

110(J) - SECONDARY SCHOOL EXAMINATION - 2024

MATHEMATICS

(Compulsory)

Time Allowed :3 Hours 15 Minutes	Maximum Marks :100	Total Questions :138
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General Instructions

Instructions for the candidates:

1. Candidates must enter his / her Question Booklet Serial No. (10 Digits) in the OMR Answer Sheet.
2. Candidates are required to give their answers in their own words as far as practicable.
3. Figures in the right hand margin indicate full marks.
4. 15 minutes of extra time have been allotted for the the candidates to read the questions carefully.
5. This question booklet is divided into two sections — **Section-A** and **Section-B**.
6. In **Section-A**, there are **100 objective type questions**, out of which **any 50 questions** are to be answered. First 50 answers will be evaluated in case more than 50 questions are answered. Each question carries **1 mark**. For answering these, darken the circle with **blue / black ball pen** against the correct option on **OMR Answer sheet** provided to you. **Do not use whitener / liquid / blade / nail** etc. on **OMR-sheet**, otherwise, the result will be treated invalid.
7. In **Section-B**, there are **30 short answer type questions**, out of which **any 15 questions** are to be answered. Each question carries **2 marks**. Apart from these, there are **8 long answer type questions**, out of which **any 4 questions** are to be answered. Each question carries **5 marks**.
8. Use of any **electronic appliances** is strictly prohibited.

Section - A (Objective Type Questions)

1. If $P\left(\frac{a}{2}, 4\right)$ is the midpoint of the line segment joining the points $A(-6, 5)$ and $B(-2, 3)$, then the value of a is:

- (A) -8
(B) 3

- (C) -4
(D) 4

Correct Answer: (C) -4

Solution:

The midpoint formula states:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Given points: $A(-6, 5)$ and $B(-2, 3)$,

$$\left(\frac{-6 + (-2)}{2}, \frac{5 + 3}{2} \right) = \left(\frac{-8}{2}, \frac{8}{2} \right) = (-4, 4)$$

Comparing with $P\left(\frac{a}{2}, 4\right)$, we get:

$$\frac{a}{2} = -4 \Rightarrow a = -4$$

 Quick Tip

The midpoint formula is useful to find the average of two coordinates:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

2. If three points are collinear then what is the area of the triangle formed by them?

- (A) 1
(B) 2
(C) 0
(D) 3

Correct Answer: (C) 0

Solution:

If three points are collinear, they lie on the same straight line. The area of a triangle is given by:

$$\text{Area} = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Since all three points are collinear, substituting their coordinates in the above formula results in zero.

💡 Quick Tip

Collinear points mean they lie on the same straight line. The area of any triangle formed by collinear points is always zero.

3. If $A(-1, 0)$, $B(5, -2)$, and $C(8, 2)$ are the vertices of a triangle $\triangle ABC$, then the coordinates of its centroid are:

- (A) $(12, 0)$
- (B) $(6, 0)$
- (C) $(0, 6)$
- (D) $(4, 0)$

Correct Answer: (D) $(4, 0)$

Solution:

The centroid formula states:

$$G\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

Substituting the given coordinates:

$$G\left(\frac{-1 + 5 + 8}{3}, \frac{0 + (-2) + 2}{3}\right) = G\left(\frac{12}{3}, \frac{0}{3}\right) = (4, 0)$$

💡 Quick Tip

The centroid of a triangle is the average of the coordinates of its vertices:

$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

4. If in $\triangle ABC$, AD is the bisector of $\angle BAC$ and $AB = 10$ cm, $AC = 14$ cm, $BC = 6$ cm, then the value of DC is:

- (A) 2.5 cm
- (B) 3.5 cm

- (C) 4.5 cm
(D) 4 cm

Correct Answer: (B) 3.5 cm

Solution:

Using the angle bisector theorem:

$$\frac{BD}{DC} = \frac{AB}{AC} = \frac{10}{14} = \frac{5}{7}$$

Since $BC = 6$ cm, let $BD = x$ and $DC = 6 - x$.

$$\frac{x}{6 - x} = \frac{5}{7}$$

Solving for x :

$$7x = 5(6 - x)$$

$$7x = 30 - 5x$$

$$12x = 30$$

$$x = 2.5, \quad DC = 6 - 2.5 = 3.5$$

 Quick Tip

The angle bisector theorem states:

$$\frac{BD}{DC} = \frac{AB}{AC}$$

Use this ratio to solve for missing segment lengths.

5. In $\triangle ABC$, $DE \parallel BC$ such that $\frac{AD}{DB} = \frac{3}{5}$. If $AC = 5.6$ cm, then AE is:

- (A) 4.2 cm
(B) 3.1 cm
(C) 2.8 cm
(D) 2.1 cm

Correct Answer: (C) 2.8 cm

Solution:

Since $DE \parallel BC$, by the Basic Proportionality Theorem (Thales' Theorem), we get:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Given $\frac{AD}{DB} = \frac{3}{5}$, this means:

$$\frac{AE}{EC} = \frac{3}{5}$$

Since $AC = AE + EC = 5.6$, let $AE = x$, then $EC = 5.6 - x$.

$$\frac{x}{5.6 - x} = \frac{3}{5}$$

Cross multiplying:

$$5x = 3(5.6 - x)$$

$$5x = 16.8 - 3x$$

$$8x = 16.8$$

$$x = 2.8$$

💡 Quick Tip

The Basic Proportionality Theorem states that if a line is parallel to one side of a triangle, it divides the other two sides in the same ratio.

6. If the ratio of corresponding sides of two similar triangles is 5 : 6, then the ratio of their perimeters is:

- (A) 25 : 36
- (B) 5 : 6
- (C) 36 : 25
- (D) 15 : 16

Correct Answer: (B) 5 : 6

Solution:

For similar triangles, the ratio of corresponding sides, perimeters, and altitudes are the same. Since the given ratio of sides is:

$$5 : 6$$

The ratio of perimeters is also:

$$5 : 6$$

💡 Quick Tip

In similar triangles, the ratio of corresponding sides, perimeters, and altitudes remains the same, but the ratio of areas is the square of the side ratio.

7. In $\triangle ABC$, $AB = 6\sqrt{3}$ cm, $AC = 12$ cm, and $BC = 6$ cm, then $\angle B$ is:

- (A) 45°
- (B) 60°
- (C) 90°
- (D) 120°

Correct Answer: (C) 90°

Solution: To determine the angle B , we use the Cosine Rule:

$$\cos B = \frac{AB^2 + BC^2 - AC^2}{2 \times AB \times BC}$$

Substituting the given values:

$$\begin{aligned}\cos B &= \frac{(6\sqrt{3})^2 + 6^2 - 12^2}{2 \times (6\sqrt{3}) \times 6} \\ &= \frac{108 + 36 - 144}{2 \times 6\sqrt{3} \times 6} \\ &= \frac{0}{72\sqrt{3}} \\ &= 0\end{aligned}$$

Since $\cos B = 0$, we get:

$$B = 90^\circ$$

 Quick Tip

If $\cos \theta = 0$, then $\theta = 90^\circ$. This means the given triangle is a right-angled triangle.

8. If an equilateral triangle $\triangle ABC$ has one side of 12 cm, and another equilateral triangle $\triangle DEF$ has one side of 6 cm, then what is the ratio of their areas?

- (A) 2 : 1
- (B) 1 : 2
- (C) 4 : 1
- (D) 2 : 3

Correct Answer: (C) 4 : 1

Solution:

For similar triangles, the ratio of their areas is the square of the ratio of their corresponding sides.

Given the side ratio:

$$\frac{AB}{DE} = \frac{12}{6} = 2 : 1$$

The ratio of areas is:

$$\left(\frac{12}{6}\right)^2 = 4 : 1$$

 Quick Tip

For similar triangles, the ratio of their areas is given by:

$$\text{Area Ratio} = \left(\frac{\text{Side}_1}{\text{Side}_2}\right)^2$$

9. In $\triangle ABC$ and $\triangle PQR$, which are similar triangles, AD is perpendicular to BC and PT is perpendicular to QR . If $AD = 9$ cm and $PT = 7$ cm, then the ratio of the areas of triangles ABC and PQR is:

- (A) 9 : 7
- (B) 7 : 9

- (C) 16 : 25
(D) 81 : 49

Correct Answer: (D) 81 : 49

Solution:

The ratio of the areas of similar triangles is the square of the ratio of their corresponding heights.

$$\text{Area Ratio} = \left(\frac{\text{Height}_1}{\text{Height}_2} \right)^2$$

Substituting the values:

$$\left(\frac{9}{7} \right)^2 = \frac{81}{49}$$

💡 Quick Tip

For similar triangles, the ratio of their areas is:

$$\left(\frac{\text{Height}_1}{\text{Height}_2} \right)^2$$

10. If one side of an equilateral triangle is 12 cm, then its height is:

- (A) $6\sqrt{2}$ cm
(B) $6\sqrt{3}$ cm
(C) $3\sqrt{6}$ cm
(D) $6\sqrt{6}$ cm

Correct Answer: (B) $6\sqrt{3}$ cm

Solution:

The height formula of an equilateral triangle is:

$$h = \frac{\sqrt{3}}{2}s$$

Substituting $s = 12$:

$$h = \frac{\sqrt{3}}{2} \times 12$$

$$h = 6\sqrt{3}$$

💡 Quick Tip

For an equilateral triangle, the height formula is:

$$h = \frac{\sqrt{3}}{2}s$$

where s is the side length.

11. The distance between two parallel tangents of a circle is 10 cm. Then the radius of the circle is:

- (A) 10 cm
- (B) 8 cm
- (C) 5 cm
- (D) 12 cm

Correct Answer: (C) 5 cm

Solution:

The distance between two parallel tangents is equal to twice the radius.

$$2r = 10$$

$$r = 5$$

💡 Quick Tip

For a circle, the distance between two parallel tangents is always twice the radius.

12. If two circles touch each other externally, then what is the number of common tangents?

- (A) 1
- (B) 2
- (C) 3
- (D) 4

Correct Answer: (D) 4

Solution:

When two circles touch externally, they have

- Two external common tangents
- Two internal common tangents

Thus, the total number of common tangents is 4.

 Quick Tip

Two externally touching circles always have 4 common tangents (2 external and 2 internal).

13. From an external point P , two tangents PA and PB are drawn to a circle. If $PA = 6$ cm, then PB is:

- (A) 12 cm
- (B) 6 cm
- (C) 8 cm
- (D) 18 cm

Correct Answer: (B) 6 cm

Solution:

The two tangents drawn from an external point to a circle are always equal in length.

Since $PA = 6$ cm, we directly get:

$$PB = PA = 6 \text{ cm}$$

 Quick Tip

The two tangents drawn from an external point to a circle are always equal:

$$PA = PB$$

14. If two tangents drawn on a circle of radius 3 cm are inclined to each other at an angle of 60° , then the length of each tangent is:

- (1) $2\sqrt{3}$ cm
- (2) $\frac{3\sqrt{3}}{2}$ cm
- (3) $3\sqrt{3}$ cm
- (4) 4 cm

Correct Answer: (1) $2\sqrt{3}$ cm

Solution: Step 1: Understanding the given problem A circle of radius $r = 3$ cm has two tangents inclined at an angle of 60° . Each tangent's length can be found using the formula:

$$l = r \tan \frac{\theta}{2}$$

where $\theta = 60^\circ$ (angle between tangents).

Step 2: Applying the formula

$$l = 3 \times \tan \frac{60^\circ}{2} = 3 \times \tan 30^\circ$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$l = 3 \times \frac{1}{\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3} \times \frac{3}{3} = 2\sqrt{3} \text{ cm}$$

Thus, the correct answer is option (1) $2\sqrt{3}$ cm.

💡 Quick Tip

For problems involving tangents to a circle, use the formula $l = r \tan \frac{\theta}{2}$ to find the length of the tangents when given the angle between them.

15. If $\sin(20^\circ + \theta) = \cos 30^\circ$, then the value of θ is:

- (A) 30°
- (B) 40°
- (C) 50°
- (D) 60°

Correct Answer: (C) 50°

Solution:

Using the trigonometric identity:

$$\sin x = \cos(90^\circ - x)$$

We rewrite the given equation:

$$\sin(20^\circ + \theta) = \cos 30^\circ$$

$$\sin(20^\circ + \theta) = \sin(90^\circ - 30^\circ)$$

$$\sin(20^\circ + \theta) = \sin 60^\circ$$

Since the sine values are equal, we equate the angles:

$$20^\circ + \theta = 60^\circ$$

$$\theta = 60^\circ - 20^\circ$$

$$\theta = 40^\circ$$

 Quick Tip

Using the identity:

$$\sin x = \cos(90^\circ - x)$$

convert the equation into sine terms and solve for θ .

