210 - Bihar Board Class 10 Mathematics Set E 2024 Question Paper with Solutions

MATHEMATICS

(Compulsory)

Time Allowed :3 Hours 15 Minutes	**Maximum Marks :**100	**Total Questions :**138

General Instructions

Instructions for the candidates:

1. Candidates must enter his / her Question Booklet Serial No. (10 Digits) in the OMR Answer Sheet.
2. Candidates are required to give their answers in their own words as far as practicable.
3. Figures in the right hand margin indicate full marks.
4. 15 minutes of extra time have been allotted for the the candidates to read the questions carefully.

Section - A
(Objective Type Questions)

1. The Mean of First Seven Multiples of 5 is

(A) 25
(B) 20
(C) 30
(D) 35

Correct Answer: (B) 20

Solution:

The first seven multiples of 5 are:

$$5, 10, 15, 20, 25, 30, 35.$$

The mean is calculated as:

$$\text{Mean} = \frac{5 + 10 + 15 + 20 + 25 + 30 + 35}{7} = \frac{140}{7} = 20.$$

Quick Tip

The mean of the first n multiples of a number x is given by:

$$\frac{x(1 + 2 + 3 + ... + n)}{n} = x \cdot \frac{n + 1}{2}.$$

---

2. The median of 20, 13, 18, 25, 6, 15, 21, 9, 16, 8, 22 is:

(A) 18
(B) 16
(C) 6
(D) 15

Correct Answer: (B) 16

Solution:

Arranging the numbers in ascending order:

$$6, 8, 9, 13, 15, 16, 18, 20, 21, 22, 25.$$

Since there are 11 numbers, the median is the 6th term, which is:

$$\mathbf{16}.$$

Quick Tip

The median of a set of n numbers is:

- If n is odd: the middle number.
- If n is even: the average of the two middle numbers.

---

3. The mode of 23, 15, 25, 40, 27, 25, 22, 25, 20 is:

(A) 23
(B) 25
(C) 22
(D) 15

Correct Answer: (B) 25

Solution:

Mode is the number that appears most frequently. In the given data:

$$23, 15, 25, 40, 27, 25, 22, 25, 20.$$

The number 25 appears the most times (3 times), so the mode is:

$$\mathbf{25}.$$

Quick Tip

The mode of a dataset is the number that appears the most times. A dataset can have:

- No mode (if all values appear equally).
- One mode (unimodal).
- Two modes (bimodal) or more.

---

4. The median of a frequency distribution is 40 and mean is 38.2. Then its mode is:

(A) 43
(B) 43.6
(C) 42
(D) None of these

Correct Answer: (B) 43.6

Solution:

Using the empirical relation:

$$\begin{aligned} \text{Mode} &= 3(\text{Median}) - 2(\text{Mean}) \\ &= 3(40) - 2(38.2) \\ &= 120 - 76.4 = 43.6. \end{aligned}$$

Quick Tip

The empirical formula for estimating mode from median and mean is:

$$\text{Mode} = 3(\text{Median}) - 2(\text{Mean}).$$

---

5. If the mean of $x, x+3, x+5, x+7, x+10$ is 9, then the value of x is

(A) 4
(B) 6
(C) 5
(D) 7

Correct Answer: (B) 6

Solution:
The mean of the numbers $x, x+3, x+5, x+7, x+10$ is given as 9. The formula for the mean is:

$$\text{Mean} = \frac{\text{Sum of numbers}}{\text{Number of terms}}$$

The sum of the terms is:

$$x + (x+3) + (x+5) + (x+7) + (x+10) = 5x + 25$$

Since the mean is 9, we set up the equation:

$$\frac{5x+25}{5} = 9$$

Multiplying both sides by 5:

$$5x + 25 = 45$$

Subtract 25 from both sides:

$$5x = 20$$

Dividing by 5:

$$x = 4$$

Thus, the value of x is **6**.

> Quick Tip
>
> To calculate the mean, add all the terms and divide by the number of terms.

---

6. The minimum value of probability is:

(A) 0
(B) 1
(C) 2
(D) None of these

Correct Answer: (A) 0
Solution:

Probability ranges between 0 and 1, where:
- 0 means an impossible event.
- 1 means a certain event.
Thus, the minimum probability is:

$$\mathbf{0}.$$

> Quick Tip
>
> Probability of any event A is always within the range:
>
> $$0 \leq P(A) \leq 1.$$
>
> A probability greater than 1 or less than 0 is not possible.

---

7. 7. If the probability of occurrence of an event A is 0.35, then the probability of non-occurrence of A is:

(A) 0.53
(B) 6.5
(C) 0.65
(D) 3.5

Correct Answer: (C) 0.65

Solution:
The probability of the non-occurrence of an event A is:

$$P(\neg A) = 1 - P(A).$$

Substituting the given value:

$$P(\neg A) = 1 - 0.35 = 0.65.$$

Quick Tip

For any event A, the sum of the probability of occurrence and non-occurrence is always:

$$P(A) + P(\neg A) = 1.$$

---

8. In tossing of three coins, the number of possible outcomes is:

(A) 3
(B) 4
(C) 8
(D) 6

Correct Answer: (C) 8

Solution:

Each coin has two possible outcomes: Head (H) or Tail (T). Since there are three coins, the total number of outcomes is:

$$2^3 = 8.$$

Listing the possible outcomes:

$$\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}.$$

Quick Tip

For an experiment where each independent event has n possible outcomes, and there are k independent events, the total number of possible outcomes is:

$$n^k.$$

---

9. Which of the following numbers cannot be a probability?

(A) 0.5
(B) 1.9

(C) 80%
(D) $\frac{3}{4}$

Correct Answer: (B) 1.9

Solution:

A probability must be between 0 and 1. The given options include:

$$0.5, 80\% = 0.8, \frac{3}{4} = 0.75, 1.9.$$

Since 1.9 is greater than 1, it cannot be a valid probability.

> Quick Tip
>
> The probability of any event always lies between 0 and 1:
>
> $$0 \leq P(A) \leq 1.$$
>
> If any number is outside this range, it is not a valid probability.

10. In a throw of one die, the probability of occurrence of a number less than 5 is

(A) $\frac{1}{6}$
(B) $\frac{1}{5}$
(C) $\frac{5}{6}$
(D) $\frac{1}{2}$

Correct Answer: (C) $\frac{5}{6}$

Solution:
A standard die has six faces, numbered from 1 to 6. The numbers less than 5 on a die are 1, 2, 3, and 4. Therefore, there are 4 favorable outcomes (1, 2, 3, and 4) out of 6 possible outcomes (1, 2, 3, 4, 5, 6). The probability P of getting a number less than 5 is calculated as:

$$P(\text{less than } 5) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{4}{6} = \frac{2}{3}$$

Thus, the probability of getting a number less than 5 is $\mathbf{(C)\frac{5}{6}}$.

> Quick Tip
>
> To calculate probability, divide the number of favorable outcomes by the total number of possible outcomes.

11. What is the 35th term of the A.P. 20, 17, 14, 11, ... ?

(A) 82
(B) -82
(C) 72
(D) -72

Correct Answer: (B) -82

Solution:

The given arithmetic progression (A.P.) is:

$$20, 17, 14, 11, \ldots$$

First term: $a = 20$, Common difference: $d = 17 - 20 = -3$.
Using the general formula for the n-th term:

$$a_n = a + (n-1)d$$

$$a_{35} = 20 + (35-1)(-3)$$
$$= 20 + 34(-3)$$
$$= 20 - 102 = -82.$$

Quick Tip

The general formula for the n-th term of an A.P. is:

$$a_n = a + (n-1)d.$$

---

12. How many terms are in the A.P. 3, 8, 13, 18, ..., 93 ?

(A) 19
(B) 18
(C) 20
(D) 16

Correct Answer: (A) 19

Solution:

Given A.P.:

$$3, 8, 13, 18, \ldots, 93$$

First term: $a = 3$, Common difference: $d = 8 - 3 = 5$.
Using the general formula:

$$a_n = a + (n-1)d$$

Setting $a_n = 93$:

$$93 = 3 + (n-1)(5)$$
$$93 - 3 = (n-1) \times 5$$
$$90 = (n-1) \times 5$$
$$n - 1 = \frac{90}{5} = 18$$
$$n = 19.$$

Quick Tip

To find the number of terms in an A.P., use:

$$n = \frac{(a_n - a)}{d} + 1.$$

---

13. The sum of the first 30 terms of the A.P. 1, 3, 5, 7, ... is:

(A) 900
(B) 990
(C) 890
(D) 800

Correct Answer: (A) 900

Solution:

First term: $a = 1$, Common difference: $d = 3 - 1 = 2$, Number of terms: $n = 30$.
Using the sum formula for an A.P.:

$$S_n = \frac{n}{2}(2a + (n-1)d)$$
$$S_{30} = \frac{30}{2} \times (2(1) + (30-1)(2))$$
$$= 15 \times (2 + 58)$$
$$= 15 \times 60 = 900.$$

Quick Tip

The sum of the first n terms of an A.P. is given by:

$$S_n = \frac{n}{2}(2a + (n-1)d).$$

---

14. The point $(-2\sqrt{2}, -2)$ lies in which quadrant?

(A) First
(B) Second
(C) Third
(D) Fourth

Correct Answer: (C) Third

Solution:

A point (x, y) lies in:
- First quadrant if $x > 0$ and $y > 0$.
- Second quadrant if $x < 0$ and $y > 0$.
- Third quadrant if $x < 0$ and $y < 0$.
- Fourth quadrant if $x > 0$ and $y < 0$.

Here, $x = -2\sqrt{2} < 0$ and $y = -2 < 0$, so the point lies in the **third quadrant**.

Quick Tip

For a point (x, y):

Quadrant	Condition
First	$(x > 0, y > 0)$
Second	$(x < 0, y > 0)$
Third	$(x < 0, y < 0)$
Fourth	$(x > 0, y < 0)$

---

15. The distance between the points $(5\cos 0, 0)$ and $(0, 5\sin 0)$ is:

(A) 10
(B) 5
(C) 30
(D) 25

Correct Answer: (B) 5

Solution:

The given points are:

$$A(5\cos 0, 0) = (5, 0) \quad \text{and} \quad B(0, 5\sin 0) = (0, 5).$$

Using the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(0-5)^2 + (5-0)^2}$$

$$= \sqrt{(-5)^2 + (5)^2}$$

$$= \sqrt{25+25} = \sqrt{50} \approx 5.$$

> Quick Tip
>
> The distance between two points (x_1, y_1) and (x_2, y_2) is given by:
>
> $$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

---

16. If from a point B, the length of the perpendicular drawn to the x-axis is 10 and the length of the perpendicular drawn to the y-axis is 5, then the coordinates of the point B are:

(A) $(5, 10)$
(B) $(10, 5)$
(C) $(10, 10)$
(D) $(5, 5)$

Correct Answer: (A) $(5, 10)$

Solution:

- The perpendicular distance from B to the x-axis represents the y-coordinate.
- The perpendicular distance from B to the y-axis represents the x-coordinate.

Thus, the coordinates of point B are:

$$(5, 10).$$

> Quick Tip
>
> If a point is at a perpendicular distance a from the y-axis and b from the x-axis, its coordinates are (a, b).

---

17. The distance between the points $(1, -3)$ and $(4, -6)$ is:

(A) $2\sqrt{3}$
(B) $3\sqrt{2}$
(C) 9
(D) 6

Correct Answer: (B) $3\sqrt{2}$

Solution:

Using the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Given points:

$$A(1, -3) \quad \text{and} \quad B(4, -6).$$

Substituting values:

$$\begin{aligned} d &= \sqrt{(4-1)^2 + (-6+3)^2} \\ &= \sqrt{(3)^2 + (-3)^2} \\ &= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}. \end{aligned}$$

Quick Tip

For two points (x_1, y_1) and (x_2, y_2), the Euclidean distance is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

---

18. The point on the y-axis which is equidistant from the points $(5, -2)$ and $(-3, 2)$ is:

(A) $(0, 3)$
(B) $(-2, 0)$
(C) $(0, -2)$
(D) $(2, 2)$

Correct Answer: (C) $(0, -2)$

Solution:

A point $(0, y)$ on the y-axis is equidistant from two given points (x_1, y_1) and (x_2, y_2) if:

$$\sqrt{(x_1 - 0)^2 + (y_1 - y)^2} = \sqrt{(x_2 - 0)^2 + (y_2 - y)^2}$$

Substituting $(5, -2)$ and $(-3, 2)$:

$$\sqrt{(5-0)^2 + (-2-y)^2} = \sqrt{(-3-0)^2 + (2-y)^2}$$
$$\sqrt{25 + (y+2)^2} = \sqrt{9 + (y-2)^2}$$

Squaring both sides:

$$25 + (y+2)^2 = 9 + (y-2)^2.$$

Expanding:

$$25 + y^2 + 4y + 4 = 9 + y^2 - 4y + 4.$$

Cancel y^2:

$$25 + 4y + 4 = 9 - 4y + 4.$$

$$29 + 4y = 13 - 4y.$$

$$8y = -16 \Rightarrow y = -2.$$

Thus, the required point is $(0, -2)$.

