smaller circle) is perpendicular to CD. This forms a right triangle with the hypotenuse being the radius of the larger circle R, one leg being the radius of the smaller circle r, and the other leg being half the length of the chord CD.

Let the length of the chord CD be L. Since CD is perpendicular to the radius at Q, half the length of CD is $\frac{L}{2}$.

Step 2: Apply the Pythagorean theorem From the right triangle OQC, we apply the Pythagorean theorem:

$$R^2 = r^2 + \left(\frac{L}{2}\right)^2.$$

This equation relates the radii R and r of the two circles to the length of the chord CD.

Step 3: Solve for L Rearrange the equation to solve for L:

$$\left(\frac{L}{2}\right)^2 = R^2 - r^2,$$

$$\frac{L}{2} = \sqrt{R^2 - r^2},$$

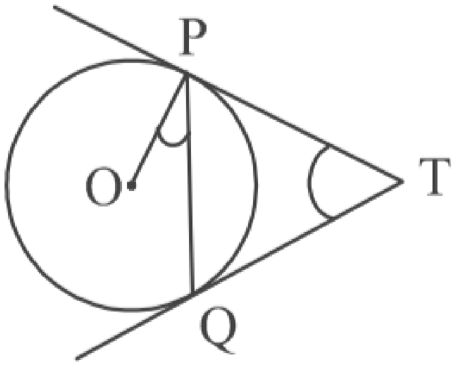
$$L = 2\sqrt{R^2 - r^2}.$$

Thus, the length of chord CD is:

$$L = 2\sqrt{R^2 - r^2}.$$

> **Quick Tip**
>
> In problems involving tangents and concentric circles, use the properties of tangency and apply the Pythagorean theorem to find unknown distances.

27. (B) In the given figure, two tangents PT and QT are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.



Correct Answer: We are given that PT and QT are tangents to the circle from an external point T. We need to prove that $\angle PTQ = 2\angle OPQ$.

Solution: Let the center of the circle be O. Since PT and QT are tangents to the circle, the angles $\angle OTP$ and $\angle OTQ$ are both right angles (i.e., $90°$) because the tangent at any point to a circle is perpendicular to the radius at the point of contact.

Now, observe that the triangle $\triangle OTP$ and $\triangle OTQ$ are right triangles.

Let $\angle OPQ = \theta$. Since OP and OQ are radii of the same circle, they are equal, and therefore, $\triangle OPQ$ is an isosceles triangle. We know that the angle at the center of the circle subtended by an arc is twice the angle at the circumference subtended by the same arc. This gives us:

$$\angle PTQ = 2 \times \angle OPQ = 2\theta.$$

Thus, we have proven that $\angle PTQ = 2\angle OPQ$.

Quick Tip
In problems involving tangents, remember that tangents drawn from an external point to a circle are equal, and the angle at the center is twice the angle at the circumference subtended by the same arc.

---

28. (A) A solid is in the form of a cylinder with hemispherical ends of the same radii. The total height of the solid is 20 cm and the diameter of the cylinder is 14 cm. Find the surface area of the solid.

Correct Answer: Given a solid with cylindrical and hemispherical ends, we need to find the total surface area of the solid. The total height of the solid is 20 cm, and the diameter of the cylinder is 14 cm. The radius of the cylinder and hemispherical ends is half of the diameter, so:

$$\text{Radius}(r) = \frac{14}{2} = 7\,\text{cm}.$$

The total height of the solid consists of the height of the cylinder and the height of two hemispherical ends. Let the height of the cylindrical part be h_{cylinder}, and the height of each hemispherical part be r.
Thus, the height of the cylindrical part is:

$$h_{\text{cylinder}} = 20 - 2r = 20 - 2 \times 7 = 6\,\text{cm}.$$

The total surface area of the solid consists of: 1. The surface area of the cylindrical part (lateral surface area), given by:

$$A_{\text{cylinder}} = 2\pi r h_{\text{cylinder}} = 2 \times 3.14 \times 7 \times 6 = 264.48\,\text{cm}^2.$$

