CAT 2025 QA Slot 2 Question Paper with Solutions

ZOLLEGE

General Instructions

(i) This booklet contains 22 questions, each provided with a complete, step-by-step solution.

(ii) It comprises 16 single-correct multiple-choice questions and 6 numerical / integer-type questions.

(iii) Attempt each question on your own before reviewing the given solution.

(iv) For numerical questions, report the answer rounded exactly as asked.

1. Let a_n be the n^{th} term of a decreasing infinite geometric progression. If $a_1 + a_2 + a_3 = 52$ and $a_1a_2 + a_2a_3 + a_3a_1 = 624$, then the sum of this geometric progression is:

(A) 57

(B) 63

(C) 54

(D) 60

Correct Answer: (C) 54

SOLUTION:

Approach: Skip the symmetric algebra entirely. Three consecutive G.P. terms are $\frac{a}{r}$, a, ar about a middle term a. This middle-term framing makes the second equation collapse instantly.

Step 1: Call the three terms $\frac{t}{r}$, t, tr, where t is the middle term. Their sum is

$$\frac{t}{r} + t + tr = 52.$$

Step 2: Now the sum of pairwise products. Pair them up:

$$\frac{t}{r} \cdot t + t \cdot tr + tr \cdot \frac{t}{r} = t^2\left(\frac{1}{r} + r + 1\right) = t \cdot \underbrace{t\left(\frac{1}{r} + 1 + r\right)}_{=52} = 52\,t = 624.$$

So $t = \dfrac{624}{52} = 12$. The middle term is 12 directly.

Step 3: Put $t = 12$ back into the sum:

$$\frac{12}{r} + 12 + 12r = 52 \;\Rightarrow\; \frac{12}{r} + 12r = 40 \;\Rightarrow\; 12 + 12r^2 = 40r \;\Rightarrow\; 3r^2 - 10r + 3 = 0.$$

This gives $r = 3$ or $r = \frac{1}{3}$; for a decreasing infinite G.P. we keep $r = \frac{1}{3}$.

Step 4: The first term is $a = \dfrac{t}{r} = \dfrac{12}{1/3} = 36$, so

$$S_\infty = \frac{36}{1 - \frac{1}{3}} = 54.$$

Final answer: 54.

Quick Tip: For an infinite geometric progression with first term a and common ratio r (where $|r| < 1$):

$$S_\infty = \frac{a}{1 - r}.$$

Also, using relationships between sums and products of initial terms can help form equations in a and r.

2. Two tangents drawn from a point P touch a circle with center O at points Q and R. Points A and B lie on PQ and PR, respectively, such that AB is also a tangent to the same circle. If $\angle AOB = 50°$, then $\angle APB$, in degrees, equals:

Correct Answer: —

SOLUTION:

Approach: Forget incircle/excircle naming and just chase the two tangent-length angle pairs around the centre. Every tangent pair from one external point is mirror-symmetric about the line to the centre, so the centre angles double up cleanly.

Step 1: Let the circle touch PQ at Q, PR at R, and the extra tangent AB at T. Point A is on PQ, so from A two tangents go out: AQ and AT. Tangents from a point are symmetric about the line joining that point to the centre, hence OA bisects $\angle QOT$:

$$\angle QOA = \angle AOT = x.$$

Step 2: Similarly B is on PR, with tangents BR and BT, so OB bisects $\angle TOR$:

$$\angle TOB = \angle BOR = y.$$

Step 3: The given angle is the inner pair:

$$\angle AOB = \angle AOT + \angle TOB = x + y = 50°.$$

The total central angle between the original contact points Q and R is

$$\angle QOR = \angle QOA + \angle AOT + \angle TOB + \angle BOR = 2x + 2y = 100^\circ.$$

Step 4: In quadrilateral $PQOR$, the radius meets each tangent at 90°, so $\angle OQP = \angle ORP = 90^\circ$. Angles of a quadrilateral sum to 360°:

$$\angle QPR = 360^\circ - 90^\circ - 90^\circ - 100^\circ = 80^\circ.$$

Since $\angle QPR$ is the same as $\angle APB$,

$$\angle APB = 80^\circ.$$

Final answer: 80°.

Quick Tip: Tangents from an external point to a circle are equal, and the line from the center to the point of tangency is perpendicular to the tangent. These facts often allow you to form congruent triangles and use angle sums in quadrilaterals involving the center.

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3. Let $ABCDEF$ be a regular hexagon and P and Q be the midpoints of AB and CD, respectively. Then, the ratio of the areas of trapezium $PBCQ$ and hexagon $ABCDEF$ is:

(A) $6:19$

(B) $5:24$

(C) $6:25$

(D) $7:24$

Correct Answer: (B) $5:24$

SOLUTION:

Approach: Pure ratio reasoning, no coordinates. The line PQ joining the two midpoints cuts the hexagon into a triangle-fraction grid; build the trapezium area from the basic equilateral triangle of side s and a small corner triangle.