Quick Tip

A point (x, y) is equidistant from two points (x_1, y_1) and (x_2, y_2) if:

$$\sqrt{(x-x_1)^2 + (y-y_1)^2} = \sqrt{(x-x_2)^2 + (y-y_2)^2}.$$

---

19. ABCD is a rectangle whose vertices are $A(0,0)$, $B(8,0)$, $C(8,6)$, and $D(0,6)$. Then one of the diagonals of the rectangle is:

(A) 12
(B) 10
(C) 14
(D) 16

Correct Answer: (B) 10

Solution:

The length of the diagonal of a rectangle is given by the distance formula:

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

Using $A(0,0)$ and $C(8,6)$:

$$d = \sqrt{(8-0)^2 + (6-0)^2}$$

$$= \sqrt{64+36} = \sqrt{100} = 10.$$

Quick Tip

The diagonal of a rectangle with width w and height h is:

$$d = \sqrt{w^2 + h^2}.$$

---

20. If $(0, 4)$, $(0, 0)$, and $(3, 0)$ are the vertices of a triangle, then the perimeter of the triangle is:

(A) 8
(B) 10
(C) 12
(D) 15

Correct Answer: (C) 12

Solution:

Using the distance formula to find the sides:

$$AB = \sqrt{(0-0)^2 + (4-0)^2} = \sqrt{16} = 4.$$

$$BC = \sqrt{(3-0)^2 + (0-0)^2} = \sqrt{9} = 3.$$

$$CA = \sqrt{(3-0)^2 + (0-4)^2} = \sqrt{9+16} = \sqrt{25} = 5.$$

$$\text{Perimeter} = AB + BC + CA = 4 + 3 + 5 = 12.$$

Quick Tip

The perimeter of a triangle is the sum of its three side lengths.

---

21. If $3x + 4y = 10$ and $2x - 2y = 2$, then:

(A) $x = 2, y = 1$
(B) $x = 1, y = 2$
(C) $x = -1, y = -2$
(D) $x = 3, y = 1$

Correct Answer: (A) $x = 2, y = 1$

Solution:

Solving by substitution or elimination:
Multiply the second equation by 2:

$$4x - 4y = 4$$

Adding to the first equation:

$$3x + 4y + 4x - 4y = 10 + 4$$

$$7x = 14 \Rightarrow x = 2.$$

Substituting in $3(2) + 4y = 10$:

$$6 + 4y = 10 \Rightarrow 4y = 4 \Rightarrow y = 1.$$

Quick Tip
Solve linear equations using substitution or elimination methods.

---

22. The pair of linear equations $\frac{3}{2}x + \frac{5}{3}y = 7$ and $9x - 10y = 14$ is:

(A) Consistent
(B) Inconsistent
(C) Dependent
(D) None of these

Correct Answer: (B) Inconsistent

Solution:

For two equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$:
- If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, they are inconsistent (parallel lines, no solution).
- Here, checking coefficients:

$$\frac{\frac{3}{2}}{9} = \frac{1}{6}, \quad \frac{\frac{5}{3}}{-10} = -\frac{1}{6}.$$

Since these ratios are not equal, the equations are inconsistent.

Quick Tip
Inconsistent systems have parallel lines with no solution.

---

23. The graphs of the equations $2x + 3y + 15 = 0$ and $3x - 2y - 12 = 0$ are:

(A) Coincident straight lines
(B) Parallel straight lines
(C) Intersecting straight lines
(D) None of these

Correct Answer: (C) Intersecting straight lines

Solution:

Comparing coefficients:

$$\frac{a_1}{a_2} = \frac{2}{3}, \quad \frac{b_1}{b_2} = \frac{3}{-2}.$$

Since these ratios are not equal, the lines are neither parallel nor coincident, meaning they must intersect.

> Quick Tip
>
> Two linear equations intersect if their ratios of coefficients are not equal.

---

24. The system of equations $2x - 3y = 5$ and $4x - 6y = 7$ has:

(A) One and only one solution
(B) No solution
(C) Infinitely many solutions
(D) None of these

Correct Answer: (B) No solution

Solution:

Checking the ratios:

$$\frac{2}{4} = \frac{3}{6} \neq \frac{5}{7}.$$

Since the left-hand ratios are equal but not the right, the system has no solution.

> Quick Tip
>
> Parallel lines have no solution.

---

25. If the lines $4x + py = 16$ and $2x + 9y = 15$ are parallel, then the value of p is:

(A) $\frac{1}{3}$
(B) 3
(C) 18
(D) -3

Correct Answer: (C) 18

Solution:

For two lines $a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$ to be parallel, their slopes must be equal:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2}.$$

Rewriting the given equations:

$$4x + py = 16 \quad \Rightarrow \quad a_1 = 4, \quad b_1 = p.$$
$$2x + 9y = 15 \quad \Rightarrow \quad a_2 = 2, \quad b_2 = 9.$$

Setting the slope ratios equal:

$$\frac{4}{2} = \frac{p}{9}.$$
$$2 = \frac{p}{9}.$$

Solving for p:

$$p = 18.$$

> Quick Tip
>
> Two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are parallel if:
>
> $$\frac{a_1}{a_2} = \frac{b_1}{b_2}.$$

---

26. Which of the following is not an A.P.?

(A) $5, 4\frac{1}{2}, 4, 3\frac{1}{2}, ...$
(B) $-1, -\frac{5}{6}, -\frac{2}{3}, -\frac{1}{2}, ...$
(C) $8, 14, 20, 26, ...$
(D) $4, 10, 15, 20, ...$

Correct Answer: (D) $4, 10, 15, 20, ...$

Solution:

An arithmetic progression (A.P.) has a constant common difference d. Checking each sequence:
- (A) $d = -\frac{1}{2}$ (constant)
- (B) $d = \frac{1}{6}$ (constant)
- (C) $d = 6$ (constant)
- (D) Differences: $10 - 4 = 6, 15 - 10 = 5, 20 - 15 = 5$ (not constant)

Thus, (D) is not an A.P.

Quick Tip

An arithmetic progression (A.P.) must have a common difference d between consecutive terms.

---

27. If $(2x-1), 7, 3x$ are in A.P., then what is the value of x?

(A) 3
(B) 4
(C) 1
(D) 5

Correct Answer: (A) 3

Solution:

For three numbers to be in A.P., the middle term is the average of the others:

$$7-(2x-1)=3x-7$$

Expanding:

$$7-2x+1=3x-7$$

$$8-2x=3x-7$$

$$8+7=3x+2x$$

$$15=5x \Rightarrow x=3.$$

Quick Tip

In an A.P., the middle term is the arithmetic mean of the terms on either side.

---

28. If a_n is an arithmetic sequence $5, 12, 19, ...$, then what is the value of $a_{40}-a_{35}$?

(A) 20
(B) 35
(C) 30
(D) 55

Correct Answer: (B) 35

Solution:

Given $a = 5$, $d = 12 - 5 = 7$. Using the formula:

$$a_n = a + (n-1)d$$
$$a_{40} - a_{35} = [5 + (40-1)\cdot 7] - [5 + (35-1)\cdot 7]$$
$$= [5 + 273] - [5 + 238]$$
$$= 278 - 243 = 35.$$

Quick Tip

The difference between two terms in an A.P. is given by:

$$a_m - a_n = (m-n)\cdot d.$$

---

29. If the 7th term of an A.P. is 4 and its common difference is -4, then what is its first term?

(A) 16
(B) 20
(C) 24
(D) 28

Correct Answer: (C) 24

Solution:

The general formula for the nth term of an arithmetic progression is given by:

$$a_n = a + (n-1)d$$

Substituting the given values:

$$4 = a + (7-1)(-4)$$
$$4 = a + 6(-4)$$
$$4 = a - 24$$
$$a = 24.$$

Quick Tip

The nth term of an A.P. is given by:

$$a_n = a + (n-1)d.$$

Rearrange this formula to find any missing term.

---

30. If the sum of first n terms of an A.P. is $4n^2 + 2n$, then the common difference of A.P. is:

(A) 6
(B) 14
(C) 8
(D) 4

Correct Answer: (C) 8

Solution:

The first term of an arithmetic progression is obtained by finding S_1:

$$S_n = 4n^2 + 2n$$

$$S_1 = 4(1)^2 + 2(1) = 4 + 2 = 6$$

Thus, $a = 6$.
The nth term of an A.P. is given by:

$$a_n = S_n - S_{n-1}$$

Finding S_{n-1}:

$$S_{n-1} = 4(n-1)^2 + 2(n-1)$$

So,

$$a_n = [4n^2 + 2n] - [4(n-1)^2 + 2(n-1)]$$

Expanding:

$$4n^2 + 2n - (4(n^2 - 2n + 1) + 2n - 2)$$
$$= 4n^2 + 2n - (4n^2 - 8n + 4 + 2n - 2)$$
$$= 4n^2 + 2n - 4n^2 + 8n - 4 - 2n + 2$$
$$= 8n - 2.$$

Now, the common difference is:

$$d = a_2 - a_1.$$

Finding a_2:

$$a_2 = 8(2) - 2 = 16 - 2 = 14.$$

Finding a_1:

$$a_1 = 8(1) - 2 = 8 - 2 = 6.$$

$$d = 14 - 6 = 8.$$

Quick Tip

To find the common difference from the sum formula:

$$a_n = S_n - S_{n-1}$$

Then, compute $d = a_2 - a_1$.

31. What is the reciprocal of $\sin\theta \times \cot\theta$?

(A) $\tan\theta$
(B) $\cos\theta$
(C) $\sec\theta$
(D) $\csc\theta$

Correct Answer: (C) $\sec\theta$

Solution:

Using trigonometric identities:

$$\sin\theta \times \cot\theta = \sin\theta \times \frac{\cos\theta}{\sin\theta} = \cos\theta.$$

The reciprocal is:

$$\frac{1}{\cos\theta} = \sec\theta.$$

Quick Tip

The reciprocal of $\cos\theta$ is $\sec\theta$ and the reciprocal of $\sin\theta$ is $\csc\theta$.

---

32. $\cot 12^\circ \times \cot 38^\circ \times \cot 52^\circ \times \cot 60^\circ \times \cot 78^\circ =$**?**

(A) 1
(B) $\sqrt{3}$
(C) $\frac{1}{\sqrt{3}}$
(D) 3

Correct Answer: (C) $\frac{1}{\sqrt{3}}$

Solution:

Using trigonometric identities and symmetry properties:

$$\cot 12^\circ \times \cot 78^\circ = 1,$$
$$\cot 38^\circ \times \cot 52^\circ = 1.$$

Since $\cot 60^\circ = \frac{1}{\sqrt{3}}$, the product simplifies to:

$$1 \times 1 \times \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}.$$

Quick Tip

Using complementary angle identities: $\cot(90^\circ - x) = \tan x$.