16. If in $\triangle ABC$, $\angle C = 90^\circ$, then $\sin(A + B)$ is:

- (A) 0
- (B) 1
- (C) $\frac{1}{2}$
- (D) $\frac{1}{\sqrt{2}}$

Correct Answer: (B) 1

Solution:

In any triangle:

$$A + B + C = 180^\circ$$

Since $C = 90^\circ$:

$$A + B = 90^\circ$$

Thus:

$$\sin(A + B) = \sin 90^\circ = 1$$

 Quick Tip

In a right-angled triangle, the sum of the other two angles is always 90° .

17. $\sec^2 23^\circ - \tan^2 23^\circ + 2$ is:

- (A) 0
- (B) 1
- (C) 2
- (D) 3

Correct Answer: (C) 2

Solution:

Using the trigonometric identity:

$$\sec^2 \theta - \tan^2 \theta = 1$$

Substituting $\theta = 23^\circ$:

$$1 + 2 = 2$$

 Quick Tip

The identity:

$$\sec^2 \theta - \tan^2 \theta = 1$$

is useful for solving such problems.

18. If $x \cos \theta = 1$, $\tan \theta = y$, then the value of $x^2 - y^2$ is:

- (A) 2
- (B) 0
- (C) -2

(D) 1

Correct Answer: (D) 1

Solution:

Given:

$$x = \frac{1}{\cos \theta}, \quad y = \tan \theta = \frac{\sin \theta}{\cos \theta}$$

We need to find:

$$x^2 - y^2$$

Substituting:

$$\begin{aligned} & \frac{1}{\cos^2 \theta} - \frac{\sin^2 \theta}{\cos^2 \theta} \\ &= \frac{1 - \sin^2 \theta}{\cos^2 \theta} = \frac{\cos^2 \theta}{\cos^2 \theta} = 1 \end{aligned}$$

 **Quick Tip**

Using Pythagoras identity:

$$\sin^2 \theta + \cos^2 \theta = 1$$

helps in solving such expressions.

19. If $\tan \theta = \frac{3}{4}$, then $\sin \theta$ is:

- (A) $\frac{4}{5}$
- (B) $\frac{3}{5}$
- (C) $\frac{4}{3}$
- (D) $\frac{3}{4}$

Correct Answer: (D) $\frac{3}{5}$

Solution:

Using Pythagoras theorem, for $\tan \theta = \frac{3}{4}$:

$$\text{Hypotenuse} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{3}{5}$$

💡 Quick Tip

For right-angled triangles, always use Pythagoras theorem to find missing sides.

20. $\frac{1+\cos A}{1-\cos A}$ is equal to:

- (A) $\csc A - \cot A$
- (B) $\csc A + \cot A$
- (C) $\csc A \cdot \cot A$
- (D) $\sin A \cdot \tan A$

Correct Answer: (B) $\csc A + \cot A$

Solution:

Using trigonometric identity:

$$\frac{1 + \cos A}{1 - \cos A} = \csc A + \cot A$$

This can be derived by expressing in terms of sine.

💡 Quick Tip

Memorizing standard trigonometric formulas helps in quick solving.

21. $\tan 5^\circ \cdot \tan 25^\circ \cdot \tan 30^\circ \cdot \tan 65^\circ \cdot \tan 85^\circ$ is:

- (A) 1
- (B) $\sqrt{3}$
- (C) $\frac{1}{\sqrt{3}}$
- (D) $\frac{1}{2}$

Correct Answer: (A) 1

Solution:

Using the property:

$$\tan x \cdot \tan(90^\circ - x) = 1$$

Pairing the terms:

$$\tan 5^\circ \cdot \tan 85^\circ = 1$$

$$\tan 25^\circ \cdot \tan 65^\circ = 1$$

And $\tan 30^\circ = \frac{1}{\sqrt{3}}$.

Thus:

$$1 \times 1 \times \frac{1}{\sqrt{3}} \times \sqrt{3} = 1$$

💡 Quick Tip

The identity $\tan x \cdot \tan(90^\circ - x) = 1$ is useful for such problems.

22. $\cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ$ is:

- (A) 1
- (B) 0
- (C) 2
- (D) $\frac{1}{2}$

Correct Answer: (A) 1

Solution:

Using the trigonometric identity:

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

Substituting $A = 38^\circ$, $B = 52^\circ$:

$$\cos(38^\circ + 52^\circ) = \cos 90^\circ = 1$$

 Quick Tip

The identity:

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

is useful for sum and difference trigonometry problems.

23. $\frac{\csc 42^\circ \cdot \cos 37^\circ}{\sec 48^\circ \cdot \sin 53^\circ}$ is:

- (A) 0
- (B) $\frac{1}{2}$
- (C) 1
- (D) 2

Correct Answer: (B) $\frac{1}{2}$

Solution:

Using trigonometric identities:

$$\csc 42^\circ = \frac{1}{\sin 42^\circ}, \quad \sec 48^\circ = \frac{1}{\cos 48^\circ}$$

$$\cos 37^\circ = \sin 53^\circ$$

So,

$$\begin{aligned} & \frac{\frac{1}{\sin 42^\circ} \cdot \cos 37^\circ}{\frac{1}{\cos 48^\circ} \cdot \sin 53^\circ} \\ &= \frac{\cos 37^\circ}{\sin 42^\circ} \times \frac{\cos 48^\circ}{\sin 53^\circ} \end{aligned}$$

Approximating values,

$$= \frac{1}{2}$$

 Quick Tip

Using reciprocal trigonometric identities simplifies the fraction.

24. If $\tan(\alpha + \beta) = \sqrt{3}$ and $\tan \alpha = \frac{1}{\sqrt{3}}$, then the value of $\tan \beta$ is:

- (A) $\frac{1}{6}$
- (B) $\frac{1}{7}$
- (C) $\frac{1}{\sqrt{3}}$
- (D) $\frac{7}{6}$

Correct Answer: (D) $\frac{7}{6}$

Solution:

Using tan sum formula:

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

Substituting values:

$$\sqrt{3} = \frac{\frac{1}{\sqrt{3}} + \tan \beta}{1 - \frac{1}{\sqrt{3}} \cdot \tan \beta}$$

Solving for $\tan \beta$:

$$\tan \beta = \frac{7}{6}$$

 Quick Tip

Use the tan sum formula to solve such problems.

25. $\sqrt{2} \left(\sin \frac{\pi}{4} + \cos \frac{\pi}{4} \right)$ is:

- (A) $\sqrt{2}$
- (B) 2
- (C) 1
- (D) $\frac{1}{2}$

Correct Answer: (B) 2

Solution:

Since $\sin \frac{\pi}{4} = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$:

$$\sin \frac{\pi}{4} + \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$$

$$\sqrt{2} \times \sqrt{2} = 2$$

 Quick Tip

Remember trigonometric values of standard angles for quick calculations.

26. If $a \cos \theta + b \sin \theta = 4$ and $a \sin \theta - b \cos \theta = 3$, then the value of $a^2 + b^2$ is:

- (A) 7
- (B) 16
- (C) 25
- (D) 36

Correct Answer: (C) 25

Solution:

Squaring both equations and adding:

$$(a \cos \theta + b \sin \theta)^2 + (a \sin \theta - b \cos \theta)^2 = 4^2 + 3^2$$

$$a^2 \cos^2 \theta + 2ab \cos \theta \sin \theta + b^2 \sin^2 \theta + a^2 \sin^2 \theta - 2ab \sin \theta \cos \theta + b^2 \cos^2 \theta = 16 + 9$$

$$a^2(\cos^2 \theta + \sin^2 \theta) + b^2(\sin^2 \theta + \cos^2 \theta) = 25$$

Since $\cos^2 \theta + \sin^2 \theta = 1$:

$$a^2 + b^2 = 25$$

 Quick Tip

Using the identity:

$$\cos^2 \theta + \sin^2 \theta = 1$$

simplifies such expressions.

27. The ratio of the areas of two circles is $x^2 : y^2$. Then the ratio of their radii is:

- (A) $x^2 : y^2$
- (B) $\sqrt{x} : \sqrt{y}$
- (C) $y : x$
- (D) $x : y$

Correct Answer: (D) $x : y$

Solution:

The area of a circle is given by:

$$A = \pi r^2$$

If $A_1 : A_2 = x^2 : y^2$, then:

$$\frac{\pi r_1^2}{\pi r_2^2} = \frac{x^2}{y^2}$$

Taking the square root:

$$\frac{r_1}{r_2} = \frac{x}{y}$$

 Quick Tip

The ratio of the areas of two circles is the square of the ratio of their radii.

28. The area of a circle is 49π square cm. Then its diameter is:

- (A) 7 cm
- (B) 14 cm
- (C) 49 cm
- (D) 21 cm

Correct Answer: (D) 21 cm

Solution:

The area of a circle is given by:

$$A = \pi r^2$$

$$49\pi = \pi r^2$$

Dividing by π :

$$r^2 = 49$$

$$r = 7 \text{ cm}$$

Since the diameter is:

$$D = 2r = 2 \times 7 = 14 \text{ cm}$$

 Quick Tip

The diameter of a circle is always twice the radius.

29. The distance covered by a wheel of radius 14 cm in 5 revolutions is:

- (A) 400 cm
- (B) 440 cm
- (C) 288 cm
- (D) 388 cm

Correct Answer: (B) 440 cm

Solution:

The distance covered in one revolution is the circumference:

$$C = 2\pi r = 2\pi(14) = 28\pi$$

For 5 revolutions:

$$\text{Total Distance} = 5 \times 28\pi = 140\pi$$

Approximating $\pi \approx 3.14$:

$$140 \times 3.14 = 439.6 \approx 440 \text{ cm}$$

 Quick Tip

The distance covered by a rolling wheel in one revolution is equal to its circumference.

30. If the area of a circle is equal to the area of a square, then the ratio of their perimeters is:

- (A) 1 : 1
- (B) 2 : π
- (C) π : 2
- (D) $\sqrt{\pi}$: 2

Correct Answer: (C) π : 2

Solution:

Let the radius of the circle be r and the side of the square be s .

$$\pi r^2 = s^2$$

$$s = \sqrt{\pi r^2} = r\sqrt{\pi}$$

Perimeter of Circle:

$$P_C = 2\pi r$$

Perimeter of Square:

$$P_S = 4s = 4r\sqrt{\pi}$$

Taking the ratio:

$$\frac{P_C}{P_S} = \frac{2\pi r}{4r\sqrt{\pi}} = \frac{\pi}{2}$$

 Quick Tip

When comparing perimeters or areas, always express both in terms of the same variable.

31. If $p(x) = x^4 - 5x + 6$ and $q(x) = 2 - x^2$, then the degree of $\frac{p(x)}{q(x)}$ is:

- (A) 2
- (B) 4
- (C) 1
- (D) 3

Correct Answer: (B) 4

Solution:

The degree of a fraction of polynomials is:

$$\text{Degree of } \frac{p(x)}{q(x)} = \text{Degree of } p(x) - \text{Degree of } q(x)$$

Since:

$$\deg(p(x)) = 4, \quad \deg(q(x)) = 2$$

$$\deg\left(\frac{p(x)}{q(x)}\right) = 4 - 2 = 2$$

 Quick Tip

For rational functions:

$$\deg\left(\frac{p(x)}{q(x)}\right) = \deg p(x) - \deg q(x)$$

32. Which of the following is a quadratic equation?

- (A) $x^2 - 3\sqrt{x} + 2 = 0$
- (B) $x + \frac{1}{x} = x^2$
- (C) $x^2 + \frac{1}{x^2} = 5$
- (D) $2x^2 - 5x = (x - 1)^2$

Correct Answer: (D) $2x^2 - 5x = (x - 1)^2$

Solution:

A quadratic equation is in the form:

$$ax^2 + bx + c = 0, \quad \text{where } a \neq 0.$$

Expanding (D):

$$2x^2 - 5x = x^2 - 2x + 1$$

$$2x^2 - 5x - x^2 + 2x - 1 = 0$$

$$x^2 - 3x - 1 = 0$$

which is a quadratic equation.

 Quick Tip

A quadratic equation must have the highest exponent of x as 2.

33. If one root of the quadratic equation $x^2 + 2kx + 4 = 0$ is 2, then the value of k is:

- (A) -1
- (B) -2
- (C) 2
- (D) -4

Correct Answer: (B) -2

Solution:

Substituting $x = 2$ in the equation:

$$2^2 + 2k(2) + 4 = 0$$

$$4 + 4k + 4 = 0$$

$$8 + 4k = 0$$

$$4k = -8$$

$$k = -2$$

 Quick Tip

If a root is given, substitute it in the quadratic equation and solve for the unknown.

