



General Instructions

- (i) **Duration:** The total duration of the examination is 3 hours (180 minutes).
- (ii) **Total Marks:** The complete paper carries a maximum of 390 marks.
- (iii) **Structure:** The paper has 4 Sections:
 - **Part 1:** 30 Multiple Choice Questions (Physics).
 - **Part 2:** 30 Multiple Choice Questions (Chemistry).
 - **Part 3:** 10 Multiple Choice Questions (English Proficiency),
20 Multiple Choice Questions (Logical Reasoning)
 - **Part 4:** 40 Multiple Choice Questions (Mathematics/Biology)
- (iv) **Compulsory Questions:** All 130 questions are compulsory, and +12 Questions (Optional Extra Questions)
- (v) Each question has four options. Only **one** option is correct.
- (vi) **Correct Answer:** +3 marks.
- (vii) **Incorrect Answer:** -1 (Negative marking).
- (viii) **Unanswered/Marked for Review:** 0 marks.

PHYSICS

1. A particle moves along a circle of radius R with a constant angular acceleration α . If the initial angular velocity is zero, the total acceleration of the particle at time t is:

- (A) $R\alpha$
- (B) $R\alpha^2 t^2$
- (C) $R\alpha\sqrt{1 + \alpha^2 t^4}$
- (D) $R\alpha t$

Correct Answer: (C) $R\alpha\sqrt{1 + \alpha^2 t^4}$

Solution:

Step 1 : Understanding the Question:

This problem asks us to determine the net or total linear acceleration of a particle transitioning along a circular trajectory of a fixed radius under a constant angular acceleration. The particle starts from rest, which means its initial angular velocity is zero. As it moves, its speed changes (giving rise to tangential acceleration) and its direction of motion constantly changes (giving rise to radial or centripetal acceleration). We need to find the vector sum of these two components at any given time t .

Step 2 : Key Formulas and Approach:

The motion of the particle involves both rotational and translational components, which are linked by the radius of the circle.

The angular velocity after time t is given by the rotational kinematic equation:

$$\omega = \omega_0 + \alpha t$$

The tangential acceleration (a_t) is responsible for changing the speed of the particle and is given by:

$$a_t = R\alpha$$

The centripetal acceleration (a_c) is responsible for keeping the particle in circular motion and is given by:

$$a_c = \omega^2 R$$

Because these two components are perpendicular to each other, the magnitude of the total linear acceleration is calculated using the Pythagorean theorem:

$$a_{\text{total}} = \sqrt{a_t^2 + a_c^2}$$

Step 3 : Detailed Explanation:

- We start by determining the angular velocity ω at time t using the initial condition. Since the particle starts from rest, the initial angular velocity ω_0 is equal to zero. Substituting this into our kinematic formula gives $\omega = \alpha t$.
- Next, we express the tangential acceleration of the particle. The tangential acceleration depends only on the constant angular acceleration and the radius of the circular path. This gives the constant value $a_t = R\alpha$.
- Then, we calculate the centripetal acceleration at time t . We substitute the expression for ω into the centripetal acceleration formula to get $a_c = (\alpha t)^2 R = R\alpha^2 t^2$.
- We now combine these two orthogonal acceleration vectors to find the magnitude of the total acceleration. Substituting the individual expressions into the magnitude formula gives $a_{\text{total}} = \sqrt{(R\alpha)^2 + (R\alpha^2 t^2)^2}$.
- Expanding the terms inside the square root yields $a_{\text{total}} = \sqrt{R^2\alpha^2 + R^2\alpha^4 t^4}$.
- To simplify the expression, we factor out the common term $R^2\alpha^2$ from both terms under the radical, giving $a_{\text{total}} = \sqrt{R^2\alpha^2(1 + \alpha^2 t^4)}$.
- Extracting the square root of $R^2\alpha^2$ outside the radical leads to the final simplified expression: $a_{\text{total}} = R\alpha\sqrt{1 + \alpha^2 t^4}$.

Step 4 : Final Answer:

The total acceleration of the particle at time t is given by the expression $R\alpha\sqrt{1 + \alpha^2 t^4}$. This corresponds to Option (C).

Quick Tip: To quickly verify your formula during an assessment, analyze the physical state of the particle at the starting instant $t = 0$. At this initial point, the particle is not rotating, so the centripetal acceleration must be zero. The only acceleration acting on the particle is the tangential start-up acceleration, which is $R\alpha$. Substituting $t = 0$ into Option (C) yields $R\alpha\sqrt{1 + 0} = R\alpha$, which is consistent with this physical limit.

2. The focal length of a convex lens is f in air. When it is completely immersed in water of refractive index $\frac{4}{3}$, its focal length becomes (take refractive index of glass = 1.5):

- (A) f
- (B) $2f$
- (C) $4f$
- (D) $\frac{f}{2}$

Correct Answer: (C) $4f$

Solution:**Step 1 : Understanding the Question:**

This problem asks us to calculate the change in the focal length of a convex lens when it is transferred from air into water. The focal length of any lens depends on the refractive index of the lens material relative to the surrounding medium, as well as the radii of curvature of the lens surfaces. Immersing the lens in a denser medium like water reduces the relative refractive index, which decreases the light-bending capability of the lens and consequently increases the focal length.

Step 2 : Key Formulas and Approach:

The quantitative relationship between the focal length of a lens, its refractive index, the

refractive index of the medium, and its curvature is given by the Lens Maker's Formula:

$$\frac{1}{f} = \left(\frac{\mu_{\text{lens}}}{\mu_{\text{medium}}} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

We will first apply this formula for the lens in air (where the medium's refractive index is 1). Next, we will apply the formula for the lens immersed in water (where the medium's refractive index is $4/3$).

By taking the ratio of these two equations, we can eliminate the geometric factor containing the radii of curvature and find the new focal length in terms of the original focal length.

