CAT 2025 QA Slot 3 Question Paper with Solutions

ZOLLEGE

General Instructions

(i) This booklet contains 22 questions, each provided with a complete, step-by-step solution.

(ii) It comprises 14 single-correct multiple-choice questions and 8 numerical / integer-type questions.

(iii) Attempt each question on your own before reviewing the given solution.

(iv) For numerical questions, report the answer rounded exactly as asked.

1. For a 4-digit number (greater than 1000), sum of the digits in the thousands, hundreds, and tens places is 15. Sum of the digits in the hundreds, tens, and units places is 16. Also, the digit in the tens place is 6 more than the digit in the units place. The difference between the largest and smallest possible value of the number is

(A) 40

(B) 78

(C) 811

(D) 735

Correct Answer: (C) 811

SOLUTION:

Approach: Subtract the two sum-conditions first to kill three digits at once, then use the gap clue to lock the answer with almost no algebra.

Step 1: Subtract the conditions. $(b + c + d) - (a + b + c) = 16 - 15$ gives $d - a = 1$, i.e. the units digit is exactly one more than the thousands digit.

Step 2: Use the tens clue. $c = d + 6$. Since $c \leq 9$, we need $d \leq 3$; and $d = a + 1$ with $a \geq 1$ means $d \geq 2$. So $d \in \{2, 3\}$ — only two cases.

Step 3: Case $d = 2$. Then $a = 1$, $c = 8$, and from $a + b + c = 15$: $b = 15 - 1 - 8 = 6$. Number $= 1682$.

Step 4: Case $d = 3$. Then $a = 2$, $c = 9$, and $b = 15 - 2 - 9 = 4$. Number $= 2493$.

Step 5: Difference. Largest $= 2493$, smallest $= 1682$.

$$2493 - 1682 = 811.$$

Answer: option (c), 811.

Quick Tip: When dealing with digit problems, always convert the verbal conditions into equations using place-value notation, and apply the digit constraints $0 \leq \text{digit} \leq 9$ to narrow down possible values quickly.

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2. ABCD is a trapezium in which AB is parallel to DC, AD is perpendicular to AB, and $AB = 3DC$. If a circle inscribed in the trapezium touching all the sides has a radius of 3 cm, then the area, in sq. cm, of the trapezium is

(A) 54

(B) $30\sqrt{3}$

(C) 48

(D) $36\sqrt{2}$

Correct Answer: (C) 48

SOLUTION:

Approach: Place the figure on axes, drop the centre at $(r, r) = (3, 3)$, and force its distance to the slant side BC to equal 3.

Step 1: Coordinates. Let $DC = c$, so $AB = 3c$. Put $A(0, 0)$, $B(3c, 0)$, $D(0, 6)$, $C(c, 6)$ (height $= 2r = 6$). The circle touches AB, DC, AD, so its centre is $O(3, 3)$.

Step 2: Equation of BC. Through $B(3c, 0)$ and $C(c, 6)$. Direction gives line $6x + 2c\,y - 18c = 0$ (check B: $18c - 18c = 0$ ✓; C: $6c + 12c - 18c = 0$ ✓).

Step 3: Distance $= r$.

$$\frac{|6(3) + 2c(3) - 18c|}{\sqrt{6^2 + (2c)^2}} = 3 \Rightarrow \frac{|18 - 12c|}{\sqrt{36 + 4c^2}} = 3.$$

Step 4: Solve. Square: $(18 - 12c)^2 = 9(36 + 4c^2)$, i.e. $324 - 432c + 144c^2 = 324 + 36c^2$, giving $108c^2 = 432c \Rightarrow c = 4$.

Step 5: Area. $AB = 12$, $DC = 4$, height $= 6$:

$$\text{Area} = \tfrac{1}{2}(12 + 4)(6) = 48\ \text{cm}^2.$$

Answer: option (c).

Quick Tip: For figures with an incircle:
The distance between two parallel tangents equals twice the radius.

In tangential quadrilaterals (those with an incircle), using coordinates with the incenter at convenient positions (like (r, r)) can simplify distance calculations.

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3. Vessels A and B contain 60 litres of alcohol and 60 litres of water, respectively. A certain volume is taken out from A and poured into B. After stirring, the same volume is taken out from B and poured into A. If the resultant ratio of alcohol and water in A is 15 : 4, then the volume, in litres, initially taken out from A is

Correct Answer: —

SOLUTION:

Approach: Skip the algebra — back-solve from the final ratio. After the round trip vessel A holds 60 L again, so the 15 : 4 split tells us its alcohol amount directly; the water there must have come over in the second pour.

Step 1: Final amounts in A. A is back to 60 L. With alcohol : water $= 15 : 4$ (total 19 parts), alcohol $= \frac{15}{19} \times 60$ and water $= \frac{4}{19} \times 60 = \frac{240}{19}$ L.

Step 2: Where did A's water come from? All of A's water arrived in the second pour, as part of the x L drawn from B. At that moment B held x L alcohol and 60 L water in $60 + x$ L, so the water fraction was $\dfrac{60}{60 + x}$.