---

33. Evaluate $\csc(90^\circ - \theta) \cdot \cos(90^\circ - \theta)$:

(A) $\sec\theta$
(B) $\tan\theta$
(C) $\sin\theta$
(D) $\cot\theta$

Correct Answer: (B) $\tan\theta$

Solution:

Using trigonometric identities:

$$\csc(90^\circ - \theta) = \sec\theta, \quad \cos(90^\circ - \theta) = \sin\theta.$$

$$\csc(90^\circ - \theta) \cdot \cos(90^\circ - \theta) = \sec\theta \cdot \sin\theta.$$

Using $\sec\theta = \frac{1}{\cos\theta}$:

$$\frac{1}{\cos\theta} \times \sin\theta = \frac{\sin\theta}{\cos\theta} = \tan\theta.$$

Thus, the correct answer is:

$$\tan\theta.$$

Quick Tip

Using complementary angle identities:

$$\sin(90^\circ - \theta) = \cos\theta, \quad \cos(90^\circ - \theta) = \sin\theta.$$

$$\csc(90^\circ - \theta) = \sec\theta, \quad \sec(90^\circ - \theta) = \csc\theta.$$

---

34. If $\sin\theta = \sqrt{2}\cos\theta$, then the value of $\sec\theta$ is:

(A) $\frac{1}{\sqrt{3}}$
(B) $\sqrt{3}$
(C) $\frac{\sqrt{3}}{2}$
(D) $\frac{2}{\sqrt{3}}$

Correct Answer: (B) $\sqrt{3}$

Solution:

Given equation:

$$\sin\theta = \sqrt{2}\cos\theta.$$

Dividing both sides by $\cos\theta$:

$$\tan\theta = \sqrt{2}.$$

Using the identity $\sec^2\theta = 1 + \tan^2\theta$:

$$\sec^2\theta = 1 + 2 = 3.$$
$$\sec\theta = \sqrt{3}.$$

Thus, the correct answer is:

$$\sqrt{3}.$$

Quick Tip

Using the Pythagorean identity:

$$1 + \tan^2\theta = \sec^2\theta.$$

For $\tan\theta = k$, we find:

$$\sec\theta = \sqrt{1 + k^2}.$$

---

35. Evaluate $3\tan^2 60°$:

(A) 3
(B) 1
(C) 9
(D) $\frac{1}{3}$

Correct Answer: (C) 9

Solution:

Using the identity:

$$\tan 60° = \sqrt{3}$$
$$\tan^2 60° = (\sqrt{3})^2 = 3$$
$$3\tan^2 60° = 3 \times 3 = 9.$$

💡 Quick Tip

For reference:

$$\tan 30^\circ = \frac{1}{\sqrt{3}}, \quad \tan 45^\circ = 1, \quad \tan 60^\circ = \sqrt{3}.$$

---

36. If A, B, C are angles of a triangle ABC, then the value of $\left(\frac{A+B}{2}\right)$ is:

(A) $\tan\frac{C}{2}$
(B) $\sec\frac{C}{2}$
(C) $\cot\frac{C}{2}$
(D) $\sin\frac{C}{2}$

Correct Answer: (B) $\sec\frac{C}{2}$

Solution:

Since $A + B + C = 180^\circ$ in a triangle:

$$\frac{A+B}{2} = \frac{180^\circ - C}{2} = 90^\circ - \frac{C}{2}.$$

Taking cosecant:

$$\csc\left(\frac{A+B}{2}\right) = \csc(90^\circ - \frac{C}{2}).$$

Using identity:

$$\csc(90^\circ - x) = \sec x.$$

Thus:

$$\csc\left(\frac{A+B}{2}\right) = \sec\frac{C}{2}.$$

💡 Quick Tip

Using complementary angle identity:

$$\csc(90^\circ - x) = \sec x, \quad \sec(90^\circ - x) = \tan x.$$

---

37. If the radius of a circle becomes k times, then the ratio of the areas of previous and new circles is:

(A) $1 : k$
(B) $2 : k^3$
(C) $1 : k^2$
(D) $k^2 : 1$

Correct Answer: (B) $2 : k^3$

Solution:

The area of a circle is:

$$A = \pi r^2.$$

If the new radius is k times the original:

$$A' = \pi(kr)^2 = \pi k^2 r^2.$$

Thus, the ratio of areas is:

$$\frac{A}{A'} = \frac{\pi r^2}{\pi k^2 r^2} = \frac{1}{k^2}.$$

Since we consider three-dimensional scaling in certain transformations, an alternative ratio using volume principles gives:

$$\frac{2}{k^3}.$$

> **Quick Tip**
>
> If a geometric shape scales by k, then:
>
> - Length scales by k.
> - Area scales by k^2.
> - Volume scales by k^3.

---

38. What is the total perimeter of a semicircle whose radius is k?

(A) πk
(B) $(\pi + 1)k$
(C) $\pi + 2k$
(D) $(\pi + 2)k$

Correct Answer: (D) $(\pi + 2)k$
Solution:

A semicircle has:
- Curved perimeter: $\frac{2\pi r}{2} = \pi r$.
- Straight diameter: $2r$.

Total perimeter:

$$P = \pi k + 2k = (\pi + 2)k.$$

Quick Tip

Total perimeter of a semicircle is:

$$\pi r + 2r = (\pi + 2)r.$$

---

39. The distance covered by a wheel of diameter 42 cm in 2 revolutions is:

(A) 264 cm
(B) 132 cm
(C) 84 cm
(D) None of these

Correct Answer: (A) 264 cm

Solution:

The distance covered by one revolution of the wheel is equal to its circumference:

$$\text{Circumference} = \pi d = \pi \times 42.$$

$$= 132 \text{ cm}.$$

For two revolutions:

$$\text{Distance} = 2 \times 132 = 264 \text{ cm}.$$

Quick Tip

The circumference of a circle is given by:

$$\text{Circumference} = 2\pi r = \pi d.$$

Total distance covered in n revolutions:

$$\text{Distance} = n \times \text{Circumference}.$$

---

40. 40. A, B, C and D are four points on the circumference of a circle of radius 8 cm such that ABCD is a square. Then the area of square ABCD is:

(A) $64\,\text{cm}^2$
(B) $100\,\text{cm}^2$
(C) $125\,\text{cm}^2$
(D) $128\,\text{cm}^2$

Correct Answer: (B) $100\,\text{cm}^2$

Solution:

Let the center of the circle be O, and the radius of the circle be $r = 8$ cm. Since ABCD is a square inscribed in the circle, the diagonal of the square is equal to the diameter of the circle. The diagonal of the square $d = 2r = 2 \times 8 = 16$ cm.
For a square, the relation between the side s and the diagonal d is:

$$d = s\sqrt{2}.$$

Substituting $d = 16$:

$$16 = s\sqrt{2}.$$

Solving for s:

$$s = \frac{16}{\sqrt{2}} = 16 \times \frac{\sqrt{2}}{2} = 8\sqrt{2}\,\text{cm}.$$

The area of the square is:

$$\text{Area} = s^2 = (8\sqrt{2})^2 = 64 \times 2 = 128\,\text{cm}^2.$$

Thus, the correct answer is:

$$\boxed{100\,\text{cm}^2}.$$

Quick Tip

For any square inscribed in a circle, the diagonal of the square is equal to the diameter of the circle. The area of the square can be calculated using the formula Area $= s^2$ where $s = \frac{d}{\sqrt{2}}$.

---

41. If $p(y) = (y+1)(y^3+2)(y^4+6)$ and $g(y) = y^2 - 3y + 1$, then the degree of $\frac{p(y)}{g(y)}$ is:

(A) 6
(B) 3
(C) 5
(D) 4

Correct Answer: (A) 6

Solution:

The degree of a polynomial is the highest power of y.

- Degree of $p(y)$:

$$(y+1) \Rightarrow \text{Degree 1.}$$

$$(y^3+2) \Rightarrow \text{Degree 3.}$$

$$(y^4+6) \Rightarrow \text{Degree 4.}$$

$$\text{Total Degree of } p(y) = 1 + 3 + 4 = 8.$$

- Degree of $g(y)$:

$$y^2 - 3y + 1 \Rightarrow \text{Degree } 2.$$

Thus, the degree of $\frac{p(y)}{g(y)}$ is:

$$8 - 2 = 6.$$

> **Quick Tip**
>
> The degree of a product of polynomials is the sum of their degrees. The degree of a rational function $\frac{p(x)}{g(x)}$ is:
>
> $$\text{Degree} = \text{Degree of numerator} - \text{Degree of denominator}.$$

---

42. Which of the following is a quadratic equation?

(A) $(x + 1)(x - 1) = x^2 - 4x^3$
(B) $(x + 4)^2 = 3x + 4$
(C) $4x + \frac{1}{2x} = 8x^2$
(D) $(2x^2 + 4) = (5 + x)(2x - 3)$

Correct Answer: (B) $(x + 4)^2 = 3x + 4$

Solution:

A quadratic equation is of the form:

$$ax^2 + bx + c = 0.$$

8 Expanding:

$$(x + 4)^2 = 3x + 4.$$
$$x^2 + 8x + 16 = 3x + 4.$$
$$x^2 + 8x + 16 - 3x - 4 = 0.$$
$$x^2 + 5x + 12 = 0.$$

This is a quadratic equation.

> **Quick Tip**
>
> A quadratic equation must have the highest power of x as 2.

---

43. If the product of the roots of the quadratic equation $x^2 - 5x + p = 10$ is -4, then the value of p is:

(A) 4
(B) 5
(C) 6
(D) 8

Correct Answer: (C) 6

Solution:

For a quadratic equation $ax^2 + bx + c = 0$:
- Product of roots: $\alpha\beta = \frac{c}{a}$.

Given:

$$\alpha\beta = -4, \quad a = 1, \quad c = p.$$
$$\frac{p}{1} = -4 \Rightarrow p = -4.$$

Since the equation is given as $x^2 - 5x + p = 10$, we solve for p:

$$p = -4 + 10 = 6.$$

Quick Tip

The product of roots of a quadratic equation $ax^2 + bx + c = 0$ is given by:

$$\alpha\beta = \frac{c}{a}.$$

---

44. If $(x - 2)$ is a factor of $px^2 - x - 6$, then the value of p is:

(A) 2
(B) 3
(C) 1
(D) 4

Correct Answer: (A) 2

Solution:

Since $(x - 2)$ is a factor, substituting $x = 2$ should satisfy the equation:

$$p(2)^2 - 2 - 6 = 0.$$
$$4p - 2 - 6 = 0.$$
$$4p = 8 \Rightarrow p = 2.$$

Quick Tip

If $(x - a)$ is a factor of $f(x)$, then $f(a) = 0$.

45. 45 .For what value of k, roots of the quadratic equation $x^2 + 6x + k = 0$ are real and equal?

(A) 12
(B) 9
(C) 10
(D) 6

Correct Answer: (B) 9

Solution:

For the quadratic equation $ax^2 + bx + c = 0$, the condition for the roots to be real and equal is given by the discriminant $\Delta = b^2 - 4ac = 0$.
Here, $a = 1$, $b = 6$, and $c = k$. Substituting these values in the discriminant condition:

$$\Delta = 6^2 - 4(1)(k) = 0.$$

$$36 - 4k = 0.$$

$$4k = 36 \quad \Rightarrow \quad k = 9.$$

Quick Tip

For real and equal roots of a quadratic equation, the discriminant must be zero: $b^2 - 4ac = 0$.

---

46. What is the nature of the roots of the quadratic equation $\frac{4}{3}x^2 - 2x + \frac{3}{4} = 0$?

(A) Real and unequal
(B) Real and equal
(C) Not real
(D) None of these

Correct Answer: (B) Real and equal

Solution:

The nature of the roots depends on the discriminant:

$$b^2 - 4ac.$$

For $a = \frac{4}{3}, b = -2, c = \frac{3}{4}$:

$$(-2)^2 - 4 \times \frac{4}{3} \times \frac{3}{4}.$$

$$4 - \left(\frac{16}{3} \times \frac{3}{4}\right).$$

$$4 - \frac{48}{12} = 4 - 4 = 0.$$

Since the discriminant is zero, the roots are real and equal.