Step 3 : Detailed Explanation:

- We begin by analyzing the initial state of the glass lens in air. Here, the refractive index of the surrounding medium is $\mu_{\text{air}} = 1$, and the refractive index of the glass is $\mu_g = 1.5 = \frac{3}{2}$.
- Substituting these values into the Lens Maker's Formula gives: $\frac{1}{f} = \left(\frac{3/2}{1} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) = \frac{1}{2} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$. We can label this as Equation 1.
- We then analyze the second state where the lens is completely immersed in water. The refractive index of the water is $\mu_w = \frac{4}{3}$, and the refractive index of the lens remains $\mu_g = \frac{3}{2}$. Let the new focal length in water be denoted as f_w .
- Substituting these values into the formula yields: $\frac{1}{f_w} = \left(\frac{3/2}{4/3} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$.
- Simplifying the relative refractive index term gives: $\frac{\mu_g}{\mu_w} = \frac{3}{2} \times \frac{3}{4} = \frac{9}{8}$.
- Substituting this back into the equation for f_w gives: $\frac{1}{f_w} = \left(\frac{9}{8} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) = \frac{1}{8} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$. We can label this as Equation 2.

- To find the relationship between f and f_w , we divide Equation 1 by Equation 2. This mathematical step cancels out the geometric term $\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ completely.
- Performing the division gives: $\frac{1/f}{1/f_w} = \frac{f_w}{f} = \frac{1/2}{1/8} = \frac{8}{2} = 4$.
- Rearranging this equation to solve for f_w gives: $f_w = 4f$.

Step 4 : Final Answer:

The focal length of the convex lens when completely immersed in water is $4f$, which corresponds to Option (C).

Quick Tip: It is helpful to remember this standard result for quick calculation. Whenever a typical glass lens with a refractive index of 1.5 is fully submerged in water with a refractive index of 1.33, its focal length always increases by a factor of exactly four. Memorizing this factor can save valuable time during examinations.

3. The stopping potential for photoelectrons emitted from a surface illuminated by light of wavelength λ is V_s . If the intensity of the incident light is doubled while keeping wavelength identical, the stopping potential will be:

- (A) $2V_s$
- (B) $\frac{V_s}{2}$
- (C) V_s
- (D) $4V_s$

Correct Answer: (C) V_s

Solution:

Step 1 : Understanding the Question:

This question asks us to identify how the stopping potential of photoelectrons changes when the intensity of the incident light is doubled, while keeping its wavelength constant. The

photoelectric effect describes the emission of electrons when light shines on a material. We need to understand whether the energy of the emitted photoelectrons is governed by the intensity of the light source or by its frequency and wavelength.

Step 2 : Key Formulas and Approach:

The relationship between the incident light energy and the kinetic energy of the emitted electrons is described by Einstein's photoelectric equation:

$$K_{\max} = eV_s = h\nu - \phi_0$$

In terms of the wavelength of the incident light, the equation can be rewritten as:

$$eV_s = \frac{hc}{\lambda} - \phi_0$$

Here, $K_{\max}$ represents the maximum kinetic energy of the emitted photoelectrons, e is the elementary charge, V_s is the stopping potential, h is Planck's constant, c is the speed of light, λ is the wavelength of the incident light, and ϕ_0 is the work function of the metal.

We will analyze the influence of light intensity on the parameters of this equation to find the new stopping potential.

Step 3 : Detailed Explanation:

- According to wave-particle duality, light consists of packets of energy called photons. The energy of each individual photon depends solely on its frequency or wavelength, as given by $E = \frac{hc}{\lambda}$.
- The intensity of light is a measure of the number of photons striking a unit area of the surface per unit time. Increasing the intensity at a constant wavelength increases the flux of photons, but does not alter the energy carried by any individual photon.
- When the intensity of the light is doubled, the number of photons striking the metal

surface per second is doubled. As a result, the number of photoelectrons ejected per second (and hence the saturation photocurrent) is also doubled, provided the frequency of the light is above the threshold frequency.

- However, because the wavelength λ is kept completely identical, the energy of each incident photon remains exactly the same. Since the work function ϕ_0 of the metal is also constant, the maximum kinetic energy $K_{\max}$ of the individual photoelectrons remains unchanged.
- Because the maximum kinetic energy of the photoelectrons does not change, the stopping potential required to halt these electrons, defined by $V_s = \frac{K_{\max}}{e}$, also remains completely unchanged.
- Therefore, doubling the intensity of the incident light has no effect on the stopping potential, and it remains equal to the original value V_s .

Step 4 : Final Answer:

The stopping potential of the photoelectrons remains unchanged at V_s , which corresponds to Option (C).

Quick Tip: A useful conceptual shortcut is to distinguish clearly between energy-related properties and quantity-related properties in the photoelectric effect. Wavelength and frequency determine the energy properties, such as photon energy, maximum kinetic energy of photoelectrons, and stopping potential. Intensity determines the quantity properties, such as the rate of photon arrival and the magnitude of the photo-electric current. These two categories do not interfere with each other.

4. The de-Broglie wavelength of an electron accelerated from rest through a potential difference of 100V is approximately:

(A) $1.227 \text{ \AA}$

- (B) 12.27 Å
(C) 0.1227 Å
(D) 122.7 Å

Correct Answer: (A) 1.227 Å

Solution:

Step 1 : Understanding the Question:

This problem asks us to determine the wave-like wavelength, known as the de-Broglie wavelength, associated with an electron that has been accelerated from a state of rest through an electric potential difference of 100 volts. According to quantum mechanics, moving particles exhibit wave-like characteristics, and their wavelengths are inversely proportional to their momentum. We need to relate the electric potential to the kinetic energy, then to the momentum, and finally to the de-Broglie wavelength of the electron.

