BITSAT 2026 May 25 Shift 1

Question Paper (Memory Based) with Solutions

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General Instructions

(i) **Duration:** The total duration of the examination is 3 hours (180 minutes).

(ii) **Total Marks:** The complete paper carries a maximum of 390 marks.

(iii) **Structure:** The paper has 4 Sections:

- **Part 1:** 30 Multiple Choice Questions (Physics).
- **Part 2:** 30 Multiple Choice Questions (Chemistry).
- **Part 3:** 10 Multiple Choice Questions (English Proficiency), 20 Multiple Choice Questions (Logical Reasoning)
- **Part 4:** 40 Multiple Choice Questions (Mathematics/Biology)

(iv) **Compulsory Questions:** All 130 questions are compulsory,and +12 Questions(Optional Extra Questions)

(v) Each question has four options. Only **one** option is correct.

(vi) **Correct Answer:** +3 marks.

(vii) **Incorrect Answer:** −1 (Negative marking).

(viii) **Unanswered/Marked for Review:** 0 marks.

PHYSICS

1. Two point charges $+q$ and $-q$ are held fixed at a distance d apart. The net electric potential at a point midway between the two charges and its net field are?

(A) $E = 0 \quad V = 0$

(B) $E \neq 0 \quad V \neq 0$

(C) $E = 0 \quad V \neq 0$

(D) $E \neq 0 \quad V = 0$

Correct Answer: (D) $E \neq 0 \quad V = 0$

Solution:

Topic of the Question:

This problem belongs to electrostatics, focusing on the properties of electric potential and electric field due to a point charge configuration, specifically an electric dipole system.

Step 1 : Understanding the Question:

We are given two equal and opposite point charges $+q$ and $-q$ separated by a fixed distance d. We need to evaluate both the net electric potential (V) and the net electric field (E) at the midpoint on the line joining these two charges.

Step 2 : Key Formulas and Approach:

- Electric potential (V) is a scalar quantity. The net potential at a point is the algebraic sum of individual potentials: $V = \sum \frac{kq_i}{r_i}$, where $k = \frac{1}{4\pi\varepsilon_0}$.

- Electric field ($\vec{E}$) is a vector quantity. The net electric field at any point is the vector sum of individual fields: $\vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 + \ldots$, where the magnitude of a point charge's field is $E = \frac{k|q|}{r^2}$.

Step 3 : Detailed Explanation:

- At the midway point, the distance from each point charge is equal to $r = \frac{d}{2}$.

- To find the net electric potential, we sum the potential due to the positive charge and the potential due to the negative charge: $V_{\text{net}} = V_+ + V_- = \frac{kq}{d/2} + \frac{k(-q)}{d/2}$.

- Simplifying this algebraic sum yields $V_{\text{net}} = \frac{2kq}{d} - \frac{2kq}{d} = 0$. Hence, the net electric potential at the center is exactly zero.

- To calculate the net electric field, we analyze the direction of the field vectors. The electric field due to the positive charge ($\vec{E}_+$) points away from it, which is towards the negative charge (to the right).

- The electric field due to the negative charge ($\vec{E}_-$) points towards it, which is also to the right. Since both vector fields point in the exact same direction, their magnitudes add constructively.

- The total field magnitude is calculated as $E_{\text{net}} = E_+ + E_- = \frac{kq}{(d/2)^2} + \frac{kq}{(d/2)^2} = \frac{4kq}{d^2} + \frac{4kq}{d^2} = \frac{8kq}{d^2}$.

- Since q and d are non-zero, this net electric field magnitude is clearly non-zero ($E \neq 0$).

Step 4 : Final Answer:

Based on the calculations, the net potential is zero ($V = 0$) and the net electric field is non-zero ($E \neq 0$). This matches the expressions provided in Option (D).

Quick Tip: When analyzing a system of two equal and opposite charges, any point along the perpendicular bisecting plane (including the midpoint) lies at an equal distance from both charges. This configuration ensures that the scalar electric potentials cancel out completely to zero, whereas the vector electric field lines run parallel to the axis, resulting in a constructive, non-zero field.