Step 1: Take side $s = 2$ so midpoints land on whole numbers; then $AP = PB = 1$ and $CQ = QD = 1$. The hexagon area is $\frac{3\sqrt{3}}{2}(2)^2 = 6\sqrt{3}$.

Step 2: Drop the trapezium $PBCQ$ onto the standard hexagon. Look at the big triangle ABC inside the hexagon: with B at the bottom corner, A up-left and C to the right, triangle $PBCQ$ is triangle-shaped region trimmed by joining the midpoints P (on AB) and Q (on CD).

Step 3: Compute directly with the trapezium formula using the two parallel chords. The lower chord is $BC = s = 2$. The upper chord PQ runs between the midpoint of AB and the midpoint of CD; by the geometry of the hexagon this length is $PQ = \frac{3}{2}s = 3$. The perpendicular distance between these two parallel chords is half the height of the band $ABCD$, namely $\frac{\sqrt{3}}{4}s = \frac{\sqrt{3}}{2}$. Then

$$\text{Area}_{PBCQ} = \frac{1}{2}(BC + PQ) \times h = \frac{1}{2}(2+3) \times \frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{4}.$$

Step 4: Form the ratio against $6\sqrt{3}$:

$$\frac{\text{Area}_{PBCQ}}{\text{Area}_{\text{hex}}} = \frac{5\sqrt{3}/4}{6\sqrt{3}} = \frac{5}{24}.$$

Final answer: $5 : 24$.

Quick Tip: For regular polygons, coordinate geometry is very handy: place convenient vertices on the axes, find key points (like midpoints) using averages of coordinates, and then use distance or area formulas to compute lengths and areas.

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4. Suppose a, b, c are three distinct natural numbers, such that $3ac = 8(a + b)$. Then, the smallest possible value of $3a + 2b + c$ is:

Correct Answer: —

SOLUTION:

Approach: Attack the cheapest building block. The cheapest legal piece is b: make $b = 1$ (its smallest natural value) and see what a, c must be. Then a one-line divisibility check pins the answer, no case ladder.

Step 1: Rewrite the constraint as

$$8b = a(3c - 8).$$

To minimise $S = 3a + 2b + c$, push every variable toward its floor. Try $b = 1$: then $a(3c - 8) = 8$.

Step 2: So a and $(3c - 8)$ are a factor pair of 8. List factor pairs $(a, 3c - 8)$ of 8 where $3c - 8$ is itself of the form $3c - 8$ for a natural c:

$3c - 8 = 1 \Rightarrow c = 3$, paired with $a = 8$ — gives $(a, b, c) = (8, 1, 3)$, $S = 24 + 2 + 3 = 29$.

$3c - 8 = 4 \Rightarrow c = 4$, paired with $a = 2$ — gives $(a, b, c) = (2, 1, 4)$, $S = 6$

$+ 2 + 4 = 12.$
$3c - 8 = 2$ or 8 give $c = \frac{10}{3}$ or $\frac{16}{3}$, not natural, so rejected.

Step 3: Both surviving triples have distinct naturals; the smaller sum is 12 from $(2, 1, 4)$. Could $b = 1$ be beaten? No — pushing b to its minimum already minimises the $2b$ term, and the $a = 2$ here is near the floor too, so any other configuration only adds.

$$S_{\min} = 3(2) + 2(1) + 4 = 12.$$

Final answer: 12.

Quick Tip: When you have a Diophantine equation (integer solutions) and need to minimize an expression, first express one variable in terms of the others, then systematically test small integer values under the constraints (like distinctness and positivity).

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5. The ratio of expenditures of Lakshmi and Meenakshi is $2:3$, and the ratio of income of Lakshmi to expenditure of Meenakshi is $6:7$. If excess of income over expenditure is saved by Lakshmi and Meenakshi, and the ratio of their savings is $4:9$, then the ratio of their incomes is:

(A) $7:8$

(B) $3:5$

(C) $2:1$

(D) $5:6$

Correct Answer: (B) $3:5$

SOLUTION:

Approach: Pin a clean money value on Meenakshi's expenditure to clear all the sevenths at once. Since $I_L : E_M = 6 : 7$, choosing $E_M = 7$ makes $I_L = 6$ a whole number, and every other quantity follows in integers.

Step 1: Let $E_M = 7$. Then from $I_L : E_M = 6 : 7$ we get $I_L = 6$.

Step 2: Expenditures are in ratio $2 : 3$, i.e. $E_L : E_M = 2 : 3$. With $E_M = 7$,

$$E_L = \frac{2}{3} \times 7 = \frac{14}{3}.$$

Step 3: Lakshmi's saving:

$$S_L = I_L - E_L = 6 - \frac{14}{3} = \frac{18 - 14}{3} = \frac{4}{3}.$$

Step 4: Savings are in ratio $4 : 9$, so

$$S_M = \frac{9}{4} S_L = \frac{9}{4} \times \frac{4}{3} = 3.$$

Meenakshi's income is then

$$I_M = E_M + S_M = 7 + 3 = 10.$$

Step 5: Income ratio:

$$I_L : I_M = 6 : 10 = 3 : 5.$$

Final answer: 3 : 5.