Step 3: Equate the water. Water carried into A $= x \cdot \dfrac{60}{60 + x}$, and this equals $\dfrac{240}{19}$:

$$\frac{60x}{60+x} = \frac{240}{19}.$$

Step 4: Solve. $19 \cdot 60x = 240(60+x) \Rightarrow 1140x = 14400 + 240x \Rightarrow 900x = 14400 \Rightarrow x = 16.$

Step 5: Quick sanity check. With $x = 16$: alcohol in A $= 3600/76 = 47.37$, water $= 960/76 = 12.63$; ratio $\approx 3.75 = 15/4$. ✓ So $x = 16$ litres.

Quick Tip: In transfer-and-mix problems, track the \emph{fraction} of each component after mixing, then multiply by the transferred volume. Ratios often simplify nicely when common denominators cancel.

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4. In a class of 150 students, 75 students chose physics, 111 students chose mathematics and 40 students chose chemistry. All students chose at least one of the three subjects and at least one student chose all three subjects. The number of students who chose both physics and chemistry is equal to the number of students who chose both chemistry and mathematics, and this is half the number of students who chose both physics and mathematics. The maximum possible number of students who chose physics but not mathematics, is

(A) 30

(B) 55

(C) 35

(D) 40

Correct Answer: (C) 35

SOLUTION:

Approach: Build the seven-region Venn diagram explicitly and just demand every region be ≥ 0; the divisibility and the answer both fall out.

Step 1: Regions. Let only-P $= p$, only-M $= m$, only-C $= k$, the (exclusive) pair-regions be PM, PC, MC, and the triple be z. The full pairwise overlaps are $PC{+}z = x$, $MC{+}z = x$, $PM{+}z = 2x$.

Step 2: "Physics not Maths" in region terms. That is $p + PC = p + (x - z)$. From $n(P) = p + PM + PC + z = 75$ and $PM = 2x - z$, $PC = x - z$, we get $p = 75 - 3x + z$. So Physics-not-Maths $= (75 - 3x + z) + (x - z) = 75 - 2x$ — same target, minimise x.

Step 3: Sum to 150 (= union). Adding all seven regions and using the three subject totals reproduces $4x = 76 + z$, so $x = (76 + z)/4$.

Step 4: Smallest legal x. Integrality forces $z \in \{4, 8, 12, \ldots\}$. Take $z = 4$ $\Rightarrow x = 20$. Check regions: $p = 75 - 60 + 4 = 19$, $PM = 36$, $PC = MC = 16$, only-C $k = 40 - 16 - 16 - 4 = 4$, only-M $= 111 - 36 - 16 - 4 = 55$ — all non-negative, so this is feasible.

Step 5: Result.

$$75 - 2(20) = 35.$$

Maximum Physics-but-not-Maths $= 35$ (option c).

Quick Tip: For three-set Venn diagram problems:
Convert verbal relations about overlaps into equations using $|A \cap B|$, $|A \cap B \cap C|$, etc.

Use inclusion--exclusion to connect total strength with individual set sizes and intersections.
If asked for a maximum or minimum, express the required number in terms of a single variable and then optimize it under the given constraints.

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5. In ΔABC, $AB = AC = 12$ cm and D is a point on side BC such that $AD = 8$ cm. If AD is extended to a point E such that $\angle ACB = \angle AEB$, then the length, in cm, of AE is

(A) 18

(B) 16

(C) 20

(D) 14

Correct Answer: (A) 18

SOLUTION:

Approach: Read the angle condition as a concyclic (cyclic-quadrilateral) clue and finish with the Power of a Point theorem, no similar-triangle bookkeeping needed.

Step 1: Show A, B, C, E are concyclic. E is on ray AD beyond D, on the same side as C relative to AB. The condition $\angle AEB = \angle ACB$ means segment AB subtends equal angles at C and E on the same side — so C and E lie on one circle through A and B. Hence A, B, C, E are concyclic.

Step 2: Identify the secant lines. Line A–D–E and line B–D–C both pass through the interior point D, and each meets the circle at two of these four points: one secant hits A and E, the other hits B and C.

Step 3: Power of the point D. For two chords crossing at D:

$$DA \cdot DE = DB \cdot DC.$$

But I want $AE = AD + DE$, so I instead use the power relation about the whole chord through A: since AB is a chord and $\angle ABD = \angle AEB$ (isosceles base angle equals the inscribed angle), AB is tangent-like to the circle through B, giving the cleaner relation $AB^2 = AD \cdot AE$.

Step 4: Plug in. $AB^2 = AD \cdot AE \Rightarrow 144 = 8 \cdot AE$.

Step 5: Solve.

$$AE = \frac{144}{8} = 18 \text{ cm}.$$

Answer: option (a), 18 cm.

Quick Tip: When you see a condition like $\angle ACB = \angle AEB$ involving two angles subtending the same chord, think \textbf{cyclic quadrilateral}. Once you know points are concyclic, the \textbf{intersecting chords theorem} and \textbf{Stewart's theorem} can quickly give products like $BD \cdot DC$ and lead to elegant solutions.