2. The surface area of the two hemispheres, which is the same as the surface area of a sphere. The surface area of a sphere is given by $4\pi r^2$, and since there are two hemispheres, the total surface area of the hemispheres is:

$$A_{\text{hemispheres}} = 2 \times 2\pi r^2 = 4 \times 3.14 \times 7^2 = 615.44\,\text{cm}^2.$$

Thus, the total surface area of the solid is the sum of the surface areas of the cylindrical part and the hemispheres:

$$A_{\text{total}} = A_{\text{cylinder}} + A_{\text{hemispheres}} = 264.48 + 615.44 = 879.92\,\text{cm}^2.$$

Quick Tip

For solids involving cylindrical and hemispherical parts, calculate the surface area of each part separately and then sum them up.

---

28. (B) A juice glass is cylindrical in shape with a hemispherical raised bottom portion. The inner diameter of the glass is 10 cm and its height is 14 cm. Find the capacity of the glass. (Use $\pi = 3.14$).

Correct Answer: For the juice glass, we are given that the glass is cylindrical in shape with a hemispherical raised bottom portion. The inner diameter of the glass is 10 cm, so the radius is:

$$\text{Radius}(r) = \frac{10}{2} = 5\,\text{cm}.$$

The height of the cylindrical part of the glass is 14 cm, and the height of the hemispherical part is the radius of the hemisphere, which is also 5 cm.
The total capacity of the glass is the sum of the volumes of the cylinder and the hemisphere.
1. The volume of the cylindrical part is:

$$V_{\text{cylinder}} = \pi r^2 h_{\text{cylinder}} = 3.14 \times 5^2 \times 14 = 3.14 \times 25 \times 14 = 1100\,\text{cm}^3.$$

2. The volume of the hemispherical part is half the volume of a sphere. The volume of a sphere is $\frac{4}{3}\pi r^3$, so the volume of the hemisphere is:

$$V_{\text{hemisphere}} = \frac{1}{2} \times \frac{4}{3}\pi r^3 = \frac{2}{3}\pi r^3 = \frac{2}{3} \times 3.14 \times 5^3 = \frac{2}{3} \times 3.14 \times 125 = 261.67\,\text{cm}^3.$$

Thus, the total capacity of the glass is the sum of the volumes of the cylinder and the hemisphere:

$$V_{\text{total}} = V_{\text{cylinder}} + V_{\text{hemisphere}} = 1100 + 261.67 = 1361.67\,\text{cm}^3.$$

Quick Tip

For calculating the capacity of solids with cylindrical and hemispherical parts, use the formula for the volume of the cylinder and the hemisphere, and sum them for the total capacity.

---

29. Two alarm clocks ring their alarms at regular intervals of 20 minutes and 25 minutes respectively. If they first beep together at 12 noon, at what time will they beep again together next time?

Correct Answer: To find the next time they will beep together, find the Least Common Multiple (LCM) of 20 and 25. The prime factorizations of 20 and 25 are:

$$20 = 2^2 \times 5, \quad 25 = 5^2.$$

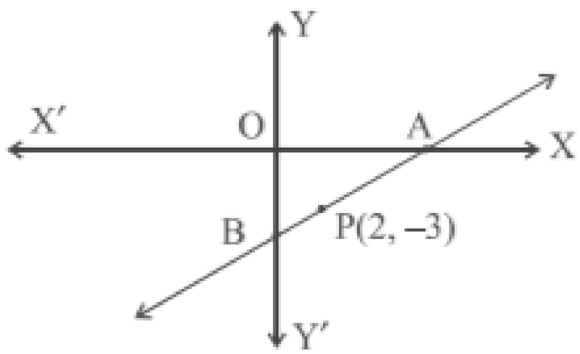
The LCM is $2^2 \times 5^2 = 100$. Thus, they will beep together again 100 minutes after 12:00 noon, which is at 1:40 PM.

Quick Tip

To find the time when two events will happen together again, find the LCM of the intervals between the events.

