BITSAT 2026 May 27 Shift 1

Question Paper (Memory Based) with Solutions

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General Instructions

(i) **Duration:** The total duration of the examination is 3 hours (180 minutes).

(ii) **Total Marks:** The complete paper carries a maximum of 390 marks.

(iii) **Structure:** The paper has 4 Sections:

- **Part 1:** 30 Multiple Choice Questions (Physics).
- **Part 2:** 30 Multiple Choice Questions (Chemistry).
- **Part 3:** 10 Multiple Choice Questions (English Proficiency), 20 Multiple Choice Questions (Logical Reasoning)
- **Part 4:** 40 Multiple Choice Questions (Mathematics/Biology)

(iv) **Compulsory Questions:** All 130 questions are compulsory,and +12 Questions(Optional Extra Questions)

(v) Each question has four options. Only **one** option is correct.

(vi) **Correct Answer:** +3 marks.

(vii) **Incorrect Answer:** −1 (Negative marking).

(viii) **Unanswered/Marked for Review:** 0 marks.

PHYSICS

1. A body of mass m is dropped from a height h. What is its kinetic energy just before it hits the ground?

(A) mgh

(B) $\frac{1}{2}mgh$

(C) $2mgh$

(D) mgh^2

Correct Answer: (A) mgh

Solution:

Step 1 : Understanding the Question:

This problem deals with the motion of an object undergoing free fall under the influence of a uniform gravitational field. We are asked to determine the final kinetic energy of a body of mass m that is dropped from rest at an initial height h, right before it reaches the ground.

Step 2 : Key Formulas and Approach:

The most straightforward approach to solve this problem is to apply the Law of Conservation of Mechanical Energy. This principle states that the total mechanical energy in an isolated system remains constant if only conservative forces (such as gravity) are acting.

The total mechanical energy E_{total} at any point is given by:

$$E_{\text{total}} = KE + PE$$

where the kinetic energy is $KE = \frac{1}{2}mv^2$ and the gravitational potential energy is $PE = mgh$.

Step 3 : Detailed Explanation:

- At the release height h, the body is at rest because it is "dropped", meaning its initial velocity is zero. Consequently, its initial kinetic energy KE_i is equal to 0.

- At this peak height, the gravitational potential energy of the mass is at its maximum, which is expressed as $PE_i = mgh$.

- Summing these individual components gives the total initial mechanical energy of the

system: $E_i = PE_i + KE_i = mgh + 0 = mgh$.

- As the body falls freely under gravity, its potential energy decreases as it is converted entirely into kinetic energy.
- Just before making contact with the ground, the height of the body is zero, which means the final potential energy PE_f becomes 0.
- The final mechanical energy is therefore composed entirely of kinetic energy: $E_f = PE_f + KE_f = 0 + KE_f$.
- Because the total mechanical energy of the system must remain constant ($E_i = E_f$), we can equate the two states: $mgh = KE_f$.

Step 4 : Final Answer:

The kinetic energy of the body just before it hits the ground is mgh, which corresponds to option (A).

Quick Tip: For any object falling freely in a conservative force field without air resistance, the potential energy lost during the descent is directly converted into the kinetic energy gained by the object.

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2. Two charges q_1 and q_2 are placed at a distance r. If the distance between them is doubled, the electrostatic force between them becomes:

(A) One-fourth of the original force

(B) Half of the original force

(C) Four times the original force

(D) Twice the original force

Correct Answer: (A) One-fourth of the original force

Solution:

Step 1 : Understanding the Question:

This problem concerns the electrostatic interaction between two static point charges separated by a certain distance. We are asked to evaluate how the electrostatic force changes when the distance of separation between these charges is doubled.

Step 2 : Key Formulas and Approach:

The electrostatic force between two stationary point charges is governed by Coulomb's Law. This fundamental law states that the force of attraction or repulsion is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.

