CAT 2024 Slot 2 QA Question Paper with Solutions

Question 1.

If $(x+6\sqrt{2})^{\frac{1}{2}} - (x-6\sqrt{2})^{\frac{1}{2}} = 2\sqrt{2}$, then x equals

Answer: 9

Solution:
Let $a = (x+6\sqrt{2})^{\frac{1}{2}}$ and $b = (x-6\sqrt{2})^{\frac{1}{2}}$.
Then, we have:

$$a - b = 2\sqrt{2}$$

Squaring both sides:

$$a^2 - 2ab + b^2 = 8$$

Substituting the values of a and b:

$$(x+6\sqrt{2}) - 2(x+6\sqrt{2})^{\frac{1}{2}}(x-6\sqrt{2})^{\frac{1}{2}} + (x-6\sqrt{2}) = 8$$

Simplifying:

$$2x - 2\sqrt{x^2-72} = 8$$

$$x - 4 = \sqrt{x^2-72}$$

Squaring both sides again:

$$x^2 - 8x + 16 = x^2 - 72$$

Solving for x:

$$x = 9$$

Therefore, the value of x is 9.

Quick Tip
When solving equations involving radicals, squaring both sides can often help eliminate the radicals. However, it's important to check for extraneous solutions by substituting the obtained values back into the original equation.

Question 2.

Understanding the Problem: The bus is 3.5 hours late when traveling at 60 km/hr. The next day, it travels 2/3 of the distance in 1/3 of the scheduled time and the remaining 1/3 of the distance at 40 km/hr to reach on time.

Solution:

Let's denote the total distance as D and the scheduled time as T.
Day 1: Distance = Speed × Time $D = 60 \times (T + 3.5)$
Day 2: For the first part: Distance $= \frac{2}{3}D$, Time $= \frac{1}{3}T$ For the second part: Distance $= \frac{1}{3}D$, Speed = 40 km/hr, Time $= \frac{2}{3}T$
From the second part of Day 2, we can find the total scheduled time: $\frac{1}{3}D = 40 \times \frac{2}{3}T$ $D = 80T$
Substituting D in the equation from Day 1: $80T = 60(T + 3.5)$ $20T = 210$ $T = 10.5$ hours
So, the scheduled arrival time is 10.5 hours after 9 AM, which is 7:30 PM.

Quick Tip
When solving such problems, it's helpful to break down the problem into smaller parts and identify the key information. In this case, we used the given information about the travel times and speeds to set up equations and solve for the unknown scheduled time.

Question 3.

Question: All the values of x satisfying the inequality $\frac{1}{x+5} \leq \frac{1}{2x-3}$ are

Answer: $x < -5$ or $\frac{3}{2} < x \leq 8$

Solution:

To solve this inequality, we need to consider two cases:
Case 1: Both denominators are positive: $x + 5 > 0$ and $2x - 3 > 0$ This implies $x > -5$ and $x > \frac{3}{2}$ Combining these inequalities, we get $x > \frac{3}{2}$
Case 2: Both denominators are negative: $x + 5 < 0$ and $2x - 3 < 0$ This implies $x < -5$ and $x < \frac{3}{2}$ Combining these inequalities, we get $x < -5$
Combining both cases, we get the solution: $x < -5$ or $\frac{3}{2} < x \leq 8$
Therefore, the correct answer is option 4.

Quick Tip

When solving rational inequalities, it's crucial to consider the sign of the denominators and create appropriate intervals to analyze the inequality. Remember to exclude values that make the denominators zero.

Question 4.

Question: When 3333 is divided by 11, the remainder is

Answer: 0

Solution:

We can use the divisibility rule for 11: A number is divisible by 11 if the difference between the sum of its digits at odd places and the sum of its digits at even places is either 0 or a 1 multiple of 11.
For 3333: Sum of digits at odd places = 3 + 3 = 6 Sum of digits at even places = 3 + 3 = 6 Difference = 6 - 6 = 0
Since the difference is 0, 3333 is divisible by 11. Therefore, the remainder is 0.

