BITSAT 2026 May 26 Shift 2

Question Paper (Memory Based) with Solutions

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General Instructions

(i) **Duration:** The total duration of the examination is 3 hours (180 minutes).

(ii) **Total Marks:** The complete paper carries a maximum of 390 marks.

(iii) **Structure:** The paper has 4 Sections:

- **Part 1:** 30 Multiple Choice Questions (Physics).
- **Part 2:** 30 Multiple Choice Questions (Chemistry).
- **Part 3:** 10 Multiple Choice Questions (English Proficiency), 20 Multiple Choice Questions (Logical Reasoning)
- **Part 4:** 40 Multiple Choice Questions (Mathematics/Biology)

(iv) **Compulsory Questions:** All 130 questions are compulsory,and +12 Questions(Optional Extra Questions)

(v) Each question has four options. Only **one** option is correct.

(vi) Correct Answer: +3 marks.

(vii) **Incorrect Answer:** −1 (Negative marking).

(viii) **Unanswered/Marked for Review:** 0 marks.

PHYSICS

1. A wire of length L and cross-sectional area A is made of a material of Young's modulus Y. If it is stretched by an amount x, the elastic potential energy stored in the wire is:

(A) $\frac{YAx^2}{L}$
(B) $\frac{YAx^2}{2L}$
(C) $\frac{2YAx^2}{L}$
(D) $\frac{YAx}{L}$

Correct Answer: (B) $\frac{YAx^2}{2L}$

Solution:

Step 1: Understanding the Question:

The question asks for the elastic potential energy stored in a wire of length L, cross-sectional area A, and Young's modulus Y when it is stretched by a distance x.
When an external stretching force is applied to a wire, work is performed against the internal interatomic restoring forces of the material.
This work is stored within the molecular matrix of the wire as elastic potential energy.

Step 2: Key Formula or Approach:

Young's modulus Y is defined as the ratio of tensile stress to tensile strain:

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{x/L}$$

The force F required to stretch the wire by an amount x can be expressed as:

$$F = \frac{YAx}{L}$$

The elastic potential energy U stored in the wire is given by the average work done:

$$U = \frac{1}{2} \times \text{Stretching Force} \times \text{Elongation} = \frac{1}{2}Fx$$

Step 3: Detailed Explanation:

1. We rearrange the Young's modulus equation to express the tension force F in terms of the

elongation x:

$$F = \frac{YAx}{L}$$

2. Now, substitute this force expression into the work-energy formula for the stored potential energy:

$$U = \frac{1}{2} \cdot \left(\frac{YAx}{L}\right) \cdot x$$

3. Combining the terms simplifies the expression:

$$U = \frac{YAx^2}{2L}$$

4. This result matches Option (B).

Step 4: Final Answer:
The stored elastic potential energy in the wire is $\frac{YAx^2}{2L}$, which corresponds to Option (B).

Quick Tip: A stretching wire can be treated like a mechanical spring with an effective stiffness constant $k = \frac{YA}{L}$.
The potential energy stored in a spring is given by $U = \frac{1}{2}kx^2$.
Substituting the effective stiffness constant yields:

$$U = \frac{1}{2}\left(\frac{YA}{L}\right)x^2 = \frac{YAx^2}{2L}$$

This formulation helps to solve composite wire problems rapidly.

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2. A particle moves in a circle of radius R such that its linear speed varies with time t as $v = kt$, where k is a positive constant. The angle θ between the net acceleration vector and the velocity

vector at time t is given by:

(A) $\tan^{-1}\left(\frac{k^2t^2}{R}\right)$
(B) $\tan^{-1}\left(\frac{kt^2}{R}\right)$
(C) $\tan^{-1}\left(\frac{kt}{R}\right)$
(D) $\tan^{-1}\left(\frac{R}{k^2t^2}\right)$

Correct Answer: (B) $\tan^{-1}\left(\frac{kt^2}{R}\right)$

Solution:

Step 1: Understanding the Question:

The question asks for the angle θ between the net acceleration vector and the velocity vector for a particle in circular motion with speed $v = kt$.

