

BSEAP Mathematics Question Paper 2023 with solution

Time Allowed :3 hours 15 minutes

Maximum Marks :100

Total Questions :33

Instructions

Read the following instructions carefully:

1. In the duration of **3 hours 15 minutes**, **15 minutes** of time is allotted to read the question paper.
2. **All** answers shall be written in the answer booklet **only**.
3. The question paper consists of **4 Sections** and **33 questions**.
4. Internal choice is available in **Section IV only**.
5. Answers shall be written **neatly and legibly**.

SECTION-I

Note :

1. Answer **all** the questions in **one word** or a **phrase**.
2. Each question carries **1 mark**.

1. Find the LCM of 12, 15 and 21.

Solution:

The prime factorization of the numbers are:

$$12 = 2^2 \times 3, \quad 15 = 3 \times 5, \quad 21 = 3 \times 7$$

Thus, the LCM = $2^2 \times 3 \times 5 \times 7 = 420$.

 Quick Tip

To find LCM, always take the highest power of all prime factors involved.

2. Write the following set in roster form:

$$A = \{x : x \text{ is a natural number less than } 6\}$$

Solution:

The roster form of the set clearly lists all elements explicitly:

$$A = \{1, 2, 3, 4, 5\}$$

 Quick Tip

Roster form explicitly lists all elements of a set without repetition.

3. Choose the correct answer satisfying the following statements:

Statement (P): The degree of the quadratic polynomial is 2.

Statement (Q): Maximum no. of zeroes of a quadratic polynomial is 2.

(A) Both (P) and (Q) are true.

(B) (P) is true, (Q) is false.

(C) (P) is false, (Q) is true.

(D) Both (P) and (Q) are false.

Correct Answer: (A) Both (P) and (Q) are true.

Solution:

Statement (P) is correct because, by definition, a quadratic polynomial always has a degree equal to 2. Statement (Q) is also correct because the maximum number of zeroes (roots) of any polynomial equals its degree, and hence, a quadratic polynomial can have a maximum of 2 zeroes. Therefore, both statements (P) and (Q) are correct.

 Quick Tip

Always remember: the degree of a polynomial determines the maximum possible number of its zeroes.

4. Assertion : $3x + 6y = 3900$, $x + 2y = 1300$ represent coincident lines and have infinite number of solutions.

Reason : If $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ and $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then, these lines are coincident lines.

Choose the correct answer.

(A) Both Assertion and Reason are true, Reason is supporting the Assertion

(B) Both Assertion and Reason are true, But Reason is not supporting the Assertion

(C) Assertion is true, but the Reason is false

(D) Assertion is false, but the Reason is true

Correct Answer: (A) Both Assertion and Reason are true, Reason is supporting the Assertion

Solution:

Both given equations are proportional, as:

$$\frac{3}{1} = \frac{6}{2} = \frac{3900}{1300} = 3$$

Thus, the lines represented by these equations coincide and have infinitely many solutions. Therefore, both Assertion and Reason are correct, and Reason correctly explains the Assertion.

💡 Quick Tip

If two linear equations have proportional coefficients, they represent coincident lines with infinitely many solutions.

5. The number of roots of the equation $5x^2 - 6 = 0$ is?

Solution:

Given the quadratic equation:

$$5x^2 - 6 = 0 \Rightarrow x^2 = \frac{6}{5}$$

Taking square roots on both sides, we get two solutions:

$$x = \sqrt{\frac{6}{5}} \quad \text{and} \quad x = -\sqrt{\frac{6}{5}}$$

Hence, the equation has exactly 2 roots.

💡 Quick Tip

A quadratic equation always has exactly two roots, which may be real or imaginary depending on the discriminant.

6. State Thales theorem.

Solution:

Thales' theorem states that if a line is drawn parallel to one side of a triangle, intersecting the other two sides at distinct points, then it divides those sides in the same ratio.

💡 Quick Tip

Thales' theorem is useful for solving problems involving proportional segments in triangles.

7. Find the number of tangents drawn at the end points of the diameter.

Solution:

At each endpoint of the diameter of a circle, exactly two tangents can be drawn, which are perpendicular to each other. Therefore, the total number of tangents at the endpoints of the diameter is 2.

 Quick Tip

Tangents drawn at the endpoints of a diameter are always perpendicular.

8. Find the volume of a cube, whose side is 4 cm.

Solution:

The volume of a cube is given by the formula:

$$V = s^3$$

Substituting $s = 4$ cm:

$$V = 4^3 = 64 \text{ cm}^3$$

Thus, the volume of the cube is 64 cm^3 .

 Quick Tip

The volume of a cube is always the cube of its side length.