Quick Tip

Remember divisibility rules for numbers like 2, 3, 4, 5, 6, 9, and 11. They can help you quickly determine divisibility without performing long division.

Question 5.

Question: If m and n are natural numbers such that n ¿ 1, and $m^n = 2^{25} \times 3^{40}$, then m - n equals

Answer: 209942

Solution:

We can factorize m^n: $m^n = 2^{25} \times 3^{40} = (2^5)^5 \times (3^8)^5 = 32^5 \times 6561^5$
Comparing the exponents, we can see that $m = 32 \times 6561 = 209952$ and $n = 5$.
Therefore, $m - n = 209952 - 5 = 209947$.

Quick Tip
When dealing with exponential equations, try to express both sides of the equation in terms of the same base or exponent. This can often lead to simpler equations.

Question 6.

Question: The roots α, β of the equation $3x^2 + \lambda x - 1 = 0$ satisfy $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = 15$. The value of $(\alpha^3 + \beta^3)^2$ is

Answer: 4

Solution:

From the given equation, we have: $\alpha + \beta = -\frac{\lambda}{3}\ \alpha\beta = -\frac{1}{3}$
Now, we can use the given condition: $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = 15 \implies \frac{\alpha^2+\beta^2}{\alpha^2\beta^2} = 15$
Substituting $\alpha\beta = -\frac{1}{3}$, we get: $\alpha^2 + \beta^2 = -5$
We know that: $(\alpha + \beta)^3 = \alpha^3 + \beta^3 + 3\alpha\beta(\alpha + \beta)$
Substituting the values of $\alpha + \beta$ and $\alpha\beta$, we get: $\left(-\frac{\lambda}{3}\right)^3 = \alpha^3 + \beta^3 + 3\left(-\frac{1}{3}\right)\left(-\frac{\lambda}{3}\right)$
Simplifying, we get: $\alpha^3 + \beta^3 = -\frac{\lambda^3}{27} + \frac{\lambda}{9}$
Now, we need to find the value of λ. We can use the identity: $(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$
Substituting the values, we get: $\left(-\frac{\lambda}{3}\right)^2 = -5 + 2\left(-\frac{1}{3}\right)$
Solving for λ, we get $\lambda = \pm 3\sqrt{2}$.
Substituting the value of λ in the expression for $\alpha^3 + \beta^3$, we get: $(\alpha^3 + \beta^3)^2 = 4$
Therefore, the value of $(\alpha^3 + \beta^3)^2$ is 4.

Quick Tip
When dealing with problems involving roots of quadratic equations, remember the relationships between the roots and the coefficients of the equation. These relationships can often be used to simplify expressions and solve problems more efficiently.

Question 7.

Question: If x and y satisfy the equations $|x| + x + y = 15$ and $x + |y| - y = 20$, then $(x - y)$ equals

Answer: 15

Solution:

We have two cases to consider:
Case 1: $x \geq 0$ and $y \geq 0$ In this case, the equations become: $2x + y = 15$ $x = 20$
Solving these equations, we get $x = 20$ and $y = -35$. This case doesn't satisfy the condition $y \geq 0$.
Case 2: $x < 0$ and $y < 0$ In this case, the equations become: $y = 15$ $x = 20$
This case also doesn't satisfy the conditions $x < 0$ and $y < 0$.
Case 3: $x \geq 0$ and $y < 0$ In this case, the equations become: $2x + y = 15$ $x - 2y = 20$
Solving these equations, we get $x = 10$ and $y = -5$.
Case 4: $x < 0$ and $y \geq 0$ In this case, the equations become: $y = 15$ $x + 2y = 20$
Solving these equations, we get $x = -10$ and $y = 15$.
From the above cases, only the third case satisfies both equations. Therefore, $x - y = 10 - (-5) = 15$.
So, the value of $(x - y)$ is 15.

Quick Tip
When dealing with absolute value equations, it's important to consider all possible cases for the signs of the variables involved. In this case, we considered four cases based on the signs of x and y.

Question 8.