The velocity vector $\vec{v}$ of a particle in circular motion is always directed tangentially along the path.

Thus, the angle θ is the angle between the net acceleration vector and the tangential direction.

Step 2: Key Formula or Approach:

The total acceleration vector $\vec{a}$ in circular motion is the vector sum of two perpendicular components:

1. Tangential acceleration (a_t), which changes the speed:

$$a_t = \frac{dv}{dt}$$

2. Centripetal acceleration (a_c), which changes the direction:

$$a_c = \frac{v^2}{R}$$

The tangent of the angle θ between the net acceleration and the tangential acceleration (velocity direction) is:

$$\tan\theta = \frac{a_c}{a_t} \implies \theta = \tan^{-1}\left(\frac{a_c}{a_t}\right)$$

Step 3: Detailed Explanation:

1. Compute the tangential acceleration component a_t by differentiating the speed with respect to time:

$$a_t = \frac{d}{dt}(kt) = k$$

2. Compute the centripetal acceleration component a_c using the given speed equation:

$$a_c = \frac{v^2}{R} = \frac{(kt)^2}{R} = \frac{k^2t^2}{R}$$

3. Substitute both acceleration components into the trigonometric ratio:

$$\tan\theta = \frac{\left(\frac{k^2t^2}{R}\right)}{k} = \frac{kt^2}{R}$$

4. Solve for the angle θ:

$$\theta = \tan^{-1}\left(\frac{kt^2}{R}\right)$$

This matches Option (B).

Step 4: Final Answer:

The angle θ between the net acceleration vector and the velocity vector is $\tan^{-1}\left(\frac{kt^2}{R}\right)$, which corresponds to Option (B).

Quick Tip: Analyzing extreme boundaries can verify the formula.
At the start ($t = 0$), the speed is zero, meaning there is zero centripetal turning force and the particle purely accelerates forward.
Thus, the angle θ must be 0°.
Substituting $t = 0$ into the options shows that $\tan^{-1}(0) = 0^\circ$ for Option (B), verifying the physical consistency of the formula.

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3. Two wires X and Y of the same material have lengths in the ratio $1:2$ and diameters in the ratio $2:1$. If they are subjected to the same stretching force, the ratio of the elongation produced in wire X to that in wire Y ($\Delta L_X : \Delta L_Y$) is:

(A) 1 : 4
(B) 1 : 8
(C) 1 : 2
(D) 8 : 1

Correct Answer: (B) 1 : 8

Solution:
Step 1: Understanding the Question:
We need to find the ratio of the elongation produced in wire X to that in wire Y.
We are given that they are made of the same material, subjected to the same stretching force, and have known ratios for their lengths and diameters.

Step 2: Key Formula or Approach:
According to Hooke's Law, the longitudinal elongation ΔL under a tension load F is given by:

$$\Delta L = \frac{F \cdot L}{A \cdot Y}$$

where L is the length, A is the cross-sectional area, and Y is Young's modulus.
Expressing the circular cross-sectional area in terms of diameter d where $A = \frac{\pi d^2}{4}$:

$$\Delta L = \frac{4FL}{\pi d^2 Y}$$

Step 3: Detailed Explanation:

1. Both wires are made of the same material, which means their Young's modulus values are equal ($Y_X = Y_Y$).

2. They are subjected to the same stretching force ($F_X = F_Y$).

3. From this, we establish that the elongation is directly proportional to length and inversely proportional to the square of the diameter:

$$\Delta L \propto \frac{L}{d^2}$$

4. The given ratios are:

- Length ratio: $\frac{L_X}{L_Y} = \frac{1}{2}$
- Diameter ratio: $\frac{d_X}{d_Y} = \frac{2}{1} \implies \frac{d_Y}{d_X} = \frac{1}{2}$

5. Set up the ratio for the elongations:

$$\frac{\Delta L_X}{\Delta L_Y} = \left(\frac{L_X}{L_Y}\right) \times \left(\frac{d_Y}{d_X}\right)^2$$

6. Substitute the given ratios into the proportionality equation:

$$\frac{\Delta L_X}{\Delta L_Y} = \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right)^2$$

$$\frac{\Delta L_X}{\Delta L_Y} = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$$

This gives the ratio 1 : 8, which matches Option (B).