Quick Tip: When working with income–expenditure–saving problems:

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6. If $\log_{64} x^2 + \log_8 \sqrt{y} + 3\log_{512}(\sqrt{y}z) = 4$, where x, y and z are positive real numbers, then the minimum possible value of $(x + y + z)$ is:

(A) 24

(B) 36

(C) 96

(D) 48

Correct Answer: (D) 48

SOLUTION:

Approach: Use the change-of-base shortcut $\log_{2^k} N = \frac{\log_8 N}{\log_8 2^k}$ by working entirely in base 8 — all three given bases are powers of 2 that relate cleanly to $8 = 2^3$. Get the product condition, then instead of quoting AM-GM, argue the symmetric minimum directly.

Step 1: Convert everything to powers of 2 in one shot. Writing $a = \log_2 x,\ b = \log_2 y,\ c = \log_2 z$:

$$\log_{64} x^2 = \frac{2a}{6} = \frac{a}{3}, \quad \log_8 \sqrt{y} = \frac{b/2}{3} = \frac{b}{6}, \quad 3\log_{512}(\sqrt{y}\,z) = 3 \cdot \frac{b/2 + c}{9} = \frac{b/2 + c}{3}.$$

Step 2: Sum to 4:

$$\frac{a}{3}+\frac{b}{6}+\frac{b/2+c}{3}=4 \Rightarrow \frac{a}{3}+\frac{b}{6}+\frac{b}{6}+\frac{c}{3}=4 \Rightarrow \frac{a+b+c}{3}=4.$$

Hence $a+b+c=12$, i.e. $\log_2(xyz)=12$ and $xyz=4096$.

Step 3: Now minimise $x+y+z$ given the fixed product $xyz=4096$. For positive numbers with a fixed product, the sum is smallest when all three are equal (any imbalance — making one larger and another smaller while keeping the product fixed — increases the sum, since the function is convex). Setting $x=y=z=t$ gives $t^3=4096$, so $t=16$.

Step 4: Therefore the minimum sum is

$$x+y+z=3t=3\times 16=48.$$

Final answer: 48.

Quick Tip: When an equation in logarithms simplifies to a fixed product like $xyz=\text{constant}$, use the AM-GM inequality to find the minimum (or maximum) of sums such as $x+y+z$. Equality in AM-GM occurs when all the variables are equal.

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7. If $9^{x^2+2x-3}-4\left(3^{x^2+2x-2}\right)+27=0$, then the product of all possible values of x is:

(A) 2

(B) 4

(C) 10

(D) 20

Correct Answer: (D) 20

SOLUTION:

Approach: Spot that the equation is quadratic in 3^{x^2+2x} if you factor out the powers cleanly, but here is a slicker route: divide the whole equation by 27 to normalise it, which makes the hidden quadratic in $t = 3^{x^2+2x}$ pop out with small coefficients.

Step 1: Let $t = 3^{x^2+2x}$. Then, using $9^{x^2+2x} = t^2$ and pulling the constant exponents out as numerical factors:

$$9^{x^2+2x-3} = \frac{t^2}{9^3} = \frac{t^2}{729}, \qquad 3^{x^2+2x-2} = \frac{t}{9}.$$

The equation becomes

$$\frac{t^2}{729} - \frac{4t}{9} + 27 = 0.$$

Step 2: Multiply through by 729 and divide by the common structure:

$$t^2 - 324\,t + 19683 = 0.$$

Factor by looking for two numbers multiplying to $19683 = 3^9$ and adding to 324: those are $243 = 3^5$ and $81 = 3^4$. So

$$(t - 243)(t - 81) = 0 \;\Rightarrow\; t = 243 \text{ or } 81.$$

Step 3: Recall $t = 3^{x^2+2x}$:

$$3^{x^2+2x} = 3^5 \Rightarrow x^2 + 2x = 5, \qquad 3^{x^2+2x} = 3^4 \Rightarrow x^2 + 2x = 4.$$

Step 4: So $x^2 + 2x - 5 = 0$ (roots multiply to -5) and $x^2 + 2x - 4 = 0$ (roots multiply to -4). Both quadratics have positive discriminant, so all roots are real and admissible. Product of all possible x:

$$(-5)(-4) = 20.$$

Final answer: 20.

Quick Tip: When exponents share a common expression (like $x^2 + 2x$ here), substitute it with a single variable to simplify. Then, look for ways to convert to a common base and reduce the equation to a quadratic in a new variable such as 3^u.

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8. The average number of copies of a book sold per day by a shopkeeper is 60 in the initial seven days and 63 in the initial eight days, after the book launch. On the ninth day, she sells 11 copies less than the eighth day, and the average number of copies sold per day from the second day to the ninth day becomes 66. The number of copies sold on the first day of the book launch is:

Correct Answer: —

SOLUTION:

Approach: Skip the day-by-day totals and work with the two overlapping 8-day windows directly. Days 1 to 8 and days 2 to 9 share days 2 to 8, so their totals differ by exactly (day 9 minus day 1).