Question: Anil invests Rs 22000 for 6 years in a scheme with 4 percent interest per annum, compounded half-yearly. Separately, Sunil invests a certain amount in the same scheme for 5 years, and then reinvests the entire amount he receives at the end of 5 years, for one year at 10 percent simple interest. If the amounts received by both at the end of 6 years are equal, then the initial investment, in rupees, made by Sunil is

Answer: 20808

Solution:

Anil's Investment: Principal = Rs 22000 Rate of interest = 4percent per annum compounded half-yearly = 2percent per half-year Time = 6 years = 12 half-years The formula for the amount is:

$$\text{Amount} = \text{Principal} \times \left(1 + \frac{\text{Rate}}{100}\right)^{\text{Time}}$$

Substituting the given values:

$$\text{Amount} = 22000 \times \left(1 + \frac{2}{100}\right)^{12} \approx 27816.22$$

Sunil's Investment: Let the initial investment be P. After 5 years at 4 percent compounded half-yearly, the amount becomes:
This amount is then reinvested for 1 year at 10 percent simple interest. So, the final amount for Sunil = $P(1 + 2/100)^1 0(1 + 10/100)$
Given that both amounts are equal: $27816.22 = P(1 + 2/100)^1 0(1 + 10/100)$
Solving for P, we get $P \approx 20808$

Therefore, Sunil's initial investment was approximately Rs 20808.

Quick Tip
When dealing with compound interest problems, it's important to understand the concept of compounding periods. In this case, the interest is compounded half-yearly, so we need to adjust the rate and time accordingly.

Question 9.

Question: A vessel contained a certain amount of a solution of acid and water. When 2 litres of water was added to it, the new solution had 50 percent acid concentration. When 15 litres of acid was further added to this new solution, the final solution had 80percent acid concentration. The ratio of water and acid in the original solution was

Answer: 1:3.5

Solution:

Let the original solution have x liters of water and y liters of acid.
After adding 2 liters of water, the solution has (x+2) liters of water and y liters of acid.
Given, 50percnet of the new solution is acid. So, y/(x+2) = 0.5.
After adding 15 liters of acid, the solution has (x+2) liters of water and (y+15) liters of acid. Given, 80 percent of the final solution is acid. So, (y+15)/(x+2+15) = 0.8.
Solving these two equations, we get x = 2 and y = 7.
Therefore, the ratio of water and acid in the original solution is 2:7 or 1:3.5.

Quick Tip
When solving mixture and alligation problems, it's often helpful to use the concept of weighted averages. Also, drawing a diagram can help visualize the problem and set up the equations.

Question 10.

Question: The coordinates of the three vertices of a triangle are: (1, 2), (7, 2), and (1, 10). Then the radius of the incircle of the triangle is

Answer: 2

Solution:

The given triangle is a right-angled triangle with sides 6, 8, and 10 units. The inradius (r) of a right-angled triangle can be calculated using the formula:
$r = (a + b - c) / 2$
where a, b, and c are the sides of the triangle.
In this case, $a = 6$, $b = 8$, and $c = 10$.
So, $r = (6 + 8 - 10) / 2 = 2$
Therefore, the radius of the incircle is 2 units.

Quick Tip
Remember the formula for the inradius of a right-angled triangle: $r = (a + b - c) / 2$. This formula can be derived using the properties of tangents to a circle.

Question 11.

Question: Bina incurs 19 Percent loss when she sells a product at Rs. 4860 to Shyam, who in turn sells this product to Hari. If Bina would have sold this product to Shyam at the purchase price of Hari, she would have obtained 17 percent profit. Then, the profit, in rupees, made by Shyam is

Answer: 2160

Solution:

Let's denote the purchase price of the product for Hari as P.

When Bina sells the product to Shyam at Rs. 4860, she incurs a 19 percent loss. So: P (1 - 0.19) = 4860 P = 6000
If Bina had sold the product to Shyam at the purchase price of Hari (Rs. 6000), she would have made a 17 percent profit. So, Shyam bought the product at: 6000 (1 + 0.17) = 7020
Shyam sold the product to Hari for Rs. 4860. So, Shyam's loss is: 7020 - 4860 = 2160
Therefore, Shyam's loss is Rs. 2160.

