CAT 2025 QA Slot 1 Question Paper with Solutions

General Instructions

(i) This booklet contains 22 questions, each provided with a complete, step-by-step solution.

(ii) It comprises 14 single-correct multiple-choice questions and 8 numerical / integer-type questions.

(iii) Attempt each question on your own before reviewing the given solution.

(iv) For numerical questions, report the answer rounded exactly as asked.

1. Kamala divided her investment of Rs 100000 between stocks, bonds, and gold. Her investment in bonds was 25% of her investment in gold. With annual returns of 10%, 6%, 8% on stocks, bonds, and gold, respectively, she gained a total amount of Rs 8200 in one year. The amount, in rupees, that she gained from the bonds, was:

Correct Answer: —

SOLUTION:

Approach: Skip the third variable entirely. Compare the actual return with a baseline where the whole 1 lakh earns the gold rate of 8%; the gaps come only from stocks (above) and bonds (below).

Step 1: Baseline at 8%.
If all Rs 100000 earned 8%, the gain would be $0.08 \times 100000 = 8000$.
The real gain is 8200, i.e. 200 extra.

Step 2: Account for the deviations.
Let bonds $ = B$, gold $ = 4B$ (since bonds are 25% of gold), so

stocks $ = 100000 − 5B$. Gold already earns the baseline $8%$, contributing nothing extra. Stocks earn $10%$ — that is $2%$ above baseline. Bonds earn $6%$ — that is $2%$ below baseline.

$$+0.02 \times (100000 - 5B) - 0.02 \times B = 200.$$

Step 3: Solve.

$$2000 - 0.10B - 0.02B = 200 \Rightarrow 0.12B = 1800 \Rightarrow B = 15000.$$

Step 4: Bond gain.

Bonds earn $6%$ of $15000 = 900$.

Gain from bonds = Rs 900.

Quick Tip: When a mixture investment problem involves fixed returns and proportional allocations, reduce variables using the percentage relationships first. This simplifies the system into a single solvable equation.

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2. If $a - 6b + 6c = 4$ and $6a + 3b - 3c = 50$, where a, b and c are real numbers, the value of $2a + 3b - 3c$ is:

(A) 18

(B) 20

(C) 15

(D) 14

Correct Answer: (A) 18

SOLUTION:

Approach: Build the target as a weighted sum of the two given equations. Find multipliers p, q so that $p\,(\mathrm{Eq1}) + q\,(\mathrm{Eq2})$ reproduces $2a + 3b - 3c$ exactly — then the answer is just $4p + 50q$, no need to find a, b, c at all.

Step 1: Match coefficients.

Eq1 is $a - 6b + 6c$; Eq2 is $6a + 3b - 3c$. We want

$$p(a - 6b + 6c) + q(6a + 3b - 3c) = 2a + 3b - 3c.$$

Coefficient of b: $-6p + 3q = 3$.
Coefficient of c: $6p - 3q = -3$ (same equation, consistent).
Coefficient of a: $p + 6q = 2$.

Step 2: Solve for p, q.

From $-6p + 3q = 3$: $q = 1 + 2p$. Substitute into $p + 6q = 2$:

$$p + 6(1 + 2p) = 2 \Rightarrow 13p + 6 = 2 \Rightarrow p = -\tfrac{4}{13},\ q = 1 + 2p = \tfrac{5}{13}.$$

Step 3: Combine the right-hand sides.

$$2a + 3b - 3c = 4p + 50q = 4\left(-\tfrac{4}{13}\right) + 50\left(\tfrac{5}{13}\right) = \tfrac{-16+250}{13} = \tfrac{234}{13} = 18.$$

Value = 18 — option (1).

Quick Tip: When several linear expressions repeat with the same structure (like $3b - 3c$ here), use substitution to reduce the system to two variables. This often simplifies the equations dramatically.

3. The (x, y) coordinates of vertices P, Q and R of a parallelogram $PQRS$ are $(-3, -2)$, $(1, -5)$ and $(9, 1)$, respectively. If the diagonal SQ intersects the x-axis at $(a, 0)$, then the value of a is:

(A) $\frac{29}{9}$

(B) $\frac{27}{9}$

(C) $\frac{29}{7}$

(D) $\frac{9}{29}$

Correct Answer: (A) $\frac{29}{9}$

SOLUTION:

Approach: Once we know $S = (5, 4)$ and $Q = (1, -5)$, use the section-formula directly. The x-axis cuts SQ at the point where the y-coordinate becomes 0; split SQ in the ratio that kills y, then read off x. No line equation needed.

Step 1: Locate S by vector shift.

In parallelogram $PQRS$, $\vec{PQ} = \vec{SR}$, so $S = R - (Q - P) = R + P - Q$. Then $S = (9, 1) + (-3, -2) - (1, -5) = (5, 4)$.

Step 2: Divide SQ so that $y = 0$.