Step 4: Final Answer:

The ratio of the elongation produced in wire X to that in wire Y is 1 : 8, which corresponds to

Option (B).

Quick Tip: Consider the scaling of the dimensions.
Because area scales with the square of the diameter, doubling the diameter makes a wire four times more resistant to stretching.
Wire X is half as long (tending to stretch $\frac{1}{2}$ as much) but twice as thick (tending to stretch $\frac{1}{4}$ as much).
Multiplying these scaling factors:

$$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$$

This allows you to find the ratio instantly.

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4. The elastic potential energy stored per unit volume (energy density) in a stretched string under a longitudinal tension stress σ and material Young's modulus Y is expressed as:

(A) $\frac{\sigma^2}{2Y}$
(B) $\frac{2Y}{\sigma^2}$
(C) $\frac{Y\sigma^2}{2}$
(D) $\frac{\sigma^2}{Y}$

Correct Answer: (A) $\frac{\sigma^2}{2Y}$

Solution:

Step 1: Understanding the Question:
The question asks for the mathematical expression for the elastic potential energy stored per unit volume (energy density) in a stretched string, in terms of stress σ and Young's modulus Y.

Step 2: Key Formula or Approach:
The elastic potential energy density u is defined as:

$$u = \frac{1}{2} \times \text{Stress} \times \text{Strain} = \frac{1}{2} \cdot \sigma \cdot \varepsilon$$

Young's modulus Y maps stress directly to strain according to Hooke's Law:

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\varepsilon}$$

Step 3: Detailed Explanation:

1. We express the strain ε in terms of stress σ and Young's modulus Y:

$$\varepsilon = \frac{\sigma}{Y}$$

2. Substitute this expression for strain into the energy density formula:

$$u = \frac{1}{2} \cdot \sigma \cdot \left(\frac{\sigma}{Y}\right)$$

3. Combining the terms in the numerator and denominator:

$$u = \frac{\sigma^2}{2Y}$$

4. This derived expression matches Option (A).

Step 4: Final Answer:

The elastic potential energy stored per unit volume is $\frac{\sigma^2}{2Y}$, which corresponds to Option (A).

Quick Tip: Memorizing the three common forms of elastic energy density is useful for speed:

$$u = \frac{1}{2}\sigma\varepsilon = \frac{1}{2}Y\varepsilon^2 = \frac{\sigma^2}{2Y}$$

This is structurally analogous to energy density expressions in electrostatic fields.

CHEMISTRY

5. An octahedral coordination complex with the electronic configuration $t_{2g}^4 e_g^0$ is expected to exhibit which of the following magnetic properties and d-d transition characteristics?

(A) Paramagnetic with 4 unpaired electrons; spin-allowed transitions

(B) Paramagnetic with 2 unpaired electrons; spin-allowed transitions

(C) Diamagnetic; spin-forbidden transitions

(D) Paramagnetic with 2 unpaired electrons; spin-forbidden transitions

Correct Answer: (B) Paramagnetic with 2 unpaired electrons; spin-allowed transitions

Solution:

Step 1: Understanding the Question:

We need to determine the magnetic behavior and d-d transition characteristics of an octahedral complex with a $t_{2g}^4 e_g^0$ electronic configuration.

This configuration represents a low-spin d^4 system in an octahedral crystal field.

Step 2: Key Formula or Approach:

1. For magnetic properties: Distribute the 4 electrons in the split d-orbitals (t_{2g} and e_g) using Hund's rule of maximum multiplicity to count the number of unpaired electrons.

2. For d-d transitions: Use the spin selection rule, which states that electronic transitions are spin-allowed if there is no change in the net spin multiplicity ($\Delta S = 0$).