Step 1: Total of days 1 to 8 $= 8 \times 63 = 504$. Total of days 2 to 9 $= 8$

$\times 66 = 528$.

Step 2: Both windows contain days 2 to 8. Subtracting the common part, the difference between the windows is $(x_9 - x_1)$:

$$x_9 - x_1 = 528 - 504 = 24.$$

Step 3: Find x_9. The 8th day comes from the first two averages: $x_8 = 8 \times 63 - 7 \times 60 = 504 - 420 = 84$, so $x_9 = 84 - 11 = 73$.

Step 4: Now solve for the first day:

$$x_1 = x_9 - 24 = 73 - 24 = 49.$$

Final answer: $x_1 = 49$.

Quick Tip: When dealing with averages over overlapping time intervals, convert each average to a total sum. Then, use differences of these sums to find individual day values and set up equations to solve for the unknowns.

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9. A loan of Rs 1000 is fully repaid by two installments of Rs 530 and Rs 594, paid at the end of the first and second year, respectively. If the interest is compounded annually, then the rate of interest, in percentage, is:

(A) 6

(B) 7

(C) 8

(D) 9

Correct Answer: (C) 8

SOLUTION:

Approach: Don't solve a quadratic at all. CAT gives you four clean rate options, so just test them by tracking the outstanding balance year by year and check which one clears the loan exactly.

Step 1: The rule: each year the balance grows by the rate, then the instalment is subtracted. The correct rate leaves a zero balance after the second payment.

Step 2: Test $r = 8\%$. After year 1 the loan becomes $1000 \times 1.08 = 1080$. Pay Rs 530, leaving $1080 - 530 = 550$.

Step 3: That Rs 550 grows for the second year: $550 \times 1.08 = 594$. Pay the second instalment of Rs 594, leaving $594 - 594 = 0$.

Step 4: The balance vanishes exactly, so 8% is correct. (Quick sanity check: at 7%, $1000 \times 1.07 - 530 = 540$, then $540 \times 1.07 = 577.8 \neq 594$, so it fails.)

Final answer: 8%.

Quick Tip: Whenever loan repayments occur in installments, convert each installment into its present value and sum them. If the interest is compounded annually, the discount factor for the n-th year is $(1+r)^n$.

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10. The set of all real values of x for which $(x^2 - |x+9| + x) > 0$ is:

(A) $(-\infty, -9) \cup (3, \infty)$

(B) $(-\infty, -3) \cup (9, \infty)$

(C) $(-\infty, -3) \cup (3, \infty)$

(D) $(-9, -3) \cup (3, 9)$

Correct Answer: (C) $(-\infty, -3) \cup (3, \infty)$

SOLUTION:

Approach: Skip the casework and use the options as a sieve. Pick one boundary value and a couple of test points; whichever option's set agrees with the true sign of the expression at those points must be the answer.

Let $E(x) = x^2 - |x+9| + x$. We need $E(x) > 0$.

Test $x = 0$ (a point all options near the middle care about): $E(0) = 0 - 9 + 0 = -9 < 0$. So 0 must be excluded. Options 1, 3, 4 all exclude 0; option 2 excludes the interval $[-3, 9]$ which contains 0 — keep watching.

Test $x = -5$: $E(-5) = 25 - |4| - 5 = 25 - 4 - 5 = 16 > 0$. So -5 must be included. Option 1 excludes $[-9, 3]$ (rejects -5) — out. Option 2 includes -5 (its excluded zone is $[-3, 9]$) — survives. Option 3 includes -5 — survives. Option 4 excludes -5 — out.

Decide between options 2 and 3 using $x = 5$: $E(5) = 25 - 14 + 5$

$= 16 > 0$, so 5 must be included. Option 2 excludes 5 (its gap is $(-3, 9)$) — out. Option 3 includes 5 — it survives every test.

Final answer: $(-\infty, -3) \cup (3, \infty)$ — option 3.

Quick Tip: When solving inequalities involving absolute values, always split into cases based on the sign of the expression inside the absolute value. Analyze each case separately, then combine intervals carefully by intersection and union.

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11. The equations $3x^2 - 5x + p = 0$ and $2x^2 - 2x + q = 0$ have one common root. The sum of the other roots of these two equations is:

(A) $\frac{5}{3} - p + q$

(B) $\frac{8}{3} + p - q$

(C) $\frac{8}{3} - p + \frac{3}{2}q$

(D) $p + q - 1$

Correct Answer: (C) $\frac{8}{3} - p + \frac{3}{2}q$

SOLUTION:

Approach: The answer must hold for any valid p, q, so manufacture one concrete pair of equations with a known common root, compute the actual sum of the other roots, and see which option reproduces that number. This dodges all the abstract algebra.

Step 1: Force a common root, say $\alpha = 1$. Plug into the first equation: $3(1) - 5(1) + p = 0 \Rightarrow p = 2$. Plug into the second: $2(1) - 2(1) + q = 0$

$\Rightarrow q = 0$.