9. Match the following:

- | | |
|------------------|--|
| P) $\sin \theta$ | i) $\frac{1}{\sec \theta}$ |
| Q) $\cos \theta$ | ii) $\sqrt{\sec^2 \theta - 1}$ |
| R) $\tan \theta$ | iii) $\frac{\sec^2 \theta - 1}{\sec^2 \theta}$ |

Choose the correct answer.

- (A) $P \rightarrow (i), Q \rightarrow (ii), R \rightarrow (iii)$
(B) $P \rightarrow (iii), Q \rightarrow (i), R \rightarrow (ii)$
(C) $P \rightarrow (iii), Q \rightarrow (ii), R \rightarrow (i)$
(D) $P \rightarrow (i), Q \rightarrow (iii), R \rightarrow (ii)$

Correct Answer: (B) $P \rightarrow (iii), Q \rightarrow (i), R \rightarrow (ii)$

Solution:

From trigonometric identities:

$$\sin \theta = \frac{\sec^2 \theta - 1}{\sec^2 \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\tan \theta = \sqrt{\sec^2 \theta - 1}$$

Thus, the correct matching is:

$$P \rightarrow (iii), \quad Q \rightarrow (i), \quad R \rightarrow (ii)$$

which corresponds to option (B).

💡 Quick Tip
Memorizing standard trigonometric identities helps in solving such matching problems quickly.

10. "You are observing the top of your school building at an angle of elevation 60° from a point which is at 20 meters distance from the foot of the building."

Draw a rough diagram to the above situation.

Solution:

The given problem represents a right-angled triangle where: - The observer's position forms the base ($AB = 20$ m). - The height of the building is to be determined (BC). - The angle of elevation is 60° .

By using the formula:

$$\tan 60^\circ = \frac{\text{opposite}}{\text{adjacent}} = \frac{BC}{AB}$$

$$\sqrt{3} = \frac{BC}{20}$$

$$BC = 20\sqrt{3} \approx 34.64 \text{ m}$$

Thus, the height of the building is approximately 34.64 meters.

💡 Quick Tip
Use trigonometric ratios like tan, sin, or cos to solve problems involving heights and distances.

11. If $P(E) = 0.05$, what is the probability of not 'E'?

Solution:

By the probability rule:

$$P(\neg E) = 1 - P(E)$$

$$P(\neg E) = 1 - 0.05 = 0.95$$

Thus, the probability of not E is 0.95.

 Quick Tip

The sum of the probability of an event and its complement is always 1.

12. Find the mean of the given data.

2, 3, 7, 6, 6, 3, 8

Solution:

The mean is calculated as:

$$\text{Mean} = \frac{\sum \text{data values}}{\text{total number of values}}$$

Summing the values:

$$2 + 3 + 7 + 6 + 6 + 3 + 8 = 35$$

Total number of values = 7.

$$\text{Mean} = \frac{35}{7} = 5$$

Thus, the mean of the given data is 5.

 Quick Tip

The mean (average) is obtained by dividing the sum of all values by the total count.

SECTION-II**Note :**

1. Answer **all** the questions
2. Each question carries **4 mark**.

13. If $A = \{3, 4, 5, 6\}$, $B = \{5, 6, 7, 8, 9\}$, then illustrate $A \cap B$ in a Venn diagram.**Solution:**

The intersection of two sets, $A \cap B$, includes elements that are common in both sets:

$$A \cap B = \{5, 6\}$$

A Venn diagram would represent A and B as overlapping circles, with $\{5, 6\}$ in the intersection region.

 Quick Tip

The intersection of two sets contains only the elements that are common to both.

14. 6 pencils and 4 pens together cost 50, whereas 5 pencils and 6 pens together cost 46. Express the above statements in the form of Linear equations.

Solution:

Let the cost of one pencil be x and the cost of one pen be y .

From the given conditions, we can form the following linear equations:

$$6x + 4y = 50$$

$$5x + 6y = 46$$

Thus, these two equations represent the given conditions in the form of linear equations.

 Quick Tip

Linear equations in two variables can be represented in the form $ax + by = c$.

15. Check whether $(x - 2)^2 + 1 = 2x - 3$ is a quadratic equation or not.

Solution:

Expanding the given equation:

$$(x - 2)^2 + 1 = 2x - 3$$

$$x^2 - 4x + 4 + 1 = 2x - 3$$

$$x^2 - 4x + 5 = 2x - 3$$

Rearrange the terms:

$$x^2 - 4x + 5 - 2x + 3 = 0$$

$$x^2 - 6x + 8 = 0$$

Since the equation is in the form $ax^2 + bx + c = 0$ where $a = 1, b = -6, c = 8$, it is a quadratic equation.

 Quick Tip

A quadratic equation always has a variable raised to the power of 2 as its highest degree.