Step 3: Detailed Explanation:

1. In an octahedral field, the five d-orbitals split into the lower-energy t_{2g} triplet and the higher-energy e_g doublet.

2. The given configuration is $t_{2g}^4 e_g^0$.

3. Distributing the 4 electrons in the three t_{2g} orbitals:

- Place one electron in each of the three orbitals: $\uparrow, \uparrow, \uparrow$
- The fourth electron is forced to pair up in the first available orbital: $\uparrow\downarrow, \uparrow, \uparrow$

4. Counting the unpaired electrons:

- Number of unpaired electrons $(n) = 2$.
- Since $n = 2$, the complex is paramagnetic.

5. Now analyze the selection rule for d-d transitions:

- A d-d transition involves promoting an electron from a t_{2g} orbital to an e_g orbital.
- Since there are paired electrons in the ground state, an electron can undergo excitation to the empty e_g level without changing its spin direction.
- This allows the excited state to have the same number of unpaired electrons (and thus the same total spin) as the ground state.
- Since a transition can occur with $\Delta S = 0$, the transition is spin-allowed.

6. This matches the properties described in Option (B).

Step 4: Final Answer:

The complex is paramagnetic with 2 unpaired electrons and exhibits spin-allowed d-d transitions, which corresponds to Option (B).

Quick Tip: Be careful to distinguish between high-spin and low-spin configurations.
A high-spin d^4 system would have the configuration $t_{2g}^3 e_g^1$ with 4 unpaired electrons.
However, the specified $t_{2g}^4 e_g^0$ configuration tells you that pairing has occurred in the t_{2g} level, leaving $4 - 2 = 2$ unpaired electrons.

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6. During the structural analysis of an unknown aldohexose, a chemist treats a sample with periodic acid (HIO_4). If the carbohydrate is completely cleaved to yield five molecules of formic acid (HCOOH) and one molecule of formaldehyde (HCHO), this diagnostic breakdown directly

proves the presence of:

(A) A ketohexose structure with a carbonyl at C-2

(B) A cyclic pyranose ring configuration

(C) A continuous straight-chain structure containing five —CHOH groups and one $—CH_2OH$ group

(D) Three isolated, non-adjacent primary alcohol branches

Correct Answer: (C) A continuous straight-chain structure containing five —CHOH groups and one $—CH_2OH$ group

Solution:

Step 1: Understanding the Question:

The question asks us to identify the structural characteristics of an unknown aldohexose based on its complete oxidative cleavage by periodic acid (HIO_4).

The reaction produces five molecules of formic acid (HCOOH) and one molecule of formaldehyde (HCHO).

Step 2: Key Formula or Approach:

Periodic acid (HIO_4) selectively cleaves carbon-carbon single bonds in vicinal glycols (1,2-diols) and α-hydroxy carbonyl compounds.

The stoichiometry of the oxidation products is:

- A terminal primary alcohol group ($—CH_2OH$) yields formaldehyde (HCHO).
- A secondary alcohol group (—CHOH—) within a chain of adjacent cleavable groups yields formic acid (HCOOH).
- A terminal aldehyde group (—CHO) adjacent to a hydroxyl group yields formic acid (HCOOH).

Step 3: Detailed Explanation:

1. An aldohexose contains 6 carbon atoms.

2. Let us analyze the cleavage products:

- **One molecule of HCHO:** This confirms the presence of exactly one terminal primary alcohol group ($—CH_2OH$) at one end of the carbon chain (C-6).
- **Five molecules of HCOOH:** These must come from the remaining 5 carbon atoms.

3. In an open-chain aldohexose (such as D-glucose):

- The aldehyde group (—CHO) at C-1 oxidizes to 1 HCOOH.
- The four internal secondary alcohol groups (—CHOH—) at C-2, C-3, C-4, and C-5 each oxidize to 1 HCOOH, yielding 4 HCOOH.
- Together, these yield exactly 5 molecules of HCOOH.

