

CAT 2007 Question Paper with Solutions

Time Allowed :2 Hours	Maximum Marks :180	Total Questions :60
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General Instructions

Read the following instructions very carefully:

1. The total duration of the test is **2 hours**, with **40 minutes** allotted per section.
2. The question paper is divided into **three sections**:
 - **Section 1:** Verbal Ability and Reading Comprehension (VARC) – 20 questions
 - **Section 2:** Data Interpretation and Logical Reasoning (DILR) – 20 questions
 - **Section 3:** Quantitative Aptitude (QA) – 20 questions
3. Each correct answer carries **+3 marks**.
4. For multiple-choice questions (MCQs), **–1 mark** will be deducted for each wrong answer.
5. There is **no negative marking** for Type-in-the-Answer (TITA) questions.
6. Once the allotted time for a section ends, you will automatically move to the next section. You cannot go back to a previous section.
7. All answers must be entered on the computer screen using the mouse and keyboard. Rough sheets will be provided.
8. The use of calculators, mobile phones, or any other electronic device is strictly prohibited. An on-screen basic calculator will be provided.

Section - I

1. Consider the set $S = \{2, 3, 4, \dots, 2n + 1\}$, where 'n' is a positive integer larger than 2007. Define X as the average of the odd integers in S and Y as the average of the even integers in S. What is the value of $X - Y$?

- (1) 0
- (2) 1
- (3) $\frac{1}{2}n$
- (4) $\frac{n+1}{2n}$
- (5) 2008

Correct Answer: (2) 1

Solution:

Step 1: Understanding the Concept: The problem asks for the difference between the

average of odd integers and the average of even integers in a given set S. Both the odd and even integers in the set form an arithmetic progression (AP). The average of an AP can be easily calculated. The condition $n > 2007$ is extraneous information.

Step 2: Key Formula or Approach: For a set of numbers in an Arithmetic Progression, the average (arithmetic mean) is given by:

$$\text{Average} = \frac{\text{First Term} + \text{Last Term}}{2}$$

We will apply this formula to find the average of the odd integers (X) and the average of the even integers (Y) in the set S.

Step 3: Calculation: The set is $S = \{2, 3, 4, \dots, 2n+1\}$.

First, identify the odd integers in S. They form the sequence: 3, 5, 7, ..., $2n+1$. This is an AP with the first term $a_{\text{odd}} = 3$ and the last term $l_{\text{odd}} = 2n+1$. The average of these odd integers is X.

$$X = \frac{3 + (2n+1)}{2} = \frac{2n+4}{2} = n+2$$

Next, identify the even integers in S. They form the sequence: 2, 4, 6, ..., $2n$. This is an AP with the first term $a_{\text{even}} = 2$ and the last term $l_{\text{even}} = 2n$. The average of these even integers is Y.

$$Y = \frac{2 + 2n}{2} = \frac{2(1+n)}{2} = n+1$$

Finally, calculate the value of $X - Y$.

$$X - Y = (n+2) - (n+1) = n+2 - n - 1 = 1$$

Step 4: Final Answer: The value of $X - Y$ is 1.

$$\boxed{X - Y = 1}$$

Step 5: Why This is the Correct Option: The calculation shows that the difference between the averages is a constant value, 1, regardless of the value of 'n' (as long as 'n' is a positive integer). The condition that $n > 2007$ does not affect the outcome. Thus, option (2) is the correct choice.

Quick Tip

- For any arithmetic progression, the average is the same as the median, and it can be quickly found by averaging the first and last terms. - Be careful with extraneous information in questions. The condition ' $n > 2007$ ' is designed to make the problem seem more complex than it is.

2. Ten years ago, the ages of the members of a joint family of eight people added up to 231 years. Three years later, one member died at the age of 60 years and a child was born during the same year. After another three years, one more member died, again at 60, and a child was born during the same year. The current average

age of this eight-member joint family is nearest to

- (1) 23 years
- (2) 22 years
- (3) 21 years
- (4) 25 years
- (5) 24 years

Correct Answer: (5) 24 years

Solution:

Step 1: Understanding the Concept: This is a problem on averages, specifically focusing on age-related calculations. The key is to track the total sum of the ages of the family members over time, accounting for the passage of years and changes in the family composition (deaths and births). The number of family members remains constant at 8.

Step 2: Key Formula or Approach: The core idea is to update the sum of ages at each stage.

1. Start with the sum of ages at a past point in time.
2. Add the total increase in age for the group over a period. If 'k' years pass for a group of 'n' people, the sum of ages increases by $n \times k$.
3. Adjust the sum for any deaths (subtract the age of the deceased) and births (add the age of the newborn, which is 0).
4. Calculate the final average: $\text{Average Age} = \frac{\text{Current Sum of Ages}}{\text{Number of Members}}$.

Step 3: Calculation: Let's track the sum of ages from "10 years ago" to the "present".

- **10 years ago:** The sum of the ages of 8 members was 231 years.
- **7 years ago (3 years later):** The sum of ages would increase by $8 \times 3 = 24$ years. Sum of ages before changes = $231 + 24 = 255$ years. In that year, one member (age 60) died and a child (age 0) was born. New sum of ages = $255 - 60 + 0 = 195$ years. The number of members is still 8.
- **4 years ago (another 3 years later):** The sum of ages would again increase by $8 \times 3 = 24$ years. Sum of ages before changes = $195 + 24 = 219$ years. In that year, another member (age 60) died and a child (age 0) was born. New sum of ages = $219 - 60 + 0 = 159$ years. The number of members is still 8.
- **Present (4 years after the last change):** The sum of ages increases by $8 \times 4 = 32$ years. Current sum of ages = $159 + 32 = 191$ years.

Now, calculate the current average age:

$$\text{Current Average Age} = \frac{\text{Current Sum of Ages}}{\text{Number of Members}} = \frac{191}{8} = 23.875 \text{ years}$$

The value 23.875 is nearest to 24 years.

Step 4: Final Answer: The current average age of the family is nearest to 24 years.

24 years

Step 5: Why This is the Correct Option: The chronological calculation of the sum of ages results in a current total of 191. The average is $191/8 = 23.875$, which rounds to 24. Therefore, option (5) is the correct choice.

💡 Quick Tip

- Break down age problems into a clear timeline. - Remember to update the total age for all members of the group as time passes. - A common mistake is to forget to add the age increase for the entire group between events.

3. A function $f(x)$ satisfies $f(1) = 3600$ and $f(1) + f(2) + \dots + f(n) = n^2 f(n)$, for all positive integers $n \geq 1$. What is the value of $f(9)$?

- (1) 80
- (2) 240
- (3) 200
- (4) 100
- (5) 120

Correct Answer: (1) 80

Solution:

Step 1: Understanding the Concept: The problem defines a function using a recursive relation involving the sum of its first 'n' terms. The goal is to find an explicit formula or a simpler recurrence relation for $f(n)$ to calculate $f(9)$.

Step 2: Key Formula or Approach:

Let $S(n) = f(1) + f(2) + \dots + f(n)$.

The given relation is $S(n) = n^2 f(n)$ for $n \geq 1$.

For $n \geq 2$, we can also write $S(n-1) = (n-1)^2 f(n-1)$.

The key insight is to use the identity $f(n) = S(n) - S(n-1)$ for $n \geq 2$.

By substituting the given relations, we can find a recurrence for $f(n)$ in terms of $f(n-1)$.

Step 3: Calculation:

Using the identity $f(n) = S(n) - S(n-1)$:

$$f(n) = n^2 f(n) - (n-1)^2 f(n-1) \quad (\text{for } n \geq 2)$$

Rearranging the terms to solve for $f(n)$:

$$(n-1)^2 f(n-1) = n^2 f(n) - f(n)$$

$$(n-1)^2 f(n-1) = (n^2 - 1) f(n)$$

$$(n-1)^2 f(n-1) = (n-1)(n+1) f(n)$$

Since $n > 1$, we can divide both sides by $(n - 1)$:

$$(n - 1)f(n - 1) = (n + 1)f(n)$$

This gives us a simpler recurrence relation:

$$f(n) = \frac{n - 1}{n + 1}f(n - 1)$$

Now we can find a general formula for $f(n)$ by expanding the recurrence:

$$f(n) = \frac{n - 1}{n + 1} \cdot f(n - 1) = \frac{n - 1}{n + 1} \cdot \frac{n - 2}{n} \cdot f(n - 2) = \dots$$

$$f(n) = \left(\frac{n - 1}{n + 1}\right) \left(\frac{n - 2}{n}\right) \left(\frac{n - 3}{n - 1}\right) \dots \left(\frac{2}{4}\right) \left(\frac{1}{3}\right) f(1)$$

Many terms cancel out (telescoping product):

$$f(n) = \frac{(n - 1)(n - 2) \dots 2 \cdot 1}{(n + 1)n(n - 1) \dots 4 \cdot 3} f(1) = \frac{1 \cdot 2}{(n + 1)n} f(1)$$

So, the explicit formula is:

$$f(n) = \frac{2f(1)}{n(n + 1)}$$

We need to find $f(9)$. We are given $f(1) = 3600$.

$$f(9) = \frac{2 \times 3600}{9 \times (9 + 1)} = \frac{7200}{9 \times 10} = \frac{7200}{90} = 80$$

Step 4: Final Answer: The value of $f(9)$ is 80.

$$\boxed{f(9) = 80}$$

Step 5: Why This is the Correct Option: The derived explicit formula for $f(n)$ is correct for all $n \geq 1$. Substituting $n=9$ and $f(1)=3600$ gives the value 80, which corresponds to option (1).

💡 Quick Tip

- For functional equations involving sums like $S(n) = \sum_{i=1}^n f(i)$, the relation $f(n) = S(n) - S(n - 1)$ is extremely useful to find a direct recurrence for $f(n)$. - Look for telescoping sums or products, where intermediate terms cancel out, simplifying the expression significantly.

4. Suppose you have a currency, named Miso, in three denominations: 1 Miso, 10 Misos and 50 Misos. In how many ways can you pay a bill of 107 Misos?

- (1) 17
- (2) 16
- (3) 18

(4) 15

(5) 19

Correct Answer: (3) 18

Solution:

Step 1: Understanding the Concept: This is a classic combinatorial problem known as the "change-making problem". We need to find the number of non-negative integer solutions to a linear Diophantine equation that represents the total value of the bill.

Step 2: Key Formula or Approach: Let x , y , and z be the number of 50 Miso, 10 Miso, and 1 Miso denominations used, respectively. The equation to solve is:

$$50x + 10y + z = 107$$

where x , y , and z are integers such that $x \geq 0, y \geq 0, z \geq 0$. The most systematic way to solve this is to iterate through the possible values of the variable with the largest coefficient (x), which constrains the possibilities for the other variables.

Step 3: Calculation: We will consider the possible values for x (number of 50 Miso coins). Since $50x \leq 107$, x can be 0, 1, or 2.

Case 1: $x = 2$ (Two 50 Miso coins) The equation becomes $50(2) + 10y + z = 107$, which simplifies to $100 + 10y + z = 107$, or $10y + z = 7$.

- If $y = 0$, then $z = 7$. This is one valid solution (2, 0, 7).
- If $y \geq 1$, then $10y \geq 10$, which is greater than 7. No other solutions in this case.

Total ways for $x = 2$: 1.

Case 2: $x = 1$ (One 50 Miso coin) The equation becomes $50(1) + 10y + z = 107$, which simplifies to $10y + z = 57$. Since $10y \leq 57$, y can take integer values from 0 to 5. For each choice of y , z is uniquely determined ($z = 57 - 10y$).

- $y = 0 \implies z = 57$
- $y = 1 \implies z = 47$
- $y = 2 \implies z = 37$
- $y = 3 \implies z = 27$
- $y = 4 \implies z = 17$
- $y = 5 \implies z = 7$

This gives 6 possible solutions. Total ways for $x = 1$: 6.

Case 3: $x = 0$ (Zero 50 Miso coins) The equation becomes $50(0) + 10y + z = 107$, which simplifies to $10y + z = 107$. Since $10y \leq 107$, y can take integer values from 0 to 10. For each choice of y , z is uniquely determined. The possible values for y are $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, which is 11 possible values. Total ways for $x = 0$: 11.

Total Number of Ways: Summing the ways from all cases: $1 + 6 + 11 = 18$.

Step 4: Final Answer: There are 18 different ways to pay a bill of 107 Misos.

18 ways

Step 5: Why This is the Correct Option: The case-by-case analysis based on the number of 50 Miso coins covers all possibilities without any overlap. The sum of the solutions from each distinct case gives the total number of ways, which is 18. This matches option (3).

 Quick Tip

- When solving Diophantine equations of the form $ax + by + cz = d$, always start by finding the possible range for the variable with the largest coefficient. This greatly simplifies the problem by reducing it to smaller subproblems.

5. A confused bank teller transposed the rupees and paise when he cashed a cheque for Shailaja. giving her rupees instead of paise and paise instead of rupees. After buying a toffee for 50 paise, Shailaja noticed that she was left with exactly three times as much as the amount on the cheque. Which of the following is a valid statement about the cheque amount?

- (1) Over Rupees 13 but less than Rupees 14
- (2) Over Rupees 7 but less than Rupees 8
- (3) Over Rupees 22 but less than Rupees 23
- (4) Over Rupees 18 but less than Rupees 19
- (5) Over Rupees 4 but less than Rupees 5

Correct Answer: (4) Over Rupees 18 but less than Rupees 19

Solution:

Step 1: Understanding the Concept: This problem involves setting up and solving a linear Diophantine equation based on a word problem. The core of the problem lies in correctly representing the value of money in terms of rupees and paise and translating the given conditions into a mathematical equation.

Step 2: Key Formula or Approach: Let the original cheque amount be R rupees and P paise. The value of the cheque in paise is $V_{original} = 100R + P$. The amount given by the teller after transposing is P rupees and R paise. The value of the amount received in paise is $V_{received} = 100P + R$. After spending 50 paise, the remaining amount is $V_{left} = V_{received} - 50 = (100P + R) - 50$. The problem states that this remaining amount is three times the original cheque amount:

$$V_{left} = 3 \times V_{original}$$

$$(100P + R) - 50 = 3(100R + P)$$

We must solve this equation for integers R and P , where $0 \leq R < 100$ and $0 \leq P < 100$.

Step 3: Calculation: Expand and rearrange the equation:

$$100P + R - 50 = 300R + 3P$$

$$100P - 3P - 300R - R = 50$$

$$97P - 299R = 50$$

We need to find integer solutions for this equation. We can express P in terms of R:

$$97P = 299R + 50$$

$$P = \frac{299R + 50}{97}$$

To simplify, we can use division: $299 = 3 \times 97 + 8$.

$$P = \frac{(3 \times 97 + 8)R + 50}{97} = 3R + \frac{8R + 50}{97}$$

For P to be an integer, $(8R + 50)$ must be an integer multiple of 97. Let this multiple be k.

$$8R + 50 = 97k$$

Since R and P are positive amounts on a cheque, k must be a positive integer. Let's test values of k:

- If $k = 1$: $8R + 50 = 97 \implies 8R = 47$. R is not an integer.
- If $k = 2$: $8R + 50 = 97 \times 2 = 194 \implies 8R = 144 \implies R = \frac{144}{8} = 18$.

So, we have a valid integer solution: $R = 18$. Now find the corresponding value of P:

$$P = 3R + \frac{8R + 50}{97} = 3(18) + \frac{8(18) + 50}{97} = 54 + \frac{144 + 50}{97} = 54 + \frac{194}{97} = 54 + 2 = 56$$

So, $P = 56$. Both $R=18$ and $P=56$ are valid as they are between 0 and 100. The original cheque amount was 18 rupees and 56 paise, or Rs 18.56.

Step 4: Final Answer: The cheque amount is Rs 18.56. This value is over Rupees 18 but less than Rupees 19.

Rs 18.56

Step 5: Why This is the Correct Option: The unique integer solution satisfying the conditions is $R=18$ and $P=56$. This corresponds to a cheque value of Rs 18.56. This amount falls in the range described by option (4): "Over Rupees 18 but less than Rupees 19".

💡 Quick Tip

- When dealing with money, it's often easiest to convert everything to the smallest unit (in this case, paise) to avoid decimals and work with integers. - The technique of expressing one variable in terms of another and separating the integer part is very effective for solving linear Diophantine equations.

6. How many pairs of positive integers m, n satisfy $\frac{1}{m} + \frac{4}{n} = \frac{1}{12}$, where, 'n' is an odd integer less than 60?

- (1) 6
- (2) 4
- (3) 7
- (4) 5
- (5) 3

Correct Answer: (5) 3

Solution:

Step 1: Understanding the Concept: This problem requires finding the number of positive integer solutions (m, n) to a given rational equation, with specific constraints on the variable 'n'. This is a type of Diophantine equation problem.

Step 2: Key Formula or Approach: The first step is to rearrange the equation to express one variable in terms of the other. Let's isolate the term with 'm'.

$$\frac{1}{m} = \frac{1}{12} - \frac{4}{n}$$

Combine the fractions on the right side:

$$\frac{1}{m} = \frac{n - 4 \times 12}{12n} = \frac{n - 48}{12n}$$

Now, solve for m:

$$m = \frac{12n}{n - 48}$$

Since m must be a positive integer, the numerator $12n$ must be positive, and the denominator $n - 48$ must also be positive. Since n is given as a positive integer, $12n > 0$. Therefore, we must have:

$$n - 48 > 0 \implies n > 48$$

Also, for m to be an integer, $(n - 48)$ must be a divisor of $12n$. We can simplify the expression for m using polynomial division (or by adding and subtracting a term):

$$m = \frac{12(n - 48) + 12 \times 48}{n - 48} = 12 + \frac{576}{n - 48}$$

This form is very useful. For m to be an integer, $(n - 48)$ must be a divisor of 576.

Step 3: Calculation: We need to find the values of 'n' that satisfy all the given conditions:

1. n is a positive integer.
2. n is odd.
3. $n \nmid 60$.
4. $n \nless 48$ (derived from the condition that m is positive).

Combining these conditions, we need to find odd integers n such that $48 < n < 60$. The possible values for n are 49, 51, 53, 55, 57, 59.

Now, we test each of these values to see if $(n - 48)$ is a divisor of 576.

- **n = 49:** $n - 48 = 1$. Is 1 a divisor of 576? Yes. $m = 12 + \frac{576}{1} = 588$. This is a valid integer solution (588, 49).
- **n = 51:** $n - 48 = 3$. Is 3 a divisor of 576? Yes ($5+7+6=18$, which is divisible by 3). $m = 12 + \frac{576}{3} = 12 + 192 = 204$. This is a valid integer solution (204, 51).
- **n = 53:** $n - 48 = 5$. Is 5 a divisor of 576? No (576 does not end in 0 or 5).
- **n = 55:** $n - 48 = 7$. Is 7 a divisor of 576? No ($576 = 7 \times 82 + 2$).

- **n = 57:** $n - 48 = 9$. Is 9 a divisor of 576? Yes ($5+7+6=18$, which is divisible by 9).
 $m = 12 + \frac{576}{9} = 12 + 64 = 76$. This is a valid integer solution (76, 57).
- **n = 59:** $n - 48 = 11$. Is 11 a divisor of 576? No ($576 = 11 \times 52 + 4$).

The pairs of positive integers (m, n) that satisfy the conditions are (588, 49), (204, 51), and (76, 57). There are 3 such pairs.

Step 4: Final Answer: There are 3 pairs of positive integers that satisfy the given equation and constraints.

3 pairs

Step 5: Why This is the Correct Option: By systematically analyzing the constraints on 'n', we found a small set of possible values. Testing each of these values against the divisibility condition revealed exactly 3 valid solutions. Therefore, option (5) is correct.

 Quick Tip

- A very useful technique for equations like $\frac{a}{x} + \frac{b}{y} = \frac{c}{d}$ is to isolate one variable and rewrite the expression to identify a divisibility condition. The form $m = k + \frac{\text{constant}}{\text{expression of } n}$ is ideal for testing integer solutions. - Remember to use all given constraints (positive, odd, less than 60) to narrow down the possibilities.

Directions for Questions 7 to 10: Each question is followed by two statements A and B. Indicate your response based on the following directives.

Mark (1) if the question can be answered using A alone but not using B alone.

Mark (2) if the question can be answered using B alone but not using A alone.

Mark (3) if the question can be answered using A and B together, but not using either A or B alone.

Mark (4) if the question cannot be answered even using A and B together.

7. The average weight of a class of 100 students is 45 kg. The class consists of two sections, I and II, each with 50 students. The average weight, W_I , of Section I is smaller than the average weight W_{II} , of the Section II. If the heaviest student say Deepak, of section II is moved to Section I, and the lightest student, say Poonam, of Section I is moved to Section II, then the average weights of the two sections are switched, i.e., the average weight of Section I becomes W_{II} and that of Section II becomes W_I . What is the weight of Poonam?

A: $W_{II} - W_I = 1.0$.

B: Moving Deepak from Section II to I (without any move I to II) makes the average weights of the two sections equal.

Correct Answer: (3) if the question can be answered using A and B together, but not using either A or B alone.

Solution:

Step 1: Understanding the Concept: This is a data sufficiency problem based on the concept of averages. The key is to translate the word problems into algebraic equations and determine if these equations can be solved for the required value (weight of Poonam).

Step 2: Key Formula or Approach: Let S_I and S_{II} be the initial total weights of Section I and II respectively. Let D be Deepak's weight and P be Poonam's weight. From the problem statement:

1. Total weight = $100 \times 45 = 4500$ kg.
2. $S_I = 50 \times W_I$; $S_{II} = 50 \times W_{II}$.
3. $S_I + S_{II} = 4500 \implies 50(W_I + W_{II}) = 4500 \implies W_I + W_{II} = 90$.
4. After swapping, new average of Section I is $\frac{S_I - P + D}{50} = W_{II}$. This simplifies to $\frac{50W_I - P + D}{50} = W_{II}$, which gives $D - P = 50(W_{II} - W_I)$.

We need to find the value of P .

Step 3: Calculation: Analysis of Statement A: " $W_{II} - W_I = 1.0$." We have two equations for W_I and W_{II} : $W_I + W_{II} = 90$ and $W_{II} - W_I = 1$. Solving them gives $W_{II} = 45.5$ and $W_I = 44.5$. Substituting into our main equation: $D - P = 50(1) = 50$. This gives a relationship between D and P , but we cannot find a unique value for P .

Statement A alone is not sufficient.

Analysis of Statement B: "Moving Deepak from Section II to I ... makes the average weights of the two sections equal."

The new number of students in Section I is 51, and in Section II is 49. New average of I = $\frac{S_I + D}{51}$. New average of II = $\frac{S_{II} - D}{49}$.

Setting them equal: $\frac{50W_I + D}{51} = \frac{50W_{II} - D}{49}$.

$$49(50W_I + D) = 51(50W_{II} - D).$$

$$2450W_I + 49D = 2550W_{II} - 51D.$$

$$100D = 2550W_{II} - 2450W_I \implies D = 25.5W_{II} - 24.5W_I.$$

This gives another relationship, but with W_I , W_{II} , D , and P , we still have too many unknowns.

Statement B alone is not sufficient.

Analysis of Statements A and B together:

From A, we have $W_I = 44.5$ and $W_{II} = 45.5$.

Substitute these into the equation from B:

$$D = 25.5(45.5) - 24.5(44.5) = 1160.25 - 1090.25 = 70.$$

So, Deepak's weight is 70 kg.

Now use the equation $D - P = 50$ (derived from A and the main problem).

$$70 - P = 50 \implies P = 20.$$

We have found a unique value for Poonam's weight.

Both statements together are sufficient.

Step 4: Final Answer: The question can be answered using A and B together, but not using either A or B alone.

Mark (3)

Step 5: Why This is the Correct Option: Statement A alone only provides the difference in weights between Deepak and Poonam. Statement B alone provides a different relationship between their weights and the average weights. Neither is sufficient on its own. When combined, they form a system of equations that can be solved to find a unique value for Poonam's weight.

 Quick Tip

For questions on averages, always convert averages back to sums (Sum = Average $\times$ Number of items) to build your equations. This avoids confusion when items are added, removed, or transferred between groups.

8. ABC Corporation is required to maintain at least 400 Kilolitres of water at all times in its factory, in order to meet safety and regulatory requirements. ABC is considering the suitability of a spherical tank with uniform wall thickness for the purpose. The outer diameter of the tank is 10 meters. Is the tank capacity adequate to meet ABC's requirements?

A: The inner diameter of the tank is at least 8 meters.

B: The tank weighs 30,000 kg when empty, and is made of a material with density of 3 gm/cc.

Correct Answer: (2) if the question can be answered using B alone but not using A alone.

Solution:

Step 1: Understanding the Concept: This is a data sufficiency question about the volume (capacity) of a hollow sphere. We need to determine if the inner volume is greater than or equal to a specified amount.

Step 2: Key Formula or Approach: The question is: Is the inner volume (V_{in}) $\geq$ 400 kilolitres? Key conversions and formulas:

- 1 kilolitre = 1 cubic meter (m^3). So, we need to check if $V_{in} \geq 400 m^3$.
- Volume of a sphere: $V = \frac{4}{3}\pi r^3$.
- Given: Outer diameter = 10 m $\implies$ Outer radius, $R_{out} = 5$ m.
- We need to find the inner radius, R_{in} , to calculate V_{in} .

Step 3: Calculation: Analysis of Statement A: "The inner diameter of the tank is at least 8 meters."

This means the inner radius, $R_{in} \geq 4$ m.

The capacity could be at its minimum if $R_{in} = 4$ m.

Minimum $V_{in} = \frac{4}{3}\pi(4)^3 = \frac{256\pi}{3} \approx \frac{256 \times 3.14}{3} \approx 268 m^3$.

Since $268 m^3 < 400 m^3$, the tank might not be adequate.

However, if $R_{in} = 4.8$ m (which is ≥ 4 m), then $V_{in} = \frac{4}{3}\pi(4.8)^3 \approx 463 m^3$, which is adequate.

Since the answer could be "No" or "Yes", Statement A is not sufficient.

Analysis of Statement B: "The tank weights 30,000 kg when empty, and is made of a material with density of 3 gm/cc." This allows us to find the volume of the material of the tank wall (V_{wall}).

- Mass (M) = 30,000 kg.
- Density (ρ) = 3 gm/cc = $3 \frac{10^{-3}\text{kg}}{(10^{-2}\text{m})^3} = 3 \frac{10^{-3}}{10^{-6}} \text{ kg/m}^3 = 3000 \text{ kg/m}^3$.
- $V_{\text{wall}} = \frac{M}{\rho} = \frac{30000}{3000} = 10 \text{ m}^3$.

The volume of the wall is the difference between the outer and inner volumes. $V_{\text{wall}} = V_{\text{out}} - V_{\text{in}}$.

$$10 = \frac{4}{3}\pi(R_{\text{out}})^3 - V_{\text{in}}.$$

$$10 = \frac{4}{3}\pi(5)^3 - V_{\text{in}} = \frac{500\pi}{3} - V_{\text{in}}.$$

$$V_{\text{in}} = \frac{500\pi}{3} - 10 \approx \frac{500 \times 3.14159}{3} - 10 \approx 523.6 - 10 = 513.6 \text{ m}^3.$$

Since 513.6 m^3 is definitely greater than 400 m^3 , we can give a definitive "Yes" answer.

Statement B alone is sufficient.

Step 4: Final Answer: The question can be answered using B alone but not using A alone.

Mark (2)

Step 5: Why This is the Correct Option: Statement A provides a range for the inner diameter, leading to uncertainty about the capacity. Statement B provides the mass and density of the material, which allows for a precise calculation of the tank's inner volume. This precise volume can be compared to the requirement, yielding a definite answer.

 Quick Tip

Be vigilant about units. This question mixes kilolitres, meters, kilograms, and gm/cc. Converting all quantities to a consistent system (like SI units: m^3 , m, kg) at the beginning is crucial to avoid errors.

9. Consider integers x, y, z. What is the minimum possible value of $x^2 + y^2 + z^2$?

A: $x + y + z = 89$.

B: Among x, y, z two are equal.

Correct Answer: (1) if the question can be answered using A alone but not using B alone.

Solution:

Step 1: Understanding the Concept: This question is about minimizing a quadratic expression ($x^2 + y^2 + z^2$) subject to certain constraints on the integer variables x, y, and z. A key mathematical principle is that for a fixed sum, the sum of squares is minimized when the numbers are as close to each other as possible.

Step 2: Key Formula or Approach: To minimize $\sum x_i^2$ subject to $\sum x_i = S$, the values of x_i should be chosen to be as close to the average S/n as possible.

Step 3: Calculation: Analysis of Statement A: " $x + y + z = 89$." The sum of the three integers is fixed at 89. To minimize the sum of their squares, we should choose x , y , and z to be as close to each other as possible. The average is $\frac{89}{3} = 29.66\dots$. The integers closest to 29.66... are 29 and 30. We need to choose three integers that sum to 89. The combination must be $\{29, 30, 30\}$. Check the sum: $29 + 30 + 30 = 89$. The minimum possible value of the sum of squares is:

$$29^2 + 30^2 + 30^2 = 841 + 900 + 900 = 2641$$

Since we can find a unique minimum value, Statement A alone is sufficient.

Analysis of Statement B: "Among x , y , z two are equal." Let's assume $y = z$. The expression to minimize is $x^2 + 2y^2$. This statement imposes a condition on the variables, but it does not constrain their values. The integers x and y could be anything. For example:

- If $x=1$, $y=1$, $z=1$, then $x^2 + y^2 + z^2 = 3$.
- If $x=0$, $y=0$, $z=0$, then $x^2 + y^2 + z^2 = 0$.
- If $x=10$, $y=10$, $z=11$, this is not allowed. If $x=11$, $y=10$, $z=10$, then sum of squares is $121 + 100 + 100 = 321$.

We cannot determine a unique minimum value for $x^2 + y^2 + z^2$ because there's no information to limit the values of x , y , and z . Statement B alone is not sufficient.

Step 4: Final Answer: The question can be answered using A alone but not using B alone.