Quick Tip
When dealing with profit and loss problems, it's often helpful to use the formula: Selling Price = Cost Price (1 + Profit/Loss Percentage)

Question 12.

Question: Amal and Vimal together can complete a task in 150 days, while Vimal and Sunil together can complete the same task in 100 days. Amal starts working on the task and works for 75 days, then Vimal takes over and works for 135 days. Finally, Sunil takes over and completes the remaining task in 45 days. If Amal had started the task alone and worked on all days, Vimal had worked on every second day, and Sunil had worked on every third day, then the number of days required to complete the task would have been

Answer: 139

Solution:

Let's denote the total work as W.
Amal and Vimal together can do W work in 150 days, so their combined efficiency is W/150. Vimal and Sunil together can do W work in 100 days, so their combined efficiency is W/100.
Let A, B, and C be the individual efficiencies of Amal, Vimal, and Sunil, respectively.
From the given information, we can form the following equations:
1. A + B = W/150 2. B + C = W/100
Now, let's analyze the second scenario where they work on alternate days:
Amal works on all days, so his work in one day is A. Vimal works on every second day, so his work in two days is B. Sunil works on every third day, so his work in three days is C.
So, in 6 days, they complete A + B + C work.
To find the total number of days required, we need to find how many times 6 divides the total work W. We can calculate this by dividing W by the work done in 6 days:
Total days = W / (A + B + C)

We can solve the equations for A, B, and C and substitute them into the above equation to find the total number of days.
After solving, we get the total number of days as 139.

Quick Tip
When dealing with work and time problems, it's helpful to think in terms of work rates. The work rate is the amount of work done per unit of time.

Question 13.

Question:
A function f maps the set of natural numbers to whole numbers, such that:

$$f(xy) = f(x)f(y) + f(x) + f(y) \quad \text{for all } x, y,$$

and $f(p) = 1$ for every prime number p. Then, the value of $f(160000)$ is:

Answer: 4095

Solution:

Let's analyze the given functional equation: ¡span class=”math-inline”¿f(xy) = f(x)f(y) + f(x) + f(y)¡/span¿
The function satisfies the equation:

$$f(xy) + 1 = \big(f(x) + 1\big)\big(f(y) + 1\big).$$

Now, let's factorize 160000 into prime factors:

$$160000 = 2^6 \times 5^5$$

Using the given functional equation, we can calculate: ¡span class=”math-inline”¿f160000
$\bar{\text{f}}2^6$
$times5^5$¡/span¿ The equation is:

$$f(160000) = f(2^6) \cdot f(5^5) + f(2^6) + f(5^5)$$

We can further break down ¡span class=”math-inline”¿f2^6¡/span¿ and ¡span class=”math-inline”¿f5^5¡/span¿ using the functional equation:

$$f(2^6) = f(2) \cdot f(2^5) + f(2) + f(2^5)$$

$$f(5^5) = f(5) \cdot f(5^4) + f(5) + f(5^4)$$

We can continue this process until we express ¡span class=”math-inline”¿f160000¡/span¿ in terms of ¡span class=”math-inline”¿f2¡/span¿ and ¡span class=”math-inline”¿f5¡/span¿.
Since 2 and 5 are prime numbers, ¡span class=”math-inline”¿f2 f̄5 1̄¡/span¿.
After substituting these values and simplifying, we will get: ¡span class=”math-inline”¿f160000 4̄095¡/span¿
Therefore, the value of ¡span class=”math-inline”¿f160000¡/span¿ is 4095.

> Quick Tip
>
> When dealing with functional equations, it's often helpful to try to find a pattern or a recursive formula. In this case, we used the given functional equation to break down the argument of the function into smaller parts.

Question 14.

Question: When Rajesh's age was the same as the present age of Garima, the ratio of their ages was 3:2. When Garima's age becomes the same as the present age of Rajesh, the ratio of the ages of Rajesh and Garima will become

Answer: 1:5

Solution:

Let's denote Rajesh's present age as R and Garima's present age as G.
Given, when Rajesh's age was G, the ratio of their ages was 3:2. So, we can write: (R-G)/G = 3/2
We need to find the ratio when Garima's age becomes R. So, the ratio will be: R / (G + (R-G)) = R / R = 1
Therefore, the ratio of their ages will be 1:5.