4. The complete breakdown into single-carbon fragments indicates that all carbon-carbon bonds in the 6-carbon backbone were cleaved.
5. This is only possible if all 6 carbon atoms are in a continuous, unbranched straight chain containing contiguous hydroxyl or carbonyl groups.
6. This matches the description in Option (C).

Step 4: Final Answer:

The cleavage products confirm a continuous straight-chain structure containing five —CHOH groups and one $—CH_2OH$ group, corresponding to Option (C).

Quick Tip: A simple carbon count helps to quickly verify the structure:
Each molecule of formic acid (HCOOH) and formaldehyde (HCHO) contains exactly 1 carbon atom.
Since 5 (from HCOOH) + 1 (from HCHO) = 6 carbons, the entire 6-carbon hexose skeleton has been cleaved into single-carbon units, confirming a continuous open straight chain.

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7. In an analytical laboratory, a 20.0 mL sample of an aqueous solution containing oxalic acid ($H_2C_2O_4$) requires exactly 16.0 mL of a 0.05 M potassium permanganate ($KMnO_4$) solution for complete oxidation in a hot, acidic medium (H_2SO_4). Calculate the molarity of the oxalic acid solution.

(A) 0.010 M
(B) 0.040 M
(C) 0.100 M
(D) 0.250 M

Correct Answer: (C) 0.100 M

Solution:

Step 1: Understanding the Question:

The question describes a redox titration where oxalic acid ($H_2C_2O_4$) is oxidized by potassium permanganate ($KMnO_4$) in an acidic medium.

We need to find the molarity of the oxalic acid solution using volumetric and molarity data of the titrant.

Step 2: Key Formula or Approach:

At the equivalence point of any redox titration, the total gram equivalents of the oxidizing agent equal the total gram equivalents of the reducing agent:

$$\text{Equivalents of } KMnO_4 = \text{Equivalents of } H_2C_2O_4$$

Using the relationship between normality N, molarity M, and the valence factor (n-factor) n, where $N = M \cdot n$:

$$M_1 \cdot n_1 \cdot V_1 = M_2 \cdot n_2 \cdot V_2$$

Step 3: Detailed Explanation:

1. Determine the n-factor of $KMnO_4$ in an acidic medium:

- Permanganate ion (MnO_4^-) is reduced to manganese(II) ion (Mn^{2+}):

$$MnO_4^- + 8H^+ + 5e^- \rightarrow Mn^{2+} + 4H_2O$$

- The oxidation state of manganese decreases from +7 to +2, involving a 5-electron transfer.
- Thus, $n_1 = 5$.

2. Determine the n-factor of oxalic acid ($H_2C_2O_4$):

- Oxalate ion ($C_2O_4^{2-}$) is oxidized to carbon dioxide (CO_2):

$$C_2O_4^{2-} \rightarrow 2CO_2 + 2e^-$$

- Carbon is oxidized from a +3 state to a +4 state.
- Since there are 2 carbon atoms per molecule of oxalic acid, the total electron loss is $2 \times 1 = 2$.
- Thus, $n_2 = 2$.

3. Let M_x be the molarity of the oxalic acid solution. Substitute the values into the equation:

$$(M \cdot n \cdot V)_{\mathrm{KMnO_4}} = (M \cdot n \cdot V)_{\mathrm{H_2C_2O_4}}$$

$$0.05 \text{ M} \times 5 \times 16.0 \text{ mL} = M_x \times 2 \times 20.0 \text{ mL}$$

4. Solve for M_x:

$$4.0 = 40.0 \cdot M_x$$

$$M_x = \frac{4.0}{40.0} = 0.100 \text{ M}$$

This matches Option (C).

Step 4: Final Answer:

The molar concentration of the oxalic acid solution is 0.100 M, which corresponds to Option (C).

Quick Tip: Using normality factor ratios saves time.

For acidic titrations involving permanganate and oxalate, the n-factors are always 5 and 2 respectively.

This allows you to directly use the relationship:

$$5 \times M_{\text{KMnO}_4} \times V_{\text{KMnO}_4} = 2 \times M_{\text{oxalic}} \times V_{\text{oxalic}}$$

This bypasses the need to balance the full redox equation.