The equation is written as:

$$F = k\frac{q_1 q_2}{r^2}$$

where k is Coulomb's constant, q_1 and q_2 represent the magnitudes of the charges, and r is the separation distance. This shows an inverse-square relationship:

$$F \propto \frac{1}{r^2}$$

Step 3 : Detailed Explanation:

- Let us define the initial state of the system where the distance between the two charges is r. The initial electrostatic force is represented by $F_1 = \frac{kq_1q_2}{r^2}$.

- According to the problem statement, the distance between the charges is doubled, which gives us a new distance of $r' = 2r$.

- We can write the expression for the new electrostatic force F_2 by substituting the new

distance r' into the formula: $F_2 = \frac{kq_1q_2}{(r')^2} = \frac{kq_1q_2}{(2r)^2}$.

- Simplifying the denominator by squaring the term gives: $F_2 = \frac{kq_1q_2}{4r^2}$.
- To find the relationship between the two forces, we can factor out the constant from the expression: $F_2 = \frac{1}{4}\left(\frac{kq_1q_2}{r^2}\right)$.
- Substituting the initial force F_1 back into this equation yields: $F_2 = \frac{1}{4}F_1$.

Step 4 : Final Answer:

The electrostatic force between the two charges becomes one-fourth of the original force, which corresponds to option (A).

Quick Tip: Any physical interaction governed by an inverse-square law will scale by the inverse square of the distance multiplier; doubling the distance reduces the force to $1/2^2 = 1/4$ of its original value.

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3. A light ray enters a glass slab of refractive index $\mu = 1.5$ from air. What is the speed of light inside the glass slab? (Speed of light in air $c = 3 \times 10^8$ m/s)

(A) 2×10^8 m/s

(B) 4.5×10^8 m/s

(C) 3×10^8 m/s

(D) 1.5×10^8 m/s

Correct Answer: (A) 2×10^8 m/s

Solution:

Step 1 : Understanding the Question:

This question deals with geometrical optics and the propagation of electromagnetic waves

through different media. We are required to determine the speed of a light ray after it transitions from air into a transparent glass medium with a given refractive index.

Step 2 : Key Formulas and Approach:

The absolute refractive index μ of a medium is defined as the ratio of the speed of light in a vacuum or air c to its speed in that specific medium v.

The formula is expressed as:

$$\mu = \frac{c}{v}$$

To find the velocity of light in the medium, we can rearrange the formula to solve for v:

$$v = \frac{c}{\mu}$$

Step 3 : Detailed Explanation:

- Identify the given physical constants from the problem text: the refractive index of the glass slab is $\mu = 1.5$, and the speed of light in air is $c = 3 \times 10^8$ m/s.
- Set up the rearranged formula to compute the speed of light inside the denser medium: $v = \frac{c}{\mu}$.
- Substitute the given values into this mathematical relation to obtain: $v = \frac{3\times10^8 \text{ m/s}}{1.5}$.
- Simplify the fraction by noting that 1.5 is equivalent to $\frac{3}{2}$, which gives: $v = \frac{3\times10^8}{\frac{3}{2}} = 2 \times 10^8$ m/s.
- This calculation confirms that the speed of light decreases when entering a medium with a higher refractive index.

Step 4 : Final Answer:

The speed of light inside the glass slab is 2×10^8 m/s, which corresponds to option (A).

Quick Tip: A higher refractive index corresponds to an optically denser medium, which means that light travels slower in that medium compared to air or a vacuum.

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4. A gas undergoes an adiabatic process. During this process:

(A) No heat is exchanged with the surroundings

(B) The temperature of the gas remains constant

(C) The pressure of the gas remains constant

(D) The volume of the gas remains constant

Correct Answer: (A) No heat is exchanged with the surroundings

Solution:

Step 1 : Understanding the Question:

This question focuses on basic thermodynamic terminology and the definitions of various processes. We must identify the core characteristic that defines an adiabatic process from the given options.

Step 2 : Key Formulas and Approach:

The processes occurring within a thermodynamic system are categorized based on physical properties that are held constant. This can be analyzed using the First Law of Thermodynamics:

$$dU = dQ - dW$$

where dU is the change in internal energy, dQ is the heat added to the system, and dW is the work done by the system.