Step 2 : Key Formulas and Approach:

The de-Broglie relationship states that the wavelength λ of a moving particle is given by:

$$\lambda = \frac{h}{p}$$

Where h is Planck's constant and p is the momentum of the particle.

The kinetic energy K of a charged particle accelerated through a potential difference V is given by:

$$K = qV$$

The relationship between kinetic energy and momentum is:

$$p = \sqrt{2mK} = \sqrt{2mqV}$$

Substituting this into the de-Broglie equation gives:

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

For an electron, we can substitute the values of the constant physical quantities to arrive at a simplified practical formula.

Step 3 : Detailed Explanation:

- We start by substituting the physical constants for an electron into the general wavelength formula. The mass of an electron is $m \approx 9.1 \times 10^{-31}$ kg, the charge is $q = e \approx 1.6 \times 10^{-19}$ C, and Planck's constant is $h \approx 6.626 \times 10^{-34}$ J s.
- Substituting these constant values into $\lambda = \frac{h}{\sqrt{2meV}}$ yields a highly simplified formula specifically for electrons: $\lambda = \frac{12.27}{\sqrt{V}}$ Å, where $1 \text{ Å} = 10^{-10}$ m.
- Next, we identify the given accelerating potential difference from the problem, which is $V = 100$ V.
- We substitute $V = 100$ into our simplified formula: $\lambda = \frac{12.27}{\sqrt{100}}$ Å.
- Since the square root of 100 is exactly 10, the expression simplifies to: $\lambda = \frac{12.27}{10}$ Å.
- Performing this division shifts the decimal point one place to the left, resulting in $\lambda = 1.227$ Å.

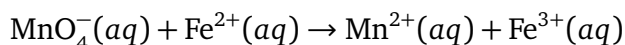
Step 4 : Final Answer:

The de-Broglie wavelength of the accelerated electron is approximately 1.227 Å, which corresponds to Option (A).

Quick Tip: An alternative version of the electron shortcut formula is $\lambda = \sqrt{\frac{150}{V}} \text{ \AA}$. This formula can sometimes make mental calculations even simpler. For instance, substituting $V = 100 \text{ V}$ into this formula gives $\lambda = \sqrt{1.5} \text{ \AA}$. Since $1.2^2 = 1.44$ and $1.3^2 = 1.69$, the square root of 1.5 must lie slightly above 1.2, pointing directly to 1.227 $\text{\AA}$ and helping you avoid decimal placement errors.

CHEMISTRY

5. Balance the following redox reaction in acidic medium and determine the stoichiometric coefficient of H_2O in the final balanced equation.



- (A) 2
- (B) 4
- (C) 6
- (D) 8

Correct Answer: (B) 4

Solution:

Step 1 : Understanding the Question:

This question asks us to balance a classic redox reaction involving permanganate ions and iron(II) ions in an acidic aqueous medium. Once the complete equation is balanced, we need to identify the stoichiometric coefficient of water molecules (H_2O) on the product side. To achieve this, we can systematically apply the half-reaction method, which ensures that both mass and charge are conserved.

Step 2 : Key Formulas and Approach:

The ion-electron method is the most reliable approach to balance redox reactions in acidic conditions. The systematic steps are as follows:

1. Separate the overall reaction into oxidation and reduction half-reactions.
2. Balance all atoms in each half-reaction except for hydrogen and oxygen.

3. Balance oxygen atoms by adding water molecules (H_2O) to the oxygen-deficient side.
4. Balance hydrogen atoms by adding hydrogen ions (H^+) to the hydrogen-deficient side.
5. Balance the electrical charges by adding electrons (e^-) to the more positive side.
6. Multiply the half-reactions by appropriate integers so that the number of electrons lost equals the number of electrons gained, and then add them together.

Step 3 : Detailed Explanation:

- We begin by identifying the oxidation states of the key elements. Manganese in MnO_4^- has an oxidation state of +7 and is reduced to +2 in Mn^{2+} . Iron goes from +2 in Fe^{2+} to +3 in Fe^{3+} , indicating oxidation.
- Next, we write down the oxidation half-reaction: $\text{Fe}^{2+}(\text{aq}) \rightarrow \text{Fe}^{3+}(\text{aq})$. Since the iron atoms are already balanced, we balance the charge by adding one electron to the product side, yielding: $\text{Fe}^{2+}(\text{aq}) \rightarrow \text{Fe}^{3+}(\text{aq}) + e^-$.
- Now, we write down the reduction half-reaction skeleton: $\text{MnO}_4^-(\text{aq}) \rightarrow \text{Mn}^{2+}(\text{aq})$.
- The manganese atoms are already balanced with one on each side. To balance the four oxygen atoms on the reactant side, we add four water molecules to the product side: $\text{MnO}_4^-(\text{aq}) \rightarrow \text{Mn}^{2+}(\text{aq}) + 4\text{H}_2\text{O}(\text{l})$.
- Adding water introduces eight hydrogen atoms on the right. To balance these, we add eight hydrogen ions (H^+) to the reactant side: $\text{MnO}_4^-(\text{aq}) + 8\text{H}^+(\text{aq}) \rightarrow \text{Mn}^{2+}(\text{aq}) + 4\text{H}_2\text{O}(\text{l})$.
- To balance the net charge in the reduction half-reaction, we calculate the charges on both sides. The reactant side has a net charge of $(-1) + 8(+1) = +7$, while the product side has a net charge of +2. We add five electrons to the reactant side to balance the charge: $\text{MnO}_4^-(\text{aq}) + 8\text{H}^+(\text{aq}) + 5e^- \rightarrow \text{Mn}^{2+}(\text{aq}) + 4\text{H}_2\text{O}(\text{l})$.