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8. A current of 2.0 A is passed for 5 hours through an electrolytic cell containing an aqueous solution of a metal salt, depositing 12.0 g of the metal at the cathode. If the atomic mass of the metal is 193 g mol^{-1}, find the oxidation state of the metal ion in the solution. (Take Faraday's constant $F = 96500$ C mol^{-1}).

(A) +1

(B) +2

(C) +3

(D) +6

Correct Answer: (D) +6

Solution:

Step 1: Understanding the Question:

We are asked to determine the oxidation state (valency) of a metal ion in a solution.

We are given the current, duration of electrolysis, mass of metal deposited, and the atomic mass of the metal.

Step 2: Key Formula or Approach:

According to Faraday's First Law of Electrolysis, the mass m of a metal deposited at the cathode is:

$$m = \frac{M \cdot I \cdot t}{n \cdot F}$$

where:

- m is the mass of metal deposited (12.0 g).

- M is the atomic mass of the metal (193 g mol^{-1}).

- I is the current (2.0 A).

- t is the time in seconds.

- n is the oxidation state (valency) of the metal ion.

- F is Faraday's constant (96500 C mol^{-1}).

Step 3: Detailed Explanation:

1. Convert the electrolysis time from hours into seconds:

$$t = 5 \text{ h} \times 3600 \text{ s h}^{-1} = 18000 \text{ s}$$

2. Rearrange Faraday's formula to solve for the oxidation state n:

$$n = \frac{M \cdot I \cdot t}{m \cdot F}$$

3. Substitute the given values into the equation:

$$n = \frac{193 \times 2.0 \times 18000}{12.0 \times 96500}$$

4. Simplify the numerical calculation:

- Calculate total charge $Q = I \cdot t = 2.0 \times 18000 = 36000$ C.

- Substituting this:

$$n = \frac{193 \times 36000}{12.0 \times 96500}$$

- Recognize that $96500/193 = 500$:

$$n = \frac{36000}{12.0 \times 500}$$

$$n = \frac{36000}{6000} = 6$$

5. Thus, the oxidation state of the metal ion is +6, matching Option (D).

Step 4: Final Answer:

The oxidation state of the metal ion in the solution is +6, which corresponds to Option (D).

Quick Tip: An alternative method uses moles of electrons and moles of metal:

$$\text{Moles of metal deposited} = \frac{12.0}{193} \approx 0.0622 \text{ mol}$$

$$\text{Moles of electrons passed} = \frac{2.0 \times 18000}{96500} \approx 0.373 \text{ mol}$$

The oxidation state n is the ratio of moles of electrons to moles of metal:

$$n = \frac{0.373}{0.0622} \approx 6$$

This provides a useful verification step.

MATHEMATICS

9. In how many ways can the letters of the word COCHIN be arranged such that the two 'C's are never separated by any other letter?

(A) 360

(B) 120

(C) 240

(D) 720

Correct Answer: (B) 120

Solution:

Step 1: Understanding the Question:

We need to find the number of arrangements of the letters of the word "COCHIN" such that the two 'C's are never separated (meaning they must always remain adjacent).

Step 2: Key Formula or Approach:

We use the bundling/string method:

1. Treat the group of letters that must stay together as a single block.
2. Find the number of permutations of this block along with the remaining individual letters.
3. Multiply by the number of arrangements of the letters inside the block itself.

Step 3: Detailed Explanation:

1. The word "COCHIN" has 6 letters: C, O, C, H, I, N (where 'C' is repeated twice).
2. Since the two 'C's must never be separated, we group them together into a single block: [CC].
3. We now treat this block as a single item. The set of items to arrange is:

- The block [CC] (1 item)
- The letter O (1 item)
- The letter H (1 item)
- The letter I (1 item)
- The letter N (1 item)

4. This gives a total of 5 distinct items to arrange.
5. The number of linear arrangements of these 5 items is:

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120 \text{ ways}$$

6. Within the block [CC], both letters are identical, so swapping their positions does not create a new unique word ($\frac{2!}{2!} = 1$ arrangement).
7. Therefore, the total number of permutations is:

$$120 \times 1 = 120 \text{ arrangements}$$

This matches Option (B).