Let the x-axis point divide $S(5, 4)$ and $Q(1, -5)$ in ratio $k : 1$ from S. Its y-coordinate is

$$\frac{k(-5)+1(4)}{k+1}=0 \Rightarrow -5k+4=0 \Rightarrow k=\tfrac{4}{5}.$$

Step 3: Get the \$x\$-coordinate at that ratio.

$$a=\frac{k(1)+1(5)}{k+1}=\frac{\frac{4}{5}+5}{\frac{4}{5}+1}=\frac{\frac{29}{5}}{\frac{9}{5}}=\frac{29}{9}.$$

\$$a=\frac{29}{9}$\$ — option (1).

Quick Tip: In any parallelogram, diagonals always bisect each other. So, to find missing vertices, equate midpoints of the diagonals. This avoids unnecessary vector calculations and simplifies coordinate geometry problems.

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4. At a certain simple rate of interest, a given sum amounts to Rs 13920 in 3 years, and to Rs 18960 in 6 years and 6 months. If the same given sum had been invested for 2 years at the same rate as before but with interest compounded every 6 months, then the total interest earned, in rupees, would have been nearest to:

(A) 3096

(B) 3221

(C) 3180

(D) 3150

Correct Answer: (B) 3221

SOLUTION:

Approach: Same $P = 9600$ and $R = 15\%$, but build the compound interest by successive-period growth, adding 7.5% four times and tracking each half-year balance. This avoids the fourth-power expansion and shows where each rupee of interest comes from.

Step 1: Recover P and R.
SI per year $= (18960 - 13920)/3.5 = 1440$. Principal $= 13920 - 3(1440) = 9600$. Rate $= 1440/9600 = 15\%$. Half-yearly rate $= 7.5\%$, i.e. multiply by 1.075 each six months.

Step 2: Grow the balance four times.
Half-year 1: $9600 \times 1.075 = 10320$.
Half-year 2: $10320 \times 1.075 = 11094$.
Half-year 3: $11094 \times 1.075 = 11926.05$.
Half-year 4: $11926.05 \times 1.075 \approx 12820.5$.

Step 3: Interest $=$ final balance $-$ principal.

$$12820.5 - 9600 = 3220.5 \approx 3221.$$

Total interest $\approx$ Rs 3221 — option (2).

Quick Tip: When you know the amounts at two different times under simple interest, the difference of amounts directly gives you the interest for the extra period. From that, you can easily find the yearly interest, then the rate and principal, and finally plug those into a compound interest calculation.

5. Let $3 \leq x \leq 6$ and $[x^2] = [x]^2$, where $[x]$ is the greatest integer not exceeding x. If set S represents all feasible values of x, then which of the following is a possible subset of S?

(A) $(3, \sqrt{10}) \cup [5, \sqrt{26}) \cup \{6\}$

(B) $(4, \sqrt{10}) \cup [5, \sqrt{27}) \cup \{6\}$

(C) $[3, \sqrt{10}] \cup [5, \sqrt{26}]$

(D) $[3, \sqrt{10}] \cup [4, \sqrt{17}] \cup \{6\}$

Correct Answer: (A) $(3, \sqrt{10}) \cup [5, \sqrt{26}) \cup \{6\}$

SOLUTION:

Approach: Instead of building S first, test the four options directly with the right-edge trap. The equality $[x^2] = [x]^2$ breaks the instant x^2 reaches the next integer above $[x]^2$; so any option containing such a boundary point is disqualified. Eliminate, and one survives.

Step 1: The danger points.
On strip with integer floor n, the condition holds only for $n \leq x < \sqrt{n^2+1}$. The closed upper ends $\sqrt{n^2+1}$ (e.g. $\sqrt{10}, \sqrt{17}, \sqrt{26}$) are NOT allowed, because there x^2 hits n^2+1 and $[x^2] = n^2 + 1 \neq n^2$.

Step 2: Knock out bad options.
Option (3) $[3, \sqrt{10}] \cup [5, \sqrt{26}]$: includes $\sqrt{10}$ and $\sqrt{26}$ — both forbidden. Out.
Option (4) $[3, \sqrt{10}] \cup [4, \sqrt{17}] \cup \{6\}$: includes $\sqrt{10}, \sqrt{17}$ — forbidden. Out.
Option (2) $(4, \sqrt{10}) \cup [5, \sqrt{27}) \cup \{6\}$: $\sqrt{27} > \sqrt{26}$, so $x \in (\sqrt{26}, \sqrt{27})$ gives $[x^2] = 26$ but $[x]^2 = 25$ — those x are not in S. Out. (Also $(4, \sqrt{10})$ is incoherent since $\sqrt{10} < 4$.)

Step 3: Confirm the survivor.

Option (1) $(3, \sqrt{10}) \cup [5, \sqrt{26}) \cup \{6\}$: open-left at 3 is fine (3 itself is also fine, but a subset need not include it), the upper ends are open at $\sqrt{10}, \sqrt{26}$, and $6 \in S$. Every element satisfies the equality.

Answer: option (1).

Quick Tip: For equations involving the floor function, always:

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6. The number of non-negative integer values of k for which the quadratic equation $x^2 - 5x + k = 0$ has only integer roots, is:

Correct Answer: —

SOLUTION:

Approach: Force the discriminant to be a perfect square. Integer roots need $25 - 4k$ to be a perfect square AND the numerator $5 \pm \sqrt{25 - 4k}$ even; scan the few allowed k.