- To combine the two half-reactions, we equalize the electron transfer. Since the oxidation half-reaction releases one electron and the reduction half-reaction consumes five, we multiply the oxidation half-reaction by five: $5\text{Fe}^{2+}(\text{aq}) \rightarrow 5\text{Fe}^{3+}(\text{aq}) + 5\text{e}^-$.
- Finally, we sum the two adjusted half-reactions. The five electrons on each side cancel out, giving the final balanced equation: $\text{MnO}_4^-(\text{aq}) + 5\text{Fe}^{2+}(\text{aq}) + 8\text{H}^+(\text{aq}) \rightarrow \text{Mn}^{2+}(\text{aq}) + 5\text{Fe}^{3+}(\text{aq}) + 4\text{H}_2\text{O}(\text{l})$. Looking at the water molecules, we see that the coefficient is 4.

Step 4 : Final Answer:

The stoichiometric coefficient of H_2O in the final balanced equation is 4, which corresponds to Option (B).

Quick Tip: For reactions involving the reduction of the permanganate ion (MnO_4^-) to the manganese(II) ion (Mn^{2+}) in an acidic medium, the coefficient of water is determined by the oxygen atoms. Since all oxygen atoms in the reactant permanganate must end up as water, and each permanganate contains four oxygen atoms, one mole of MnO_4^- will always produce exactly four moles of H_2O , regardless of the species being oxidized. This can help you find the coefficient of water instantly.

6. Titration of 0.1467 g of primary standard $\text{Na}_2\text{C}_2\text{O}_4$ required 28.85 mL of KMnO_4 solution. Calculate the molar concentration of KMnO_4 solution.

- (A) 0.01518 M
 (B) 0.001518 M
 (C) 0.15180 M
 (D) 1.5180 M

Correct Answer: (A) 0.01518 M

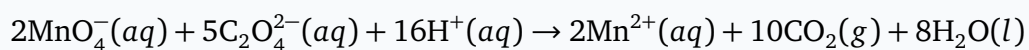
Solution:

Step 1 : Understanding the Question:

This problem presents a quantitative redox titration scenario where a primary standard substance, sodium oxalate ($\text{Na}_2\text{C}_2\text{O}_4$), is used to standardize a solution of potassium permanganate (KMnO_4). We are given the mass of sodium oxalate and the volume of permanganate solution required to reach the end point. Our goal is to determine the molarity of the potassium permanganate solution by using the stoichiometric relationships of the chemical reaction.

Step 2 : Key Formulas and Approach:

The reaction occurs in an acidic medium, where permanganate ions oxidize oxalate ions. The balanced chemical equation for this redox reaction is:



From this balanced equation, we establish the stoichiometric ratio between the reacting species:

$$\text{Moles of KMnO}_4 = \frac{2}{5} \times \text{Moles of Na}_2\text{C}_2\text{O}_4$$

We can calculate the moles of sodium oxalate using its mass and molar mass:

$$\text{Moles} = \frac{\text{Mass (g)}}{\text{Molar Mass (g/mol)}}$$

Once the moles of permanganate are found, the molarity of the permanganate solution is calculated using the volume in liters:

$$\text{Molarity (M)} = \frac{\text{Moles of Solute}}{\text{Volume of Solution (L)}}$$

Step 3 : Detailed Explanation:

- First, we calculate the molar mass of sodium oxalate ($\text{Na}_2\text{C}_2\text{O}_4$). Using the atomic weights of sodium (23.00 g/mol), carbon (12.01 g/mol), and oxygen (16.00 g/mol),

we get: Molar Mass = $2(23.00) + 2(12.01) + 4(16.00) = 46.00 + 24.02 + 64.00 = 134.02 \text{ g/mol}$.

- Next, we find the number of moles of sodium oxalate used in the titration by dividing the given mass by its molar mass: Moles of $\text{Na}_2\text{C}_2\text{O}_4 = \frac{0.1467 \text{ g}}{134.02 \text{ g/mol}} \approx 1.0946 \times 10^{-3} \text{ moles}$.
- Using the stoichiometric ratio from our balanced chemical equation, we find the moles of potassium permanganate that reacted: Moles of $\text{KMnO}_4 = \frac{2}{5} \times (1.0946 \times 10^{-3} \text{ moles}) \approx 4.3784 \times 10^{-4} \text{ moles}$.
- We then convert the volume of the potassium permanganate solution from milliliters to liters: Volume = $28.85 \text{ mL} = \frac{28.85}{1000} \text{ L} = 0.02885 \text{ L}$.
- Finally, we calculate the molarity of the potassium permanganate solution by dividing the moles of permanganate by the volume in liters: Molarity = $\frac{4.3784 \times 10^{-4} \text{ moles}}{0.02885 \text{ L}} \approx 0.015176 \text{ M}$.
- Rounding this result to four significant figures gives the final molar concentration of 0.01518 M .

Step 4 : Final Answer:

The molar concentration of the KMnO_4 solution is 0.01518 M , which matches Option (A).

Quick Tip: To perform these analytical calculations rapidly without writing out the full balanced molecular equation, you can use the principle of equivalents: $N_1 V_1 = N_2 V_2$. Under acidic conditions, the valence factor (n-factor) of the oxidizing agent KMnO_4 is always 5 (as Mn changes from +7 to +2), and the n-factor of the reducing agent $\text{Na}_2\text{C}_2\text{O}_4$ is always 2 (as two carbons change from +3 to +4). This allows you to set up the direct equation: $M_{\text{permanganate}} \times 5 \times V_{\text{permanganate}} = \frac{\text{Mass of oxalate}}{\text{Molar mass of oxalate}} \times 2$, which simplifies the calculation steps.

7. A current of 4.0 A is passed through 0.5 L of 0.2 M NaCl solution for 1200s. Calculate the pH of the solution after electrolysis.

- (A) 1.3
- (B) 13
- (C) 7.0
- (D) 2.0

Correct Answer: (B) 13

Solution:

Step 1 : Understanding the Question:

This problem asks us to determine the pH of a sodium chloride (brine) solution after a specific current has been passed through it for a given duration. During the electrolysis of aqueous sodium chloride, water is reduced at the cathode, producing hydrogen gas and hydroxide ions, while chloride ions are oxidized at the anode to produce chlorine gas. The generation of hydroxide ions increases the basicity of the solution, and we need to calculate the resulting pH.