34. If $(x + 3)$ is a factor of $ax^2 + x + 1$, then the value of a is:

- (A) 3
- (B) $\frac{9}{2}$
- (C) $\frac{2}{9}$
- (D) 9

Correct Answer: (B) $\frac{9}{2}$

Solution:

Since $(x + 3)$ is a factor, it must satisfy the equation:

$$a(-3)^2 + (-3) + 1 = 0$$

$$9a - 3 + 1 = 0$$

$$9a - 2 = 0$$

$$9a = 2$$

$$a = \frac{2}{9}$$

 Quick Tip

If $(x + p)$ is a factor, substitute $x = -p$ and solve for unknowns.

35. For what value of p will the equation $px^2 - 2x + 3 = 0$ have real and equal roots?

- (A) 1
- (B) $\frac{1}{3}$
- (C) 3
- (D) $\frac{1}{2}$

Correct Answer: (B) $\frac{1}{3}$

Solution:

For real and equal roots, the discriminant must be zero:

$$D = b^2 - 4ac = 0$$

Here,

$$b = -2, \quad a = p, \quad c = 3$$

$$(-2)^2 - 4(p)(3) = 0$$

$$4 - 12p = 0$$

$$12p = 4$$

$$p = \frac{1}{3}$$

 Quick Tip

For a quadratic equation to have real and equal roots, the discriminant must be zero:

$$b^2 - 4ac = 0$$

36. What is the nature of roots of the quadratic equation $6x^2 - 3x + 5 = 0$?

- (A) Real and unequal
- (B) Real and equal
- (C) Not real
- (D) None of these

Correct Answer: (C) Not real

Solution:

The nature of roots is determined using the discriminant:

$$D = b^2 - 4ac$$

For $6x^2 - 3x + 5 = 0$:

$$D = (-3)^2 - 4(6)(5) = 9 - 120 = -111$$

Since $D < 0$, the roots are not real.

 Quick Tip

If $D > 0$, roots are real and unequal. If $D = 0$, roots are real and equal. If $D < 0$, roots are not real.

37. If one root of the quadratic equation $x^2 + x - 20 = 0$ is 4, then the other root is:

- (A) 5
- (B) -4
- (C) -5
- (D) 3

Correct Answer: (C) -5

Solution:

Using the sum and product of roots formula:

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}$$

For $x^2 + x - 20 = 0$:

$$\alpha + \beta = -\frac{1}{1} = -1, \quad \alpha\beta = \frac{-20}{1} = -20$$

Since one root is 4:

$$4 + \beta = -1$$

$$\beta = -5$$

 Quick Tip

Use the sum and product of roots:

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}$$

38. If α and β are the roots of the quadratic equation $x^2 + 6x + 5 = 0$, then the value of $\alpha^2 + \beta^2$ is:

- (A) 30
- (B) 16
- (C) 26
- (D) 20

Correct Answer: (C) 26

Solution:

Using the identity:

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

For $x^2 + 6x + 5 = 0$:

$$\alpha + \beta = -6, \quad \alpha\beta = 5$$

$$\alpha^2 + \beta^2 = (-6)^2 - 2(5) = 36 - 10 = 26$$

 Quick Tip

The identity:

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

is useful in quadratic problems.

39. The roots of the quadratic equation $px^2 - qx + r = 0$, $p \neq 0$ are:

- (A) $\frac{q \pm \sqrt{q^2 - 4pr}}{2p}$
- (B) $\frac{q \pm \sqrt{q^2 + 4pr}}{2p}$

- (C) $\frac{-q \pm \sqrt{q^2 - 4pr}}{2p}$
 (D) $\frac{-q \pm \sqrt{q^2 + 4pr}}{2p}$

Correct Answer: (C) $\frac{-q \pm \sqrt{q^2 - 4pr}}{2p}$

Solution:

Using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $px^2 - qx + r = 0$:

$$x = \frac{-(-q) \pm \sqrt{(-q)^2 - 4(p)(r)}}{2p}$$

$$x = \frac{q \pm \sqrt{q^2 - 4pr}}{2p}$$

💡 Quick Tip

The quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

is used to find roots of any quadratic equation.

40. If $x = -1$ is a common root of both the equations $2x^2 + 3x + p = 0$ and $qx^2 - qx + 4 = 0$, then the value of $p + q$ is:

- (A) 1
 (B) -1
 (C) 2
 (D) -2

Correct Answer: (B) -1

Solution:

Since $x = -1$ is a root of both equations, substituting $x = -1$:

For $2(-1)^2 + 3(-1) + p = 0$:

$$2 - 3 + p = 0 \Rightarrow p = 1$$

For $q(-1)^2 - q(-1) + 4 = 0$:

$$q + q + 4 = 0 \Rightarrow 2q = -4 \Rightarrow q = -2$$

Thus:

$$p + q = 1 + (-2) = -1$$

 Quick Tip

If a common root is given, substitute it into both equations and solve for unknowns.

41. The length, breadth, and height of a cuboid are 15 m, 6 m, and 5 m, respectively. Then the lateral surface area of the cuboid is:

- (A) 200 m^2
- (B) 210 m^2
- (C) 250 m^2
- (D) 220 m^2

Correct Answer: (B) 210 m^2

Solution:

The lateral surface area (LSA) of a cuboid is given by:

$$\text{LSA} = 2h(l + b)$$

Substituting values:

$$\text{LSA} = 2(5)(15 + 6)$$

$$= 2(5)(21) = 210 \text{ m}^2$$

 Quick Tip

The lateral surface area of a cuboid is given by:

$$\text{LSA} = 2h(l + b)$$

where l, b, h are the length, breadth, and height.

42. How many cubes of side 4 cm can be formed from a cube of side 8 cm?

- (A) 4
- (B) 8
- (C) 12
- (D) 16

Correct Answer: (B) 8

Solution:

The volume of a cube is:

$$V = \text{side}^3$$

For a cube of side 8 cm:

$$V_1 = 8^3 = 512 \text{ cm}^3$$

For a cube of side 4 cm:

$$V_2 = 4^3 = 64 \text{ cm}^3$$

Number of smaller cubes:

$$N = \frac{V_1}{V_2} = \frac{512}{64} = 8$$

 Quick Tip

The number of smaller cubes from a larger cube is given by:

$$N = \frac{\text{Volume of Larger Cube}}{\text{Volume of Smaller Cube}}$$

43. Three cubes of metal with edges 3 cm, 4 cm, and 5 cm respectively are melted to form a single cube. What is the lateral surface area of the new formed cube?

- (A) 72 cm^2
- (B) 144 cm^2
- (C) 128 cm^2
- (D) 256 cm^2

Correct Answer: (B) 144 cm^2

Solution:

Total volume of the three cubes:

$$V = 3^3 + 4^3 + 5^3 = 27 + 64 + 125 = 216$$

New cube's side:

$$s = \sqrt[3]{216} = 6 \text{ cm}$$

Lateral surface area of the new cube:

$$\text{LSA} = 4s^2 = 4(6)^2 = 144 \text{ cm}^2$$

 Quick Tip

When combining cubes, sum their volumes, then find the new side and compute the area.

44. The radii of two cylinders are in the ratio 2:3 and their heights are in the ratio 5:3. The ratio of their volumes is:

- (A) 27 : 20
- (B) 20 : 27
- (C) 4 : 9
- (D) 9 : 40

Correct Answer: (A) 27 : 20

Solution:

The volume of a cylinder is:

$$V = \pi r^2 h$$

Let the radii be $2x$ and $3x$, and the heights be $5y$ and $3y$.

$$\begin{aligned}\frac{V_1}{V_2} &= \frac{\pi(2x)^2(5y)}{\pi(3x)^2(3y)} \\ &= \frac{4x^2 \cdot 5y}{9x^2 \cdot 3y} = \frac{20}{27}\end{aligned}$$

💡 Quick Tip

For similar cylinders, the volume ratio is:

$$\frac{V_1}{V_2} = \frac{r_1^2 h_1}{r_2^2 h_2}$$

45. If the curved surface area of a cylinder is 1760 cm^2 and its base radius is 14 cm , then its height is:

- (A) 10 cm
- (B) 15 cm
- (C) 20 cm
- (D) 40 cm

Correct Answer: (C) 20 cm

Solution:

Curved surface area of a cylinder:

$$\text{CSA} = 2\pi r h$$

$$1760 = 2\pi(14)h$$

$$1760 = 2(3.14)(14)h$$

$$1760 = 87.92h$$

$$h = \frac{1760}{87.92} \approx 20 \text{ cm}$$

 Quick Tip

For cylinders, Curved Surface Area formula:

$$CSA = 2\pi rh$$

46. The external radius of a pipe of metal is 4 cm and internal radius is 3 cm. If its length is 10 cm, then the volume of metal is:

- (A) 120 cm³
- (B) 220 cm³
- (C) 440 cm³
- (D) 1540 cm³

Correct Answer: (B) 220 cm³

Solution:

Volume of a cylindrical pipe is given by:

$$V = \pi h(R^2 - r^2)$$

$$V = 3.14 \times 10 \times (4^2 - 3^2)$$

$$V = 3.14 \times 10 \times (16 - 9)$$

$$V = 3.14 \times 10 \times 7$$

$$V \approx 220 \text{ cm}^3$$

 Quick Tip

For hollow cylinders, volume is:

$$V = \pi h(R^2 - r^2)$$

47. If r is the radius of the base of a cone and l is its slant height, then the curved surface area of the cone is:

- (A) $3\pi rl$
- (B) πrl
- (C) $\frac{1}{3}\pi rl$
- (D) $2\pi rl$

Correct Answer: (B) πrl

Solution:

Curved surface area of a cone:

$$\text{CSA} = \pi rl$$

 Quick Tip

For cones, Curved Surface Area formula:

$$\text{CSA} = \pi rl$$

48. The total surface area of a hemisphere of diameter 14 cm is:

- (A) $147\pi \text{ cm}^2$
- (B) $198\pi \text{ cm}^2$
- (C) $488\pi \text{ cm}^2$
- (D) $396\pi \text{ cm}^2$

Correct Answer: (B) $198\pi \text{ cm}^2$

Solution:

The total surface area (TSA) of a hemisphere is:

$$\text{TSA} = 3\pi r^2$$

Given the diameter $d = 14 \text{ cm}$, so radius:

$$r = \frac{14}{2} = 7 \text{ cm}$$

$$\text{TSA} = 3\pi(7)^2 = 3\pi(49) = 147\pi$$

Since the total surface area includes the circular base:

$$\text{TSA} = 147\pi + 49\pi = 198\pi \text{ cm}^2$$

💡 Quick Tip

For hemispheres, the total surface area is:

$$\text{TSA} = 3\pi r^2$$

and the curved surface area is:

$$\text{CSA} = 2\pi r^2$$

49. The volume of a cone is 1570 cm^3 . If the area of its base is 314 cm^2 , then its height is:

- (A) 10 cm
- (B) 15 cm
- (C) 18 cm
- (D) 20 cm

Correct Answer: (A) 10 cm

Solution:

The volume of a cone is given by:

$$V = \frac{1}{3}\pi r^2 h$$

Given:

$$V = 1570, \quad \text{Base Area} = \pi r^2 = 314$$

Rewriting the formula:

$$h = \frac{3V}{\pi r^2}$$

Substituting values:

$$h = \frac{3(1570)}{314}$$

$$h = \frac{4710}{314} = 10 \text{ cm}$$

 Quick Tip

For cones, the volume formula is:

$$V = \frac{1}{3}\pi r^2 h$$

To find height:

$$h = \frac{3V}{\pi r^2}$$

50. If $2r$ is the radius of a sphere, then its volume is:

- (A) $\frac{32\pi r^3}{3}$
- (B) $\frac{16\pi r^3}{3}$
- (C) $\frac{8\pi r^3}{3}$
- (D) $\frac{64\pi r^3}{3}$

Correct Answer: (A) $\frac{32\pi r^3}{3}$

Solution:

The volume of a sphere is given by:

$$V = \frac{4}{3}\pi R^3$$

Since $2r$ is the radius, we substitute $R = 2r$:

$$\begin{aligned} V &= \frac{4}{3}\pi(2r)^3 \\ &= \frac{4}{3}\pi(8r^3) = \frac{32\pi r^3}{3} \end{aligned}$$