Mark (1)

Step 5: Why This is the Correct Option: Statement A provides a fixed sum, which is a strong constraint that allows for the determination of a unique minimum for the sum of squares. Statement B only provides a relationship between the variables without constraining their magnitude, making it impossible to find a specific minimum value.

 Quick Tip

Remember the principle of minimizing sum of squares: distribute the total sum as evenly as possible among the variables. For integers, this means using values that are consecutive or identical.

10. Rahim plans to draw a square JKLM with point O on the side JK but is not successful. Why is Rahim unable to draw the square?

A: The length of OM is twice that of OL.

B: The length of OM is 4 cm.

Correct Answer: (1) if the question can be answered using A alone but not using B alone.

Solution:

Step 1: Understanding the Concept: This is a data sufficiency question based on coordinate geometry and the properties of a square. A statement is sufficient if it demonstrates a geometric impossibility, thus explaining why the square cannot be drawn.

Step 2: Key Formula or Approach: Let's set up a coordinate system. Let the vertices of the square JKLM be J(0, s), K(s, s), L(s, 0), and M(0, 0), where 's' is the side length. Point O is on the side JK, so its coordinates are O(x, s), where $0 \leq x \leq s$. We can now write the squared distances from O to L and M using the distance formula $d^2 = (\Delta x)^2 + (\Delta y)^2$.

$$OL^2 = (s - x)^2 + (0 - s)^2 = (s - x)^2 + s^2$$

$$OM^2 = (0 - x)^2 + (0 - s)^2 = x^2 + s^2$$

The question asks for a reason for the impossibility. A statement is sufficient if it leads to a mathematical contradiction.

Step 3: Calculation: Analysis of Statement A: "The length of OM is twice that of OL." This means $OM = 2(OL)$, or $OM^2 = 4(OL^2)$. Substitute our expressions:

$$x^2 + s^2 = 4[(s - x)^2 + s^2]$$

$$x^2 + s^2 = 4[s^2 - 2sx + x^2 + s^2]$$

$$x^2 + s^2 = 4[2s^2 - 2sx + x^2]$$

$$x^2 + s^2 = 8s^2 - 8sx + 4x^2$$

Rearrange into a quadratic equation in terms of x:

$$3x^2 - 8sx + 7s^2 = 0$$

For a real point O to exist, this quadratic equation (for the variable x) must have a real solution. Let's check the discriminant ($\Delta = b^2 - 4ac$):

$$\Delta = (-8s)^2 - 4(3)(7s^2) = 64s^2 - 84s^2 = -20s^2$$

Since the side length 's' must be positive, s^2 is positive. Therefore, the discriminant Δ is negative. A negative discriminant means there are no real solutions for x. This implies that it's geometrically impossible for a point O on JK to satisfy $OM = 2(OL)$. This provides a valid reason for Rahim's failure. *Statement A alone is sufficient.*

Analysis of Statement B: "The length of OM is 4 cm." This gives the equation $OM^2 = 16$.

$$x^2 + s^2 = 16$$

This is a single equation with two variables, x and s. We need to check if a valid solution exists. For O to be on JK, we need $0 \leq x \leq s$. Can we find such s and x? Let $s = 3$. Then $x^2 + 9 = 16 \implies x^2 = 7 \implies x = \sqrt{7}$. Since $2 < \sqrt{7} < 3$, the condition $0 \leq \sqrt{7} \leq 3$ is met. So, a square with side length 3 and O at $(\sqrt{7}, 3)$ is possible. Since it is possible to draw such a square, this statement does not explain why Rahim failed. *Statement B alone is not sufficient.*

Step 4: Final Answer: The question can be answered using A alone but not using B alone.

Mark (1)

Step 5: Why This is the Correct Option: Statement A creates a geometric condition that leads to an algebraic equation with no real solutions, thus proving the impossibility of drawing the square under that condition. Statement B provides a condition for which a valid square can be constructed, so it fails to explain the failure.

💡 Quick Tip

To prove a geometric construction is impossible, a powerful method is to translate it into an algebraic system and show that this system has no real solutions. A common way this occurs is with a quadratic equation having a negative discriminant.

Directions for Questions 11 and 12: Answer the following questions based on the information given below:

Cities A and B are in different time zones. A is located 3000 km east of B. The table below describes the schedule of an airline operating non-stop flights between A and B. All the times indicated are local and on the same day.

Departure		Arrival	
City	Time	City	Time
B	8:00 am	A	3:00 pm
A	4:00 pm	B	8:00 pm

Assume that planes cruise at the same speed in both directions. However, the effective speed is influenced by a steady wind blowing from east to west at 50 km per hour.

11. What is the time difference between A and B?

- (A) 1 hour and 30 minutes
- (B) 2 hours
- (C) 2 hours and 30 minutes
- (D) 1 hour
- (E) Cannot be determined

Correct Answer: (D) 1 hour

Solution:

Step 1: Understanding the Concept: This problem involves calculations with relative speed and time zones. The key is to set up equations for the two journeys, accounting for the plane's speed, wind speed, and the time zone difference. Since City A is east of City B, its local time will be ahead of City B's local time.

Step 2: Key Formula or Approach: Let p be the plane's cruising speed (km/hr) and w be the wind speed (50 km/hr). Let t be the time difference between A and B in hours. Let D be the distance (3000 km). The wind blows from East to West.

- Journey from B to A (West to East): The plane flies against the wind. Effective speed = $p - w$.
- Journey from A to B (East to West): The plane flies with the wind. Effective speed = $p + w$.

The actual travel time can be related to local departure/arrival times and the time zone difference t .

$$\text{Actual Time} = \frac{\text{Distance}}{\text{Effective Speed}}$$

For flight B $\rightarrow$ A: Actual time $T_{BA} = (15:00 - 8:00) - t = 7 - t$ hours. For flight A $\rightarrow$ B: Actual time $T_{AB} = (20:00 - 16:00) + t = 4 + t$ hours.

Step 3: Calculation: Using the formulas from Step 2, we can set up two equations:

$$T_{BA} = \frac{D}{p - w} \implies 7 - t = \frac{3000}{p - 50} \quad \dots (1)$$

$$T_{AB} = \frac{D}{p + w} \implies 4 + t = \frac{3000}{p + 50} \quad \dots (2)$$

To solve for t , we first need to find p . We can eliminate t by adding the two equations:

$$(7 - t) + (4 + t) = \frac{3000}{p - 50} + \frac{3000}{p + 50}$$

$$11 = 3000 \left(\frac{(p + 50) + (p - 50)}{(p - 50)(p + 50)} \right)$$

$$11 = 3000 \left(\frac{2p}{p^2 - 2500} \right)$$

$$11(p^2 - 2500) = 6000p$$

$$11p^2 - 6000p - 27500 = 0$$

Solving this quadratic equation for p , we find $p = 550$ km/hr (the other root is negative and thus not physically possible). Now, substitute $p = 550$ into equation (2):

$$4 + t = \frac{3000}{550 + 50}$$

$$4 + t = \frac{3000}{600}$$

$$4 + t = 5$$

$$t = 5 - 4 = 1$$

Step 4: Final Answer: The time difference t is 1 hour.

$$\boxed{t = 1 \text{ hour}}$$

Step 5: Why This is the Correct Option: The calculation yields a time difference of 1 hour, which corresponds to option (D). Verifying with equation (1): $7 - 1 = 6$, and $\frac{3000}{550 - 50} = \frac{3000}{500} = 6$. The results are consistent.

 Quick Tip

When dealing with time zone and travel problems:

- Clearly define variables for unknown quantities like speed and time difference.
- Remember that travel time is affected by both speed and time zones. For eastbound travel, you 'lose' time, so subtract the time difference. For westbound travel, you 'gain' time, so add the time difference.
- Adding or subtracting the travel time equations is often a quick way to eliminate one of the variables.

12. What is the plane's cruising speed in km per hour?

- (A) 700
(B) 550
(C) 600
(D) 500
(E) Cannot be determined.

Correct Answer: (B) 550

Solution:

Step 1: Understanding the Concept: This question is a continuation of the previous problem. The core concepts are relative speed (accounting for wind) and the relationship between distance, speed, and time. The goal is to calculate the plane's own speed in still air, referred to as its cruising speed.

Step 2: Key Formula or Approach: We use the same setup as in Question 11. Let p be the plane's cruising speed. The two main equations are:

$$7 - t = \frac{3000}{p - 50} \quad \dots (1)$$

$$4 + t = \frac{3000}{p + 50} \quad \dots (2)$$

To find p , we can add the two equations to eliminate t , which leads to a quadratic equation in terms of p .

Step 3: Calculation: Adding equation (1) and (2):

$$(7 - t) + (4 + t) = \frac{3000}{p - 50} + \frac{3000}{p + 50}$$

$$11 = 3000 \left(\frac{1}{p - 50} + \frac{1}{p + 50} \right)$$

$$11 = 3000 \left(\frac{p + 50 + p - 50}{(p - 50)(p + 50)} \right)$$

$$11 = 3000 \left(\frac{2p}{p^2 - 50^2} \right)$$

$$11(p^2 - 2500) = 6000p$$

$$11p^2 - 6000p - 27500 = 0$$

We solve this quadratic equation using the formula $p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$:

$$p = \frac{6000 \pm \sqrt{(-6000)^2 - 4(11)(-27500)}}{2(11)}$$

$$p = \frac{6000 \pm \sqrt{36000000 + 1210000}}{22}$$

$$p = \frac{6000 \pm \sqrt{37210000}}{22}$$

$$p = \frac{6000 \pm 6100}{22}$$

Since speed must be a positive value, we take the positive root:

$$p = \frac{6000 + 6100}{22} = \frac{12100}{22} = 550$$

Step 4: Final Answer: The plane's cruising speed is 550 km per hour.

$$p = 550 \text{ km/hr}$$

Step 5: Why This is the Correct Option: The solution to the derived quadratic equation for the plane's speed is 550 km/hr. This matches option (B).

💡 Quick Tip

For round trip problems with constant headwind/tailwind:

- The total time for the trip is always longer than if there were no wind.
- A useful shortcut is recognizing that the average speed over the round trip is NOT the plane's cruising speed.
- Let T_1 and T_2 be the actual travel times. Then $T_1 + T_2 = D/(p - w) + D/(p + w)$. In this problem, $T_1 + T_2$ is found by adding the local time equations, which gave 11 hours.

Directions for Questions 13 and 14: Answer the following questions based on the information given below:

Shabnam is considering three alternatives to invest her surplus cash for a week. She wishes to guarantee maximum returns on her investment. She has three options, each of which can be

utilized fully or partially in conjunction with others.

Option A: Invest in a public sector bank. It promises a return of +0.10%.

Option B: Invest in mutual funds of ABC Ltd. A rise in the stock market will result in a return of + 5% while a fall will entail a return of -3%.

Option C: Invest in mutual funds of CBA Ltd. A rise in the stock market will result in a return of -2.5%, while a fall will entail a return of +2%.

13. The maximum guaranteed return to Shabnam is

- (A) 0.25%
- (B) 0.10%
- (C) 0.20%
- (D) 0.15%
- (E) 0.30%

Correct Answer: (C) 0.20%

Solution:

Step 1: Understanding the Concept: The goal is to find the "maximum guaranteed return". This implies a strategy that yields the same return regardless of the market's movement (rise or fall). This is a portfolio hedging problem where we combine risky assets (B and C) to create a risk-free return, and then compare this return to the already risk-free asset (A).

Step 2: Key Formula or Approach: Let x , y , and z be the proportions of the total investment allocated to options A, B, and C, respectively, where $x + y + z = 1$. The total return R depends on the market scenario:

- If market rises: $R_{\text{rise}} = 0.10x + 5y - 2.5z$
- If market falls: $R_{\text{fall}} = 0.10x - 3y + 2z$

For the return to be guaranteed, it must be the same in both scenarios: $R_{\text{rise}} = R_{\text{fall}}$.

Step 3: Calculation: Set the returns equal to each other to find the hedging condition:

$$0.10x + 5y - 2.5z = 0.10x - 3y + 2z$$

The term with x cancels out, which means the guaranteed return from combining B and C is independent of A.

$$5y - 2.5z = -3y + 2z$$

$$8y = 4.5z$$

$$8y = \frac{9}{2}z \implies 16y = 9z$$

This implies that to get a guaranteed return from Options B and C, the investment amounts must be in the ratio $\frac{y}{z} = \frac{9}{16}$. Now, let's calculate the value of this guaranteed return. We can create a new "combined asset" from B and C with this ratio. Let's assume we invest 9 parts in B and 16 parts in C. The total investment is $9 + 16 = 25$ parts. The return on these 25 parts is:

- If market rises: $5 \times 9 - 2.5 \times 16 = 45 - 40 = 5$

- If market falls: $-3 \times 9 + 2 \times 16 = -27 + 32 = 5$

The return is 5 units for an investment of 25 units. The percentage return is:

$$\text{Return from B+C mix} = \frac{5}{25} = 0.20\%$$

So, any money invested in a portfolio of B and C with a 9:16 ratio will yield a guaranteed return of 0.20%. Now, Shabnam has two choices for a guaranteed return:

1. Invest in Option A for a return of 0.10%.
2. Invest in the B+C combination for a return of 0.20%.

To maximize her guaranteed return, she should choose the option with the higher return, which is 0.20%.

Step 4: Final Answer: The maximum guaranteed return Shabnam can achieve is 0.20%.

0.20%

Step 5: Why This is the Correct Option: By combining the two risky assets B and C in a specific ratio (9:16), we can eliminate the market risk and create a synthetic risk-free asset that yields 0.20%. This return is higher than the 0.10% offered by the risk-free bank deposit (Option A). Therefore, the highest possible guaranteed return is 0.20%, which corresponds to option (C).

💡 Quick Tip

In portfolio problems asking for a "guaranteed" return from risky assets, the first step is always to set the returns from different scenarios equal to each other. This gives the "hedging ratio" that eliminates risk. Once you have this synthetic risk-free asset, compare its return to any other explicit risk-free options available.

14. What strategy will maximize the guaranteed return to Shabnam?

- (A) 100% in option A
- (B) 36% in option B and 64% in option C
- (C) 64% in option B and 36% in option C
- (D) 1/3 in each of the three options
- (E) 30% in option A, 32% in option B and 38% in option C

Correct Answer: (B) 36% in option B and 64% in option C

Solution:

Step 1: Understanding the Concept: This question asks for the specific investment allocation (the strategy) that produces the maximum guaranteed return calculated in the previous

question. We need to find the percentage of total capital allocated to each option (A, B, and C).

Step 2: Key Formula or Approach: From the solution to Question 13, we determined that the maximum guaranteed return is 0.20%, which is achieved by investing in a combination of options B and C. This return is higher than the 0.10% from option A. Therefore, to maximize the guaranteed return, the entire investment should be placed in the B and C combination, meaning the allocation to option A is zero. The conditions for the optimal strategy are:

$$x = 0$$

$$y + z = 1 \quad (\text{or } 100\%)$$

$$16y = 9z \quad (\text{the hedging ratio})$$

Step 3: Calculation: We have a system of two equations with two variables, y and z :

$$y + z = 1 \quad \dots (1)$$

$$16y = 9z \implies y = \frac{9}{16}z \quad \dots (2)$$

Substitute (2) into (1):

$$\frac{9}{16}z + z = 1$$

$$\frac{9z + 16z}{16} = 1$$

$$25z = 16$$

$$z = \frac{16}{25} = 0.64$$

So, 64% of the investment should be in option C. Now find y :

$$y = 1 - z = 1 - \frac{16}{25} = \frac{9}{25} = 0.36$$

So, 36% of the investment should be in option B. The final strategy is:

- 0% in Option A
- 36% in Option B
- 64% in Option C

Step 4: Final Answer: The strategy that maximizes the guaranteed return is to invest 36% in option B and 64% in option C.

Strategy: 0% in A, 36% in B, 64% in C

Step 5: Why This is the Correct Option: The calculated allocation (36% in B, 64% in C) perfectly matches the description in option (B). Any other allocation would either result in a lower guaranteed return (e.g., investing in A) or would not provide a guaranteed return at all (i.e., the return would depend on the market's movement).

 Quick Tip

Once you find the optimal ratio for a part of the portfolio (like $y/z = 9/16$), remember that the proportions must sum to 1 (or 100)

Directions for Questions 15 and 16: Answer the following questions based on the information given below:

Let S be the set of all pairs (i, j) where, $1 \leq i < j \leq n$ and $n \geq 4$. Any two distinct members of S are called "friends" if they have one constituent of the pairs in common and "enemies" otherwise. For example, if $n = 4$, then $S = \{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)\}$. Here, $(1, 2)$ and $(1, 3)$ are friends, $(1, 2)$ and $(2, 3)$ are also friends, but $(1, 4)$ and $(2, 3)$ are enemies.

15. For general 'n', how many enemies will each member of S have?

- (A) $n - 3$
- (B) $\frac{1}{2}(n^2 - 3n - 2)$
- (C) $2n - 7$
- (D) $\frac{1}{2}(n^2 - 5n + 6)$
- (E) $\frac{1}{2}(n^2 - 7n + 14)$

Correct Answer: (D) $\frac{1}{2}(n^2 - 5n + 6)$

Solution:

Step 1: Understanding the Concept: This is a combinatorial problem. We need to find the number of "enemies" for an arbitrary pair in the set S . The number of enemies is the total number of pairs in S minus the number of "friends" of the chosen pair, and also subtracting the pair itself.

Step 2: Key Formula or Approach: 1. Calculate the total number of pairs in the set S . 2. Pick an arbitrary member of S , say (i, j) , and count its number of friends. 3. The number of enemies will be: Total pairs - (Number of friends + 1).

The total number of pairs in S is the number of ways to choose 2 distinct elements from n elements, which is given by the combination formula:

$$|S| = \binom{n}{2} = \frac{n(n-1)}{2}$$

Step 3: Calculation: Let's consider an arbitrary pair (i, j) from the set S . First, we find the number of friends of (i, j) . Friends are pairs that share exactly one element with (i, j) .

Friends of (i, j) can be of two types:

1. Pairs containing 'i' but not 'j'. These are of the form (i, k) where $k \neq j$, or (k, i) where $k \neq j$. Since the condition for S is $k < i$ or $k > i$, we can count them systematically. The elements other than i and j are $\{1, 2, \dots, n\} \setminus \{i, j\}$. There are $n - 2$ such elements. Any of these can form a pair with 'i'. So, there are $n - 2$ friends containing 'i'.

2. Pairs containing 'j' but not 'i'. Similarly, these pairs are formed with 'j' and one of the $n - 2$ elements from $\{1, 2, \dots, n\} \setminus \{i, j\}$. So, there are $n - 2$ friends containing 'j'.

The total number of friends of (i, j) is the sum of these two counts:

$$\text{Number of friends} = (n - 2) + (n - 2) = 2n - 4$$

Now, we can find the number of enemies. The total number of pairs in S is $\binom{n}{2}$. For a given pair (i, j), every other pair is either a friend or an enemy.

$$\text{Number of enemies} = (\text{Total pairs in S}) - 1 - (\text{Number of friends of (i, j)})$$

$$\begin{aligned} \text{Number of enemies} &= \frac{n(n-1)}{2} - 1 - (2n - 4) \\ &= \frac{n^2 - n}{2} - 1 - 2n + 4 \\ &= \frac{n^2 - n - 2(1) - 2(2n) + 2(4)}{2} \\ &= \frac{n^2 - n - 2 - 4n + 8}{2} \\ &= \frac{n^2 - 5n + 6}{2} \end{aligned}$$

Step 4: Final Answer: The number of enemies for each member of S is $\frac{1}{2}(n^2 - 5n + 6)$.

$$\boxed{\frac{1}{2}(n^2 - 5n + 6)}$$

Step 5: Why This is the Correct Option: The derived expression exactly matches option (D). The logic of counting the total pairs and subtracting the friends (and the pair itself) is sound.

💡 Quick Tip

In counting problems, it's often easier to count the complement. Here, instead of directly counting pairs with no common elements, we counted the pairs with a common element (friends) and subtracted from the total. Remember to also subtract the element itself from the total count.

16. For general 'n', consider any two members of S that are friends. How many other members of S will be common friends of both these members?

- (A) $\frac{1}{2}(n^2 - 5n + 8)$
 (B) $2n - 6$
 (C) $\frac{1}{2}n(n - 3)$

- (D) $n - 2$
 (E) $\frac{1}{2}(n^2 - 7n + 16)$

Correct Answer: (D) $n - 2$

Solution:

Step 1: Understanding the Concept: This problem asks for the size of the intersection of the "friend" sets of two friendly pairs. We need to pick two pairs that are friends and then find all other pairs that are friends with both of them simultaneously.

Step 2: Key Formula or Approach: Let's consider two members of S that are friends. Since they are friends, they share one element. Let the two pairs be $P_1 = (i, j)$ and $P_2 = (i, k)$, where i, j, k are distinct. We are looking for a third pair, $P_3 = (a, b)$, which is a friend to both P_1 and P_2 (and P_3 is not P_1 or P_2).

A common friend P_3 must satisfy:

- P_3 is a friend of $P_1 = (i, j) \implies P_3$ contains 'i' or 'j'.
- P_3 is a friend of $P_2 = (i, k) \implies P_3$ contains 'i' or 'k'.

Combining these conditions using logical AND: P_3 must contain ('i' or 'j') AND ('i' or 'k'). This simplifies to: P_3 contains 'i' OR (P_3 contains 'j' AND 'k').

Step 3: Calculation: We count the common friends based on the two cases derived above.

Case 1: The common friend P_3 contains the element 'i'. The pair P_3 will be of the form (i, m) or (m, i) . The element 'm' must be different from i, j , and k for P_3 to be distinct from P_1 and P_2 . There are n numbers from 1 to n . The numbers i, j, k are three distinct numbers. So, 'm' can be any number from the set $\{1, 2, \dots, n\} \setminus \{i, j, k\}$. The number of choices for 'm' is $n - 3$. So there are $n - 3$ common friends of this type.

Case 2: The common friend P_3 contains both 'j' and 'k'. The pair P_3 must be (j, k) (assuming $j \nmid k$, or (k, j) if $k \nmid j$). This pair, (j, k) , is a friend of (i, j) because they share 'j'. It is also a friend of (i, k) because they share 'k'. So, the pair (j, k) is a common friend. There is exactly 1 such pair.

Total Common Friends: The total number of common friends is the sum of the counts from both disjoint cases.

$$\text{Total common friends} = (\text{from Case 1}) + (\text{from Case 2}) = (n - 3) + 1 = n - 2$$

The result would be the same if we had chosen pairs like (i, k) and (j, k) initially.

Step 4: Final Answer: There are $n - 2$ other members of S that are common friends to both members.

$$\boxed{n - 2}$$

Step 5: Why This is the Correct Option: The logical breakdown of the properties of a common friend leads to two distinct cases. Summing the number of possibilities in each case gives $n - 2$, which corresponds to option (D).

💡 Quick Tip

For problems involving relationships between sets or objects, it is helpful to choose specific but general examples. Here, picking two friendly pairs as (i, j) and (i, k) simplifies the logic. Always check if other configurations (e.g., (i, j) and (k, j)) would change the result. In this case, the logic and the answer remain the same.

17. In a tournament, there are n teams $T_1, T_2, \dots, T_n$, with $n \geq 5$. Each team consists of ' k ' players, $k \geq 3$. The following pairs of teams have one player in common: $T_1 \& T_2, T_2 \& T_3, \dots, T_{n-1} \& T_n$, and $T_n \& T_1$. No other pair of teams has any player in common. How many players are participating in the tournament, considering all the ' n ' teams together?

- (A) $n(k-1)$
- (B) $k(n-1)$
- (C) $n(k-2)$
- (D) $k(n-2)$
- (E) $(n-1)(k-1)$

Correct Answer: (A) $n(k-1)$

Solution:

Step 1: Understanding the Concept: This is a counting problem that can be solved using the Principle of Inclusion-Exclusion or by direct logical counting. We need to find the total number of unique players across all teams, given a specific pattern of player overlaps between the teams.

Step 2: Key Formula or Approach: Let $P(T_i)$ be the set of players in team T_i . We want to find $|\bigcup_{i=1}^n P(T_i)|$. **Method 1: Direct Counting** We can sum the players team by team, adding only the new, uncounted players at each step. **Method 2: Formulaic Approach (Inclusion-Exclusion)** Total Players = (Sum of players in all teams) - (Sum of players in intersections of pairs) + ... Given the problem constraints, intersections of three or more teams are empty.

Step 3: Calculation: Using Method 2 (Inclusion-Exclusion): The total number of players if there were no overlaps would be $n \times k$.

$$\sum_{i=1}^n |P(T_i)| = nk$$

The problem states that there are ' n ' pairs of teams with a common player, and each of these intersections has exactly 1 player. The pairs are $(T_1, T_2), (T_2, T_3), \dots, (T_{n-1}, T_n), (T_n, T_1)$. The sum of players in these intersections is:

$$\sum_{\text{adjacent pairs}} |P(T_i) \cap P(T_j)| = n \times 1 = n$$

No other pair of teams has a player in common, so for all non-adjacent pairs (T_i, T_j) , $|P(T_i) \cap P(T_j)| = 0$. Also, consider any three teams, e.g., T_1, T_2, T_3 . A player in $P(T_1) \cap P(T_2) \cap P(T_3)$ would imply a player common to T_1 and T_3 . But this is not allowed. Therefore, all intersections of three or more teams are empty. Using the Principle of Inclusion-Exclusion:

$$\text{Total Players} = \sum |P(T_i)| - \sum |P(T_i) \cap P(T_j)|$$

$$\text{Total Players} = nk - n = n(k - 1)$$

Using Method 1 (Direct Counting): Let's count the number of unique players contributed by each role.

1. **Shared Players:** There is one player shared between T_1 and T_2 , one between T_2 and T_3 , and so on, in a cycle. This gives 'n' shared players. These 'n' players must all be distinct. If the player between T_1, T_2 was the same as the one between T_2, T_3 , then that player would be on all three teams, which would mean T_1 and T_3 have a common player, which is forbidden. So, we have 'n' distinct shared players.
2. **Non-shared Players:** Each team T_i has k players. It shares one player with T_{i-1} (or T_n for $i = 1$) and one player with T_{i+1} (or T_1 for $i = n$). These two shared players are distinct. Therefore, each team has $k - 2$ players who are unique to that team.

Total number of players = (Total unique players in all teams) + (Total shared players)

$$\text{Total Players} = n \times (k - 2) + n = nk - 2n + n = nk - n = n(k - 1)$$

Step 4: Final Answer: The total number of players participating in the tournament is $n(k - 1)$.

$$\boxed{n(k - 1)}$$

Step 5: Why This is the Correct Option: Both methods of calculation consistently yield the result $n(k - 1)$. This corresponds directly to option (A).

💡 Quick Tip

Visualizing the problem can help. Think of the teams as vertices of an n-sided polygon, and the shared players as being on the edges connecting the vertices. This makes it clear that there are 'n' teams and 'n' unique shared players in a cyclical structure.

18. Consider four-digit numbers for which the first two digits are equal and the last two digits are also equal. How many such numbers are perfect squares?

- (A) 3
- (B) 2
- (C) 4

- (D) 0
(E) 1

Correct Answer: (E) 1

Solution:

Step 1: Understanding the Concept: This is a number theory problem. We need to find numbers of a specific form (AABB) that are also perfect squares. The approach is to express the number algebraically and use the properties of perfect squares to constrain the possible values for the digits A and B.

Step 2: Key Formula or Approach: A four-digit number with the first two digits equal to A and the last two digits equal to B can be written as:

$$N = 1000A + 100A + 10B + B$$

We can simplify this expression and then set it equal to k^2 for some integer k .

$$N = 1100A + 11B = 11(100A + B)$$

For N to be a perfect square, since it has a factor of 11, it must have a factor of 11^2 . This means the term $(100A + B)$ must also have a factor of 11.

Step 3: Calculation: We have the condition that $100A + B$ must be divisible by 11. Using the divisibility rule for 11, or modular arithmetic:

$$100A + B \equiv 0 \pmod{11}$$

Since $100 = 9 \times 11 + 1$, we have $100 \equiv 1 \pmod{11}$.

$$1 \cdot A + B \equiv 0 \pmod{11} \implies A + B \text{ is a multiple of 11.}$$

The digits A and B have constraints: A must be in $\{1, 2, \dots, 9\}$ (since it's a four-digit number). B must be in $\{0, 1, \dots, 9\}$. The sum $A + B$ must therefore be in the range $[1+0, 9+9]$, i.e., $[1, 18]$. The only multiple of 11 in this range is 11 itself. So, we must have:

$$A + B = 11$$

Now, let's go back to our main condition: $N = 11(100A + B) = k^2$. Let $100A + B = 11m$. Then $N = 11(11m) = 121m$. For N to be a perfect square, m must also be a perfect square. Let $m = p^2$. Let's find the value of m :

$$m = \frac{100A + B}{11} = \frac{99A + A + B}{11} = \frac{99A + 11}{11} = 9A + 1$$

So, we need $9A + 1$ to be a perfect square.

$$9A + 1 = p^2$$

We can now test the possible values for A (from 1 to 9):

- $A = 1$: $9(1) + 1 = 10$ (Not a square)
- $A = 2$: $9(2) + 1 = 19$ (Not a square)

- $A = 3: 9(3) + 1 = 28$ (Not a square)
- $A = 4: 9(4) + 1 = 37$ (Not a square)
- $A = 5: 9(5) + 1 = 46$ (Not a square)
- $A = 6: 9(6) + 1 = 55$ (Not a square)
- $A = 7: 9(7) + 1 = 63 + 1 = 64 = 8^2$ (This is a perfect square!)
- $A = 8: 9(8) + 1 = 73$ (Not a square)
- $A = 9: 9(9) + 1 = 82$ (Not a square)

The only value of A that works is $A = 7$. If $A = 7$, we find B from $A + B = 11$, which gives $B = 11 - 7 = 4$. The number is $AABB = 7744$. Let's verify: $7744 = 11 \times 704 = 11 \times (11 \times 64) = 121 \times 64 = (11 \times 8)^2 = 88^2$. So, 7744 is a perfect square. This is the only solution.