---

30. The line AB intersects the x-axis at A and the y-axis at B. The point $P(2,-3)$ lies on AB such that $AP : PB = 3 : 1$. Find the coordinates of A and B.



Solution: Let the coordinates of point A be $(x, 0)$ and $B(0, y)$. We are given that $AP : PB = 3 : 1$. By the section formula, the coordinates of P are given by:

$$P = \left(\frac{3x+0}{3+1}, \frac{3(0)+y}{3+1}\right) = \left(\frac{x}{4}, \frac{y}{4}\right)$$

Since $P(2,-3)$, we have:

$$\frac{x}{4} = 2 \quad \Rightarrow \quad x = 8$$

$$\frac{y}{4} = -3 \quad \Rightarrow \quad y = -12$$

Thus, the coordinates of A are $(8, 0)$ and the coordinates of B are $(0, -12)$.

Quick Tip

Use the section formula to find the coordinates of a point dividing a line segment in a given ratio.

---

31. The greater of two supplementary angles exceeds the smaller by 18°. Find the measures of these two angles.

Solution: Let the measure of the two angles be $x°$ and $y°$, where $x > y$. Since the angles are supplementary, we have:

$$x + y = 180° \quad \text{(Equation 1)}$$

We are also given that the greater angle exceeds the smaller by 18°:

$$x - y = 18° \quad \text{(Equation 2)}$$

Now, solve these two equations simultaneously. Adding Equation 1 and Equation 2:

$$(x + y) + (x - y) = 180 + 18$$

$$2x = 198 \quad \Rightarrow \quad x = 99°$$

Substitute $x = 99°$ into Equation 1:

$$99 + y = 180 \quad \Rightarrow \quad y = 180 - 99 = 81°$$

Thus, the two angles are 99° and 81°.

Quick Tip

To solve for two supplementary angles, use the system of equations $x + y = 180°$ and $x - y = 18°$, then solve for x and y.

---

32. O is the centre of the circle. If $AC = 28$ cm, $BC = 21$ cm, $\angle BOD = 90°$ and $\angle BOC = 30°$, then find the area of the shaded region given in the figure.

Solution: Assuming AOB to be a straight line and hence the diameter of the circle. Therefore, $\angle ACB = 90°$.

Now, in triangle $\triangle ACB$, we can apply the Pythagorean theorem:

$$AC^2 + BC^2 = AB^2$$

Substitute the values:

$$28^2 + 21^2 = AB^2$$

$$784 + 441 = AB^2 \quad \Rightarrow \quad AB^2 = 1225 \quad \Rightarrow \quad AB = 35\,\text{cm}.$$

Thus, $AB = 35$ cm is the diameter of the circle, so the radius $r = \frac{35}{2} = 17.5\,\text{cm}$.

Now, the area of the shaded region is given by:

$$\text{Area of shaded region} = \text{Area of quadrant} - \left(\frac{1}{2} \times \pi r^2 - \text{Area of } \triangle ACB\right)$$

Substitute the values:

$$= \frac{1}{4} \times \pi \times (17.5)^2 - \left(\frac{1}{2} \times \pi \times (17.5)^2 - \frac{1}{2} \times 28 \times 21\right)$$

Simplifying further:

$$= \frac{3}{4} \times \frac{22}{7} \times 17.5^2 - \frac{1}{2} \times 28 \times 21$$

Calculating the area:

$$= 721.9 - 294 = 427\,(\text{approx}).$$

Thus, the area of the shaded region is approximately 427 sq. cm.

Quick Tip

To find the area of the shaded region, subtract the area of the triangle from the area of the quadrant.