16. Write the formula to find n^{th} term of A.P. and explain the terms in it.

Solution:

The general formula for finding the n^{th} term of an arithmetic progression (A.P.) is given by:

$$a_n = a + (n - 1)d$$

where:

- a_n represents the n^{th} term of the A.P.
- a is the **first term** of the sequence.
- d is the **common difference**, which is the difference between any two consecutive terms.
- n is the **number of terms** in the sequence.

This formula helps in finding any term of an arithmetic sequence without listing all preceding terms.

 Quick Tip

In an arithmetic progression, each term increases or decreases by a fixed amount called the **common difference** (d).

17. Find the distance between the two points $(7, 8)$ and $(-2, 3)$.

Solution:

To calculate the distance between two points (x_1, y_1) and (x_2, y_2) , we use the **distance formula**:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substituting $(x_1, y_1) = (7, 8)$ and $(x_2, y_2) = (-2, 3)$:

$$\begin{aligned} d &= \sqrt{(-2 - 7)^2 + (3 - 8)^2} \\ &= \sqrt{(-9)^2 + (-5)^2} = \sqrt{81 + 25} = \sqrt{106} \end{aligned}$$

Thus, the **distance between the two points** is $\sqrt{106}$.

 Quick Tip

The **distance formula** is derived from the **Pythagorean theorem** in coordinate geometry.

18. From a point Q , the length of the tangent to a circle is 24 cm, and the distance of Q from the centre is 25 cm. Find the radius of the circle.

Solution:

The given problem forms a **right-angled triangle** where: - OQ is the **hypotenuse** (distance from the centre to point Q) = 25 cm. - PQ is the **tangent length** = 24 cm. - OP is the **radius** of the circle.

Using **Pythagoras' theorem**:

$$OQ^2 = OP^2 + PQ^2$$

Substituting values:

$$25^2 = r^2 + 24^2$$

$$625 = r^2 + 576$$

$$r^2 = 49 \quad \Rightarrow \quad r = 7 \text{ cm}$$

Thus, the **radius of the circle** is **7 cm**.

 Quick Tip

The **tangent to a circle** is always **perpendicular to the radius** at the point of contact.

19. If $\cos A = \frac{12}{13}$, then find $\sin A$ and $\tan A$.

Solution:

Using the **Pythagorean identity**:

$$\sin^2 A + \cos^2 A = 1$$

Substituting $\cos A = \frac{12}{13}$:

$$\sin^2 A = 1 - \left(\frac{12}{13}\right)^2$$

$$\sin^2 A = 1 - \frac{144}{169} = \frac{25}{169}$$

$$\sin A = \frac{5}{13}$$

Now, calculating $\tan A$:

$$\tan A = \frac{\sin A}{\cos A} = \frac{\frac{5}{13}}{\frac{12}{13}} = \frac{5}{12}$$

Thus, the **values are**: $\sin A = \frac{5}{13}$ and $\tan A = \frac{5}{12}$.

 Quick Tip

The **Pythagorean identity** helps find missing trigonometric values using known ratios.

20. A die is thrown once, find the probability of getting:

i) A prime number

ii) An odd number

Solution:

A die has six faces: $\{1, 2, 3, 4, 5, 6\}$.

i) Probability of getting a prime number:

Prime numbers in this range: $\{2, 3, 5\}$

Total favorable outcomes = 3, total possible outcomes = 6.

$$P(\text{Prime}) = \frac{3}{6} = \frac{1}{2}$$

ii) Probability of getting an odd number:

Odd numbers in this range: $\{1, 3, 5\}$

Total favorable outcomes = 3, total possible outcomes = 6.

$$P(\text{Odd}) = \frac{3}{6} = \frac{1}{2}$$

Thus, the **probability of getting a prime number** is $\frac{1}{2}$ and the **probability of getting an odd number** is $\frac{1}{2}$.

 Quick Tip

The **probability of an event** is calculated as:

$$P(E) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}}$$

SECTION—III

Note :

1. Answer **all** the questions
2. Each question carries **4 mark**.

21. Find x , if $2 \log 5 + \frac{1}{2} \log 9 - \log 3 = \log x$.

Solution:

We start with the given equation:

$$2 \log 5 + \frac{1}{2} \log 9 - \log 3 = \log x$$

Using the logarithmic identity $a \log b = \log b^a$, we rewrite:

$$\log 5^2 + \log 9^{\frac{1}{2}} - \log 3 = \log x$$

$$\log 25 + \log 3 - \log 3 = \log x$$

Since $\log 3 - \log 3 = 0$, we are left with:

$$\log 25 = \log x$$

By comparing, we get:

$$x = 25$$

Thus, the value of x is **25**.