Step 1: Bound k.

Roots $x = \dfrac{5 \pm \sqrt{25 - 4k}}{2}$ are real only if $25 - 4k \geq 0$, i.e. $k \leq 6.25$. With k a non-negative integer: $k \in \{0, 1, 2, 3, 4, 5, 6\}$.

Step 2: Which give a perfect-square discriminant?

$D = 25 - 4k$: for $k = 0, 1, 2, 3, 4, 5, 6$ we get $25, 21, 17, 13, 9, 5, 1$. Perfect squares: $25\,(k{=}0)$, $9\,(k{=}4)$, $1\,(k{=}6)$.

Step 3: Parity check on the numerator.
$\sqrt{D}$ is $5, 3, 1$ respectively — all odd, so $5 \pm \sqrt{D}$ is even, and dividing by 2 gives integers. All three pass: $k = 0 \to \{0, 5\}$, $k = 4 \to \{1, 4\}$, $k = 6 \to \{2, 3\}$.

Number of values = 3.

Quick Tip: For quadratics with integer coefficients, to ensure integer roots:

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7. A shopkeeper offers a discount of 22 on the marked price of each chair, and gives 13 chairs to a customer for the discounted price of 12 chairs to earn a profit of 26 on the transaction. If the cost price of each chair is Rs 100, then the marked price, in rupees, of each chair is:

Correct Answer: —

SOLUTION:

Approach: Work entirely on a per-chair basis using the "13-for-12" idea as an effective extra discount. The customer pays the 12-chair amount but receives 13 chairs, so the effective rate per chair handed over carries the profit margin directly.

Step 1: Effective selling price per chair handed over. The shopkeeper hands over 13 chairs and is paid for 12 at $0.78M$ each. So the effective SP per chair is

$$\frac{12 \times 0.78M}{13} = \frac{9.36M}{13}.$$

Step 2: Use the per-chair profit relation. Cost per chair is 100 and profit is 26%, so each chair must effectively sell for

$$100 \times 1.26 = 126.$$

Therefore

$$\frac{9.36M}{13} = 126.$$

Step 3: Solve.

$$9.36M = 126 \times 13 = 1638 \implies M = \frac{1638}{9.36} = 175.$$

Same answer, reached by thinking in rupees-per-chair rather than totals.

$$\boxed{M = 175}$$

Quick Tip: When a shopkeeper gives more items than charged (like "13 for the price of 12"), treat all given items as contributing to total cost but only the charged items as contributing to revenue.

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8. A cafeteria offers 5 types of sandwiches. Moreover, for each type of sandwich, a customer can choose one of 4 breads and opt for either small or large sized sandwich. Optionally, the customer may also add up to 2 out of 6 available sauces. The number of different ways in which an order can be placed for a sandwich, is:

(A) 600

(B) 840

(C) 880

(D) 800

Correct Answer: (C) 880

SOLUTION:

Approach: Replace the $\binom{6}{0}+\binom{6}{1}+\binom{6}{2}$ count with a per-sauce on/off model, restricted to at most 2 chosen. This shows where the number 22 really comes from.

Step 1: Fixed part. Type, bread and size give $5 \times 4 \times 2 = 40$ base orders.

Step 2: Count sauce subsets of size at most 2. Each of the 6 sauces is either in or out, but we keep only subsets with 0, 1 or 2 sauces.
Empty subset: 1.
Exactly one sauce: pick the single sauce, 6 ways.
Exactly two sauces: pick an unordered pair, $\frac{6\times5}{2} = 15$ ways.
Total valid sauce subsets: $1 + 6 + 15 = 22$.

Step 3: Multiply.

$$40 \times 22 = 880.$$

Counting the pair as $\frac{6\times5}{2}$ (not 6×5) reminds us the two chosen sauces are interchangeable.

$$\boxed{880}$$

confirming option (3).

Quick Tip: When a question says "up to k items" from n options, sum the combinations:

$$\sum_{r=0}^{k}\binom{n}{r}.$$

Then multiply by the number of ways for all other independent choices.

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9. The number of distinct integers n for which $\log_{\left(\frac{1}{4}\right)}(n^2 - 7n + 11) > 0$ is:

(A) 1

(B) 2

(C) 20

(D) infinite

Correct Answer: (B) 2

SOLUTION:

We must solve:

$$\log_{\left(\frac{1}{4}\right)}(n^2 - 7n + 11) > 0.$$

Two conditions must hold: --- **Step 1:** Domain condition The logarithm's argument must be positive:

$$n^2 - 7n + 11 > 0.$$

Solve the quadratic inequality. Roots:

$$n = \frac{7 \pm \sqrt{49 - 44}}{2} = \frac{7 \pm \sqrt{5}}{2} \approx 2.38,\ 4.62.$$

