

General Instructions

- (i) This question paper contains 19 questions. All questions are compulsory.
- (ii) It comprises 12 single-correct multiple-choice questions and 7 numerical / integer-type questions.
- (iii) Attempt every question; detailed solutions are provided in the companion solutions booklet.
- (iv) For numerical questions, report the answer rounded exactly as asked.

1. If m and n are natural numbers such that $n > 1$, and $m^n = 2^{25} \times 3^{40}$, then $m - n$ equals

- (A) 209942
- (B) 209947
- (C) 209932
- (D) 209937

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2. The roots α, β of the equation $3x^2 + \lambda x - 1 = 0$, satisfy $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = 15$. The value of $(\alpha^3 + \beta^3)^2$ is

- (A) 16
- (B) 9
- (C) 1
- (D) 4

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3. If x and y satisfy the equations $|x| + x + y = 15$ and $x + |y| - y = 20$, then $(x - y)$ equals

- (A) 5
- (B) 10
- (C) 20
- (D) 15

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4. Anil invests Rs 22000 for 6 years in a scheme with 4% interest per annum, compounded half-yearly. Separately, Sunil invests a certain amount in the same scheme for 5 years, and then reinvests the entire amount he receives at the end of 5 years, for one year at 10% simple interest. If the amounts received by both at the end of 6 years are equal, then the initial investment, in rupees, made by Sunil is

- (A) 20640
- (B) 20808
- (C) 20860
- (D) 20480

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5. A vessel contained a certain amount of a solution of acid and water. When 2 litres of water was added to it, the new solution had 50% acid concentration. When 15 litres of acid was further added to this new solution, the final solution had 80% acid concentration. The ratio of water and acid in the original solution was

- (A) 3 : 5
- (B) 5 : 3
- (C) 4 : 5
- (D) 5 : 4

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6. A bus leaves the bus stand at 9 am and travels at a constant speed of 60 km/h. It reaches its destination 3.5 hours later than its original time. The next day, it travels $\frac{2}{3}$ of its route within $\frac{1}{3}$ of its initial time and travels the rest of the route within 40 minutes. At what time does it reach its destination normally?

- (A) 1:15 PM
- (B) 1:17 PM
- (C) 3:12 PM
- (D) 1:25 PM

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7. The coordinates of the three vertices of a triangle are: (1, 2), (7, 2), and (1, 10). Then the radius of the incircle of the triangle is

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8. Bina incurs 19% loss when she sells a product at Rs. 4860 to Shyam, who in turn sells this product to Hari. If Bina would have sold this product to Shyam at the purchase price of Hari, she would have obtained 17% profit. Then, the profit, in rupees, made by Shyam is

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9. Amal and Vimal together can complete a task in 150 days, while Vimal and Sunil together can complete the same task in 100 days. Amal starts working on the task and works for 75 days, then Vimal takes over and works for 135 days. Finally, Sunil takes over and completes the remaining task in 45 days. If Amal had started the task alone and worked on all days, Vimal had worked on every second day, and Sunil had worked on every third day, then the number of days required to complete the task would have been

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10. A function f maps the set of natural numbers to whole numbers, such that $f(xy) = f(x)f(y) + f(x) + f(y)$ for all x, y and $f(p) = 1$ for every prime number p . Then, the value of $f(160000)$ is

- (A) 4095
- (B) 8191
- (C) 2047
- (D) 1023

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11. When Rajesh's age was same as the present age of Garima, the ratio of their ages was 3 : 2. When Garima's age becomes the same as the present age of Rajesh, the ratio of the ages of Rajesh and Garima will become

- (A) 5 : 4
- (B) 2 : 1
- (C) 4 : 3
- (D) 3 : 2

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12. The sum of the infinite series $\frac{1}{5} \left(\frac{1}{5} - \frac{1}{7} \right) + \left(\frac{1}{5} \right)^2 \left(\frac{1}{5} - \frac{1}{7} \right)^2 - \left(\frac{1}{7} \right)^2 + \left(\frac{1}{5} \right)^3 \left(\frac{1}{5} - \frac{1}{7} \right)^3 + \dots$ is equal to

- (A) $\frac{7}{408}$
- (B) $\frac{5}{408}$
- (C) $\frac{7}{816}$
- (D) $\frac{5}{816}$

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13. A fruit seller has a stock of mangoes, bananas, and apples with at least one fruit of each type. At the beginning of a day, the number of mangoes makes up 40% of his stock. That day, he sells half of the mangoes, 96 bananas, and 40% of the apples. At the end of the day, he ends up selling 50% of the fruits. What is the smallest possible total number of fruits in the stock at the beginning of the day?

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14. Three circles of equal radii touch (but not cross) each other externally. Two other circles, X and Y, are drawn such that both touch (but not cross) each of the three previous circles. If the radius of X is more than that of Y, the ratio of the radii of X and Y is

- (A) $4 + \sqrt{3} : 1$
- (B) $2 + \sqrt{3} : 1$
- (C) $4 + 2\sqrt{3} : 1$
- (D) $7 + 4\sqrt{3} : 1$

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15. A company has 40 employees whose names are listed in a certain order. In the year 2022, the average bonus of the first 30 employees was Rs. 40000, of the last 30 employees was Rs. 60000, and of the first 10 and last 10 employees together was Rs. 50000. Next year, the average bonus of the first 10 employees increased by 100%, of the last 10 employees increased by 200% and of the remaining employees was unchanged. Then, the average bonus, in rupees, of all the 40 employees together in the year 2023 was

- (A) 90000
- (B) 95000
- (C) 85000
- (D) 80000

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16. ABCD is a trapezium in which AB is parallel to CD. The sides AD and BC when extended, intersect at point E. If $AB = 2$ cm, $CD = 1$ cm, and perimeter of ABCD is 6 cm, then the perimeter, in cm, of $\triangle AEB$ is

- (A) 10
- (B) 9
- (C) 8
- (D) 7

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17. If x and y are real numbers such that $4x^2 + 4y^2 - 4xy - 6y + 3 = 0$, then the value of $(4x + 5y)$ is

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18. P, Q, R and S are four towns. One can travel between P and Q along 3 direct paths, between Q and S along 4 direct paths, and between P and R along 4 direct paths. There is no direct path between P and S, while there are few direct paths between Q and R, and between R and S. One can travel from P to S either via Q, or via R, or via Q followed by R, respectively, in exactly 62 possible ways. One can also travel from Q to R either directly, or via P, or via S, in exactly 27 possible ways. Then, the number of direct paths between Q and R is

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19. If a , b , and c are positive real numbers such that $a > 10 \geq b \geq c$ and

$$\frac{\log_8(a+b)}{\log_2 c} + \frac{\log_{27}(a-b)}{\log_3 c} = \frac{2}{3}$$

then the greatest possible integer value of a is