 Quick Tip

Use logarithmic properties: $a \log b = \log b^a$ and $\log A + \log B = \log(A \times B)$ to simplify expressions.

22. Check whether -3 and 3 are the zeroes of the polynomial $x^4 - 81$.

Solution:

To check whether -3 and 3 are zeroes of the given polynomial:

$$f(x) = x^4 - 81$$

Step 1: Substitute $x = -3$

$$\begin{aligned} f(-3) &= (-3)^4 - 81 \\ &= 81 - 81 = 0 \end{aligned}$$

Step 2: Substitute $x = 3$

$$\begin{aligned} f(3) &= (3)^4 - 81 \\ &= 81 - 81 = 0 \end{aligned}$$

Since $f(-3) = 0$ and $f(3) = 0$, both -3 and 3 satisfy the polynomial equation. Thus, -3 and 3 are zeroes of the polynomial $x^4 - 81$.

 Quick Tip

To check if a number is a zero of a polynomial, substitute it into the equation. If the result is zero, it is a valid zero.

23. Solve the pair of linear equations using the elimination method.

$$3x + 2y = -1$$

$$2x + 3y = -9$$

Solution:

We use the **elimination method** to solve the given equations.

Step 1: Multiply both equations to make the coefficients of one variable equal.

Multiply the first equation by 3 and the second by 2 to equalize the coefficients of y :

$$(3x + 2y) \times 3 = (-1) \times 3 \Rightarrow 9x + 6y = -3$$

$$(2x + 3y) \times 2 = (-9) \times 2 \Rightarrow 4x + 6y = -18$$

Step 2: Subtract the equations.

$$(9x + 6y) - (4x + 6y) = (-3) - (-18)$$

$$5x = 15$$

$$x = 3$$

Step 3: Substitute $x = 3$ into the first equation.

$$3(3) + 2y = -1$$

$$9 + 2y = -1$$

$$2y = -10$$

$$y = -5$$

Thus, the solution is $\mathbf{x = 3, y = -5}$.

 Quick Tip

The elimination method is useful when the coefficients of a variable can be made equal by multiplication, allowing for easier cancellation.

24. Rohan's mother is 26 years older than him. The product of their ages after 3 years will be 360 years. Write the quadratic equation to find Rohan's present age.

Solution:

Let Rohan's present age be x years. Then his mother's age is:

$$(x + 26) \text{ years}$$

After 3 years, their ages will be:

$$(x + 3) \text{ and } (x + 26 + 3) = (x + 29)$$

According to the given condition:

$$(x + 3)(x + 29) = 360$$

Expanding:

$$x^2 + 29x + 3x + 87 = 360$$

$$x^2 + 32x + 87 - 360 = 0$$

$$x^2 + 32x - 273 = 0$$

Thus, the required quadratic equation is:

$$\mathbf{x^2 + 32x - 273 = 0}$$

 Quick Tip

To form a quadratic equation, express unknown values in terms of a single variable and use given conditions to set up an equation.

25. Draw a tangent to a given circle with centre 'O' from a point 'R' outside the circle. How many tangents can be drawn to the circle from that point?

Solution:

From a point **outside** a given circle, exactly **two** tangents can be drawn to the circle.

A tangent is a line that touches the circle at exactly one point without crossing it. Since an external point can have two such contact points on the circle, the total number of tangents that can be drawn is:

💡 Quick Tip

From any point outside a circle, exactly two tangents can be drawn to the circle.

26. An oil drum is in the shape of a cylinder having the following dimensions:

Diameter = 2 m, Height = 7 m.

The painter charges 3 per m² to paint the drum. Find the total charges to be paid to the painter for 10 drums.

Solution:

The total surface area of a cylinder is given by the formula:

$$\text{Total Surface Area} = 2\pi r(r + h)$$

Given:

$$\text{Radius} = \frac{\text{Diameter}}{2} = \frac{2}{2} = 1 \text{ m}, \quad \text{Height} = 7 \text{ m}$$

$$\text{Surface Area} = 2\pi(1)(1 + 7) = 2\pi(8) = 16\pi \text{ m}^2$$

Approximating $\pi = 3.14$:

$$\text{Surface Area} = 16 \times 3.14 = 50.24 \text{ m}^2$$

The cost per square meter is 3, so the cost to paint one drum is:

$$50.24 \times 3 = 150.72$$

For 10 drums:

$$150.72 \times 10 = 1507.2$$

Thus, the total charge to be paid is **1507.20**.

💡 Quick Tip

To find the total surface area of a cylinder, use the formula: $2\pi r(r + h)$.