Since the parabola opens upward:

$$n < \frac{7 - \sqrt{5}}{2} \quad \text{or} \quad n > \frac{7 + \sqrt{5}}{2}.$$

Thus the integer values allowed by the domain are:

$$\{\dots, 0, 1, 2\} \cup \{5, 6, 7, \dots\}.$$

Step 2: Inequality from the logarithm Since the base $\frac{1}{4}$ is less than 1, the inequality reverses when removing the log:

$$\log_{\left(\frac{1}{4}\right)}(A) > 0 \quad \iff \quad A < 1.$$

Thus:

$$n^2 - 7n + 11 < 1,$$

$$n^2 - 7n + 10 < 0,$$

$$(n - 2)(n - 5) < 0.$$

Thus:

$$2 < n < 5,$$

with integer candidates:

$$n = 3, 4.$$

Step 3: Combine with domain condition We test whether $n = 3, 4$ satisfy the \emph{positivity} of the argument:

$$f(n) = n^2 - 7n + 11.$$

$$f(3) = 9 - 21 + 11 = -1 \quad \text{(invalid)}.$$

$$f(4) = 16 - 28 + 11 = -1 \quad \text{(invalid)}.$$

Thus no integer satisfies both conditions if the inequality is strict > 0. --- **Interpretation used in the answer key** The official answer key indicates that the intended inequality was effectively:

$$\log_{\left(\frac{1}{4}\right)}(n^2 - 7n + 11) \geq 0.$$

This gives:

$$n^2 - 7n + 11 \leq 1,$$

$$n^2 - 7n + 10 \leq 0,$$

$$(n - 2)(n - 5) \leq 0,$$

$$2 \leq n \leq 5.$$

Testing domain condition: - $n = 2 : f(2) = 1 > 0 \Rightarrow \log = 0$ (valid) - $n = 3 : f(3) = -1$ invalid - $n = 4 : f(4) = -1$ invalid - $n = 5 : f(5) = 1 > 0 \Rightarrow \log = 0$ (valid) Valid integers:

$$n = 2, 5.$$

Thus there are exactly:

$\boxed{2}$

solutions.

Quick Tip: For logarithms with base between 0 and 1, the inequality reverses when removing the logarithm. Always check both: 1. Domain ($A > 0$), 2. Result of transformed inequality. Boundary cases where the argument becomes 1 often produce equality of the log to zero.

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10.

For any natural number k, let $a_k = 3^k$. The smallest natural number m for which

$$(a_1)^1 \times (a_2)^2 \times \cdots \times (a_{20})^{20} < a_{21} \times a_{22} \times \cdots \times a_{20+m}$$

is:

(A) 58

(B) 59

(C) 57

(D) 56

Correct Answer: (A) 58

SOLUTION:

Approach: Instead of summing the RHS as one AP, think of it as "how many more terms of the 3-power product do we need to overtake the left side" and frame the comparison as a difference that must turn positive. Solve the resulting quadratic by completing the comparison rather than the discriminant formula.

Step 1: Reduce to exponents. Taking $\log_3$, the LHS exponent is $\sum_{k=1}^{20} k^2$ $= \frac{20\cdot 21\cdot 41}{6} = 2870$, and the RHS exponent is $21 + 22 + \cdots + (20 + m)$ $= \frac{m(m+41)}{2}$. We need

$$\frac{m(m+41)}{2} > 2870 \iff m^2 + 41m - 5740 > 0.$$

Step 2: Estimate the threshold by a near-square. $m^2 + 41m$ is close to $(m + 20.5)^2$. We want $(m + 20.5)^2 > 5740 + 20.5^2 = 5740 + 420.25$ $= 6160.25$. Since $\sqrt{6160.25} \approx 78.5$, we get $m > 78.5 - 20.5 = 58.0$ (just above 58).

Step 3: Pin it down with a clean integer check. The estimate sits right at 58, so test the two neighbours exactly:

$m = 57: \; 57^2 + 41(57) = 3249 + 2337 = 5586 < 5740$ (fails).

$m = 58: \; 58^2 + 41(58) = 3364 + 2378 = 5742 > 5740$ (works).

Step 4: Conclude. The first m that pushes the RHS exponent past 2870 is

$$\boxed{m = 58}.$$

Option (1).

Quick Tip: When both sides of an inequality are powers of the same base greater than 1, you can drop the base and compare the exponents directly. For products of powers with patterned indices, convert to sums using exponent rules and arithmetic/progression formulas.

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11. The number of distinct pairs of integers (x, y) satisfying the inequalities $x > y \geq 3$ and $x + y < 14$ is:

Correct Answer: —

SOLUTION:

Approach: Use a substitution to make the strict inequalities easier, then count lattice points in a clean triangular region. Let the gap between x and y be a positive integer; the sum condition then caps that gap.

Step 1: Shift the variables. Let $y = 3 + a$ with $a \geq 0$, and let $x = y + b$ $= 3 + a + b$ with $b \geq 1$ (so $x > y$). All of a, b are integers.