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2. A charge Q is placed at the center of an uncharged conducting spherical shell of inner radius R_1 and outer radius R_2. The surface charge density on the outer surface of the shell is:

(A) $\frac{Q}{(4\pi R_1^2)}$

(B) $\frac{Q}{(4\pi R_2^2)}$

(C) $\frac{-Q}{(4\pi R_1^2)}$

(D) Zero

Correct Answer: (B) $\frac{Q}{(4\pi R_2^2)}$

Solution:

Topic of the Question:

This question deals with electrostatic induction in conductors and the calculation of surface charge density on a conducting spherical shell.

Step 1 : Understanding the Question:

We are given an uncharged, conducting spherical shell with an inner radius R_1 and an outer radius R_2. A point charge Q is positioned at the exact center of this shell. We need to find the resulting surface charge density on the outer surface of the conducting shell.

Step 2 : Key Formulas and Approach:

- Electrostatic induction: Under electrostatic equilibrium, the electric field inside the metallic bulk of a conductor must be zero.
- Gauss's Law states that the net charge enclosed by any Gaussian surface drawn completely inside the conducting material must be zero.
- Surface charge density (σ) represents charge per unit area: $\sigma = \frac{\text{Charge on the surface}}{\text{Total surface area}} = \frac{q}{4\pi R^2}$.

Step 3 : Detailed Explanation:

- When a point charge $+Q$ is placed at the center, it produces an electric field pointing outwards. To shield the interior of the conducting material and maintain a zero electric field inside the bulk of the shell, electrostatic induction occurs.
- An equal and opposite charge of $-Q$ is drawn to the inner surface of the shell at radius R_1. This inner charge completely cancels out the field of the central charge inside the

conducting material.

- Because the conducting shell was initially uncharged and is electrically isolated, the total net charge on the shell must remain zero by the law of conservation of charge.
- Consequently, a charge of $+Q$ must be distributed over the outer boundary of the shell at radius R_2 to balance the $-Q$ induced on the inner surface.
- The outer boundary of the shell is a sphere of radius R_2, which has a total surface area of $4\pi R_2^2$.
- We calculate the surface charge density on this outer boundary by dividing the outer surface charge by the outer surface area: $\sigma_{\text{outer}} = \frac{+Q}{4\pi R_2^2}$.

Step 4 : Final Answer:

The induced charge on the outer surface is $+Q$, which gives a surface charge density of $\frac{Q}{4\pi R_2^2}$. This corresponds to Option (B).

Quick Tip: To satisfy the condition of zero electric field inside the material of a conductor, the inner boundary of a cavity must always acquire an induced charge that is equal in magnitude and opposite in sign to any enclosed charge. For an isolated uncharged conductor, an equivalent charge of the same sign is repelled to the outer surface, where it distributes uniformly.

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3. A parallel plate capacitor is charged and then disconnected from the charging battery. If the plates are now pulled further apart using insulating handles:

(A) The charge increases

(B) The voltage decreases

(C) The capacitance increases

(D) The electrostatic energy stored increases

Correct Answer: (D) The electrostatic energy stored increases

Solution:

Topic of the Question:

The topic of this question is the behavior of a parallel plate capacitor under physical changes, specifically when the plates are separated after being disconnected from a power source.

Step 1 : Understanding the Question:

We are analyzing a parallel plate capacitor that has been fully charged and then completely disconnected from its charging battery. We must determine the effects on charge, voltage, capacitance, and stored electrostatic potential energy when the distance between the plates is increased using insulating handles.

Step 2 : Key Formulas and Approach:

- Capacitance of a parallel plate capacitor: $C = \frac{\varepsilon_0 A}{d}$, where A is the area of the plates and d is their separation distance.
- Relationship between charge, capacitance, and potential difference: $V = \frac{Q}{C}$.
- Electrostatic potential energy stored in the capacitor: $U = \frac{Q^2}{2C}$.
- Conservation of charge: When a capacitor is disconnected from a circuit, its plates are electrically isolated, meaning the net charge Q on the plates must remain constant.

Step 3 : Detailed Explanation:

- Since the capacitor is disconnected from the battery, there is no conductive path for charge to enter or leave the plates. Therefore, the charge Q remains constant, making Option (A) incorrect.