Step 4: Final Answer: There is exactly one such four-digit number.

1

Step 5: Why This is the Correct Option: The algebraic simplification and application of number theory properties correctly narrowed down the possibilities to a single case ($A=7$, $B=4$). The exhaustive check confirms that no other digits work. Therefore, there is only one such number.

 Quick Tip

When a problem involves perfect squares, look for prime factors. If a number $N = p \cdot X$ is a perfect square, and p is a prime, then X must have p as a factor. In this case, $N = 11 \cdot (100A + B)$, which immediately told us that $(100A + B)$ must be a multiple of 11. This is a powerful starting point.

Directions for Questions 19 and 20: Answer the following questions based on the information given below:

Mr. David manufactures and sells a single product at a fixed price in a niche market. The selling price of each unit is Rs. 30. On the other hand, the cost, in rupees, of producing 'x' units is $240 + bx + cx^2$, where 'b' and 'c' are some constants. Mr. David noticed that doubling the daily production from 20 to 40 units increases the daily production cost by $66\frac{2}{3}\%$. However, an increase in daily production from 40 to 60 units results in an increase of only 50% in the daily production cost. Assume that demand is unlimited and that Mr. David can sell as much as he can produce. His objective is to maximize the profit.

19. How many units should Mr. David produce daily?

- (A) 130
(B) 100

- (C) 70
 (D) 150
 (E) Cannot be determined

Correct Answer: (B) 100

Solution:

Step 1: Understanding the Concept: This problem involves profit maximization. The profit function is the difference between the revenue function and the cost function. Since the cost function is quadratic, the profit function will also be quadratic. The maximum of a quadratic function $P(x) = ax^2 + bx + c$ (with $a < 0$) occurs at the vertex, $x = -b/(2a)$.

Step 2: Key Formula or Approach: 1. Define the Cost function $C(x) = 240 + bx + cx^2$. 2. Use the given information to create two equations and solve for b and c. 3. Define the Revenue function $R(x) = 30x$. 4. Define the Profit function $P(x) = R(x) - C(x)$. 5. Find the value of x that maximizes $P(x)$.

Information provided:

- $C(40) = C(20) + \frac{2}{3}C(20) = \frac{5}{3}C(20)$
- $C(60) = C(40) + \frac{1}{2}C(40) = \frac{3}{2}C(40)$

Step 3: Calculation: First, let's form the equations for b and c. From $3C(40) = 5C(20)$:

$$3(240 + 40b + 1600c) = 5(240 + 20b + 400c)$$

$$720 + 120b + 4800c = 1200 + 100b + 2000c$$

$$20b + 2800c = 480$$

Dividing by 20: $b + 140c = 24 \quad \dots (1)$

From $2C(60) = 3C(40)$:

$$2(240 + 60b + 3600c) = 3(240 + 40b + 1600c)$$

$$480 + 120b + 7200c = 720 + 120b + 4800c$$

The '120b' terms cancel out.

$$2400c = 240 \implies c = \frac{240}{2400} = 0.1$$

Substitute $c = 0.1$ into equation (1):

$$b + 140(0.1) = 24$$

$$b + 14 = 24 \implies b = 10$$

So, the cost function is $C(x) = 240 + 10x + 0.1x^2$.

Now, let's find the profit function $P(x)$:

$$P(x) = R(x) - C(x) = 30x - (240 + 10x + 0.1x^2)$$

$$P(x) = -0.1x^2 + 20x - 240$$

This is a downward-opening parabola. The profit is maximized at the vertex. The x-coordinate of the vertex is given by $x = \frac{-(\text{coefficient of } x)}{2(\text{coefficient of } x^2)}$.

$$x = \frac{-20}{2(-0.1)} = \frac{-20}{-0.2} = 100$$

Step 4: Final Answer: To maximize profit, Mr. David should produce 100 units daily.

100 units

Step 5: Why This is the Correct Option: The calculations correctly determine the cost function parameters and then find the vertex of the resulting quadratic profit function. The x-value of the vertex, 100, represents the production level for maximum profit, matching option (B).

 Quick Tip

For any quadratic profit function $P(x) = ax^2 + bx + c$, the maximum (if $a < 0$) or minimum (if $a > 0$) is always at $x = -b/2a$. This is a faster way to find the optimal quantity than using calculus (taking the derivative and setting it to zero), although both methods yield the same result.

20. What is the maximum daily profit, in rupees, that Mr. David can realize from his business?

- (A) 620
- (B) 920
- (C) 840
- (D) 760
- (E) Cannot be determined

Correct Answer: (D) 760

Solution:

Step 1: Understanding the Concept: This question is a direct continuation of the previous one. Having found the optimal production quantity that maximizes profit, we now need to calculate the profit at that level by substituting the value back into the profit function.

Step 2: Key Formula or Approach: We use the profit function derived in the previous question:

$$P(x) = -0.1x^2 + 20x - 240$$

And the optimal number of units, also found previously:

$$x_{max} = 100$$

We need to calculate $P(100)$.

Step 3: Calculation: Substitute $x = 100$ into the profit function:

$$P(100) = -0.1(100)^2 + 20(100) - 240$$

$$P(100) = -0.1(10000) + 2000 - 240$$

$$P(100) = -1000 + 2000 - 240$$

$$P(100) = 1000 - 240$$

$$P(100) = 760$$

Step 4: Final Answer: The maximum daily profit is Rs. 760.

Rs. 760

Step 5: Why This is the Correct Option: The calculation shows that the maximum value of the profit function is 760, which corresponds to option (D). This confirms the result from the previous problem is correct.

 Quick Tip

After finding the optimal quantity x , you can calculate the profit either by plugging x into $P(x)$, or by calculating $R(x)$ and $C(x)$ separately and then finding the difference. Sometimes, calculating $R(x) = 30 \times 100 = 3000$ and $C(x) = 240 + 10(100) + 0.1(100)^2 = 240 + 1000 + 1000 = 2240$, then $P = 3000 - 2240 = 760$, can reduce calculation errors.

21. The price of Darjeeling tea (in rupees per kilogram) is $100 + 0.10n$, on the n^{th} day of 2007 ($n = 1, 2, \dots, 100$), and then remains constant. On the other hand, the price of Ooty tea (in rupees per kilogram) is $89 + 0.15n$, on the n^{th} day of 2007 ($n = 1, 2, \dots, 365$). On which date in 2007 will the prices of these two varieties of tea be equal?

- (A) May 21
- (B) April 11
- (C) May 20
- (D) April 10
- (E) June 30

Correct Answer: (C) May 20

Solution:

Step 1: Understanding the Concept: This problem requires us to find the point of intersection of two price functions. One function is piecewise: it's linear for a certain period and

then becomes constant. The other function is linear throughout the year. We need to set the two price functions equal and solve for the day number 'n', then convert 'n' to a calendar date.

Step 2: Key Formula or Approach: Let $P_D(n)$ be the price of Darjeeling tea and $P_O(n)$ be the price of Ooty tea on day 'n'.

$$P_D(n) = \begin{cases} 100 + 0.10n & \text{if } 1 \leq n \leq 100 \\ \text{constant} & \text{if } n > 100 \end{cases}$$

$$P_O(n) = 89 + 0.15n \quad \text{for } 1 \leq n \leq 365$$

First, we need to find the constant price of Darjeeling tea for $n > 100$. This is the price on day 100. Then, we set $P_D(n) = P_O(n)$ and solve for n.

Step 3: Calculation: Calculate the price of Darjeeling tea on day 100:

$$P_D(100) = 100 + 0.10(100) = 100 + 10 = 110$$

So, for any day $n > 100$, $P_D(n) = 110$.

Now, we need to find the day 'n' when the price of Ooty tea becomes 110. Since the Ooty tea price is continuously increasing, this will happen after day 100. Set $P_O(n) = 110$:

$$89 + 0.15n = 110$$

$$0.15n = 110 - 89$$

$$0.15n = 21$$

$$n = \frac{21}{0.15} = \frac{2100}{15} = 140$$

So the prices will be equal on the 140th day of 2007.

Now, we convert the 140th day to a calendar date. The year 2007 is not a leap year.

- January: 31 days
- February: 28 days
- March: 31 days
- April: 30 days

Total days up to the end of April = $31 + 28 + 31 + 30 = 120$ days. The 140th day is $140 - 120 = 20$ days into the next month, which is May. The date is May 20th.

Step 4: Final Answer: The prices will be equal on May 20, 2007.

May 20

Step 5: Why This is the Correct Option: The calculation correctly handles the piecewise nature of the Darjeeling tea price function. After determining the constant price, we solve for the day 'n' and convert it to the correct calendar date, which is May 20, matching option (C).

💡 Quick Tip

When dealing with piecewise functions, always check which piece of the function is relevant for the solution. A quick check here: if we had set $100 + 0.1n = 89 + 0.15n$, we would get $11 = 0.05n \implies n = 220$, which is outside the range $n \leq 100$. This immediately tells you that the intersection must occur when the Darjeeling price is constant.

22. Two circles with centres P and Q cut each other at two distinct points A and B. The circles have the same radii and neither P nor Q falls within the intersection of the circles. What is the smallest range that includes all possible values of the angle AQP in degrees?

- (A) Between 0 and 90
- (B) Between 0 and 30
- (C) Between 0 and 60
- (D) Between 0 and 75
- (E) Between 0 and 45

Correct Answer: (C) Between 0 and 60

Solution:

Step 1: Understanding the Concept: This is a geometry problem involving the properties of intersecting circles and the relationship between side lengths and angles in a triangle (specifically, the Law of Cosines). The constraints on the positions of the circle centers relative to each other define the possible shapes of the triangle formed by the two centers and an intersection point.

Step 2: Key Formula or Approach: Let the radius of both circles be r . Let the distance between the centers P and Q be d . Consider the triangle $\triangle PAQ$. The vertices are the two centers (P, Q) and one intersection point (A). The side lengths are:

- $PA = r$ (radius of the circle with center P)
- $QA = r$ (radius of the circle with center Q)
- $PQ = d$ (distance between centers)

The constraints given are: 1. The circles intersect at two distinct points: This means $d < r + r \implies d < 2r$. 2. Neither P nor Q falls within the intersection of the circles. This means Q is not inside the circle centered at P, and P is not inside the circle centered at Q. This implies the distance between the centers must be at least the radius: $d \geq r$. Combining these, we have the range for the distance between centers: $r \leq d < 2r$. We want to find the range for $\angle AQP$. We can use the Law of Cosines in $\triangle PAQ$.

$$PA^2 = AQ^2 + PQ^2 - 2(AQ)(PQ) \cos(\angle AQP)$$

Step 3: Calculation: Substitute the side lengths into the Law of Cosines formula:

$$r^2 = r^2 + d^2 - 2(r)(d) \cos(\angle AQP)$$

$$0 = d^2 - 2rd \cos(\angle AQP)$$

Since the circles are distinct, $d \neq 0$. We can divide by d :

$$0 = d - 2r \cos(\angle AQP)$$

$$2r \cos(\angle AQP) = d$$

$$\cos(\angle AQP) = \frac{d}{2r}$$

Now, we use the inequality for d that we established: $r \leq d < 2r$. Divide all parts of the inequality by $2r$ (which is a positive number):

$$\frac{r}{2r} \leq \frac{d}{2r} < \frac{2r}{2r}$$

$$\frac{1}{2} \leq \cos(\angle AQP) < 1$$

The cosine function is a decreasing function for angles between 0° and 90° .

- When $\cos(\angle AQP)$ approaches 1, $\angle AQP$ approaches 0° .
- When $\cos(\angle AQP) = 1/2$, $\angle AQP = 60^\circ$.

Therefore, the inequality $\frac{1}{2} \leq \cos(\angle AQP) < 1$ translates to:

$$0^\circ < \angle AQP \leq 60^\circ$$

The range of possible values for the angle is $(0, 60]$. This range is contained within "Between 0 and 60".

Step 4: Final Answer: The smallest range that includes all possible values is between 0 and 60 degrees.

$$\boxed{0^\circ < \angle AQP \leq 60^\circ}$$

Step 5: Why This is the Correct Option: The geometric constraints on the distance between the centers were correctly translated into an inequality. Applying the Law of Cosines and solving for the angle showed that its value must be greater than 0 and less than or equal to 60 degrees. Option (C) is the smallest range provided that contains this interval.

💡 Quick Tip

Visualizing the extreme cases helps.

- Case 1 ($d = r$): P is on the circumference of the circle centered at Q. $\triangle PAQ$ has all sides equal to r , making it equilateral. All angles are 60° . So $\angle AQP = 60^\circ$.
- Case 2 (d approaches $2r$): The circles are almost touching externally. The triangle $\triangle PAQ$ becomes almost flat, with A almost on the line segment PQ. $\angle AQP$ approaches 0° .

These extreme cases confirm the range $(0, 60]$.

23. A quadratic function $f(x)$ attains a maximum of 3 at $x = 1$. The value of the function at $x = 0$ is 1. What is the value $f(x)$ at $x = 10$?

- (A) -119
- (B) -159
- (C) -110
- (D) -180
- (E) -105

Correct Answer: (B) -159

Solution:

Step 1: Understanding the Concept: This question involves finding the equation of a specific quadratic function given its properties. When the vertex (maximum or minimum point) of a parabola is known, the vertex form of the quadratic equation is the most efficient way to represent the function.

Step 2: Key Formula or Approach: The vertex form of a quadratic function is:

$$f(x) = a(x - h)^2 + k$$

where (h, k) is the vertex of the parabola. The problem gives us: 1. The maximum value is 3, which occurs at $x = 1$. This means the vertex (h, k) is $(1, 3)$. 2. The function passes through the point $(0, 1)$, i.e., $f(0) = 1$. We can use this information to determine the value of the constant 'a' and thus find the complete function.

Step 3: Calculation: Using the vertex $(h, k) = (1, 3)$, the function can be written as:

$$f(x) = a(x - 1)^2 + 3$$

Now, use the second piece of information, $f(0) = 1$, to find 'a':

$$f(0) = a(0 - 1)^2 + 3$$

$$1 = a(-1)^2 + 3$$

$$1 = a(1) + 3$$

$$1 = a + 3$$

$$a = 1 - 3 = -2$$

Since a is negative, the parabola opens downwards, which is consistent with having a maximum value. The complete function is:

$$f(x) = -2(x - 1)^2 + 3$$

Finally, we need to calculate the value of the function at $x = 10$:

$$f(10) = -2(10 - 1)^2 + 3$$

$$f(10) = -2(9)^2 + 3$$

$$f(10) = -2(81) + 3$$

$$f(10) = -162 + 3$$

$$f(10) = -159$$

Step 4: Final Answer: The value of $f(x)$ at $x = 10$ is -159.

$f(10) = -159$

Step 5: Why This is the Correct Option: Using the vertex form of a quadratic equation allowed for a straightforward determination of the function's formula. Plugging in $x = 10$ yields -159, which matches option (B).

Quick Tip

The vertex form $f(x) = a(x - h)^2 + k$ is incredibly useful.

- (h, k) is the vertex.
- If $a > 0$, the parabola opens up (minimum at k).
- If $a < 0$, the parabola opens down (maximum at k).
- The line $x = h$ is the axis of symmetry.

Always use this form when the vertex is given; it's much faster than using the standard form $ax^2 + bx + c$.

Directions for Questions 24 and 25: Answer the following questions based on the information given below:

Let $a_1 = p$ and $b_1 = q$, where p and q are positive quantities. Define

$a_n = pb_{n-1}$, $b_n = qb_{n-1}$, for even $n \geq 1$,

and

$a_n = pa_{n-1}$, $b_n = qa_{n-1}$, for odd $n \geq 1$.

24. Which of the following best describes $a_n + b_n$ for even 'n'?

- (A) $q(pq)^{\frac{n}{2}-1} (p + q)$
- (B) $qp^{\frac{1}{2}n-1} (p + q)$
- (C) $q^{\frac{1}{2}n} (p + q)$
- (D) $q^{2^{\frac{1}{2}n}} (p + q)^{1^{\frac{1}{2}n}}$
- (E) $q(pq)^{\frac{1}{2}n-1} (p + q)^{2^{\frac{1}{2}n}}$

Correct Answer: (A) $q(pq)^{\frac{n}{2}-1} (p + q)$

Solution:

Step 1: Understanding the Concept: The problem defines a sequence with recursive rules that depend on whether the index 'n' is even or odd. We need to find a closed-form expression for the sum $a_n + b_n$ specifically for even values of n. This requires us to combine the recursive definitions to find a pattern.

Step 2: Key Formula or Approach: Our goal is to express a_n and b_n in terms of p, q, and n for an even n. 1. For an even n , the index $n - 1$ is odd. We can use the rule for odd indices to express a_{n-1} and b_{n-1} . 2. Then, substitute these into the rules for even n to find a recurrence relation for even-indexed terms only. 3. Solve this recurrence relation to get a general formula.

Step 3: Calculation: For any even $n > 1$:

$$a_n = pb_{n-1} \quad \text{and} \quad b_n = qa_{n-1}$$

For the term b_{n-1} , the index $n - 1$ is odd. So we use the rule for odd n :

$$b_{n-1} = qa_{n-2}$$

Now substitute this back into the equations for a_n and b_n :

$$a_n = p(qa_{n-2}) = (pq)a_{n-2}$$

$$b_n = q(qa_{n-2}) = q^2a_{n-2}$$

The equation $a_n = (pq)a_{n-2}$ shows that the sequence of even terms $a_2, a_4, a_6, \dots$ forms a geometric progression with a common ratio of pq .

Let's find the first term of this sequence, a_2 . For $n = 2$:

$$a_2 = pb_1 = pq$$

The general formula for a_n where $n = 2k$ is $a_{2k} = a_2 \cdot (pq)^{k-1} = (pq) \cdot (pq)^{k-1} = (pq)^k$. Since $n = 2k$, we have $k = n/2$. So, for even n :

$$a_n = (pq)^{n/2}$$

Now we find b_n for even n :

$$b_n = q^2a_{n-2}$$

Since $n - 2$ is also even, we can use our new formula for a_{n-2} :

$$a_{n-2} = (pq)^{(n-2)/2} = (pq)^{n/2-1}$$

So,

$$b_n = q^2(pq)^{n/2-1} = q^2p^{n/2-1}q^{n/2-1} = p^{n/2-1}q^{n/2+1}$$

Finally, we find the sum $a_n + b_n$:

$$\begin{aligned} a_n + b_n &= (pq)^{n/2} + p^{n/2-1}q^{n/2+1} \\ &= p^{n/2}q^{n/2} + p^{n/2-1}q^{n/2+1} \end{aligned}$$

Factor out the common terms, which are $p^{n/2-1}$ and $q^{n/2}$:

$$= p^{n/2-1}q^{n/2}(p^1 + q^1) = p^{n/2-1}q^{n/2}(p + q)$$

To match the format of the options, we can rewrite this as:

$$= q \cdot (p^{n/2-1} q^{n/2-1})(p+q) = q(pq)^{n/2-1}(p+q)$$

Step 4: Final Answer: The expression for $a_n + b_n$ for even 'n' is $q(pq)^{\frac{n}{2}-1}(p+q)$.

$$\boxed{q(pq)^{\frac{n}{2}-1}(p+q)}$$

Step 5: Why This is the Correct Option: The step-by-step derivation of the general formulas for a_n and b_n for even n and their subsequent summation leads directly to the expression in option (A).

💡 Quick Tip

For complex recursive sequences, calculating the first few terms can help identify a pattern or verify a derived formula. For $n=2$: $a_2 + b_2 = pq + q^2 = q(p+q)$. The formula gives $q(pq)^{2/2-1}(p+q) = q(pq)^0(p+q) = q(p+q)$. It matches. For $n=4$: $a_3 = p^2q, b_3 = pq^2 \implies a_4 = pb_3 = p^2q^2, b_4 = qb_3 = pq^3 \implies a_4 + b_4 = p^2q^2 + pq^3 = pq^2(p+q)$. The formula gives $q(pq)^{4/2-1}(p+q) = q(pq)^1(p+q) = pq^2(p+q)$. It matches.

25. If $p = \frac{1}{3}$ and $q = \frac{2}{3}$, then what is the smallest odd 'n' such that $a_n + b_n < 0.01$?

- (A) 7
- (B) 13
- (C) 11
- (D) 9
- (E) 15

Correct Answer: (D) 9

Solution:

Step 1: Understanding the Concept: This problem requires us to first find a closed-form expression for $a_n + b_n$ for odd values of 'n', similar to the previous question. Then, we must substitute the given values of p and q into this expression and solve an inequality to find the smallest odd integer n that satisfies the condition.

Step 2: Key Formula or Approach: 1. Derive the formula for $a_n + b_n$ for odd $n > 1$. 2. Substitute $p = 1/3$ and $q = 2/3$. 3. Set up the inequality $a_n + b_n < 0.01$. 4. Solve the inequality for the smallest odd integer n.

Step 3: Calculation: First, let's find the formula for $a_n + b_n$ when n is odd. For any odd $n > 1$:

$$a_n = pa_{n-1} \quad \text{and} \quad b_n = qa_{n-1}$$

Summing these gives:

$$a_n + b_n = pa_{n-1} + qa_{n-1} = (p + q)a_{n-1}$$

For the term a_{n-1} , the index $n - 1$ is even. From the previous question, we know the formula for a_k when k is even is $a_k = (pq)^{k/2}$. Substituting $k = n - 1$:

$$a_{n-1} = (pq)^{(n-1)/2}$$

Therefore, for odd n :

$$a_n + b_n = (p + q)(pq)^{(n-1)/2}$$

Now, substitute the given values $p = 1/3$ and $q = 2/3$:

$$p + q = \frac{1}{3} + \frac{2}{3} = 1$$

$$pq = \left(\frac{1}{3}\right)\left(\frac{2}{3}\right) = \frac{2}{9}$$

The expression for $a_n + b_n$ becomes:

$$a_n + b_n = (1)\left(\frac{2}{9}\right)^{(n-1)/2} = \left(\frac{2}{9}\right)^{(n-1)/2}$$

Now, we solve the inequality $a_n + b_n < 0.01$:

$$\left(\frac{2}{9}\right)^{(n-1)/2} < \frac{1}{100}$$

Let $x = (n - 1)/2$. Since n is an odd integer, $n - 1$ is an even integer, so x is an integer. We need to find the smallest integer x such that $(2/9)^x < 1/100$. Let's test integer values for x :

- If $x = 1$: $(2/9)^1 = 2/9 \approx 0.222$, which is not less than 0.01.
- If $x = 2$: $(2/9)^2 = 4/81 \approx 0.049$, which is not less than 0.01.
- If $x = 3$: $(2/9)^3 = 8/729$. Is $8/729 < 1/100$? Cross-multiply: $8 \times 100 < 1 \times 729 \implies 800 < 729$, which is false.
- If $x = 4$: $(2/9)^4 = 16/6561$. Is $16/6561 < 1/100$? Cross-multiply: $16 \times 100 < 1 \times 6561 \implies 1600 < 6561$, which is true.

The smallest integer value for x is 4. Now we find the corresponding value of n :

$$x = \frac{n - 1}{2} = 4$$

$$n - 1 = 8$$

$$n = 9$$

Since 9 is an odd number, this is our answer.

Step 4: Final Answer: The smallest odd 'n' such that $a_n + b_n < 0.01$ is 9.

$$\boxed{n = 9}$$

Step 5: Why This is the Correct Option: The derivation of the formula for the sum for odd 'n' is correct. Substituting the given values and solving the resulting exponential inequality by testing integer powers correctly identifies that the condition is first met when the exponent $(n - 1)/2$ is equal to 4, which corresponds to $n=9$.

💡 Quick Tip

When solving an inequality like $(a/b)^x < c$ where $a/b < 1$, you can often avoid using logarithms by testing small integer values for x . This is usually faster in competitive exams, especially when the options are integers.

Section - II

Directions for Questions 26 to 29: Answer the following questions based on the information given below:

A health-drink company's R&D department is trying to make various diet formulations, which can be used for certain specific purposes. It is considering a choice of 5 alternative ingredients (O, P, Q, R, and S), which can be used in different proportions in the formulations. The table below gives the composition of these ingredients. The cost per unit of each of these ingredients is O: 150, P: 50, Q: 200, R: 500, S: 100.

Composition				
Ingredient	Carbohydrate%	Protein%	Fat%	Minerals%
O	50	30	10	10
P	80	20	0	0
Q	10	30	50	10
R	5	50	40	5
S	45	50	0	5

26. For a recuperating patient, the doctor recommended a diet containing 10% minerals and at least 30% protein. In how many different ways can we prepare this diet by mixing at least two ingredients?

- (A) One
- (B) Two
- (C) Three
- (D) Four
- (E) None

Correct Answer: (A) One

Solution:

Step 1: Understanding the Concept: This is a mixture problem with specific constraints. We need to find combinations of ingredients that, when mixed, result in a final product with

exactly 10% minerals and at least 30% protein. The key principle is that the percentage of a component in a mixture must lie between the minimum and maximum percentages of that component in the ingredients being mixed.

Step 2: Key Formula or Approach: Let the mineral percentages of the ingredients be $M_1, M_2, \dots, M_k$. For a mixture of these ingredients to have a final mineral percentage of M_{final} , it must be that $\min(M_i) \leq M_{final} \leq \max(M_i)$. The target mineral percentage is exactly 10%. Let's list the mineral content of each ingredient:

- O: 10%
- P: 0%
- Q: 10%
- R: 5%
- S: 5%

Step 3: Calculation: To obtain a mixture with exactly 10% minerals, we can only use ingredients with mineral percentages that "bracket" 10% (i.e., some below/at 10% and some above/at 10%). The maximum mineral percentage available in any ingredient is 10%. This means we cannot mix any ingredient with less than 10% minerals, because doing so would inevitably result in a final mixture with less than 10% minerals. For example, mixing Q (10% min) and R (5% min) will result in a mineral percentage between 5% and 10%. Therefore, the only way to achieve exactly 10% minerals is to use only those ingredients that themselves contain exactly 10% minerals. The ingredients fitting this criterion are O and Q.

The question requires mixing at least two ingredients. So, we must use a combination of O and Q. Let's check the protein constraint for a mixture of O and Q.

- Protein% of O = 30%
- Protein% of Q = 30%

Since both ingredients have 30% protein, any mixture of O and Q will also have exactly 30% protein. This satisfies the "at least 30% protein" condition.

Thus, any mixture of O and Q is a valid diet. The question asks for the number of "ways", which refers to the number of combinations of ingredients. The only combination of at least two ingredients that works is {O, Q}. Therefore, there is only one way to prepare the diet.

Step 4: Final Answer: There is only one combination of ingredients, {O, Q}, that can be used to prepare the diet.

One

Step 5: Why This is the Correct Option: The analysis shows that to achieve exactly 10% minerals, given that no ingredient has more than 10%, only ingredients with exactly 10% minerals can be used. This restricts the choice to O and Q. This combination also meets the protein requirement. Hence, there is only one such combination of two or more ingredients.

💡 Quick Tip

The rule of alligation is a powerful tool. It states that to get a mixture with an average value C , from ingredients with values A and B (where $A < C < B$), they must be mixed in the ratio $(B-C) : (C-A)$. A key consequence is that the average must be between the values of the components. If the target value is an extreme (the max or min available), you can only use ingredients that have that exact value.

27. Which among the following is the formulation having the lowest cost per unit for a diet having 10% fat and at least 30% protein? (The diet has to be formed by mixing two ingredients).

- (A) P and Q
- (B) P and S
- (C) P and R
- (D) Q and S
- (E) R and S

Correct Answer: (D) Q and S

Solution:

Step 1: Understanding the Concept: This is a mixture problem involving optimization. We need to check several pairs of ingredients, determine the mixing ratio required to meet an exact fat percentage using alligation, check if this mix meets a minimum protein requirement, and if so, calculate its cost. Finally, we compare the costs of all valid formulations.

Step 2: Key Formula or Approach: For each pair of ingredients (say Ingredient 1 and Ingredient 2), with proportions w_1 and w_2 :

1. Use alligation on fat content to find the ratio $w_1 : w_2$ needed to achieve 10% fat.
2. Calculate the protein percentage of the mix: $\text{Protein}_{\text{mix}} = \frac{w_1 \cdot \text{Prot}_1 + w_2 \cdot \text{Prot}_2}{w_1 + w_2}$. Check if $\text{Protein}_{\text{mix}} \geq 30\%$.
3. If the protein condition is met, calculate the cost of the mix: $\text{Cost}_{\text{mix}} = \frac{w_1 \cdot \text{Cost}_1 + w_2 \cdot \text{Cost}_2}{w_1 + w_2}$.
4. Compare the costs of all valid mixes.

Step 3: Calculation: Let's evaluate each option:

- **(A) P and Q:** $\text{Fat}(P)=0\%$, $\text{Fat}(Q)=50\%$. To get 10% fat, the ratio P:Q is $(50-10):(10-0) = 40:10 = 4:1$.
 $\text{Protein} = \frac{4(20)+1(30)}{4+1} = \frac{80+30}{5} = \frac{110}{5} = 22\%$. This is less than 30%, so this formulation is not valid.
- **(B) P and S:** $\text{Fat}(P)=0\%$, $\text{Fat}(S)=0\%$. A mix of P and S will always have 0% fat. It's impossible to get 10% fat. Invalid.
- **(C) P and R:** $\text{Fat}(P)=0\%$, $\text{Fat}(R)=40\%$. To get 10% fat, the ratio P:R is $(40-10):(10-0) = 30:10 = 3:1$.

Protein = $\frac{3(20)+1(50)}{3+1} = \frac{60+50}{4} = \frac{110}{4} = 27.5\%$. This is less than 30%, so this formulation is not valid.

- **(D) Q and S:** Fat(Q)=50%, Fat(S)=0%. To get 10% fat, the ratio Q:S is (10-0):(50-10) = 10:40 = 1:4.
Protein = $\frac{1(30)+4(50)}{1+4} = \frac{30+200}{5} = \frac{230}{5} = 46\%$. This is $\geq 30\%$, so this is a valid formulation.
Cost = $\frac{1(200)+4(100)}{1+4} = \frac{200+400}{5} = \frac{600}{5} = 120$.
- **(E) R and S:** Fat(R)=40%, Fat(S)=0%. To get 10% fat, the ratio R:S is (10-0):(40-10) = 10:30 = 1:3.
Protein = $\frac{1(50)+3(50)}{1+3} = \frac{200}{4} = 50\%$. This is $\geq 30\%$, so this is a valid formulation. Cost = $\frac{1(500)+3(100)}{1+3} = \frac{500+300}{4} = \frac{800}{4} = 200$.