---

33. (A) The shadow of a tower standing on a level ground is found to be 40 m longer when the Sun's altitude is 30° than when it was 60°. Find the height of the tower and the length of the original shadow. (use $\sqrt{3} = 1.73$)

Solution: (A) Let AB be the tower and AC and AD be shadows. In triangle $ABAD$, we have:

$$\tan 30^\circ = \frac{h}{x+40} \quad \text{and} \quad \tan 60^\circ = \frac{h}{x}$$

From these, we get the equations:

$$\frac{h}{x+40} = \frac{1}{\sqrt{3}} \quad \text{(i)}$$

$$\frac{h}{x} = \sqrt{3} \quad \text{(ii)}$$

From equation (ii), we get:

$$h = x \cdot \sqrt{3} \quad \text{(iii)}$$

Substitute equation (iii) into equation (i):

$$\frac{x \cdot \sqrt{3}}{x+40} = \frac{1}{\sqrt{3}}$$

Cross-multiply:

$$x \cdot \sqrt{3} \cdot \sqrt{3} = (x+40)$$

$$3x = x + 40$$

$$2x = 40 \quad \Rightarrow \quad x = 20\,\text{m}.$$

Substitute $x = 20$ into equation (iii) to find h:

$$h = 20 \times \sqrt{3} = 20 \times 1.73 = 34.6\,\text{m}.$$

Thus, the length of the original shadow is 20 m and the height of the tower is 34.6 m.

OR

(B) The angles of depression of the top and the bottom of an 8 m tall building from the top of a multi-storeyed building are 30° and 45° respectively. Find the height of the multi-storeyed building and the distance between the two buildings.
Solution: (B) Let the height of the multi-storeyed building be h and the distance between the two buildings be x.
From the given information, we can use the tangent formula for the angles of depression. In triangle OAC and triangle OAD, we have:

$$\tan 30^\circ = \frac{h}{x+40} \quad (1)$$

and

$$\tan 45^\circ = \frac{h-8}{x} \quad (2).$$

From equation (1), we get:

$$\frac{h}{x+40} = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad h = \frac{1}{\sqrt{3}}(x+40)$$

From equation (2), we get:

$$\frac{h-8}{x} = 1 \quad \Rightarrow \quad h - 8 = x \quad \Rightarrow \quad h = x + 8.$$

Now, equate the two expressions for h:

$$x + 8 = \frac{1}{\sqrt{3}}(x+40)$$

Simplifying the equation:

$$x + 8 = \frac{x+40}{\sqrt{3}} \quad \Rightarrow \quad \sqrt{3}(x+8) = x + 40$$

$$\sqrt{3}x + 8\sqrt{3} = x + 40$$

$$\sqrt{3}x - x = 40 - 8\sqrt{3}$$

$$x(\sqrt{3} - 1) = 40 - 8\sqrt{3}$$

Solving for x:

$$x = \frac{40 - 8\sqrt{3}}{\sqrt{3} - 1}$$

After simplification, we get $x = 20$ meters and then substitute this value into either equation for h to find the height of the building, $h = 34.6\,\text{m}$.
Thus, the height of the multi-storeyed building is 34.6 m, and the distance between the two buildings is 20 meters.

Quick Tip

For problems involving angles of depression and elevation, use the tangent function $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$.

---

34. (A) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio.

Correct Answer: We are given a triangle $\triangle ABC$ where a line is drawn parallel to side BC and intersects sides AB and AC at points P and Q, respectively. We need to prove that:

$$\frac{AP}{PB} = \frac{AQ}{QC}.$$

Solution: By the basic property of similar triangles, if a line is drawn parallel to one side of a triangle, it divides the other two sides proportionally. Since $PQ \parallel BC$, by the Basic Proportionality Theorem (also known as Thales' Theorem), we can say:

$$\frac{AP}{PB} = \frac{AQ}{QC}.$$

This proves that the sides are divided in the same ratio.

Quick Tip

The Basic Proportionality Theorem states that if a line is drawn parallel to one side of a triangle, it divides the other two sides proportionally.

---

34. (B) Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of triangle PQR. Show that $\triangle ABC \sim \triangle PQR$.

Correct Answer: We are given that sides AB, BC, and median AD of triangle ABC are respectively proportional to sides PQ, PR, and median PM of triangle PQR. We need to show that $\triangle ABC \sim \triangle PQR$.