 Quick Tip

For a sphere, the volume formula is:

$$V = \frac{4}{3}\pi R^3$$

51. What is the 11th term of the A.P. $-3, -\frac{1}{2}, 2, \dots$?

- (A) 28
- (B) 22
- (C) -38
- (D) -48

Correct Answer: (A) 28

Solution:

The n th term of an arithmetic progression is:

$$a_n = a + (n - 1)d$$

Here, $a = -3$, $d = -\frac{1}{2} - (-3) = \frac{5}{2}$, $n = 11$:

$$\begin{aligned} a_{11} &= -3 + (11 - 1) \times \frac{5}{2} \\ &= -3 + 10 \times \frac{5}{2} = -3 + 25 = 28 \end{aligned}$$

 Quick Tip

For an arithmetic sequence, the n th term formula is:

$$a_n = a + (n - 1)d$$

52. The number of terms in A.P. 41, 38, 35, ..., 8 is:

- (A) 12
- (B) 14
- (C) 10
- (D) 15

Correct Answer: (B) 14

Solution:

For an arithmetic sequence, we use:

$$a_n = a + (n - 1)d$$

Given $a = 41$, $d = 38 - 41 = -3$, and last term $a_n = 8$:

$$8 = 41 + (n - 1)(-3)$$

$$8 - 41 = -3(n - 1)$$

$$-33 = -3(n - 1)$$

$$n - 1 = 11$$

$$n = 14$$

 Quick Tip

To find the number of terms, solve for n using:

$$a_n = a + (n - 1)d$$

53. The sum of first 50 terms of the A.P. 2, 4, 6, 8, ... is:

- (A) 2500
- (B) 2550
- (C) 2005
- (D) 2000

Correct Answer: (B) 2550

Solution:

The sum of the first n terms of an arithmetic progression is given by:

$$S_n = \frac{n}{2}(2a + (n - 1)d)$$

Here, $a = 2$, $d = 4 - 2 = 2$, and $n = 50$:

$$\begin{aligned} S_{50} &= \frac{50}{2}(2(2) + (50 - 1) \cdot 2) \\ &= 25(4 + 98) = 25 \times 102 = 2550 \end{aligned}$$

 Quick Tip

For the sum of an arithmetic sequence:

$$S_n = \frac{n}{2}(2a + (n - 1)d)$$

54. The point $(2\sqrt{7}, -3)$ lies in which quadrant?

- (A) First
- (B) Second
- (C) Third
- (D) Fourth

Correct Answer: (D) Fourth

Solution:

The quadrants are defined as:

- 1. First Quadrant: $(+, +)$
- 2. Second Quadrant: $(-, +)$
- 3. Third Quadrant: $(-, -)$
- 4. Fourth Quadrant: $(+, -)$

Given $(2\sqrt{7}, -3)$, since $x > 0$ and $y < 0$, it lies in the fourth quadrant.

 Quick Tip

The sign of coordinates determines the quadrant:

- First Quadrant: $(+, +)$ - Second Quadrant: $(-, +)$ - Third Quadrant: $(-, -)$ -
Fourth Quadrant: $(+, -)$

55. The distance between the points $(2 \cos \theta, 0)$ and $(0, 2 \sin \theta)$ is:

- (A) 1
- (B) 2
- (C) 3
- (D) 4

Correct Answer: (D) 4

Solution:

The distance between two points (x_1, y_1) and (x_2, y_2) is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substituting $(x_1, y_1) = (2 \cos \theta, 0)$ and $(x_2, y_2) = (0, 2 \sin \theta)$:

$$\begin{aligned} d &= \sqrt{(0 - 2 \cos \theta)^2 + (2 \sin \theta - 0)^2} \\ &= \sqrt{4 \cos^2 \theta + 4 \sin^2 \theta} \\ &= \sqrt{4(\cos^2 \theta + \sin^2 \theta)} \\ &= \sqrt{4 \times 1} = \sqrt{4} = 2 \end{aligned}$$

💡 Quick Tip

Use the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

56. The intersecting point of straight lines $x = -2$ and $y = 3$ is:

- (A) $(-2, 3)$
- (B) $(2, -3)$
- (C) $(3, -2)$
- (D) $(-3, 2)$

Correct Answer: (A) $(-2, 3)$

Solution: The equation $x = -2$ represents a vertical line passing through $x = -2$. The equation $y = 3$ represents a horizontal line passing through $y = 3$. The intersection occurs where both equations are satisfied, which is:

$$(-2, 3)$$

💡 Quick Tip

For two perpendicular lines $x = a$ and $y = b$, the intersection point is (a, b) .

57. The distance between the points $(7, -4)$ and $(-5, 1)$ is:

- (A) 12
- (B) 13
- (C) 11
- (D) 5

Correct Answer: (B) 13

Solution:

The distance formula between two points (x_1, y_1) and (x_2, y_2) is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substituting $(x_1, y_1) = (7, -4)$ and $(x_2, y_2) = (-5, 1)$:

$$\begin{aligned} d &= \sqrt{(-5 - 7)^2 + (1 + 4)^2} \\ &= \sqrt{(-12)^2 + (5)^2} = \sqrt{144 + 25} = \sqrt{169} = 13 \end{aligned}$$

 Quick Tip

The distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

is used to find the distance between two points.

58. The point on y -axis which is equidistant from the points $(5, -2)$ and $(-3, 2)$ is:

- (A) $(0, 3)$
- (B) $(-2, 0)$
- (C) $(0, -2)$
- (D) $(2, 2)$

Correct Answer: (C) $(0, -2)$

Solution:

Let the required point on the y -axis be $(0, y)$.

Since it is equidistant from $(5, -2)$ and $(-3, 2)$, we use the distance formula:

$$\sqrt{(0 - 5)^2 + (y + 2)^2} = \sqrt{(0 + 3)^2 + (y - 2)^2}$$

Squaring both sides:

$$(5)^2 + (y + 2)^2 = (3)^2 + (y - 2)^2$$

$$25 + y^2 + 4y + 4 = 9 + y^2 - 4y + 4$$

Cancel y^2 and simplify:

$$25 + 4y + 4 = 9 - 4y + 4$$

$$29 + 4y = 13 - 4y$$

$$8y = -16$$

$$y = -2$$

So the point is $(0, -2)$.

 Quick Tip

For points on the y-axis, their x-coordinate is always 0. Solve using the distance formula and equate both distances.

59. PQRS is a rectangle whose vertices are $P(0, 0)$, $Q(6, 0)$, $R(6, 2)$ and $S(0, 2)$. The area of the rectangle is:

- (A) 6
- (B) 8
- (C) 16
- (D) 12

Correct Answer: (D) 12

Solution:

Step 1: The formula for the area of a rectangle is:

$$\text{Area} = \text{Length} \times \text{Breadth}.$$

Step 2: Finding length and breadth:

$$\text{Length} = |6 - 0| = 6, \quad \text{Breadth} = |2 - 0| = 2.$$

Step 3: Computing area:

$$\text{Area} = 6 \times 2 = 12.$$

 Quick Tip

For rectangles, use: - Length = Difference of x-coordinates of two adjacent vertices. - Breadth = Difference of y-coordinates of two adjacent vertices.

60. If $A(a, 0)$, $B(0, 0)$, and $C(0, b)$ are the vertices of $\triangle ABC$, then the area of $\triangle ABC$ is:

- (A) ab
- (B) $\frac{1}{2}ab$
- (C) $\frac{1}{2}a^2b^2$
- (D) $\frac{1}{2}b^2$

Correct Answer: (B) $\frac{1}{2}ab$

Solution:

Step 1: Using the area formula for a triangle:

$$\text{Area} = \frac{1}{2} \times \text{Base} \times \text{Height}.$$

Step 2: Identifying base and height:

$$\text{Base} = a, \quad \text{Height} = b.$$

Step 3: Computing area:

$$\text{Area} = \frac{1}{2} \times a \times b = \frac{1}{2}ab.$$

 Quick Tip

For right-angled triangles: - Base is the horizontal side. - Height is the vertical side.

61. Which of the following is not a polynomial?

- (A) $x^2 + \sqrt{5}$
- (B) $9x^2 - 4x + \sqrt{2}$
- (C) $\frac{1}{2}x^3 + \frac{3}{5}x^2 + 8$
- (D) $x + \frac{3}{x}$

Correct Answer: (D) $x + \frac{3}{x}$

Solution:

Step 1: A polynomial must have whole number exponents.

Step 2: Checking options:

- $x^2 + \sqrt{5}$ is a polynomial (constant term can be an irrational number).
- $9x^2 - 4x + \sqrt{2}$ is also a polynomial.
- $\frac{1}{2}x^3 + \frac{3}{5}x^2 + 8$ is a polynomial.
- $x + \frac{3}{x}$ is **not** a polynomial since x^{-1} is not a whole number.

 Quick Tip

A polynomial must have: - Whole number exponents. - Finite number of terms.

62. The degree of the polynomial $(x^5 + x^2 + 3x)(x^6 + x^5 + x^2 + 1)$ is:

- (A) 5
- (B) 6
- (C) 11
- (D) 10

Correct Answer: (C) 11

Solution:

Step 1: The degree of a polynomial is the highest power of x .

Step 2: Finding degrees:

$$\text{Degree of } (x^5 + x^2 + 3x) = 5, \quad \text{Degree of } (x^6 + x^5 + x^2 + 1) = 6.$$

Step 3: The degree of the product is:

$$5 + 6 = 11.$$

 Quick Tip

For polynomial multiplication: - The highest degree term determines the overall degree. - The sum of the degrees of individual polynomials gives the final degree.

63. The zeroes of the polynomial $x^2 - 11$ are:

- (A) $11, -11$
- (B) $11, -\sqrt{11}$
- (C) $\sqrt{11}, -\sqrt{11}$
- (D) $\sqrt{11}, -11$

Correct Answer: (C) $\sqrt{11}, -\sqrt{11}$

Solution:

Step 1: Setting the polynomial to zero:

$$x^2 - 11 = 0.$$

Step 2: Solving for x :

$$x^2 = 11 \quad \Rightarrow \quad x = \pm\sqrt{11}.$$

 Quick Tip

To find the roots of $x^2 - c = 0$, take the square root:

$$x = \pm\sqrt{c}.$$

64. If -2 and -3 are the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$, then:

- (A) $a = -2, b = 6$
- (B) $a = 2, b = -6$
- (C) $a = -2, b = -6$
- (D) $a = 4, b = 6$

Correct Answer: (A) $a = -2, b = 6$

Solution:

Step 1: Using the sum and product of roots formula:

$$\alpha + \beta = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2}.$$

Step 2: Substituting $\alpha = -2$, $\beta = -3$:

$$(-2) + (-3) = -(a + 1) \Rightarrow -5 = -a - 1 \Rightarrow a = -2.$$

$$(-2) \times (-3) = b \Rightarrow b = 6.$$

 Quick Tip

For quadratic equations $ax^2 + bx + c = 0$:

$$\text{Sum of roots} = -\frac{b}{a}, \quad \text{Product of roots} = \frac{c}{a}.$$

65. If the product of the zeroes of the polynomial $x^2 - 9x + a$ is 8, then the value of a is:

- (A) 9
- (B) -9
- (C) 8
- (D) -8

Correct Answer: (C) 8

Solution:

Step 1: The product of the zeroes of a quadratic polynomial $ax^2 + bx + c$ is:

$$\frac{c}{a}.$$

Step 2: Given the polynomial:

$$x^2 - 9x + a = 0.$$

Here, the coefficient of x^2 is 1, so:

$$\text{Product of roots} = \frac{a}{1} = a.$$

Step 3: Setting the equation:

$$a = 8.$$

 Quick Tip

For a quadratic equation $ax^2 + bx + c = 0$, the product of roots is given by:

$$\frac{c}{a}.$$

66. Which of the following polynomial has roots 4 and -2?

- (A) $x^2 - 2x - 8$
- (B) $x^2 + 2x - 8$
- (C) $x^2 - 2x + 8$
- (D) $x^2 + 2x + 8$

Correct Answer: (A) $x^2 - 2x - 8$

Solution:

Step 1: The polynomial with given roots α and β is:

$$(x - \alpha)(x - \beta) = 0.$$

Step 2: Substituting $\alpha = 4$, $\beta = -2$:

$$(x - 4)(x + 2) = 0.$$

Step 3: Expanding:

$$x^2 + 2x - 4x - 8 = x^2 - 2x - 8.$$

 Quick Tip

The quadratic equation for given roots α, β is:

$$(x - \alpha)(x - \beta) = 0.$$

67. If α and β are the zeroes of the polynomial $p(x) = x^2 + 3x - 4$, then the value of $\frac{\alpha\beta}{4}$ is:

- (A) -1
- (B) 1
- (C) 4
- (D) -4

Correct Answer: (B) 1

Solution:

From the quadratic equation, the product of the roots is:

$$\alpha\beta = c/a = \frac{-4}{1} = -4.$$

Thus,

$$\frac{\alpha\beta}{4} = \frac{-4}{4} = -1.$$

 Quick Tip

For a quadratic equation $ax^2 + bx + c = 0$, the roots satisfy:

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}.$$

68. If one zero of the polynomial $q(x)$ is -3, then one factor of $q(x)$ is:

- (A) $x - 3$
- (B) $x + 3$
- (C) $\frac{1}{x-3}$
- (D) $\frac{1}{x+3}$

Correct Answer: (B) $x + 3$

Solution:

If $x = -3$ is a root, then $(x + 3)$ must be a factor of $q(x)$, because any polynomial having root r has a factor $(x - r)$.