Step 2: Apply the sum bound.

$$x + y = (3 + a) + (3 + a + b) = 6 + 2a + b < 14 \implies 2a + b < 8 \implies 2a + b \leq 7.$$

Step 3: Count (a, b) with $a \geq 0,\ b \geq 1,\ 2a + b \leq 7.$

$a = 0 :\ b \in \{1, \dots, 7\} \Rightarrow 7.$

$a = 1 :\ b \leq 5 \Rightarrow b \in \{1, \dots, 5\} \Rightarrow 5.$

$a = 2 :\ b \leq 3 \Rightarrow b \in \{1, 2, 3\} \Rightarrow 3.$

$a = 3 :\ b \leq 1 \Rightarrow b \in \{1\} \Rightarrow 1.$

$a \geq 4 :\ 2a \geq 8 > 7 \Rightarrow$ none.

Step 4: Total.

$$7 + 5 + 3 + 1 = 16.$$

Same triangular count, reached by reducing to a single clean inequality $2a + b \leq 7$.

$\boxed{16}$

Quick Tip: When given two-variable inequalities, fix one variable and count valid values of the other. Stopping conditions appear naturally when inequalities become impossible to satisfy.

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12. In a circle with center C and radius $6\sqrt{2}$ cm, PQ and SR are two parallel chords separated by one of the diameters. If $\angle PQC = 45°$, and the ratio of the perpendicular distance of PQ and SR from C is $3:2$, then the area, in sq. cm, of the quadrilateral $PQRS$ is:

(A) $20(3+\sqrt{14})$

(B) $20(3\sqrt{2}+\sqrt{7})$

(C) $4(3+\sqrt{14})$

(D) $4(3\sqrt{2}+\sqrt{7})$

Correct Answer: (A) $20(3+\sqrt{14})$

SOLUTION:

Approach: Drop the chords onto a coordinate grid centred at C. Then the area of the (isosceles) trapezium is just the integral-style "average width times height" — but computing it via coordinates of the four corners with the shoelace idea makes every length explicit.

Step 1: Place C at the origin. Let the diameter separating the chords be horizontal (the x-axis). PQ is a horizontal chord at height $+d_1$ and SR a horizontal chord at depth $-d_2$.

Step 2: Find d_1 from the $45°$ angle. The perpendicular from C to PQ meets it at its midpoint M, giving right triangle CMQ with hypotenuse $R = 6\sqrt{2}$ and base angle $45°$ at Q. So $d_1 = R\sin 45° = 6$ and the half-chord $MQ = R\cos 45° = 6$. Hence $PQ = 12$, and its endpoints are $(\pm 6,\ 6)$.

Step 3: Find d_2 and SR. Ratio $d_1 : d_2 = 3:2 \Rightarrow d_2 = 4$. Half-chord of SR: $\sqrt{R^2 - d_2^2} = \sqrt{72-16} = \sqrt{56} = 2\sqrt{14}$. So $SR = 4\sqrt{14}$, endpoints $(\pm 2\sqrt{14},\ -4)$.

Step 4: Area by the trapezium height $=$ vertical gap. The two parallel sides are 12 and $4\sqrt{14}$; the vertical separation is $6-(-4) = 10$.

$$\text{Area} = \frac{12 + 4\sqrt{14}}{2} \times 10 = (6 + 2\sqrt{14}) \times 10 = 60 + 20\sqrt{14} = 20(3+\sqrt{14}).$$

$$\boxed{20(3+\sqrt{14})}$$

matching option (1).

Quick Tip: For chords in a circle:

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13. Stocks A, B and C are priced at rupees 120, 90 and 150 per share, respectively. A trader holds a portfolio consisting of 10 shares of stock A, and 20 shares of stocks B and C put together. If the total value of her portfolio is rupees 3300, then the number of shares of stock B that she holds is:

Correct Answer: —

SOLUTION:

Approach: Use weighted-average (alligation) thinking. After stripping stock A, the 20 shares of B and C together must average a known price per share, and alligation reads off the split immediately without solving simultaneous equations.

Step 1: Strip stock A. A contributes $10 \times 120 = 1200$, so B and C together are worth $3300 - 1200 = 2100$ across 20 shares.

Step 2: Required average price of the B–C bundle.

$$\frac{2100}{20} = 105 \text{ per share.}$$

Step 3: Alligation between B (90) and C (150). The mixture price is 105. Distances from the mean:

$$\text{B side: } 150 - 105 = 45, \qquad \text{C side: } 105 - 90 = 15.$$

So shares of B to shares of C $= 45 : 15 = 3 : 1$.

Step 4: Split the 20 shares. With ratio $3 : 1$ over 20 shares, B gets

$$\frac{3}{4} \times 20 = 15, \qquad C \text{ gets } 5.$$

$$\boxed{15 \text{ shares of B}}$$

Alligation turns the whole system into a one-line ratio split.

Quick Tip: When the total number of shares of two assets is fixed, express one in terms of the other and substitute into the value equation. This reduces the problem to a simple linear equation.

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14. A value of c for which the minimum value of $f(x) = x^2 - 4cx + 8c$ is greater than the maximum value of $g(x) = -x^2 + 3cx - 2c$, is:

(A) $\frac{1}{2}$

(B) $-\frac{1}{2}$

(C) -2

(D) 2

Correct Answer: (A) $\frac{1}{2}$

SOLUTION:

Approach (plug each option and test directly): The whole problem is one inequality, $f_{\min}(c) > g_{\max}(c)$. Get clean formulas for both, then just substitute each candidate c.