Step 2 : Key Formulas and Approach:

First, we find the total electrical charge Q passed through the electrolyte using:

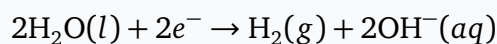
$$Q = I \times t$$

Where I is the current in amperes and t is the time in seconds.

Next, we calculate the number of moles of electrons transferred using Faraday's constant ($F \approx 96500 \text{ C/mol}$):

$$\text{Moles of } e^- = \frac{Q}{F}$$

The reduction of water at the cathode is represented by the half-reaction:



This shows that 1 mole of electrons produces 1 mole of hydroxide ions.

We will check whether the chloride ions are the limiting reactant. If not, the amount of OH^- formed is determined by the total charge. Finally, we calculate the concentration of OH^- , find the pOH, and then find the pH using:

$$\text{pH} = 14 - \text{pOH}$$

Step 3 : Detailed Explanation:

- We start by calculating the total electric charge passed through the solution:
 $Q = 4.0 \text{ A} \times 1200 \text{ s} = 4800 \text{ Coulombs}.$
- Next, we determine the number of moles of electrons that this charge represents:
 $\text{Moles of } e^- = \frac{4800 \text{ C}}{96500 \text{ C/mol}} \approx 0.04974 \text{ moles} \approx 0.05 \text{ moles}.$
- Now, we check the initial quantity of sodium chloride in the solution to ensure it is not a limiting reactant: Initial moles of NaCl = Molarity $\times$ Volume = $0.2 \text{ mol/L} \times 0.5 \text{ L} = 0.1 \text{ moles}.$
- Since the reaction requires 0.05 moles of electrons to oxidize 0.05 moles of chloride ions, and we have 0.1 moles available, the chloride ions are in excess, and the reaction is limited only by the electricity passed.
- From the stoichiometry of the cathode reaction, the moles of hydroxide ions produced are equal to the moles of electrons transferred. Therefore, Moles of OH^- produced = 0.05 moles.
- We calculate the molar concentration of the hydroxide ions in the 0.5 L solution:
 $[\text{OH}^-] = \frac{\text{Moles of OH}^-}{\text{Volume of Solution}} = \frac{0.05 \text{ moles}}{0.5 \text{ L}} = 0.1 \text{ M} = 10^{-1} \text{ M}.$

- Using the definition of pOH, we get: $\text{pOH} = -\log_{10}[\text{OH}^-] = -\log_{10}(10^{-1}) = 1$.
- Finally, we calculate the pH of the solution at standard temperature (25°C):
 $\text{pH} = 14 - \text{pOH} = 14 - 1 = 13$.

Step 4 : Final Answer:

The pH of the solution after electrolysis is 13, which corresponds to Option (B).

Quick Tip: When evaluating multiple-choice questions on this topic, you can often narrow down the options by noting that the electrolysis of brine generates a strong base, sodium hydroxide. This means the final solution must be basic, so the pH must be significantly greater than 7. This qualitative observation immediately rules out Options (A), (C), and (D), leaving Option (B) as the only plausible choice without requiring any calculations.

8. Using the standard electrode potential, find out the pair between which redox reaction is not feasible.

$E^\ominus$ values: $\text{Fe}^{3+}/\text{Fe}^{2+} = +0.77 \text{ V}$; $\text{I}_2/\text{I}^- = +0.54 \text{ V}$; $\text{Cu}^{2+}/\text{Cu} = +0.34 \text{ V}$; $\text{Ag}^+/\text{Ag} = +0.80 \text{ V}$

- (A) Fe^{3+} and I^-
- (B) Ag^+ and Cu
- (C) Fe^{3+} and Cu
- (D) Ag and Fe^{3+}

Correct Answer: (D) Ag and Fe^{3+}

Solution:

Step 1 : Understanding the Question:

This problem asks us to determine which of the given pairs of chemical species cannot undergo

a spontaneous redox reaction under standard conditions. A reaction is thermodynamically feasible if its standard cell potential is positive. We are given the standard reduction potentials for several half-cells, and we need to evaluate the cell potential for the proposed reactions to find the one with a negative cell potential.

Step 2 : Key Formulas and Approach:

For any redox reaction, the standard cell potential (E_{cell}°) is calculated using the standard reduction potentials (SRP) of the cathode (where reduction occurs) and the anode (where oxidation occurs):

$$E_{\text{cell}}^{\circ} = E_{\text{cathode (reduction)}}^{\circ} - E_{\text{anode (oxidation)}}^{\circ}$$

A reaction is spontaneous and thermodynamically feasible if:

$$E_{\text{cell}}^{\circ} > 0$$

If $E_{\text{cell}}^{\circ} < 0$, the reaction is non-spontaneous and not feasible.

Alternatively, a species with a higher reduction potential will spontaneously oxidize a species with a lower reduction potential. We will test the feasibility of the options using this principle.

Step 3 : Detailed Explanation:

- We begin by analyzing the reaction between Ag and Fe^{3+} proposed in Option (D). In this potential reaction, Fe^{3+} is in its oxidized state and must be reduced to Fe^{2+} , while Ag is in its reduced state and must be oxidized to Ag^+ .
- The reduction half-reaction is: $\text{Fe}^{3+} + e^{-} \rightarrow \text{Fe}^{2+}$ with $E_{\text{reduction}}^{\circ} = +0.77 \text{ V}$.
- The oxidation half-reaction is: $\text{Ag} \rightarrow \text{Ag}^{+} + e^{-}$ with $E_{\text{reduction}}^{\circ} = +0.80 \text{ V}$.
- We designate the iron half-cell as the cathode and the silver half-cell as the anode to

calculate the standard cell potential: $E_{\text{cell}}^{\circ} = E_{\text{Fe}^{3+}/\text{Fe}^{2+}}^{\circ} - E_{\text{Ag}^{+}/\text{Ag}}^{\circ} = +0.77 \text{ V} - (+0.80 \text{ V}) = -0.03 \text{ V}$.