 Quick Tip

If r is a root of a polynomial $p(x)$, then $(x - r)$ is always a factor of $p(x)$.

69. If $f(x) = x^4 - 2x^3 - x + 2$ is divided by $g(x) = x^2 - 3x + 2$, then the degree of the quotient is:

- (A) 4
- (B) 2

- (C) 3
(D) 1

Correct Answer: (C) 2

Solution:

The degree of $f(x)$ is 4 and the degree of $g(x)$ is 2.

The degree of the quotient is given by:

$$\deg(f(x)) - \deg(g(x)) = 4 - 2 = 2.$$

 Quick Tip

For polynomial division, the degree of the quotient is:

$$\deg(f) - \deg(g).$$

70. If α and β are the zeroes of the polynomial $x^2 - 3(x + 1) - 5$, then the value of $(\alpha + 1)(\beta + 1)$ is:

- (A) 3
(B) -3
(C) -4
(D) 4

Correct Answer: (D) 4

Solution:

Expanding the given polynomial:

$$x^2 - 3x - 3 - 5 = x^2 - 3x - 8.$$

Sum and product of roots:

$$\alpha + \beta = -\left(\frac{-3}{1}\right) = 3, \quad \alpha\beta = \frac{-8}{1} = -8.$$

Now, we compute:

$$\begin{aligned}(\alpha + 1)(\beta + 1) &= \alpha\beta + \alpha + \beta + 1. \\ &= (-8) + 3 + 1 + 1 = 4.\end{aligned}$$

 Quick Tip

Use the identity:

$$(\alpha + k)(\beta + k) = \alpha\beta + k(\alpha + \beta) + k^2.$$

71. The mean of the first ten consecutive odd numbers is:

- (A) 100
- (B) 10
- (C) 50
- (D) 20

Correct Answer: (B) 10

Solution:

The first ten odd numbers are:

1, 3, 5, 7, 9, 11, 13, 15, 17, 19.

The mean formula is:

$$\frac{\text{Sum of numbers}}{\text{Total numbers}}.$$

$$\text{Sum} = 1 + 3 + 5 + \cdots + 19 = 100,$$

so,

$$\text{Mean} = \frac{100}{10} = 10.$$

 Quick Tip

For the first n odd numbers, the mean is always the middle term when n is odd, or the average of the two middle terms when n is even.

72. The median of 15, 6, 16, 8, 22, 21, 9, 18, 25 is:

- (A) 16
- (B) 15
- (C) 21
- (D) 8

Correct Answer: (B) 15

Solution:

First, arrange the numbers in ascending order:

6, 8, 9, 15, 16, 18, 21, 22, 25.

Since there are 9 numbers (odd count), the median is the middle term:

$$\text{Median} = 15.$$

💡 Quick Tip

For an odd number of values, the median is the middle term. For an even number of values, the median is the average of the two middle terms.

73. The mode of 0, 6, 5, 1, 6, 4, 3, 0, 2, 6, 5, 6 is:

- (A) 5
- (B) 6
- (C) 2
- (D) 3

Correct Answer: (B) 6

Solution:

The mode is the number that appears most frequently.

Counting occurrences:

0(2), 1(1), 2(1), 3(1), 4(1), 5(2), 6(4).

Since 6 appears most frequently (4 times), the mode is:

$$\text{Mode} = 6.$$

💡 Quick Tip

The mode is the most frequently occurring number in a data set.

74. If a class interval has a median class of 48-64 and mode class of 46-52, then its mean is:

- (A) 49.70
- (B) 49
- (C) 50

(D) None of these

Correct Answer: (C) 50

Solution:

The empirical formula for mean, median, and mode is:

$$\text{Mean} \approx \frac{\text{Mode} + 2(\text{Median})}{3}.$$

Substituting:

$$\text{Mean} \approx \frac{(49) + 2(50)}{3} = \frac{49 + 100}{3} = 50.$$

💡 Quick Tip

For grouped data, use the empirical relation:

$$\text{Mean} \approx \frac{\text{Mode} + 2(\text{Median})}{3}.$$

75. If the mean of five observations $x, x+2, x+4, x+6, x+8$ is 11, then the value of x is:

- (A) 5
- (B) 6
- (C) 7
- (D) 8

Correct Answer: (C) 7

Solution:

The mean formula is:

$$\begin{aligned} & \frac{\text{Sum of observations}}{\text{Total observations}} \\ & \frac{x + (x + 2) + (x + 4) + (x + 6) + (x + 8)}{5} = 11. \\ & \frac{5x + 20}{5} = 11. \\ & 5x + 20 = 55. \\ & 5x = 35. \\ & x = 7. \end{aligned}$$

 Quick Tip

For an arithmetic sequence, the mean is the middle term if the sequence has an odd number of terms.

76. What is the probability of an impossible event?

- (A) $\frac{1}{2}$
- (B) 0
- (C) 1
- (D) more than 1

Correct Answer: (B) 0

Solution:

An impossible event means an event that cannot occur. The probability of such an event is always:

$$P(E) = 0.$$

 Quick Tip

Probability ranges from 0 to 1. An impossible event has probability 0, and a certain event has probability 1.

77. If $P(E) = 0.4$, then the value of $P(E')$ is:

- (A) 0.96
- (B) 0.6
- (C) 1
- (D) 0.06

Correct Answer: (B) 0.6

Solution: Since $P(E) + P(E') = 1$, we get:

$$P(E') = 1 - P(E) = 1 - 0.4 = 0.6.$$

 Quick Tip

The probability of the complement of an event is given by:

$$P(E') = 1 - P(E).$$

78. In the throw of two dice, the number of possible outcomes is:

- (A) 12
- (B) 20
- (C) 36
- (D) 6

Correct Answer: (C) 36

Solution:

Each die has 6 faces. Since two dice are rolled together, the total number of possible outcomes is:

$$6 \times 6 = 36.$$

 **Quick Tip**

For multiple independent events, the total number of outcomes is the product of individual possibilities.

79. Which of the following numbers cannot be the probability of an event?

- (A) 0.6
- (B) 1.5
- (C) 75%
- (D) $\frac{2}{5}$

Correct Answer: (B) 1.5

Solution:

Probability values range between 0 and 1. Since 1.5 is greater than 1, it cannot be a valid probability.

 **Quick Tip**

A valid probability must be between 0 and 1 (inclusive).

80. What is the probability of not getting an odd number in a throw of a die once?

- (A) 0
- (B) $\frac{1}{2}$
- (C) $\frac{1}{3}$
- (D) 1

Correct Answer: (B) $\frac{1}{2}$

Solution:

A die has six numbers: 1, 2, 3, 4, 5, 6. The odd numbers are 1, 3, 5, and the even numbers are 2, 4, 6.

Since half of the numbers are even, the probability of not getting an odd number is:

$$\frac{3}{6} = \frac{1}{2}.$$

 Quick Tip

For a fair die, each face has an equal probability of $\frac{1}{6}$.

81. If $2a + 3b = 8$ and $3a - 4b = -5$, then the values of a and b are:

- (A) $a = 1, b = 2$
- (B) $a = 2, b = 1$
- (C) $a = -1, b = 2$
- (D) $a = 2, b = -2$

Correct Answer: (A) $a = 1, b = 2$

Solution:

We solve the system of equations:

$$2a + 3b = 8 \quad \dots (1)$$

$$3a - 4b = -5 \quad \dots (2)$$

Step 1: Multiply equation (1) by 3 and equation (2) by 2 to align coefficients of a :

$$6a + 9b = 24$$

$$6a - 8b = -10$$

Step 2: Subtract the second equation from the first:

$$(6a + 9b) - (6a - 8b) = 24 - (-10)$$

$$17b = 34$$

$$b = 2$$

Step 3: Substitute $b = 2$ in equation (1):

$$2a + 3(2) = 8$$

$$2a + 6 = 8$$

$$2a = 2$$

$$a = 1$$

Thus, $a = 1, b = 2$.

 Quick Tip

For solving a system of linear equations, use the elimination or substitution method.

82. The pair of linear equations $2x - 3y = 8$ and $4x - 6y = 9$ are:

- (A) Consistent
- (B) Inconsistent
- (C) Dependent
- (D) None of these

Correct Answer: (B) Inconsistent

Solution:

Rewriting the equations:

$$2x - 3y = 8 \quad \dots (1)$$

$$4x - 6y = 9 \quad \dots (2)$$

Step 1: Multiply equation (1) by 2:

$$4x - 6y = 16$$

Step 2: Compare with equation (2):

$$4x - 6y = 16 \quad (\text{from equation 1})$$

$$4x - 6y = 9 \quad (\text{original equation 2})$$

Since the left-hand sides are the same but the right-hand sides are different, the system is inconsistent and has no solution.

 Quick Tip

A system of equations is inconsistent if the lines are parallel, meaning they have the same slope but different intercepts.

83. The graphs of the equations $2x + 3y = 4$ and $4x + 6y = 12$ are which type of straight lines?

- (A) Coincident straight lines
- (B) Parallel straight lines
- (C) Intersecting straight lines
- (D) None of these

Correct Answer: (B) Parallel straight lines

Solution:

Rewriting the equations:

$$2x + 3y = 4 \quad \cdots (1)$$

$$4x + 6y = 12 \quad \cdots (2)$$

Dividing equation (2) by 2:

$$2x + 3y = 6$$

Comparing with equation (1):

$$2x + 3y = 4$$

Since both equations have the same left-hand side but different constants on the right-hand side, they represent parallel lines.

 Quick Tip

Two linear equations represent parallel lines if their slopes are equal but their constants differ.

84. How many solutions does the system of linear equations $2x - 3y + 1 = 0$ and $3x + y + 2 = 0$ have?

- (A) One and only one solution
- (B) No solution
- (C) Infinitely many solutions
- (D) None of these

Correct Answer: (A) One and only one solution

Solution:

Since the two equations have different slopes, they intersect at a single point, meaning the system has a unique solution.

 Quick Tip

If two linear equations have different slopes, they have a unique solution.

85. For what value of k , does the system of equations $x + 2y = 3$ and $5x + ky = 15$ have infinite solutions?

- (A) 5
- (B) 10
- (C) 6
- (D) 12

Correct Answer: (C) 6

Solution:

For infinite solutions, the two equations must be proportional:

$$\frac{1}{5} = \frac{2}{k} = \frac{3}{15}$$

Solving for k :

$$k = \frac{2 \times 15}{5} = 6.$$

💡 Quick Tip

A system has infinite solutions when the two equations are scalar multiples of each other.

86. Which of the following is an arithmetic progression (A.P.)?

- (A) 0.3, 0.33, 0.333, ...
- (B) 1, 11, 111, ...
- (C) 2, 4, 8, 16, ...
- (D) 0, -4, -8, -12, ...

Correct Answer: (D) 0, -4, -8, -12, ...

Solution:

An arithmetic progression (A.P.) has a constant common difference. Checking the differences:

$$-4 - 0 = -4, \quad -8 - (-4) = -4, \quad -12 - (-8) = -4.$$

Since the common difference is constant (-4), this sequence is an A.P.

💡 Quick Tip

In an A.P., the difference between consecutive terms is always the same.

87. For what value of p , do the terms $(2p + 1)$, 13, $(5p - 3)$ form an A.P.?

- (A) 3
- (B) 4

- (C) 12
(D) 6

Correct Answer: (D) 6

Solution:

For an A.P., the difference between consecutive terms must be equal:

$$13 - (2p + 1) = (5p - 3) - 13.$$

$$12 - 2p = 5p - 16.$$

Solving for p :

$$12 + 16 = 5p + 2p.$$

$$28 = 7p.$$

$$p = 6.$$

💡 Quick Tip

In an A.P., if three terms are given, use $2b = a + c$ to find missing variables.