Step 3 : Detailed Explanation:

- By definition, an adiabatic process is one in which the system is thermally isolated from its surroundings, meaning there is no exchange of heat energy. This condition is represented mathematically as $dQ = 0$.
- Let us evaluate each option to find the correct match based on thermodynamic definitions.
- Option (A) states that no heat is exchanged with the surroundings, which perfectly matches the condition $dQ = 0$ for an adiabatic process.
- Option (B) describes an isothermal process, where the temperature T of the system remains constant throughout ($dT = 0$).
- Option (C) describes an isobaric process, where the pressure P of the system remains constant ($dP = 0$).
- Option (D) describes an isochoric (or isometric) process, where the volume V of the system remains constant ($dV = 0$).

Step 4 : Final Answer:

The correct characteristic of an adiabatic process is that no heat is exchanged with the surroundings, which corresponds to option (A).

Quick Tip: Adiabatic processes typically occur when a system is very well-insulated or when a process happens so rapidly that there is no time for heat transfer to take place.

5. Using the standard electrode potential, find out the pair between which redox reaction is not feasible. $E^{\ominus}$ values: $Fe^{3+}/Fe^{2+} = +0.77V$; $I_2/I^- = +0.54V$; $Cu^{2+}/Cu = +0.34V$; $Ag^+/Ag = +0.80V$.

(A) Fe^{3+} and I^-

(B) Ag^+ and Cu

(C) Fe^{3+} and Cu

(D) Ag and Fe^{3+}

Correct Answer: (D) Ag and Fe^{3+}

Solution:

Step 1 : Understanding the Question:

This problem concerns electrochemistry and the spontaneity of redox reactions. We need to analyze which combination of the given species cannot undergo a feasible redox reaction based on their standard reduction potentials ($E^{\ominus}$).

Step 2 : Key Formulas and Approach:

For any redox reaction to be spontaneous and feasible, the standard cell potential $E^{\ominus}_{cell}$ must be positive. This relates directly to the change in standard Gibbs free energy:

$$\Delta G^{\ominus} = -nFE^{\ominus}_{cell}$$

For a positive $E^{\ominus}_{cell}$ (and a spontaneous reaction), the species with the higher standard reduction potential must undergo reduction at the cathode, while the species with the lower reduction potential must undergo oxidation at the anode:

$$E^{\ominus}_{cell} = E^{\ominus}_{reduction} - E^{\ominus}_{oxidation} = E^{\ominus}_{cathode} - E^{\ominus}_{anode} > 0$$

Step 3 : Detailed Explanation:

- Let us analyze the feasibility of Option (A), involving the reaction between Fe^{3+} and I^-.

Here, Fe^{3+} is reduced to Fe^{2+} ($E^{\ominus} = 0.77V$) and I^- is oxidized to I_2 ($E^{\ominus} = 0.54V$). The cell potential is $E^{\ominus}_{cell} = 0.77 - 0.54 = +0.23V$. Since this value is positive, the reaction is feasible.

- Let us analyze the feasibility of Option (B), involving the reaction between Ag^+ and Cu. Here, Ag^+ is reduced to Ag ($E^{\ominus} = 0.80V$) and Cu is oxidized to Cu^{2+} ($E^{\ominus} = 0.34V$). The cell potential is $E^{\ominus}_{cell} = 0.80 - 0.34 = +0.46V$. Since this value is positive, the reaction is feasible.

- Let us analyze the feasibility of Option (C), involving the reaction between Fe^{3+} and Cu. Here, Fe^{3+} is reduced to Fe^{2+} ($E^{\ominus} = 0.77V$) and Cu is oxidized to Cu^{2+} ($E^{\ominus} = 0.34V$). The cell potential is $E^{\ominus}_{cell} = 0.77 - 0.34 = +0.43V$. Since this value is positive, the reaction is feasible.