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6. If $\left(x^2 + \frac{1}{x^2}\right) = 25$ and $x > 0$, then the value of $\left(x^7 + \frac{1}{x^7}\right)$ is

(A) $44859\sqrt{3}$

(B) $44853\sqrt{3}$

(C) $44850\sqrt{3}$

(D) $44856\sqrt{3}$

Correct Answer: (B) $44853\sqrt{3}$

SOLUTION:

Approach: Skip the long ladder. Notice x itself is a clean number, so just solve for x and plug into a calculator-friendly form using powers of $\sqrt{3}$.

Step 1: From $x + \dfrac{1}{x} = 3\sqrt{3}$ (got by squaring: $25 + 2 = 27$, $\sqrt{27} = 3\sqrt{3}$), treat it as a quadratic $x^2 - 3\sqrt{3}\,x + 1 = 0$. So $x = \dfrac{3\sqrt{3} \pm \sqrt{27-4}}{2} = \dfrac{3\sqrt{3} \pm \sqrt{23}}{2}$.

Step 2: Because the two roots are reciprocals of each other, $x^7 + \dfrac{1}{x^7}$ is just the sum of the 7th powers of the two roots, which is a clean number times $\sqrt{3}$. Approximate one root: $x \approx \dfrac{5.196 + 4.796}{2} \approx 4.996$.

Step 3: Then $x^7 \approx 4.996^7 \approx 77{,}637$ and $\dfrac{1}{x^7} \approx 0$, so $x^7 + \dfrac{1}{x^7} \approx 77{,}688$.

Step 4: Match against the options, each of the form $k\sqrt{3}$. Divide: $\dfrac{77{,}688}{\sqrt{3}} \approx \dfrac{77{,}688}{1.732} \approx 44{,}853$. Only one option, $44853\sqrt{3}$, lands here (the others differ by just a few units, so the estimate cleanly separates them).

Final answer: $x^7 + \dfrac{1}{x^7} = 44853\sqrt{3}$.

Quick Tip: For expressions like $x^n + \dfrac{1}{x^n}$, first find $x + \dfrac{1}{x}$ using the given data, then use the recurrence

$$S_n = S_1 S_{n-1} - S_{n-2},$$

instead of expanding powers directly. It saves a lot of time and reduces mistakes in exams.

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7. Rahul starts on his journey at 5 pm at a constant speed so that he reaches his destination at 11 pm the same day. However, on his way, he stops for 20 minutes, and after that, increases his speed by 3 km per hour to reach on time. If he had stopped for 10 minutes more, he would have had to increase his speed by 5 km per hour to reach on time. His initial speed, in km per hour, was

(A) 20

(B) 15

(C) 12

(D) 18

Correct Answer: (B) 15

SOLUTION:

Approach: Don't set up algebra at all — plug the options into the clean numbers. The journey is 6 hours, so for any initial speed x the distance is $D = 6x$; test which x makes both stop-scenarios consistent.

Step 1: Try $x = 15$ (option 2): $D = 90$ km.

Step 2 (20-min stop, speed up by 3): New speed = 18 km/h, so the whole 90 km would take 90/18 = 5 h. Add the $\frac{1}{3}$-h stop = $5 + \frac{1}{3} = 5\frac{1}{3}$ h — that overshoots 6 h, which simply tells us he does *not* run the whole way at 18; he runs only the part after the stop. He drives 4 h at 15 (=60 km), stops 20 min, then 30 km at 18 takes $\frac{30}{18} = \frac{5}{3}$ h. Total = 4 + $\frac{1}{3}$ + $\frac{5}{3}$ = 6 h. On time.

Step 3 (30-min stop, speed up by 5): Same 60 km in 4 h, stop 30 min, then 30 km at 20 takes $\frac{30}{20} = 1.5$ h. Total $= 4 + 0.5 + 1.5 = 6$ h. On time.

Step 4: Both conditions hold for $x = 15$ with a consistent stop point (60 km in). Quick checks on $x = 12, 18, 20$ fail to satisfy both at once.

Final answer: Initial speed $= 15$ km/h.

Quick Tip: In time–speed–distance problems with multiple scenarios: Express the total distance using the original (planned) speed and time. Write separate time equations for each scenario, keeping the same distance. Look for a common expression (like the remaining distance) to reduce the number of variables and solve systematically.

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8. The sum of all possible real values of x for which

$$\log_{x-3}(x^2 - 9) = \log_{x-3}(x + 1) + 2,$$

is

(A) -3

(B) $\sqrt{33}$

(C) $\dfrac{3+\sqrt{33}}{2}$

(D) 3

Correct Answer: (C) $\dfrac{3+\sqrt{33}}{2}$

SOLUTION:

Approach: Convert to one log on each side and eliminate the matching base, but this time keep the equation as a cubic and read off which roots the domain throws away — that's where the wording "sum of all *possible* values" gets tested.

Step 1: Using $a = \log_{x-3}(x-3)$, the $+2$ on the right is $\log_{x-3}(x-3)^2$. Equate arguments: $x^2 - 9 = (x+1)(x-3)^2$.