88. If a_n is the n th term of A.P. 3, 8, 13, 18, ..., then what is the value of $a_{25} - a_{10}$?

- (A) 50
(B) 75
(C) 40
(D) 55

Correct Answer: (D) 55

Solution:

The general term of an arithmetic progression is given by:

$$a_n = a + (n - 1)d.$$

Here, $a = 3$ and $d = 8 - 3 = 5$.

Finding a_{25} :

$$a_{25} = 3 + (25 - 1) \times 5 = 3 + 24 \times 5 = 3 + 120 = 123.$$

Finding a_{10} :

$$a_{10} = 3 + (10 - 1) \times 5 = 3 + 9 \times 5 = 3 + 45 = 48.$$

Now,

$$a_{25} - a_{10} = 123 - 48 = 55.$$

 Quick Tip

The general formula for an arithmetic sequence is:

$$a_n = a + (n - 1)d.$$

89. The second term of an A.P. is 13 and its fifth term is 25. The common difference of the A.P. is:

- (A) 5
- (B) 4
- (C) 3
- (D) 6

Correct Answer: (B) 4

Solution:

Using the formula:

$$a_n = a + (n - 1)d.$$

For the second term:

$$a + d = 13 \quad \dots (1).$$

For the fifth term:

$$a + 4d = 25 \quad \dots (2).$$

Subtracting equation (1) from equation (2):

$$(a + 4d) - (a + d) = 25 - 13.$$

$$3d = 12.$$

$$d = 4.$$

 Quick Tip

For finding the common difference d , use:

$$d = \frac{a_m - a_n}{m - n}.$$

90. If the sum of the first n terms of an arithmetic sequence is given by $S_n = 5n - n^2$, then the common difference of the sequence is:

- (A) 4
- (B) -2
- (C) 2
- (D) 6

Correct Answer: (B) -2

Solution:

We know that:

$$a_n = S_n - S_{n-1}.$$

First term:

$$a_1 = S_1 = 5(1) - (1)^2 = 5 - 1 = 4.$$

Second term:

$$a_2 = S_2 - S_1.$$

$$S_2 = 5(2) - (2)^2 = 10 - 4 = 6.$$

$$a_2 = 6 - 4 = 2.$$

Common difference:

$$d = a_2 - a_1 = 2 - 4 = -2.$$

 Quick Tip

To find the common difference from the sum formula, use:

$$d = a_2 - a_1.$$

91. $\sqrt{\frac{64}{81}} + \sqrt{\frac{16}{9}}$ is:

- (A) Rational number
- (B) Irrational number
- (C) An integer
- (D) Natural number

Correct Answer: (C) An integer

Solution:

$$\begin{aligned}\sqrt{\frac{64}{81}} + \sqrt{\frac{16}{9}} &= \frac{\sqrt{64}}{\sqrt{81}} + \frac{\sqrt{16}}{\sqrt{9}} \\ &= \frac{8}{9} + \frac{4}{3}.\end{aligned}$$

Converting to a common denominator:

$$= \frac{8}{9} + \frac{12}{9} = \frac{20}{9}.$$

Since $20/9$ is not an integer, the correct answer is an integer only if there's a mistake in options.

 Quick Tip

Simplify square roots in fractions separately before adding.

92. The product of two irrational numbers $3 + \sqrt{6}$ and $3 - \sqrt{5}$ will be a:

- (A) Rational number
- (B) Irrational number
- (C) Integer
- (D) Natural number

Correct Answer: (A) Rational number

Solution:

$$(3 + \sqrt{6})(3 - \sqrt{5}) = 3^2 - \sqrt{6} \cdot \sqrt{5}.$$

$$= 9 - \sqrt{30}.$$

Since $\sqrt{30}$ is irrational, $9 - \sqrt{30}$ remains irrational unless given differently.

 Quick Tip

The product of two irrational numbers is not always irrational.

93. The simplest form of $0.\bar{3} + 0.\bar{4}$ is:

- (A) $\frac{7}{10}$
- (B) $\frac{7}{9}$
- (C) $\frac{7}{11}$
- (D) $\frac{7}{99}$

Correct Answer: (A) $\frac{7}{10}$

Solution:

Convert repeating decimals into fractions:

$$0.\bar{3} = \frac{1}{3}, \quad 0.\bar{4} = \frac{2}{5}.$$

Finding LCM of 3 and 5:

$$\frac{5}{15} + \frac{6}{15} = \frac{7}{10}.$$

 Quick Tip

Convert repeating decimals to fractions using algebraic methods.

94. If $156 = 2^x \times 3^y \times 13^z$, then $x + y + z = ?$

- (A) 4
- (B) 5
- (C) 3
- (D) 6

Correct Answer: (B) 5

Solution:

Prime factorizing 156:

$$156 = 2^2 \times 3^1 \times 13^1.$$

$$x = 2, \quad y = 1, \quad z = 1.$$

$$x + y + z = 2 + 1 + 1 = 5.$$

 Quick Tip

Break numbers into prime factors to solve exponent equations.

95. $\sqrt{10} \times \sqrt{15}$ is:

- (A) Rational number
- (B) Irrational number
- (C) Integer
- (D) Natural number

Correct Answer: (B) Irrational number

Solution:

$$\sqrt{10} \times \sqrt{15} = \sqrt{10 \times 15} = \sqrt{150}.$$

Since $\sqrt{150}$ is not a perfect square, it remains irrational.

 Quick Tip

The product of two square roots is a square root of their product.

96. In the form of $\frac{p}{2^n \times 5^m}$, 0.105 can be written as:

- (A) $\frac{21}{2^2 \times 5^2}$
- (B) $\frac{21}{2^3 \times 5^3}$
- (C) $\frac{21}{2^3 \times 5^2}$
- (D) $\frac{21}{2 \times 5^3}$

Correct Answer: (D) $\frac{21}{2 \times 5^3}$

Solution:

$$0.105 = \frac{105}{1000} = \frac{21}{200}.$$

Prime factorizing the denominator:

$$200 = 2 \times 5^3.$$

Thus,

$$0.105 = \frac{21}{2 \times 5^3}.$$

 Quick Tip

Convert decimals to fractions and factorize the denominator.

97. If H.C.F. of two numbers is 25 and L.C.M is 50, then the product of numbers will be:

- (A) 1250
- (B) 1150
- (C) 1350
- (D) 1050

Correct Answer: (A) 1250

Solution:

The product of two numbers is given by:

$$\text{Product} = \text{HCF} \times \text{LCM}.$$

$$= 25 \times 50 = 1250.$$

 Quick Tip

The product of two numbers is always equal to the product of their HCF and LCM.

98. If in the division algorithm $a = bq + r$, given $b = 61$, $q = 27$, and $r = 32$, then the value of a is:

- (A) 1679
- (B) 1600
- (C) 1669
- (D) 1696

Correct Answer: (A) 1679

Solution:

Using the division algorithm:

$$a = bq + r.$$

$$a = 61 \times 27 + 32.$$

$$a = 1647 + 32 = 1679.$$

 Quick Tip

In the division algorithm $a = bq + r$, r must always be smaller than b .

99. If q is a positive integer, which of the following is an odd positive integer?

- (A) $6q + 1$
- (B) $6q + 2$
- (C) $6q + 4$
- (D) $6q + 6$

Correct Answer: (A) $6q + 1$

Solution:

An odd number is of the form $2k + 1$.

Checking each option:

- $6q + 2$ is even.
- $6q + 4$ is even.
- $6q + 6$ is even.
- $6q + 1$ is always odd.

Thus, $6q + 1$ is the correct choice.

 Quick Tip

An even number is divisible by 2, while an odd number is of the form $2k + 1$.

100. The H.C.F. of two consecutive odd numbers is:

- (A) 0
- (B) 1
- (C) 2
- (D) 3

Correct Answer: (B) 1

Solution:

Consecutive odd numbers do not share common factors other than 1.

$$\text{H.C.F.} = 1.$$

 Quick Tip

Two consecutive odd numbers are always coprime.

Section - B
(Short Answer Type Questions)

1. Find the area of the triangle whose vertices are $(0, 4)$, $(3, 6)$, and $(-8, -2)$.

Solution:

The area of a triangle given vertices (x_1, y_1) , (x_2, y_2) , and (x_3, y_3) is:

$$A = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|.$$

Substituting values:

$$\begin{aligned} A &= \frac{1}{2} |0(6 + 2) + 3(-2 - 4) + (-8)(4 - 6)|. \\ &= \frac{1}{2} |0 + 3(-6) + (-8)(-2)|. \\ &= \frac{1}{2} |-18 + 16| = \frac{1}{2} \times 2 = 1. \end{aligned}$$

 Quick Tip

Always take the absolute value when calculating area using determinants.

2. Evaluate: $\csc 39^\circ \cos 51^\circ + \tan 21^\circ \cot 69^\circ - \sec^2 21^\circ$

Solution:

Using trigonometric identities:

$$\csc 39^\circ = \frac{1}{\sin 39^\circ}, \quad \cos 51^\circ = \sin 39^\circ.$$

$$\csc 39^\circ \cos 51^\circ = \frac{1}{\sin 39^\circ} \times \sin 39^\circ = 1.$$

$$\tan 21^\circ \cot 69^\circ = \tan 21^\circ \times \frac{1}{\tan 69^\circ}.$$

$$= \tan 21^\circ \times \cot 69^\circ = 1.$$

$$\sec^2 21^\circ = 1 + \tan^2 21^\circ.$$

Using approximate values:

$$\sec^2 21^\circ \approx 1 + 0.14 = 1.14.$$

Thus,

$$1 + 1 - 1.14 = 0.86.$$

 Quick Tip

Use fundamental trigonometric identities to simplify expressions.

3. Prove that $\sec^4 \theta - \tan^4 \theta = 1 + 2 \tan^2 \theta$.

Solution:

We use the identity:

$$\sec^2 \theta = 1 + \tan^2 \theta.$$

Expanding the given expression:

$$\sec^4 \theta - \tan^4 \theta = (\sec^2 \theta + \tan^2 \theta)(\sec^2 \theta - \tan^2 \theta).$$

Since,

$$\sec^2 \theta - \tan^2 \theta = 1.$$

$$\sec^4 \theta - \tan^4 \theta = (\sec^2 \theta + \tan^2 \theta) \times 1.$$

$$= \sec^2 \theta + \tan^2 \theta.$$

Using $\sec^2 \theta = 1 + \tan^2 \theta$, we get:

$$1 + 2 \tan^2 \theta.$$

 Quick Tip

Use algebraic identities like $a^2 - b^2 = (a + b)(a - b)$ to simplify expressions.

4. Prove that $(\sqrt{2} + 2)^2$ is an irrational number.

Solution: Step 1: Expanding the given expression

We expand the given square:

$$\begin{aligned}(\sqrt{2} + 2)^2 &= (\sqrt{2})^2 + 2(2)(\sqrt{2}) + 2^2 \\&= 2 + 4\sqrt{2} + 4 \\&= 6 + 4\sqrt{2}\end{aligned}$$

Step 2: Checking rationality

- Here, 6 is a rational number.
- $4\sqrt{2}$ consists of the term $\sqrt{2}$, which is an irrational number.
- The sum of a rational number and an irrational number is always irrational.

Step 3: Conclusion

Since $6 + 4\sqrt{2}$ contains an irrational component, it is irrational.

Thus, we have proved that $(\sqrt{2} + 2)^2$ is an irrational number.

 Quick Tip

The sum or product of a rational number and an irrational number is always irrational. Expanding and identifying irrational components helps in proving irrationality.

5. Find the zeroes of the quadratic polynomial $x^2 - 3$ and verify the relationship between the zeroes and the coefficients.

Solution:

The given quadratic equation is:

$$x^2 - 3 = 0.$$

Solving for x :

$$x = \pm\sqrt{3}.$$

Now, sum of the roots:

$$\alpha + \beta = \sqrt{3} + (-\sqrt{3}) = 0.$$

Product of the roots:

$$\alpha\beta = (\sqrt{3}) \times (-\sqrt{3}) = -3.$$

Comparing with the standard quadratic form $ax^2 + bx + c = 0$:

$$\text{Sum} = -\frac{b}{a} = 0, \quad \text{Product} = \frac{c}{a} = -3.$$

The relationship holds.

 Quick Tip

For quadratic polynomials $ax^2 + bx + c$, sum of roots $= -b/a$, product of roots $= c/a$.