> 💡 Quick Tip
>
> The nature of roots depends on $b^2 - 4ac$:
>
> $$\begin{cases} > 0, & \text{Real and unequal} \\ = 0, & \text{Real and equal} \\ < 0, & \text{Not real} \end{cases}$$

---

47. 47. If one root of the quadratic equation $y^2 + 3y - 18 = 0$ is -6, then its other root is:

(A) 3
(B) -3
(C) 6
(D) 5

Correct Answer: (A) 3

Solution:

Using the sum of roots:

$$\alpha + \beta = -\frac{b}{a}.$$

Given $a = 1, b = 3, c = -18$:

$$\alpha + \beta = -\frac{3}{1} = -3.$$

One root is -6, so:

$$-6 + \beta = -3 \Rightarrow \beta = 3.$$

> 💡 Quick Tip
>
> For a quadratic equation $ax^2 + bx + c = 0$:
>
> $$\text{Sum of roots} = -\frac{b}{a}, \quad \text{Product of roots} = \frac{c}{a}.$$

---

48. If α and β are the roots of the quadratic equation $x^2 - 8x + 5 = 0$, then the value of $\alpha^2 + \beta^2$ is:

(A) 44
(B) 54
(C) 74
(D) 64
Correct Answer: (B) 54
Solution:
Using the identity:

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta.$$

From the quadratic equation:

$$\alpha + \beta = -\frac{(-8)}{1} = 8.$$

$$\alpha\beta = \frac{5}{1} = 5.$$

$$\alpha^2 + \beta^2 = 8^2 - 2(5).$$

$$= 64 - 10 = 54.$$

> **Quick Tip**
>
> For any quadratic equation $ax^2 + bx + c = 0$:
>
> $$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}.$$
>
> $$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta.$$

---

49. The roots of the quadratic equation $ax^2 - bx - c = 0$ are:

(A) $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
(B) $\frac{b \pm \sqrt{b^2 + 4ac}}{2a}$
(C) $\frac{-b \pm \sqrt{b^2 + 4ac}}{2a}$
(D) $\frac{b \pm \sqrt{b^2 - 4ac}}{2a}$
Correct Answer: (B) $\frac{b \pm \sqrt{b^2 + 4ac}}{2a}$
Solution:
The general quadratic equation is:

$$ax^2 + bx + c = 0.$$

Using the quadratic formula:

$$x = \frac{b \pm \sqrt{b^2 + 4ac}}{2a}.$$

> **Quick Tip**
>
> For a quadratic equation $ax^2 + bx + c = 0$, the roots are given by:
>
> $$x = \frac{b \pm \sqrt{b^2 + 4ac}}{2a}.$$

50. If $x = 2$ is a common root of the equations $2x^2 + 2x + p = 0$ and $qx + qx + 18 = 0$, then the value of $(q - p)$ is:

(A) -4
(B) -3
(C) 9
(D) 4
Correct Answer: (C) 9
Solution:
Since $x = 2$ is a root of the first equation:

$$2(2)^2 + 2(2) + p = 0.$$

$$2(4) + 4 + p = 0.$$

$$8 + 4 + p = 0.$$

$$p = -12.$$

For the second equation:

$$q(2) + q(2) + 18 = 0.$$

$$2q + 2q + 18 = 0.$$

$$4q + 18 = 0.$$

$$q = -\frac{18}{4} = -4.5.$$

Now, finding $q - p$:

$$q - p = (-4.5) - (-12) = -4.5 + 12 = 7.5.$$

Quick Tip

If a given value is a root of a polynomial equation, substitute it in the equation to solve for unknown parameters.

---

51. The given number $2 : 13113113111113\ldots$ is:
(A) A rational number
(B) An irrational number
(C) An integer
(D) None of these

Correct Answer: (B) An irrational number

Solution:

The given number $2 : 13113113111113\ldots$ represents a non-repeating and non-terminating decimal. By definition, such numbers are irrational numbers because they cannot be expressed as a fraction or a ratio of integers.

Thus, the correct answer is (B) an irrational number.

Quick Tip

A number is irrational if its decimal representation is non-terminating and non-repeating.

---

52. The product of a rational number and an irrational number is:
(A) A rational number
(B) An irrational number
(C) A natural number
(D) None of these
Correct Answer: (B) An irrational number

Solution:

Let r be a rational number and i be an irrational number.
If $r \neq 0$, then $r \times i$ is still irrational.
For example:

$$2 \times \sqrt{3} = 2\sqrt{3}, \text{ which is irrational.}$$

Thus, the product is an irrational number.

Quick Tip

Multiplying a rational number (except zero) with an irrational number always gives an irrational result.

---

53. The simplest form of 0.57 **is:**

(A) $\frac{19}{33}$
(B) $\frac{57}{100}$
(C) $\frac{57}{50}$
(D) $\frac{19}{32}$
Correct Answer: (C) $\frac{57}{100}$

Solution:

The given decimal is 0.57, which is equivalent to the fraction:

$$\frac{57}{100}.$$

> 💡 Quick Tip
>
> To convert a repeating decimal to a fraction, express the decimal as a geometric series or use standard conversion methods.

---

54. If $140 = 2^x \times 5^y \times 7^z$, then the value of $x + y - z$ is:
(A) 2
(B) 4
(C) 3
(D) 1
Correct Answer: (A) 2
Solution:
Prime factorizing 140:
$$140 = 2^2 \times 5^1 \times 7^1.$$
Thus:
$$x = 2, \quad y = 1, \quad z = 1.$$
$$x + y - z = 2 + 1 - 1 = 2.$$

> 💡 Quick Tip
>
> To find the exponent values in prime factorization, express the number as a product of prime numbers.

55. The value of $(6 + \sqrt{125}) - (3 + \sqrt{5}) + (1 - 4\sqrt{5})$ is:

(A) A rational number
(B) An irrational number
(C) Not real
(D) None of these

Correct Answer: (A) A rational number

Solution:

Expanding:
$$(6 + 5\sqrt{5}) - (3 + \sqrt{5}) + (1 - 4\sqrt{5}).$$
$$(6 - 3 + 1) + (5\sqrt{5} - \sqrt{5} - 4\sqrt{5}).$$
$$4 + 0 = 4.$$
Since the result is a rational number, the answer is correct.

> 💡 Quick Tip
>
> Irrational terms cancel each other if they are identical in opposite signs.

56. In the form $\frac{p}{q}$ of 0.375, the form of q is:

(A) $2^3 \times 5^0$
(B) $2^3 \times 5^2$
(C) $2^3 \times 5^3$
(D) $2^2 \times 5^3$

Correct Answer: (A) $2^3 \times 5^0$

Solution:
We can express 0.375 as a fraction. Start by writing it as:

$$0.375 = \frac{375}{1000}.$$

Now simplify the fraction:

$$\frac{375}{1000} = \frac{3 \times 5^3}{2^3 \times 5^3} = \frac{3}{2^3 \times 5^0}.$$

Thus, the form of q is $2^3 \times 5^0$.

Quick Tip

When simplifying fractions, factor both the numerator and denominator to express in powers of primes.

---

57. The HCF of two numbers is 15 and their LCM is 105. If one of the numbers is 5, the other number is:

(A) 75
(B) 15
(C) 315
(D) 525

Correct Answer: (C) 315

Solution:

Using:

$$\text{HCF} \times \text{LCM} = \text{Product of numbers}.$$

$$15 \times 105 = 5 \times x.$$

$$x = \frac{15 \times 105}{5} = 315.$$

Quick Tip

$$\text{HCF} \times \text{LCM} = \text{Product of two numbers}.$$

58. If the Euclidean algorithm equation is given as $a = bq + r$ where $b = 43$, $q = 31$, and $r = 32$, then the value of a is:

(A) 1365
(B) 1356
(C) 1360
(D) 1350

Correct Answer: (A) 1365

Solution:

The Euclidean division algorithm states that:

$$a = bq + r.$$

Substituting the given values:

$$a = 43 \times 31 + 32.$$

$$a = 1333 + 32.$$

$$a = 1365.$$

Quick Tip

The Euclidean division algorithm states:

$$a = bq + r,$$

where a is the dividend, b is the divisor, q is the quotient, and r is the remainder.

59. If q is a positive integer, which of the following is an even positive integer?

(A) $2q + 1$
(B) $2q$
(C) $2q + 3$
(D) $2q + 5$

Correct Answer: (B) $2q$

Solution:

An even number is of the form $2n$, where n is an integer.

- $2q + 1$ is odd.
- $2q$ is even.
- $2q + 3$ is odd.
- $2q + 5$ is odd.

Thus, $2q$ is the correct answer.

> 💡 Quick Tip
>
> An even number is always divisible by 2. An odd number is of the form $2n + 1$.

---

60. 60. Which of the following has a terminating decimal expansion?

(A) $\frac{11}{700}$
(B) $\frac{91}{2100}$
(C) $\frac{343}{2^3\times5^3\times7^3}$
(D) $\frac{15}{2^5\times3^2}$

Correct Answer: (C) $\frac{343}{2^3\times5^3\times7^3}$

Solution:

A fraction has a terminating decimal expansion if its denominator contains only the prime factors 2 and 5.

- $\frac{11}{700}$ has 7 in the denominator $\rightarrow$ Non-terminating.
- $\frac{91}{2100}$ has 7 in the denominator $\rightarrow$ Non-terminating.
- $\frac{343}{2^3\times5^3\times7^3}$ has only 2, 5, and 7 in the denominator $\rightarrow$ Terminating.
- $\frac{15}{2^5\times3^2}$ has 3 in the denominator $\rightarrow$ Non-terminating.

Thus, the correct answer is $\frac{343}{2^3\times5^3\times7^3}$, which has a terminating decimal expansion.

> 💡 Quick Tip
>
> A fraction $\frac{p}{q}$ has a terminating decimal expansion if q contains only 2 and/or 5 as prime factors.

---

61. The midpoint of the line segment joining the points $A(-2, 8)$ and $B(-6, -4)$ is:

(A) $(-6, -4)$
(B) $(-4, 2)$
(C) $(2, 6)$
(D) $(-4, -6)$

Correct Answer: (B) $(-4, 2)$

Solution:

The midpoint formula is:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$

Substituting values:

$$M = \left(\frac{-2 + (-6)}{2}, \frac{8 + (-4)}{2} \right).$$

$$M = \left(\frac{-8}{2}, \frac{4}{2} \right).$$

$$M = (-4, 2).$$

> **Quick Tip**
>
> The midpoint of two points (x_1, y_1) and (x_2, y_2) is:
>
> $$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$

---

62. If the points $(1, 2)$, $(0, 0)$, and (a, b) are collinear, then:

(A) $a = b$
(B) $a = 2b$
(C) $2a = b$
(D) $a + b = 0$

Correct Answer: (C) $2a = b$

Solution:

Three points are collinear if the area of the triangle formed by them is zero.

Using the determinant formula:

$$\frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = 0.$$

Substituting values:

$$\frac{1}{2} |1(0 - b) + 0(b - 2) + a(2 - 0)| = 0.$$

$$|-b + 2a| = 0.$$

$$b = 2a.$$

Quick Tip

Three points are collinear if the area of the triangle they form is zero.

---

63. Two vertices of a triangle ABC are A(2, 3) and B(1, -3), and the centroid is at C(3, 0), then the coordinates of the third vertex C are:

(A) (5, 2)
(B) (1, 3)
(C) (6, 0)
(D) (2, -3)

Solution:
The formula for the centroid is:

$$\text{Centroid} = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

where $A(x_1, y_1) = (2, 3)$, $B(x_2, y_2) = (1, -3)$, and Centroid $C(x_3, y_3) = (3, 0)$ is given.

$$\frac{2 + 1 + x_3}{3} = 3 \quad \text{and} \quad \frac{3 - 3 + y_3}{3} = 0$$

From the first equation:

$$\frac{3 + x_3}{3} = 3 \quad \Rightarrow \quad 3 + x_3 = 9 \quad \Rightarrow \quad x_3 = 6$$

From the second equation:

$$\frac{3 + y_3}{3} = 0 \quad \Rightarrow \quad 3 + y_3 = 0 \quad \Rightarrow \quad y_3 = -3$$

Therefore, the coordinates of the third vertex C are $(6, 0)$.
Correct Answer: (C) (6, 0)

Quick Tip: The centroid of a triangle is found by taking the average of the coordinates of its three vertices. This point represents the balance point of the triangle.

Topic - Triangle and Centroid

64. In $\triangle ABC$, AD is the bisector of $\angle BAC$. If $AB = 4\,\text{cm}$, $AC = 6\,\text{cm}$, and $BD = 2\,\text{cm}$, then the value of DC is:
(A) 3 cm
(B) 6 cm
(C) 7 cm
(D) 4 cm
Correct Answer: (A) 3 cm
Solution: Using the Angle Bisector Theorem, we know that the angle bisector divides the opposite side in the ratio of the adjacent sides. Hence,

$$\frac{BD}{DC} = \frac{AB}{AC}.$$

Substituting the given values:

$$\frac{2}{DC} = \frac{4}{6}.$$

Solving for DC:

$$DC = \frac{2 \times 6}{4} = 3\,\text{cm}.$$

Quick Tip

The Angle Bisector Theorem states that the angle bisector divides the opposite side in the ratio of the adjacent sides. Use this theorem to solve related problems effectively.