- When the plates are pulled further apart, the separation distance d increases. According to the formula $C = \frac{\varepsilon_0 A}{d}$, the capacitance C is inversely proportional to d. An increase in separation leads to a decrease in capacitance, making Option (C) incorrect.

- Using the potential relationship $V = \frac{Q}{C}$, we can determine how the voltage changes. Since the charge Q is constant and the capacitance C decreases, the potential difference V across the plates must increase. Thus, Option (B) is incorrect.

- Now we evaluate the stored electrostatic potential energy using the formula $U = \frac{Q^2}{2C}$. Since Q is constant and C decreases, the energy U must increase.

- This increase in electrostatic energy is physically supplied by the mechanical work performed by an external agent pulling the plates apart against the attractive electrostatic forces between the oppositely charged plates.

Step 4 : Final Answer:

The separation of the plates causes the stored electrostatic energy of the system to increase. This corresponds to the statement in Option (D).

Quick Tip: When analyzing capacitor modifications, remember that disconnecting the battery isolates the system, keeping the charge Q constant while letting the voltage V vary. Conversely, if the battery remains connected, the voltage V is held constant by the source while the charge Q adjusts to any geometric changes.

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4. The magnetic field at the center of a circular current-carrying loop of radius R is B_0. The magnetic field at an axial point at a distance $x = R$ from the center of the loop is:

(A) $B_0/2$

(B) $B_0/2\sqrt{2}$

(C) $B_0/4$

(D) $B_0\sqrt{2}$

Correct Answer: (B) $B_0/2\sqrt{2}$

Solution:

Topic of the Question:

This question deals with electromagnetism, specifically the calculation and comparison of the magnetic field generated by a circular current-carrying loop at its center versus along its axis.

Step 1 : Understanding the Question:

We are given that a circular loop of radius R carries a current and produces a magnetic field of magnitude B_0 at its center. We need to determine the magnitude of the magnetic field at a point on the axis of the loop at a distance $x = R$ from the center.

Step 2 : Key Formulas and Approach:

- The magnetic field at the center of a circular loop carrying current I is: $B_{\text{center}} = \frac{\mu_0 I}{2R}$.
- The magnetic field at an axial point located at a distance x from the center of the loop is: $B_{\text{axial}} = \frac{\mu_0 I R^2}{2(R^2+x^2)^{3/2}}$.
- We can solve this by expressing the axial magnetic field formula in terms of the central magnetic field B_0 and substituting $x = R$.

Step 3 : Detailed Explanation:

- We begin by writing the expression for the magnetic field at the center of the loop, which is given as B_0: $B_0 = \frac{\mu_0 I}{2R}$.
- Next, we write down the general equation for the magnetic field along the axis of the loop: $B_{\text{axial}} = \frac{\mu_0 I R^2}{2(R^2+x^2)^{3/2}}$.

- We substitute the given axial distance $x = R$ into this equation: $B_{\text{axial}} = \frac{\mu_0 I R^2}{2(R^2+R^2)^{3/2}}$.
- Simplifying the term in the denominator: $(R^2+R^2)^{3/2} = (2R^2)^{3/2} = 2^{3/2} \cdot (R^2)^{3/2} = 2\sqrt{2}R^3$.
- We substitute this back into our expression for the axial field: $B_{\text{axial}} = \frac{\mu_0 I R^2}{2 \cdot 2\sqrt{2}R^3} = \frac{\mu_0 I}{2R \cdot 2\sqrt{2}}$.
- By comparing this simplified expression to our initial equation for B_0, we can substitute $B_0 = \frac{\mu_0 I}{2R}$ directly into the relation.
- This substitution yields the final axial field expression: $B_{\text{axial}} = \frac{B_0}{2\sqrt{2}}$.

Step 4 : Final Answer:

The magnetic field at the axial distance $x = R$ is equal to $B_0/(2\sqrt{2})$, which matches Option (B).