- Since the calculated cell potential is negative ($E_{\text{cell}}^{\circ} = -0.03 \text{ V}$), this reaction is non-spontaneous under standard conditions and is not feasible.
- For comparison, let us verify Option (A) involving Fe^{3+} and I^{-} . Here, iron is reduced (+0.77 V) and iodide is oxidized (+0.54 V): $E_{\text{cell}}^{\circ} = 0.77 \text{ V} - 0.54 \text{ V} = +0.23 \text{ V}$, which is positive and feasible.
- For Option (B) involving Ag^{+} and Cu, silver is reduced (+0.80 V) and copper is oxidized (+0.34 V): $E_{\text{cell}}^{\circ} = 0.80 \text{ V} - 0.34 \text{ V} = +0.46 \text{ V}$, which is positive and feasible.
- For Option (C) involving Fe^{3+} and Cu, iron is reduced (+0.77 V) and copper is oxidized (+0.34 V): $E_{\text{cell}}^{\circ} = 0.77 \text{ V} - 0.34 \text{ V} = +0.43 \text{ V}$, which is positive and feasible.

Step 4 : Final Answer:

The pair between which the redox reaction is not feasible is Ag and Fe^{3+} , which corresponds to Option (D).

Quick Tip: To determine the feasibility of a reaction quickly, arrange the standard reduction potentials in decreasing order: Ag^{+}/Ag (0.80 V), $\text{Fe}^{3+}/\text{Fe}^{2+}$ (0.77 V), I_2/I^{-} (0.54 V), Cu^{2+}/Cu (0.34 V). Any species on the left (oxidized form) can spontaneously oxidize any species on the right (reduced form) that lies below it in this list. Since silver is above iron, Fe^{3+} cannot oxidize Ag, making this reaction non-feasible.

9. If p and q be the longest and the shortest distance respectively of the point $(-7, 2)$ from any point (α, β) on the curve whose equation is $x^2 + y^2 - 10x - 14y - 51 = 0$, then find the Geometric Mean (G.M.) of p and q .

- (A) $2\sqrt{11}$
- (B) $5\sqrt{5}$
- (C) 13
- (D) 11

Correct Answer: (A) $2\sqrt{11}$

Solution:

Step 1 : Understanding the Question:

This coordinate geometry problem asks us to find the geometric mean of the maximum and minimum distances from an external point to a circle. The circle is defined by a quadratic equation in two variables. We need to identify the geometric center and radius of this circle, calculate the distance from the given external point to the center, and use these values to express the longest and shortest distances. Finally, we will compute the geometric mean of these two distances.

Step 2 : Key Formulas and Approach:

For a circle given by the general equation $x^2 + y^2 + 2gx + 2fy + c = 0$:

The center C is located at $(-g, -f)$ and the radius R is given by:

$$R = \sqrt{g^2 + f^2 - c}$$

For any point $P(x_1, y_1)$ lying outside the circle:

The shortest distance q from P to the circle is along the line passing through the center:

$$q = PC - R$$

The longest distance p from P to the circle is also along this normal line:

$$p = PC + R$$

The Geometric Mean (G.M.) of two numbers p and q is defined as:

$$\text{G.M.} = \sqrt{p \times q}$$

By substituting the expressions for p and q , we can simplify the product before taking the square root.

Step 3 : Detailed Explanation:

- We start by finding the center and radius of the circle given by the equation $x^2 + y^2 - 10x - 14y - 51 = 0$. By comparing this with the general equation of a circle, we find $2g = -10 \implies g = -5$, and $2f = -14 \implies f = -7$, with constant term $c = -51$.
- This gives the coordinates of the center C of the circle as $(-g, -f) = (5, 7)$.
- We calculate the radius of the circle using the formula: $R = \sqrt{(-5)^2 + (-7)^2 - (-51)} = \sqrt{25 + 49 + 51} = \sqrt{125} = 5\sqrt{5}$.
- Next, we calculate the distance PC between the given point $P(-7, 2)$ and the center of the circle $C(5, 7)$ using the distance formula: $PC = \sqrt{(5 - (-7))^2 + (7 - 2)^2} = \sqrt{(12)^2 + (5)^2} = \sqrt{144 + 25} = \sqrt{169} = 13$.
- Since the distance to the center $PC = 13$ is greater than the radius $R = \sqrt{125} \approx 11.18$, the point P lies outside the circle, which validates our formulas for p and q .
- We express the longest distance p and shortest distance q as: $p = 13 + R$ and $q = 13 - R$.
- The product of these two distances is: $p \times q = (13 + R)(13 - R) = 13^2 - R^2 = 169 - 125 =$

44.

- Finally, we find the geometric mean by taking the square root of this product:

$$\text{G.M.} = \sqrt{p \times q} = \sqrt{44} = \sqrt{4 \times 11} = 2\sqrt{11}.$$

Step 4 : Final Answer:

The Geometric Mean of the longest and shortest distances is $2\sqrt{11}$, which corresponds to Option (A).

Quick Tip: This problem can be solved more rapidly by applying the Power of a Point theorem. For any point P outside a circle, the product of the longest and shortest distances from P to the circle is equal to the power of the point with respect to the circle, denoted as S_1 . The power of the point is found by substituting the coordinates of $P(-7, 2)$ directly into the circle's equation: $S_1 = (-7)^2 + (2)^2 - 10(-7) - 14(2) - 51 = 49 + 4 + 70 - 28 - 51 = 44$. Since $\text{G.M.} = \sqrt{p \times q} = \sqrt{S_1}$, we immediately get $\sqrt{44} = 2\sqrt{11}$.