- Let us analyze the feasibility of Option (D), involving the reaction between Ag and Fe^{3+}. In this scenario, Fe^{3+} is intended to act as the cathode (reduction, $E^{\ominus} = 0.77V$) and Ag is intended to act as the anode (oxidation, $E^{\ominus} = 0.80V$). The standard cell potential is $E^{\ominus}_{cell} = 0.77 - 0.80 = -0.03V$. Since the cell potential is negative, this reaction is not feasible.

Step 4 : Final Answer:

The redox reaction between Ag and Fe^{3+} is not feasible, which corresponds to option (D).

Quick Tip: To quickly assess feasibility, remember that a species can only oxidize another species if it has a higher (more positive) reduction potential than that species.

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6. The reaction $A(g) \rightarrow P(g) + Q(g) + R(g)$ follows first-order kinetics with a half-life of 69.3 s at 500°C. Starting with pure A in a container at 500°C and a pressure of 0.4 atm, what will be the total pressure of the system after 230 s?

(A) 1.15 atm

(B) 1.32 atm

(C) 1.22 atm

(D) 1.12 atm

Correct Answer: (D) 1.12 atm

Solution:

Step 1 : Understanding the Question:

This problem belongs to chemical kinetics, focusing specifically on a first-order gas-phase decomposition reaction. We are required to find the total pressure of the gas mixture after a given time interval, starting with a known initial pressure of pure reactant A.

Step 2 : Key Formulas and Approach:

For a first-order reaction, the rate constant k is related to the half-life $t_{1/2}$ by the equation:

$$k = \frac{\ln(2)}{t_{1/2}} = \frac{0.693}{t_{1/2}}$$

The pressure of the remaining reactant over time is determined using the integrated rate law:

$$P_A = P_0 e^{-kt}$$

Using stoichiometry, we can relate the remaining pressure of A to the pressures of the products and find the total pressure.

Step 3 : Detailed Explanation:

- Step 1: Calculate the first-order rate constant k. Given that $t_{1/2} = 69.3$ s, we have:

$$k = \frac{0.693}{69.3} = 0.01 \text{ s}^{-1}$$

- Step 2: Determine the remaining partial pressure of reactant A at $t = 230$ s. Using the

integrated rate law:

$$P_A = 0.4 \times e^{-0.01 \times 230} = 0.4 \times e^{-2.3}$$

- Using the approximation $e^{-2.3} \approx 0.1$, we find the partial pressure of A remaining:

$$P_A = 0.4 \times 0.1 = 0.04 \text{ atm}$$

- Step 3: Establish the pressure stoichiometry of the reaction. Let x be the decrease in the partial pressure of A:

$$A(g) \rightarrow P(g) + Q(g) + R(g)$$

Initial pressures: $P_0 \quad 0 \quad 0 \quad 0$
Pressures at time t: $(P_0 - x) \quad x \quad x \quad x$

- Since $P_A = 0.4 - x = 0.04$ atm, it follows that $x = 0.36$ atm.

- Calculate the total pressure of the system, which is the sum of all individual partial pressures:

$$P_T = (0.4 - x) + x + x + x = 0.4 + 2x$$

$$P_T = 0.4 + 2(0.36) = 0.4 + 0.72 = 1.12 \text{ atm}$$

Step 4 : Final Answer:

The total pressure of the system after 230 seconds is 1.12 atm, which corresponds to option (D).

Quick Tip: For gas-phase reactions of type $A(g) \rightarrow nP(g)$, you can express the total pressure in terms of the initial pressure and the degree of reaction to avoid multiple algebraic steps.

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7. Electron affinity is positive, when:

(A) $O^- \rightarrow O^-$

(B) $O^- \rightarrow O^{2-}$

(C) $O \rightarrow O^+$

(D) $O \rightarrow O^{2+}$

Correct Answer: (B) $O^- \rightarrow O^{2-}$

Solution:

Step 1 : Understanding the Question:

This question covers periodic properties, specifically focusing on the energetics of electron gain processes. We need to identify which of the given processes corresponds to a positive electron affinity, which represents an endothermic process where energy is absorbed.