Quick Tip: To find the magnetic field at any axial point of a circular loop quickly, use the ratio relation: $\frac{B_{\text{axial}}}{B_{\text{center}}} = \left(1 + \frac{x^2}{R^2}\right)^{-3/2}$. Substituting the condition $x = R$ directly gives the factor $(1+1)^{-3/2} = 2^{-3/2} = \frac{1}{2\sqrt{2}}$, bypassing the need to write out the full physical constants.

CHEMISTRY

5. The value of enthalpy change (ΔH) for the reaction $C_2H_5OH(l) + 3O_2(g) \rightarrow 2CO_2(g) + 3H_2O(l)$, at 27°C is -1366.5 kJ mol^{-1}. The value of internal energy change for the above reaction at this temperature will be

(A) -1371.5 kJ mol^{-1}

(B) -1369.0 kJ mol^{-1}

(C) -1364.0 kJ mol^{-1}

(D) -1361.5 kJ mol^{-1}

Correct Answer: (C) -1364.0 kJ mol^{-1}

Solution:

Topic of the Question:

The topic of this question is chemical thermodynamics, focusing on the relationship between enthalpy change and internal energy change during a chemical reaction.

Step 1 : Understanding the Question:

We are given the enthalpy of reaction (ΔH) for the combustion of liquid ethanol at 27°C as -1366.5 kJ mol^{-1}. We are asked to calculate the corresponding change in internal energy (ΔU) for the reaction at this same temperature.

Step 2 : Key Formulas and Approach:

- The thermodynamic relation between the enthalpy change (ΔH) and the internal energy change (ΔU) at a constant temperature T is: $\Delta H = \Delta U + \Delta n_g RT$.
- Rearranging the formula to solve for the internal energy change gives: $\Delta U = \Delta H - \Delta n_g RT$.
- The change in the number of moles of gaseous species is: $\Delta n_g = \sum n_g(\text{gaseous products}) - \sum n_g(\text{gaseous reactants})$.
- The universal gas constant R is 8.314 J mol^{-1} K^{-1}, which must be converted to kJ mol^{-1} K^{-1} by dividing by 1000.
- Temperature must be converted from Celsius to Kelvin: $T(\text{K}) = T(^\circ\text{C}) + 273$.

Step 3 : Detailed Explanation:

- We begin by analyzing the balanced chemical equation to calculate the change in

gaseous moles: $C_2H_5OH(l) + 3O_2(g) \rightarrow 2CO_2(g) + 3H_2O(l)$.

- The only gaseous product is 2 moles of $CO_2(g)$, and the only gaseous reactant is 3 moles of $O_2(g)$. The liquid species, $C_2H_5OH(l)$ and $H_2O(l)$, are excluded.
- This gives: $\Delta n_g = 2 - 3 = -1$.
- Next, we convert the temperature to Kelvin: $T = 27 + 273 = 300$ K.
- We express the gas constant in terms of kilojoules: $R = 8.314 \times 10^{-3}$ kJ mol^{-1} K^{-1}.
- Now we substitute these values into our rearranged equation for internal energy change: $\Delta U = -1366.5 - [(-1) \times (8.314 \times 10^{-3}) \times 300]$.
- Simplifying the terms: $\Delta U = -1366.5 + [8.314 \times 10^{-3} \times 300] = -1366.5 + 2.4942$.
- This calculation yields: $\Delta U \approx -1364.0$ kJ mol^{-1}.

Step 4 : Final Answer:

The internal energy change for the reaction is -1364.0 kJ mol^{-1}, which matches Option (C).

Quick Tip: When calculating Δn_g, always double-check the physical states indicated in the reaction equation. Only species labeled as gas (g) are counted, while liquids (l) and solids (s) must be omitted. Additionally, always convert R to units of kJ to match the units of ΔH.

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6. For the complete combustion of ethanol, $C_2H_5OH(l) + 3O_2(g) \rightarrow 2CO_2(g) + 3H_2O(l)$, the amount of heat produced as measured in bomb calorimeter is 1364.47 kJ mol^{-1} at 25°C. Assum-

ing ideality the enthalpy of combustion, ΔH_C, for the reaction will be ($R = 8.314\ JK^{-1}mol^{-1}$)

(A) -1366.95 kJ mol^{-1}

(B) -1361.95 kJ mol^{-1}

(C) -1460.50 kJ mol^{-1}

(D) -1350.50 kJ mol^{-1}

Correct Answer: (A) -1366.95 kJ mol^{-1}

Solution:

Topic of the Question:

This question deals with thermochemistry, focusing on the interpretation of calorimetric measurements and the calculation of reaction enthalpy from constant-volume heat data.