Comparing the valid formulations, the mix of Q and S has a cost of 120, while the mix of R and S has a cost of 200. The lowest cost is 120.

Step 4: Final Answer: The formulation with the lowest cost per unit is the mixture of Q and S.

Q and S

Step 5: Why This is the Correct Option: After systematically checking all pairs, only (Q and S) and (R and S) are valid formulations that meet both the fat and protein requirements. Between these two, the (Q and S) mixture has a significantly lower cost (120 vs 200).

💡 Quick Tip

When applying alligation, draw a diagram to avoid errors:

Value of Cheaper (A)		Value of Dearer (B)
	Mean Value (C)	
(B-C)	:	(C-A)

This visual aid helps ensure you subtract in the correct order to get the ratio of A to B.

28. In what proportion P, Q and S should be mixed to make a diet having at least 60% carbohydrate at the lowest cost per unit?

- (A) 2:1:3
- (B) 4:1:2
- (C) 2:1:4
- (D) 3:1:2
- (E) 4:1:1

Correct Answer: (E) 4:1:1

Solution:

Step 1: Understanding the Concept: This is a three-component mixture problem with an inequality constraint (Carbohydrate % $\geq 60\%$) and an optimization goal (minimize cost). Since the options provide specific ratios, the most direct method is to check the validity and cost of each option.

Step 2: Key Formula or Approach: For each given ratio $P:Q:S = p:q:s$: 1. Calculate the carbohydrate percentage of the mixture: $\text{Carb}_{mix} = \frac{p \cdot \text{Carb}_P + q \cdot \text{Carb}_Q + s \cdot \text{Carb}_S}{p+q+s}$. Check if $\text{Carb}_{mix} \geq 60\%$.

2. If the carbohydrate condition is met, calculate the cost of the mixture:

$$\text{Cost}_{mix} = \frac{p \cdot \text{Cost}_P + q \cdot \text{Cost}_Q + s \cdot \text{Cost}_S}{p+q+s}.$$

3. Compare the costs of all valid mixtures to find the lowest. Ingredient Data:

- P: Carb=80%, Cost=50
- Q: Carb=10%, Cost=200
- S: Carb=45%, Cost=100

Step 3: Calculation: Let's evaluate each option:

- **(A) 2:1:3:** Total parts = 6. Carb = $\frac{2(80)+1(10)+3(45)}{6} = \frac{160+10+135}{6} = \frac{305}{6} \approx 50.8\%$. Fails ($< 60\%$).
- **(B) 4:1:2:** Total parts = 7. Carb = $\frac{4(80)+1(10)+2(45)}{7} = \frac{320+10+90}{7} = \frac{420}{7} = 60\%$. Valid ($\geq 60\%$). Cost = $\frac{4(50)+1(200)+2(100)}{7} = \frac{200+200+200}{7} = \frac{600}{7} \approx 85.71$.
- **(C) 2:1:4:** Total parts = 7. Carb = $\frac{2(80)+1(10)+4(45)}{7} = \frac{160+10+180}{7} = \frac{350}{7} = 50\%$. Fails ($< 60\%$).
- **(D) 3:1:2:** Total parts = 6. Carb = $\frac{3(80)+1(10)+2(45)}{6} = \frac{240+10+90}{6} = \frac{340}{6} \approx 56.7\%$. Fails ($< 60\%$).
- **(E) 4:1:1:** Total parts = 6. Carb = $\frac{4(80)+1(10)+1(45)}{6} = \frac{320+10+45}{6} = \frac{375}{6} = 62.5\%$. Valid ($\geq 60\%$). Cost = $\frac{4(50)+1(200)+1(100)}{6} = \frac{200+200+100}{6} = \frac{500}{6} \approx 83.33$.

Comparing the two valid options, (B) and (E):

- Cost of (B) is ≈ 85.71 .
- Cost of (E) is ≈ 83.33 .

The lowest cost is achieved with the proportion 4:1:1.

Step 4: Final Answer: The proportion 4:1:1 gives a valid diet at the lowest cost among the options.

$$\boxed{4 : 1 : 1}$$

Step 5: Why This is the Correct Option: By checking each option against the carbohydrate constraint, we eliminated options (A), (C), and (D). Of the remaining valid options, (B) and (E), the calculation shows that (E) results in a lower cost per unit.

 Quick Tip

To minimize cost while meeting a minimum nutritional requirement, you generally want to maximize the use of the ingredient that is cheapest per unit of that nutrient. Here, P is the best source of carbohydrates (80%) and is also the cheapest ingredient (50). Therefore, the optimal solution will likely have a high proportion of P, allowing for quick elimination of options with low proportions of P.

29. The company is planning to launch a balanced diet required for growth needs of adolescent children. This diet must contain at least 30% each of carbohydrate and protein, no more than 25% fat and at least 5% minerals. Which one of the following combinations of equally mixed ingredients is feasible?

- (A) O and P
- (B) R and S
- (C) P and S
- (D) Q and R
- (E) O and S

Correct Answer: (E) O and S

Solution:

Step 1: Understanding the Concept: This is a feasibility problem. We need to check if a 1:1 mixture of the given pairs of ingredients satisfies a set of four nutritional constraints simultaneously.

Step 2: Key Formula or Approach: For an equal mix (1:1 ratio) of two ingredients, the composition of the mix is the simple average of their individual compositions. For each pair, we will calculate the average for Carbohydrate, Protein, Fat, and Minerals and check against the given constraints:

- Carbohydrate % $\geq 30\%$
- Protein % $\geq 30\%$
- Fat % $\leq 25\%$
- Minerals % $\geq 5\%$

Step 3: Calculation: Let's check each option:

- (A) O and P: Carb = $\frac{50+80}{2} = 65\%$ ($\geq 30\%$, OK). Protein = $\frac{30+20}{2} = 25\%$ ($< 30\%$, **FAIL**).
- (B) R and S: Carb = $\frac{5+45}{2} = 25\%$ ($< 30\%$, **FAIL**).
- (C) P and S: Carb = $\frac{80+45}{2} = 62.5\%$ ($\geq 30\%$, OK). Protein = $\frac{20+50}{2} = 35\%$ ($\geq 30\%$, OK). Fat = $\frac{0+0}{2} = 0\%$ ($\leq 25\%$, OK). Minerals = $\frac{0+5}{2} = 2.5\%$ ($< 5\%$, **FAIL**).
- (D) Q and R: Carb = $\frac{10+5}{2} = 7.5\%$ ($< 30\%$, **FAIL**).
- (E) O and S: Carb = $\frac{50+45}{2} = 47.5\%$ ($\geq 30\%$, OK). Protein = $\frac{30+50}{2} = 40\%$ ($\geq 30\%$, OK). Fat = $\frac{10+0}{2} = 5\%$ ($\leq 25\%$, OK). Minerals = $\frac{10+5}{2} = 7.5\%$ ($\geq 5\%$, OK). All conditions are met. This combination is feasible.

Step 4: Final Answer: The combination of O and S, when mixed equally, is feasible.

O and S

Step 5: Why This is the Correct Option: This is the only pair that satisfies all four conditions simultaneously when mixed in a 1:1 ratio. All other pairs fail on at least one of the nutritional requirements.

 Quick Tip

In feasibility questions with multiple constraints, check the constraints one by one for each option. As soon as an option fails a single constraint, you can eliminate it and move to the next. This saves time and prevents unnecessary calculations.

Directions for Questions 30 to 33: Each question is followed by two statements, A and B. Answer each question using the following instructions:

Mark (1) if the question can be answered by using the statement A alone but not by using the statement B alone.

Mark (2) if the question can be answered by using the statement B alone but not by using the statement A alone.

Mark (3) if the question can be answered by using either of the statements alone.

Mark (4) if the question can be answered by using both the statements together but not by either of the statements alone.

Mark (5) if the question cannot be answered on the basis of the two statements.

30. In a particular school, sixty students were athletes. Ten among them were also among the top academic performers. How many top academic performers were in the school?

A. Sixty per cent of the top academic performers were not athletes.

B. All the top academic performers were not necessarily athletes.

Correct Answer: (1) if the question can be answered by using the statement A alone but not by using the statement B alone.

Solution:

Step 1: Understanding the Concept: This is a data sufficiency question based on set theory. We need to determine if the given statements provide enough information to calculate the total number of top academic performers.

Step 2: Key Formula or Approach: Let T be the set of top academic performers and A be the set of athletes. We are given:

- Number of athletes, $|A| = 60$.
- Number of students who are both athletes and top performers, $|A \cap T| = 10$.

The question asks for the total number of top academic performers, $|T|$. We know that the total number of top performers is the sum of those who are athletes and those who are not:

$$|T| = |A \cap T| + |A^c \cap T|$$

where $|A^c \cap T|$ is the number of top performers who are not athletes. We need to find $|T|$.

Step 3: Calculation: Analysis of Statement A: "Sixty per cent of the top academic performers were not athletes." This can be written as:

$$|A^c \cap T| = 0.60 \times |T|$$

Using the formula from Step 2:

$$|T| = |A \cap T| + |A^c \cap T|$$

$$|T| = 10 + 0.60 \times |T|$$

$$|T| - 0.60 \times |T| = 10$$

$$0.40 \times |T| = 10$$

$$|T| = \frac{10}{0.40} = 25$$

Statement A provides a specific value for $|T|$. Thus, Statement A alone is sufficient.

Analysis of Statement B: "All the top academic performers were not necessarily athletes." This statement is qualitative. It only tells us that the set of top performers who are not athletes is not necessarily empty ($|A^c \cap T| \geq 0$). It does not give us a specific number or a percentage that would allow us to calculate $|T|$. For example, there could be 1 or 100 or any other number of non-athlete top performers. Thus, Statement B alone is not sufficient.

Step 4: Final Answer: The question can be answered using statement A alone, but not by using statement B alone.

Mark (1)

Step 5: Why This is the Correct Option: Statement A provides a quantitative relationship between a subset of top performers and the total, allowing us to form a solvable equation. Statement B provides only a vague, qualitative piece of information that is insufficient to find a unique numerical answer. Therefore, only A is sufficient.

 Quick Tip

In data sufficiency, be wary of statements that use words like "not necessarily," "some," or "could be." These often indicate a lack of specific, quantitative information, making the statement insufficient on its own.

31. Five students Atul, Bala, Chetan, Dev and Ernesto were the only ones who participated in a quiz contest. They were ranked based on their scores in the contest. Dev got a higher rank as compared to Ernesto, while Bala got a higher rank as compared to Chetan. Chetan's rank was lower than the median. Who among the five got the highest rank?

A. Atul was the last rank holder.

B. Bala was not among the top two rank holders.

Correct Answer: (4) if the question can be answered by using both the statements together but not by either of the statements alone.

Solution:

Step 1: Understanding the Concept: This is a data sufficiency question based on ordering and ranking. We need to determine if the statements provide enough constraints to uniquely identify the student with rank 1 (the highest rank).

Step 2: Key Formula or Approach: Let's denote the rank of a student X as $R(X)$. A higher rank means a smaller number (e.g., $R=1$ is the highest). There are 5 ranks: 1, 2, 3, 4, 5. The median rank is 3. From the question stem, we have:

1. $R(\text{Dev}) \leq R(\text{Ernesto})$
2. $R(\text{Bala}) \leq R(\text{Chetan})$
3. Chetan's rank was lower than the median: $R(\text{Chetan}) \leq 3$. So, $R(\text{Chetan})$ is either 4 or 5.

Step 3: Calculation: Analysis of Statement A: "Atul was the last rank holder." This means $R(\text{Atul}) = 5$. From the question, we know $R(\text{Chetan})$ can be 4 or 5. Since Atul is 5, we must have $R(\text{Chetan}) = 4$. From $R(\text{Bala}) \leq R(\text{Chetan})$, we have $R(\text{Bala}) \leq 4$. So $R(\text{Bala})$ can be 1, 2, or 3. The remaining ranks are 1, 2, 3 for Bala, Dev, and Ernesto. From $R(\text{Dev}) \leq R(\text{Ernesto})$, they will occupy two of these ranks. The highest rank (1) could go to either Bala or Dev. We cannot be certain. Thus, Statement A alone is not sufficient.

Analysis of Statement B: "Bala was not among the top two rank holders." This means $R(\text{Bala}) \neq 1$ and $R(\text{Bala}) \neq 2$. So $R(\text{Bala})$ can be 3, 4, or 5. From the question, $R(\text{Bala}) \leq R(\text{Chetan})$ and $R(\text{Chetan}) \leq 3$. Let's combine these:

- If $R(\text{Chetan})=4$, then $R(\text{Bala})$ must be ≤ 4 . With this statement's constraint, $R(\text{Bala}) = 3$. This is a possible scenario.
- If $R(\text{Chetan})=5$, then $R(\text{Bala})$ must be ≤ 5 . With this statement's constraint, $R(\text{Bala})$ can be 3 or 4. This is also possible.

We don't have enough information to fix the ranks. Rank 1 is still open for Dev, Atul, or Ernesto. Thus, Statement B alone is not sufficient.

Analysis of Statements A and B together: From A, we have: $R(\text{Atul}) = 5$ and $R(\text{Chetan}) = 4$. From the question, this implies $R(\text{Bala}) \leq 4$, so $R(\text{Bala})$ can be 1, 2, or 3. From B, we have: $R(\text{Bala})$ is not 1 or 2. Combining these, the only possibility for Bala's rank is $R(\text{Bala}) = 3$. So far, we have fixed:

- $R(\text{Atul}) = 5$
- $R(\text{Chetan}) = 4$
- $R(\text{Bala}) = 3$

The remaining ranks are 1 and 2, and the remaining students are Dev and Ernesto. From the question, $R(\text{Dev}) \leq R(\text{Ernesto})$. Therefore, we must have $R(\text{Dev}) = 1$ and $R(\text{Ernesto}) = 2$. We can now definitively say that Dev got the highest rank. Thus, both statements together are sufficient.

Step 4: Final Answer: The question can be answered using both statements together, but not by either of the statements alone.

Mark (4)

Step 5: Why This is the Correct Option: Each statement alone leaves too many possibilities open to identify the rank 1 student. However, when combined, the constraints from both statements work together to eliminate all ambiguities, leading to a unique ranking for all five students.

 Quick Tip

For ranking problems, create a list of available slots (e.g., 1, 2, 3, 4, 5). As you process each piece of information, fill in the slots or cross out possibilities. This systematic approach helps manage the complexity.

32. Thirty per cent of the employees of a call centre are males. Ten per cent of the female employees have an engineering background. What is the percentage of male employees with engineering background?

A. Twenty five per cent of the employees have engineering background.

B. Number of male employees having an engineering background is 20% more than the number of female employees having an engineering background.

Correct Answer: (3) if the question can be answered by using either of the statements alone.

Solution:

Step 1: Understanding the Concept: This is a data sufficiency question involving percentages and demographic data. We need to determine if the statements provide enough information to calculate a specific sub-group's percentage.

Step 2: Key Formula or Approach: Let T be the total number of employees. From the question stem:

- Number of male employees (M) = 0.30T
- Number of female employees (F) = (1 - 0.30)T = 0.70T
- Number of female employees with an engineering background (FE) = 10% of F = 0.10 × 0.70T = 0.07T

Let ME be the number of male employees with an engineering background. The question asks for the percentage of male employees with an engineering background, which is $\frac{ME}{M} \times 100\%$. Since M=0.3T, this is equivalent to finding the value of $\frac{ME}{0.3T} \times 100\%$. This means we need to find ME in terms of T.

Step 3: Calculation: Analysis of Statement A: "Twenty five per cent of the employees have engineering background." Total employees with engineering background = ME + FE = 0.25T. We know from the question stem that FE = 0.07T.

$$ME + 0.07T = 0.25T$$

$$ME = 0.25T - 0.07T = 0.18T$$

Now we can calculate the required percentage:

$$\frac{ME}{M} \times 100\% = \frac{0.18T}{0.30T} \times 100\% = 0.6 \times 100\% = 60\%$$

We found a unique value. Thus, Statement A alone is sufficient.

Analysis of Statement B: "Number of male employees having an engineering background is 20% more than the number of female employees having an engineering background." This translates to:

$$ME = FE + 0.20 \times FE = 1.20 \times FE$$

We know from the question stem that FE = 0.07T.

$$ME = 1.20 \times (0.07T) = 0.084T$$

Now we can calculate the required percentage:

$$\frac{ME}{M} \times 100\% = \frac{0.084T}{0.30T} \times 100\% = 0.28 \times 100\% = 28\%$$

We found a unique value. Thus, Statement B alone is sufficient.

Step 4: Final Answer: The question can be answered by using either of the statements alone.

Mark (3)

Step 5: Why This is the Correct Option: Both statements provide a different, but equally valid, piece of quantitative information that allows us to find the number of male engineers

(ME) in terms of the total number of employees (T). This is enough to calculate the required percentage. Since each statement is sufficient on its own, the correct option is (3).

💡 Quick Tip

In percentage problems, you can often assume a total of 100 to simplify calculations. Let $T=100$. Then $M=30$, $F=70$. $FE = 10\%$ of $70 = 7$. A: Total Engg = 25% of $100 = 25$. $ME = 25 - FE = 25 - 7 = 18$. $\% \text{ Male Engg} = (18/30)*100 = 60\%$. B: $ME = 1.2 * FE = 1.2 * 7 = 8.4$. $\% \text{ Male Engg} = (8.4/30)*100 = 28\%$. Since both A and B give a definite number, each is sufficient.

33. In a football match, at the half-time, Mahindra and Mahindra Club was trailing by three goals. Did it win the match?

A. In the second-half Mahindra and Mahindra Club scored four goals.

B. The opponent scored four goals in the match.

Correct Answer: (5) if the question cannot be answered on the basis of the two statements.

Solution:

Step 1: Understanding the Concept: This is a data sufficiency question that requires a definitive "Yes" or "No" answer. If the given information leads to a situation where the team might have won or might not have won, the information is insufficient.

Step 2: Key Formula or Approach: Let M&M's score be M and the opponent's score be O . Let M_1 , O_1 be the scores in the first half, and M_2 , O_2 be the scores in the second half. Final scores are $M_{final} = M_1 + M_2$ and $O_{final} = O_1 + O_2$. From the question stem, M&M was trailing by 3 goals at half-time:

$$O_1 = M_1 + 3$$

The question is: Did M&M win? This means, is $M_{final} > O_{final}$? Substituting the known relations: Is $M_1 + M_2 > O_1 + O_2$? Is $M_1 + M_2 > (M_1 + 3) + O_2$? Simplifying, the question is: Is $M_2 > O_2 + 3$? (Did M&M score more than 3 goals more than the opponent in the second half?)

Step 3: Calculation: Analysis of Statement A: "In the second-half Mahindra and Mahindra Club scored four goals." This gives us $M_2 = 4$. The winning condition becomes: Is $4 > O_2 + 3$? This simplifies to: Is $1 > O_2$? This means M&M won only if the opponent scored 0 goals in the second half ($O_2 = 0$). If the opponent scored 1 goal ($O_2=1$), it was a draw. If $O_2 \geq 1$, M&M lost. Since we do not know O_2 , we cannot give a definitive yes/no answer. Thus, Statement A alone is not sufficient.

Analysis of Statement B: "The opponent scored four goals in the match." This means $O_{final} = O_1 + O_2 = 4$. From the question stem, $O_1 = M_1 + 3$. Since scores are non-negative integers, M_1 can be 0 or 1.

- Case 1: If $M_1 = 0$, then $O_1 = 3$. Since $O_1+O_2=4$, then $O_2 = 1$.
- Case 2: If $M_1 = 1$, then $O_1 = 4$. Since $O_1+O_2=4$, then $O_2 = 0$.

We still do not know M2, the number of goals M&M scored in the second half. Thus, Statement B alone is not sufficient.

Analysis of Statements A and B together: From A, we have $M2 = 4$. From B, we have two possible scenarios for the opponent's score:

- **Scenario 1:** $M1=0$, $O1=3$, $O2=1$. Final M&M score = $M1 + M2 = 0 + 4 = 4$. Final Opponent score = $O1 + O2 = 3 + 1 = 4$. Result: Draw. M&M did not win.
- **Scenario 2:** $M1=1$, $O1=4$, $O2=0$. Final M&M score = $M1 + M2 = 1 + 4 = 5$. Final Opponent score = $O1 + O2 = 4 + 0 = 4$. Result: M&M won.

Since combining both statements leads to two different outcomes (a win or a draw), we cannot definitively answer the question "Did it win the match?". Thus, both statements together are not sufficient.

Step 4: Final Answer: The question cannot be answered on the basis of the two statements.

Mark (5)

Step 5: Why This is the Correct Option: Even with complete information about second-half scores (from A) and the opponent's total score (from B), the ambiguity in the first-half score ($M1=0$, $O1=3$ vs $M1=1$, $O1=4$) leads to different final outcomes. Since a definite "Yes" or "No" cannot be reached, the data is insufficient.

 Quick Tip

For a "Yes/No" data sufficiency question, a statement is sufficient only if it leads to a definite "Yes" in all possible cases or a definite "No" in all possible cases. If it can be "Yes" in some cases and "No" in others, it is insufficient.

Directions for Questions 34 to 37: Answer the following questions based on the information given below:

The following table shows the break-up of actual costs incurred by a company in last five years (year 2002 to year 2006) to produce a particular product:

	Year 2002	Year 2003	Year 2004	Year 2005	Year 2006
Volume of production and sale (units)	1000	900	1100	1200	1200
Costs (Rs.)					
Material	50,000	45,100	55,200	59,900	60,000
Labour	20,000	18,000	22,100	24,150	24,000
Consumables	2,000	2,200	1,800	1,600	1,400
Rent of building	1,000	1,000	1,100	1,100	1,200
Rates and taxes	400	400	400	400	400
Repair and maintenance expenses	800	820	780	790	800
Operating cost of machines	30,000	27,000	33,500	36,020	36,000
Selling and marketing expenses	5,750	5,800	5,800	5,750	5,800

The production capacity of the company is 2000 units. The selling price for the year 2006 was Rs. 125 per unit. Some costs change almost in direct proportion to the change in volume of production, while others do not follow any obvious pattern of change with respect to the volume of production and hence are considered fixed. Using the information provided for the year 2006 as the basis for projecting the figures for the year 2007, answer the following questions:

Analysis of Costs (Common for Questions 34-37)

Before solving the questions, we must classify costs into variable and fixed based on the 2006 data (1200 units).

- **Variable Costs** are proportional to production volume.
 - Material: Rs. 60,000 / 1200 units = Rs. 50/unit.
 - Labour: Rs. 24,000 / 1200 units = Rs. 20/unit.
 - Operating cost of machines: Rs. 36,000 / 1200 units = Rs. 30/unit.

Total Variable Cost (VC) per unit = 50 + 20 + 30 = Rs. 100.

- **Fixed Costs** are treated as constant based on 2006 figures.
 - Consumables: Rs. 1,400
 - Rent of building: Rs. 1,200
 - Rates and taxes: Rs. 400
 - Repair and maintenance: Rs. 800
 - Selling and marketing: Rs. 5,800

Total Fixed Cost (FC) = 1400 + 1200 + 400 + 800 + 5800 = Rs. 9,600.

Cost Function for x units: $TC(x) = FC + VC \cdot x = 9600 + 100x$

Selling Price (SP): Rs. 125 per unit.

34. What is the approximate cost per unit in rupees, if the company produces and sells 1400 units in the year 2007?

- (A) 104
- (B) 107
- (C) 110
- (D) 115
- (E) 116

Correct Answer: (B) 107

Solution:

Step 1: Understanding the Concept: The question asks for the average cost (cost per unit) of production for a specific volume (1400 units). This involves calculating the total cost for that volume and then dividing by the number of units.

Step 2: Key Formula or Approach: We will use the total cost function derived from the 2006 data.

$$\text{Total Cost (TC)} = \text{Fixed Costs} + (\text{Variable Cost per unit} \times \text{Number of units})$$

$$\text{Cost per unit} = \frac{\text{Total Cost}}{\text{Number of units}}$$

Using our derived cost function: $\text{TC}(x) = 9600 + 100x$.

Step 3: Calculation: First, calculate the total cost for producing 1400 units ($x = 1400$).

$$\text{TC}(1400) = 9600 + 100 \times 1400$$

$$\text{TC}(1400) = 9600 + 140000 = 149600$$

Next, calculate the cost per unit by dividing the total cost by the number of units.

$$\text{Cost per unit} = \frac{149600}{1400} = \frac{1496}{14} \approx 106.857$$

The approximate cost per unit is Rs. 107.

Step 4: Final Answer: The approximate cost per unit is Rs. 107.

107

Step 5: Why This is the Correct Option: The calculation, based on the correct classification of fixed and variable costs from the 2006 data, yields a cost per unit of approximately 107.

 Quick Tip

Remember that average cost per unit is not constant. Due to fixed costs, the average cost decreases as production volume increases. This is known as economies of scale. Don't simply use the variable cost per unit as the answer.

35. What is the minimum number of units that the company needs to produce and sell to avoid any loss?

- (A) 313
- (B) 350
- (C) 384
- (D) 747
- (E) 928

Correct Answer: (C) 384

Solution:

Step 1: Understanding the Concept: The question asks for the break-even point in units. The break-even point is the level of production at which total revenue equals total costs, resulting in zero profit and zero loss.

Step 2: Key Formula or Approach: At the break-even point (BEP), Profit = 0, which means:

$$\text{Total Revenue (TR)} = \text{Total Cost (TC)}$$

$$(\text{Selling Price per unit} \times x) = \text{Fixed Costs} + (\text{Variable Cost per unit} \times x)$$

Let x be the number of units.

$$SP \cdot x = FC + VC \cdot x$$

We can solve for x :

$$x_{BEP} = \frac{FC}{SP - VC}$$

The term $(SP - VC)$ is called the contribution margin per unit.

Step 3: Calculation: Using the values derived earlier:

- $SP = \text{Rs. } 125$ per unit
- $VC = \text{Rs. } 100$ per unit
- $FC = \text{Rs. } 9,600$

$$x_{BEP} = \frac{9600}{125 - 100} = \frac{9600}{25}$$
$$x_{BEP} = 384$$

The company needs to produce and sell 384 units to avoid any loss.

Step 4: Final Answer: The minimum number of units is 384.

384

Step 5: Why This is the Correct Option: The break-even formula correctly applied with the derived cost structure and selling price yields exactly 384 units.

 Quick Tip

The break-even formula $FC/(SP - VC)$ is fundamental in cost-volume-profit analysis and is extremely useful for quick calculations in exams. Always identify these three components (Fixed Cost, Selling Price, Variable Cost) first.

36. If the company reduces the price by 5%, it can produce and sell as many units as it desires. How many units the company should produce to maximize its profit?

- (A) 1400
(B) 1600

- (C) 1800
- (D) 1900
- (E) 2000

Correct Answer: (E) 2000

Solution:

Step 1: Understanding the Concept: This is a profit maximization problem. We need to find the production level that yields the highest profit. The profit function is linear, and we need to consider the company's production capacity.

Step 2: Key Formula or Approach: First, calculate the new selling price. Then, write the profit function:

$$\text{Profit}(x) = \text{Total Revenue}(x) - \text{Total Cost}(x)$$

$$P(x) = (\text{New SP} \cdot x) - (FC + VC \cdot x)$$

$$P(x) = (\text{New SP} - VC)x - FC$$

This is a linear function of x . If the contribution margin (New SP - VC) is positive, profit increases with every unit sold. Therefore, profit will be maximized at the highest possible production level, which is the company's capacity.

Step 3: Calculation: Calculate the new selling price (New SP):

$$\text{New SP} = 125 - (5\% \text{ of } 125) = 125 - (0.05 \times 125) = 125 - 6.25 = 118.75$$

Calculate the contribution margin per unit:

$$\text{Contribution Margin} = \text{New SP} - VC = 118.75 - 100 = 18.75$$

Since the contribution margin is positive (Rs. 18.75), each unit sold increases the total profit. To maximize profit, the company should produce and sell the maximum number of units possible. The production capacity is given as 2000 units. Therefore, the company should produce 2000 units.

Step 4: Final Answer: The company should produce 2000 units to maximize profit.

2000

Step 5: Why This is the Correct Option: With a linear profit function and a positive contribution margin, profit is directly proportional to the number of units sold. The maximum profit is achieved at the maximum possible sales volume, which is the production capacity of 2000 units.

💡 Quick Tip

For a linear profit function $P(x) = mx + c$, if the slope m (the contribution margin) is positive, the function is always increasing. The maximum value will always occur at the highest allowed value of x . If m were negative, the maximum would be at the lowest allowed value of x .

37. Given that the company cannot sell more than 1700 units, and it will have to reduce the price by Rs.5 for all units, if it wants to sell more than 1400 units, what is the maximum profit, in rupees, that the company can earn?

- (A) 25,400
- (B) 24,400
- (C) 31,400
- (D) 32,900
- (E) 32,000

Correct Answer: (A) 25,400

Solution:

Step 1: Understanding the Concept: This problem involves profit maximization with a piecewise pricing strategy. We need to calculate the maximum profit in each price tier and then compare them to find the overall maximum profit.

Step 2: Key Formula or Approach: We must analyze two distinct scenarios based on the sales volume.

- **Scenario 1: Sales volume $x \leq 1400$ units.**

- Selling Price (SP1) = Rs. 125
- Profit Function: $P_1(x) = (SP1 - VC)x - FC = (125 - 100)x - 9600 = 25x - 9600$
- Find the maximum profit in this range.

- **Scenario 2: Sales volume $1400 < x \leq 1700$ units.**

- Selling Price (SP2) = Rs. 125 - Rs. 5 = Rs. 120
- Profit Function: $P_2(x) = (SP2 - VC)x - FC = (120 - 100)x - 9600 = 20x - 9600$
- Find the maximum profit in this range.