Step 2: First equation $3x^2 - 5x + 2 = 0$ has roots summing to $5/3$, so the other root $\beta = 5/3 - 1 = 2/3$. Second equation $2x^2 - 2x = 0$ has roots 0 and 1, so the other root $\gamma = 0$. Actual sum of other roots $= \frac{2}{3} + 0 = \frac{2}{3}$.

Step 3: Now evaluate each option at $p = 2,\ q = 0$ and match $2/3$: option 1 gives $5/3 - 2 = -1/3$; option 2 gives $8/3 + 2 = 14/3$; option 3 gives $8/3 - 2 + 0 = 2/3$ — match; option 4 gives $2 + 0 - 1 = 1$. Only option 3 hits $2/3$.

Final answer: $\frac{8}{3} - p + \frac{3}{2}q$ — option 3.

Quick Tip: When two quadratics share a common root, equate the root expressions by eliminating the squared term. Using Vieta's formulas then makes it easy to compute required expressions involving the other roots.

12. An item with a cost price of Rs.1650 is sold at a certain discount on a fixed marked price to earn a profit of 20% on the cost price. If the discount was doubled, the profit would have been Rs.110. The rate of discount, in percentage, at which the profit percentage would be equal to the rate of discount, is nearest to:

(A) 12

(B) 13

(C) 14

(D) 15

Correct Answer: (C) 14

SOLUTION:

Approach: Once you have the marked price (Rs 2200) and cost (Rs 1650), reframe the last condition as a ratio. Discount% acts on MP, profit% acts on CP, and they must be equal — so compare what one rupee of percentage buys on each side.

Step 1: Pin down MP. With 20% profit, $SP_1 = 1.2 \times 1650 = 1980$. Doubling the discount drops profit to Rs 110, i.e. $SP_2 = 1760$. The extra discount D caused a drop of $1980 - 1760 = 220$, so $D = 220$ and $MP = 1980 + 220 = 2200$.

Step 2: Let the equal rate be x%. Reading SP two ways: from the marked side $SP = MP(1 - x/100) = 2200 - 22x$; from the cost side $SP = CP(1 + x/100) = 1650 + 16.5x$.

Step 3: Set the two expressions for SP equal:

$$2200 - 22x = 1650 + 16.5x.$$

So $2200 - 1650 = 22x + 16.5x$, giving $550 = 38.5x$ and

$$x = \frac{550}{38.5} = \frac{100}{7} \approx 14.29.$$

Step 4: The nearest whole number is 14.

Final answer: 14 — option 3.

Quick Tip: When the same marked price is used under different discount and profit conditions, set up equations using $SP = MP - \text{Discount}$ and $SP = CP + \text{Profit}$ for each scenario. Once the marked price is known, you can introduce a variable discount rate and equate the profit percentage to that rate to solve such "rate equals rate" problems.

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13. A certain amount of money was divided among Pinu, Meena, Rinu, and Seema. Pinu received 20% of the total amount and Meena received 40% of the remaining amount. If Seema received 20% less than Pinu, the ratio of the amounts received by Pinu and Rinu is:

(A) 4 : 5

(B) 5 : 8

(C) 3 : 5

(D) 2 : 3

Correct Answer: (B) 5 : 8

SOLUTION:

Approach: Keep the total as a variable x and chain the fractions of x symbolically; Rinu is the closure of the equation (everything must add to x). This shows the answer is independent of the actual amount.

Step 1: Pinu $= 0.2x$. The amount left after Pinu is $0.8x$, and Meena takes 40% of that: Meena $= 0.4 \times 0.8x = 0.32x$.

Step 2: Seema is 20% below Pinu, so Seema $= 0.8 \times 0.2x = 0.16x$.

Step 3: Since the four shares exhaust the whole amount,

$$\text{Rinu} = x - (0.2x + 0.32x + 0.16x) = x - 0.68x = 0.32x.$$

Step 4: The x cancels in the ratio:

$$\frac{\text{Pinu}}{\text{Rinu}} = \frac{0.2x}{0.32x} = \frac{20}{32} = \frac{5}{8}.$$

Final answer: 5 : 8 — option 2.

Quick Tip: In distribution problems, it often helps to assume a convenient total (like 100) when only percentages are involved. This makes computations easy and does not affect the final ratios.

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14. Let $f(x) = \dfrac{x}{2x-1}$ and $g(x) = \dfrac{x}{x-1}$. Then, the domain of the function

$$h(x) = f(g(x)) + g(f(x))$$

is all real numbers except:

(A) $\frac{1}{2}, 1, \frac{3}{2}$

(B) $\frac{1}{2}, 1$

(C) $-\frac{1}{2}, \frac{1}{2}, 1$

(D) $-1, \frac{1}{2}, 1$

Correct Answer: (D) $-1, \frac{1}{2}, 1$

SOLUTION:

Approach: Instead of hunting forbidden points abstractly, simplify each composite into a single clean fraction. The denominator of the simplified form, together with the inner restriction, exposes exactly which x-values are banned.