Step 1 : Understanding the Question:

The combustion of liquid ethanol is studied in a bomb calorimeter at 25°C, yielding a heat output of 1364.47 kJ mol^{-1}. We need to determine the standard enthalpy of combustion (ΔH_C) for this reaction.

Step 2 : Key Formulas and Approach:

- A bomb calorimeter operates under rigid, constant-volume conditions. Therefore, the heat released in this device is equal to the internal energy change of the reaction: $q_v = \Delta U$.
- Since the reaction is exothermic (heat is produced), the internal energy change is negative: $\Delta U = -1364.47$ kJ mol^{-1}.
- We relate the enthalpy change to the internal energy change using the formula: $\Delta H_C = \Delta U + \Delta n_g RT$.
- Δn_g is the difference between the stoichiometric coefficients of gaseous products and gaseous reactants.

- The temperature must be converted to Kelvin: $T = 25^{\circ}\text{C} + 273.15 = 298.15$ K.

Step 3 : Detailed Explanation:

- We analyze the balanced equation for the complete combustion of ethanol: $C_2H_5OH(l) + 3O_2(g) \rightarrow 2CO_2(g) + 3H_2O(l)$.

- The products include 2 moles of $CO_2(g)$ and 3 moles of $H_2O(l)$. The reactants include 1 mole of $C_2H_5OH(l)$ and 3 moles of $O_2(g)$.

- Only the gaseous species are considered for calculating Δn_g: $\Delta n_g = n_g(\text{products}) - n_g(\text{reactants}) = 2 - 3 = -1$.

- We write the gas constant in terms of kilojoules: $R = 8.314 \times 10^{-3}$ kJ K^{-1} mol^{-1}.

- We substitute these parameters into the main relationship: $\Delta H_C = -1364.47 + [(-1) \times (8.314 \times 10^{-3}) \times 298.15]$.

- Evaluating the second term: $\Delta n_g RT = -1 \times 0.008314 \times 298.15 \approx -2.4788$ kJ mol^{-1}.

- Adding this to the internal energy change gives: $\Delta H_C = -1364.47 - 2.4788 \approx -1366.95$ kJ mol^{-1}.

Step 4 : Final Answer:

The standard enthalpy of combustion for ethanol under these conditions is -1366.95 kJ mol^{-1}, which matches Option (A).

Quick Tip: Always distinguish between constant-volume and constant-pressure conditions. Heat measured in a bomb calorimeter corresponds to constant-volume conditions (ΔU), whereas open-vessel calorimeter experiments measure heat at constant pressure (ΔH).

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7. What is $[NH_4^+]$ in a solution that is 0.02M NH_3 and 0.01 M KOH? $[K_b(NH_3) = 1.8 \times 10^{-5}]$

(A) 3.6×10^{-5} M

(B) 1.8×10^{-5} M

(C) 0.9×10^{-5} M

(D) 7.2×10^{-5} M

Correct Answer: (A) 3.6×10^{-5} M

Solution:

Topic of the Question:

The topic of this question is ionic equilibrium, specifically focusing on the common ion effect on the dissociation of a weak base in the presence of a strong base.

Step 1 : Understanding the Question:

We are asked to calculate the equilibrium concentration of ammonium ions ($[NH_4^+]$) in an aqueous solution that contains both a weak base (0.02 M NH_3) and a strong base (0.01 M KOH). The base dissociation constant (K_b) for ammonia is given as 1.8×10^{-5}.

Step 2 : Key Formulas and Approach:

- The strong base KOH dissociates completely in solution: $KOH(aq) \rightarrow K^+(aq) + OH^-(aq)$.
- The weak base ammonia undergoes partial dissociation in water: $NH_3(aq) + H_2O(l) \rightleftharpoons NH_4^+(aq) + OH^-(aq)$.
- The equilibrium expression for ammonia dissociation is: $K_b = \frac{[NH_4^+][OH^-]}{[NH_3]}$.