Step 1: Completing the square (or vertex formula): for the upward parabola f, $f_{\min} = 8c - 4c^2$; for the downward parabola g, $g_{\max} = \frac{9c^2}{4} - 2c$.

Step 2: Test $c = \frac{1}{2}$:

$$f_{\min} = 8(0.5) - 4(0.25) = 4 - 1 = 3, \qquad g_{\max} = \frac{9(0.25)}{4} - 2(0.5) = 0.5625 - 1 = -0.4375.$$

Here $3 > -0.4375$ holds. This option works.

Step 3 (rule out the rest quickly): For any $c \leq 0$ (options $-\frac{1}{2}$ and -2), $f_{\min} = 8c - 4c^2 \leq 0$ while $g_{\max} = \frac{9c^2}{4} - 2c \geq 0$, so the inequality fails. For $c = 2$: $f_{\min} = 16 - 16 = 0$ but $g_{\max} = 9 - 4 = 5$, and $0 > 5$ is false.

Answer: Only $c = \frac{1}{2}$ satisfies the condition.

Quick Tip: For quadratic functions, compare minimum and maximum values by:

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15. Shruti travels a distance of 224 km in four parts for a total travel time of 3 hours. Her speeds in these four parts follow an arithmetic progression, and the corresponding time taken to cover these four parts follow another arithmetic progression. If she travels at a speed of 960 meters per minute for 30 minutes to cover the first part, then the distance, in meters, she travels in the fourth part is:

(A) 72000

(B) 80000

(C) 86400

(D) 90000

Correct Answer: (C) 86400

SOLUTION:

Approach (use the middle-term shortcut for distance): For a 4-term time AP and 4-term speed AP, write each leg's distance and use total distance = 224 with a symmetric average, avoiding fraction juggling.

Step 1: First leg gives $v_1 = 57.6$ km/h, $t_1 = 0.5$ h. Total time 3 h with times in AP starting at 0.5: sum $= 4(0.5) + 6d = 3 \Rightarrow d = \frac{1}{6}$, so times are $\frac{3}{6}, \frac{4}{6}, \frac{5}{6}, \frac{6}{6}$ h.

Step 2: Let the four speeds be $a - 3p$, $a - p$, $a + p$, $a + 3p$ (a symmetric AP, common difference $2p$). Matching $v_1 = a - 3p = 57.6$ keeps things tidy.

Step 3: Total distance:

$$\frac{3}{6}(a - 3p) + \frac{4}{6}(a - p) + \frac{5}{6}(a + p) + \frac{6}{6}(a + 3p) = 224.$$

Multiply by 6: $18a + (-9 - 4 + 5 + 18)p = 1344 \Rightarrow 18a + 10p = 1344$.

Step 4: With $a - 3p = 57.6 \Rightarrow a = 57.6 + 3p$, substitute: $18(57.6 + 3p) + 10p = 1344 \Rightarrow 1036.8 + 64p = 1344 \Rightarrow p = 4.8$. Then $v_4 = a + 3p = 57.6 + 6(4.8) = 86.4$ km/h.

Step 5: Distance in part 4 $= 86.4 \times 1 = 86.4$ km $= 86400$ m.

Answer: 86400 metres.

Quick Tip: When both speeds and times form arithmetic progressions, express each term in terms of the first term and common difference, then use:

$$\text{Total distance} = \sum(\text{speed} \times \text{time})$$

to solve for the unknown common difference.

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16. In a 3-digit number N, the digits are non-zero and distinct such that none of the digits is a perfect square, and only one of the digits is a prime number. Then, the number of factors of the minimum possible value of N is:

Correct Answer: —

SOLUTION:

Approach (count non-primes first to force the digit set): Instead of starting from primes, notice the binding scarcity is non-prime digits.

Step 1: Usable digits (non-zero, not a perfect square) are $\{2, 3, 5, 6, 7, 8\}$. The non-prime ones are exactly 6 and 8 — just two of them.

Step 2: The rule "only one digit is prime" means two of the three digits are non-prime. Since only 6 and 8 qualify, both are forced into N. The remaining digit is a single prime from $\{2, 3, 5, 7\}$.

Step 3: To minimise N, the hundreds digit should be as small as possible. The smallest digit available across $\{6, 8, \text{prime}\}$ is the prime 2, so put 2 in the hundreds place, then 6, then 8: $N = 268$.

Step 4: Factorise: $268 = 4 \times 67 = 2^2 \times 67^1$. Total divisors $= (2+1)(1+1) = 6$.

Answer: The minimum N is 268, which has 6 factors.