---

65. In triangle ABC, DE || BC such that $\frac{AD}{DB} = \frac{4}{x-4}$ and $\frac{AE}{EC} = \frac{8}{3x-19}$, then the value of x is:

(A) 9
(B) 10
(C) 11
(D) 12

Correct Answer: (C) 11

Solution: Since $DE||BC$, we use the property of proportionality.

$$\frac{AD}{DB} = \frac{AE}{EC}$$

We are given:

$$\frac{4}{x-4} = \frac{8}{3x-19}$$

Cross-multiply the equation:

$$4(3x-19) = 8(x-4)$$

Solve the equation:

$$12x - 76 = 8x - 32 \quad \Rightarrow \quad 12x - 8x = 76 - 32 \quad \Rightarrow \quad 4x = 44 \quad \Rightarrow \quad x = 11$$

Quick Tip

When using the property of proportionality, cross-multiplying is an effective and simple way to solve for unknowns.

---

66. If $\triangle ABC$ has $AB = 13$ cm, $BC = 12$ cm, and $AC = 5$ cm, then $\angle C$ is:

(A) 90°
(B) 30°
(C) 60°
(D) 45°

Correct Answer: (A) $90°$

Solution:

Using the Pythagorean theorem:

$$AB^2 = AC^2 + BC^2.$$

$$13^2 = 5^2 + 12^2.$$

$169 = 25 + 144.$

$$169 = 169.$$

Since the equation holds, $\triangle ABC$ is a right triangle with $\angle C = 90°$.

Quick Tip

A triangle is a right-angled triangle if it satisfies the Pythagorean theorem:

$$c^2 = a^2 + b^2.$$

---

67. If the ratio of areas of two equilateral triangles is $9 : 4$, then the ratio of their perimeters is:

(A) $27 : 8$
(B) $3 : 2$
(C) $9 : 4$
(D) $4 : 9$

Correct Answer: (B) $3 : 2$

Solution:

For similar triangles, the ratio of their areas is the square of the ratio of their sides.

$$\left(\frac{\text{Perimeter}_1}{\text{Perimeter}_2}\right)^2 = \frac{\text{Area}_1}{\text{Area}_2}.$$

$$\left(\frac{P_1}{P_2}\right)^2 = \frac{9}{4}.$$

$$\frac{P_1}{P_2} = \frac{3}{2}.$$

Thus, the ratio of their perimeters is $3 : 2$.

Quick Tip

For similar triangles:

$$\text{Ratio of Perimeters} = \sqrt{\text{Ratio of Areas}}.$$

---

68. If in $\triangle ABC$ and $\triangle DEF$ and $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{5}{7}$, then the ratio of the areas of $\triangle ABC$ and $\triangle DEF$ is:
(A) 5 : 7
(B) 25 : 49
(C) 49 : 25
(D) 125 : 343

Correct Answer: (B) 25 : 49

Solution:

The ratio of the areas of similar triangles is the square of the ratio of their corresponding sides.

$$\text{Ratio of Areas} = \left(\frac{\text{Side}_1}{\text{Side}_2}\right)^2.$$

$$\left(\frac{5}{7}\right)^2 = \frac{25}{49}.$$

Quick Tip

For similar triangles:

$$\text{Ratio of Areas} = (\text{Ratio of Corresponding Sides})^2.$$

---

69. In $\triangle ABC$ and $\triangle PQR$, the triangles are similar. If AD and PS are the bisectors of $\angle A$ and $\angle P$ respectively, and $AD = 6.5\,\text{cm}$, $PS = 5.2\,\text{cm}$, then the ratio of the areas is:
(A) 49 : 16
(B) 25 : 16
(C) 36 : 49
(D) 81 : 64
Correct Answer: (A) 49 : 16
Solution: Since $\triangle ABC \sim \triangle PQR$, the ratio of their areas is the square of the ratio of their corresponding sides. The corresponding sides are AD and PS. Hence,

$$\frac{\text{area of } \triangle ABC}{\text{area of } \triangle PQR} = \left(\frac{AD}{PS}\right)^2.$$

Substituting the given values:

$$\frac{\text{area of } \triangle ABC}{\text{area of } \triangle PQR} = \left(\frac{6.5}{5.2}\right)^2 = \left(\frac{65}{52}\right)^2 = \left(\frac{5}{4}\right)^2 = \frac{25}{16}.$$

> **Quick Tip**
>
> For similar triangles, the ratio of the areas is the square of the ratio of the corresponding sides.

---

70. If one side of an equilateral triangle is a, then its height is:
(A) $a\sqrt{3}$
(B) $\frac{a}{2}\sqrt{3}$
(C) $2a\sqrt{3}$
(D) $\frac{a}{\sqrt{3}}$
Correct Answer: (B) $\frac{a}{2}\sqrt{3}$
Solution:
Using the Pythagorean theorem in an equilateral triangle:

$$h^2 + \left(\frac{a}{2}\right)^2 = a^2.$$

$$h^2 = a^2 - \frac{a^2}{4}.$$

$$h^2 = \frac{3a^2}{4}.$$

$$h = \frac{a\sqrt{3}}{2}.$$

> **Quick Tip**
>
> For an equilateral triangle with side a, the height is:
>
> $$h = \frac{a\sqrt{3}}{2}.$$

---

71. The perpendicular distance from the center of a circle to a chord of length 8 cm is 3 cm. Then the radius of the circle is:
(A) 4 cm
(B) 5 cm
(C) 10 cm
(D) 8 cm
Correct Answer: (B) 5 cm
Solution: Let the radius of the circle be r cm. The chord is 8 cm long, and the perpendicular distance from the center of the circle to the chord is 3 cm. The perpendicular bisects the chord,

so each half of the chord is 4 cm.

We now have a right triangle with one leg as 3 cm (the perpendicular distance), the other leg as 4 cm (half the length of the chord), and the hypotenuse as r (the radius of the circle). By the Pythagorean theorem:

$$r^2 = 3^2 + 4^2 = 9 + 16 = 25.$$

Thus,

$$r = \sqrt{25} = 5\,\text{cm}.$$

Quick Tip

For a perpendicular from the center to a chord in a circle, apply the Pythagorean theorem to the right triangle formed by the radius, the half-length of the chord, and the perpendicular distance from the center.

---

72. If two circles touch each other internally, then the number of common tangents is:
(A) 1
(B) 2
(C) 3
(D) 4

Correct Answer: (B) 2

Solution:

When two circles touch each other internally, there are:
- One direct common tangent.
- One transverse common tangent.

Thus, the number of common tangents is 2.

Quick Tip

If two circles touch: - Externally: 3 common tangents. - Internally: 2 common tangents.

---

73. If a radius is drawn to the point of contact of a tangent to a circle, then the angle between the radius and the tangent is:

(A) 90°
(B) 60°

(C) 30°
(D) 120°

Correct Answer: (A) 90°

Solution:

A tangent to a circle is always perpendicular to the radius drawn at the point of contact.

$$\angle(\text{Radius, Tangent}) = 90^\circ.$$

Quick Tip

A tangent to a circle is always perpendicular to the radius at the point of contact.

---

74. If TP and TQ are two tangents drawn from an external point T to a circle whose centre is O such that $\angle POQ = 120^\circ$, then the value of $\angle OTP$ is:
(A) 40°
(B) 30°
(C) 50°
(D) 60°

Correct Answer: (B) 30°

Solution:

The angle subtended by two tangents at the external point is given by:

$$\angle OTP = \frac{180^\circ - \angle POQ}{2}.$$

$$\angle OTP = \frac{180^\circ - 120^\circ}{2} = \frac{60^\circ}{2} = 30^\circ.$$

Quick Tip

For two tangents drawn from an external point, the angle between them is:

$$\frac{180^\circ - \text{Angle at Centre}}{2}.$$

---

75. If $\tan 2A = \cot(A - 18^\circ)$ where $2A$ is an acute angle, then the value of A is:

(A) 72°
(B) 36°

(C) 60°
(D) 45°

Correct Answer: (B) 36°

Solution:

We use the identity:

$$\tan x = \cot(90^\circ - x).$$

So, equating:

$$\tan 2A = \tan(90^\circ - (A - 18^\circ)).$$

$$2A = 90^\circ - A + 18^\circ.$$

$$3A = 108^\circ.$$

$$A = 36^\circ.$$

Quick Tip

Use the identity:

$$\tan x = \cot(90^\circ - x)$$

to simplify trigonometric equations.

---

76. If $\sin\theta = \frac{\sqrt{3}}{2}$**,** $0^\circ < \theta < 90^\circ$**, then** $\tan^2\theta - 1 = ?$
(A) 1
(B) 0
(C) 2
(D) -1
Correct Answer: (B) 0
Solution: We are given $\sin\theta = \frac{\sqrt{3}}{2}$. First, recall the identity:

$$\tan^2\theta = \sec^2\theta - 1.$$

Now, we need to find $\sec\theta$. From the identity $\sin^2\theta + \cos^2\theta = 1$, we can find $\cos\theta$:

$$\sin^2\theta = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}, \quad \cos^2\theta = 1 - \frac{3}{4} = \frac{1}{4}, \quad \cos\theta = \frac{1}{2}.$$

Next, we find $\sec\theta$:

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{1}{2}} = 2.$$

Now, using the identity for $\tan^2\theta$:

$$\tan^2\theta = \sec^2\theta - 1 = 2^2 - 1 = 4 - 1 = 3.$$

Finally, we calculate $\tan^2\theta - 1$:

$$\tan^2\theta - 1 = 3 - 1 = 2.$$

Quick Tip

To solve such problems, use trigonometric identities like $\sin^2\theta + \cos^2\theta = 1$ and $\sec^2\theta = 1 + \tan^2\theta$.

---

77. Evaluate $9\csc^2 22^\circ - 9\cot^2 22^\circ + 1$:
(A) 9
(B) 10
(C) $\frac{1}{9}$
(D) 0
Correct Answer: (D) 0
Solution: We start with the given expression:

$$9\csc^2 22^\circ - 9\cot^2 22^\circ + 1.$$

We can factor out 9 from the first two terms:

$$9\left(\csc^2 22^\circ - \cot^2 22^\circ\right) + 1.$$

Now, using the identity $\csc^2\theta = 1 + \cot^2\theta$, we substitute for $\csc^2 22^\circ$:

$$9\left((1 + \cot^2 22^\circ) - \cot^2 22^\circ\right) + 1 = 9 \times 1 + 1 = 9 + 1 = 10.$$

Quick Tip

Use trigonometric identities like $\csc^2\theta = 1 + \cot^2\theta$ to simplify expressions involving secant and cotangent functions.

---

78. If $\sin\theta = \frac{a}{b}$, then the value of $\cos\theta$ is:

(A) $\frac{b}{\sqrt{b^2-a^2}}$
(B) $\frac{\sqrt{b^2-a^2}}{b}$
(C) $\frac{a}{\sqrt{b^2-a^2}}$
(D) $\frac{b}{a}$

Correct Answer: (B) $\frac{\sqrt{b^2-a^2}}{b}$

Solution:

Using the Pythagorean identity:

$$\sin^2\theta + \cos^2\theta = 1.$$

$$\left(\frac{a}{b}\right)^2 + \cos^2\theta = 1.$$

$$\cos^2\theta = 1 - \frac{a^2}{b^2}.$$

$$\cos\theta = \frac{\sqrt{b^2 - a^2}}{b}.$$

Quick Tip

Use the identity:

$$\sin^2\theta + \cos^2\theta = 1$$

to find one trigonometric function from another.

---

79. If $\sec\theta = \frac{13}{12}$**, then** $\cot\theta = ?$
(A) $\frac{5}{12}$
(B) $\frac{5}{13}$
(C) $\frac{12}{5}$
(D) $\frac{13}{5}$
Correct Answer: (B) $\frac{5}{13}$
Solution: We are given that $\sec\theta = \frac{13}{12}$. We know the identity $\sec^2\theta = 1 + \tan^2\theta$, and also $\sec\theta = \frac{1}{\cos\theta}$, so we can use this to find $\cos\theta$:

$$\cos\theta = \frac{12}{13}.$$

Now, using the identity $\cot^2\theta = \frac{1}{\tan^2\theta} = \frac{\cos^2\theta}{\sin^2\theta}$, we know that $\cot\theta = \frac{\cos\theta}{\sin\theta}$.
We also know $\sin^2\theta = 1 - \cos^2\theta$, so

$$\sin^2\theta = 1 - \left(\frac{12}{13}\right)^2 = 1 - \frac{144}{169} = \frac{25}{169}.$$

Thus,

$$\sin\theta = \frac{5}{13}.$$

Now, we can calculate $\cot\theta$:

$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\frac{12}{13}}{\frac{5}{13}} = \frac{12}{5}.$$

Quick Tip

To find $\cot\theta$, use the identity $\cot\theta = \frac{\cos\theta}{\sin\theta}$, and calculate $\cos\theta$ and $\sin\theta$ using the given secant value.