- Due to the common ion effect of OH^- ions from KOH, the dissociation of ammonia is highly suppressed, allowing us to use simplifying approximations.

Step 3 : Detailed Explanation:

- Since KOH is a strong electrolyte, it dissociates completely, contributing 0.01 M of OH^- ions to the solution.

- We set up the equilibrium concentration table for ammonia. Let the concentration of ammonia that dissociates be x M.

- At equilibrium, the concentrations of the species are: $[NH_3] = 0.02 - x$ M, $[NH_4^+] = x$ M, and $[OH^-] = 0.01 + x$ M.

- Because K_b is small (1.8×10^{-5}) and the presence of the common ion OH^- further suppresses the dissociation, x is extremely small.

- We can apply the following approximations: $0.02 - x \approx 0.02$ M and $0.01 + x \approx 0.01$ M.

- Substituting these approximations into the equilibrium constant expression: $K_b = \frac{(x)(0.01)}{0.02}$.

- We set this equal to the given value of K_b: $1.8 \times 10^{-5} = \frac{x \cdot 0.01}{0.02} = 0.5 \cdot x$.

- Solving for x: $x = 1.8 \times 10^{-5} \times 2 = 3.6 \times 10^{-5}$ M.

- Since x represents the equilibrium concentration of ammonium ions, we have: $[NH_4^+] = 3.6 \times 10^{-5}$ M.

Step 4 : Final Answer:

The equilibrium concentration of ammonium ions is 3.6×10^{-5} M, which corresponds to Option (A).

Quick Tip: For a system containing a weak base mixed with a strong base, the dissociation of the weak base is suppressed so heavily that the total hydroxide concentration is essentially determined by the strong base alone. You can find the conjugate acid concentration directly using: $[\text{Conjugate Acid}] \approx K_b \times \frac{[\text{Weak Base}]}{[\text{Strong Base}]}$.

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8. 1.1 mole of A mixed with 2.2 moles of B and the mixture is kept in a 1 L flask and the equilibrium, $A + 2B \rightleftharpoons 2C + D$ is reached. If at equilibrium 0.2 mole of C is formed then the value of K_c will be

(A) 0.1

(B) 0.01

(C) 0.001

(D) 1

Correct Answer: (C) 0.001

Solution:

Topic of the Question:

This question belongs to the topic of chemical equilibrium, specifically focusing on the calculation of the equilibrium constant (K_c) using stoichiometry and initial concentrations of reactants.

Step 1 : Understanding the Question:

We are given that 1.1 moles of reactant A and 2.2 moles of reactant B are mixed in a 1 L flask. They react according to the equation: $A + 2B \rightleftharpoons 2C + D$. At equilibrium, the flask contains 0.2 moles of product C. We need to find the value of the equilibrium constant K_c.

Step 2 : Key Formulas and Approach:

- The equilibrium constant (K_c) for the given reaction is expressed as: $K_c = \frac{[C]^2[D]}{[A][B]^2}$.

- We use an ICE (Initial, Change, Equilibrium) table to determine the equilibrium concentrations of all species in terms of a change variable x.

- Since the volume of the container is 1 L, the molar concentration of each species is numerically equal to its number of moles: $[X] = \frac{\text{moles of X}}{1\text{ L}}$.

Step 3 : Detailed Explanation:

- We construct the ICE table to track the moles of each reactant and product throughout the reaction:

- Initial state: Moles of A $= 1.1$, Moles of B $= 2.2$, Moles of C $= 0$, Moles of D $= 0$.

- Change state: Let x moles of A react. According to stoichiometry, $2x$ moles of B react, producing $2x$ moles of C and x moles of D.

- Equilibrium state: Moles of A $= 1.1 - x$, Moles of B $= 2.2 - 2x$, Moles of C $= 2x$, Moles of D $= x$.

- We are given that the moles of C at equilibrium is 0.2. Therefore, we can write: $2x = 0.2 \implies x = 0.1$.

- We substitute $x = 0.1$ back into our expressions to find the equilibrium moles of each species:

- Moles of A $= 1.1 - 0.1 = 1.0$ mole.