The overall maximum profit will be the larger of the two maximums found.

Step 3: Calculation: Calculate Maximum Profit for Scenario 1: The profit function $P_1(x) = 25x - 9600$ is linear and increasing. Therefore, profit is maximized at the highest sales volume in this tier, which is $x = 1400$.

$$\text{Max Profit}_1 = P_1(1400) = 25 \times 1400 - 9600 = 35000 - 9600 = 25400$$

Calculate Maximum Profit for Scenario 2: The profit function $P_2(x) = 20x - 9600$ is also linear and increasing. Therefore, profit is maximized at the highest sales volume in this tier, which is $x = 1700$.

$$\text{Max Profit}_2 = P_2(1700) = 20 \times 1700 - 9600 = 34000 - 9600 = 24400$$

Compare the Maximums: Compare the maximum profit from both scenarios:

$$\text{Max}(\text{Max Profit}_1, \text{Max Profit}_2) = \text{Max}(25400, 24400) = 25400$$

The overall maximum profit the company can earn is Rs. 25,400.

Step 4: Final Answer: The maximum profit is Rs. 25,400.

25,400

Step 5: Why This is the Correct Option: Calculating the maximum possible profit for each pricing tier separately shows that the profit at 1400 units (Rs. 25,400) is higher than the profit at 1700 units with the reduced price (Rs. 24,400). Therefore, the optimal strategy is to sell exactly 1400 units.

💡 Quick Tip

When dealing with tiered pricing or discounts, always calculate the profit at the boundary points of the tiers. In this case, the profit at the end of the first tier (1400 units) was higher than the profit at the end of the second tier (1700 units). The price drop was too significant to be offset by the increased volume.

Directions for Questions 38 to 41: Answer the following questions based on the information given below:

The proportion of male students and the proportion of vegetarian students in a school are given below. The school has a total of 800 students, 80% of whom are in the Secondary Section and rest are equally divided between Class 11 and 12.

	Male (M)	Vegetarian (V)
Class 12	0.6	
Class 11	0.55	0.5
Secondary Section		0.55
Total	0.475	0.53

Initial Data Analysis (Common for Questions 38-41)

1. Student Distribution:

- Total students = 800.
- Secondary Section students = 80% of 800 = 640.
- Remaining students = $800 - 640 = 160$.
- Class 11 students = $160 / 2 = 80$.
- Class 12 students = $160 / 2 = 80$.

2. Absolute Numbers from Proportions:

- Total Males = $0.475 \times 800 = 380$.
- Total Vegetarians = $0.53 \times 800 = 424$.

- Class 11 Males = $0.55 \times 80 = 44$.
- Class 11 Vegetarians = $0.5 \times 80 = 40$.
- Class 12 Males = $0.6 \times 80 = 48$.
- Secondary Section Vegetarians = $0.55 \times 640 = 352$.

3. Calculating Missing Values:

- Males in Secondary Section = Total Males - (Class 11 Males + Class 12 Males) = $380 - (44 + 48) = 380 - 92 = 288$.
- Vegetarians in Class 12 = Total Vegetarians - (Class 11 Veg + Secondary Veg) = $424 - (40 + 352) = 424 - 392 = 32$.

4. Summary Table of Absolute Numbers:

Category	Total	Males	Vegetarians
Class 12	80	48	32
Class 11	80	44	40
Secondary Section	640	288	352
Total School	800	380	424

38. What is the percentage of male students in the secondary section?

- (A) 40
- (B) 45
- (C) 50
- (D) 55
- (E) 60

Correct Answer: (B) 45

Solution:

Step 1: Understanding the Concept: The question asks for the proportion of male students within the Secondary Section, expressed as a percentage. This requires finding the number of male students in the Secondary Section and dividing it by the total number of students in that section.

Step 2: Key Formula or Approach:

$$\text{Percentage of Males in Secondary} = \frac{\text{Number of Males in Secondary}}{\text{Total Students in Secondary}} \times 100\%$$

We will use the values calculated in the initial data analysis.

Step 3: Calculation: From the initial analysis:

- Number of Males in Secondary Section = 288.
- Total Students in Secondary Section = 640.

$$\begin{aligned}\text{Percentage} &= \frac{288}{640} \times 100\% \\ &= \frac{2880}{640}\% = \frac{288}{64}\% = 4.5 \times 10\% = 45\%\end{aligned}$$

Step 4: Final Answer: The percentage of male students in the secondary section is 45%.

45%

Step 5: Why This is the Correct Option: The calculation based on the given school demographics correctly yields 45%, which corresponds to option (B).

 Quick Tip

In data interpretation sets, it's often beneficial to do a preliminary calculation to find all the base numbers first. This creates a master table that makes answering subsequent questions much faster and less error-prone.

39. In Class 12, twenty five per cent of the vegetarians are male. What is the difference between the number of female vegetarians and male non-vegetarians?

- (A) less than 8
- (B) 10
- (C) 12
- (D) 14
- (E) 16

Correct Answer: (E) 16

Solution:

Step 1: Understanding the Concept: This question requires us to use the base data for Class 12 and a new piece of information to calculate the sizes of two specific subgroups (female vegetarians and male non-vegetarians) and then find the difference between them.

Step 2: Key Formula or Approach: 1. Find the total number of vegetarians in Class 12. 2. Calculate the number of male vegetarians using the given 25%. 3. Calculate female vegetarians by subtraction: Total Vegetarians - Male Vegetarians. 4. Find the total number of males in Class 12. 5. Calculate male non-vegetarians by subtraction: Total Males - Male Vegetarians. 6. Find the difference between the two calculated groups.

Step 3: Calculation: From the initial analysis for Class 12:

- Total students = 80.
- Total Males = 48.

- Total Vegetarians = 32.

Using the new information: "twenty five per cent of the vegetarians are male".

- Number of Male Vegetarians = 25% of 32 = $0.25 \times 32 = 8$.

Now, we can find the required subgroups:

- Number of Female Vegetarians = Total Vegetarians - Male Vegetarians = $32 - 8 = 24$.
- Number of Male Non-Vegetarians = Total Males - Male Vegetarians = $48 - 8 = 40$.

Finally, calculate the difference:

$$\text{Difference} = |\text{Female Vegetarians} - \text{Male Non-Vegetarians}| = |24 - 40| = 16$$

Step 4: Final Answer: The difference is 16.

16

Step 5: Why This is the Correct Option: The step-by-step calculation correctly determines the size of each subgroup, and the difference between them is 16, matching option (E).

💡 Quick Tip

For problems involving overlapping categories (like Male/Female and Vegetarian/Non-Veg), a 2x2 table is a very effective tool for organizing information and finding unknown values.

40. What is the percentage of vegetarian students in Class 12?

- (A) 40
- (B) 45
- (C) 50
- (D) 55
- (E) 60

Correct Answer: (A) 40

Solution:

Step 1: Understanding the Concept: The question asks for the proportion of vegetarian students within Class 12, expressed as a percentage. This requires finding the number of vegetarian students in Class 12 and dividing it by the total number of students in that class.

Step 2: Key Formula or Approach:

$$\text{Percentage of Vegetarians in Class 12} = \frac{\text{Number of Vegetarians in Class 12}}{\text{Total Students in Class 12}} \times 100\%$$

We will use the values calculated in the initial data analysis.

Step 3: Calculation: From the initial analysis:

- Number of Vegetarians in Class 12 = 32.
- Total Students in Class 12 = 80.

$$\begin{aligned}\text{Percentage} &= \frac{32}{80} \times 100\% \\ &= \frac{320}{8}\% = 40\%\end{aligned}$$

Step 4: Final Answer: The percentage of vegetarian students in Class 12 is 40%.

40%

Step 5: Why This is the Correct Option: The calculation, derived from filling the gaps in the provided table using the total values, shows that 32 out of 80 students in Class 12 are vegetarian, which is 40%. This matches option (A).

💡 Quick Tip

Double-check what the percentage is relative to. This question asks for the percentage *of students in Class 12*, not the percentage of total school students or total vegetarians. Misinterpreting the "base" of the percentage is a common error.

41. In the Secondary Section, 50% of the students are vegetarian males. Which of the following statements is correct?

- (A) Except vegetarian males, all other groups have same number of students.
- (B) Except non-vegetarian males, all other groups have same number of students.
- (C) Except vegetarian females, all other groups have same number of students.
- (D) Except non-vegetarian females, all other groups have same number of students.
- (E) All of the above groups have the same number of students.

Correct Answer: (B) Except non-vegetarian males, all other groups have same number of students.

Solution:

Step 1: Understanding the Concept: This question introduces new information about the composition of the Secondary Section and asks us to determine the relative sizes of four subgroups: vegetarian males (VM), vegetarian females (VF), non-vegetarian males (NVM), and non-vegetarian females (NVF). The original wording "50% of the students are vegetarian males" is ambiguous. It could mean 50% of total students, 50% of males, or 50% of vegetarians.

Given the numbers, "50% of vegetarians are male" is the only interpretation that doesn't lead to a contradiction. We will proceed with this assumption.

Step 2: Key Formula or Approach: 1. Use the base data for the Secondary Section (Total students, Males, Females, Vegetarians, Non-Vegetarians). 2. Apply the new information: Number of VM = 50% of Total Vegetarians in the section. 3. Construct a 2x2 contingency table to find the numbers for the other three groups (VF, NVM, NVF). 4. Compare the sizes of the four groups to evaluate the given statements.

Step 3: Calculation: From the initial analysis for the Secondary Section:

- Total Students = 640
- Total Males (M) = 288
- Total Females (F) = 352
- Total Vegetarians (V) = 352
- Total Non-Vegetarians (NV) = 288

Applying the interpretation "50% of the vegetarians are male":

- Number of Vegetarian Males (VM) = 50% of 352 = $0.5 \times 352 = 176$.

Now, we find the other groups:

- Number of Vegetarian Females (VF) = Total Vegetarians - VM = $352 - 176 = 176$.
- Number of Non-Vegetarian Males (NVM) = Total Males - VM = $288 - 176 = 112$.
- Number of Non-Vegetarian Females (NVF) = Total Females - VF = $352 - 176 = 176$.

The sizes of the four groups are: VM=176, VF=176, NVM=112, NVF=176.

Now, we check the options:

- (A) False. NVM (112) is different from VF and NVF (both 176).
- (B) Correct. The other groups are VM, VF, and NVF. Their numbers are 176, 176, and 176, which are the same.
- (C) False. NVM (112) is different from VM and NVF (both 176).
- (D) False. NVM (112) is different from VM and VF (both 176).
- (E) False. NVM (112) is different from the others.

Step 4: Final Answer: The statement "Except non-vegetarian males, all other groups have same number of students" is correct.

Statement (B) is correct.

Step 5: Why This is the Correct Option: Based on the most logical interpretation of the ambiguous information provided, the calculated sizes of the four subgroups are 176, 176, 112, and 176. This distribution of numbers makes statement (B) the only true statement among the choices.

💡 Quick Tip

When a question in a DI set seems ambiguous or contradictory, try to work through different possible interpretations. Often, only one interpretation will lead to a sensible answer that matches one of the options. Note that in a real exam, such a question might be flagged as erroneous.

Directions for Questions 42 to 45: Answer the following questions based on the information given below:

The Table below shows the comparative costs, in US Dollars, of major surgeries in USA and a select few Asian countries. The second table shows the cost of poor quality, which needs to be added to the procedure cost.

Table 1: Comparative Costs (in US Dollar)

Procedure	USA	India	Thailand	Singapore	Malaysia
Heart Bypass	130000	10000	11000	18500	9000
Heart Valve Replacement	160000	9000	10000	12500	9000
Angioplasty	57000	11000	13000	13000	11000
Hip Replacement	43000	9000	12000	12000	10000
Hysterectomy	20000	3000	4500	6000	3000
Knee Replacement	40000	8500	10000	13000	8000
Spinal Fusion	62000	5500	7000	9000	6000

Table 2: Cost of Poor Quality (in US Dollars '000)

Procedure	USA	India	Thailand	Singapore	Malaysia
Heart Bypass	0	3	3	2	4
Heart Valve Replacement	0	5	4	5	5
Angioplasty	0	5	5	4	6
Hip Replacement	0	7	5	5	8
Hysterectomy	0	5	6	5	4
Knee Replacement	0	9	6	4	4
Spinal Fusion	0	5	6	5	6

Exchange Rate: 1 US Dollar = 32.89 Bahts.

Initial Data Analysis (Common for Questions 42-45)

Let's create a combined table for the **Total Cost (Procedure Cost + Poor Quality Cost)** in USD.

Table 3: Total Cost (in US Dollar)

Procedure	USA	India	Thailand	Singapore	Malaysia
Heart Bypass	130000	13000	14000	20500	13000
Heart Valve Replacement	160000	14000	14000	17500	14000
Angioplasty	57000	16000	18000	17000	17000
Hip Replacement	43000	16000	17000	17000	18000
Hysterectomy	20000	8000	10500	11000	7000
Knee Replacement	40000	17500	16000	17000	12000
Spinal Fusion	62000	10500	13000	14000	12000

42. A US citizen is hurt in an accident and requires an angioplasty, hip replacement and a knee replacement. Cost of foreign travel and stay is not a consideration since the government will take care of it. Which country will result in the cheapest package, taking cost of poor quality into account?

- (A) India
- (B) Thailand
- (C) Malaysia
- (D) Singapore
- (E) USA

Correct Answer: (C) Malaysia

Solution:

Step 1: Understanding the Concept: The question asks for the cheapest country for a package of three specific surgeries. This requires summing the total cost (procedure cost + cost of poor quality) for these three surgeries in each country and then comparing the total package costs.

Step 2: Key Formula or Approach: For each country, calculate the total package cost:

$$\text{Package Cost} = \text{TotalCost(Angio)} + \text{TotalCost(Hip)} + \text{TotalCost(Knee)}$$

We will use the pre-calculated values from our 'Total Cost' table (Table 3).

Step 3: Calculation: Let's sum the costs for the three procedures for each country (values are bolded in Table 3):

- **India:** 16000 (Angio) + 16000 (Hip) + 17500 (Knee) = 49,500 USD
- **Thailand:** 18000 (Angio) + 17000 (Hip) + 16000 (Knee) = 51,000 USD
- **Malaysia:** 17000 (Angio) + 18000 (Hip) + 12000 (Knee) = 47,000 USD
- **Singapore:** 17000 (Angio) + 17000 (Hip) + 17000 (Knee) = 51,000 USD
- **USA:** 57000 (Angio) + 43000 (Hip) + 40000 (Knee) = 140,000 USD

Comparing the total package costs, Malaysia has the lowest cost at 47,000 USD.

Step 4: Final Answer: The country resulting in the cheapest package is Malaysia.

Malaysia

Step 5: Why This is the Correct Option: By summing the total costs for the required procedures in each country, Malaysia is clearly the most economical choice.

 Quick Tip

For questions involving multi-item comparisons from a table, creating a summary table first (like Table 3) can prevent errors from repeatedly adding numbers. It organizes the data needed for all subsequent questions.

43. Taking the cost of poor quality into account, which country/countries will be the most expensive for knee replacement?

- (A) India
- (B) Thailand
- (C) Malaysia
- (D) Singapore
- (E) India and Singapore

Correct Answer: (A) India

Solution:

Step 1: Understanding the Concept: The question asks to identify the most expensive country for a single procedure, Knee Replacement, considering the total cost. The options only list the Asian countries, implying the comparison should be made among them.

Step 2: Key Formula or Approach: We need to compare the total cost (Procedure + Poor Quality) for Knee Replacement across the listed Asian countries using the values from Table 3.

Step 3: Calculation: Let's list the total costs for Knee Replacement from Table 3 for the Asian countries:

- **India:** 17,500 USD
- **Thailand:** 16,000 USD
- **Singapore:** 17,000 USD
- **Malaysia:** 12,000 USD

Comparing these values, the highest cost is 17,500 USD in India.

Step 4: Final Answer: The most expensive country among the given options for a knee replacement is India.

India

Step 5: Why This is the Correct Option: A direct comparison of the total costs shows that India has the highest cost of 17,500 USD for a knee replacement among the Asian countries listed in the options.

 Quick Tip

Pay close attention to the options provided. The absence of "USA" in the options for this question is a strong hint that the comparison should be limited to the other countries, even if the USA has the highest absolute cost in the table.

44. Approximately, what difference in amount in Bahts will it make to a Thai citizen if she were to get a hysterectomy done in India instead of in her native country, taking into account the cost of poor quality? (It costs 7500 Bahts for one-way travel between Thailand and India).

- (A) 23500
- (B) 40500
- (C) 57500
- (D) 67500
- (E) 75000

Correct Answer: (D) 67500

Solution:

Step 1: Understanding the Concept: This is a cost-comparison problem involving currency conversion and additional costs (travel). We need to calculate the total cost for the procedure in both countries in the same currency (Bahts) and find the difference.

Step 2: Key Formula or Approach:

1. Calculate Total Cost in Thailand (in USD) and convert to Bahts.
2. Calculate Total Cost in India (in USD).
3. Calculate total travel cost in Bahts.
4. Add travel cost to the Indian procedure cost to find the total expense for the India option, all in Bahts.
5. Find the difference between the cost in Thailand and the total cost for the India option.

Step 3: Calculation: Cost in Thailand:

- Total USD cost from Table 3 = 10,500 USD.
- Cost in Bahts = $10,500 \text{ USD} \times 32.89 \text{ Bahts/USD} = 345,345 \text{ Bahts}$.

Cost of India Option:

- Total USD cost from Table 3 = 8,000 USD.
- Cost in Bahts = $8,000 \text{ USD} \times 32.89 \text{ Bahts/USD} = 263,120 \text{ Bahts}$.
- Travel Cost = $2 \times 7500 \text{ Bahts (round trip)} = 15,000 \text{ Bahts}$.
- Total cost for India option = $263,120 + 15,000 = 278,120 \text{ Bahts}$.

Difference in Amount:

$$\text{Difference} = \text{Cost in Thailand} - \text{Total Cost for India option}$$

$$\text{Difference} = 345,345 - 278,120 = 67,225 \text{ Bahts}$$

This is approximately 67,500 Bahts.

Step 4: Final Answer: The approximate difference in amount is 67,500 Bahts.

67,500 Bahts

Step 5: Why This is the Correct Option: The comprehensive calculation, including the conversion of all costs into a single currency (Bahts) and adding the necessary travel expenses, yields a difference of approximately 67,500 Bahts.

💡 Quick Tip

When a problem involves multiple currencies and additional costs, it's safest to convert all figures to the target currency (in this case, Bahts) before performing the final additions or subtractions. This minimizes rounding errors and confusion.

45. The rupee value increases to Rs.35 for a US Dollar, and all other things including quality, remain the same. What is the approximate difference in cost, in US Dollars, between Singapore and India for a Spinal Fusion, taking this change into account?

- (A) 700
- (B) 2500
- (C) 4500
- (D) 8000
- (E) No difference

Correct Answer: (B) 2500

Solution:

Step 1: Understanding the Concept: This question tests the understanding of how exchange rate fluctuations affect costs presented in a foreign currency. The key is to determine which cost components are based on the local currency (and are thus affected) and which are not. The phrase "all other things including quality, remain the same" suggests the poor quality cost, an international benchmark, might remain fixed in USD.

Step 2: Key Formula or Approach: 1. Assume the 'Procedure' cost for India is originally set in Rupees and was converted to USD for Table 1 using the old rate (1 USD = 40.928 Rs). 2. Assume the 'Cost of Poor Quality' is a benchmark fixed in USD and does not change. 3. Convert the Indian Rupee procedure cost back to USD using the new exchange rate (1 USD

= 35 Rs). 4. Calculate the new total cost for India in USD by adding the new procedure cost and the unchanged poor quality cost. 5. Calculate the difference between the total cost in Singapore (which is unaffected) and the new total cost in India.

Step 3: Calculation: Original Indian Procedure Cost in Rupees:

- From Table 1, Procedure cost = 5,500 USD.
- Original Rupee cost = 5,500 USD $\times$ 40.928 Rs/USD = 225,104 Rs.

New Indian Cost in USD:

- New Procedure cost in USD = 225,104 Rs $\div$ 35 Rs/USD $\approx$ 6,431.5 USD.
- Cost of Poor Quality for India remains = 5,000 USD.
- New Total Cost for India = 6,431.5 + 5,000 = 11,431.5 USD.

Cost in Singapore (unchanged):

- Total Cost from Table 3 = 14,000 USD.

New Difference in Cost:

$$\text{Difference} = \text{Cost in Singapore} - \text{New Cost in India}$$

$$\text{Difference} = 14,000 - 11,431.5 = 2,568.5 \text{ USD}$$

This is approximately 2,500 USD.

Step 4: Final Answer: The approximate new difference in cost is 2,500 USD.

2,500 USD

Step 5: Why This is the Correct Option: The only logical interpretation that leads to one of the given options is that the base procedure cost is subject to exchange rate changes while the quality cost is not. This calculation results in a difference of approximately 2,500 USD.

💡 Quick Tip

In DI questions, if your initial logical approach doesn't match any option, re-read the question carefully for subtle clues. Here, "all other things including quality, remain the same" was the hint that the quality cost might be treated differently from the procedure cost.

Directions for Questions 46 to 50: Answer the following questions based on the information given below:

A low-cost airline company connects ten Indian cities, A to J. The table below gives the distance between a pair of airports and the corresponding price charged by the company. Travel is permitted only from a departure airport to an arrival airport. The customers do not travel by a route where they have to stop at more than two intermediate airports.

Sector No	Airport of Departure	Airport of Arrival	Distance between the Airports (km)	Price (Rs.)
1	A	B	560	670
2	A	C	790	1350
3	A	D	850	1250
4	A	E	1245	1600
5	A	F	1345	1700
6	A	G	1350	2450
7	A	H	1950	1850
8	B	C	1650	2000
9	B	H	1750	1900
10	B	I	2100	2450
11	B	J	2300	2275
12	C	D	460	450
13	C	F	410	430
14	C	G	910	1100
15	D	E	540	590
16	D	F	625	700
17	D	G	640	750
18	D	H	950	1250
19	D	J	1650	2450
20	E	F	1250	1700
21	E	G	970	1150
22	E	H	850	875
23	F	G	900	1050
24	F	I	875	950
25	F	J	970	1150
26	G	I	510	550
27	G	J	830	890
28	H	I	790	970
29	H	J	400	425
30	I	J	460	540

46. What is the lowest price, in rupees, a passenger has to pay for travelling by the shortest route from A to J?

- (A) 2275
- (B) 2850
- (C) 2890
- (D) 2930
- (E) 3340

Correct Answer: (D) 2930

Solution:

Step 1: Understanding the Concept: The question asks for the price of the "shortest route". This means we must first identify the route from city A to city J that has the minimum total distance, and then find the total price for that specific route. The journey can have at most two intermediate stops.

Step 2: Key Formula or Approach: We need to list all possible routes from A to J, calculate the total distance for each, identify the route with the minimum distance, and then find its corresponding total price.

$$\text{Total Distance} = \sum (\text{Distances of individual flight legs})$$

$$\text{Total Price} = \sum (\text{Prices of individual flight legs})$$

Step 3: Calculation: Let's find the distances of all possible routes from A to J.

One-stop routes (A-X-J):

- $A \rightarrow B \rightarrow J$: Distance = $560 + 2300 = 2860$ km
- $A \rightarrow D \rightarrow J$: Distance = $850 + 1650 = 2500$ km
- $A \rightarrow F \rightarrow J$: Distance = $1345 + 970 = 2315$ km
- $A \rightarrow G \rightarrow J$: Distance = $1350 + 830 = 2180$ km
- $A \rightarrow H \rightarrow J$: Distance = $1950 + 400 = 2350$ km

Two-stop routes (A-X-Y-J):

- $A \rightarrow C \rightarrow F \rightarrow J$: Distance = $790 + 410 + 970 = \mathbf{2170}$ km
- $A \rightarrow D \rightarrow G \rightarrow J$: Distance = $850 + 640 + 830 = 2320$ km
- $A \rightarrow D \rightarrow H \rightarrow J$: Distance = $850 + 950 + 400 = 2200$ km
- ... (and other longer routes)

Comparing the distances, the shortest route is $A \rightarrow C \rightarrow F \rightarrow J$ with a total distance of 2170 km.

Now, we calculate the price for this shortest route:

- Price($A \rightarrow C$) = Rs. 1350
- Price($C \rightarrow F$) = Rs. 430
- Price($F \rightarrow J$) = Rs. 1150

$$\text{Total Price} = 1350 + 430 + 1150 = 2930$$

Step 4: Final Answer: The price for the shortest route ($A \rightarrow C \rightarrow F \rightarrow J$) is Rs. 2930.

Rs. 2930

Step 5: Why This is the Correct Option: The shortest route is identified as $A \rightarrow C \rightarrow F \rightarrow J$ with a distance of 2170 km. The cumulative price for this route is Rs. 2930. Other routes might be cheaper (like $A \rightarrow H \rightarrow J$ for Rs. 2275), but they are not the shortest in terms of distance.

💡 Quick Tip

In such questions, carefully distinguish between "shortest route" (minimum distance) and "cheapest route" (minimum price). They are often not the same. It's crucial to first find the route that satisfies the primary condition (shortest distance) and then calculate its price.

47. The company plans to introduce a direct flight between A and J. The market research results indicate that all its existing passengers travelling between A and J will use this direct flight if it is priced 5% below the minimum price that they pay at present. What should the company charge approximately, in rupees, for this direct flight?

- (A) 1991
- (B) 2161
- (C) 2707
- (D) 2745
- (E) 2783

Correct Answer: (B) 2161

Solution:

Step 1: Understanding the Concept: This question requires finding the current "minimum price" for a journey from A to J using any valid combination of existing flights. Then, a new price needs to be calculated which is 5

Step 2: Key Formula or Approach: First, find the minimum price among all possible routes from A to J.

$$P_{\min} = \min(\text{Total Price of all routes from A to J})$$

Then, calculate the new price for the direct flight.

$$P_{\text{new}} = P_{\min} \times (1 - 0.05) = P_{\min} \times 0.95$$

Step 3: Calculation: Let's find the prices of all possible routes from A to J to identify the minimum.

One-stop routes (A-X-J):

- A → B → J: Price = 670 + 2275 = Rs. 2945
- A → D → J: Price = 1250 + 2450 = Rs. 3700
- A → F → J: Price = 1700 + 1150 = Rs. 2850
- A → G → J: Price = 2450 + 890 = Rs. 3340
- A → H → J: Price = 1850 + 425 = **Rs. 2275**

Two-stop routes (A-X-Y-J):

- A → C → F → J: Price = 1350 + 430 + 1150 = Rs. 2930
- A → D → G → J: Price = 1250 + 750 + 890 = Rs. 2890
- A → E → H → J: Price = 1600 + 875 + 425 = Rs. 2900
- ... (and other more expensive routes)

Comparing the total prices, the minimum price for travelling from A to J is Rs. 2275 for the route $A \rightarrow H \rightarrow J$.

$$P_{\min} = \text{Rs. } 2275$$

Now, we calculate the price of the new direct flight, which is 5

$$P_{\text{new}} = 2275 \times (1 - 0.05) = 2275 \times 0.95$$

$$P_{\text{new}} = 2161.25$$

The question asks for the approximate charge, which is Rs. 2161.

Step 4: Final Answer: The company should charge approximately Rs. 2161.

Rs. 2161

Step 5: Why This is the Correct Option: The cheapest existing route from A to J costs Rs. 2275. A price 5

 Quick Tip

To solve this efficiently, you don't need to calculate the price for every single possible route. Start with routes that seem intuitively cheap (e.g., using legs with lower prices listed in the table). Once you find a low price, you can quickly eliminate other routes that are clearly more expensive.

48. If the airports C, D and H are closed down owing to security reasons, then what would be the minimum price, in rupees, to be paid by a passenger travelling from A to J?

- (A) 2275
- (B) 2615
- (C) 2850
- (D) 2945
- (E) 3190

Correct Answer: (C) 2850

Solution:

Step 1: Understanding the Concept: We need to find the minimum price for a journey from A to J under a new constraint: airports C, D, and H cannot be used as departure, arrival, or intermediate stops. This means we must re-evaluate the possible routes and find the cheapest one among the valid options.

Step 2: Key Formula or Approach: List all routes from A to J that do not involve airports C, D, or H. Calculate the total price for each of these valid routes and identify the minimum price.

$$P_{\min_{\text{new}}} = \min(\text{Total Price of valid routes from A to J})$$

Step 3: Calculation: We need to check our previously identified routes and eliminate any that pass through C, D, or H.

Original Routes and their Validity:

- $A \rightarrow B \rightarrow J$: Valid. Price = $670 + 2275 = \text{Rs. } 2945$
- $A \rightarrow D \rightarrow J$: Invalid (involves D).
- $A \rightarrow F \rightarrow J$: Valid. Price = $1700 + 1150 = \text{Rs. } 2850$
- $A \rightarrow G \rightarrow J$: Valid. Price = $2450 + 890 = \text{Rs. } 3340$
- $A \rightarrow H \rightarrow J$: Invalid (involves H).
- $A \rightarrow B \rightarrow H \rightarrow J$: Invalid (involves H).
- $A \rightarrow C \rightarrow F \rightarrow J$: Invalid (involves C).
- $A \rightarrow D \rightarrow G \rightarrow J$: Invalid (involves D).
- $A \rightarrow F \rightarrow I \rightarrow J$: Valid. Price = $1700 + 950 + 540 = \text{Rs. } 3190$
- $A \rightarrow G \rightarrow I \rightarrow J$: Valid. Price = $2450 + 550 + 540 = \text{Rs. } 3540$
- $A \rightarrow E \rightarrow F \rightarrow J$: Valid. Price = $1600 + 1700 + 1150 = \text{Rs. } 4450$

Now, we compare the prices of the valid routes:

- Rs. 2945 (for $A \rightarrow B \rightarrow J$)
- Rs. 2850 (for $A \rightarrow F \rightarrow J$)
- Rs. 3340 (for $A \rightarrow G \rightarrow J$)
- Rs. 3190 (for $A \rightarrow F \rightarrow I \rightarrow J$)
- etc.