Step 2 : Key Formulas and Approach:

The first electron affinity is the energy change when an electron is added to a neutral gaseous atom. This process is generally exothermic (negative change in enthalpy):

$$X(g) + e^- \rightarrow X^-(g) \quad (\Delta H < 0)$$

When an additional electron is added to a negatively charged ion, strong electrostatic repulsion occurs between the incoming electron and the negatively charged ion. Work must be done to overcome this repulsion, making the process endothermic:

$$X^{-}(g) + e^{-} \rightarrow X^{2-}(g) \quad (\Delta H > 0)$$

Step 3 : Detailed Explanation:

- Let us analyze the formation of oxide ions. Adding the first electron to a neutral oxygen atom releases energy because the attractive nuclear force dominates:

$$O(g) + e^{-} \rightarrow O^{-}(g) \quad (\Delta H < 0)$$

- To form the divalent oxide ion (O^{2-}), a second electron must be added to the monovalent oxygen anion (O^{-}):

$$O^{-}(g) + e^{-} \rightarrow O^{2-}(g)$$

- The incoming electron experiences significant electrostatic repulsion from the existing electron cloud of the O^{-} anion.

- External energy must be supplied to force the electron into the anion against this repulsive force, making the process endothermic. Therefore, the electron affinity is positive.

- The other processes listed in options (C) and (D) describe the removal of electrons from oxygen, which represents ionization energy rather than electron affinity.

Step 4 : Final Answer:

The electron affinity is positive for the process $O^{-} \rightarrow O^{2-}$, which corresponds to option (B).

Quick Tip: While the first electron affinity for most non-metals is negative (exothermic), the second and subsequent electron affinities are always positive (endothermic) due to inter-electronic repulsion.

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8. The ionic radii in Å of N^{3-}, O^{2-}, and F^{-} are respectively:

(A) $1.71, 1.40$ and 1.36

(B) $1.71, 1.36$ and 1.40

(C) $1.36, 1.40$ and 1.71

(D) $1.36, 1.71$ and 1.40

Correct Answer: (A) $1.71, 1.40$ and 1.36

Solution:

Step 1 : Understanding the Question:

This question belongs to the topic of periodic properties, specifically dealing with the ionic radii of isoelectronic species. We are asked to identify the correct set of ionic radii for the ions N^{3-}, O^{2-}, and F^{-}.

Step 2 : Key Formulas and Approach:

Isoelectronic species are atoms or ions that possess the same number of electrons. For these species, the size of the ion is determined by the effective nuclear charge.

The nuclear charge is given by the atomic number Z. As Z increases, the ratio of protons to electrons increases:

$$\text{Ionic Radius} \propto \frac{1}{Z}$$

A higher nuclear charge exerts a stronger attractive pull on the electron cloud, pulling it closer to the nucleus and decreasing the ionic radius.

Step 3 : Detailed Explanation:

- Step 1: Count the total number of electrons in each given ion.

N^{3-} has $7+3=10$ electrons.

O^{2-} has $8+2=10$ electrons.

F^- has $9+1=10$ electrons. Since all three have 10 electrons, they are isoelectronic.

- Step 2: Compare their atomic numbers (nuclear charge, Z).
  The atomic numbers are: $Z(N)=7$, $Z(O)=8$, and $Z(F)=9$.

- Step 3: Analyze the trend in ionic radii. Since nitrogen has the lowest nuclear charge ($Z=7$), its nucleus exerts the weakest pull on the 10 electrons, making N^{3-} the largest ion.

- Conversely, fluorine has the highest nuclear charge ($Z=9$), exerting the strongest pull on the electron cloud, making F^- the smallest ion.

- The descending order of their ionic radii is: $N^{3-} > O^{2-} > F^-$.

- Step 4: Match this trend with the values given in the options. The values must follow a decreasing sequence: 1.71 Å > 1.40 Å > 1.36 Å.

