

General Instructions

- (i) This question paper contains 22 questions. All questions are compulsory.
- (ii) It comprises 14 single-correct multiple-choice questions and 8 numerical / type-in-the-answer questions.
- (iii) Attempt every question; detailed solutions are provided in the companion solutions booklet.
- (iv) For numerical questions, report the answer rounded exactly as asked.

1. Working alone, the times taken by Anu, Tanu and Manu to complete any job are in the ratio $5 : 8 : 10$. They accept a job which they can finish in 4 days if they all work together for 8 hours per day. However, Anu and Tanu work together for the first 6 days, working 6 hours 40 minutes per day. Then, the number of hours that Manu will take to complete the remaining job working alone is

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2. Mr. Pinto invests one-fifth of his capital at 6%, one-third at 10% and the remaining at 1%, each rate being simple interest per annum. Then, the minimum number of years required for the cumulative interest income from these investments to equal or exceed his initial capital is

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3. Regular polygons A and B have number of sides in the ratio $1 : 2$ and interior angles in the ratio $3 : 4$. Then the number of sides of B equals

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4. The number of distinct integer values of n satisfying $4 - \log \frac{2n}{3} - \log 4n < 0$, is

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5. The average of a non-decreasing sequence of N numbers $a_1, a_2, \dots, a_N$ is 300. If a_1 is replaced by $6a_1$, the new average becomes 400. Then, the number of possible values of a_1 is

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6. If a and b are non-negative real numbers such that $a + 2b = 6$, then the average of the maximum and minimum possible values of $(a + b)$ is

- (A) 4
- (B) 4.5
- (C) 3.5
- (D) 3

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7. The length of each side of an equilateral triangle ABC is 3 cm. Let D be a point on BC such that the area of triangle ADC is half the area of triangle ABD . Then the length of AD , in cm, is

- (A) $\sqrt{6}$
- (B) $\sqrt{5}$
- (C) $\sqrt{8}$
- (D) $\sqrt{7}$

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8. The number of integers greater than 2000 that can be formed with the digits 0, 1, 2, 3, 4, 5 using each digit at most once, is

- (A) 1440
- (B) 1200
- (C) 1420
- (D) 1480

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9. Let $f(x)$ be a quadratic polynomial in x such that $f(x) \geq 0$ for all real numbers x . If $f(2) = 0$ and $f(4) = 6$, then $f(-2)$ is equal to

- (A) 12
- (B) 36
- (C) 24
- (D) 6

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10. Manu earns ₹4000 per month and wants to save an average of ₹550 per month in a year. In the first nine months, his monthly expense was ₹3500, and he foresees that, tenth month onward, his monthly expense will increase to ₹3700. In order to meet his yearly savings target, his monthly earnings, in rupees, from the tenth month onward should be

- (A) 4200
- (B) 4400
- (C) 4300
- (D) 4350

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11. In an election, there were four candidates and 80% of the registered voters casted their votes. One of the candidates received 30% of the casted votes while the other three candidates received the remaining casted votes

in the proportion $1:\text{ratio2}:\text{ratio3}$. If the winner of the election received 2512 votes more than the candidate with the second highest votes, then the number of registered voters was

- (A) 40192
- (B) 60288
- (C) 50240
- (D) 62800

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12. On day one, there are 100 particles in a laboratory experiment. On day n , where $n \geq 2$, one out of every n particles produces another particle. If the total number of particles in the laboratory experiment increases to 1000 on day m , then m equals

- (A) 19
- (B) 16
- (C) 17
- (D) 18

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13. There are two containers of the same volume, first container half-filled with sugar syrup and the second container half-filled with milk. Half the content of the first container is transferred to the second container, and then the half of this mixture is transferred back to the first container. Next, half the content of the first container is transferred back to the second container. Then the ratio of sugar syrup and milk in the second container is

- (A) $5\text{:}6$
- (B) $5\text{:}4$
- (C) $6\text{:}5$
- (D) $4\text{:}5$

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14. Five students, including Amit, appear for an examination in which possible marks are integers between 0 and 50, both inclusive. The average marks for all the students is 38 and exactly three students got more than 32. If no two students got the same marks and Amit got the least marks among the five students, then the difference between the highest and lowest possible marks of Amit is

- (A) 21
- (B) 24
- (C) 20
- (D) 22

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15. Two ships meet mid-ocean, and then, one ship goes south and the other ship goes west, both travelling at constant speeds. Two hours later, they are 60 km apart. If the speed of one of the ships is 6 km per hour more than the other one, then the speed, in km per hour, of the slower ship is

- (A) 12
- (B) 18
- (C) 20
- (D) 24

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16. For some natural number n , assume that $(15,000)!$ is divisible by $(n!)!$. The largest possible value of n is

- (A) 5
- (B) 7
- (C) 4
- (D) 6

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17. Suppose for all integers x , there are two functions f and g such that $f(x) + f(x-1) - 1 = 0$ and $g(x) = x^2$. If $f(x^2 - x) = 5$, then the value of the sum $f(g(5)) + g(f(5))$ is

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18. In triangle ABC , altitudes AD and BE are drawn to the corresponding bases. If $\angle BAC = 45^\circ$ and $\angle ABC = \theta$, then $\frac{AD}{BE}$ equals

- (A) $\sqrt{2}\sin\theta$
- (B) $\sqrt{2}\cos\theta$
- (C) $\frac{(\sin\theta + \cos\theta)}{\sqrt{2}}$
- (D) 1

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19. The number of integer solutions of the equation $(x^2 - 10)^{(x^2 - 3x - 10)} = 1$ is

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20. Let r and c be real numbers. If r and $-r$ are roots of $5x^3 + cx^2 - 10x + 9 = 0$, then c equals

- (A) $\frac{-9}{2}$
- (B) $\frac{9}{2}$
- (C) -4
- (D) 4

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21. Consider the arithmetic progression 3,7,11,...and let A_n denote the sum of the first n terms of this progression.Then the value of $\frac{1}{25} \sum_{n=1}^{25} A_n$ is

- (A) 404
- (B) 442
- (C) 455
- (D) 415

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22. In an examination,there were 75 questions.3 marks were awarded for each correct answer,1 mark was deducted for each wrong answer and 1 mark was awarded for each unattempted question.Rayan scored a total of 97 marks in the examination.If the number of unattempted questions was higher than the number of attempted questions,then the maximum number of correct answers that Rayan could have given in the examination is