10. The distance from the origin to the image of $(1, 1)$ with respect to the line $x + y + 5 = 0$ is:

- (A) $7\sqrt{2}$
- (B) $3\sqrt{2}$
- (C) $6\sqrt{2}$
- (D) $4\sqrt{2}$

Correct Answer: (C) $6\sqrt{2}$

Solution:

Step 1 : Understanding the Question:

This 2D coordinate geometry problem asks us to find the distance from the origin to the reflection (image) of the point $(1, 1)$ across the mirror line $x + y + 5 = 0$. To solve this, we must first determine the coordinates of the image point. Once the coordinates of this reflected point are established, we can calculate its straight-line distance to the origin $(0, 0)$.

Step 2 : Key Formulas and Approach:

The coordinates of the image point (x_2, y_2) of a given point (x_1, y_1) with respect to a line $ax + by + c = 0$ can be determined using the standard transformation formula:

$$\frac{x_2 - x_1}{a} = \frac{y_2 - y_1}{b} = -2 \left(\frac{ax_1 + by_1 + c}{a^2 + b^2} \right)$$

We will substitute the coordinates of the point and the coefficients of the line into this formula to solve for x_2 and y_2 .

After finding the image coordinates, we will compute the distance from the origin $(0, 0)$ to (x_2, y_2) using the standard Euclidean distance formula:

$$D = \sqrt{x_2^2 + y_2^2}$$

Step 3 : Detailed Explanation:

- We start by identifying the parameters from the problem: the given point is $(x_1, y_1) = (1, 1)$, and the equation of the line is $1x + 1y + 5 = 0$. This gives the coefficients $a = 1$, $b = 1$, and $c = 5$.
- Next, we calculate the term on the right-hand side of the image formula:
$$-2 \left(\frac{1(1) + 1(1) + 5}{1^2 + 1^2} \right) = -2 \left(\frac{7}{2} \right) = -7.$$
- We now set up two linear equations to find the coordinates of the image point (x_2, y_2) .
- For the x-coordinate, we have: $\frac{x_2 - 1}{1} = -7 \implies x_2 - 1 = -7 \implies x_2 = -6.$
- For the y-coordinate, we have: $\frac{y_2 - 1}{1} = -7 \implies y_2 - 1 = -7 \implies y_2 = -6.$

- This gives the coordinates of the reflected image point as $(-6, -6)$.
- Finally, we calculate the distance from the origin $(0, 0)$ to the image point $(-6, -6)$:

$$D = \sqrt{(-6-0)^2 + (-6-0)^2} = \sqrt{(-6)^2 + (-6)^2} = \sqrt{36 + 36} = \sqrt{72}.$$
- Simplifying the square root of 72 gives: $D = \sqrt{36 \times 2} = 6\sqrt{2}$ units.

Step 4 : Final Answer:

The distance from the origin to the image point is $6\sqrt{2}$, which corresponds to Option (C).

Quick Tip: You can use a symmetry-based approach to solve this problem more quickly. The given line $x + y + 5 = 0$ is perpendicular to the line $y = x$. Since the starting point $(1, 1)$ lies on the line of symmetry $y = x$, its image across the line $x + y + 5 = 0$ must also lie on the line $y = x$. This means the coordinates of the image will be of the form (k, k) . Knowing that $x_2 = -6$ immediately tells us that $y_2 = -6$. The distance of any point (k, k) from the origin is simply $|k|\sqrt{2}$, which directly yields $6\sqrt{2}$ without requiring the full calculation.

11. General solution of $\tan 5\theta = \cot 2\theta$ is:

- (A) $\theta = \frac{n\pi}{7} + \frac{\pi}{14}$
 (B) $\theta = \frac{n\pi}{7} + \frac{\pi}{5}$
 (C) $\theta = \frac{n\pi}{7} + \frac{\pi}{2}$
 (D) $\theta = \frac{n\pi}{7} + \frac{\pi}{3}$

Correct Answer: (A) $\theta = \frac{n\pi}{7} + \frac{\pi}{14}$

Solution:

Step 1 : Understanding the Question:

This problem asks us to find the general solution for θ in the trigonometric equation $\tan 5\theta = \cot 2\theta$. The equation contains two different trigonometric functions, tangent and cotangent, acting on different multiples of θ . To find the solution, we need to convert both

sides of the equation into the same trigonometric function and then apply the standard general solution formula.

Step 2 : Key Formulas and Approach:

We can convert the cotangent function to the tangent function using the complementary angle identity:

$$\cot \phi = \tan \left(\frac{\pi}{2} - \phi \right)$$

This conversion allows us to write the equation in the standard form:

$$\tan A = \tan B$$

The general solution for this standard trigonometric equation is given by:

$$A = n\pi + B, \quad \text{where } n \in \mathbb{Z}$$

We will apply this general formula to our transformed equation and isolate the variable θ .

Step 3 : Detailed Explanation:

- We start with the given trigonometric equation: $\tan 5\theta = \cot 2\theta$.
- To express both sides in terms of the tangent function, we apply the complementary angle identity to the right-hand side: $\cot 2\theta = \tan \left(\frac{\pi}{2} - 2\theta \right)$.
- Substituting this back into the original equation gives: $\tan 5\theta = \tan \left(\frac{\pi}{2} - 2\theta \right)$.
- Next, we apply the general solution formula for the tangent function, which leads to the equation: $5\theta = n\pi + \left(\frac{\pi}{2} - 2\theta \right)$, where n is any integer.