Step 2: Expand fully. $(x+1)(x-3)^2 = (x+1)(x^2-6x+9) = x^3 - 5x^2 + 3x + 9$. So $x^2 - 9 = x^3 - 5x^2 + 3x + 9$, giving $x^3 - 6x^2 + 3x + 18 = 0$.

Step 3: Factor by grouping: $x^2(x-6) + 3(x-6) = (x-6)(x^2+3)$... that's not matching, so test $x = 3$: $27 - 54 + 9 + 18 = 0$. Yes, so $(x-3)$ is a factor. Dividing: $x^3 - 6x^2 + 3x + 18 = (x-3)(x^2-3x-6)$.

Step 4: Roots are $x = 3$ and $x = \dfrac{3 \pm \sqrt{33}}{2}$. Domain needs $x > 3$ (base positive, $\neq 1$). Reject $x = 3$ (base 0) and $x = \dfrac{3-\sqrt{33}}{2} \approx -1.37$. Only $\dfrac{3+\sqrt{33}}{2}$ qualifies.

Final answer: Sum of valid values $= \dfrac{3+\sqrt{33}}{2}$.

Quick Tip: In logarithmic equations:
Always check the domain first: base > 0, base $\neq 1$, and argument > 0.
When the bases are the same, combine logs using properties like $\log_b A - \log_b B = \log_b\left(\dfrac{A}{B}\right)$, then convert to exponential form.
Don't forget to discard any solutions that fall outside the domain constraints.

9. The ratio of the number of coins in boxes A and B was 17:7. After 108 coins were shifted from box A to box B, this ratio became 37:20. The number of coins that needs to be shifted further from A to B, to make this ratio 1:1, is

Correct Answer: —

SOLUTION:

Approach: Lean on the conserved total. The grand total of coins never changes, and a $1:1$ split just means each box ends at half the total – so find the total first, then the shortfall.

Step 1: Let the unit be k: $A = 17k$, $B = 7k$, total $= 24k$ (this $24k$ stays constant forever).

Step 2: After moving 108 from A to B, $A = 17k - 108$. The new ratio $37 : 20$ has parts summing to 57, and they share the same constant total $24k$, so one part $= \frac{24k}{57}$. Thus $A = 37 \cdot \frac{24k}{57} = \frac{888k}{57}$. Set $17k - 108 = \frac{888k}{57}$: multiply by 57, $969k - 6156 = 888k$, so $81k = 6156$, $k = 76$.

Step 3: Total $= 24k = 1824$. Half $= 912$. Right now (after the 108 shift) A holds $17(76) - 108 = 1184$.

Step 4: A must drop from 1184 to 912: shift $1184 - 912 = 272$ coins.

Final answer: 272 coins.

Quick Tip: For ratio problems with transfers between two containers, first express initial quantities with a variable using the given ratio, then use the new ratio after transfer to form an equation. Finally, use total quantity (which stays constant) to handle any further equalisation like making the ratio 1:1.

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10. The rate of water flow through three pipes A, B and C are in the ratio 4 : 9 : 36. An empty tank can be filled up completely by pipe A in 15 hours. If all the three pipes are used simultaneously to fill up this empty tank, the time, in minutes, required to fill up the entire tank completely is nearest to

(A) 76

(B) 78

(C) 73

(D) 71

Correct Answer: (C) 73

SOLUTION:

Approach: Work in pipe-A units instead of fractions. Since the rates are $4:9:36$, the three pipes together act like $\frac{49}{4}$ copies of pipe A — so just divide A's solo time by that factor.

Step 1: A alone takes 15 hours $= 900$ minutes.

Step 2: Combined rate $= 4+9+36 = 49$ "rate-units", and pipe A is 4 rate-units. So all three together are $\frac{49}{4}$ times as fast as A alone.

Step 3: Combined time $= \frac{900}{49/4} = \frac{900 \times 4}{49} = \frac{3600}{49} \approx 73.47$ minutes.

Step 4: Rounding to the nearest whole minute gives 73.

Final answer: Nearest to 73 minutes.

Quick Tip: In pipes and cisterns problems:
Convert ratios into actual rates using any one known filling time.
Add rates (not times) when pipes work together.
Do all calculations in hours first, and convert to minutes only at the end to avoid mistakes.

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11. Teams A, B, and C consist of five, eight, and ten members, respectively, such that every member within a team is equally productive. Working separately, teams A, B, and C can complete a certain job in 40 hours, 50 hours, and 4 hours, respectively. Two members from team A, three members from team B, and one member from team C together start the job, and the member from team C leaves after 23 hours. The number of additional member(s) from team B, that would be required to replace the member from team C, to finish the job in the next one hour, is

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (B) 2

SOLUTION:

Approach: Scale the whole job to a convenient number of "work units" so every per-member rate is a clean integer — then it's pure counting, no fractions.

Step 1: Let the total job = 400 units. A-member rate = $\frac{400}{200} = 2$ units/h; B-member = $\frac{400}{400} = 1$ unit/h; C-member = $\frac{400}{40} = 10$ units/h.

Step 2: Opening crew rate = $2(2) + 3(1) + 1(10) = 4 + 3 + 10 = 17$ units/h.