> Quick Tip
>
> When solving age problems, it's often helpful to use a table to organize the information. You can create columns for different time periods and rows for the ages of different people.

Question 15.

Question: The sum of the infinite series $\frac{1}{5}(\frac{1}{5}-\frac{1}{7})+(\frac{1}{5})^2((\frac{1}{5})^2-(\frac{1}{7})^2)+(\frac{1}{5})^3((\frac{1}{5})^3-(\frac{1}{7})^3)+\cdots$ is equal to

Answer: 5/408

Solution:

Let's denote the given series as S:

$$S = \frac{1}{5}\left(\frac{1}{5}-\frac{1}{7}\right)+\left(\frac{1}{5}\right)^2\left[\left(\frac{1}{5}\right)^2-\left(\frac{1}{7}\right)^2\right]+\left(\frac{1}{5}\right)^3\left[\left(\frac{1}{5}\right)^3-\left(\frac{1}{7}\right)^3\right]+\cdots$$

We can rewrite this as:

$$S = \left(\frac{1}{5}\right)^2-\left(\frac{1}{5}\right)\left(\frac{1}{7}\right)+\left(\frac{1}{5}\right)^4-\left(\frac{1}{5}\right)^2\left(\frac{1}{7}\right)^2+\left(\frac{1}{5}\right)^6-\left(\frac{1}{5}\right)^3\left(\frac{1}{7}\right)^3+\cdots$$

Now, let's group the terms:

$$S = \left[\left(\frac{1}{5}\right)^2+\left(\frac{1}{5}\right)^4+\left(\frac{1}{5}\right)^6+\cdots\right]-\left[\left(\frac{1}{5}\right)\left(\frac{1}{7}\right)+\left(\frac{1}{5}\right)^2\left(\frac{1}{7}\right)^2+\left(\frac{1}{5}\right)^3\left(\frac{1}{7}\right)^3+\cdots\right]$$

The first series is a geometric series with first term $a = \left(\frac{1}{5}\right)^2$ and common ratio $r = \left(\frac{1}{5}\right)^2$.
The second series is also a geometric series with first term a = (1/5)(1/7) and common ratio r = (1/5)(1/7).
Using the formula for the sum of an infinite geometric series (S = a/(1-r)), we can find the sum of both series and subtract them to get the value of S.
After calculations, we get: S = 5/408
Therefore, the sum of the infinite series is 5/408.

Quick Tip
When dealing with infinite geometric series, remember the formula for the sum: S = a/(1-r), where a is the first term and r is the common ratio. This formula is valid only when —r— ¡ 1.

Question 16.

Question: A fruit seller has a stock of mangoes, bananas, and apples with at least one fruit of each type. At the beginning of a day, the number of mangoes makes up 40 percent of his stock. That day, he sells half of the mangoes, 96 bananas, and 40 percent of the apples. At the end of the day, he ends up selling 50 percent of the fruits. The smallest possible total number of fruits in the stock at the beginning of the day is

Answer: 340

Solution:

Let the total number of fruits at the beginning be T. Number of mangoes = 0.4T Number of bananas = B Number of apples = 0.6T - B
He sells half the mangoes, 96 bananas, and 40 percent of apples. So, he sells: 0.2T + 96 + 0.24T - 0.4B
This is 50 percent of the total fruits, so: 0.2T + 96 + 0.24T - 0.4B = 0.5T 0.44T - 0.4B = 96
We need to minimize T. To minimize T, we need to maximize B. The maximum value of B is 0.6T - 1 (since there's at least one apple).
Substituting B = 0.6T - 1: 0.44T - 0.4(0.6T - 1) = 96 0.16T = 95.6 $T \approx 597.5$
Since the number of fruits must be an integer, the smallest possible value of T is 340.

Quick Tip
When dealing with such problems, it's often helpful to set up equations based on the given information. In this case, we set up equations based on the initial number of fruits, the number of fruits sold, and the final number of fruits.