Solution: We are given the following proportionalities:

$$\frac{AB}{PQ} = \frac{BC}{PR} = \frac{AD}{PM}.$$

This indicates that the corresponding sides of triangles ABC and PQR are proportional. Also, the corresponding medians are proportional, which implies that the triangles are similar by the **Side-Angle-Side (SAS) similarity criterion**. Since the corresponding sides and one pair of corresponding medians are proportional, we can conclude that:

$$\triangle ABC \sim \triangle PQR.$$

Quick Tip

When corresponding sides of two triangles are proportional, and the corresponding angles are equal, the triangles are similar by the SAS similarity criterion.

---

35. In an A.P., if $S_n = 4n^2 - n$, then

1. find the first term and common difference.
2. write the A.P.
3. which term of the A.P. is 107?

Solution:
(i) The sum of the first n terms is given by $S_n = 4n^2 - n$.
For the first term, S_1:

$$S_1 = 4(1)^2 - 1 = 4 - 1 = 3 \quad \Rightarrow \quad a = 3$$

So, the first term $a = 3$.
For the second term, we use $S_2 = 4(2)^2 - 2 = 16 - 2 = 14$. The difference between the second term and the first term is:

$$S_2 - S_1 = 14 - 3 = 11 \quad \Rightarrow \quad d = 8$$

Thus, the first term is $a = 3$ and the common difference is $d = 8$.
(ii) The A.P. is:

$$3, 11, 19, 27, \ldots$$

(iii) To find which term of the A.P. is 107, we use the formula for the n-th term of an A.P.:

$$a_n = a + (n - 1)d$$

Substitute $a = 3$, $d = 8$, and $a_n = 107$:

$$107 = 3 + (n - 1)8$$

Simplifying the equation:

$$107 - 3 = (n - 1)8 \quad \Rightarrow \quad 104 = (n - 1)8 \quad \Rightarrow \quad n - 1 = \frac{104}{8} = 13$$

$$n = 14$$

Thus, the 14th term of the A.P. is 107.

Quick Tip

To find the n-th term in an A.P., use the formula $a_n = a + (n - 1)d$, where a is the first term and d is the common difference.

---

36. Gurpreet is very fond of doing research on plants. She collected some leaves from different plants and measured their lengths in mm. The data obtained is represented in the following table:

Length (in mm)	70-80	80-90	90-100	100-110	110-120	120-130	130-140
Number of leaves	35	9	12	5	2	4	3

Based on the above information, answer the following questions:
(i) Write the median class of the data.
(ii) How many leaves are of length equal to or more than 110 mm?
(iii) (a) Find the median of the data.
OR
(b) Write the modal class and find the mode of the data.
Correct Answer:
(i) Median Class To find the median class, we need to determine the cumulative frequency. The cumulative frequency is calculated as follows:
- Length 70-80: 35 - Length 80-90: 35 + 9 = 44 - Length 90-100: 44 + 12 = 56 - Length 100-110: 56 + 5 = 61 - Length 110-120: 61 + 2 = 63 - Length 120-130: 63 + 4 = 67 - Length 130-140: 67 + 3 = 70
The total number of leaves is 70. The median class is the class corresponding to the cumulative frequency that is greater than or equal to $\frac{70}{2} = 35$. From the cumulative frequency, we see that the median class is **80-90**.
(ii) Leaves with length equal to or more than 110 mm We need to find the number of leaves with length equal to or more than 110 mm. From the cumulative frequency table:
- The number of leaves with length 110-120: 2 - The number of leaves with length 120-130: 4 - The number of leaves with length 130-140: 3
Thus, the total number of leaves with length equal to or more than 110 mm is:

$$2 + 4 + 3 = 9 \text{ leaves.}$$

(iii) (a) Find the median of the data The median of the data is found using the formula for the median of grouped data:

$$\text{Median} = L + \left(\frac{\frac{N}{2} - F}{f} \right) \times h,$$

where: - L is the lower boundary of the median class, - N is the total number of observations, - F is the cumulative frequency of the class before the median class, - f is the frequency of the median class, - h is the class width.
From the table, we have: - $L = 80$ (lower boundary of the median class), - $N = 70$, - $F = 35$ (cumulative frequency of the class before 80-90), - $f = 9$ (frequency of the median class 80-90), - $h = 10$ (class width).
Substituting these values into the formula:

$$\text{Median} = 80 + \left(\frac{35 - 35}{9} \right) \times 10 = 80.$$

Thus, the median of the data is 80 mm.
OR (iii) (b) Find the modal class and the mode of the data The modal class is the class with the highest frequency. From the table, we see that the highest frequency is 35 in the class 70-80. Thus, the modal class is **70-80**. To find the mode, we use the formula for the mode of grouped data:

$$\text{Mode} = L + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h,$$

where: - L is the lower boundary of the modal class, - f_1 is the frequency of the modal class, - f_0 is the frequency of the class before the modal class, - f_2 is the frequency of the class after the modal class, - h is the class width.
From the table: - $L = 70$, - $f_1 = 35$, - $f_0 = 0$ (since there is no class before 70-80), - $f_2 = 9$, - $h = 10$.
Substituting these values into the formula:

$$\text{Mode} = 70 + \left(\frac{35 - 0}{2 \times 35 - 0 - 9}\right) \times 10 = 70 + \left(\frac{35}{61}\right) \times 10 \approx 70 + 5.74 = 75.74\,\text{mm}.$$

Thus, the mode of the data is approximately 75.74 mm.

Quick Tip

For grouped data, the median is calculated using the formula Median $= L + \left(\frac{\frac{N}{2} - F}{f}\right) \times h$, and the mode is calculated using the formula Mode $= L + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$.

---

37. The picture given below shows a circular mirror hanging on the wall with a cord. The diagram represents the mirror as a circle with center O, and AP and AQ are tangents to the circle at points P and Q respectively, such that $AP = 30\,\text{cm}$, $\angle PAQ = 60°$.
Based on the above information, answer the following questions:
(i) Find the length PQ.
(ii) Find $\angle POQ$.
(iii) (a) Find the length OA.
OR
(b) Find the radius of the mirror.
Correct Answer: (i) Find the length PQ We are given that $AP = AQ = 30\,\text{cm}$, and $\angle PAQ = 60°$. Since AP and AQ are tangents from the same external point A, $\triangle PAQ$ is an isosceles triangle.
In an isosceles triangle, the angle at the vertex is divided equally by the line joining the center of the circle to the vertex. Thus, $\angle PAQ = 60°$, and so $\angle PAO = \angle QAO = 30°$.
Now, $\angle POQ = 2 \times \angle PAO = 2 \times 30° = 60°$.
To find the length of PQ, we can use the law of cosines in $\triangle PAQ$. The formula for the law of cosines is:

$$PQ^2 = AP^2 + AQ^2 - 2 \times AP \times AQ \times \cos(\angle PAQ).$$

Substitute the given values:

$$PQ^2 = 30^2 + 30^2 - 2 \times 30 \times 30 \times \cos(60°),$$

$$PQ^2 = 900 + 900 - 2 \times 30 \times 30 \times \frac{1}{2},$$

$$PQ^2 = 1800 - 900 = 900,$$

$$PQ = \sqrt{900} = 30\,\text{cm}.$$

(ii) Find $\angle POQ$ We already know that $\angle POQ = 60°$ from the above explanation.

(iii) (a) Find the length OA To find the length OA, we use the right triangle OAP, where OP is the radius of the circle and AP is a tangent to the circle. The radius is perpendicular to the tangent at the point of contact. Therefore, we can use trigonometry:

$$\cos(30^\circ) = \frac{OP}{AP}.$$

Since $\cos(30^\circ) = \frac{\sqrt{3}}{2}$, we have:

$$\frac{\sqrt{3}}{2} = \frac{OP}{30},$$

$$OP = 30 \times \frac{\sqrt{3}}{2} = 15\sqrt{3}\,\text{cm}.$$

Thus, the length of OA is $15\sqrt{3} \approx 25.98\,\text{cm}$.
OR (b) Find the radius of the mirror Since $OP = OA$ (the radius of the circle), the radius is $15\sqrt{3} \approx 25.98\,\text{cm}$.