Step 1 — simplify $f(g(x))$: With $g(x) = \dfrac{x}{x-1}$,

$$f(g(x)) = \frac{g}{2g-1} = \frac{\frac{x}{x-1}}{\frac{2x}{x-1}-1} = \frac{x}{2x-(x-1)} = \frac{x}{x+1}.$$

This simplified form bans $x = -1$; and the inner step used $x \neq 1$ (else g itself is undefined). So $f(g(x))$ excludes $x = -1$ and $x = 1$.

Step 2 — simplify $g(f(x))$: With $f(x) = \dfrac{x}{2x-1}$,

$$g(f(x)) = \frac{f}{f-1} = \frac{\frac{x}{2x-1}}{\frac{x}{2x-1}-1} = \frac{x}{x-(2x-1)} = \frac{x}{1-x}.$$

This bans $x = 1$; and the inner step used $x \neq \frac{1}{2}$ (else f is undefined). So $g(f(x))$ excludes $x = \frac{1}{2}$ and $x = 1$.

Step 3 — union of all bans: $\{-1,\ 1\} \cup \{\frac{1}{2},\ 1\} = \{-1,\ \frac{1}{2},\ 1\}$.

Final answer: All reals except $-1,\ \dfrac{1}{2},\ 1$ — option 4.

Quick Tip: When dealing with compositions of rational functions, 1.~Exclude values that make any denominator zero, and 2.~Also exclude values that make the inner function produce an invalid input for the outer function. Always combine all such restrictions at the end.

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15. The number of divisors of $(2^6 \times 3^5 \times 5^3 \times 7^2)$, which are of the form $(3r+1)$, where r is a non-negative integer, is:

(A) 42

(B) 36

(C) 56

(D) 24

Correct Answer: (A) 42

SOLUTION:

Approach: Instead of parity-counting, build the answer multiplicatively. Group the divisors as (part from $2^a 5^c$) $\times$ (part from $3^b 7^d$) and track each piece's residue mod 3.

Step 1: A divisor is $2^a 3^b 5^c 7^d$. Mod 3 the primes reduce to $2 \equiv 2,\ 3 \equiv 0,\ 5 \equiv 2,\ 7 \equiv 1$. Any factor of 3 kills the residue, so $b = 0$ is compulsory and $7^d \equiv 1$ always.

Step 2: So the residue depends only on $2^a 5^c \equiv 2^{a+c} \pmod 3$. The powers of 2 cycle $2^0 \equiv 1,\ 2^1 \equiv 2,\ 2^2 \equiv 1, \ldots$ i.e. $2^{a+c} \equiv 1$ when $a+c$ is even, $\equiv 2$ when odd.

Step 3 (clean counting trick): Total divisors with $b = 0$ are $(7 \text{ values of } a) \times (4 \text{ values of } c) = 28$, ignoring d. Among these I need $a + c$ even. List by a-parity: a even (4 ways) needs c even (2 ways) $= 8$; a odd (3 ways) needs c odd (2 ways) $= 6$. So 14 of the 28 qualify.

Step 4: Re-attach the free 7^d with $d \in \{0, 1, 2\}$, each keeping the residue 1: multiply by 3.

$$14 \times 3 = 42.$$

Answer: 42.

Quick Tip: When counting divisors with a condition like "of the form $3r + 1$", work modulo 3:

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16. The sum of digits of the number $(625)^{65} \times (128)^{36}$ is:

Correct Answer: —

SOLUTION:

Approach: Same idea, but pull out the factor of 10^{252} first using a single power-balancing step, then just square 625.

Step 1: $N = 625^{65} \times 128^{36} = 5^{260} \times 2^{252}$. Notice the powers of 2 and 5 differ by exactly $260 - 252 = 8$ in favour of 5.

Step 2: Factor out the common power $\min(260, 252) = 252$ from both:

$$N = (2 \cdot 5)^{252} \times 5^8 = 10^{252} \times 5^8.$$

Multiplying by 10^{252} only appends zeros, so it cannot change the digit sum.

Step 3: So the digit sum of N equals the digit sum of 5^8. Build it up: 5^2

$= 25,\ 5^4 = 625,\ 5^8 = 625 \times 625 = 390625.$

Step 4: Add the digits: $3 + 9 + 0 + 6 + 2 + 5 = 25.$

Answer: 25.

Quick Tip: When multiplying large powers of 2 and 5, group them into powers of 10. This isolates a manageable non-zero block of digits followed by many zeros, simplifying digit-sum problems.

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17. Ankita is twice as efficient as Bipin, while Bipin is twice as efficient as Chandan. All three of them start together on a job, and Bipin leaves the job after 20 days. If the job got completed in 60 days, the number of days needed by Chandan to complete the job alone, is:

(A) 240

(B) 260

(C) 300

(D) 340

Correct Answer: (D) 340

SOLUTION:

Approach: Assign total work as a convenient LCM-style number so the rates are whole, then solve for the job size from the two phases directly.

Step 1: Take efficiencies in the ratio Ankita : Bipin : Chandan $= 4 : 2 : 1$. Let the whole job be W units (unknown for now), with Chandan's rate = c units/day, Bipin = $2c$, Ankita = $4c$.