Quick Tip: When minimizing a multi-digit number under digit constraints:

Unknown environment 'itemize'

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17. The ratio of the number of students in the morning shift and afternoon shift of a school was $13 : 9$. After 21 students moved from the morning shift to the afternoon shift, this ratio became $19 : 14$. Next, some new students joined the morning and afternoon shifts in the ratio $3 : 8$ and then the ratio of the number of students in the morning shift and the afternoon shift became $5 : 4$. The number of new students who joined is:

(A) 88

(B) 12

(C) 11

(D) 99

Correct Answer: (D) 99

SOLUTION:

Approach (track the total head-count, which never changes during a transfer): A transfer between shifts does not change the school total, so the total is constant until new students arrive — use that to anchor the numbers.

Step 1: Original total is a multiple of $13 + 9 = 22$. After transfer the total is a multiple of $19 + 14 = 33$. The same total must be divisible by both 22 and 33, so by $\mathrm{lcm} = 66$. The smallest workable total: $13x + 9x = 22x$ equals the post-transfer total $33t$. From $22x = 33t \Rightarrow 2x = 3t$. Taking $x = 63$ gives total 1386 $(= 66 \times 21)$, and post-transfer parts $\frac{19}{33}(1386) = 798$ and $\frac{14}{33}(1386) = 588$. (Check the 21-shift: $13(63) = 819 \rightarrow 798$, a drop of 21. Correct.)

Step 2: Now N new students join in ratio $3 : 8$, so morning gains $\frac{3}{11}N$ and afternoon gains $\frac{8}{11}N$. Final ratio $5 : 4$ means final total $= 1386 + N$ splits as $\frac{5}{9}$ and $\frac{4}{9}$.

Step 3: Morning equation: $798 + \frac{3}{11}N = \frac{5}{9}(1386 + N)$. Note $\frac{5}{9}(1386) = 770$, so $798 + \frac{3}{11}N = 770 + \frac{5}{9}N \Rightarrow 28 = \left(\frac{5}{9} - \frac{3}{11}\right)N = \frac{55-27}{99}N = \frac{28}{99}N$.

Step 4: Hence $N = 99$.

Answer: 99 new students joined.

Quick Tip: In ratio problems with movements or new additions:

Unknown environment 'itemize'

18. If the length of a side of a rhombus is 36 cm and the area of the rhombus is 396 sq. cm, then the absolute value of the difference between the lengths, in cm, of the diagonals of the rhombus is:

Correct Answer: —

SOLUTION:

Approach (solve for the two half-diagonals as a sum-and-product pair): Let the half-diagonals be $p = d_1/2$ and $q = d_2/2$. They satisfy a neat right-triangle relation, and you can recover them as roots of a quadratic.

Step 1: Right triangle of the half-diagonals with the side: $p^2 + q^2 = 36^2 = 1296$. Area $= \frac{1}{2}d_1 d_2 = 2pq = 396 \Rightarrow pq = 198$.

Step 2: Find $|p - q|$ directly: $(p-q)^2 = p^2 + q^2 - 2pq = 1296 - 396 = 900 \Rightarrow |p-q| = 30$.

Step 3: The full diagonals' difference is $|d_1 - d_2| = 2\,|p-q| = 2 \times 30 = 60$.

(Sanity check): $(p+q)^2 = 1296 + 396 = 1692$, so $p + q = \sqrt{1692} \approx 41.13$; with $p - q = 30$ this gives $p \approx 35.6$, $q \approx 5.6$, both real and positive — consistent.

Answer: The diagonals differ by 60 cm.

Quick Tip: For any rhombus with side length a and diagonals d_1, d_2, use:

$$d_1^2 + d_2^2 = 4a^2, \qquad \text{and} \qquad \text{Area} = \frac{1}{2}d_1 d_2.$$

These two equations quickly give sums and products of diagonal lengths, letting you compute their difference using algebraic identities.

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19. In the set of consecutive odd numbers $\{1, 3, 5, \ldots, 57\}$, there is a number k such that the sum of all the elements less than k is equal to the sum of all the elements greater than k. Then, k equals?

(A) 37

(B) 41

(C) 39

(D) 43

Correct Answer: (B) 41

SOLUTION:

Approach (guess-and-verify the balance using n^2 chunks): Each option is one of the listed odd numbers; for each, the part below it is a block of the first few odds, whose sum is a perfect square. Test which split is symmetric.

Step 1: Total of all 29 odds is $29^2 = 841$. For k to balance, the numbers strictly below k must sum to $\frac{841-k}{2}$ (half of what's left after removing k). So I need $\frac{841-k}{2}$ to equal the sum of the odds below k.

Step 2: Try $k = 41$ (the 21st odd number). Numbers below it are the first 20 odds, summing to $20^2 = 400$. Required value: $\frac{841-41}{2} = \frac{800}{2}$ $= 400$. Match.

Step 3 (why the others fail): For $k = 39$ (20th odd), below-sum $= 19^2$ $= 361$, but $\frac{841-39}{2} = 401$ — no. For $k = 43$ (22nd odd), below-sum

$= 21^2 = 441$, but $\frac{841-43}{2} = 399$ — no. For $k = 37$ (19th odd), below-sum $= 18^2 = 324$ vs $\frac{841-37}{2} = 402$ — no.

Answer: Only $k = 41$ splits the set evenly into 400 and 400.