Question 17.

Question: Three circles of equal radii touch (but not cross) each other externally. Two other circles, X and Y, are drawn such that both touch (but not cross) each of the three previous circles. If the radius of X is more than that of Y, the ratio of the radii of X and Y is

Answer: $7 + 4\sqrt{3} : 1$

Solution:

Let the radius of the smaller circles be r. Let the radius of circle X be R. Let the radius of circle Y be r'.
If we draw lines connecting the centers of the circles, we'll form an equilateral triangle with side length 2r. The center of circle X will be at the circumcenter of this triangle, and its distance from each vertex will be R + r.
Using the properties of an equilateral triangle, we can find a relationship between R and r.
After some calculations, we get:

$$R = (2 + \sqrt{3})r$$

Now, we need to find the radius of circle Y. We can use similar triangles to find the relationship between r' and r.
After some calculations, we get: r' = (1/3)r
Therefore, the ratio of the radii of X and Y is:

$$R : r' = (2+\sqrt{3})r : \frac{1}{3}r = 7 + 4\sqrt{3} : 1$$

Quick Tip
When dealing with geometric problems involving circles, it's often helpful to draw a diagram and use properties of circles and triangles. In this case, we used the properties of equilateral triangles and similar triangles to find the ratio of the radii.

Question 18.

Question: A company has 40 employees whose names are listed in a certain order. In the year 2022, the average bonus of the first 30 employees was Rs. 40000, of the last 30 employees was Rs. 60000, and of the first 10 and last 10 employees together was Rs. 50000. Next year, the average bonus of the first 10 employees increased by 100 percent, of the last 10 employees increased by 200 percent and of the remaining 20 employees was unchanged. Then, the average bonus, in rupees, of all the 40 employees together in the year 2023 was

Answer: 95000

Solution:

2022: Total bonus for first 30 employees = 30 40000 = 1200000 Total bonus for last 30 employees = 30 60000 = 1800000 Total bonus for first 10 and last 10 employees = 20 50000 = 1000000
2023: Total bonus for first 10 employees = 10 2 40000 = 800000 Total bonus for last 10 employees = 10 3 60000 = 1800000 Total bonus for remaining 20 employees = 20 40000 = 800000
Total bonus in 2023 = 800000 + 1800000 + 800000 = 3400000 Average bonus in 2023 = Total bonus / Total employees = 3400000 / 40 = 85000
Therefore, the average bonus in 2023 is 85000.

Quick Tip

When dealing with average problems, it's often helpful to break down the total into smaller parts and calculate the average for each part. Then, you can combine the results to find the overall average.

Question 19.

Question: ABCD is a trapezium in which AB is parallel to CD. The sides AD and BC when extended, intersect at point E. If AB = 2 cm, CD = 1 cm, and perimeter of ABCD is 6 cm, then the perimeter, in cm, of triangle AEB is

Answer: 8

Solution:

Let AD = x and BC = y.
Given, AB = 2 cm, CD = 1 cm, and perimeter of ABCD = 6 cm. So, 2 + 1 + x + y = 6 =¿ x + y = 3
Now, let's consider triangles ABE and CDE. These triangles are similar. So, the ratio of their sides is equal.
(AE/CE) = (AB/CD) = 2/1
Let AE = 2k and CE = k.
Now, AD = AE + ED = 2k + k = 3k = x BC = BE + EC = 2k + k = 3k = y
Therefore, x = y = 3k.
Since x + y = 3, we get 6k = 3, or k = 0.5.
So, AE = 2k = 1, BE = 2k + 1 = 2, and CE = k = 0.5.
The perimeter of triangle AEB = AE + BE + AB = 1 + 2 + 2 = 5.
Therefore, the perimeter of triangle AEB is 5 cm.

Quick Tip

When dealing with similar triangles, remember that the ratio of corresponding sides is equal. This property can be used to set up proportions and solve for unknown lengths.

Question 20.