Step 2: Work in phase 1 (all three, 20 days) $= 20(4c + 2c + c) = 140c$. Work in phase 2 (Ankita + Chandan, 40 days) $= 40(4c + c) = 200c$.

Step 3: The whole job is done: $140c + 200c = W$, so $W = 340c$.

Step 4: Chandan alone works at c units/day, so time $= \dfrac{W}{c} = \dfrac{340c}{c}$ $= 340$ days. The unknown unit c cancels, which is exactly why we never needed its actual value.

Answer: 340 days.

Quick Tip:

In work and time problems with different efficiencies, it is often easiest to: Assume one person's efficiency as a variable, Express others' efficiencies in terms of that variable using the given ratios, Compute total work done, then divide by an individual’s efficiency for “alone” time.

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18. If m and n are integers such that $(m + 2n)(2m + n) = 27$, then the maximum possible value of $2m - 3n$ is:

(A) 9

(B) 13

(C) 15

(D) 17

Correct Answer: (D) 17

SOLUTION:

Approach: Express the target $2m - 3n$ as a linear combination of the two factors, so we maximize it without ever solving for m, n separately.

Step 1: Let $u = m + 2n$ and $v = 2m + n$, with $uv = 27$. Add and subtract: $u + v = 3m + 3n = 3(m + n)$ and $v - u = m - n$. So $m = \frac{2v - u}{3}$ and $n = \frac{2u - v}{3}$.

Step 2: Then $2m - 3n = \frac{2(2v - u) - 3(2u - v)}{3} = \frac{4v - 2u - 6u + 3v}{3} = \frac{7v - 8u}{3}$. To maximize this with $uv = 27$, we want v large and positive and u negative.

Step 3: Integer factor pairs of 27 that also keep m, n integers (need $7v - 8u$ divisible by 3): testing $(u, v) = (-9, -3)$ gives $\frac{7(-3) - 8(-9)}{3} = \frac{-21 + 72}{3} = \frac{51}{3} = 17$. Testing $(u, v) = (3, 9)$ gives $\frac{63 - 24}{3} = 13$; $(-3, -9)$ gives $\frac{-63 + 24}{3} = -13$; $(9, 3)$ gives $\frac{21 - 72}{3} = -17$.

Step 4: The largest value is 17, at $(u, v) = (-9, -3)$, i.e. $m = 1,\ n = -5$.

Answer: 17.

Quick Tip: When given a product condition like $(m + 2n)(2m + n) = 27$ with integer solutions,

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19. In a $\triangle ABC$, points D and E are on the sides BC and AC, respectively. BE and AD intersect at point T such that $AD : AT = 4 : 3$, and $BE : BT = 5 : 4$. Point F lies on AC such that DF is parallel to BE. Then, $BD : CD$ is:

(A) $15 : 4$

(B) $11 : 4$

(C) $9 : 4$

(D) $7 : 4$

Correct Answer: (B) $11 : 4$

SOLUTION:

Approach: Drop the synthetic geometry and use coordinates. Place the triangle conveniently, write D and E with unknown ratios, force T to sit on both cevians with the given splits, and solve.

Step 1: Let $A = (0, 0),\ B = (1, 0),\ C = (0, 1)$. Put D on BC with $BD : DC = t : (1 - t)$, so $D = (1 - t,\ t)$. Put E on AC with $AE : EC = s : (1 - s)$, so $E = (0,\ s)$.

Step 2: T is on AD with $AT : TD = 3 : 1$, i.e. $AT = \frac{3}{4}AD$:

$$T = A + \tfrac{3}{4}(D - A) = \left(\tfrac{3}{4}(1 - t),\ \tfrac{3}{4}t\right).$$

Step 3: T is also on BE with $BT : TE = 4 : 1$, i.e. $BT = \frac{4}{5}BE$:

$$T = B + \tfrac{4}{5}(E - B) = \left(1 - \tfrac{4}{5},\ \tfrac{4}{5}s\right) = \left(\tfrac{1}{5},\ \tfrac{4}{5}s\right).$$

Step 4: Match x-coordinates: $\frac{3}{4}(1-t) = \frac{1}{5} \Rightarrow 1-t = \frac{4}{15} \Rightarrow t = \frac{11}{15}$.

Step 5: Therefore $BD : DC = t : (1-t) = \frac{11}{15} : \frac{4}{15} = 11 : 4$. (Matching y-coordinates gives $s = \frac{11}{16}$, consistent, but we already have the answer.)

Answer: $BD : CD = 11 : 4$.

Quick Tip: When internal cevians intersect (like AD and BE meeting at T) and you need a side ratio, Menelaus' Theorem on carefully chosen triangles with transversals through T can be more direct than coordinate or mass-point geometry.

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20. A mixture of coffee and cocoa, 16% of which is coffee, costs Rs 240 per kg. Another mixture of coffee and cocoa, of which 36% is coffee, costs Rs 320 per kg. If a new mixture of coffee and cocoa costs Rs 376 per kg, then the quantity, in kg, of coffee in 10 kg of this new mixture is:

(A) 2.5

(B) 5

(C) 4

(D) 6

Correct Answer: (B) 5

SOLUTION:

Approach: Skip pure-ingredient prices entirely. Treat the two given mixtures themselves as the two ingredients and apply alligation on price to find how to blend them into a Rs 376 mixture; the coffee fraction then follows from blending their coffee percentages.