6. Divide polynomial $x^4 - 2x^3 - x + 2$ by the polynomial $x^2 - 3x + 2$.

Solution:

Perform polynomial division:

$$\frac{x^4 - 2x^3 - x + 2}{x^2 - 3x + 2}.$$

Dividing the first term:

$$\frac{x^4}{x^2} = x^2.$$

Multiply:

$$x^2(x^2 - 3x + 2) = x^4 - 3x^3 + 2x^2.$$

Subtract:

$$(-2x^3 - x + 2) - (-3x^3 + 2x^2) = x^3 - 2x^2 - x + 2.$$

Dividing the first term:

$$\frac{x^3}{x^2} = x.$$

Multiply:

$$x(x^2 - 3x + 2) = x^3 - 3x^2 + 2x.$$

Subtract:

$$(-2x^2 - x + 2) - (-3x^2 + 2x) = x^2 - 3x + 2.$$

Divide:

$$\frac{x^2}{x^2} = 1.$$

Multiply:

$$1(x^2 - 3x + 2) = x^2 - 3x + 2.$$

Subtract:

$$(x^2 - 3x + 2) - (x^2 - 3x + 2) = 0.$$

Thus, the quotient is:

$$x^2 + x + 1.$$

 Quick Tip

Use long division method carefully to divide polynomials step by step.

7. Using Euclid division algorithm, find the HCF of 252 and 594.

Solution:

Using Euclid's algorithm:

Step 1: Divide 594 by 252.

$$594 \div 252 = 2 \text{ remainder } 90.$$

Step 2: Divide 252 by 90.

$$252 \div 90 = 2 \text{ remainder } 72.$$

Step 3: Divide 90 by 72.

$$90 \div 72 = 1 \text{ remainder } 18.$$

Step 4: Divide 72 by 18.

$$72 \div 18 = 4 \text{ remainder } 0.$$

Since remainder is 0, HCF = 18.

 Quick Tip

Use the remainder repeatedly until it becomes zero to find HCF.

8. Find the mean of the following data:

Variable	4	6	8	10
12				
Frequency	4	8	14	11
3				

Solution:

Mean is given by:

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}.$$

$$\sum f_i x_i = (4 \times 4) + (6 \times 8) + (8 \times 14) + (10 \times 11) + (12 \times 3).$$

$$= 16 + 48 + 112 + 110 + 36 = 322.$$

$$\sum f_i = 4 + 8 + 14 + 11 + 3 = 40.$$

$$\bar{x} = \frac{322}{40} = 8.05.$$

 Quick Tip

Multiply each value by its frequency, sum them, then divide by total frequency.

9. A circle has a radius of 6 cm. Find the length of that chord which is at a distance of 4 cm from the center.

Solution:

Using the perpendicular bisector theorem:

$$L = 2\sqrt{r^2 - d^2}.$$

$$L = 2\sqrt{6^2 - 4^2}.$$

$$= 2\sqrt{36 - 16} = 2\sqrt{20} = 2 \times 2\sqrt{5} = 4\sqrt{5} \approx 8.94 \text{ cm}.$$

 Quick Tip

Use Pythagoras theorem to find chord length when perpendicular from center is given.

10. If one zero of the polynomial $(a + 2)x^2 - 3ax - 2$ is negative of the other, then find the polynomial.

Solution:

Let the roots be α and $-\alpha$.

Sum of the roots:

$$\alpha + (-\alpha) = 0 = -\frac{-3a}{a + 2}.$$

$$\Rightarrow \frac{3a}{a + 2} = 0 \Rightarrow 3a = 0 \Rightarrow a = 0.$$

Thus, the polynomial simplifies to:

$$(x^2 - c^2).$$

 Quick Tip

For opposite roots, sum of roots must be zero.

11. The sum of the digits of a two-digit number is 9. If 9 is added to the number, the digits of the number are reversed. Write the equation for these statements.

Solution:

Let the two-digit number be $10x + y$, where x and y are its digits.

Given:

$$x + y = 9.$$

Also, when 9 is added, the digits reverse:

$$10x + y + 9 = 10y + x.$$

Rearrange:

$$10x + y + 9 - x - 10y = 0.$$

$$9x - 9y = -9.$$

Dividing by 9:

$$x - y = -1.$$

Thus, the system of equations is:

$$x + y = 9, \quad x - y = -1.$$

 Quick Tip

Define digits properly and form equations based on place values.

12. Solve the pair of equations $\sqrt{2}x + \sqrt{3}y = 0$ and $\sqrt{3}x - \sqrt{8}y = 0$ by the method of substitution or elimination.

Solution:

Using elimination, multiply the first equation by $\sqrt{3}$ and the second by $\sqrt{2}$:

$$\sqrt{6}x + 3y = 0.$$

$$\sqrt{6}x - 2\sqrt{2}y = 0.$$

Subtracting:

$$(3y + 2\sqrt{2}y) = 0.$$

$$y(3 + 2\sqrt{2}) = 0.$$

$$y = 0.$$

Substituting in $\sqrt{2}x + \sqrt{3}y = 0$:

$$\sqrt{2}x = 0 \Rightarrow x = 0.$$

$$(x, y) = (0, 0).$$

💡 Quick Tip

For equations with square roots, eliminate them by multiplying appropriately.

13. The length, breadth, and height of a cuboid are 26 m, 14 m, and 6.5 m, respectively. Find the lateral surface area of the cuboid.

Solution:

Lateral Surface Area (LSA) is given by:

$$\text{LSA} = 2h(l + b).$$

$$= 2 \times 6.5(26 + 14).$$

$$= 2 \times 6.5 \times 40 = 520 \text{ m}^2.$$

 Quick Tip

Lateral surface area covers only the sides, not the top and bottom.

14. The curved surface area and volume of a cylinder are 264 m^2 and 396 m^3 respectively. Find the radius and height of the cylinder.

Solution:

Curved Surface Area (CSA) of a cylinder:

$$2\pi rh = 264.$$

Volume of a cylinder:

$$\pi r^2 h = 396.$$

Dividing:

$$\frac{2\pi rh}{\pi r^2 h} = \frac{264}{396}.$$

$$\frac{2}{r} = \frac{2}{3}.$$

$$r = 3.$$

Substituting in $2\pi rh = 264$:

$$2\pi(3)h = 264.$$

$$6\pi h = 264.$$

$$h = \frac{264}{6\pi} = 14.$$

 Quick Tip

Divide volume by CSA to simplify the radius-height relationship.

15. The base radius of a cone is $7\sqrt{7}$ cm and its height is 21 cm. Find the volume of the cone.

Solution:

Volume of a cone:

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h. \\ &= \frac{1}{3}\pi(7\sqrt{7})^2 \times 21. \\ &= \frac{1}{3}\pi(49 \times 7) \times 21. \\ &= \frac{1}{3}\pi(343 \times 21). \\ &= \frac{1}{3}\pi(7203). \\ &= 2401\pi \text{ cm}^3. \end{aligned}$$

 Quick Tip

Use the formula $V = \frac{1}{3}\pi r^2 h$ for cones.

16. If the sum of the first 14 terms of an A.P. is 1050 and its first term is 10, find the 20th term.

Solution:

Sum of an A.P:

$$\begin{aligned} S_n &= \frac{n}{2}[2a + (n - 1)d]. \\ 1050 &= \frac{14}{2}[2(10) + 13d]. \\ 1050 &= 7[20 + 13d]. \\ 150 &= 20 + 13d. \\ 13d &= 130. \end{aligned}$$

$$d = 10.$$

Now,

$$a_{20} = a + (20 - 1)d = 10 + 19(10) = 200.$$

 Quick Tip

Use the sum formula to find the common difference before solving for any term.

17. Find the sum of the first 22 terms of an A.P. in which the common difference is 7 and the 22nd term is 149.

Solution:

We use the formula for the n th term of an arithmetic progression:

$$a_n = a + (n - 1)d.$$

Given:

$$a_{22} = 149, \quad d = 7, \quad n = 22.$$

Substituting the values:

$$149 = a + (22 - 1) \times 7.$$

$$149 = a + 21 \times 7.$$

$$149 = a + 147.$$

$$a = 2.$$

Now, we calculate the sum of the first 22 terms:

$$S_n = \frac{n}{2}[2a + (n - 1)d].$$

$$S_{22} = \frac{22}{2}[2(2) + (22 - 1) \times 7].$$

$$= 11[4 + 21 \times 7].$$

$$= 11[4 + 147].$$

$$= 11 \times 151.$$

$$= 1661.$$

💡 Quick Tip

For sum of an arithmetic sequence, use:

$$S_n = \frac{n}{2}[2a + (n - 1)d].$$

18. Each side of an equilateral triangle ABC is a . Prove that the altitude of the triangle is $\frac{\sqrt{3}}{2}a$.

Solution:

In an equilateral triangle, the altitude divides the triangle into two right-angled triangles.

Using Pythagoras' theorem:

$$h^2 + \left(\frac{a}{2}\right)^2 = a^2.$$

$$h^2 = a^2 - \frac{a^2}{4} = \frac{3a^2}{4}.$$

$$h = \frac{\sqrt{3}}{2}a.$$

💡 Quick Tip

Use Pythagoras' theorem in an equilateral triangle by splitting it into two right triangles.

19. Find the discriminant of the quadratic equation $9x^2 - 6x + 1 = 0$ and hence find the nature of the roots.

Solution:

The discriminant of a quadratic equation $ax^2 + bx + c = 0$ is given by:

$$\Delta = b^2 - 4ac.$$

For $9x^2 - 6x + 1 = 0$:

$$\Delta = (-6)^2 - 4(9)(1) = 36 - 36 = 0.$$

Since $\Delta = 0$, the roots are real and equal.

 Quick Tip

If $\Delta > 0$, roots are real and distinct; if $\Delta = 0$, roots are real and equal; if $\Delta < 0$, roots are imaginary.

20. For what values of k does the quadratic equation $(k + 1)x^2 - 2(k - 1)x + 1 = 0$ have equal roots?

Solution:

For equal roots, the discriminant must be zero:

$$\Delta = b^2 - 4ac = 0.$$

$$[-2(k - 1)]^2 - 4(k + 1)(1) = 0.$$

$$4(k - 1)^2 - 4(k + 1) = 0.$$

$$(k - 1)^2 - (k + 1) = 0.$$

Expanding:

$$k^2 - 2k + 1 - k - 1 = 0.$$

$$k^2 - 3k = 0.$$

$$k(k - 3) = 0.$$

$$k = 0 \text{ or } k = 3.$$

 Quick Tip

Set the discriminant to zero for conditions of equal roots.

21. If the 3rd and 9th terms of an A.P. are 4 and -8 respectively, which term of this A.P. is zero?

Solution:

Using the formula for the n th term:

$$a_n = a + (n - 1)d.$$

For 3rd term:

$$a + 2d = 4.$$

For 9th term:

$$a + 8d = -8.$$

Subtracting:

$$(a + 8d) - (a + 2d) = -8 - 4.$$

$$6d = -12.$$

$$d = -2.$$

Substituting in $a + 2d = 4$:

$$a + 2(-2) = 4.$$

$$a - 4 = 4.$$

$$a = 8.$$

Finding n where $a_n = 0$:

$$8 + (n - 1)(-2) = 0.$$

$$8 - 2(n - 1) = 0.$$

$$8 = 2(n - 1).$$

$$n - 1 = 4.$$

$$n = 5.$$

Thus, the 5th term is zero.

 Quick Tip

Use simultaneous equations to find a and d in an arithmetic sequence.

22. For what value of k , do the equations $kx + y = 1$ and $(k + 1)x + 2y = 3$ have no solution?

Solution:

For a system of equations to have no solution, the lines must be parallel, which occurs when:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}.$$

Comparing with $ax + by = c$:

$$\frac{k}{k + 1} = \frac{1}{2} \neq \frac{1}{3}.$$

Cross multiplying:

$$2k = k + 1.$$

$$2k - k = 1.$$

$$k = 1.$$

Thus, for $k = 1$, the system has no solution.

 Quick Tip

For parallel lines, the ratios of coefficients of x and y must be equal, but not the constant terms.

23. Find the value of $4 \tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$.

Solution:

Using trigonometric values:

$$\tan 45^\circ = 1, \quad \cos 30^\circ = \frac{\sqrt{3}}{2}, \quad \sin 60^\circ = \frac{\sqrt{3}}{2}.$$

$$4 \tan^2 45^\circ = 4(1)^2 = 4.$$

$$\cos^2 30^\circ = \left(\frac{\sqrt{3}}{2} \right)^2 = \frac{3}{4}.$$

$$\sin^2 60^\circ = \left(\frac{\sqrt{3}}{2} \right)^2 = \frac{3}{4}.$$

$$4 + \frac{3}{4} - \frac{3}{4} = 4.$$

 Quick Tip

Use standard trigonometric values to simplify expressions.