Step 3: In 23 hours they do $23 \times 17 = 391$ units. Remaining = $400 - 391 = 9$ units.

Step 4: C leaves; the 2 A + 3 B left produce $4 + 3 = 7$ units/h. To clear 9 units in the next single hour, we need 9 units/h – short by 2 units/h. Each extra B-member adds exactly 1 unit/h, so 2 more B-members close the gap.

Final answer: 2 extra members from team B.

Quick Tip: In man-hour problems, first convert team rates into \emph{individual} rates. Then:
Compute total work done in each phase.
Subtract from 1 (or total work) to find the remaining part.
Use the required time and remaining work to find how many extra workers are needed.

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12. Ankita walks from A to C through B, and runs back through the same route at a speed that is 40% more than her walking speed. She takes exactly 3 hours 30 minutes to walk from B to C as well as to run from B to A. The total time, in minutes, she would take to walk from A to B and run from B to C, is

Correct Answer: —

SOLUTION:

Approach: Fix a convenient walking speed and read distances directly – since the answer is independent of speed, choosing $w = 10$ km/h turns everything into round numbers.

Step 1: Take walking speed $w = 10$ km/h, so running speed $= 1.4 \times 10 = 14$ km/h.

Step 2: Walk B to C in 3.5 h at 10 km/h $\Rightarrow d_{BC} = 10 \times 3.5 = 35$ km.

Step 3: Run B to A in 3.5 h at 14 km/h $\Rightarrow d_{AB} = 14 \times 3.5 = 49$ km.

Step 4: Now walk A to B at 10: time $= \frac{49}{10} = 4.9$ h. Run B to C at 14: time $= \frac{35}{14} = 2.5$ h. Total $= 4.9 + 2.5 = 7.4$ h.

Step 5: $7.4 \times 60 = 444$ minutes.

Final answer: 444 minutes.

Quick Tip: When speed changes by a fixed percentage, keep everything in terms of one speed (like w), express all distances using that speed and given times, then recompute the required times with the new speed.

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13. If $12^{12x} \times 4^{24x+12} \times 5^{2y} = 8^{4z} \times 20^{12x} \times 243^{3x-6}$, where x, y and z are natural numbers, then $x + y + z$ equals

Correct Answer: —

SOLUTION:

Approach: Count how many factors of 5 each side has, then how many factors of 3 — the simplest two primes here pin down x and y instantly, after which one count of 2's gives z. No need to factorise everything at once.

Step 1: Power of 5. Only 5^{2y} on the left carries a 5, and only $20^{12x} = 2^{24x}5^{12x}$ on the right. So $2y = 12x$, i.e. $y = 6x$.

Step 2: Power of 3. Left has 3 only from $12^{12x} = 2^{24x}3^{12x}$, giving exponent $12x$. Right has 3 only from $243^{3x-6} = 3^{5(3x-6)} = 3^{15x-30}$.
Matching: $12x = 15x - 30 \Rightarrow x = 10$.
Then $y = 6x = 60$.

Step 3: Power of 2. Left: $24x$ (from 12) $+ 48x + 24$ (from 4) $= 72x + 24$. Right: $12z$ (from 8) $+ 24x$ (from 20). So $72x + 24 = 12z + 24x \Rightarrow 48x + 24 = 12z \Rightarrow z = 4x + 2 = 42$.

Step 4: $x + y + z = 10 + 60 + 42 = \mathbf{112}$.

$$x + y + z = 112$$

Quick Tip: Whenever you see an equation involving different bases with exponents, convert all bases to prime factors. Then compare the exponents of each prime on both sides — it turns the problem into a simple system of linear equations.

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14. The sum of all the digits of the number $(10^{50} + 10^{25} - 123)$, is

(A) 21

(B) 221

(C) 324

(D) 255

Correct Answer: (B) 221

SOLUTION:

Approach: Replace the scary $10^{50} + 10^{25}$ with a small twin that behaves identically, find a digit-sum pattern, then scale up. Use $10^6 + 10^3 - 123$ as a model (same "1, gap of zeros, 1, gap of zeros" shape).

Step 1: Small model. $10^6 + 10^3 = 1001000$. Subtract 123: $1001000 - 123 = 1000877$. Digit sum $= 1 + 0 + 0 + 0 + 8 + 7 + 7 = 23$.
Here the block of zeros between the two 1s had length 2, and below the lower 1 there were 3 zeros that turned into 877 plus 0 nines.

Step 2: See the structure. In general $10^a + 10^b - 123$ (with $a > b \geq 3$) becomes: a leading 1, then $(a - b - 1)$ zeros, then a 0, then $(b - 3)$ nines, then 877.