Step 1: Call mixture 1 (16% coffee, Rs 240) cheaper, and mixture 2 (36% coffee, Rs 320) costlier. We want a blend costing Rs 376 — note this is **dearer than both**, so a plain positive blend of the two cannot reach 376. That signals the new mixture is not a mix of these two; instead use them only to find pure prices via alligation logic.

Step 2: Alligation between the two mixtures gives the price gap per 20% coffee: going from 16% to 36% coffee (a rise of 20%) raises price by $320 - 240 = 80$. So each 1% of coffee adds $\frac{80}{20} = 4$ Rs/kg.

Step 3: Extrapolate linearly: price as a function of coffee percentage p is $\text{price} = 240 + 4(p - 16)$. Set this to 376: $240 + 4(p - 16) = 376 \Rightarrow 4(p - 16) = 136 \Rightarrow p - 16 = 34 \Rightarrow p = 50$.

Step 4: So the new mixture is 50% coffee. In 10 kg: $0.50 \times 10 = 5$ kg of coffee.

Answer: 5 kg.

Quick Tip: When mixtures of the same two ingredients are given with different compositions and costs, set up linear equations using the percentage of each ingredient and solve for the individual prices. Then use those prices to find the composition of any new mixture.

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21. Rita and Sneha can row a boat at 5 km/h and 6 km/h in still water, respectively. In a river flowing with a constant velocity, Sneha takes 48 minutes more to row 14 km upstream than to row the same distance downstream. If Rita starts from a certain location in the river, and returns downstream to the same location, taking a total of 100 minutes, then the total distance, in km, Rita will cover is:

Correct Answer: —

SOLUTION:

Approach: Same physics, but get the current from a product-of-speeds shortcut, then get Rita's distance from her average round-trip speed (harmonic mean) instead of summing two times.

Step 1: For Sneha, the time difference for a fixed distance $L = 14$ satisfies $L\left(\frac{1}{6-c} - \frac{1}{6+c}\right) = 0.8$, i.e. $\frac{14 \cdot 2c}{36 - c^2} = 0.8$. Clearing: $28c = 28.8 - 0.8c^2 \Rightarrow 0.8c^2 + 28c - 28.8 = 0$, and dividing by 0.8, $c^2 + 35c - 36 = 0$, so $c = 1$ (rejecting $c = -36$).

Step 2: Rita's two speeds are $5 - 1 = 4$ (up) and $5 + 1 = 6$ (down). For an equal-distance up-and-down trip, her overall average speed is the **harmonic mean**:

$$v_{\text{avg}} = \frac{2 \cdot 4 \cdot 6}{4 + 6} = \frac{48}{10} = 4.8 \text{ km/h}.$$

Step 3: Total time is $100 \text{ min} = \frac{5}{3}$ h, so total distance $= v_{\text{avg}} \times \text{time} = 4.8 \times \frac{5}{3} = \frac{24}{3} = 8$ km.

Step 4: Quick check: that is 4 km up at 4 km/h ($= 1$ h $= 60$ min) and 4 km down at 6 km/h (= 40 min), totalling 100 min. Consistent.

Answer: 8 km.

Quick Tip: In upstream–downstream problems:

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22. If a, b, c and d are integers such that their sum is 46, then the minimum possible value of $(a-b)^2 + (a-c)^2 + (a-d)^2$ is:

Correct Answer: —

SOLUTION:

Approach: Argue a hard lower bound first, then hit it with one example. The squares can only be 0 when a number equals a; a parity argument shows you cannot get three zeros.

Step 1: Lower bound from parity.
If all three squares were 0, then $b = c = d = a$, giving sum $4a = 46$. But 46 is not a multiple of 4, so this is impossible. Hence at least one of b, c, d is unequal to a, and that square is at least 1.

Step 2: Can exactly one square be non-zero?
Suppose only $d \neq a$, so $b = c = a$. Then sum $= 3a + d = 46$. With the single square $(a-d)^2 = 1$, we need $d = a \pm 1$. Take $d = a + 1$: then $3a + a + 1 = 46 \Rightarrow 4a = 45$, not an integer. Take $d = a - 1$: $4a - 1 = 46 \Rightarrow 4a = 47$, not an integer. So a single gap of 1 is impossible — we need at least two of the squares to be ≥ 1.

Step 3: Therefore the sum is at least 2.

At least two squares are ≥ 1, so the expression ≥ 2.

Step 4: Achieve 2.

Pick $a = 12,\ b = 12,\ c = 11,\ d = 11$: sum $= 46$ and the expression $= 0 + 1 + 1 = 2$.

Hence the minimum is 2.

Quick Tip: For minimizing sums of squares with a fixed sum constraint, keep the numbers as close together as possible. Here, that meant taking a, b, c, d near the average $\frac{46}{4} = 11.5$, and distributing small integer deviations evenly.