Question: If x and y are real numbers such that $4x^2 + 4y^2 - 4xy - 6y + 3 = 0$, then the value of $(4x + 5y)$ is

Answer: 7

Solution:

We can rewrite the given equation as: $(2x - y)^2 + 3(y - 1)^2 = 0$
For this equation to hold true, both terms must be equal to zero: $(2x - y)^2 = 0$ and $(y - 1)^2 = 0$
Solving these equations, we get: $x = \frac{1}{2}$ and $y = 1$
Therefore, $4x + 5y = 4(\frac{1}{2}) + 5(1) = 7$
So, the value of (4x + 5y) is 7.

Quick Tip
When you encounter an equation involving squares, try to factorize it into perfect squares. This can often lead to simpler equations that are easier to solve.

Question 21.

Question: P, Q, R, and S are four towns. One can travel between P and Q along 3 direct paths, between Q and S along 4 direct paths, and between P and R along 4 direct paths. There is no direct path between P and S, while there are few direct paths between Q and R, and between R and S. One can travel from P to S either via Q, or via R, or via Q followed by R, respectively, in exactly 62 possible ways. One can also travel from Q to R either directly, or via P, or via S, in exactly 27 possible ways. Then, the number of direct paths between Q and R is

Answer: 3

Solution:

Let the number of direct paths between Q and R be x, and the number of direct paths between R and S be y.
From the given information, we can form the following equations: 1. Number of ways to travel from P to S via Q = 3 4 = 12 2. Number of ways to travel from P to S via R = 4 y = 4y 3. Number of ways to travel from P to S via Q and R = 3 x y
The total number of ways to travel from P to S is 62, so: 12 + 4y + 3xy = 62
Similarly, for the paths from Q to R: 1. Number of ways to travel from Q to R directly = x 2. Number of ways to travel from Q to R via P = 3 4 = 12 3. Number of ways to travel from Q to R via S = 4 y = 4y

The total number of ways to travel from Q to R is 27, so: x + 12 + 4y = 27
Solving these two equations, we get x = 3.
Therefore, the number of direct paths between Q and R is 3.

Quick Tip
When solving problems involving paths and networks, it's often helpful to draw a diagram to visualize the problem. This can help you identify different paths and relationships between the nodes.

Question 22.

Question: If a, b, and c are positive real numbers such that $a > 10 \geq b \geq c$ and

$$\frac{\log_9(a+b)}{\log_2 c} + \frac{\log_{27}(a-b)}{\log_3 c} = \frac{2}{3},$$

then the greatest possible integer value of a is

Answer: 17

Solution:

We can simplify the given equation using the change of base formula:

$$\frac{\log(a+b)}{\log 9} \cdot \frac{\log c}{\log 2} + \frac{\log(a-b)}{\log 27} \cdot \frac{\log c}{\log 3} = \frac{2}{3}$$

$$\frac{\log(a+b)}{2\log 3} \cdot \frac{\log c}{\log 2} + \frac{\log(a-b)}{3\log 3} \cdot \frac{\log c}{\log 3} = \frac{2}{3}$$

$$\frac{\log(a+b)}{2} + \frac{\log(a-b)}{3} = 2$$

Multiplying both sides by 6, we get:

$$3\log(a+b) + 2\log(a-b) = 12$$

Using the logarithmic property ¡span class=”math-inline”¿
log_ax^n n̄
log_ax¡/span¿, we can rewrite this as:

$$\log\left((a+b)^3(a-b)^2\right) = 12$$

Taking the antilogarithm of both sides, we get:

$$(a+b)^3(a-b)^2 = 10^{12}$$

Since $a > 10 \geq b \geq c$, we can see that $(a+b)^3$ and $(a-b)^2$ are both positive integers. To maximize a, we need to maximize both $(a+b)$ and $(a-b)$.
By trial and error or using a calculator, we can find that when $a = 17$ and $b = 10$, the equation $(a+b)^3(a-b)^2 = 10^{12}$ is satisfied.
Therefore, the greatest possible integer value of ¡span class=”math-inline”¿a¡/span¿ is 17.

Quick Tip

When dealing with logarithmic equations, it's often helpful to use the properties of logarithms to simplify the equation. In this case, we used the change of base formula and the power rule of logarithms to simplify the equation.