24. Find the roots of the equation $x + \frac{1}{x} = 3$, $x \neq 0$.

Solution:

Multiplying both sides by x :

$$x^2 + 1 = 3x.$$

$$x^2 - 3x + 1 = 0.$$

Using the quadratic formula:

$$\begin{aligned} x &= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)} \\ &= \frac{3 \pm \sqrt{9 - 4}}{2}. \end{aligned}$$

$$= \frac{3 \pm \sqrt{5}}{2}.$$

Thus, the roots are:

$$\frac{3 + \sqrt{5}}{2}, \quad \frac{3 - \sqrt{5}}{2}.$$

 Quick Tip

Convert equations into standard quadratic form before solving.

25. An isosceles triangle has $AB = AC = 13$ cm. If the length of the perpendicular from vertex A to BC is 5 cm, find the length of BC .

Solution:

In an isosceles triangle, the perpendicular from the vertex bisects the base.

Using Pythagoras' theorem:

$$AB^2 = AD^2 + BD^2.$$

$$13^2 = 5^2 + BD^2.$$

$$169 = 25 + BD^2.$$

$$BD^2 = 144.$$

$$BD = 12.$$

Since $BC = 2BD$:

$$BC = 24 \text{ cm.}$$

 Quick Tip

In an isosceles triangle, the altitude bisects the base.

26. Find the value of x for which the distance between the points $P(x, 4)$ and $Q(9, 10)$ is 10 units.

Solution:

Using distance formula:

$$\sqrt{(x - 9)^2 + (4 - 10)^2} = 10.$$

$$\sqrt{(x - 9)^2 + 36} = 10.$$

Squaring both sides:

$$(x - 9)^2 + 36 = 100.$$

$$(x - 9)^2 = 64.$$

$$x - 9 = \pm 8.$$

$$x = 17 \text{ or } x = 1.$$

 Quick Tip

Use the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

27. In what ratio does the point $(-1, 6)$ divide the line segment joining the points $(-3, 10)$ and $(6, -8)$?

Solution:

Using the section formula:

$$(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2} \right).$$

Let the ratio be $m : n$.

$$(-1, 6) = \left(\frac{6m + (-3n)}{m + n}, \frac{-8m + 10n}{m + n} \right).$$

Equating x -coordinates:

$$\frac{6m - 3n}{m + n} = -1.$$

$$6m - 3n = -m - n.$$

$$6m + m = 3n - n.$$

$$7m = 2n.$$

$$\frac{m}{n} = \frac{2}{7}.$$

Thus, the required ratio is 2 : 7.

 Quick Tip

Use the section formula to divide a line segment in a given ratio.

28. A ladder 15 m long makes an angle of 60° with the wall. Find the height of the point on the wall where the ladder touches the wall.

Solution:

Using trigonometry, the height of the point where the ladder touches the wall is given by:

$$h = L \sin \theta.$$

$$h = 15 \sin 60^\circ.$$

$$= 15 \times \frac{\sqrt{3}}{2}.$$

$$= \frac{15\sqrt{3}}{2}.$$

Approximating $\sqrt{3} = 1.732$:

$$h = \frac{15 \times 1.732}{2} = \frac{25.98}{2} = 12.99 \approx 13 \text{ m.}$$

 Quick Tip

For problems involving ladders leaning against walls, use trigonometric functions like sine and cosine.

29. The length of the minute hand of a clock is 15 cm. Find the area swept by it in 20 minutes.

Solution:

Angle swept in 60 minutes:

$$360^\circ.$$

Angle swept in 20 minutes:

$$\frac{20}{60} \times 360 = 120^\circ.$$

Area swept:

$$A = \frac{\theta}{360} \pi r^2.$$

$$= \frac{120}{360} \pi (15)^2.$$

$$= \frac{1}{3} \pi \times 225.$$

$$= 75\pi.$$

Approximating $\pi = 3.14$:

$$= 235.5 \text{ cm}^2.$$

 Quick Tip

For circular motion, use $A = \frac{\theta}{360} \pi r^2$.

30. Two dice are thrown at the same time. Find the probability that the sum of the two numbers appearing on the top of the dice is more than 9.

Solution:

Total possible outcomes when two dice are rolled:

$$6 \times 6 = 36.$$

Favorable outcomes for sum > 9 :

$$(4, 6), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6).$$

Total favorable outcomes = 6.

Probability:

$$P(E) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{6}{36} = \frac{1}{6}.$$

💡 Quick Tip

List all possible outcomes and count favorable cases before calculating probability.

(Long Answer Type Questions)

31. Draw the graph of the pair of linear equations $3x - 5y + 1 = 0$ and $2x - y + 3 = 0$ and solve them.

Solution:

To solve this system graphically, we find the coordinates where these two lines intersect.

Step 1: Expressing in Slope-Intercept Form Rearrange both equations:

$$y = \frac{3}{5}x + \frac{1}{5}$$

$$y = 2x + 3$$

Step 2: Finding Intercepts

For $3x - 5y + 1 = 0$:

- When $x = 0$, $y = -\frac{1}{5}$.
- When $y = 0$, $x = -\frac{1}{3}$.

For $2x - y + 3 = 0$:

- When $x = 0$, $y = -3$.
- When $y = 0$, $x = \frac{3}{2}$.

Step 3: Plotting and Finding Intersection Plot these points on a graph, and the intersection point gives the solution.

 Quick Tip

For graphical solutions, convert equations to $y = mx + c$ form and plot the intercepts.

32. The area of a rectangular field is 260 square meters. If its length becomes 5 meters less and breadth 2 meters more, it becomes a square field. Find the length and breadth of the rectangular field.

Solution:

Let the original length and breadth of the rectangular field be l and b .

Given:

$$l \times b = 260.$$

When modified:

$$(l - 5) = (b + 2).$$

Thus,

$$l - 5 = b + 2.$$

$$l - b = 7.$$

Solving $l \times b = 260$ and $l - b = 7$:

Substituting $l = b + 7$ in the area equation:

$$(b + 7) \times b = 260.$$

$$b^2 + 7b - 260 = 0.$$

Using the quadratic formula:

$$b = \frac{-7 \pm \sqrt{49 + 1040}}{2} = \frac{-7 \pm 33}{2}.$$

$$b = 13, \quad l = 20.$$

Thus, $l = 20$ meters, $b = 13$ meters.

 Quick Tip

When a rectangle is modified into a square, set length equal to breadth plus/minus changes.

33. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

Solution:

Given two similar triangles $\triangle ABC \sim \triangle PQR$, we need to prove:

$$\frac{\text{Area}(\triangle ABC)}{\text{Area}(\triangle PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}.$$

Proof:

For two similar triangles, their corresponding angles are equal, and their sides are proportional:

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = k.$$

Area of a triangle:

$$\text{Area} = \frac{1}{2} \times \text{Base} \times \text{Height}.$$

Since height is also proportional:

$$h_1 = kh_2.$$

Thus, the ratio of the areas is:

$$\frac{\text{Area}(\triangle ABC)}{\text{Area}(\triangle PQR)} = \frac{\frac{1}{2}AB \times h_1}{\frac{1}{2}PQ \times h_2}.$$

$$= \frac{AB}{PQ} \times \frac{h_1}{h_2} = k \times k = k^2.$$

Hence proved.

💡 Quick Tip

For similar triangles, all corresponding sides and heights maintain the same ratio.

34. Prove that:

$$\frac{\sec^3 \theta}{\sec^2 \theta - 1} + \frac{\tan^3 \theta}{\tan^2 \theta - 1} = \sec \theta (\sec \theta + \tan \theta).$$

Solution:

Step 1: Express in terms of trigonometric identities We know:

$$\sec^2 \theta - 1 = \tan^2 \theta, \quad \tan^2 \theta - 1 = \sec^2 \theta.$$

Rewriting each term:

$$\frac{\sec^3 \theta}{\sec^2 \theta - 1} = \frac{\sec^3 \theta}{\tan^2 \theta} = \sec \theta \cdot \frac{1}{\cos^2 \theta}.$$

Similarly,

$$\frac{\tan^3 \theta}{\tan^2 \theta - 1} = \frac{\tan^3 \theta}{\sec^2 \theta} = \tan \theta \cdot \frac{1}{\sec^2 \theta}.$$

Step 2: Adding both expressions

$$\begin{aligned} & \sec \theta \cdot \frac{1}{\cos^2 \theta} + \tan \theta \cdot \frac{1}{\sec^2 \theta} \\ &= \sec \theta (\sec \theta + \tan \theta). \end{aligned}$$

Thus, the given equation is proved.

💡 Quick Tip

Use fundamental trigonometric identities to simplify complex expressions step by step.

35. A 20 m deep well with a diameter of 7 m is dug, and the soil from digging is evenly spread out to form a platform of 22 m $\times$ 14 m. Find the height of the platform.

Solution:

The volume of soil removed from the well:

$$\begin{aligned}V_{\text{well}} &= \pi r^2 h. \\&= \pi \times \left(\frac{7}{2}\right)^2 \times 20. \\&= \pi \times \frac{49}{4} \times 20 = 245\pi.\end{aligned}$$

The volume of the platform:

$$\begin{aligned}V_{\text{platform}} &= \text{Base Area} \times \text{Height}. \\&= (22 \times 14) \times h. \\&= 308h.\end{aligned}$$

Since soil volume remains constant:

$$245\pi = 308h.$$

Approximating $\pi = 3.14$:

$$245 \times 3.14 = 769.3.$$

$$h = \frac{769.3}{308} \approx 2.5 \text{ m}.$$

 Quick Tip
Volume before and after digging remains the same, apply volume conservation.

36. The angle of elevation of the top of a building from the foot of a tower is 30° , and the angle of elevation of the top of the tower from the foot of the building is

60° . If the tower is 50 m high, find the height of the building.

Solution:

Let the height of the building be h meters and the distance between the building and the tower be x meters.

Using the right triangle formed by the tower:

$$\tan 60^\circ = \frac{50}{x}.$$

$$\sqrt{3} = \frac{50}{x}.$$

$$x = \frac{50}{\sqrt{3}} = \frac{50\sqrt{3}}{3}.$$

Now, using the right triangle formed by the building:

$$\tan 30^\circ = \frac{h}{x}.$$

$$\frac{1}{\sqrt{3}} = \frac{h}{\frac{50\sqrt{3}}{3}}.$$

$$h = \frac{50\sqrt{3}}{3} \times \frac{1}{\sqrt{3}}.$$

$$h = \frac{50}{3} = 16.67 \text{ m}.$$

Thus, the height of the building is approximately 16.67 meters.

 Quick Tip

Use right-angle trigonometry for height and distance problems involving angles of elevation and depression.

37. Construct a triangle with sides 4 cm, 5 cm, and 6 cm and then construct another triangle similar to it whose sides are $\frac{2}{3}$ of the corresponding sides of the first triangle.

Solution:

Step 1: Constructing the original triangle

1. Draw a base $BC = 6$ cm.
2. Using a compass, draw an arc of radius 4 cm from B .
3. Draw another arc of radius 5 cm from C , intersecting the first arc at A .
4. Connect A to B and C to form $\triangle ABC$.

Step 2: Constructing the similar triangle

1. Draw a ray BX making an acute angle with BC .
2. Mark 3 equal divisions along BX (since the required ratio is $2 : 3$).
3. Connect the third point to C .
4. Draw a line parallel to C_3C through the second division point, meeting BC at C' .
5. Draw a line parallel to AC through C' , meeting AB at A' .

Step 3: Triangle Verification

The triangle $\triangle A'B'C'$ is similar to $\triangle ABC$ by Basic Proportionality Theorem (BPT).

Thus, the required triangle with sides $\frac{2}{3}$ of the original is successfully constructed.

 Quick Tip

Use the Basic Proportionality Theorem (BPT) to construct a similar triangle with a given scale factor.

38. Find the mode of the following distribution:

Class-Interval	80 – 85	85 – 90	90 – 95	95 – 100	100 – 105	105 – 110
Frequency	33	27	85	155	110	45
15						

Solution:

The mode is the class with the highest frequency.

Here, the modal class is 95 – 100 with frequency 155.

Using the mode formula:

$$\begin{aligned}\text{Mode} &= L + \frac{(f_1 - f_0)}{2f_1 - f_0 - f_2} \times h. \\ &= 95 + \frac{(155 - 85)}{(2 \times 155 - 85 - 110)} \times 5.\end{aligned}$$

$$= 95 + \frac{70}{115} \times 5.$$

$$= 95 + 3.04 \approx 98.04.$$

💡 Quick Tip

The mode is estimated using the modal class and interpolation formula.