27. Show that $\frac{1 - \tan^2 A}{\cot^2 A - 1} = \tan^2 A$.

Solution:

Rewriting $\cot^2 A$ in terms of $\tan^2 A$:

$$\cot^2 A = \frac{1}{\tan^2 A}$$

Substituting in the given equation:

$$\begin{aligned} & \frac{1 - \tan^2 A}{\frac{1}{\tan^2 A} - 1} \\ &= \frac{1 - \tan^2 A}{\frac{1 - \tan^2 A}{\tan^2 A}} \\ &= \tan^2 A \end{aligned}$$

Thus, the given equation holds true.

💡 Quick Tip

Use trigonometric identities like $\cot A = \frac{1}{\tan A}$ to simplify expressions.

28. A survey conducted on 20 households in a locality by a group of students resulted in the following frequency table for the number of family members in a household.

Family size 9 – 11	1 – 3	3 – 5	5 – 7	7 – 9
No. of families 1	7	8	2	2

Find the mode of the data.

Solution:

To find the mode, we use the **mode formula** for grouped data:

$$\text{Mode} = L + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

where:

- L = Lower boundary of the modal class
- f_1 = Frequency of the modal class
- f_0 = Frequency of the class preceding the modal class
- f_2 = Frequency of the class succeeding the modal class
- h = Class width

Step 1: Identify the modal class

The modal class is the class with the highest frequency. From the table, the highest frequency is 8, corresponding to the class 3 – 5.

$$L = 3, \quad f_1 = 8, \quad f_0 = 7, \quad f_2 = 2, \quad h = 2$$

Step 2: Apply the formula

$$\begin{aligned}\text{Mode} &= 3 + \left(\frac{8 - 7}{(2 \times 8) - 7 - 2} \right) \times 2 \\ &= 3 + \left(\frac{1}{16 - 9} \right) \times 2 \\ &= 3 + \left(\frac{1}{7} \right) \times 2 \\ &= 3 + 0.29 = 3.29\end{aligned}$$

Thus, the **mode of the data is approximately 3.29**.

 Quick Tip

The modal class is the class with the highest frequency. Use the formula for grouped data to estimate the mode accurately.

SECTION-IV

Note :

1. Answer **all** the questions
2. Each question carries **8 marks**.
3. Each Question has internal choice

29. a) Prove that $6 + \sqrt{2}$ is irrational.

Solution:

We will prove this by contradiction. Assume that $6 + \sqrt{2}$ is a rational number.

Step 1: Express it as a rational number.

Let $6 + \sqrt{2} = r$, where r is rational.

Rearrange:

$$\sqrt{2} = r - 6$$

Since r is rational, $r - 6$ is also rational. However, we know that $\sqrt{2}$ is irrational, and a rational number cannot be equal to an irrational number.

Step 2: Contradiction

Since our assumption leads to a contradiction, $6 + \sqrt{2}$ must be irrational.

 Quick Tip

The sum or difference of a rational and an irrational number is always irrational.

OR

b) Show that $a_1, a_2, a_3, \dots, a_n$ form an AP where a_n is defined as below.

i) $a_n = 3 + 4n$

ii) $a_n = 9 - 5n$

Also find the sum of the first 15 terms in each case.

Solution:

To prove that a_n forms an arithmetic progression (AP), we check if the difference between consecutive terms is constant.

For $a_n = 3 + 4n$:

First term: $a_1 = 3 + 4(1) = 7$

Second term: $a_2 = 3 + 4(2) = 11$

Third term: $a_3 = 3 + 4(3) = 15$

Common difference:

$$d = a_2 - a_1 = 11 - 7 = 4$$

Since the difference is constant, it forms an AP.

Sum of first 15 terms:

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{15} = \frac{15}{2}(2(7) + 14(4))$$

$$= \frac{15}{2}(14 + 56) = \frac{15}{2}(70) = 525$$

For $a_n = 9 - 5n$:

First term: $a_1 = 9 - 5(1) = 4$

Second term: $a_2 = 9 - 5(2) = -1$

Third term: $a_3 = 9 - 5(3) = -6$

Common difference:

$$d = a_2 - a_1 = -1 - 4 = -5$$

Since the difference is constant, it forms an AP.

Sum of first 15 terms:

$$S_{15} = \frac{15}{2}(2(4) + 14(-5))$$

$$= \frac{15}{2}(8 - 70) = \frac{15}{2}(-62) = -465$$

Thus, both sequences form an AP, and the sums of the first 15 terms are **525** and **-465** respectively.

💡 Quick Tip

In an arithmetic progression (AP), the sum of n terms is given by:

$$S_n = \frac{n}{2}(2a + (n - 1)d)$$

30. a) Find the volume of the largest right circular cone that can be cut out of a cube whose edge is 7 cm.

Solution:

The largest right circular cone that can be cut from a cube has its base inscribed within one face of the cube, and its height equals the edge length of the cube.