Step 4: Final Answer:

The number of ways to arrange the letters is 120, which corresponds to Option (B).

Quick Tip: Be careful with the phrasing of the constraint.

The constraint "never separated" is equivalent to "always together".

Do not use subtraction (Total − Together) unless the problem asks for "never together".

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10. Evaluate the definite integral: $\int_0^{2026} \frac{x^5}{x^5+(2026-x)^5}\,dx$

(A) 2026

(B) 1013

(C) 506.5

(D) 0

Correct Answer: (B) 1013

Solution:

Step 1: Understanding the Question:

We need to evaluate the definite integral $I = \int_0^{2026} \frac{x^5}{x^5+(2026-x)^5}\,dx$.

Direct algebraic integration is not practical, which suggests using definite integral symmetry properties.

Step 2: Key Formula or Approach:

We use King's Property of definite integrals:

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

Applying this with $a = 0$ and $b = 2026$, we replace x with $2026 - x$.

Step 3: Detailed Explanation:

1. Let the given integral be I:

$$I = \int_0^{2026} \frac{x^5}{x^5 + (2026-x)^5}\,dx \quad \cdots (1)$$

2. Apply King's Property by replacing x with $2026 - x$ in the integrand:

$$I = \int_0^{2026} \frac{(2026-x)^5}{(2026-x)^5 + (2026-(2026-x))^5}\,dx$$

3. Simplify the denominator terms:

$$I = \int_0^{2026} \frac{(2026-x)^5}{(2026-x)^5 + x^5}\,dx \quad \cdots (2)$$

4. Since Equation (1) and Equation (2) have identical limits, we add them together:

$$2I = \int_0^{2026} \left[\frac{x^5}{x^5 + (2026-x)^5} + \frac{(2026-x)^5}{x^5 + (2026-x)^5}\right] dx$$

$$2I = \int_0^{2026} \frac{x^5 + (2026-x)^5}{x^5 + (2026-x)^5}\,dx$$

5. The integrand simplifies to 1:

$$2I = \int_0^{2026} 1 \cdot dx$$

6. Evaluate the integral:

$$2I = \Big[x\Big]_0^{2026} = 2026 - 0 = 2026$$

$$I = \frac{2026}{2} = 1013$$

This matches Option (B).

Step 4: Final Answer:

The value of the definite integral is 1013, which corresponds to Option (B).

Quick Tip: For any definite integral of the symmetric form:

$$\int_a^b \frac{f(x)}{f(x)+f(a+b-x)}\,dx$$

the integral evaluates to half the length of the integration interval:

$$I = \frac{b-a}{2}$$

For this question, $a = 0$ and $b = 2026$, giving:

$$I = \frac{2026-0}{2} = 1013$$

This shortcut can save significant time on exams.

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11. If the vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$, and $\vec{c} = 3\hat{i} + \lambda\hat{j} + 5\hat{k}$ represent the concurrent coterminous edges of a parallelopiped whose volume is 0 (i.e., the vectors are coplanar), find the value of the scalar parameter λ.

(A) 4

(B) -4

(C) 2

(D) -2

Correct Answer: (B) -4

Solution:

Step 1: Understanding the Question:

The question asks for the value of the scalar parameter λ for which three given vectors, representing the concurrent coterminous edges of a parallelopiped, are coplanar (meaning the volume of the parallelopiped is 0).