- To solve for θ , we first gather all terms containing θ on the left-hand side of the equation. We add 2θ to both sides: $5\theta + 2\theta = n\pi + \frac{\pi}{2}$.
- Simplifying the left-hand side yields: $7\theta = n\pi + \frac{\pi}{2}$.
- Finally, we isolate θ by dividing the entire equation by 7: $\theta = \frac{n\pi}{7} + \frac{\pi}{2 \times 7}$.
- This simplifies to the general solution: $\theta = \frac{n\pi}{7} + \frac{\pi}{14}$.

Step 4 : Final Answer:

The general solution of the equation is $\theta = \frac{n\pi}{7} + \frac{\pi}{14}$, which corresponds to Option (A).

Quick Tip: For equations of the form $\tan A = \cot B$, you can use a direct shortcut. Since $\tan A \cdot \tan B = 1$, the general relationship between the angles can be written directly as $A + B = n\pi + \frac{\pi}{2}$. For this problem, substituting $A = 5\theta$ and $B = 2\theta$ gives $5\theta + 2\theta = n\pi + \frac{\pi}{2} \implies 7\theta = n\pi + \frac{\pi}{2}$, which allows you to find the final answer in a single step.

12. The sum of the series $(x + \frac{1}{x})^2 + (x^2 + \frac{1}{x^2})^2 + (x^3 + \frac{1}{x^3})^2 \dots \dots \dots$ up to n terms is:

- (A) $\frac{x^{2n}-1}{x^2-1} \times \frac{x^{2n+2}+1}{x^{2n}} + 2n$
 (B) $\frac{x^{2n}+1}{x^2+1} \times \frac{x^{2n+2}-1}{x^{2n}} - 2n$
 (C) $\frac{x^{2n}-1}{x^2-1} \times \frac{x^{2n}-1}{x^{2n}} - 2n$
 (D) None of these

Correct Answer: (A) $\frac{x^{2n}-1}{x^2-1} \times \frac{x^{2n+2}+1}{x^{2n}} + 2n$

Solution:

Step 1 : Understanding the Question:

This problem asks us to calculate the sum of a series up to n terms, where each term is the square of a binomial of the form $(x^k + \frac{1}{x^k})$. To find the sum, we can expand each binomial

term, group similar algebraic terms together, and identify the resulting geometric progressions. By summing these individual parts, we can find the simplified expression for the total sum of the series.

Step 2 : Key Formulas and Approach:

The expansion of each term in the series uses the algebraic identity:

$$(A + B)^2 = A^2 + 2AB + B^2$$

Applying this to the k -th term of the series, we get:

$$\left(x^k + \frac{1}{x^k}\right)^2 = x^{2k} + 2 + \frac{1}{x^{2k}}$$

This allows us to split the sum of n terms into three separate sub-series:

1. A geometric progression with term x^{2k} .
2. A geometric progression with term $\frac{1}{x^{2k}}$.
3. A constant term of 2 summed n times.

The sum of a geometric progression with first term a and common ratio r is given by:

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

We will evaluate these three sums separately and then combine and simplify them.

Step 3 : Detailed Explanation:

- We expand each term of the series individually.
- The first term is: $\left(x + \frac{1}{x}\right)^2 = x^2 + 2 + \frac{1}{x^2}$.
- The second term is: $\left(x^2 + \frac{1}{x^2}\right)^2 = x^4 + 2 + \frac{1}{x^4}$.

- The third term is: $\left(x^3 + \frac{1}{x^3}\right)^2 = x^6 + 2 + \frac{1}{x^6}$, and so on.
- We write the total sum S of n terms by grouping the expanded terms into three separate series: $S = \sum_{k=1}^n x^{2k} + \sum_{k=1}^n \frac{1}{x^{2k}} + \sum_{k=1}^n 2$.
- The first sub-series is a geometric progression: $S_1 = x^2 + x^4 + x^6 + \dots + x^{2n}$. The first term is $a_1 = x^2$ and the common ratio is $r_1 = x^2$. Using the GP sum formula, we get:

$$S_1 = \frac{x^2(x^{2n}-1)}{x^2-1}.$$
- The second sub-series is also a geometric progression: $S_2 = \frac{1}{x^2} + \frac{1}{x^4} + \dots + \frac{1}{x^{2n}}$. The first term is $a_2 = \frac{1}{x^2}$ and the common ratio is $r_2 = \frac{1}{x^2}$. Applying the GP sum formula gives:

$$S_2 = \frac{\frac{1}{x^2}\left(1 - \frac{1}{x^{2n}}\right)}{1 - \frac{1}{x^2}} = \frac{x^{2n}-1}{x^{2n}(x^2-1)}.$$
- The third sub-series is the constant 2 summed n times: $S_3 = 2n$.
- Now, we add the three sums together: $S = \frac{x^2(x^{2n}-1)}{x^2-1} + \frac{x^{2n}-1}{x^{2n}(x^2-1)} + 2n$.
- We factor out the common term $\frac{x^{2n}-1}{x^2-1}$ from the first two terms: $S = \frac{x^{2n}-1}{x^2-1} \left[x^2 + \frac{1}{x^{2n}} \right] + 2n$.
- Simplifying the expression inside the brackets by finding a common denominator gives:

$$S = \frac{x^{2n}-1}{x^2-1} \left[\frac{x^{2n+2}+1}{x^{2n}} \right] + 2n.$$

Step 4 : Final Answer:

The sum of the series is $\frac{x^{2n}-1}{x^2-1} \times \frac{x^{2n+2}+1}{x^{2n}} + 2n$, which corresponds to Option (A).

Quick Tip: To solve complex series summation problems quickly in exams, you can use the substitution method by testing $n = 1$. For $n = 1$, the sum of the series must equal just the first term, which is $\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$. Substituting $n = 1$ into Option (A) gives: $\frac{x^2-1}{x^2-1} \times \frac{x^4+1}{x^2} + 2 = 1 \times \left(x^2 + \frac{1}{x^2}\right) + 2 = x^2 + \frac{1}{x^2} + 2$. This matches the first term, confirming Option (A) is correct.