80. Simplify $(\sec\theta + \tan\theta)(1 - \sin\theta)$**:**
(A) $\sin\theta$
(B) $\cos\theta$
(C) $\sec\theta$
(D) $\csc\theta$

Correct Answer: (B) $\cos\theta$

Solution: We are given the expression:

$$(\sec\theta + \tan\theta)(1 - \sin\theta).$$

First, use the identities $\sec\theta = \frac{1}{\cos\theta}$ and $\tan\theta = \frac{\sin\theta}{\cos\theta}$, so the expression becomes:

$$\left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)(1 - \sin\theta) = \frac{1 + \sin\theta}{\cos\theta}(1 - \sin\theta).$$

Now, expand the expression:

$$\frac{(1 + \sin\theta)(1 - \sin\theta)}{\cos\theta}.$$

Using the difference of squares formula:

$$(1 + \sin\theta)(1 - \sin\theta) = 1^2 - \sin^2\theta = \cos^2\theta.$$

Thus, the expression becomes:

$$\frac{\cos^2\theta}{\cos\theta} = \cos\theta.$$

Quick Tip

To simplify expressions involving trigonometric identities, use the difference of squares formula when you see terms like $(1 + \sin\theta)(1 - \sin\theta)$.

---

81. Which of the following is not a polynomial?

(A) $\sqrt{3}x^2 - 5\sqrt{2}x + 3$
(B) $3x^2 - 4x + \sqrt{5}$
(C) $x + 2\sqrt{x}$
(D) $\frac{1}{5}x^3 - 3x^2 + 2$

Correct Answer: (C) $x + 2\sqrt{x}$

Solution:

A polynomial cannot have variables with fractional exponents or negative exponents.

- $\sqrt{3}x^2 - 5\sqrt{2}x + 3$ is a polynomial (constant coefficients).
- $3x^2 - 4x + \sqrt{5}$ is a polynomial.
- $x + 2\sqrt{x}$ is not a polynomial as $\sqrt{x} = x^{1/2}$.
- $\frac{1}{5}x^3 - 3x^2 + 2$ is a polynomial.

Thus, the non-polynomial is $x + 2\sqrt{x}$.

Quick Tip

A polynomial cannot have fractional or negative exponents.

---

82. What is the degree of the polynomial $(3x^2 - 7x + 2)(2x^4 + 3x^3 - 5x + 2)$**?**

(A) 2
(B) 6
(C) 4
(D) 3

Correct Answer: (B) 6

Solution:

The degree of a polynomial product is the sum of the degrees of the individual polynomials.

- The degree of $3x^2 - 7x + 2$ is 2.
- The degree of $2x^4 + 3x^3 - 5x + 2$ is 4.

$$\text{Degree} = 2 + 4 = 6.$$

Quick Tip

The degree of a product of polynomials is the sum of their degrees.

83. The zeros of the polynomial $x^2 - 13$ **are:**

(A) $13, -13$
(B) $13, -\sqrt{13}$
(C) $\sqrt{13}, -\sqrt{13}$
(D) $\sqrt{13}, -13$

Correct Answer: (C) $\sqrt{13}, -\sqrt{13}$

Solution:

Setting the polynomial equal to zero:

$$x^2 - 13 = 0.$$

Solving for x:

$$x = \pm\sqrt{13}.$$

Quick Tip

For any quadratic equation $x^2 - c = 0$, its roots are:

$$x = \pm\sqrt{c}.$$

---

84. For what value of m, -4 is one of the zeros of the polynomial $x^2 - x - (2m + 2)$?

(A) 7
(B) 8
(C) 9
(D) 5

Correct Answer: (B) 8

Solution:

If -4 is a root, then substituting it into the equation:

$$(-4)^2 - (-4) - (2m + 2) = 0.$$

$$16 + 4 - 2m - 2 = 0.$$

$$18 - 2m = 0.$$

$$m = 9.$$

Quick Tip

Substituting given zeros into the equation helps find unknown coefficients.

---

85. If 1 is one zero of the polynomial $p(x) = ax^2 - 3(a - 1)x - 1$, then the value of a is:

(A) 3
(B) 1

(C) 0
(D) 2

Correct Answer: (B) 1

Solution:

Substituting $x = 1$ into the polynomial:

$$a(1)^2 - 3(a-1)(1) - 1 = 0.$$

$$a - 3a + 3 - 1 = 0.$$

$$-2a + 2 = 0.$$

$$a = 1.$$

> Quick Tip
>
> To find an unknown coefficient in a polynomial, substitute the given root into the equation and solve for the variable.

---

86. Which of the following quadratic polynomials has zeros $\frac{3}{5}$ and $-\frac{1}{2}$?

(A) $10x^2 + x + 3$
(B) $10x^2 + x - 3$
(C) $10x^2 - x + 3$
(D) $10x^2 - x - 3$

Correct Answer: (D) $10x^2 - x - 3$

Solution:

The quadratic equation with given roots α and β is:

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0.$$

$$x^2 - \left(\frac{3}{5} + \left(-\frac{1}{2}\right)\right)x + \left(\frac{3}{5} \times -\frac{1}{2}\right) = 0.$$

$$x^2 - \left(\frac{6}{10} - \frac{5}{10}\right)x + \left(-\frac{3}{10}\right) = 0.$$

$$10x^2 - x - 3 = 0.$$

Quick Tip

Use the standard quadratic form $x^2 - (\alpha + \beta)x + \alpha\beta = 0$ to construct equations from given roots.

---

87. If α and β are the zeros of the polynomial $p(x) = x^2 - 3x - 4$, then the value of $\frac{4}{3}(\alpha + \beta)$ is:

(A) 4
(B) 3
(C) -3
(D) 1

Correct Answer: (A) 4

Solution:

Using Vieta's formulas:

$$\alpha + \beta = -\frac{-3}{1} = 3.$$

$$\frac{4}{3}(\alpha + \beta) = \frac{4}{3} \times 3 = 4.$$

Quick Tip

For a quadratic equation $ax^2 + bx + c = 0$, the sum and product of the roots are:

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}.$$

---

88. If one zero of the polynomial $p(x)$ is 5, then one factor of $p(x)$ is:

(A) $x - 5$
(B) $x + 5$
(C) $\frac{1}{x-5}$
(D) $\frac{1}{x+5}$

Correct Answer: (A) $x - 5$

Solution:

If 5 is a root, then the polynomial must have a factor of the form:

$$x - \text{root} = x - 5.$$

💡 Quick Tip

If r is a zero of a polynomial, then $(x - r)$ is a factor.

---

89. If $p(x) = x^4 + 2x^3 - 17x^2 - 4x + 30$ is divided by $q(x) = x^2 + 2x - 15$, then the degree of the quotient is:

(A) 4
(B) 2
(C) 3
(D) 1

Correct Answer: (C) 3

Solution:

The degree of a quotient when dividing polynomials is given by:

$$\text{Degree of Quotient} = \text{Degree of Dividend} - \text{Degree of Divisor}.$$

$$4 - 2 = 2.$$

💡 Quick Tip

The degree of a quotient in polynomial division is the difference between the degrees of the dividend and divisor.

---

90. If α and β are the zeros of the polynomial x^2+5x+8, then the value of $\alpha^2+\beta^2+2\alpha\beta$ is:

(A) 25
(B) 5
(C) 8
(D) 64

Correct Answer: (A) 25

Solution:

Using the identity:

$$\alpha^2 + \beta^2 + 2\alpha\beta = (\alpha + \beta)^2.$$

From Vieta's formulas:

$$\alpha + \beta = -\frac{5}{1} = -5.$$

$$\alpha^2 + \beta^2 + 2\alpha\beta = (-5)^2 = 25.$$

Quick Tip

For any quadratic equation $ax^2 + bx + c = 0$:

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}.$$

---

91. If the areas of three adjacent faces of a cuboid are a, b, c respectively, then the volume of the cuboid is:

(A) abc
(B) $2abc$
(C) $\sqrt{abc}$
(D) $3\sqrt{abc}$

Correct Answer: (C) $\sqrt{abc}$

Solution:

The volume V of a cuboid with three adjacent faces having areas a, b, c is given by:

$$V = \sqrt{abc}$$

Quick Tip

For a cuboid, the volume can be calculated if three adjacent face areas are known using $V = \sqrt{abc}$.

---

92. The total surface area of a cube is 216 cm^2. Find its volume.

(A) 144 cm^3
(B) 196 cm^3

(C) 212 cm^3
(D) 216 cm^3

Correct Answer: (D) 216 cm^3

Solution:

Let the side length of the cube be s. The total surface area is given by:

$$6s^2 = 216$$
$$s^2 = 36 \Rightarrow s = 6$$
$$\text{Volume} = s^3 = 6^3 = 216$$

> 💡 Quick Tip
>
> For a cube, total surface area is $6s^2$, and volume is s^3.

---

93. If the ratio of volumes of two cubes is 1:64, then the ratio of their total surface areas is:

(A) 1:4
(B) 1:16
(C) 1:18
(D) 1:8

Correct Answer: (B) 1:16

Solution:
Since volume ratio $V_1 : V_2 = 1 : 64$, side length ratio is:

$$s_1 : s_2 = \sqrt[3]{1 : 64} = 1 : 4$$

Total surface area ratio:

$$s_1^2 : s_2^2 = (1^2) : (4^2) = 1 : 16$$

> 💡 Quick Tip
>
> Volume ratio of cubes is the cube of side ratio, and surface area ratio is the square of side ratio.

---

94. Two circular cylinders of equal volume have their heights in the ratio 1:2. The ratio of their radii is:
(A) $1 : \sqrt{2}$
(B) $\sqrt{2} : 1$

(C) 1:2
(D) 1:4

Correct Answer: (B) $\sqrt{2} : 1$

Solution:

Volume of cylinder:

$$V = \pi r^2 h$$

Since $h_1 : h_2 = 1 : 2$, keeping the volume constant:

$$r_1^2 : r_2^2 = 2 : 1$$
$$r_1 : r_2 = \sqrt{2} : 1$$

> Quick Tip
>
> For equal volume, if height ratio is given, square root of the inverse ratio gives the radius ratio.

---

95. If the curved surface area of a cylinder is 1760 cm^2 and its base diameter is 28 cm, find its height.

(A) 10 cm
(B) 15 cm
(C) 20 cm
(D) 40 cm

Correct Answer: (C) 20 cm

Solution:

Curved Surface Area (CSA) of a cylinder:

$$2\pi rh = 1760$$

Given $d = 28 \Rightarrow r = 14$,

$$2\pi(14)h = 1760$$
$$h = \frac{1760}{2\pi \times 14} = 20 \text{ cm}$$

> Quick Tip
>
> CSA of a cylinder is given by $2\pi rh$, solve for h when CSA and r are known.

---

96. If O is the center and R is the radius of a circle, and $\angle AOB = \theta$, then the length of arc AB is:

(A) $\frac{2\pi R\theta}{180}$
(B) $\frac{2\pi R\theta}{360}$
(C) $\frac{\pi R^2\theta}{180}$
(D) $\frac{\pi R^2\theta}{360}$

Correct Answer: (B) $\frac{2\pi R\theta}{360}$

Solution:

Arc length formula:

$$\ell = \frac{\theta}{360} \times 2\pi R$$

💡 Quick Tip

Arc length is a fraction of the circumference based on the given angle.

---

97. If l is slant height and r is the radius of a cone, the total surface area is:

(A) $\pi rl + r$
(B) $\pi rl + \pi r^2$
(C) $\pi rl + r^2$
(D) $\pi rl + 2r^2$

Correct Answer: (B) $\pi rl + \pi r^2$

Solution:

Total surface area of a cone:

$$\pi rl + \pi r^2$$

💡 Quick Tip

Total surface area of a cone = Curved surface area + Base area.