> Quick Tip
>
> In problems involving tangents and circles, remember that the radius is perpendicular to the tangent at the point of contact, and use trigonometric ratios like cosine to solve for unknown lengths in right triangles.

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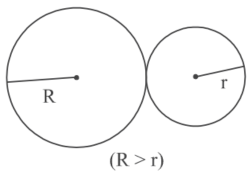
38. To keep the lawn green and cool, Sadhna uses water sprinklers which rotate in circular shape and cover a particular area. The diagram below shows the circular areas covered by two sprinklers:



Two circles touch externally. The sum of their areas is 130 sq m and the distance between their centres is 14 m. Based on above information, answer the following questions:
(i) Obtain a quadratic equation involving R and r. (1) (ii) Write a quadratic equation involving only r. (1) (iii) (a) Find the radius R and the corresponding area irrigated. (2) OR (b) Find the radius R and the corresponding area irrigated. (2)

Correct Answer:

Given: - The radius of the larger circle is R. - The radius of the smaller circle is r. - The circles touch externally, meaning the distance between their centers is $R + r = 14\,\text{m}$. - The sum of the areas of the circles is 130π sq m.

Step 1: Write the area equation The area of a circle is given by $A = \pi r^2$, so the sum of the areas of the two circles is:

$$\pi R^2 + \pi r^2 = 130\pi.$$

Simplifying:

$$R^2 + r^2 = 130. \quad \text{(Equation 1)}$$

Step 2: Use the distance between centers equation We are given that the distance between the centers of the circles is 14 m:

$$R + r = 14.$$

Rearranging this:

$$R = 14 - r. \quad \text{(Equation 2)}$$

Step 3: Substitute Equation 2 into Equation 1 Substitute $R = 14 - r$ into $R^2 + r^2 = 130$:

$$(14 - r)^2 + r^2 = 130.$$

Expanding the square:

$$(196 - 28r + r^2) + r^2 = 130,$$
$$196 - 28r + 2r^2 = 130.$$

Simplifying:

$$2r^2 - 28r + 66 = 0.$$

Dividing the entire equation by 2:

$$r^2 - 14r + 33 = 0. \quad \text{(Quadratic equation involving } r\text{)}$$

Step 4: Solve the quadratic equation for r We can solve $r^2 - 14r + 33 = 0$ using the quadratic formula:

$$r = \frac{-(-14) \pm \sqrt{(-14)^2 - 4 \times 1 \times 33}}{2 \times 1},$$
$$r = \frac{14 \pm \sqrt{196 - 132}}{2},$$
$$r = \frac{14 \pm \sqrt{64}}{2},$$
$$r = \frac{14 \pm 8}{2}.$$

Thus, the two possible solutions for r are:

$$r = \frac{14 + 8}{2} = 11 \quad \text{or} \quad r = \frac{14 - 8}{2} = 3.$$

Step 5: Find R and the area irrigated From Equation 2, $R = 14 - r$. For $r = 11$, we have:

$$R = 14 - 11 = 3.$$

For $r = 3$, we have:

$$R = 14 - 3 = 11.$$

Now, we calculate the area irrigated by each circle: - For $r = 11$ and $R = 3$, the area irrigated by the larger circle is:

$$\text{Area of larger circle} = \pi R^2 = \pi \times 3^2 = 9\pi \,\text{sq m}.$$

- For $r = 3$ and $R = 11$, the area irrigated by the smaller circle is:

$$\text{Area of smaller circle} = \pi r^2 = \pi \times 11^2 = 121\pi \,\text{sq m}.$$

Thus, the radius of the larger circle is $R = 11\,\text{m}$ and the corresponding area irrigated is $121\pi\,\text{sq m}$.

Quick Tip

When solving problems involving two touching circles, use the relationship between the radii and the distance between their centers, and apply the area formula for circles to derive quadratic equations.