- Moles of B $= 2.2 - 2(0.1) = 2.0$ moles.
- Moles of C $= 0.2$ mole.
- Moles of D $= 0.1$ mole.
- Since the volume of the flask is 1 L, these mole values are equal to the molar concentrations: $[A] = 1.0$ M, $[B] = 2.0$ M, $[C] = 0.2$ M, and $[D] = 0.1$ M.
- We substitute these concentrations into the K_c expression: $K_c = \frac{(0.2)^2 \times (0.1)}{(1.0) \times (2.0)^2}$.
- Simplifying the numerator and denominator: $K_c = \frac{0.04 \times 0.1}{1.0 \times 4.0} = \frac{0.004}{4.0} = 0.001$.

Step 4 : Final Answer:

The calculated value of the equilibrium constant K_c is 0.001, which corresponds to Option (C).

Quick Tip: Always keep track of the volume of the reaction container when solving chemical equilibrium problems. For a 1 L container, the number of moles directly equals the molar concentration, but for any other volume, failing to divide the equilibrium moles by the volume will lead to incorrect values in the K_c calculation.

MATHEMATICS

9. If $y^x = e^{y-x}$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{1+\log y}{y \log y}$

(B) $\frac{(1+\log y)^2}{y \log y}$

(C) $\frac{1+\log y}{(\log y)^2}$

(D) $\frac{(1+\log y)^2}{\log y}$

Correct Answer: (D) $\frac{(1+\log y)^2}{\log y}$

Solution:

Topic of the Question:

The topic of this question is differential calculus, specifically focusing on logarithmic differentiation and the application of differentiation rules to implicit functions.

Step 1 : Understanding the Question:

We are given an equation where variables appear in both the base and the exponent: $y^x = e^{y-x}$. We are asked to find the derivative of y with respect to x, denoted as $\frac{dy}{dx}$.

Step 2 : Key Formulas and Approach:

- Logarithmic simplification: We take the natural logarithm (log) of both sides of the equation to simplify the exponent using the identity $\log(a^b) = b \log a$.
- Explicit expression: We rearrange the simplified equation to write x explicitly as a function of y, i.e., $x = f(y)$.
- Quotient rule: To find the derivative $\frac{dx}{dy}$, we use the formula: $\frac{d}{dy}\left(\frac{u}{v}\right) = \frac{v \cdot u' - u \cdot v'}{v^2}$.
- Reciprocal derivative identity: We obtain the final derivative using the relation $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$.

Step 3 : Detailed Explanation:

- We start with the given implicit equation: $y^x = e^{y-x}$.
- Taking the natural logarithm on both sides of this equation yields: $\log(y^x) = \log(e^{y-x})$.

- Using the power rule of logarithms, we simplify this to: $x \log y = (y - x) \log e$.

- Since the natural logarithm of e is equal to 1 ($\log e = 1$), the equation becomes: $x \log y = y - x$.

- To isolate x, we collect all terms containing x on the left side of the equation: $x \log y + x = y$.

- Factoring out x gives: $x(1 + \log y) = y \implies x = \frac{y}{1+\log y}$.

- Next, we differentiate x with respect to y using the quotient rule, where $u = y$ and $v = 1 + \log y$:

- This gives: $\frac{dx}{dy} = \frac{(1+\log y)\cdot \frac{d}{dy}(y) - y \cdot \frac{d}{dy}(1+\log y)}{(1+\log y)^2}$.

- Evaluating the individual derivatives: $\frac{dx}{dy} = \frac{(1+\log y)\cdot 1 - y \cdot \left(\frac{1}{y}\right)}{(1+\log y)^2}$.

- Simplifying the numerator: $\frac{dx}{dy} = \frac{1+\log y - 1}{(1+\log y)^2} = \frac{\log y}{(1+\log y)^2}$.

- To find $\frac{dy}{dx}$, we take the reciprocal of our expression for $\frac{dx}{dy}$: $\frac{dy}{dx} = \frac{(1+\log y)^2}{\log y}$.

Step 4 : Final Answer:

The derivative $\frac{dy}{dx}$ is equal to $\frac{(1+\log y)^2}{\log y}$, which corresponds to Option (D).