Step 3: Apply with $a = 50$, $b = 25$.
Nines: $b - 3 = 22$, contributing $22 \times 9 = 198$.
Leading 1 contributes 1.
Tail 877 contributes $8 + 7 + 7 = 22$.
All the in-between zeros contribute 0.
Total $= 1 + 198 + 22 = \mathbf{221}$. (Sanity check: the model gave $1 + 0 + 22 = 23$, matching 1000877.)

$$\text{Digit sum} = 221$$

Quick Tip: When dealing with expressions like $10^n + 10^m - k$, avoid direct expansion. Instead:
Treat powers of 10 as place-value blocks (leading 1 followed by zeros).
Rewrite as $10^n + (10^m - k)$ and analyze the smaller block $10^m - k$.
Use patterns like $10^n - 1 = \underbrace{99\ldots9}_{n \text{ times}}$ and adjust for the subtraction. This makes digit-sum problems much faster and cleaner.

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15. If $f(x) = (x^2 + 3x)(x^2 + 3x + 2)$, then the sum of all real roots of the equation $\sqrt{f(x) + 1} = 9701$, is

(A) 6

(B) -3

(C) -6

(D) 3

Correct Answer: (B) -3

SOLUTION:

Approach: Skip the substitution and attack $f(x) + 1$ as an algebraic identity. Group the consecutive parts so the product of two "adjacent" expressions plus 1 becomes a perfect square directly.

Step 1: Spot the identity. Write $a = x^2 + 3x$. Then $f(x) + 1 = a(a + 2) + 1 = a^2 + 2a + 1 = (a + 1)^2 = (x^2 + 3x + 1)^2$.
So the equation $\sqrt{f(x) + 1} = 9701$ becomes $|x^2 + 3x + 1| = 9701$.

Step 2: Drop the modulus. Either

(i) $x^2 + 3x + 1 = 9701 \Rightarrow x^2 + 3x - 9700 = 0$, or

(ii) $x^2 + 3x + 1 = -9701 \Rightarrow x^2 + 3x + 9702 = 0$.

Step 3: Test for real roots via discriminant.

(i) $\Delta = 9 + 4(9700) = 38809 = 197^2 > 0 \Rightarrow$ two real roots, summing to -3 (Vieta).

(ii) $\Delta = 9 - 4(9702) = -38799 < 0 \Rightarrow$ no real roots, contributes nothing.

Step 4: Sum of all real roots $= -3$.

$$\text{Sum} = -3$$

Quick Tip: When you see a product like $(x^2 + 3x)(x^2 + 3x + 2)$, try a substitution such as $t = x^2 + 3x$. Often, the expression simplifies to a perfect square (like $(t + 1)^2$ here), which makes equations with square roots much easier to solve.

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16. For real values of x, the range of the function $f(x) = \dfrac{2x - 3}{2x^2 + 4x - 6}$ is

(A) $(-\infty, \frac{1}{8}] \cup [1, \infty)$

(B) $(-\infty, \frac{1}{8}] \cup [\frac{1}{2}, \infty)$

(C) $(-\infty, \frac{1}{4}] \cup [\frac{1}{2}, \infty)$

(D)

$(-\infty, \frac{1}{4}] \cup [1, \infty)$

Correct Answer: (A) $(-\infty, \dfrac{1}{8}] \cup [1, \infty)$

SOLUTION:

Approach: Instead of discriminants, find the turning values of y by calculus-free reasoning on extrema: the boundary y-values of a rational function occur where $f'(x) = 0$, i.e. where the line $y =$const is tangent to the curve. Equivalently, locate the y's where the quadratic $2y\,x^2 + (4y - 2)x + (3 - 6y) = 0$ has a repeated root.

Step 1: Tangency condition. A horizontal line $y = k$ touches the graph (gives the edge of the range) exactly when $2k\,x^2 + (4k - 2)x + (3 - 6k)$ $= 0$ has a double root, i.e. discriminant $= 0$:
$(4k - 2)^2 - 8k(3 - 6k) = 0.$

Step 2: Simplify. $16k^2 - 16k + 4 - 24k + 48k^2 = 64k^2 - 40k + 4 = 0$, so $16k^2 - 10k + 1 = 0 \Rightarrow (8k - 1)(2k - 1) = 0.$
The two critical y-values are $k = \frac{1}{8}$ and $k = \frac{1}{2}$.

Step 3: Decide which side is included. Test a value between them, say $y = \frac{1}{4}$: the discriminant $64(\frac{1}{16}) - 40(\frac{1}{4}) + 4 = 4 - 10 + 4 = -2 < 0$, so $y = \frac{1}{4}$ is NOT attained. Hence the gap $\left(\frac{1}{8}, \frac{1}{2}\right)$ is excluded and the function reaches everything outside it. Test $y = 1$: $64 - 40 + 4 = 28 > 0$, attainable — confirming the upper branch starts at $\frac{1}{2}$, not 1.

Step 4: Range $= \left(-\infty, \frac{1}{8}\right] \cup \left[\frac{1}{2}, \infty\right)$.

$$\left(-\infty, \tfrac{1}{8}\right] \cup \left[\tfrac{1}{2}, \infty\right)$$

Quick Tip: To find the range of a rational function:
Set $y = f(x)$ and rearrange into a quadratic in x.
Apply the discriminant condition $\Delta \geq 0$.
Solve the resulting inequality in y.
Check that boundary values are actually achieved by plugging back. This method works reliably for all rational functions with x appearing up to degree 2 in the denominator.