---

98. If the volume ratio of two spheres is 125:27, then their surface area ratio is:

(A) 9:25
(B) 25:9
(C) 5:3

(D) 3:5

Correct Answer: (B) 25:9

Solution:

$$r_1 : r_2 = \sqrt[3]{125 : 27} = 5 : 3$$
$$\text{Surface area ratio} = (5^2 : 3^2) = 25 : 9$$

Quick Tip

For spheres, surface area ratio is the square of the radius ratio.

---

99. A sphere of radius 8 cm is melted to form a cone of height 32 cm. The radius of the base of the cone is:
(A) 8 cm
(B) 9 cm
(C) 10 cm
(D) 12 cm

Correct Answer: (A) 8 cm

Solution:

Since the volume remains the same, equating the volumes:

$$\frac{4}{3}\pi r_s^3 = \frac{1}{3}\pi r_c^2 h$$

Given, $r_s = 8$ cm and $h = 32$ cm:

$$\frac{4}{3}\pi(8)^3 = \frac{1}{3}\pi r_c^2(32)$$

Canceling $\frac{1}{3}\pi$ from both sides:

$$4(512) = 32r_c^2$$

$$2048 = 32r_c^2$$

$$r_c^2 = \frac{2048}{32} = 64$$

$$r_c = \sqrt{64} = 8 \text{ cm}$$

Quick Tip

When a solid is reshaped, its volume remains the same. Use volume formulas to find unknown dimensions.

---

100. If the surface area of a sphere is 616 cm^2, then the diameter of the sphere is:

(A) 7 cm
(B) 14 cm
(C) 28 cm
(D) 56 cm

Correct Answer: (B) 14 cm

Solution:

The surface area of a sphere is given by:

$$4\pi r^2 = 616$$

Solving for r:

$$r^2 = \frac{616}{4\pi}$$

Approximating $\pi = 3.14$:

$$r^2 = \frac{616}{12.56} = 49$$

$$r = \sqrt{49} = 7 \text{ cm}$$

$$\text{Diameter} = 2r = 2(7) = 14 \text{ cm}$$

Quick Tip

For spheres, surface area is $4\pi r^2$. Solve for r, then find diameter as $2r$.

Section - B
(Short Answer Type Questions)

1. Using Euclid division algorithm find the HCF of 148 and 185.

Solution:
Using Euclid's division algorithm, we divide 185 by 148:

$$185 = 148 \times 1 + 37$$

Now, divide 148 by 37:

$$148 = 37 \times 4 + 0$$

Since the remainder is 0, the HCF is **37**.

> Quick Tip
>
> HCF using Euclid's Algorithm: Divide the larger number by the smaller number, take the remainder, and repeat until the remainder is 0.

---

2. Find the mean of the following data:

Variable	2	4	6	10	12
Frequency	3	2	3	1	2

Solution:
Mean is given by:

$$\text{Mean} = \frac{\sum(x_i \cdot f_i)}{\sum f_i}$$

Calculating:

$$\sum(x_i \cdot f_i) = (2 \times 3) + (4 \times 2) + (6 \times 3) + (10 \times 1) + (12 \times 2) = 6 + 8 + 18 + 10 + 24 = 66$$

$$\sum f_i = 3 + 2 + 3 + 1 + 2 = 11$$

$$\text{Mean} = \frac{66}{11} = 6$$

Thus, the mean is **6**.

> Quick Tip
>
> Mean of Data: Use the formula Mean $= \frac{\sum(x_i \cdot f_i)}{\sum f_i}$.

3. If the angle between two tangents drawn from an external point P to a circle of radius 3 cm and centre O is $60°$, find the length of OP.

Solution:
Using the formula:

$$OP = \frac{r}{\cos\frac{\theta}{2}}$$

$$OP = \frac{3}{\cos 30°}$$

$$= \frac{3}{\frac{\sqrt{3}}{2}} = \frac{6}{\sqrt{3}} = 2\sqrt{3} \text{ cm}$$

Thus, the length of OP is $\mathbf{2\sqrt{3}}$ cm.

> Quick Tip
>
> Length of OP in Circle: Use $OP = \frac{r}{\cos(\theta/2)}$.

---

4. For what value of k equations $kx + y = 1$ and $(k-1)x + 2y = 3$ have no solution?

Solution:
For no solution, the system of equations should be inconsistent, i.e.,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

From the given equations:

$$\frac{k}{k-1} = \frac{1}{2} \neq \frac{1}{3}$$

Solving:

$$2k = k - 1$$

$$2k - k = -1$$

$$k = -1$$

Thus, for $k = -1$, the system has no solution.

💡 Quick Tip

Inconsistent Linear Equations: If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then the system has no solution.

---

5. Write the solution of the equation $3x + y = 11$ in natural numbers.

Solution:
We need integer solutions where x, y are natural numbers.

Rearrange:

$$y = 11 - 3x$$

For natural y, $11 - 3x > 0 \Rightarrow x < \frac{11}{3} \approx 3.67$.

Possible integer values:

$$x = 1, y = 8 \quad (1, 8)$$

$$x = 2, y = 5 \quad (2, 5)$$

$$x = 3, y = 2 \quad (3, 2)$$

Thus, the natural number solutions are $(\mathbf{1}, \mathbf{8}), (\mathbf{2}, \mathbf{5}), (\mathbf{3}, \mathbf{2})$.

💡 Quick Tip

Natural Number Solutions: Solve for integer values that satisfy the equation.

---

6. Find the roots of the equation $\frac{1}{x} - \frac{1}{x-2} = 3$, $x \neq 0, 2$.

Solution:
Rewriting the equation:

$$\frac{(x-2) - x}{x(x-2)} = 3$$

$$\frac{-2}{x(x-2)} = 3$$

$$-2 = 3x(x-2)$$

$$3x^2 - 6x + 2 = 0$$

Using the quadratic formula:

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(2)}}{2(3)}$$

$$x = \frac{6 \pm \sqrt{36 - 24}}{6}$$

$$x = \frac{6 \pm \sqrt{12}}{6}$$

$$x = \frac{6 \pm 2\sqrt{3}}{6}$$

$$x = \frac{3 \pm \sqrt{3}}{3}$$

Thus, the roots are $\boldsymbol{\frac{3+\sqrt{3}}{3}}, \boldsymbol{\frac{3-\sqrt{3}}{3}}$.

> 💡 Quick Tip
>
> Roots of a Rational Equation: Convert fractions to quadratic equations and use the quadratic formula.

---

7. In $\triangle ABC$, $\angle C = 90^\circ$ and P, Q are midpoints of CA and CB respectively. Prove that $4AQ^2 = 4AC^2 + BC^2$.

Solution:
Since P and Q are midpoints,

$$AQ = \frac{1}{2}AB, \quad PQ = \frac{1}{2}BC$$

Applying the midpoint theorem:

$$4AQ^2 = AB^2 + 4PQ^2$$

Since $AB^2 = AC^2 + BC^2$ (Pythagoras theorem),

$$4AQ^2 = 4AC^2 + BC^2$$

Thus, the given equation is proved.

💡 Quick Tip

Midpoint Theorem in Right Triangle: If P, Q are midpoints of right triangle sides, use $4AQ^2 = 4AC^2 + BC^2$.

---

8. If point $A(x, 2)$ is equidistant from the points $B(8, -2)$ and $C(2, -2)$, find the value of x.

Solution:
Using the distance formula:

$$AB^2 = AC^2$$

$$(x-8)^2 + (2+2)^2 = (x-2)^2 + (2+2)^2$$

$$(x-8)^2 + 16 = (x-2)^2 + 16$$

Cancel 16 from both sides:

$$(x-8)^2 = (x-2)^2$$

$$x^2 - 16x + 64 = x^2 - 4x + 4$$

$$-16x + 64 = -4x + 4$$

$$-12x = -60$$

$$x = 5$$

Thus, $x = \mathbf{5}$.

💡 Quick Tip

Distance Formula: $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

---

9. In what ratio does the point $(-4, 6)$ divide the line segment joining $A(-6, 10)$ and $B(3, -8)$?

Solution:
Using section formula:

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \quad y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

Let the ratio be $k : 1$, so

$$-4 = \frac{3k + (-6)}{k + 1}, \quad 6 = \frac{-8k + 10}{k + 1}$$

Solving for k,

$$-4(k + 1) = 3k - 6$$

$$-4k - 4 = 3k - 6$$

$$-7k = -2$$

$$k = \frac{2}{7}$$

Thus, the required ratio is $\mathbf{2 : 7}$.

Quick Tip

Section Formula: If a point divides a line in ratio $m : n$, use $x = \frac{mx_2+nx_1}{m+n}$.

---

10. Find the area of the triangle whose vertices are $A(2, 1)$, $B(4, 5)$ and $C(0, 3)$.

Solution:
Using the formula:

$$\text{Area} = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$= \frac{1}{2} |2(5 - 3) + 4(3 - 1) + 0(1 - 5)|$$

$$= \frac{1}{2} |2(2) + 4(2) + 0|$$

$$= \frac{1}{2} |4 + 8| = \frac{1}{2} \times 12 = 6$$

Thus, the area is **6** square units.

💡 Quick Tip

Area of Triangle: Use determinant formula $A = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$.

---

11. Evaluate $\frac{\sec(90^\circ - \theta)\csc\theta - \tan(90^\circ - \theta)\cot\theta + \cos^2 25^\circ + \cos^2 65^\circ}{3\tan 27^\circ \tan 63^\circ}$.

Solution:
Using trigonometric identities:

$$\sec(90^\circ - \theta) = \csc\theta, \quad \tan(90^\circ - \theta) = \cot\theta$$

$$\Rightarrow \frac{\csc\theta\csc\theta - \cot\theta\cot\theta + \cos^2 25^\circ + \cos^2 65^\circ}{3\tan 27^\circ \tan 63^\circ}$$

Since,

$$\cos^2 25^\circ + \cos^2 65^\circ = 1$$

$$\Rightarrow \frac{1}{3(1)} = \frac{1}{3}$$

Thus, the value is $\mathbf{\frac{1}{3}}$.

💡 Quick Tip

Trigonometric Identities: Convert using $\sec(90^\circ - \theta) = \csc\theta$ and $\tan(90^\circ - \theta) = \cot\theta$.

---

12. Prove that $\frac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta} = \left(\frac{1 + \sin\theta}{\cos\theta}\right)^2$.

Solution:
Using identities:

$$\sec\theta + \tan\theta = \frac{1 + \sin\theta}{\cos\theta}, \quad \sec\theta - \tan\theta = \frac{1 - \sin\theta}{\cos\theta}$$

$$\frac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta} = \frac{\frac{1+\sin\theta}{\cos\theta}}{\frac{1-\sin\theta}{\cos\theta}}$$

$$= \frac{1+\sin\theta}{1-\sin\theta}$$

Multiplying numerator and denominator by $1+\sin\theta$:

$$= \frac{(1+\sin\theta)^2}{(1-\sin\theta)(1+\sin\theta)}$$

$$= \frac{(1+\sin\theta)^2}{1-\sin^2\theta}$$

Since $1-\sin^2\theta = \cos^2\theta$:

$$= \left(\frac{1+\sin\theta}{\cos\theta}\right)^2$$

Thus, the identity is proved.

Quick Tip

Quadratic Discriminant: If $D > 0$, roots are real and distinct; if $D = 0$, equal; if $D < 0$, complex.

---

13. Find the discriminant of the quadratic equation $\sqrt{2}x^2 - x - \sqrt{2} = 0$ and hence find the nature of the roots.

Solution:
Discriminant formula:

$$D = b^2 - 4ac$$

For $\sqrt{2}x^2 - x - \sqrt{2} = 0$,

$$a = \sqrt{2}, \quad b = -1, \quad c = -\sqrt{2}$$

$$D = (-1)^2 - 4(\sqrt{2})(-\sqrt{2})$$

$$= 1 - 4(2)$$

$$= 1 - 8 = -7$$

Since $D < 0$, the roots are complex and conjugate.

Quick Tip

Checking Term in A.P.: Use $a_n = a + (n-1)d$ and check if n is an integer.

---

14. For what value of k will the equation $kx(x-2)+6=0$ have equal roots?

Solution:
Expanding:

$$kx^2 - 2kx + 6 = 0$$

For equal roots:

$$D = 0$$

$$(-2k)^2 - 4(k)(6) = 0$$

$$4k^2 - 24k = 0$$

$$4k(k-6) = 0$$

$$k = 0 \quad \text{or} \quad k = 6$$

Thus, $k = 6$ ensures equal roots.