Quick Tip: For consecutive odd numbers, remember: sum of first n terms $= n^2$. This simplifies balance-sum problems significantly.

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20. A container holds 200 litres of a solution of acid and water, having 30% acid by volume. Atul replaces 20% of this solution with water, then replaces 10% of the resulting solution with acid, and finally replaces 15% of the solution thus obtained, with water. The percentage of acid by volume in the final solution obtained after these three replacements, is nearest to?

(A) 25

(B) 27

(C) 29

(D) 23

Correct Answer: (B) 27

SOLUTION:

Approach (work with the acid fraction and multipliers in one chain): Since volume is fixed at 200 L, follow the acid concentration as a fraction; a water-replacement just scales it, an acid-replacement scales then adds.

Step 1: Start at fraction 0.30 acid.

Step 2: Replace 20% with water → fraction $\times 0.8 = 0.30 \times 0.8 = 0.24$.

Step 3: Replace 10% with acid. Removing 10% scales acid to $0.24 \times 0.9 = 0.216$; adding 10% pure acid raises the fraction by 0.10: $0.216 + 0.10 = 0.316$.

Step 4: Replace 15% with water → fraction $\times 0.85 = 0.316 \times 0.85 = 0.2686$.

Step 5: That is 26.86%, nearest to 27%.

Answer: About 27% acid in the final solution.

Quick Tip: In replacement problems, always track the \emph{amount} of the substance (here, acid) after each step. The total volume usually returns to the original, which makes the percentage calculation easier at the end.

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21. Arun, Varun and Tarun, if working alone, can complete a task in 24, 21, and 15 days, respectively. They charge Rs 2160, Rs 2400, and Rs 2160 per day, respectively, even if they are employed for a partial day. On any given day, any of the workers may or may not be employed to work. If the task needs to be completed in 10 days or less, then the minimum possible amount, in rupees, required to be paid for the entire task is?

(A) 34400

(B) 38400

(C) 47040

(D) 38880

Correct Answer: (B) 38400

SOLUTION:

Approach: Instead of cost-per-unit, reason directly in terms of "days of work" and check the few sensible combinations, since each man can give at most 10 days and the total job is 840 units.

Step 1: Set up. Total work $= \text{LCM}(24, 21, 15) = 840$ units; daily outputs are Arun 35, Varun 40, Tarun 56. A day employed costs the full wage even for a partial day, so each man contributes (days worked) $\times$ (his rate) units and is paid (days worked) $\times$ (his wage).

Step 2: Tarun must be maxed out. Even at his ceiling Tarun gives only $56 \times 10 = 560 < 840$ units, so he should work all 10 days — he is the cheapest per unit and we still fall short, so there is no reason to cut him. That fixes 560 units done and Rs 21600 spent, leaving 280 units.

Step 3: Pick the helper for 280 units. Two clean choices:
Varun: $280/40 = 7$ days $\Rightarrow 7 \times 2400 =$ Rs 16800.
Arun: $280/35 = 8$ days $\Rightarrow 8 \times 2160 =$ Rs 17280.
Varun is cheaper, so use Varun.

Step 4: Add up.

$$21600 + 16800 = \text{Rs } 38400.$$

Trying any split of the 280 units across both helpers only raises cost, so Rs 38400 is the minimum. Answer: option (b).

Quick Tip: In mixed worker problems with a time limit and different wages, first compute \emph{cost per unit of work}. Then, to minimize total cost, allocate as much work as possible to the worker with the lowest cost per unit, subject to the time constraints.

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22. In a class, there were more than 10 boys and a certain number of girls. After 40% of the girls and 60% of the boys left the class, the remaining number of girls was 8 more than the remaining number of boys. Then, the minimum possible number of students initially in the class was?

Correct Answer: —

SOLUTION:

Approach: Work straight from the remaining (post-exit) counts, which must themselves be whole numbers, and scan upward for the first feasible case.

Step 1: Name the survivors. Let remaining boys $= b = 0.4B$ and remaining girls $= g = 0.6G$. The clue gives $g = b + 8$.

Step 2: Translate back to integers. $B = b/0.4 = 2.5b$ must be a whole number, so b is even. $G = g/0.6 = 5g/3$ must be a whole number, so g is a multiple of 3.

Step 3: Apply $B > 10$. $B = 2.5b > 10 \Rightarrow b > 4$, so $b \geq 5$; combined with

"b even", the smallest candidate is $b = 6$.

Step 4: Check the smallest cases. $b = 6 \Rightarrow g = 14$, but 14 is not a multiple of 3 — reject. $b = 8 \Rightarrow g = 16$, not a multiple of 3 — reject. $b = 10 \Rightarrow g = 18$, and 18 is a multiple of 3 — accept.

Step 5: Recover originals. $B = 2.5 \times 10 = 25\ (> 10)$, $G = \frac{5}{3} \times 18 = 30$.

$$T = 25 + 30 = 55.$$

Minimum possible initial strength $= 55$.

Quick Tip: When percentages of people leave, ensure that those percentages give integer counts. Often this forces variables to be multiples of certain numbers (like 5, 10, etc.), which helps in solving and minimizing or maximizing totals.