Step 1: Identify given values.

Edge of the cube = 7 cm

$$\text{Radius of the base} = \frac{\text{Edge}}{2} = \frac{7}{2} = 3.5 \text{ cm}$$

Height of the cone = 7 cm

Step 2: Use the volume formula for a cone.

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi(3.5)^2(7)$$

$$V = \frac{1}{3}\pi(12.25)(7)$$

$$V = \frac{85.75\pi}{3} \approx 89.92 \text{ cm}^3$$

Thus, the volume of the cone is approximately **89.92 cm³**.

💡 Quick Tip

The largest cone inside a cube has a height equal to the cube's edge and a base radius equal to half the edge.

OR

b) If $A = \{1, 2, 3, 4, 5\}$, $B = \{3, 4, 5, 6, 7\}$, $C = \{1, 3, 5, 7\}$, $D = \{2, 4, 6, 8\}$, find the following:

i) $A \cup B$

Solution:

The union of two sets A and B consists of all elements present in either A or B or both.

$$\begin{aligned} A \cup B &= \{1, 2, 3, 4, 5\} \cup \{3, 4, 5, 6, 7\} \\ &= \{1, 2, 3, 4, 5, 6, 7\} \end{aligned}$$

💡 Quick Tip

Union ($\cup$) of two sets includes all unique elements from both sets.

ii) $B \cup C$

Solution:

$$\begin{aligned} B \cup C &= \{3, 4, 5, 6, 7\} \cup \{1, 3, 5, 7\} \\ &= \{1, 3, 4, 5, 6, 7\} \end{aligned}$$

💡 Quick Tip

The union of two sets combines all elements, removing any duplicates.

iii) $A \cup D$

Solution:

$$\begin{aligned} A \cup D &= \{1, 2, 3, 4, 5\} \cup \{2, 4, 6, 8\} \\ &= \{1, 2, 3, 4, 5, 6, 8\} \end{aligned}$$

💡 Quick Tip

The union operation does not repeat elements. It only includes distinct values.

iv) $B - D$

Solution:

The difference $B - D$ consists of elements in B that are not in D .

$$\begin{aligned} B - D &= \{3, 4, 5, 6, 7\} - \{2, 4, 6, 8\} \\ &= \{3, 5, 7\} \end{aligned}$$

💡 Quick Tip

The difference ($-$) between two sets includes elements present in the first set but not in the second.

v) $A \cap B$

Solution:

The intersection $A \cap B$ consists of elements that are common in both A and B .

$$\begin{aligned} A \cap B &= \{1, 2, 3, 4, 5\} \cap \{3, 4, 5, 6, 7\} \\ &= \{3, 4, 5\} \end{aligned}$$

💡 Quick Tip

The intersection ($\cap$) operation finds elements that are present in both sets.

vi) $B \cap D$

Solution:

$$\begin{aligned} B \cap D &= \{3, 4, 5, 6, 7\} \cap \{2, 4, 6, 8\} \\ &= \{4, 6\} \end{aligned}$$

💡 Quick Tip

The intersection of two sets contains only the elements that are found in both sets.

vii) $C \cap D$

Solution:

$$C \cap D = \{1, 3, 5, 7\} \cap \{2, 4, 6, 8\}$$

$$= \emptyset$$

Since there are no common elements, the result is the empty set $\emptyset$.

💡 Quick Tip

If two sets have no common elements, their intersection is the empty set ($\emptyset$).

viii) $A - D$

Solution:

The difference $A - D$ consists of elements in A that are not in D .

$$A - D = \{1, 2, 3, 4, 5\} - \{2, 4, 6, 8\}$$

$$= \{1, 3, 5\}$$

💡 Quick Tip

Set difference ($-$) includes elements in the first set that are not in the second set.

31. a) The distribution below gives the weights of 30 students of a class. Find the median weight of the students.

Weight (in kg)	40 – 45	45 – 50	50 – 55	55 – 60	60 – 65	65 – 70	70 – 75
Number of students	2	3	8	6	6	3	2

Solution:

To find the **median class**, we use the following steps:

Step 1: Compute cumulative frequency (CF)

Class Interval	Frequency (f)	Cumulative Frequency (CF)
40 – 45	2	2
45 – 50	3	5
50 – 55	8	13
55 – 60	6	19
60 – 65	6	25
65 – 70	3	28
70 – 75	2	30

Step 2: Identify the median class

The total number of students = 30. The median class corresponds to the cumulative frequency just greater than $\frac{30}{2} = 15$.

$$\text{Median class} = 55 - 60$$

Step 3: Apply the median formula

$$\text{Median} = L + \left(\frac{\frac{n}{2} - CF}{f} \right) \times h$$

where:

- $L = 55$ (Lower boundary of median class)
- $n = 30$ (Total frequency)
- $CF = 13$ (Cumulative frequency before median class)
- $f = 6$ (Frequency of median class)
- $h = 5$ (Class width)

$$\begin{aligned}\text{Median} &= 55 + \left(\frac{15 - 13}{6} \right) \times 5 \\ &= 55 + \left(\frac{2}{6} \times 5 \right) \\ &= 55 + \left(\frac{10}{6} \right) \\ &= 55 + 1.67 = 56.67\end{aligned}$$

Thus, the **median weight is approximately 56.67 kg**.