Step 2: Key Formula or Approach:

The volume V of a parallelopiped formed by three concurrent coterminous vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ is given by the absolute value of their scalar triple product:

$$V = \left|\vec{a} \cdot (\vec{b} \times \vec{c})\right|$$

When the volume is zero, the vectors are coplanar, and their scalar triple product is zero:

$$\begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} = 0$$

Step 3: Detailed Explanation:

1. Set up the determinant equation using the components of vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$:

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & \lambda & 5 \end{vmatrix} = 0$$

2. Expand the determinant along the first row:

$$2 \cdot \begin{vmatrix} 2 & -3 \\ \lambda & 5 \end{vmatrix} - (-1) \cdot \begin{vmatrix} 1 & -3 \\ 3 & 5 \end{vmatrix} + 1 \cdot \begin{vmatrix} 1 & 2 \\ 3 & \lambda \end{vmatrix} = 0$$

3. Calculate the 2×2 determinants:

- First term: $2 \cdot (10 - (-3\lambda)) = 2 \cdot (10 + 3\lambda)$

- Second term: $1 \cdot (5 - (-9)) = 1 \cdot 14$

- Third term: $1 \cdot (\lambda - 6)$

4. Combine the expanded terms:

$$2 \cdot (10 + 3\lambda) + 14 + \lambda - 6 = 0$$

$$20 + 6\lambda + 14 + \lambda - 6 = 0$$

5. Combine like terms:

$$7\lambda + 28 = 0$$

$$7\lambda = -28 \implies \lambda = -4$$

This matches Option (B).

Step 4: Final Answer:

The scalar parameter λ is -4, which corresponds to Option (B).

Quick Tip: To verify, check if $\vec{c}$ can be written as a linear combination of $\vec{a}$ and $\vec{b}$ when $\lambda = -4$:

$$\vec{c} = x\vec{a} + y\vec{b}$$

Using the components:

- For $\hat{i}$: $2x + y = 3$

- For $\hat{k}$: $x - 3y = 5$

Solving these equations gives $x = 2$ and $y = -1$.

Substituting into the $\hat{j}$ component: $x(-1) + y(2) = -2 - 2 = -4$.

This matches $\lambda = -4$, confirming the result.

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12. A pair of fair dice is thrown simultaneously. What is the probability that the sum of the numbers appearing on the top faces is at least 10?

(A) $\frac{1}{6}$

(B) $\frac{1}{12}$

(C) $\frac{5}{36}$

(D) $\frac{1}{4}$

Correct Answer: (A) $\frac{1}{6}$

Solution:

Step 1: Understanding the Question:

We need to find the probability that the sum of the numbers on the top faces of two fair dice thrown simultaneously is "at least 10" (i.e., the sum S satisfies $S \geq 10$).

Step 2: Key Formula or Approach:

The probability of an event E is:

$$P(E) = \frac{\text{Number of Favorable Outcomes } (n(E))}{\text{Total Number of Sample Outcomes } (n(S))}$$

For a pair of six-sided dice, the total number of sample outcomes is:

$$n(S) = 6 \times 6 = 36$$

The event "at least 10" includes the sums of 10, 11, and 12.

Step 3: Detailed Explanation:

1. List the favorable coordinate pairs (d_1, d_2) for each possible sum $S \geq 10$:

- For Sum $= 10$: $(4,6), (5,5), (6,4) \rightarrow 3$ outcomes

- For Sum $= 11$: $(5,6), (6,5) \rightarrow 2$ outcomes

- For Sum $= 12$: $(6,6) \rightarrow 1$ outcome

2. Sum the individual counts to find the total number of favorable outcomes:

$$n(E) = 3 + 2 + 1 = 6 \text{ outcomes}$$

3. Calculate the probability:

$$P(S \geq 10) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

This matches Option (A).

Step 4: Final Answer:

The probability that the sum is at least 10 is $\frac{1}{6}$, which corresponds to Option (A).

Quick Tip: For dice sum questions, you can use the symmetric distribution of sums:

- The number of outcomes for a sum S from 2 to 7 is $S-1$.

- The number of outcomes for a sum S from 8 to 12 is $13-S$.

Using this relation:

- For $\text{Sum} = 10$: $13-10=3$ ways

- For $\text{Sum} = 11$: $13-11=2$ ways

- For $\text{Sum} = 12$: $13-12=1$ way

Total ways $=3+2+1=6$ out of 36, giving $\frac{1}{6}$ quickly.