Step 4 : Final Answer:

The ionic radii of N^{3-}, O^{2-}, and F^- are 1.71, 1.40, and 1.36 Å respectively, which corresponds to option (A).

Quick Tip: For isoelectronic species, the ionic radius always decreases as the atomic number (nuclear charge) increases, or as the negative charge decreases.

9. Find the sum of the series $\left(x+\frac{1}{x}\right)^2+\left(x^2+\frac{1}{x^2}\right)^2+\left(x^3+\frac{1}{x^3}\right)^2+\cdots$ up to n terms.

(A) $\frac{x^{2n}-1}{x^2-1}\times\frac{x^{2n+2}+1}{x^{2n}}+2n$

(B) $\frac{x^{2n}+1}{x^2+1}\times\frac{x^{2n+2}-1}{x^{2n}}-2n$

(C) $\frac{x^{2n}-1}{x^2-1}\times\frac{x^{2n}-1}{x^{2n}}-2n$

(D) None of these

Correct Answer: (A) $\frac{x^{2n}-1}{x^2-1}\times\frac{x^{2n+2}+1}{x^{2n}}+2n$

Solution:

Step 1 : Understanding the Question:

This problem asks us to find the sum of a series where each term is the square of a sum containing a variable x and its reciprocal raised to consecutive integer powers. We need to sum this series up to n terms.

Step 2 : Key Formulas and Approach:

The general term of the series can be expanded algebraically using the identity:

$$(a+b)^2=a^2+b^2+2ab$$

The expanded terms can then be grouped into separate geometric progressions (G.P.). The formula for the sum of the first n terms of a G.P. is:

$$S_n=\frac{a(r^n-1)}{r-1}$$

where a is the first term and r is the common ratio.

Step 3 : Detailed Explanation:

- Write down the general term T_r of the given series:

$$T_r = \left(x^r + \frac{1}{x^r}\right)^2$$

- Expand T_r using the algebraic square identity:

$$T_r = (x^r)^2 + \left(\frac{1}{x^r}\right)^2 + 2(x^r)\left(\frac{1}{x^r}\right) = x^{2r} + \frac{1}{x^{2r}} + 2$$

- Express the total sum of n terms S_n by summing T_r from $r = 1$ to n:

$$S_n = \sum_{r=1}^{n}\left(x^{2r} + \frac{1}{x^{2r}} + 2\right)$$

- Split the summation into three individual parts:

$$S_n = \sum_{r=1}^{n} x^{2r} + \sum_{r=1}^{n} \frac{1}{x^{2r}} + \sum_{r=1}^{n} 2$$

- Evaluate the first summation, which is a G.P. with first term $a = x^2$ and common ratio $r = x^2$:

$$\sum_{r=1}^{n} x^{2r} = \frac{x^2(x^{2n} - 1)}{x^2 - 1}$$

- Evaluate the second summation, which is a G.P. with first term $a = \frac{1}{x^2}$ and common ratio $r = \frac{1}{x^2}$:

$$\sum_{r=1}^{n} \frac{1}{x^{2r}} = \frac{1}{x^2}\frac{1 - (\frac{1}{x^2})^n}{1 - \frac{1}{x^2}} = \frac{x^{2n} - 1}{x^{2n}(x^2 - 1)}$$

- Evaluate the third constant summation: $\sum_{r=1}^{n} 2 = 2n$.

- Combine the three individual sums together:

$$S_n = \frac{x^2(x^{2n}-1)}{x^2-1} + \frac{x^{2n}-1}{x^{2n}(x^2-1)} + 2n$$

- Factor out the common term $\frac{x^{2n}-1}{x^2-1}$ to simplify:

$$S_n = \frac{x^{2n}-1}{x^2-1}\left(x^2 + \frac{1}{x^{2n}}\right) + 2n = \frac{x^{2n}-1}{x^2-1}\left(\frac{x^{2n+2}+1}{x^{2n}}\right) + 2n$$

Step 4 : Final Answer:

The sum of the series is $\frac{x^{2n}-1}{x^2-1} \times \frac{x^{2n+2}+1}{x^{2n}} + 2n\times$, which corresponds to option (A).