💡 Quick Tip

To find the median class, locate the class interval where the cumulative frequency exceeds $\frac{n}{2}$.

OR

b) Find the value of ‘b’ for which the points $A(1, 2)$, $B(3, b)$, and $C(5, -4)$ are collinear.

Solution:

Three points are **collinear** if the area of the triangle formed by them is zero. The area of a triangle formed by points (x_1, y_1) , (x_2, y_2) , (x_3, y_3) is given by:

$$\text{Area} = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Substituting the given points:

$$\frac{1}{2} |1(b - (-4)) + 3(-4 - 2) + 5(2 - b)| = 0$$

$$\frac{1}{2} |1(b + 4) + 3(-6) + 5(2 - b)| = 0$$

$$\frac{1}{2} |(b + 4) - 18 + 10 - 5b| = 0$$

$$\frac{1}{2} |-4b - 4| = 0$$

$$|-4b - 4| = 0$$

$$-4b - 4 = 0$$

$$-4b = 4$$

$$b = -1$$

Thus, the value of b for which the points are collinear is **-1**.

 Quick Tip

Three points are collinear if the area of the triangle formed by them is zero.

32. a) A 1.5 m tall boy is looking at the top of a temple which is 30 meters in height from a point at a certain distance. The angle of elevation from his eye to the top of the crown of the temple increases from 30° to 60° as he walks towards the temple. Find the distance he walked towards the temple.

Solution:

Let the initial distance of the boy from the temple be x meters and the distance he walked be d meters.

The height of the temple above the boy's eye level:

$$\text{Effective height} = 30 - 1.5 = 28.5 \text{ m}$$

Using the tan function in the right-angled triangles:

$$\tan 30^\circ = \frac{28.5}{x + d}, \quad \tan 60^\circ = \frac{28.5}{d}$$

Substituting values:

$$\frac{28.5}{x + d} = \frac{1}{\sqrt{3}}, \quad \frac{28.5}{d} = \sqrt{3}$$

Solving for $x + d$ and d :

$$x + d = 28.5\sqrt{3}, \quad d = \frac{28.5}{\sqrt{3}}$$

$$d = \frac{28.5 \times \sqrt{3}}{3} = 9.5\sqrt{3}$$

Approximating:

$$d \approx 9.5 \times 1.732 = 16.45 \text{ m}$$

Thus, the boy walked approximately 16.45 meters towards the temple.

 Quick Tip

Use trigonometric ratios such as $\tan$ for height-distance problems involving right triangles.

OR

b) One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting:

- i) A king of red color**
- ii) A face card**
- iii) A jack of hearts**
- iv) A spade**

Solution:

The total number of cards in a deck = 52.

i) Probability of getting a king of red color: There are 2 red kings (King of hearts and King of diamonds).

$$P(\text{Red King}) = \frac{2}{52} = \frac{1}{26}$$

ii) Probability of getting a face card: There are 12 face cards (4 kings, 4 queens, 4 jacks).

$$P(\text{Face Card}) = \frac{12}{52} = \frac{3}{13}$$

iii) Probability of getting a jack of hearts: There is only one jack of hearts in a deck.

$$P(\text{Jack of Hearts}) = \frac{1}{52}$$

iv) Probability of getting a spade: There are 13 spades in a deck.

$$P(\text{Spade}) = \frac{13}{52} = \frac{1}{4}$$

 Quick Tip

Probability is calculated as:

$$P(E) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}}$$

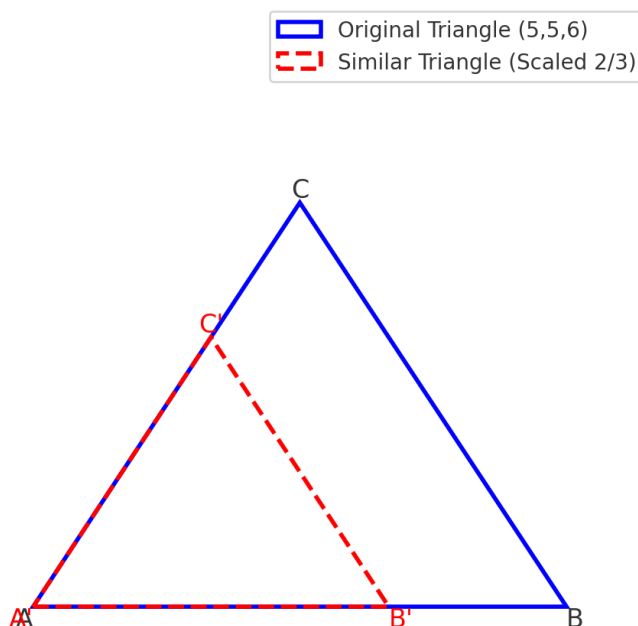
33. a) Construct a triangle of sides 5 cm, 5 cm, and 6 cm. Then, construct a triangle similar to it, whose sides are $\frac{2}{3}$ of the corresponding sides of the triangle.

Solution:

Step 1: Construct the given triangle. 1. Draw a base of 6 cm. 2. From each endpoint of the base, draw two arcs of 5 cm radius and mark the intersection as the third vertex. 3. Join all points to form the isosceles triangle.

Step 2: Construct the similar triangle. 1. Draw a ray from one vertex making an acute angle with the base. 2. Mark three equal segments (since $3 > 2$ in $\frac{2}{3}$). 3. Connect the second division point to the original vertex. 4. Draw a parallel line to the opposite side of the triangle using a compass. 5. The new triangle is similar to the original one.

Triangle Construction and Similar Triangle (Scaling 2/3)

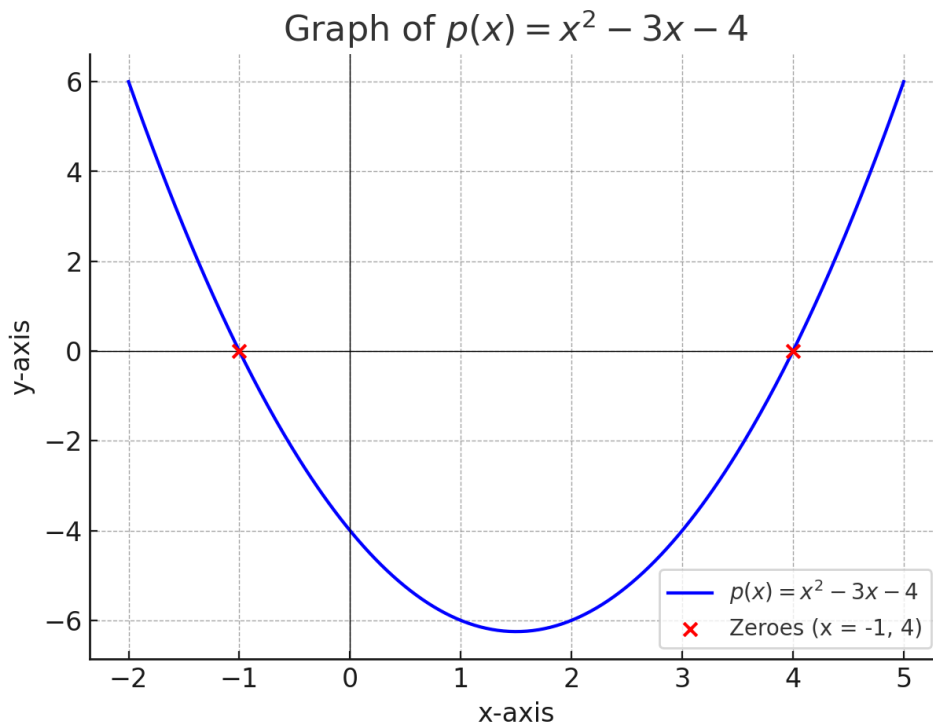


 Quick Tip

To construct a similar triangle, divide the base into equal parts according to the given ratio.

OR

b) Draw a graph of $p(x) = x^2 - 3x - 4$ and hence find the zeroes of the polynomial.



Solution:

To find the zeroes of the polynomial $p(x) = x^2 - 3x - 4$, we solve:

$$x^2 - 3x - 4 = 0$$

Step 1: Factorize the equation.

$$(x - 4)(x + 1) = 0$$

Solving:

$$x - 4 = 0 \Rightarrow x = 4, \quad x + 1 = 0 \Rightarrow x = -1$$

Thus, the zeroes of the polynomial are $x = 4$ and $x = -1$.

Step 2: Plot the graph. 1. Construct a table of values for $y = x^2 - 3x - 4$. 2. Plot points and draw a parabolic curve. 3. The points where the curve crosses the x-axis give the zeroes at $x = 4$ and $x = -1$.

💡 Quick Tip

The zeroes of a polynomial correspond to the x-values where the curve intersects the x-axis.