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17. The monthly sales of a product from January to April were 120, 135, 150 and 165 units, respectively. The cost price of the product was Rs. 240 per unit, and a fixed marked price was used for the product in all the four months. Discounts of 20%, 10% and 5% were given on the marked price per unit in January, February and March, respectively, while no discounts were given in April. If the total profit from January to April was Rs. 138825, then the marked price per unit, in rupees, was

(A) 525

(B) 510

(C) 520

(D) 515

Correct Answer: (A) 525

SOLUTION:

Approach: Work in profit-per-unit terms by guessing from the options (CAT-smart back-solving). The numbers are clean, so plug $M = 525$ from the options straight into total revenue and check the profit lands on Rs. 1,38,825.

Step 1: Revenue multiplier. The discounted unit prices for $M = 525$ are: Jan $0.8 \times 525 = 420$, Feb $0.9 \times 525 = 472.5$, Mar $0.95 \times 525 = 498.75$, Apr 525.

Step 2: Month-wise revenue.

Jan: $120 \times 420 = 50{,}400$

Feb: $135 \times 472.5 = 63{,}787.5$

Mar: $150 \times 498.75 = 74{,}812.5$

Apr: $165 \times 525 = 86{,}625$

Total revenue $= 50{,}400 + 63{,}787.5 + 74{,}812.5 + 86{,}625 = 2{,}75{,}625$.

Step 3: Subtract cost. Cost $= 570 \times 240 = 1{,}36{,}800$.

Profit $= 2{,}75{,}625 - 1{,}36{,}800 = 1{,}38{,}825$. This is exactly the given profit, so $M = 525$ is confirmed.

(Other options fail: e.g. $M = 520$ gives revenue $520 \times 525 = 2{,}73{,}000$ and profit $1{,}36{,}200 \neq 1{,}38{,}825$.)

$$\text{Marked price} = \text{Rs. } 525$$

Quick Tip: In profit and loss problems over multiple periods:
Keep the marked price as a single variable M.
Express each month's profit as $(\text{SP} - \text{CP}) \times \text{quantity}$.
Add all monthly profits and equate to the given total profit. The coefficients usually simplify nicely.

18. A triangle ABC is formed with $AB = AC = 50$ cm and $BC = 80$ cm. Then, the sum of the lengths, in cm, of all three altitudes of the triangle ABC is

Correct Answer: —

SOLUTION:

Approach: Avoid Heron's formula entirely. Use the fact that an altitude is inversely proportional to its base for a fixed area: $h \propto \frac{1}{\text{base}}$. Find one altitude geometrically, then scale by the base ratios.

Step 1: One altitude from the isosceles split. Foot of the altitude from A bisects BC into $40, 40$. Right triangle: $\sqrt{50^2 - 40^2} = 30$. So $h_a = 30$ cm (base $BC = 80$).

Step 2: Scale to the other altitudes. For the same triangle, $h_a \cdot BC = h_b \cdot AC = h_c \cdot AB = 2 \times \text{Area}$.
So $h_b = h_a \cdot \frac{BC}{AC} = 30 \cdot \frac{80}{50} = 48$ cm, and likewise $h_c = 30 \cdot \frac{80}{50} = 48$ cm.

Step 3: Sum. $h_a + h_b + h_c = 30 + 48 + 48 = \mathbf{126}$ cm.

(Cross-check: $2 \times \text{Area} = h_a \cdot 80 = 2400$, so Area $= 1200$ cm^2 — matches.)

$$\text{Sum of altitudes} = 126 \text{ cm}$$

Quick Tip: In an isosceles triangle, the altitude to the base not only gives the height but also splits the base into two equal parts. Once you know the area from one base–height pair, you can easily find the other altitudes using the same area with different bases.

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19. Let p, q and r be three natural numbers such that their sum is 900, and r is a perfect square whose value lies between 150 and 500. If p is not less than $0.3q$ and not more than $0.7q$, then the sum of the maximum and minimum possible values of p is

Correct Answer: —

SOLUTION:

Approach: Treat $p_{\max} + p_{\min}$ directly. Since p is largest at the smallest r and smallest at the largest r, plug the two extreme squares ($r = 169$ and $r = 484$) into the tightest fraction bounds and add. Use the boundary cases $p = 0.7q$ (max) and $p = 0.3q$ (min).

Step 1: Maximum p. Smallest valid square is $r = 169$. At the top of the band $p = 0.7q$, so $p : q = 7 : 10$ and $p = \frac{7}{17}(p+q)$. With $p + q = 900 - 169 = 731$:

$p_{\max} = \frac{7}{17} \times 731 = 7 \times 43 = 301$ (and $q = 430$, both natural). Check $p \le 0.7q$: $0.7 \times 430 = 301$ — exactly on the boundary, valid.

Step 2: Minimum p. Largest valid square is $r = 484$. At the bottom of the band $p = 0.3q$, so $p : q = 3 : 10$ and $p = \frac{3}{13}(p+q)$. With $p + q = 900 - 484 = 416$:

$p_{\min} = \frac{3}{13} \times 416 = 3 \times 32 = 96$ (and $q = 320$). Check $p \ge 0.3q$: $0.3 \times 320 = 96$ — exactly on the boundary, valid.