Quick Tip: When faced with complex terms in a series, expanding the powers first will often reveal simpler component series, such as geometric or arithmetic progressions.

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10. A person invites 10 friends to dinner and places them such that 4 are at one round table and 6 are at another round table. The total number of ways in which he can arrange the guests is:

(A) 10!/6!

(B) 10!/24

(C) 9!/24

(D) None of these

Correct Answer: (B) 10!/24

Solution:

Step 1 : Understanding the Question:

This problem is in the area of combinatorics, involving both grouping (combinations) and circular permutations. We need to determine the total number of ways to distribute 10 friends into two groups of 4 and 6, and then arrange them around two distinct circular tables.

Step 2 : Key Formulas and Approach:

First, we select which friends sit at which table using the combination formula:

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Second, we use the circular permutation formula. The number of unique ways to arrange m distinct items around a circular table is given by:

$$P_{\text{circle}} = (m-1)!$$

Finally, we apply the fundamental counting principle by multiplying the number of ways to group the friends by the number of ways to arrange them at each table.

Step 3 : Detailed Explanation:

- Step 1: Select the 4 friends who will sit at the first table. Out of 10 friends, the number of ways to choose 4 is:

  $$\binom{10}{4} = \frac{10!}{4! \times 6!}$$

  The remaining 6 friends are automatically assigned to the second table in $\binom{6}{6} = 1$ way.

- Step 2: Calculate the circular arrangements at each table.
  The 4 friends at the first round table can be arranged in $(4-1)! = 3! = 6$ ways.

The 6 friends at the second round table can be arranged in $(6-1)! = 5! = 120$ ways.

- Step 3: Multiply the selection ways and the arrangement ways to find the total:

$$\text{Total} = \binom{10}{4} \times 3! \times 5!$$

$$\text{Total} = \frac{10!}{4! \times 6!} \times 3! \times 5!$$

- Step 4: Simplify the factorial expression:

$$\text{Total} = \frac{10!}{(4 \times 3!) \times (6 \times 5!)} \times 3! \times 5!$$

$$\text{Total} = \frac{10!}{24 \times 6 \times 5!} \times 6 \times 5! = \frac{10!}{24}$$

Step 4 : Final Answer:

The total number of ways to arrange the guests is $\frac{10!}{24}$, which corresponds to option (B).

Quick Tip: For problems involving multiple tables, perform the grouping of individuals first using combinations, and then apply circular permutations to each isolated table.

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11. If $z_1, z_2, \ldots, z_n$ are complex numbers such that $|z_1| = |z_2| = \ldots = |z_n| = 1$, then $|z_1 + z_2 + \ldots + z_n|$ is equal to:

(A) $|z_1 z_2 z_3 \ldots z_n|$

(B) $|z_1| + |z_2| + \ldots + |z_n|$

(C) $\left|\frac{1}{z_1} + \frac{1}{z_2} + \ldots + \frac{1}{z_n}\right|$

(D) n

Correct Answer: (C) $\left|\frac{1}{z_1} + \frac{1}{z_2} + \ldots + \frac{1}{z_n}\right|$

Solution:

Step 1 : Understanding the Question:

This problem explores the properties of complex numbers that lie on the unit circle in the complex plane. Given that all the complex numbers have a modulus of 1, we want to find an equivalent expression for the modulus of their sum.