The minimum price among the valid routes is Rs. 2850.

Step 4: Final Answer: With airports C, D, and H closed, the minimum price to travel from A to J is Rs. 2850.

Rs. 2850

Step 5: Why This is the Correct Option: After eliminating all routes passing through the closed airports C, D, and H, the cheapest remaining route is $A \rightarrow F \rightarrow J$, with a total cost of Rs. 2850. The previous cheapest route ($A \rightarrow H \rightarrow J$) is no longer available.

 Quick Tip

When constraints are added (like airport closures), methodically go through your list of possible routes and strike out the ones that are no longer valid. Then, find the optimum (minimum or maximum) value from the remaining set. Do not start from scratch if you have already listed the routes for a previous question.

49. If the prices include a margin of 10% over the total cost that the company incurs, then what is the minimum cost per kilometer that the company incurs in flying from A to J?

- (A) 0.77
- (B) 0.88
- (C) 0.99
- (D) 1.06
- (E) 1.08

Correct Answer: (B) 0.88

Solution:

Step 1: Understanding the Concept: The question asks for the minimum "cost per kilometer" for the company. The given prices include a 10

Step 2: Key Formula or Approach: For any given route:

$$\begin{aligned}\text{Price} &= \text{Cost} + 10\% \text{ of Cost} = 1.1 \times \text{Cost} \\ \implies \text{Cost} &= \frac{\text{Price}}{1.1}\end{aligned}$$

The cost per kilometer is given by:

$$\text{Cost per km} = \frac{\text{Total Cost}}{\text{Total Distance}} = \frac{\text{Price}/1.1}{\text{Distance}} = \frac{1}{1.1} \times \left(\frac{\text{Price}}{\text{Distance}} \right)$$

To minimize the cost per km, we need to find the route that minimizes the ratio of Price to Distance (P/D).

Step 3: Calculation: Let's calculate the Price/Distance (P/D) ratio for the most promising routes from A to J.

- Route A → F → J: P = 2850, D = 2315. Ratio P/D = 2850 / 2315 ≈ 1.231
- Route A → H → J: P = 2275, D = 2350. Ratio P/D = 2275 / 2350 ≈ 0.968
- Route A → C → F → J: P = 2930, D = 2170. Ratio P/D = 2930 / 2170 ≈ 1.350
- Route A → D → G → J: P = 2890, D = 2320. Ratio P/D = 2890 / 2320 ≈ 1.246
- Route A → B → J: P = 2945, D = 2860. Ratio P/D = 2945 / 2860 ≈ 1.030

The minimum P/D ratio is approximately 0.968, which corresponds to the route A → H → J. Now, we can calculate the minimum cost per kilometer using this ratio.

$$\begin{aligned}\text{Min. Cost per km} &= \frac{1}{1.1} \times (\text{Min. P/D ratio}) \\ \text{Min. Cost per km} &= \frac{1}{1.1} \times 0.968 \approx 0.88\end{aligned}$$

Step 4: Final Answer: The minimum cost per kilometer that the company incurs is approximately Rs. 0.88.

Rs. 0.88

Step 5: Why This is the Correct Option: The route $A \rightarrow H \rightarrow J$ has the lowest price-to-distance ratio. After removing the 10

 Quick Tip

To find the minimum or maximum of a derived quantity like cost/km, you first need to express it in terms of the given data (Price, Distance). Here, minimizing $(\text{Price}/1.1)/\text{Distance}$ is the same as minimizing $\text{Price}/\text{Distance}$. Focus on finding the route with the best ratio, rather than calculating the final value for every single route.

50. If the prices include a margin of 15% over the total cost that the company incurs, then which among the following is the distance to be covered in flying from A to J that minimizes the total cost per kilometer for the company?

- (A) 2170
- (B) 2180
- (C) 2315
- (D) 2350
- (E) 2390

Correct Answer: (D) 2350

Solution:

Step 1: Understanding the Concept: Similar to the previous question, we need to find the route that minimizes the cost per kilometer. The only difference is the profit margin, which is now 15

Step 2: Key Formula or Approach: The relationship between price and cost is now:

$$\text{Price} = \text{Cost} + 15\% \text{ of Cost} = 1.15 \times \text{Cost}$$

The cost per kilometer is:

$$\text{Cost per km} = \frac{\text{Total Cost}}{\text{Total Distance}} = \frac{\text{Price}/1.15}{\text{Distance}} = \frac{1}{1.15} \times \left(\frac{\text{Price}}{\text{Distance}} \right)$$

To minimize the cost per km, we again need to find the route that minimizes the Price to Distance (P/D) ratio. The change in the margin percentage does not change the identity of the route with the minimum P/D ratio.

Step 3: Calculation: From the analysis in the solution for Question 49, we have already determined the route that minimizes the P/D ratio. We calculated the P/D ratio for various routes and found the minimum value.

- Route $A \rightarrow H \rightarrow J$: $P = 2275$, $D = 2350$. Ratio $P/D = 2275 / 2350 \approx \mathbf{0.968}$ (This was the minimum)
- Route $A \rightarrow B \rightarrow J$: $P = 2945$, $D = 2860$. Ratio $P/D \approx 1.030$
- Route $A \rightarrow F \rightarrow J$: $P = 2850$, $D = 2315$. Ratio $P/D \approx 1.231$

The route with the minimum cost per kilometer is $A \rightarrow H \rightarrow J$. The question asks for the distance of this route. Let's calculate the total distance for the route $A \rightarrow H \rightarrow J$:

$$\text{Distance } (A \rightarrow H) + \text{Distance } (H \rightarrow J) = 1950 \text{ km} + 400 \text{ km} = 2350 \text{ km}$$

Step 4: Final Answer: The distance of the route that minimizes the total cost per kilometer is 2350 km.

2350 km

Step 5: Why This is the Correct Option: The minimization of cost per kilometer depends entirely on minimizing the Price/Distance ratio. As established in the previous question, the route $A \rightarrow H \rightarrow J$ has the lowest P/D ratio. The total distance of this route is $1950 \text{ km} + 400 \text{ km} = 2350 \text{ km}$.

💡 Quick Tip

Recognize when a question builds upon a previous one. The core task here—finding the route with the minimum P/D ratio—is identical to that in Question 49. The change in profit margin is irrelevant for identifying the optimal route. This saves you from re-calculating all the ratios.

Section - III

Directions for Questions 51 to 53: The passage given below is followed by a set of three questions. Choose the most appropriate answer to each question.

Human Biology does nothing to structure human society: age may enfeeble us all, but cultures vary considerably in the prestige and power they accord to the elderly. Giving birth is a necessary condition for being a mother, but it is not sufficient. We expect mothers to behave in maternal ways and to display appropriately maternal sentiments. We prescribe a clutch of norms or rules that govern the **role** of a mother. That the social role is independent of the biological base can be demonstrated by going back three sentences. (giving birth is certainly not sufficient to be a mother but, as adoption and fostering show, it is not even necessary!)

The fine detail of what is expected of a mother or a father or a dutiful son differs from culture to culture, but everywhere behaviour is coordinated by the **reciprocal** nature of roles.

Husbands and wives, parents and children, employers and employees, waiters and customers, teachers and pupils, warlords and followers: each makes sense only in its relation to the other. The term 'role' is an appropriate one, because the metaphor of an actor in a play neatly expresses the rule-governed nature of much of social life and the sense that society is a joint production. Social life occurs only because people play their parts (and that is as true for war and conflicts as for peace and love) and those parts make sense only in the context of the overall show. The drama metaphor also reminds us of the artistic licence available to the players. We can play a part straight or, as the following from J.P. Sartre conveys, we can ham it up.

Let us consider this waiter in the café. His movement is quick and forward, a little too precise, a little too rapid. He comes towards the patrons with a step a little too quick. He bends forward a little too eagerly: his voice, his eyes express an interest a little too solicitous for the order of the customer. Finally there he returns, trying to imitate in his walk the inflexible stiffness of some kind of automation while carrying the tray with the recklessness of a tightropewalker. . . . All his behaviour seems to us a game. . . . But what is he playing? We need not watch long before we can explain it: he is playing at being a waiter in a café.

The American sociologist Erving Goffman built an influential body of social analysis on elaborations of the metaphor of social life as drama. Perhaps his most telling point was that it is only through acting out a part that we express character. It is not enough to be evil or virtuous: we have to be seen to be evil or virtuous.

There is distinction between the roles we play and some underlying self. Here we might note that some roles are more absorbing than others. We would not be surprised by the waitress who plays the part in such a way as to signal to us that she is much more than her occupation. We would be surprised and offended by the father who played his part 'tongue in cheek'. Some roles are broader and more far-reaching than others. Describing someone as a clergyman or faith healer would say far more about that person than describing someone as a bus driver.

51. What is the thematic highlight of the passage?

- (A) In the absence of strong biological linkages, reciprocal roles provide the mechanism for coordinating human behaviour.
- (B) In the absence of reciprocal roles, biological linkages provide the mechanism for coordinating human behaviour.
- (C) Human behaviour is independent of biological linkages and reciprocal roles.
- (D) Human behaviour depends on biological linkages and reciprocal roles.
- (E) Reciprocal roles determine normative human behaviour in society.

Correct Answer: (E) Reciprocal roles determine normative human behaviour in society.

Solution:

Step 1: Understanding the Concept: The question asks for the "thematic highlight," which means identifying the central idea or the main argument of the passage. We need to synthesize the key points to find the most accurate summary of the author's message.

Step 2: Key Formula or Approach: The approach is to analyze the passage's structure

and argument.

1. The passage begins by refuting the idea that biology structures human society ("Human Biology does nothing to structure human society"). It uses motherhood as an example, showing that biological birth is neither sufficient nor necessary.
2. It then introduces the concept of "roles" as the primary mechanism for coordinating social behavior. These roles are "reciprocal" (e.g., husband-wife, waiter-customer) and are governed by norms and rules ("We prescribe a clutch of norms or rules that govern the role of a mother").
3. The passage extends this idea using the metaphor of life as a drama, where individuals are actors playing "scripted" parts. This emphasizes the rule-governed or normative nature of social life.
4. The conclusion is that social life functions because people play these defined roles according to societal expectations.

Step 3: Calculation: Let's evaluate the options based on the analysis:

- **(A) In the absence of strong biological linkages, reciprocal roles provide the mechanism for coordinating human behaviour.** This is a correct statement supported by the passage, but it focuses more on the cause-and-effect relationship. It describes **why** roles are important (because biology isn't) rather than what their primary function **is**.
- **(B) In the absence of reciprocal roles, biological linkages provide the mechanism...** This directly contradicts the passage's main argument.
- **(C) Human behaviour is independent of biological linkages and reciprocal roles.** This is incorrect. The passage explicitly states that behavior is coordinated by reciprocal roles.
- **(D) Human behaviour depends on biological linkages and reciprocal roles.** The passage argues against the dependency on biological linkages for structuring society.
- **(E) Reciprocal roles determine normative human behaviour in society.** This option captures the essence of the passage most effectively. The passage repeatedly emphasizes the "rule-governed," "scripted," and "norm"-based nature of social roles. Phrases like "prescribe a clutch of norms or rules" and "rule-governed nature" directly support the idea that roles determine "normative" (i.e., conforming to a standard or norm) behavior. It is the most encompassing statement about the function of roles as described in the text.

Comparing (A) and (E), option (E) is a more direct and powerful summary of the theme. The passage is less about the "absence of biology" and more about the "presence and function of social roles" in dictating norms.

Step 4: Final Answer: The most accurate thematic highlight of the passage is that reciprocal roles establish the norms for social behavior.

Theme → Social Roles dictate Norms → Option (E)

The final answer is (E) Reciprocal roles determine normative human behaviour in society.

Step 5: Why This is the Correct Option: This option is correct because it best summarizes the central argument that social behavior is not instinctual or biological but is learned and performed according to a set of socially prescribed rules and expectations (norms) associated with different roles. The other options either contradict the passage or capture only a partial aspect of the main theme.

 Quick Tip

To find the thematic highlight of a passage, look for the central argument that connects all the paragraphs. Pay attention to the introductory and concluding sentences, as they often state or restate the main idea. Identify keywords that are repeated or emphasized, such as "role," "norms," "rules," and "scripted" in this passage.

52. Which of the following would have been true if biological linkages structured human society?

- (A) The role of mother would have been defined through her reciprocal relationship with her children.
- (B) We would not have been offended by the father playing his role 'tongue in cheek'.
- (C) Women would have adopted and fostered children rather than giving birth to them.
- (D) Even if warlords were physically weaker than their followers, they would still dominate them.
- (E) Waiters would have stronger motivation to serve their customers.

Correct Answer: (B) We would not have been offended by the father playing his role 'tongue in cheek'.

Solution:

Step 1: Understanding the Concept: This is a hypothetical question based on a counterfactual premise. The passage argues that society is structured by social roles, not biology. The question asks us to imagine the opposite: a world where biology is the primary structuring force. We must then evaluate which of the options would logically follow in such a world.

Step 2: Key Formula or Approach: The approach is to apply the principle of "biological determinism" to each option. If biology structures society, then actions, status, and relationships are determined by innate, physical, and genetic factors rather than social constructs, performances, or norms.

Step 3: Calculation: Let's analyze each option under the assumption that biology structures society:

- **(A) The role of mother would have been defined through her reciprocal relationship with her children.** A "reciprocal relationship" is a social construct. If biology were a key, the role of a mother would be defined purely by the biological act of giving birth and instinct, not by social interaction. This is incorrect.

- **(B) We would not have been offended by the father playing his role 'tongue in cheek'.** The passage states we *would* be offended because the father's role is a serious social performance with strong expectations ("some roles are more absorbing"). If biology were the sole determinant, the "role" would cease to be a performance. The biological fact of being a father would be all that matters. How he "plays" the part would be irrelevant, so we would not be offended by a poor performance. This statement is logically consistent with the hypothetical premise.
- **(C) Women would have adopted and fostered children rather than giving birth to them.** This is the opposite of what would happen. If biology were paramount, the biological act of giving birth and passing on one's genes would be the defining feature of motherhood. Adoption and fostering, which are social constructs, would be less significant.
- **(D) Even if warlords were physically weaker than their followers, they would still dominate them.** In a biologically determined world, physical attributes like strength would likely be a primary determinant of dominance and leadership. A physically weaker warlord would probably be overthrown. Their dominance is a social construct, not a biological one.
- **(E) Waiters would have stronger motivation to serve their customers.** The role of a waiter is a purely social and economic construct. It has no biological basis. Biological determinism doesn't provide any logical reason for a waiter's motivation to change.

Step 4: Final Answer: Based on the analysis, the only logical outcome in a biologically structured society is that the social performance of a role would become irrelevant.

Biological Society → Performance is Irrelevant → No Offense at Poor Performance

The final answer is

(B) We would not have been offended by the father playing his role 'tongue in cheek'.

Step 5: Why This is the Correct Option: This option is correct because it correctly identifies a key consequence of shifting from a social-role-based society to a biology-based one. In the latter, the social "performance" of fatherhood would lose its significance, and only the biological link would matter, thus removing the basis for being "offended" by a nonchalant performance.

💡 Quick Tip

For hypothetical "what if" questions, first clearly define the hypothetical premise (e.g., "society is structured by biology"). Then, for each option, ask: "Is this statement a logical consequence of my premise?" Discard any options that rely on the original assumptions of the passage (in this case, social roles).

53. It has been claimed in the passage that "some roles are more absorbing than others". According to the passage, which of the following seem(s) appropriate reason(s) for such a claim?

A. Some roles carry great expectations from the society preventing manifestation

of the true self.

B. Society ascribes so much importance to some roles that the conception of self may get aligned with the roles being performed.

C. Some roles require development of skill and expertise leaving little time for manifestation of self.

- (A) A only
- (B) B only
- (C) C only
- (D) A & B
- (E) B & C

Correct Answer: (D) A & B

Solution:

Step 1: Understanding the Concept: The question asks for the reasons provided *in the passage* to support the claim that "some roles are more absorbing than others." We must find direct evidence in the text to validate the given statements (A, B, C).

Step 2: Key Formula or Approach: The approach is to locate the specific part of the passage where this claim is made and analyze the examples and explanations given by the author. This claim appears in the final paragraph. We must carefully read this paragraph and map its content to statements A, B, and C.

Relevant Text: "Here we might note that some roles are more absorbing than others. We would not be surprised by the waitress who plays the part in such a way as to signal to us that she is much more than her occupation. We would be surprised and offended by the father who played his part 'tongue in cheek'. Some roles are broader and more far-reaching than others. Describing someone as a clergyman or faith healer would say far more about that person than describing someone as a bus driver."

Step 3: Calculation: Let's evaluate each statement against the evidence from the passage:

- **Statement A: Some roles carry great expectations from the society preventing manifestation of the true self.** The passage states we would be "surprised and offended by the father who played his part 'tongue in cheek'". This implies that the role of a father has strong societal expectations attached to it, which do not permit a casual or detached performance. This directly supports statement A. The expectation is so great that you are not supposed to show a "true self" that is detached from the role.
- **Statement B: Society ascribes so much importance to some roles that the conception of self may get aligned with the roles being performed.** The passage states that some roles are "broader and more far-reaching" and gives the example that "Describing someone as a clergyman or faith healer would say far more about that person" than a bus driver. This suggests that these roles are so important and all-encompassing that they fuse with the person's identity or "conception of self." This supports statement B.
- **Statement C: Some roles require development of skill and expertise leaving little time for manifestation of self.** The passage does not mention skill, expertise,

or the amount of time required for a role as a reason for it being "absorbing." The examples used (father, clergyman) relate to social significance and identity, not technical skill. Therefore, there is no textual evidence to support statement C.

Since both A and B are supported by the text, the correct option is (D).

Step 4: Final Answer: The passage provides evidence for reasons A and B but not for C.

Evidence for A → Example of the father

Evidence for B → Example of the clergyman

Evidence for C → None

The final answer is (D) A & B.

Step 5: Why This is the Correct Option: This is the correct option because the final paragraph of the passage explicitly provides examples and reasoning that align perfectly with statements A and B. The "offended father" example supports the idea of high societal expectations (A), and the "clergyman vs. bus driver" example supports the idea of the role merging with the self (B). Statement C introduces an external idea (skill and time) not mentioned in the text.

 Quick Tip

For questions that ask "According to the passage," always go back to the text to find direct proof for each statement. Be wary of options that seem plausible or logical in the real world but are not explicitly mentioned or implied by the author. The answer must be based solely on the provided text.

54. In each question, there are five sentences or parts of sentences that form a paragraph. Identify the sentence(s) or part(s) of sentence(s) that is/are correct in terms of grammar and usage. Then, choose the most appropriate option.

- A. When I returned to home, I began to read
- B. everything I could get my hand on about Israel.
- C. That same year Israel's Jewish Agency sent
- D. a Shaliach a sort of recruiter to Minneapolis.
- E. I became one of his most active devotees.

- (A) C & E
- (B) C only
- (C) E only
- (D) B, C & E
- (E) C, D & E

Correct Answer: (A) C & E

Solution:

Step 1: Understanding the Concept: The question requires identifying which of the five given parts of sentences are grammatically correct and use appropriate language. We need to analyze each part for errors in grammar, punctuation, and idiomatic usage.

Step 2: Key Formula or Approach: The approach is to analyze each sentence part individually for correctness.

- **Part A:** "When I returned to home, I began to read" - The phrase "returned to home" is idiomatically incorrect. The correct usage is "returned home." The preposition "to" is not used with "home" when it indicates the destination of a movement verb like return, go, come, etc.
- **Part B:** "everything I could get my hand on about Israel." - This is a noun clause and a fragment, not a complete sentence. It is meant to be the object of "read" in part A. Since part A contains an error, the context for B is flawed.
- **Part C:** "That same year Israel's Jewish Agency sent" - This is an incomplete sentence (a fragment) because the transitive verb "sent" lacks an object. However, as a "part of a sentence," it contains no internal grammatical error.
- **Part D:** "a Shaliach a sort of recruiter to Minneapolis." - This part is intended to be the object of the verb in C. The structure "a Shaliach a sort of recruiter" is an awkward appositive construction that would be better written with a comma ("a Shaliach, a sort of recruiter"). This makes its usage questionable.
- **Part E:** "I became one of his most active devotees." - This is a complete and grammatically correct sentence with a subject ("I"), verb ("became"), and predicate complement ("one of his most active devotees").

Step 3: Calculation: Based on the analysis, we can classify each part:

- A: Incorrect (wrong preposition).
- B: Fragment, contextually linked to an incorrect part.
- C: A fragment, but free of internal grammatical errors. Considered a correct "part".
- D: A fragment with awkward usage/punctuation.
- E: A correct and complete sentence.

Following a strict interpretation often seen in competitive exams where fragments without internal errors are considered correct "parts", and complete sentences are correct, C and E are the correct choices. Part A is definitively wrong due to the prepositional error.

Step 4: Final Answer: The correct parts are C and E. The final answer is (A) C & E.

Step 5: Why This is the Correct Option: Option (A) is correct because Part E is a grammatically flawless sentence, and Part C, while a fragment, contains no internal grammatical or usage errors, making it a "correct part of a sentence." Part A is incorrect due to the unidiomatic use of "to home." Parts B and D are fragments, and D's construction is stylistically poor.

💡 Quick Tip

In grammar-based questions, pay close attention to:

- **Prepositions:** Incorrect prepositions are a common error (e.g., 'to home' instead of 'home').
- **Fragments vs. Errors:** Distinguish between a sentence part that is merely incomplete (a fragment) and one that contains an active grammatical error. Sometimes, error-free fragments are considered "correct parts".
- **Idioms:** Check for correct idiomatic expressions.

55. In each question, there are five sentences or parts of sentences that form a paragraph. Identify the sentence(s) or part(s) of sentence(s) that is/are correct in terms of grammar and usage. Then, choose the most appropriate option.

- A. So once an economy is actually in a recession,
B. the authorities can, in principle, move the economy
C. out of slump - assuming hypothetically
D. that they know how to - by a temporary stimuli.
E. In the longer term, however, such policies have no effect on the overall behaviour of the economy.

- (A) A, B & E
(B) B, C & E
(C) C & D
(D) E only
(E) B only

Correct Answer: (E) B only

Solution:

Step 1: Understanding the Concept: The question requires us to identify the grammatically correct parts from a list of five sentence fragments that are supposed to form a paragraph. We must check for errors in grammar, word choice, and punctuation.

Step 2: Key Formula or Approach: We will examine each part meticulously for any errors.

- **Part A:** "So once an economy is actually in a recession," - Starting a formal clause with "So" followed by another conjunction "once" can be considered poor style or grammatically informal. In many grammar tests, this construction is marked as incorrect.
- **Part B:** "the authorities can, in principle, move the economy" - This is a grammatically correct and complete main clause. The subject is "the authorities," the modal verb is "can move," and the adverbial phrase "in principle" is correctly placed.
- **Part C:** "out of slump - assuming hypothetically" - The phrase "assuming hypothetically" is redundant, as "assuming" already implies a hypothetical situation. This is a stylistic error.

- **Part D:** "that they know how to - by a temporary stimuli." - The word "stimuli" is the plural form of "stimulus." The article "a" is used for singular nouns. The correct phrase would be "a temporary stimulus" or "temporary stimuli". Therefore, "a temporary stimuli" is a number agreement error.
- **Part E:** "In the longer term, however, such policies have no affect on the overall behaviour of the economy." - This sentence has a common word choice error. "Affect" is typically a verb meaning "to influence," while "effect" is a noun meaning "a result." The correct phrase should be "have no effect."

Step 3: Calculation: Let's summarize the errors found:

- A: Stylistically incorrect (redundant conjunction "So").
- B: Correct.
- C: Redundancy ("assuming hypothetically").
- D: Grammatical error (number agreement: "a stimuli").
- E: Word choice error ("affect" instead of "effect").

Only part B is free from any grammatical or usage errors.

Step 4: Final Answer: The only correct part is B. The final answer is (E) B only.

Step 5: Why This is the Correct Option: Option (E) is correct because part B is the only statement that is grammatically sound and free of errors in usage or style. All other parts contain distinct mistakes: A has poor conjunction use, C is redundant, D has a number agreement error, and E confuses "affect" with "effect."

Quick Tip

Common errors tested in these questions include:

- **Affect vs. Effect:** 'Affect' is a verb; 'Effect' is a noun. (A affects B; A has an effect on B).
- **Singular/Plural Nouns:** Be careful with Latin or Greek plurals like stimulus/stimuli, criterion/criteria, phenomenon/phenomena.
- **Redundancy:** Avoid using words with similar meanings together (e.g., 'repeat again', 'assuming hypothetically').

56. In each question, there are five sentences or parts of sentences that form a paragraph. Identify the sentence(s) or part(s) of sentence(s) that is/are correct in terms of grammar and usage. Then, choose the most appropriate option.

- A. It is sometimes told that democratic
- B. government originated in the city-states
- C. of ancient Greece. Democratic ideals have been handed to us from that time.

- D. In truth, however, this is an unhelpful assertion.**
E. The Greeks gave us the word, hence did not provide us with a model.

- (A) A, B & D
(B) B, C & D
(C) B & D
(D) B only
(E) D only

Correct Answer: (C) B & D

Solution:

Step 1: Understanding the Concept: This question requires the identification of grammatically correct sentences or sentence parts from a given set of five. The analysis should cover word choice, sentence structure, and punctuation.

Step 2: Key Formula or Approach: We will scrutinize each part for grammatical and usage errors.

- **Part A:** "It is sometimes told that democratic" - The verb "told" is used incorrectly. In this context, for stating a general belief or fact, "said," "claimed," or "thought" should be used. "Told" typically requires an indirect object (e.g., "He told *me* that..."). Thus, this part is incorrect.
- **Part B:** "government originated in the city-states" - This is a grammatically correct clause. It can correctly follow Part A (if A were correct) to form the clause "...that democratic government originated in the city-states." As a part, it is free of errors.
- **Part C:** "of ancient Greece. Democratic ideals have been handed to us from that time." - This part contains two distinct pieces. The first, "of ancient Greece.", is a prepositional phrase punctuated as a sentence, which is a grammatical error (a fragment). The second part is a complete sentence, but the overall structure of C makes it incorrect.
- **Part D:** "In truth, however, this is an unhelpful assertion." - This is a complete sentence. The introductory phrase "In truth" and the conjunctive adverb "however" are correctly punctuated with commas. The subject-verb agreement is correct, and the statement is clear. This part is correct.
- **Part E:** "The Greeks gave us the word, hence did not provide us with a model." - This sentence contains a comma splice. "Hence" is a conjunctive adverb, not a coordinating conjunction (like 'and', 'but', 'or'). It cannot join two independent clauses with only a comma. The correct punctuation would be a semicolon before "hence" ('; hence, did not...') or rewriting the sentence ('...and hence did not...').

Step 3: Calculation: Let's list the status of each part:

- A: Incorrect (wrong verb choice).
- B: Correct (error-free clause).
- C: Incorrect (contains a fragment punctuated as a sentence).

- D: Correct (a grammatically sound sentence).
- E: Incorrect (comma splice).

The only parts that are correct are B and D.

Step 4: Final Answer: The correct parts are B and D. The final answer is (C) B & D.

Step 5: Why This is the Correct Option: Option (C) is correct because B and D are the only two parts that are grammatically and structurally sound. Part B is a correct clause, and Part D is a well-formed sentence. Part A uses the wrong verb, Part C contains a fragment error, and Part E uses incorrect punctuation to join clauses, creating a comma splice.

💡 Quick Tip

To avoid comma splices, remember that conjunctive adverbs (like 'however', 'therefore', 'hence', 'consequently') require a semicolon, not a comma, to connect two independent clauses.

- **Incorrect:** I am tired, hence I will sleep.
- **Correct:** I am tired; hence, I will sleep.
- **Correct:** I am tired, and hence I will sleep.

Directions for Questions 57 to 59: The passage given below is followed by a set of three questions. Choose the most appropriate answer to each question.

Every civilized society lives and thrives on a silent but profound agreement as to what is to be accepted as the valid mould of experience. Civilization is a complex system of dams, dykes, and canals warding off, directing, and articulating the influx of the surrounding fluid element: a fertile fenland, elaborately drained and protected from the high tides of chaotic, unexercised, and inarticulate experience. In such a culture, stable and sure of itself within the frontiers of 'naturalized' experience, the arts wield their creative power not so much in width as in depth. They do not create new experience, but deepen and purify the old. Their works do not differ from one another like a new horizon from a new horizon, but like a madonna from a madonna.

The periods of art which are most vigorous in creative passion seem to occur when the established pattern of experience loosens its rigidity without as yet losing its force. Such a period was the Renaissance, and Shakespeare its poetic consummation. Then it was as though the discipline of the old order gave depth to the excitement of the breaking away, the depth of job and tragedy, of incomparable conquests and irredeemable losses. Adventurers of experience set out as though in lifeboats to rescue and bring back to the shore treasures of knowing and feeling which the old order had left floating on the high seas. The works of the early Renaissance and the poetry of Shakespeare vibrate with the compassion for live experience in danger of dying from exposure and neglect. In this compassion was the creative genius of the age. Yet, it was a genius of courage, not of desperate audacity. For, however elusively, it still knew of harbours and anchors, of homes to which to return, and of barns in which to store the harvest. The

exploring spirit of art was in the depths of its consciousness still aware of a scheme of things into which to fit its exploits and creations.

But the more this scheme of things loses its stability, the more boundless and uncharted appears the ocean of potential exploration. In the blank confusion of infinite potentialities flotsam of significance gets attached to jetsam of experience: for everything is sea, everything is at sea-

... The sea is all about us;
The sea is the land's edge also, the granite
Into which it reaches, the beaches where it tosses
Its hints of earlier and other creation...

57. In the passage, the expression "like a madonna from a madonna" alludes to

- (A) The difference arising as a consequence of artistic license.
- (B) The difference between two artistic interpretations.
- (C) The difference between 'life' and 'interpretation of life'.
- (D) The difference between 'width' and 'depth' of creative power.
- (E) The difference between the legendary character and the modern day singer.