Quick Tip

Irrational Proofs: Show that the given number has a term involving an irrational number.

---

15. Check whether 301 is a term of the sequence $5, 11, 17, 23, \ldots$.

Solution:
Given A.P.:

$$a = 5, \quad d = 11 - 5 = 6$$

General term:

$$a_n = a + (n-1)d$$

$$301 = 5 + (n-1)6$$

$$301 - 5 = (n-1)6$$

$$296 = (n-1)6$$

$$n - 1 = \frac{296}{6} = 49.33$$

Since n is not an integer, 301 is not a term.

> 💡 Quick Tip
>
> To check if a number is part of an arithmetic progression, use the formula for the n-th term of the A.P., and solve for n. If n is not an integer, the number is not in the sequence.

---

16. Prove that $(2+\sqrt{3})^2$ is not a rational number.

Solution:
Expanding:

$$(2+\sqrt{3})^2 = 4 + 4\sqrt{3} + 3 = 7 + 4\sqrt{3}$$

Since $4\sqrt{3}$ is irrational, $7 + 4\sqrt{3}$ is also irrational.

Thus, it is not a rational number.

> 💡 Quick Tip
>
> Zeroes of Quadratic Equation: Use the quadratic formula and verify sum-product properties.

---

17. Find the zeroes of the quadratic polynomial $3x^2 - x - 4$ and verify the relationship between the zeroes and the coefficients.

Solution:
Using the quadratic formula:

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(-4)}}{2(3)}$$

$$x = \frac{1 \pm \sqrt{1+48}}{6}$$

$$x = \frac{1 \pm \sqrt{49}}{6}$$

$$x = \frac{1 \pm 7}{6}$$

$$x = \frac{8}{6} = \frac{4}{3}, \quad x = \frac{-6}{6} = -1$$

Verifying:

Sum of zeroes:

$$\frac{4}{3} + (-1) = \frac{4}{3} - \frac{3}{3} = \frac{1}{3} = -\frac{b}{a} = -\frac{-1}{3}$$

Product of zeroes:

$$\frac{4}{3} \times (-1) = -\frac{4}{3} = \frac{c}{a} = \frac{-4}{3}$$

Thus, the relationship is verified.

Quick Tip

Zeroes of Quadratic Equation:Use the quadratic formula and verify sum-product properties.

---

18. Divide polynomial $2x^4 + 3x^3 - 2x^2 - 9x - 12$ by polynomial $x^2 - 3$.

Solution: Using polynomial division:

$$\frac{2x^4 + 3x^3 - 2x^2 - 9x - 12}{x^2 - 3}$$

1. Divide $2x^4$ by x^2:

$$2x^2$$

Multiply:

$$2x^4 - 6x^2$$

Subtract:

$$(2x^4 + 3x^3 - 2x^2 - 9x - 12) - (2x^4 - 6x^2)$$

$$3x^3 + 4x^2 - 9x - 12$$

2. Divide $3x^3$ by x^2:

$$3x$$

Multiply:

$$3x^3 - 9x$$

Subtract:

$$(3x^3 + 4x^2 - 9x - 12) - (3x^3 - 9x)$$

$$4x^2 - 12$$

3. Divide $4x^2$ by x^2:

$$4$$

Multiply:

$$4x^2 - 12$$

Subtract:

$$(4x^2 - 12) - (4x^2 - 12) = 0$$

Thus, the quotient is:

$$2x^2 + 3x + 4$$

Quick Tip

Polynomial Division: Divide each term successively using $x^2 - 3$.

---

19. Find the sum of 10 terms of A.P. 9, 17, 25, ...

Solution:
Given A.P.:

$$a = 9, \quad d = 17 - 9 = 8, \quad n = 10$$

Sum of first n terms:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{10} = \frac{10}{2}[2(9) + (10-1)(8)]$$

$$= 5[18 + 72] = 5 \times 90 = 450$$

Thus, $S_{10} = \mathbf{450}$.

Quick Tip

Sum of A.P.: Use $S_n = \frac{n}{2}[2a + (n-1)d]$.

---

20. Find the sum of the first 15 multiples of 8.

Solution:
Multiples of 8 form an A.P.:

$$8, 16, 24, \ldots$$

$$a = 8, \quad d = 8, \quad n = 15$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{15} = \frac{15}{2}[2(8) + (15-1)(8)]$$

$$= \frac{15}{2}[16 + 112] = \frac{15}{2} \times 128 = 960$$

Thus, $S_{15} = \mathbf{960}$.

Quick Tip

Basic Proportionality Theorem: If $\frac{PS}{SQ} = \frac{PT}{TR}$, then $ST \parallel QR$.

---

21. In triangle $\triangle PQR$, two points S and T are on sides PQ and PR such that $\frac{PS}{SQ} = \frac{PT}{TR}$ and $\angle PST = \angle PRQ$. Prove that $\triangle PQR$ is an isosceles triangle.

Solution:
Since $\frac{PS}{SQ} = \frac{PT}{TR}$, by the Basic Proportionality Theorem,

$$ST \parallel QR$$

Since $\angle PST = \angle PRQ$, corresponding angles are equal, proving $\triangle PST \sim \triangle PRQ$.

From similarity:

$$\frac{PS}{PQ} = \frac{PT}{PR}$$

Since proportions hold, $PQ = PR$, proving $\triangle PQR$ is isosceles.

> **Quick Tip**
>
> Reciprocal Roots Condition: If one root is reciprocal of the other, use $\alpha\beta = 1$.

---

22. If one zero of the polynomial $(a^2+9)x^2+13x+6a$ is reciprocal of the other, find a.

Solution:
If roots are reciprocal:

$$\alpha \cdot \beta = 1$$

Using the product of roots formula:

$$\frac{c}{a} = 1$$

$$\frac{6a}{a^2+9} = 1$$

$$6a = a^2 + 9$$

$$a^2 - 6a + 9 = 0$$

$$(a-3)^2 = 0$$

$$a = 3$$

Thus, $a = \mathbf{3}$.

> **Quick Tip**
>
> Equation Formation from Word Problems: Convert conditions into equations and solve them.

---

23. Write the equation for: On adding 1 to the numerator of a fraction it becomes $\frac{1}{2}$, and on adding 1 to the denominator it becomes $\frac{1}{3}$.

Solution:
Let fraction be $\frac{x}{y}$.

$$\frac{x+1}{y} = \frac{1}{2}$$

Cross multiplying:

$$2(x+1) = y$$

$$2x + 2 = y$$

$$y = 2x + 2$$

For second condition:

$$\frac{x}{y+1} = \frac{1}{3}$$

$$3x = y + 1$$

$$y = 3x - 1$$

Solving equations:

$$2x + 2 = 3x - 1$$

$$x = 3, \quad y = 8$$

Thus, fraction is $\frac{3}{8}$.

> **Quick Tip**
>
> Substitution Method for Linear Equations: Express one variable in terms of another, then substitute.

---

24. Solve the pair of equations $8x + 5y = 9$ and $3x + 2y = 4$ by substitution method.

Solution:
From $3x + 2y = 4$:

$$y = \frac{4 - 3x}{2}$$

Substituting in $8x + 5y = 9$:

$$8x + 5\left(\frac{4 - 3x}{2}\right) = 9$$

$$16x + 20 - 15x = 18$$

$$x = -2$$

Substituting $x = -2$ in $y = \frac{4-3x}{2}$:

$$y = \frac{4 - 3(-2)}{2} = \frac{4 + 6}{2} = 5$$

Thus, $x = \mathbf{-2}, y = \mathbf{5}$.

Quick Tip

Quick Tips for Solving Linear Equations by Substitution:

- **Step 1:** Solve one equation for one variable in terms of the other.
- **Step 2:** Substitute this expression into the second equation.
- **Step 3:** Solve for the remaining variable.
- **Step 4:** Substitute the obtained value back into the first equation to find the second variable.
- **Step 5:** Always check the solution by substituting it into both original equations.
- **Tip:** Choose the equation where solving for a variable is easiest to simplify calculations.

---

34. Prove that

$$\left(\frac{\sin A}{1 + \cos A} + \frac{1 + \cos A}{\sin A}\right)\left(\frac{\sin A}{1 - \cos A} + \frac{1 - \cos A}{\sin A}\right) = 4 \csc A \cot A.$$

Solution:
Consider the first term:

$$\frac{\sin A}{1+\cos A}+\frac{1+\cos A}{\sin A}$$

Taking LCM:

$$=\frac{\sin^2 A+(1+\cos A)^2}{\sin A(1+\cos A)}$$

Expanding:

$$=\frac{\sin^2 A+1+2\cos A+\cos^2 A}{\sin A(1+\cos A)}$$

$$=\frac{2+2\cos A}{\sin A(1+\cos A)}$$

$$=\frac{2(1+\cos A)}{\sin A(1+\cos A)}=\frac{2}{\sin A}$$

Similarly,

$$\frac{\sin A}{1-\cos A}+\frac{1-\cos A}{\sin A}$$

Solving similarly, we get:

$$=\frac{2}{\sin A}$$

Multiplying both terms:

$$\frac{2}{\sin A}\times\frac{2}{\sin A}=\frac{4}{\sin^2 A}$$

$$=4\csc A\cot A$$

Thus, the identity is proved.

Quick Tip

Trigonometric Proofs: Use trigonometric identities and simplifications.

---

35. Metallic spheres of radii 6 cm, 8 cm, and 10 cm are melted to form a single solid sphere. Find the radius of the resulting sphere.

Solution:
Volume of a sphere:

$$V = \frac{4}{3}\pi r^3$$

Total volume:

$$V_1 + V_2 + V_3 = V_{\text{new}}$$

$$\frac{4}{3}\pi(6^3) + \frac{4}{3}\pi(8^3) + \frac{4}{3}\pi(10^3) = \frac{4}{3}\pi R^3$$

$$\frac{4}{3}\pi(216 + 512 + 1000) = \frac{4}{3}\pi R^3$$

$$\frac{4}{3}\pi(1728) = \frac{4}{3}\pi R^3$$

$$R^3 = 1728$$

$$R = 12 \text{ cm}$$

Thus, the radius of the new sphere is **12** cm.

Quick Tip
Volume Conservation in Sphere Problems: Total initial volume = final volume.

---

36. The angles of elevation of the top of a tower from two points at distances of 4 m and 9 m from the base of the tower, which are in the same straight line, are complementary. Prove that the height of the tower is 6 m.

Solution:
Let the height of the tower be h.

Given angles:

$$\theta \text{ and } 90^\circ - \theta$$

Using tan formula:

$$\tan\theta = \frac{h}{4}, \quad \tan(90^\circ - \theta) = \cot\theta = \frac{h}{9}$$

$$\frac{h}{4} \times \frac{h}{9} = 1$$

$$\frac{h^2}{36} = 1$$

$$h^2 = 36$$

$$h = 6$$

Thus, the height of the tower is **6** m.

> Quick Tip
>
> Height and Distance Problems:Use tan and cot formulas.

---

37. Draw a circle of radius 6 cm. From a point 10 cm away from its center, construct the pair of tangents to the circle and measure their lengths.

Solution:
Using tangent length formula:

$$\text{Length of tangent} = \sqrt{d^2 - r^2}$$

$$= \sqrt{10^2 - 6^2}$$

$$= \sqrt{100 - 36} = \sqrt{64} = 8 \text{ cm}$$

Thus, the length of each tangent is **8** cm.

> Quick Tip
>
> Tangent Length Formula: $\sqrt{d^2 - r^2}$ where d is distance from center.

---

38. Find the median of the following data:

Class Interval 70-75	40-45	45-50	50-55	55-60	60-65	65-70
Frequency 2	2	3	8	6	6	3

Solution:
Cumulative Frequency (CF):

Class Interval	CF
40-45	2
45-50	5
50-55	13
55-60	19
60-65	25
65-70	28
70-75	30

Median class: $50 - 55$ (since $N/2 = 30/2 = 15$)

Using median formula:

$$\text{Median} = L + \frac{\frac{N}{2} - CF}{f} \times h$$

$$= 50 + \frac{15 - 5}{8} \times 5$$

$$= 50 + \frac{10}{8} \times 5$$

$$= 50 + 6.25$$

$$= 56.25$$

Thus, the median is **56.25**.

Quick Tip

Finding Median in Class Data: Use cumulative frequency and median class formula.