Step 2 : Key Formulas and Approach:

We can solve this by utilizing the properties of complex conjugates and moduli. For any complex number z:

$$|z|^2 = z\bar{z}$$

If a complex number lies on the unit circle, then $|z| = 1$, which implies:

$$z\bar{z} = 1 \implies \bar{z} = \frac{1}{z}$$

We also use the property that the modulus of a complex number is equal to the modulus of its conjugate:

$$|z| = |\bar{z}|$$

Step 3 : Detailed Explanation:

- Given that $|z_k| = 1$ for all $k = 1, 2, \ldots, n$, we can write the conjugate of each number as its reciprocal:

$$\bar{z}_k = \frac{1}{z_k}$$

- We want to analyze the expression $|z_1 + z_2 + \ldots + z_n|$. Since the modulus of a complex sum is equal to the modulus of its overall conjugate:

$$|z_1 + z_2 + \ldots + z_n| = |\overline{z_1 + z_2 + \ldots + z_n}|$$

- Use the property that the conjugate of a sum is equal to the sum of the individual conjugates:

$$|\overline{z_1 + z_2 + \ldots + z_n}| = |\bar{z}_1 + \bar{z}_2 + \ldots + \bar{z}_n|$$

- Substitute $\bar{z}_k = \frac{1}{z_k}$ into this expression:

$$|\bar{z}_1 + \bar{z}_2 + \ldots + \bar{z}_n| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \ldots + \frac{1}{z_n}\right|$$

- This shows that $|z_1 + z_2 + \ldots + z_n|$ is mathematically equivalent to $\left|\frac{1}{z_1} + \frac{1}{z_2} + \ldots + \frac{1}{z_n}\right|$.

Step 4 : Final Answer:

The given expression is equal to $\left|\frac{1}{z_1} + \frac{1}{z_2} + \ldots + \frac{1}{z_n}\right|$, which corresponds to option (C).

Quick Tip: For any complex number on the unit circle, its conjugate and its reciprocal are identical. This property is highly useful for simplifying expressions involving sums of unimodular complex numbers.

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12. Evaluate the definite integral: $\int_0^{\pi/2} \sin^2(x)\,dx$.

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{2}$

(C) 1

(D) 0

Correct Answer: (A) $\frac{\pi}{4}$

Solution:

Step 1 : Understanding the Question:

This problem asks us to compute the value of a definite integral involving the square of the sine function over the interval from 0 to $\pi/2$.

Step 2 : Key Formulas and Approach:

Integrating power-based trigonometric terms is simplified by using double-angle identities to reduce the degree of the integrand. The power-reduction identity for sine is:

$$\sin^2(x) = \frac{1-\cos(2x)}{2}$$

Alternatively, we can use the definite integral symmetry property (King's Property):

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

Step 3 : Detailed Explanation:

- Step 1: Substitute the power-reduction identity into the definite integral:

$$\int_0^{\pi/2} \sin^2(x)\,dx = \int_0^{\pi/2} \frac{1-\cos(2x)}{2}\,dx$$

- Step 2: Factor out the constant term $\frac{1}{2}$ to simplify the integrand:

$$= \frac{1}{2}\int_0^{\pi/2} (1-\cos(2x))\,dx$$

- Step 3: Find the antiderivative of the terms inside the integral:

$$= \frac{1}{2}\left[x - \frac{\sin(2x)}{2}\right]_0^{\pi/2}$$

- Step 4: Evaluate the antiderivative at the upper limit $x = \pi/2$:

$$\frac{1}{2}\left(\frac{\pi}{2} - \frac{\sin(\pi)}{2}\right) = \frac{1}{2}\left(\frac{\pi}{2} - 0\right) = \frac{\pi}{4}$$

- Step 5: Evaluate the antiderivative at the lower limit $x = 0$:

$$\frac{1}{2}\left(0 - \frac{\sin(0)}{2}\right) = 0$$

- Subtracting the lower limit evaluation from the upper limit evaluation yields:

$$\frac{\pi}{4} - 0 = \frac{\pi}{4}$$

Step 4 : Final Answer:

The value of the definite integral is $\frac{\pi}{4}$, which corresponds to option (A).

Quick Tip: Using the symmetry property $\int_0^{\pi/2} \sin^2(x)\,dx = \int_0^{\pi/2} \cos^2(x)\,dx$, you can add the two integrals together to get $\int_0^{\pi/2} 1\,dx = \pi/2$, meaning each individual integral is half of that value, or $\pi/4$.