Correct Answer: (B) The difference between two artistic interpretations.

Solution:

Step 1: Understanding the Concept: The question asks for the meaning of a figurative expression used in the passage. To understand it, we must analyze the context in which it appears.

Step 2: Key Formula or Approach: The approach is to locate the expression in the first paragraph and examine the author's argument. The passage states: "In such a culture, stable and sure of itself... the arts wield their creative power not so much in width as in depth. They do not create new experience, but deepen and purify the old. Their works do not differ from one another like a new horizon from a new horizon, but like a madonna from a madonna."

Step 3: Calculation: The author draws a contrast.

- **"A new horizon from a new horizon":** This represents creating entirely new experiences or subject matter (width).
- **"A madonna from a madonna":** This represents working within an established tradition or subject matter. The Madonna is a classic subject in Western art. Many artists have painted her, and while the subject is the same, each painting is a unique artistic interpretation, differing in style, emotion, and technique (depth).

Therefore, the expression alludes to the subtle but profound differences that arise when different artists interpret the same established subject. It's not about creating something new from scratch, but about bringing a unique vision to something old. This directly corresponds to the difference between two artistic interpretations of the same theme.

Step 4: Final Answer: The expression contrasts the creation of new subjects with the reinterpretation of existing ones.

Madonna from a madonna $\equiv$ Different artistic visions of the same subject

The final answer is (B) The difference between two artistic interpretations.

Step 5: Why This is the Correct Option: This option accurately captures the meaning of the metaphor. The passage uses it to illustrate how art in a stable culture deepens existing themes rather than creating new ones. Each new "madonna" is a distinct interpretation of a familiar subject. The other options are less precise: (A) is too general, (D) states the contrast itself but not what the expression alludes to, and (C) and (E) are not relevant to the context.

 Quick Tip

When analyzing a figurative expression or a metaphor in a reading comprehension passage, always look at the contrast or comparison the author is making in the surrounding sentences. The author often defines the metaphor through the ideas it is being compared with.

58. The sea and 'other creation' leads Rilke to

- (A) Define the place of the poet in his culture.
- (B) Reflect on the role of the oarsman and the singer.
- (C) Muse on artistic labour and its aimlessness.
- (D) Understand the elements that one has to deal with.
- (E) Delve into natural experience and real waves.

Correct Answer: (A) Define the place of the poet in his culture.

Solution:

Step 1: Understanding the Concept: This question asks for the conclusion or realization that the poet Rilke reaches as a result of the metaphorical story involving the sea, oarsmen, and a singer.

Step 2: Key Formula or Approach: The key is to read the final part of the passage carefully, as it explicitly states Rilke's conclusion derived from the story. The passage describes a scene on the sea and then quotes Rilke's final thought.

Step 3: Calculation: The last paragraph describes an anecdote from Rilke. In the story, the practical labor of the oarsmen seems meaningless, while the seemingly "idle" singer's song is what "magically conquers the despair of apparent aimlessness" and connects the boat to its destination. This entire story is an allegory. The passage concludes with Rilke's own words about what he understood from this scene: "'I don't know why and how,' is Rilke's conclusion, 'but suddenly I understood the situation of the poet, his place and function in this age. It does not matter if one denies him every place — except this one. There one must tolerate him.'" This statement is a direct and unambiguous conclusion. The experience of witnessing the scene

on the sea leads Rilke to a profound understanding of the poet's essential role and position within society.

Step 4: Final Answer: Rilke's conclusion, as stated in the text, is about the poet's role.

Allegory of the sea → Rilke's understanding → "the situation of the poet, his place and function"

The final answer is (A) Define the place of the poet in his culture.

Step 5: Why This is the Correct Option: This option is a direct paraphrase of the conclusion explicitly stated in the passage. Rilke's story serves as a metaphor to illuminate the "place and function" of the poet. The other options describe intermediate parts of the story (B, C, E) or are too general (D), but they are not the ultimate point or conclusion Rilke reaches.

 Quick Tip

In questions that ask what an author or a character concludes, look for explicit concluding phrases like "The conclusion is...", "This led him to understand...", or direct quotes that summarize a realization. The answer is often stated directly at the end of the relevant section.

59. According to the passage, the term "adventurers of experience" refers to

- (A) Poets and artists who are driven by courage.
- (B) Poets and artists who create their own genre.
- (C) Poets and artists of the Renaissance.
- (D) Poets and artists who revitalize and enrich the past for us.
- (E) Poets and artists who delve in flotsam and jetsam in sea.

Correct Answer: (D) Poets and artists who revitalize and enrich the past for us.

Solution:

Step 1: Understanding the Concept: The question asks for the definition of the term "adventurers of experience" as it is used in the passage. We need to analyze the description of these figures in the second paragraph.

Step 2: Key Formula or Approach: The approach is to examine the context around the term. The passage says: "Such a period was the Renaissance... Adventurers of experience set out as though in lifeboats to rescue and bring back to the shore treasures of knowing and feeling which the old order had left floating on the high seas."

Step 3: Calculation: Let's break down the metaphor:

- **"Adventurers of experience":** These are the artists and poets of the period (like Shakespeare).
- **"Lifeboats":** Their artistic works.

- **”Treasures... which the old order had left floating”:** These are the values, feelings, and knowledge from the past tradition.
- **”Rescue and bring back to shore”:** This implies they are not creating something completely new but are saving, reinterpreting, and giving new life to existing cultural treasures. They are revitalizing the old in a new context.

The passage further clarifies that this was a ”genius of courage, not of desperate audacity,” meaning they were bold but still connected to a ”scheme of things” (the past). Their job was to enrich and give new depth to the old, not to completely break from it. This aligns perfectly with the idea of revitalizing and enriching the past.

Step 4: Final Answer: The role of these ”adventurers” is to save and reinterpret the past.

Rescue treasures of the old order → Revitalize and enrich the past

The final answer is (D) Poets and artists who revitalize and enrich the past for us.

Step 5: Why This is the Correct Option: This option best summarizes the metaphorical description given in the passage. Option (C) is too specific; while the Renaissance is the example, the term describes a type of artist, not just those of one period. Option (B) is incorrect because they work with the ”old order,” not create entirely new genres. Option (A) is partially true but misses the main function of their work. Option (E) is a literal and incorrect interpretation of the metaphor.

💡 Quick Tip

When a passage uses a metaphor to define a term, break the metaphor down into its components. Identify the subject (who is being described?), the action (what are they doing?), and the object (what are they acting upon?). This will clarify the author’s intended meaning.

Directions for Questions 60 to 62: Each of the following questions has a paragraph from which the last sentence has been deleted. From the given options, choose the sentence that completes the paragraph in the most appropriate way.

60. Each of the following questions has a paragraph from which the last sentence has been deleted. From the given options, choose the sentence that completes the paragraph in the most appropriate way.

Characters are also part of deep structure. Characters tie events in a story together and provide a thread of continuity and meaning. Stories can be about individuals, groups, projects, or whole organizations, so from an organizational studies perspective, the focal actor(s) determine the level and unit of analysis used in a study. Stories of mergers and acquisitions, for example, are commonplace. In these stories whole organizations are personified as actors. But these macro-level stories usually are not told from the perspective of the macro-level participants, because whole organizations cannot narrate their experiences in the first person.

- (A) More generally, data concerning the identities and relationships of the characters in the story are required, if one is to understand role structure and social networks in which that process is embedded.
- (B) Personification of a whole organization abstracts away from the particular actors and from traditional notions of level of analysis.
- (C) The personification of a whole organization is important because stories differ depending on who is enacting various events.
- (D) Every story is told from a particular point of view, with a particular narrative voice, which is not regarded as part of the deep structure.
- (E) The personification of a whole organization is a textual device we use to make macro-level theories more comprehensible.

Correct Answer: (E) The personification of a whole organization is a textual device we use to make macro-level theories more comprehensible.

Solution:

Step 1: Understanding the Concept: This is a paragraph completion question. The task is to find the sentence that provides the most logical and coherent conclusion to the paragraph's line of reasoning.

Step 2: Key Formula or Approach: The approach is to first summarize the main argument of the paragraph and then identify the logical gap or question that the final sentence should address.

Paragraph Summary: Stories need characters. In organizational studies, entire organizations can be treated as characters (personified). The paragraph ends by highlighting a problem with this approach: organizations can't actually speak or narrate their own stories in the first person.

Step 3: Calculation: The paragraph sets up a practice (personifying organizations) and then points out a logical flaw or problem (they can't speak for themselves). The concluding sentence should logically resolve this tension by explaining **why** we use this flawed device. Let's analyze the options:

- (A) This sentence shifts the topic to data collection and social networks, which is not the main focus.
- (B) This explains **what** personification does (abstracts away), which is relevant but doesn't explain **why** we use it despite its limitations.
- (C) This states the importance of personification but doesn't connect logically to the final problem raised about narration.
- (D) This introduces the concept of "point of view" and "deep structure," which complicates the issue rather than concluding it.
- (E) This option provides a clear reason for the practice. It frames personification as a "textual device" and explains its purpose: to make complex, "macro-level theories more comprehensible." This directly answers the unstated question, "If organizations can't really be narrators, why do we treat them as such?" It's a functional explanation that resolves the problem stated in the previous sentence.

Step 4: Final Answer: The paragraph needs a sentence that explains the purpose of personification.

Problem: Organizations cannot narrate →

Solution/Reason: We use personification as a device to simplify complex ideas

The final answer is

(E) The personification of a whole organization is a textual device we use to make macro-level theories more relatable.

Step 5: Why This is the Correct Option: This is the best conclusion because it provides a rationale for the narrative strategy described in the paragraph, directly addressing the paradox raised in the final sentence. It concludes the logical flow by explaining the utility of the concept being discussed.

 Quick Tip

For paragraph completion, identify the paragraph's core argument and direction. Ask yourself: "Where is this argument going?" The correct answer will often be a summary, a logical consequence, or an explanation that resolves a tension introduced in the paragraph. Look for keywords that connect back to the main topic.

61. Nevertheless, photographs still retain some of the magical allure that the earliest daguerreotypes inspired. As objects, our photographs have changed; they have become physically flimsier as they have become more technologically sophisticated. Daguerre produced pictures on copper plates: today many of our photographs never become tangible things, but instead remain filed away on computers and cameras, part of the digital ether that envelops the modern world. At the same time, our patience for the creation of images has also eroded. Children today are used to being tracked from birth by digital cameras and video recorders and they expect to see the results of their poses and performances instantly. The space between life as it is being lived and life as it is being displayed shrinks to a mere second.

- (A) Yet, despite these technical developments, photographs still remain powerful because they are reminders of the people and things we care about.
- (B) Images, after all, are surrogates carried into battle by a soldier or by a traveller on holiday.
- (C) Photographs, be they digital or traditional, exist to remind us of the absent, the beloved, and the dead.
- (D) In the new era of the digital image, the images also have a greater potential for fostering falsehood and trickery, perpetuating fictions that seem so real we cannot tell the difference.
- (E) Anyway, human nature being what it is, little time has passed after photography's invention became means of living life through images.

Correct Answer: (A) Yet, despite these technical developments, photographs still remain powerful because they are reminders of the people and things we care about.

Solution:

Step 1: Understanding the Concept: The question requires us to find the most suitable concluding sentence for the given paragraph. The sentence should logically follow the preceding arguments and provide a sense of closure.

Step 2: Key Formula or Approach: Analyze the structure and flow of the paragraph. The paragraph starts with a positive assertion ("still retain magical allure"), then details a series of negative changes (flimsier, less tangible, less patience), and should logically conclude by returning to the initial positive point, creating a balanced argument.

Step 3: Calculation:

- **Opening Idea:** Photographs are still magical.
- **Supporting Details (all negative):** They've become physically flimsy, non-tangible (digital), and our patience for them has eroded due to instant digital culture.
- **Logical Conclusion Needed:** The conclusion should reconcile the negative developments with the initial positive statement. It should explain *why* photographs retain their allure despite these changes.

Let's evaluate the options:

- (A) This sentence does exactly what is needed. It starts with "Yet, despite these technical developments," which explicitly acknowledges all the negative points just made. It then reaffirms the power of photographs and provides the core reason: their function as "reminders of the people and things we care about." This brings the argument full circle.
- (B) and (C) These are true statements about photographs but are less effective as conclusions. They don't explicitly link back to or contrast with the "technical developments" discussed in the paragraph.
- (D) This introduces a new, negative aspect (falsehood), which disrupts the paragraph's structure and doesn't resolve the initial point.
- (E) This is a vague, philosophical statement that does not directly connect to the specific points made about digital photography and eroding patience.

Step 4: Final Answer: The paragraph contrasts the technical decline of photography with its enduring emotional power. The conclusion must bridge this gap.

(Negative Developments) → "Yet, despite this..." → (Enduring Power)

The final answer is

(A) Yet, despite these technical developments, photographs still remain powerful
because they are reminders of the people and things we care about.

Step 5: Why This is the Correct Option: This option is the strongest choice because it effectively summarizes the preceding points ("these technical developments") and provides a concluding counter-argument ("Yet... photographs still remain powerful") that resolves the tension in the paragraph and reinforces the opening sentence.

 Quick Tip

Look for transition words in the options. Words like "Yet," "However," "Therefore," and "Despite" signal specific logical relationships. A sentence starting with "Yet, despite..." is designed to present a contrast, making it a strong candidate for concluding a paragraph that has detailed a series of problems or changes.

62. Mma Ramotswe had a detective agency in Africa, at the foot of Kgale Hill. These were its assets: a tiny white van, two desks, two chairs, a telephone, and an old typewriter. Then there was a teapot, in which Mma Ramotswe - the only private lady detective in Botswana - brewed redbush tea. And three mugs - one for herself, one for her secretary, and one for the client. What else does a detective agency really need? Detective agencies rely on human intuition and intelligence, both of which Mma Ramotswe had in abundance.

- (A) But there was also the view, which again would appear on no inventory.
- (B) No inventory would ever include those, of course.
- (C) She had an intelligent secretary too.
- (D) She was a good detective and a good woman.
- (E) What she lacked in possessions was more than made up by a natural shrewdness.

Correct Answer: (B) No inventory would ever include those, of course.

Solution:

Step 1: Understanding the Concept: The task is to choose the most fitting final sentence for a descriptive paragraph. The best choice will continue the established theme and tone while providing a strong sense of closure.

Step 2: Key Formula or Approach: Identify the central contrast or theme of the paragraph. The paragraph meticulously lists the agency's tangible, physical assets (van, desks, etc.), which are modest. It then contrasts this with the essential, intangible assets ("human intuition and intelligence"), which are abundant. The concluding sentence should emphasize this contrast.

Step 3: Calculation: The paragraph ends by stating that the most crucial assets are "intuition and intelligence." The next sentence should logically follow this statement. Let's analyze the options:

- (A) This introduces another intangible asset ("the view"). While it fits the theme of unlisted assets, it slightly distracts from the powerful statement just made about intuition and intelligence.
- (B) This sentence directly comments on the immediately preceding words, "intuition and intelligence." The pronoun "those" refers to them. By stating that "No inventory would ever include those," it powerfully links the main intangible assets back to the initial list of tangible assets (which *would* be on an inventory). It provides a final, conclusive emphasis on the central contrast.

- (C) This adds a new piece of information about the secretary. It's a detail, not a concluding thought.
- (D) This is a general summary of her character, but it lacks the specific connection to the asset-listing theme of the paragraph.
- (E) This explicitly states the paragraph's theme. While correct, it is less subtle and artful than option (B). Option (B) reinforces the theme by commenting on the final point, making for a more elegant conclusion.

Considering the direct flow of logic, option (B) serves as the most immediate and fitting postscript to the statement about intuition and intelligence.

Step 4: Final Answer: The paragraph contrasts listed, tangible assets with unlisted, intangible assets. The conclusion should reinforce this.

Final point: "intuition and intelligence" →

Concluding comment on this point: "No inventory would ever include those"

The final answer is (B) No inventory would ever include those, of course.

Step 5: Why This is the Correct Option: This option is the most appropriate because it directly and effectively reinforces the paragraph's central theme. It serves as a final, reflective comment on the most important assets just mentioned ("intuition and intelligence"), contrasting them with the inventoried items listed at the beginning and thus bringing the paragraph to a neat and satisfying close.

Quick Tip

In narrative or descriptive paragraph completion, pay attention to pronouns in the options. A pronoun like "those" or "it" must have a clear and logical antecedent in the preceding sentence. The option that creates the smoothest and most direct connection is often the best choice.

Directions for Questions 63 to 65: The passage given below is followed by a set of three questions. Choose the most appropriate answer to each question.

To discover the relation between rules, paradigms, and normal science, consider first how the historian isolates the particular loci of commitment that have been described as accepted rules. Close historical investigation of a given specialty at a given time discloses a set of recurrent and quasi-standard illustrations of various theories in their conceptual, observational, and instrumental applications. These are the community's paradigms, revealed in its textbooks, lectures, and laboratory exercises. By studying them and by practicing with them, the members of the corresponding community learn their trade. The historian, of course, will discover in addition a penumbral area occupied by achievements whose status is still in doubt, but the core of solved problems and techniques will usually be clear. Despite occasional ambiguities, the paradigms of a mature scientific community can be determined with relative ease.

That demands a second step and one of a somewhat different kind. When undertaking it, the historian must compare the community's paradigms with each other and with its current research reports. In doing so, his object is to discover what isolable elements, explicit or implicit, the members of that community may have abstracted from their more global paradigms and deploy it as rules in their research. Anyone who has attempted to describe or analyze the evolution of a particular scientific tradition will necessarily have sought accepted principles and rules of this sort. Almost certainly, he will have met with at least partial success. But, if his experience has been at all like my own, he will have found the search for rules both more difficult and less satisfying than the search for paradigms. Some of the generalizations he employs to describe the community's shared beliefs will present more problems. Others, however, will seem a shade too strong. Phrased in just that way, or in any other way he can imagine, they would almost certainly have been rejected by some members of the group he studies. Nevertheless, if the coherence of the research tradition is to be understood in terms of rules, some specification of common ground in the corresponding area is needed. As a result, the search for a body of rules competent to constitute a given normal research tradition becomes a source of continual and deep frustration.

Recognizing that frustration, however, makes it possible to diagnose its source. Scientists can agree that a Newton, Lavoisier, Maxwell, or Einstein has produced an apparently permanent solution to a group of outstanding problems and still disagree, sometimes without being aware of it, about the particular abstract characteristics that make those solutions permanent. They can, that is, agree in their identification of a paradigm without agreeing on, or even attempting to produce, a full interpretation or rationalization of it. Lack of a standard interpretation or of an agreed reduction to rules will not prevent a paradigm from guiding research. Normal science can be determined in part by the direct inspection of paradigms, a process that is often aided by but does not depend upon the formulation of rules and assumption. Indeed, the existence of a paradigm need not even imply that any full set of rules exists.

63. What is the author attempting to illustrate through this passage?

- (A) Relationships between rules, paradigms, and normal science
- (B) How a historian would isolate a particular 'loci of commitment'
- (C) How a set of shared beliefs evolves into a paradigm
- (D) Ways of understanding a scientific tradition
- (E) The frustrations of attempting to define a paradigm of a tradition

Correct Answer: (D) Ways of understanding a scientific tradition

Solution:

Step 1: Understanding the Concept:

This question asks for the primary purpose or main idea of the entire passage. To answer this, we need to synthesize the arguments presented in all three paragraphs. The passage explores how a scientific tradition functions and how it can be analyzed.

Step 2: Detailed Explanation:

The first paragraph introduces the idea of identifying a scientific community's paradigms through historical investigation of its textbooks, lectures, and solved problems. This is presented as the first step in understanding the tradition.

The second paragraph contrasts this with a second, more difficult step: trying to distill a concrete set of "rules" from these paradigms. The author describes this search for rules as frustrating and often less successful.

The third paragraph explains why this frustration occurs. It argues that scientists can unite around a paradigm (like Newton's work) without agreeing on a precise set of rules that defines it. The paradigm itself, not a set of explicit rules, guides the research.

Considering all three paragraphs, the author is discussing the methodology for analyzing a scientific tradition, weighing the effectiveness of looking for paradigms versus looking for explicit rules. Therefore, the passage is about the "ways of understanding a scientific tradition."

Step 3: Evaluating the Options:

(A) This is a central theme, but the passage does more than just describe the relationship; it evaluates different approaches to understanding it. So, it's part of a larger purpose.

(B) This is only the starting point mentioned in the first paragraph, not the overall theme of the passage.

(C) The passage discusses how to analyze an *existing* tradition, not how a paradigm evolves from beliefs.

(D) This option accurately captures the broad scope of the passage. The author is illustrating different methods (analyzing paradigms, searching for rules) to comprehend how a scientific tradition operates. This is the most encompassing and accurate choice.

(E) The frustration mentioned is about defining the *rules* derived from a paradigm, not the paradigm itself. The passage states that identifying paradigms is "with relative ease."

Step 4: Final Answer:

The most appropriate answer is (D) because the passage as a whole is an exploration of the methods and challenges involved in analyzing and understanding a scientific tradition.

💡 Quick Tip

For "main purpose" or "primary concern" questions, look for an answer choice that synthesizes the key ideas from all paragraphs, rather than one that focuses on a detail from a single paragraph. The correct answer should reflect the overall argument of the author.

64. The term 'loci of commitment' as used in the passage would most likely correspond with which of the following?

- (A) Loyalty between a group of scientists in a research laboratory
- (B) Loyalty between groups of scientists across research laboratories
- (C) Loyalty to a certain paradigm of scientific inquiry
- (D) Loyalty to global patterns of scientific inquiry
- (E) Loyalty to evolving trends of scientific inquiry

Correct Answer: (C) Loyalty to a certain paradigm of scientific inquiry

Solution:

Step 1: Understanding the Concept:

This question requires you to determine the meaning of a specific phrase, 'loci of commitment', based on its context within the passage. The best approach is to find the phrase in the text and analyze the surrounding sentences for clues and definitions.

Step 2: Detailed Explanation:

The phrase appears in the first sentence of the passage: "...how the historian isolates the particular loci of commitment that have been described as accepted rules." The passage immediately clarifies what these 'loci of commitment' are in the following sentences: "Close historical investigation... discloses a set of recurrent and quasi-standard illustrations of various theories... These are the community's paradigms, revealed in its textbooks, lectures, and laboratory exercises."

The text directly equates the "loci of commitment" with the "community's paradigms." A paradigm represents the set of concepts, theories, and methods to which the scientific community is committed. Therefore, the 'commitment' is a professional and intellectual loyalty to the established paradigm that guides their scientific work.

Step 3: Evaluating the Options:

(A) and (B) are incorrect as they refer to interpersonal loyalty among scientists, whereas the passage discusses commitment to scientific ideas and frameworks.

(C) This option correctly identifies the commitment as being to a "paradigm of scientific inquiry." This aligns perfectly with the passage's explanation that the 'loci of commitment' are the community's paradigms.

(D) This is too broad. The passage specifies the investigation of a "given specialty at a given time," not necessarily "global patterns."

(E) This is inaccurate. The passage emphasizes paradigms as "recurrent and quasi-standard illustrations" and "solved problems," which represent established, rather than merely "evolving," frameworks.

Step 4: Final Answer:

The context clearly shows that 'loci of commitment' refers to the paradigms that a scientific community accepts and works within. Thus, it corresponds to a loyalty to that paradigm.

💡 Quick Tip

When a question asks for the meaning of a specific term or phrase from the passage, always locate it in the text. The definition or a strong contextual clue is often provided in the sentence it's in or the one immediately following it.

65. The author of this passage is likely to agree with which of the following?

- (A) Paradigms almost entirely define a scientific tradition.
- (B) A group of scientists investigating a phenomenon would benefit by defining a set of rules.
- (C) Acceptance by the giants of a tradition is a sine qua non for a paradigm to emerge.

(D) Choice of isolation mechanism determines the type of paradigm that may emerge from a tradition.

(E) Paradigms are a general representation of rules and beliefs of a scientific tradition.

Correct Answer: (E) Paradigms are a general representation of rules and beliefs of a scientific tradition.

Solution:

Step 1: Understanding the Concept:

This is an inference question. You need to identify which statement aligns with the author's arguments and perspective presented throughout the passage, even if it is not stated verbatim.

Step 2: Detailed Explanation:

The author's central argument is that paradigms are more fundamental to science than a strict set of rules. He describes paradigms as "global paradigms" from which "isolable elements" or rules might be abstracted. However, he emphasizes that a paradigm can function effectively even without a fully articulated set of rules. The final sentence states, "the existence of a paradigm need not even imply that any full set of rules exists." This suggests that a paradigm is a broader, more holistic concept that encompasses the shared beliefs, techniques, and solved problems of a community. The rules are just one potential, and often incomplete, expression of this broader paradigm.

Step 3: Evaluating the Options:

(A) The passage states, "Normal science can be determined in part by the direct inspection of paradigms," which suggests paradigms are a major, but not necessarily the *entire*, definition of a tradition. The word "almost entirely" is too strong.

(B) The author explicitly argues against this. He states that the search for rules is "more difficult and less satisfying" and a "source of continual and deep frustration," and that a paradigm can guide research without an "agreed reduction to rules."

(C) The passage uses "giants" like Newton and Einstein as examples of those who *produced* paradigms, but it doesn't state that their personal acceptance is a necessary condition for a paradigm to emerge. The focus is on the community's adoption of the paradigm.

(D) The passage discusses how a historian *isolates* (identifies) existing paradigms; it does not suggest that the act of isolation determines or creates the paradigm.

(E) This statement accurately reflects the author's view. A paradigm is a "global" concept that represents the shared commitments. Rules and beliefs are components or abstractions from this larger paradigm. The paradigm is the general representation, while rules are specific, and not always fully definable, components.

Step 4: Final Answer:

Based on the author's argument that paradigms are "global" and that rules are abstracted from them, the most accurate statement is that paradigms are a general representation of the rules and beliefs of a tradition.

💡 Quick Tip

For inference questions ("the author is likely to agree"), eliminate options that directly contradict the text or make absolute claims (e.g., using words like "entirely" or "always") that are not supported. The correct answer is often a more nuanced statement that captures the spirit of the author's argument.

66. The cricket council that *was* [A] / *were* [B] elected last March *is* [A] / *are* [B] at sixes and sevens over new rules.

The critics *censored* [A] / *censured* [B] the new movie because of its social unacceptability.

Amit's explanation for missing the meeting was *credulous* [A] / *credible* [B].

She coughed *discreetly* [A] / *discretely* [B] to announce her presence.

- (A) BBAAA
- (B) AAABA
- (C) BBBBA
- (D) AABBA
- (E) BBBA

Correct Answer: (D) AABBA

Solution:

Step 1: Understanding the Concept:

This question tests the ability to distinguish between commonly confused words and to apply the correct rules of subject-verb agreement with collective nouns. Each sentence requires selecting the most appropriate word from the given pair.

Step 3: Detailed Explanation:

Let's analyze each sentence to determine the correct word choice.

Sentence 1: "The cricket council that *was* [A] / *were* [B] elected last March *is* [A] / *are* [B] at sixes and sevens over new rules."

- **was / were:** "The cricket council" is a collective noun. When referring to the council as a single entity being elected, the singular verb **was** [A] is correct.
- **is / are:** The phrase "at sixes and sevens" implies confusion or disarray within the group. While treating the council as a single unit with the verb "is" is possible, the context suggests internal disagreement among members. However, in formal usage, treating the collective noun as a single unit is often preferred. In this case, **is** [A] is the better choice, viewing the council as a single body in a state of confusion.

Sentence 2: "The critics *censored* [A] / *censured* [B] the new movie because of its social unacceptability."

- **Censored** means to suppress or remove objectionable material.
- **Censured** means to criticize harshly or express strong disapproval.
- Critics express opinions; they don't typically have the authority to remove content. Therefore, they **censured** [B] the movie.

Sentence 3: "Amit's explanation for missing the meeting was *credulous* [A] / *credible* [B]."

- **Credulous** means gullible or too ready to believe things. It describes a person.
- **Credible** means believable or convincing. It describes something like a story or an explanation.
- The explanation was believable, so the correct word is **credible** [B].

Sentence 4: "She coughed *discreetly* [A] / *discretely* [B] to announce her presence."

- **Discreetly** means in a careful, subtle manner to avoid embarrassment or drawing too much attention.
- **Discretely** means separately or distinctly.
- She coughed subtly, so the correct word is **discreetly** [A].

Step 4: Final Answer:

The correct sequence of choices is A for the first verb ('was'), A for the second verb ('is'), B for the second sentence ('censured'), B for the third sentence ('credible'), and A for the fourth sentence ('discreetly'). This gives the sequence **AABBA**.

Step 5: Why This is Correct:

The sequence AABBA correctly applies grammatical rules and the precise meanings of the words in each context. The council is treated as a singular entity, 'censured' correctly describes criticism, 'credible' correctly describes a believable explanation, and 'discreetly' correctly describes a subtle action. This corresponds to option (D).

💡 Quick Tip

When dealing with collective nouns like 'council' or 'team', use a singular verb if the group acts as a single unit, and a plural verb if the members are acting as individuals. For confusing word pairs (homophones/homonyms), focus on their precise definitions and the context of the sentence.

67. The *further* [A] / *farther* [B] he pushed himself, the more disillusioned he grew.

For the crowds it was more of a *historical* [A] / *historic* [B] event; for their leader, it was just another day.

The old man has a healthy *distrust* [A] / *mistrust* [B] for all new technology.

This film is based on a *real* [A] / *true* [B] story.

One suspects that the *compliment* [A] / *complement* [B] was backhanded.

- (A) BABAB
- (B) ABBBA
- (C) BAABA
- (D) BBAAB
- (E) ABABA

Correct Answer: (E) ABABA

Solution:

Step 1: Understanding the Concept:

This question assesses your vocabulary, specifically your ability to differentiate between words that are often confused due to similar sounds or meanings.

Step 3: Detailed Explanation:

Let's analyze each sentence to select the appropriate word.

Sentence 1: "The *further* [A] / *farther* [B] he pushed himself, the more disillusioned he grew."