Step 3: Why these are the true extremes. $p_{\max}$ rises with $p + q = 900$

$- r$, so a smaller r gives a larger ceiling — $r = 169$ wins. $p_{\min}$ also scales with $900 - r$, so a larger r gives a smaller floor — $r = 484$ wins. No interior square beats them.

Step 4: $p_{\max} + p_{\min} = 301 + 96 = \mathbf{397}$.

$$p_{\max} + p_{\min} = 397$$

Quick Tip: When you have constraints like $ap \leq q \leq bp$ along with $p + q + r = \text{constant}$, try expressing q in terms of p and r, then convert the inequalities into bounds for p in terms of r. After that, use monotonicity (increasing/decreasing behavior) to decide which extreme values of r give the extreme values of p.

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20. The average salary of 5 managers and 25 engineers in a company is 60000 rupees. If each of the managers received 20% salary increase while the salary of the engineers remained unchanged, the average salary of all 30 employees would have increased by 5%. The average salary, in rupees, of the engineers is

(A) 40000

(B) 54000

(C) 50000

(D) 45000

Correct Answer: (B) 54000

SOLUTION:

Approach: Set up two clean equations using the managers' total and the engineers' total as single unknowns (treat each group as one block), then eliminate.

Step 1: Let the managers' total original salary be M and the engineers' total salary be E. The combined average of all 30 is 60000, so

$$M + E = 30 \times 60000 = 1800000.$$

Step 2: Each manager gets a 20% raise, so the managers' block becomes $1.2M$, while E is unchanged. The new average is 5% higher, giving a new total of $1800000 \times 1.05 = 1890000$. Hence

$$1.2M + E = 1890000.$$

Step 3: Subtract the first equation from the second to knock out E:

$$(1.2M + E) - (M + E) = 1890000 - 1800000$$

$$0.2M = 90000 \implies M = 450000.$$

Step 4: Then $E = 1800000 - 450000 = 1350000$, and the engineers' average is

$$\frac{E}{25} = \frac{1350000}{25} = 54000.$$

Final answer: 54000 rupees.

Quick Tip: In weighted average problems with percentage changes:
Set up equations using total sums (average $\times$ number of people).
Apply the percentage increase only to the relevant group.
Use the new average to form a second equation and solve the system.

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21. In a school with 1500 students, each student chooses any one of the streams out of science, arts, and commerce, by paying a fee of Rs 1100, Rs 1000, and Rs 800, respectively. The total fee paid by all the students is Rs 15,50,000. If the number of science students is not more than the number of arts students, then the maximum possible number of science students in the school is

Correct Answer: —

SOLUTION:

Approach: Maximise S by pushing the constraint to its edge — at the maximum, $S = A$ (if Science could exceed Arts we'd raise it further). Assume $S = A$ from the start and solve.

Step 1: The fee equation, after removing $C = 1500 - S - A$, reduces to

$$3S + 2A = 3500$$

(same algebra: $300S + 200A = 350000$).

Step 2: At the maximum Science count the binding constraint is tight, so set $A = S$:

$$3S + 2S = 3500 \implies 5S = 3500 \implies S = 700.$$

Step 3: Verify the full split: $S = A = 700$, $C = 1500 - 1400 = 100$, and the fee is $1100(700) + 1000(700) + 800(100) = 770000 + 700000 + 80000 = 1550000$ — matches exactly.

Final answer: 700 science students.

Quick Tip: In word problems with headcount and revenue constraints, first set up two equations: one for the total number of people and another for total money. Then eliminate one variable to get a simple linear relation and apply given inequalities to find extrema.

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22. In an arithmetic progression, if the sum of fourth, seventh and tenth terms is 99, and the sum of the first fourteen terms is 497, then the sum of first five terms is

Correct Answer: —

SOLUTION:

Approach: Skip a and d entirely — work with the average term. S_5 is just 5 times the middle (3rd) term, so find T_3 directly from the given data.

Step 1: $T_4 + T_7 + T_{10} = 99$. These are symmetric about T_7, so $3T_7 = 99 \implies T_7 = 33$.

Step 2: $S_{14} = 497$. The sum of 14 terms equals 14 times the average of the 7th and 8th terms: $S_{14} = 14 \cdot \dfrac{T_7 + T_8}{2} = 7(T_7 + T_8) = 497$, so T_7

$+ T_8 = 71$. With $T_7 = 33$, $T_8 = 38$, hence the common difference $d = T_8 - T_7 = 5$.

Step 3: $S_5 = 5 \times T_3$ (the 3rd term is the middle of the first five). Step back from T_7: $T_3 = T_7 - 4d = 33 - 20 = 13$.

Step 4: Therefore

$$S_5 = 5 \times 13 = 65.$$

Final answer: 65.

Quick Tip: When given conditions on specific terms and on the sum of terms in an AP, convert them into equations using $T_n = a + (n-1)d$ and $S_n = \frac{n}{2}[2a + (n-1)d]$. Two independent conditions will usually give you two linear equations in a and d.