- **Farther** refers to physical distance.
- **Further** refers to metaphorical or figurative distance, degree, or extent.
- "Pushing himself" is a figurative concept, so **further** [A] is the correct choice.

Sentence 2: "For the crowds it was more of a *historical* [A] / *historic* [B] event; for their leader, it was just another day."

- **Historical** means relating to history or past events.
- **Historic** means famous, important, or significant in history.
- An event of great importance is a **historic** [B] event.

Sentence 3: "The old man has a healthy *distrust* [A] / *mistrust* [B] for all new technology."

- **Distrust** implies a lack of trust based on reason or experience. The phrase "healthy distrust" is a common idiom suggesting a rational wariness.
- **Mistrust** often implies a gut feeling or suspicion without concrete evidence.
- In the context of a "healthy" or justifiable feeling, **distrust** [A] is more appropriate.

Sentence 4: "This film is based on a *real* [A] / *true* [B] story."

- **Real** means actual or existing.

- **True** means in accordance with fact.
- The standard, idiomatic expression is "based on a **true** [B] story."

Sentence 5: "One suspects that the *compliment* [A] / *complement* [B] was backhanded."

- A **compliment** is an expression of praise. A "backhanded compliment" is an insult disguised as praise.
- A **complement** is something that completes or enhances another thing.
- The context of "backhanded" clearly points to **compliment** [A].

Step 4: Final Answer:

The correct sequence of choices is A ('further'), B ('historic'), A ('distrust'), B ('true'), and A ('compliment'). This forms the sequence **ABABA**.

Step 5: Why This is Correct:

Each selected word fits the specific context of its sentence. 'Further' is used for figurative distance, 'historic' for an important event, 'distrust' for rational suspicion, 'true' for factual stories, and 'compliment' for an expression of (backhanded) praise. This sequence matches option (E).

 Quick Tip

Create mnemonic devices to remember the difference between tricky words. For example, 'farther' has 'far' in it, linking it to physical distance. 'Compliment' has an 'i', just like 'I' like to receive praise. 'Complement' helps complete something.

68. *Regrettably* [A] / *Regretfully* [B] I have to decline your invitation.

I am drawn to the poetic, *sensual* [A] / *sensuous* [B] quality of her paintings.

He was *besides* [A] / *beside* [B] himself with rage when I told him what I had done.

After brushing against a *stationary* [A] / *stationery* [B] truck my car turned turtle.

As the water began to rise *over* [A] / *above* [B] the danger mark, the signs of an imminent flood were clear.

- (A) BAABA
- (B) BBBAB
- (C) AAABA
- (D) BBAAB
- (E) BABAB

Correct Answer: (B) BBBAB

Solution:

Step 1: Understanding the Concept:

This question again tests vocabulary and the ability to choose the correct word from pairs that are frequently confused. The choice often depends on subtle differences in meaning or established idiomatic usage.

Step 3: Detailed Explanation:

Let's analyze each sentence individually.

Sentence 1: " *Regrettably* [A] / *Regretfully* [B] I have to decline your invitation."

- **Regrettably** is a sentence adverb meaning "it is unfortunate that...".
- **Regretfully** describes the manner in which an action is performed, meaning "in a way full of regret."
- When declining an invitation, it is common to express personal sorrow, making **Regretfully** [B] a suitable choice to emphasize the speaker's feelings.

Sentence 2: "I am drawn to the poetic, *sensual* [A] / *sensuous* [B] quality of her paintings."

- **Sensual** primarily relates to the gratification of physical appetites, often with sexual connotations.
- **Sensuous** relates to affecting the senses in a pleasing, aesthetic way, and is typically used for art, music, or literature.
- For the "quality of her paintings," the appropriate word is **sensuous** [B].

Sentence 3: "He was *besides* [A] / *beside* [B] himself with rage when I told him what I had done."

- **Besides** means "in addition to" or "apart from."
- **Beside** means "next to."
- The idiom "to be beside oneself with an emotion" means to be overcome by that emotion. The correct word is **beside** [B].

Sentence 4: "After brushing against a *stationary* [A] / *stationery* [B] truck my car turned turtle."

- **Stationary** means not moving.
- **Stationery** refers to writing materials like paper and envelopes.
- A truck that is not in motion is **stationary** [A].

Sentence 5: "As the water began to rise *over* [A] / *above* [B] the danger mark, the signs of an imminent flood were clear."

- **Over** can mean covering or flowing across something.
- **Above** means at a higher level than something else.
- When referring to a level rising higher than a specific point or mark, **above** [B] is the more precise and standard term.

Step 4: Final Answer:

The correct sequence of choices is B ('Regretfully'), B ('sensuous'), B ('beside'), A ('stationary'), and B ('above'). This results in the sequence **BBBAB**.

Step 5: Why This is Correct:

The chosen sequence BBBAB uses the most appropriate words for each sentence's context. 'Regretfully' expresses personal feeling, 'sensuous' describes aesthetic appeal, 'beside' is used in the correct idiom, 'stationary' means not moving, and 'above' correctly indicates a level. This sequence matches option (B).

💡 Quick Tip

To remember 'stationary' vs. 'stationery', think of 'stationary' as in stay or stand still. Think of 'stationery' as in letters or envelopes.

Directions for Questions 69 to 71: The passage given below is followed by a set of three questions. Choose the most appropriate answer to each question.

The difficulties historians face in establishing cause-and-effect relations in the history of human societies are broadly similar to the difficulties facing astronomers, climatologists, ecologists, evolutionary biologists; geologists, and palaeontologists. To varying degrees each of these fields is plagued by the impossibility of performing replicated, controlled experimental interventions, the complexity arising from enormous numbers of variables, the resulting uniqueness of each system, the consequent impossibility of formulating universal laws, and the difficulties of predicting emergent properties and future behaviour. Prediction in history, as in other historical sciences, is most feasible on large spatial scales and over long times, when the unique features of millions of small brief events become averaged out. Just as I could predict the sex ratio of the next 1,000 newborns but not the sexes of my own two children, the historian can recognize factors that made inevitable the broad outcome of the collision between American and Eurasian societies after 13,000 years of separate developments, but not the outcome of the 1960 U.S. presidential election. The details of which candidate said what during a single televised debate in October 1960 could have given the electoral victory to Nixon instead of to Kennedy, but no details of who said what could have blocked the European conquest of Native Americans.

How can students of human history profit from the experience of scientists in other historical sciences? A methodology that has proved useful involves the comparative method and so-called natural experiments. While neither astronomers studying galaxy formation nor human historians can manipulate their systems in controlled laboratory experiments, they both can take advantage of natural experiments, by comparing systems differing in the presence or absence (or in the strong or weak effect) of some putative causative factor. For example, epidemiolo-

gists, forbidden to feed large amounts of salt to people experimentally, have still been able to identify effects of high salt intake by comparing groups of humans who already differ greatly in their salt intake: and cultural anthropologists, unable to provide human groups experimentally with varying resource abundances for many centuries, still study long-term effects of resource abundance on human societies by comparing recent Polynesian populations living on islands differing naturally in resource abundance.

The student of human history can draw on many more natural experiments than just comparisons among the five inhabited continents. Comparisons can also utilize large islands that have developed complex societies in a considerable degree of isolation (such as Japan, Madagascar, Native American Hispaniola, New Guinea, Hawaii, and many others), as well as societies on hundreds of smaller islands and regional societies within each of the continents. Natural experiments in any field, whether in ecology or human history, are inherently open to potential methodological criticisms. Those include confounding effects of natural variation in additional variables besides the one of interest, as well as problems in inferring chains of causation from observed correlations between variables. Such methodological problems have been discussed in great detail for some of the historical sciences. In particular, epidemiology, the science of drawing inferences about human diseases by comparing groups of people (often by retrospective historical studies), has for a long time successfully employed formalized procedures for dealing with problems similar to those facing historians of human societies.

In short, I acknowledge that it is much more difficult to understand human history than to understand problems in fields of science where history is unimportant and where fewer individual variables operate. Nevertheless, successful methodologies for analyzing historical problems have been worked out in several fields. As a result, the histories of dinosaurs, nebulae, and glaciers are generally acknowledged to belong to fields of science rather than to the humanities.

69. Why do islands with considerable degree of isolation provide valuable insights into human history?

- (A) Isolated islands may evolve differently and this difference is of interest to us.
- (B) Isolated islands increase the number of observations available to historians.
- (C) Isolated islands, differing in their endowments and size may evolve differently and this difference can be attributed to their endowments and size.
- (D) Isolated islands, differing in their endowments and size, provide a good comparison to large islands such as Eurasia, Africa, Americas and Australia.
- (E) Isolated islands, in so far as they are inhabited, arouse curiosity about how human beings evolved there.

Correct Answer: (C) Isolated islands, differing in their endowments and size may evolve differently and this difference can be attributed to their endowments and size.

Solution:

Step 1: Understanding the Concept:

The question asks for the primary reason, according to the passage, that isolated islands are valuable for studying human history. We need to locate the relevant section of the passage and identify the author's main argument on this topic.

Step 2: Detailed Explanation:

The passage introduces the concept of "natural experiments" as a methodology for historical sciences. It explains this method as "comparing systems differing in the presence or absence... of some putative causative factor."

The passage then gives a specific example: "...cultural anthropologists... study long-term effects of resource abundance on human societies by comparing recent Polynesian populations living on islands differing naturally in resource abundance."

This highlights the core idea: islands act like controlled environments. Because they are isolated, differences in their development can be linked to specific initial variables (like resource availability, size, etc.) with fewer confounding factors from outside contact.

Let's evaluate the options based on this understanding:

- (A) is too general. It doesn't explain *why* the difference is of interest or what causes it.
- (B) is a side effect, not the main reason for their value. The value is in the *quality* of the observations (comparability), not just the quantity.
- (C) perfectly captures the essence of the "natural experiment" concept. It states that islands have different starting conditions ("endowments and size"), they evolve differently, and crucially, the difference in evolution can be linked back ("attributed to") those starting conditions. This is the central argument of the passage.
- (D) is partially true but misses the main point. The value is not just in comparing them *to* continents, but in comparing them *among themselves* to isolate variables.
- (E) is too vague. "Arousing curiosity" is not the scientific reason for their value.

Step 3: Final Answer:

Option (C) provides the most accurate and complete summary of the author's reasoning for why isolated islands are useful for historical study.

Step 4: Why This is Correct:

This option correctly identifies the core methodology described in the passage: using isolated islands as "natural experiments" where differences in outcomes can be attributed to differences in initial variables like endowments and size.

💡 Quick Tip

In reading comprehension, always look for the option that captures the central argument or mechanism described by the author, rather than a statement that is merely true or mentioned in passing. The "why" and "how" are often more important than the "what."

70. According to the author, why is prediction difficult in history?

- (A) Historical explanations are usually broad so that no prediction is possible.
- (B) Historical outcomes depend upon a large number of factors and hence prediction is difficult for each case.
- (C) Historical sciences, by their very nature, are not interested in a multitude of minor factors, which might be important in a specific historical outcome.
- (D) Historians are interested in evolution of human history and hence are only interested in long-term predictions.
- (E) Historical sciences suffer from the inability to conduct controlled experiments and therefore have explanations based on a few long-term factors.

Correct Answer: (B) Historical outcomes depend upon a large number of factors and hence prediction is difficult for each case.

Solution:

Step 1: Understanding the Concept:

The question asks for the author's explanation of why prediction in history is difficult. We need to find the specific reasons cited in the first paragraph of the passage.

Step 2: Detailed Explanation:

The first paragraph explicitly lists the difficulties faced by historians and other historical sciences. Key phrases include:

- "...the complexity arising from enormous numbers of variables..."
- "...the resulting uniqueness of each system..."
- The passage contrasts predicting the sex ratio of 1,000 newborns (possible because variables average out) with predicting the outcome of the 1960 U.S. presidential election (not possible because specific details, like "who said what during a single televised debate," have a huge impact).

This shows that the core problem is the vast number of variables and the unique influence of small-scale events on specific outcomes.

Now let's examine the options:

- (A) is incorrect. The author states that prediction is actually *more feasible* for broad, large-scale outcomes.
- (B) accurately summarizes the main point. The "large number of factors" corresponds to the "enormous numbers of variables," and the difficulty "for each case" reflects the "uniqueness of each system."
- (C) is the opposite of what the passage implies. The passage suggests that historical sciences *are* affected by a multitude of minor factors, and the inability to control or account for them is what makes specific prediction so hard.

- (D) is a misrepresentation. While historians can make long-term predictions, the passage does not claim this is their only interest.
- (E) states a true fact from the passage (inability to conduct controlled experiments), but it's an incomplete reason. Option (B) provides a more direct and comprehensive explanation for the difficulty of prediction itself. The number of variables is the inherent property of the system that makes prediction hard, whereas the inability to experiment is a methodological limitation.

Step 3: Final Answer:

Option (B) is the best answer as it directly reflects the author's argument that the complexity and "enormous numbers of variables" make specific historical prediction difficult.

Step 4: Why This is Correct:

This option correctly identifies the central reason provided by the author for the difficulty of prediction in history: the complexity arising from a large number of influential variables in any given historical event.

Quick Tip

When a question asks "why," look for the option that explains the cause, not just one that states a related fact. Several options might be true statements based on the passage, but only one will be the primary reason answering the question.

71. According to the author, which of the following statements would be true?

- (A) Students of history are missing significant opportunities by not conducting any natural experiments.
- (B) Complex societies inhabiting large islands provide great opportunities for natural experiments.
- (C) Students of history are missing significant opportunities by not studying an adequate variety of natural experiments.
- (D) A unique problem faced by historians is their inability to establish cause and effect relationships.
- (E) Cultural anthropologists have overcome the problem of confounding variables through natural experiments.

Correct Answer: (C) Students of history are missing significant opportunities by not studying an adequate variety of natural experiments.

Solution:

Step 1: Understanding the Concept:

The question asks us to identify which statement is supported by the author's arguments

throughout the passage. This requires a holistic understanding of the author's main points and tone.

Step 2: Detailed Explanation:

The author's main argument is that history can be more scientific by adopting methodologies like "natural experiments" from other historical sciences. The author actively promotes this approach and points out its potential.

Let's analyze the key sentence in the third paragraph: "The student of human history can draw on many more natural experiments than just comparisons among the five inhabited continents." This sentence directly implies that there is a broad range of available "experiments" (large islands, small islands, societies within continents) that are potentially being underutilized if historians are only focusing on continent-level comparisons.

Let's evaluate the options based on this:

- (A) is too strong. The passage doesn't say historians are conducting *no* natural experiments, but suggests they could be doing more or using a wider variety.
- (B) is a true statement directly from the passage ("Comparisons can also utilize large islands..."). However, it's a specific supporting detail rather than the main conclusion or a broader true statement representing the author's overall point.
- (C) captures the implication of the key sentence perfectly. By stating that students "can draw on many more natural experiments," the author implies that if they are not doing so, they are missing opportunities. This aligns with the author's goal of advocating for a broader application of this methodology.
- (D) is explicitly contradicted in the first sentence, which states the difficulties are "broadly similar" to those in other sciences like astronomy and geology, meaning the problem is not unique to historians.
- (E) is an overstatement. The passage says epidemiology has "successfully employed formalized procedures for dealing with problems," but also states that natural experiments are "inherently open to potential methodological criticisms," including "confounding effects." This means the problem is managed, not completely "overcome."

Step 3: Final Answer:

Option (C) is the most accurate inference from the author's argument. The author highlights the vast potential of natural experiments beyond simple continental comparisons, thereby implying that not exploring this variety is a missed opportunity.

Step 4: Why This is Correct:

This option correctly interprets the author's argumentative stance. The passage is not just describing methods but advocating for their wider use, and pointing out the untapped variety of natural experiments is a key part of this advocacy.

 Quick Tip

Differentiate between a supporting detail and a main conclusion. A statement can be factually correct according to the text (like option B) but still not be the best answer if another option (like option C) better captures the author's overall argument or a key inference.

Directions for Questions 72 to 75: In each question, there are five sentences/paragraphs. The sentence/ paragraph labelled A is in its correct place. The four that follow are labelled B, C, D and E, and need to be arranged in the logical order to form a coherent paragraph/passage. From the given options, choose the most appropriate one.

72. In America, highly educated women, who are in stronger position in the labour market than less qualified ones, have higher rates of marriage than other groups. The four that follow are labelled B, C, D and E, and need to be arranged in the logical order to form a coherent paragraph/passage.

- A. In America, highly educated women, who are in stronger position in the labour market than less qualified ones, have higher rates of marriage than other groups.
B. Some work supports the Becker thesis, and some appears to contradict it.
C. And, as with crime, it is equally inconclusive.
D. But regardless of the conclusion of any particular piece of work, it is hard to establish convincing connections between family changes and economic factors using conventional approaches.
E. Indeed, just as with crime, an enormous academic literature exists on the validity of the pure economic approach to the evolution of family structures.

- (A) BCDE
(B) DBEC
(C) BDCE
(D) ECB D
(E) EBCD

Correct Answer: (D) ECB D

Solution:

Step 1: Understanding the Concept:

This is a paragraph jumbling question. The first sentence (A) is fixed. We need to arrange the remaining four sentences (B, C, D, E) to create a logical and coherent paragraph. The key is to identify connections, such as transition words, pronouns, and the flow of ideas from general to specific.

Step 2: Detailed Explanation:

Let's analyze the sentences and their connections:

- **A (Fixed):** Presents a specific, somewhat counter-intuitive fact about educated women and marriage in America.

- **E:** Introduces the broad academic context for explaining such facts: the "pure economic approach to the evolution of family structures." The phrase "Indeed, just as with crime" links a general academic field to the topic. Placing E after A sets the stage for the academic discussion of the phenomenon presented in A. (A → E)
- **C:** Starts with "And, as with crime, it is equally inconclusive." The phrase "as with crime" directly connects to the comparison introduced in E. This makes the E → C link very strong and logical. C makes a general claim that this field of study is inconclusive.
- **B:** Gives a specific example of the inconclusiveness mentioned in C. "Some work supports the Becker thesis, and some appears to contradict it." This is a perfect example to follow the general statement in C, moving from a general claim (inconclusive) to a specific instance (conflicting support for a thesis). So, C → B.
- **D:** Functions as a concluding sentence for the entire paragraph. It starts with "But regardless of the conclusion..." and makes a broad summary statement about the overall difficulty of establishing connections in this field. It naturally follows the specific discussion of inconclusive work.

Step 3: Final Answer:

The logical flow is as follows:

1. **A:** State a specific phenomenon.
2. **E:** Introduce the broad academic field that studies this phenomenon and compare it to the study of crime.
3. **C:** Continue the comparison with crime to state that the academic field is inconclusive.
4. **B:** Provide a specific example of this inconclusiveness (the Becker thesis).
5. **D:** Offer a final, summarizing thought on the inherent difficulty of the field.

This creates the sequence A-E-C-B-D. The correct order for the jumbled part is ECB D.

Step 4: Why This is Correct:

The sequence ECB D creates the most logical flow, moving from a broad introduction of the academic literature (E), to a statement about its inconclusive nature (C), followed by a specific example (B), and ending with a concluding summary (D). The transition phrases ("just as with crime" in E, "as with crime" in C) create clear and strong links between the sentences.

Quick Tip

In paragraph jumbles, pay close attention to transition words ("Indeed," "And," "But"), pronouns ("it," "this"), and repeated key phrases ("crime"). These are the signposts that guide the logical order of the sentences. Often, the structure flows from a general statement to specific examples and ends with a conclusion.

73. Personal experience of mothering and motherhood are largely framed in relation to two discernible or "official" discourses: the "medical discourse and natural childbirth discourse". Both of these tend to focus on the "optimistic stories" of birth and mothering and underpin stereotypes of the "good mother". The four that follow are labelled B, C, D and E, and need to be arranged in the logical order to form a coherent paragraph/passage.

A. Personal experience of mothering and motherhood are largely framed in relation to two discernible or "official" discourses: the "medical discourse and natural childbirth discourse". Both of these tend to focus on the "optimistic stories" of birth and mothering and underpin stereotypes of the "good mother".

B. At the same time, the need for medical expert guidance is also a feature for contemporary reproduction and motherhood. But constructions of good mothering have not always been so conceived - and in different contexts may exist in parallel to other equally dominant discourses.

C. Similarly, historical work has shown how what are now taken-for-granted aspects of reproduction and mothering practices result from contemporary "pseudoscientific directives" and "managed constructs". These changes have led to a reframing of modern discourses that pattern pregnancy and motherhood leading to an acceptance of the need for greater expert management.

D. The contrasting, overlapping, and ambiguous strands within these frameworks focus to varying degrees on a woman's biological tie to her child and predisposition to instinctively know and be able to care for her child.

E. In addition, a third, "unofficial popular discourse" comprising "old wives" tales and based on maternal experiences of childbirth has also been noted. These discourses have also been acknowledged in work exploring the experiences of those who apparently do not "conform" to conventional stereotypes of the "good mother".

- (A) EDBC
- (B) BCED
- (C) DBCE
- (D) EDCB
- (E) BCDE

Correct Answer: (A) EDBC

Solution:

Step 1: Understanding the Concept:

This is a paragraph jumbling question where the opening sentence (A) is fixed. The task is to arrange the subsequent sentences to form a coherent passage about discourses on motherhood.

Step 2: Detailed Explanation:

Let's break down the logical connections between the sentences:

- **A (Fixed):** Introduces two "official" discourses on motherhood.
- **E:** Starts with "In addition," introducing a "third, 'unofficial popular discourse'." This is a natural continuation from sentence A, completing the list of discourses being discussed. Thus, the A -> E link is very strong.

- **D:** Mentions "these frameworks." This pronoun refers to the discourses mentioned previously. After A and E have introduced all three discourses (two official, one unofficial), D can effectively elaborate on their common themes (e.g., a woman's biological tie to her child). So, the sequence A -> E -> D is logical.
- **B:** This sentence pivots the discussion. It first acknowledges the "need for medical expert guidance" (linking back to the medical discourse in A) but then introduces a historical contrast: "But constructions of good mothering have not always been so conceived." This sets up a new line of thought about the historical development of these ideas.
- **C:** Starts with "Similarly, historical work has shown...". The word "Similarly" directly links to the historical point made in B. Sentence C expands on how historical changes led to the modern need for "expert management." This makes B and C a tight, logical pair (B -> C) that should follow the initial description of the discourses.

Step 3: Final Answer:

The most logical arrangement builds upon the fixed sentence A:

1. **A:** Introduce two official discourses.
2. **E:** Add the third, unofficial discourse.
3. **D:** Elaborate on the content of "these frameworks" (all three discourses).
4. **B:** Pivot to a historical perspective on the development of these ideas.
5. **C:** Continue the historical point introduced in B.

This creates the sequence A-E-D-B-C. The correct order for the jumbled part is EDBC.

Step 4: Why This is Correct:

The sequence EDBC is the most coherent. It first lists and describes all the relevant discourses (A, E, D) before moving on to a separate but related topic: the historical evolution of these discourses (B, C). This thematic grouping creates a clear and easy-to-follow argument.

Quick Tip

Look for logical grouping of ideas. In this case, the paragraph first defines and describes the present-day discourses (A, E, D) and then provides historical context (B, C). Identifying these thematic sub-sections can help you solve the jumble quickly.

74. Indonesia has experienced dramatic shifts in its formal governance arrangements since the fall of President Soeharto and the close of his centralized, authoritarian "New Order" regime in 1997. The four that follow are labelled B, C, D and E, and need to be arranged in the logical order to form a coherent paragraph/passage.

A. Indonesia has experienced dramatic shifts in its formal governance arrangements since the fall of President Soeharto and the close of his centralized, authoritarian

"New Order" regime in 1997.

B. The political system has taken its place in the nearly 10 years since Reformasi began. It has featured the active contest for political office among a proliferation of parties at central, provincial and district levels; direct elections for the presidency (since 2004); and radical changes in centre-local government relations towards administrative, fiscal, and political decentralization.

C. The mass media, once tidily under Soeharto's thumb, has experienced significant liberalization, as has the legal basis for non-governmental organizations, including many dedicated to such controversial issues as corruption control and human rights.

D. Such developments are seen optimistically by a number of donors and some external analysts, who interpret them as signs of Indonesia's political normalization.

E. A different group of analysts paint a picture in which the institutional forms have changed, but power relations have not. Vedi Hadiz argues that Indonesia's "democratic transition" has been anything but linear.

- (A) BDEC
- (B) CBDE
- (C) CEBD
- (D) DEBC
- (E) BCDE

Correct Answer: (E) BCDE

Solution:

Step 1: Understanding the Concept:

This is a paragraph jumbling question with a fixed opening sentence. The goal is to arrange the sentences to create a logical narrative about Indonesia's political changes.

Step 2: Detailed Explanation:

Let's establish the logical connections:

- **A (Fixed):** Sets the topic: "dramatic shifts" in Indonesia's governance since 1997.
- **B:** Provides specific examples of these shifts within the core political system (elections, parties, decentralization). This is a direct elaboration of the point made in A. So, A -> B is a strong start.
- **C:** Provides further examples of the shifts, focusing on media and NGOs. This is parallel to B and continues to provide evidence for the claim in A. It makes sense to discuss the core political system (B) before related areas like media (C). Thus, A -> B -> C is a very logical sequence of examples.
- **D:** Refers to "Such developments." This phrase logically refers to the list of changes detailed in both B and C. This sentence introduces one interpretation of these developments: an optimistic one. So, (B+C) -> D is the correct structure.
- **E:** Introduces a contrasting viewpoint: "A different group of analysts paint a picture..." This sentence presents the pessimistic or critical view of the transition. This point-counterpoint structure means E should logically follow D. (D -> E).

Step 3: Final Answer:

The paragraph follows a clear and logical structure:

1. **A:** Main idea (political shifts).
2. **B:** Example 1 (political system).
3. **C:** Example 2 (media and NGOs).
4. **D:** Interpretation 1 (optimistic view).
5. **E:** Interpretation 2 (contrasting/pessimistic view).

This structure yields the sequence A-B-C-D-E. The correct arrangement of the jumbled sentences is BCDE.

Step 4: Why This is Correct:

The BCDE order creates a perfectly structured argument. It begins by providing evidence (B, C) for the initial claim (A), then presents two opposing interpretations of that evidence (D, E). This logical progression from fact to interpretation is a hallmark of coherent writing.

💡 Quick Tip

Many paragraphs are structured as: Claim -> Evidence -> Interpretation/Analysis. Identify which sentences serve each function. In this case, A is the claim, B and C are evidence, and D and E are analysis. This can help you determine the correct order.

75. I had six thousand acres of land, and had thus got much spare land besides the coffee plantation. Part of the farm was native forest, and about one thousand acres were squatters' land, what [the Kikuyu] called their shambas. The four that follow are labelled B, C, D and E, and need to be arranged in the logical order to form a coherent paragraph/passage.

A. I had six thousand acres of land, and had thus got much spare land besides the coffee plantation. Part of the farm was native forest, and about one thousand acres were squatters' land, what [the Kikuyu] called their shambas.

B. The squatters' land was more intensely alive than the rest of the farm, and was changing with the seasons the year round. The maize grew up higher than your head as you walked on the narrow hard-trampled footpaths in between the tall green rustling regiments.

C. The squatters are Natives, who with their families hold a few acres on a white man's farm, and in return have to work for him a certain number of days in the year. My squatters, I think, saw the relationship in a different light, for many of them were born on the farm, and their fathers before them, and they very likely regarded me as a sort of superior squatter on their estates.

D. The Kikuyu also grew the sweet potatoes that have a vine like leaf and spread over the ground like a dense entangled mat, and many varieties of big yellow and green speckled pumpkins.

E. The beans ripened in the fields, were gathered and thrashed by the women, and the maize stalks and coffee pods were collected and burned, so that in certain seasons thin blue columns of smoke rose here and there all over the farm.

- (A) CBDE
- (B) BCDE
- (C) CBED
- (D) DBCE
- (E) EDBC

Correct Answer: (C) CBED

Solution:

Step 1: Understanding the Concept:

This is a paragraph jumbling question that requires arranging sentences to form a descriptive and narrative passage. The key is to follow the flow from general description to specific details and actions.

Step 2: Detailed Explanation:

Let's trace the logical flow starting from the fixed sentence A.

- **A (Fixed):** Introduces the setting and a key term: "squatters' land."
- **C:** This sentence defines who the "squatters" are and describes their relationship with the landowner. It is most logical to define a key group of people immediately after they are introduced. So, A -> C is a strong opening.
- **B:** This sentence begins to describe the "squatters' land," which was mentioned in A. After explaining who the people are (C), it makes sense to describe their land. It specifically mentions the maize. So, A -> C -> B.
- **E:** This sentence describes activities related to the crops, including the maize mentioned in B ("the maize stalks...were collected and burned"). This creates a thematic link between the description of the growing maize (B) and the subsequent actions related to it (E). So, B -> E is a plausible connection, focusing on the life cycle of the farm's activities.
- **D:** Starts with "The Kikuyu also grew...". The word "also" indicates that this is additional information. After describing the main crop (maize in B) and the activities (E), this sentence adds details about other crops (sweet potatoes, pumpkins). Placing it after E makes it feel like an addendum, providing more descriptive detail after the main action has been described.

Step 3: Final Answer:

The most coherent narrative follows this order:

1. **A:** Set the scene, introduce "squatters' land."
2. **C:** Define who the "squatters" are.
3. **B:** Describe their land and its primary crop, maize.

4. **E:** Describe the seasonal activities on the land, including what happens to the maize stalks.
5. **D:** Add information about other crops the squatters "also" grew.

This creates the sequence A-C-B-E-D. The correct order for the jumbled part is CBED.

Step 4: Why This is Correct:

The sequence CBED creates a logical flow. It moves from setting the scene (A) to defining the people (C), then describing their land and its main feature (B), followed by the activities on that land (E), and concluding with additional details about other produce (D). This progression from general to specific, and from description to action, makes the passage vivid and easy to follow.

 Quick Tip

In descriptive passages, look for a logical flow from introduction (people, place) to specific details (what they do, what grows there). Words like "also" often signal that a sentence is adding a parallel piece of information and should follow a related point.