Quick Tip: For equations where variables are present in the exponents, expressing x as an explicit function of y ($x = f(y)$) and then finding $\frac{dx}{dy}$ is often much simpler and less prone to algebraic errors than performing implicit differentiation with respect to x directly.

10. The domain of the function $f(x)=\sqrt{x-\sqrt{1-x^2}}$ is

(A) $\left[-1,-\frac{1}{\sqrt{2}}\right]\cup\left[\frac{1}{\sqrt{2}},1\right]$

(B) $[-1,1]$

(C) $\left(-\infty,-\frac{1}{\sqrt{2}}\right]\cup\left[\frac{1}{\sqrt{2}},+\infty\right]$

(D) $\left[\frac{1}{\sqrt{2}},1\right]$

Correct Answer: (D) $\left[\frac{1}{\sqrt{2}},1\right]$

Solution:

Topic of the Question:

The topic of this question is set theory and functions, specifically focusing on finding the domain of a nested real-valued square root function.

Step 1 : Understanding the Question:

We are given the real-valued function $f(x)=\sqrt{x-\sqrt{1-x^2}}$. We need to determine the set of all real values of x for which this function is mathematically defined.

Step 2 : Key Formulas and Approach:

- For a square root function of the form $\sqrt{g(x)}$ to yield real values, the expression inside the radical must be non-negative: $g(x)\geq 0$.
- For a nested radical function, we apply this non-negativity constraint systematically from the innermost square root to the outermost square root.
- The final domain of the function is determined by taking the intersection of all individual constraint intervals.

Step 3 : Detailed Explanation:

- First, we look at the innermost radical expression, $\sqrt{1-x^2}$. For this term to be defined,

the expression inside must be non-negative: $1-x^2 \geq 0$.

- This simplifies to: $x^2 \leq 1 \implies -1 \leq x \leq 1$. This gives our first constraint interval: $x \in [-1, 1]$.

- Next, we consider the outer radical, $\sqrt{x-\sqrt{1-x^2}}$. For the outer square root to be defined, its entire inner expression must be non-negative: $x-\sqrt{1-x^2} \geq 0 \implies x \geq \sqrt{1-x^2}$.

- Because the principal square root $\sqrt{1-x^2}$ is always non-negative, the inequality $x \geq \sqrt{1-x^2}$ requires x to be non-negative as well: $x \geq 0$.

- Combining the conditions $x \in [-1, 1]$ and $x \geq 0$, we restrict our working interval to $x \in [0, 1]$.

- Since both sides of the inequality $x \geq \sqrt{1-x^2}$ are non-negative on the interval $[0, 1]$, we can safely square both sides without introducing extraneous solutions: $x^2 \geq 1-x^2$.

- Rearranging this inequality gives: $2x^2 \geq 1 \implies x^2 \geq \frac{1}{2}$.

- Taking the square root, and keeping in mind that $x \geq 0$, we obtain: $x \geq \frac{1}{\sqrt{2}}$. This gives our second constraint interval: $x \in \left[\frac{1}{\sqrt{2}}, \infty\right)$.

- Finally, we find the intersection of the two constraint intervals: $[-1, 1] \cap \left[\frac{1}{\sqrt{2}}, \infty\right) = \left[\frac{1}{\sqrt{2}}, 1\right]$.

Step 4 : Final Answer:

The domain of the function is the interval $\left[\frac{1}{\sqrt{2}}, 1\right]$, which corresponds to Option (D).

Quick Tip: When evaluating inequalities that involve principal square roots, such as $x \geq \sqrt{g(x)}$, remember that the square root is always non-negative. This constraint immediately forces $x \geq 0$, allowing you to quickly eliminate any options that contain negative numbers.

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11. Let A $= \{1,2,3,4,5\}$ and R be a relation defined by R $= \{(x,y) : x, y \in A, x+y=5\}$. Then, R is

(A) reflexive and symmetric but not transitive

(B) an equivalence relation

(C) neither reflexive nor transitive

(D) neither reflexive nor symmetric but transitive

Correct Answer: (C) neither reflexive nor transitive

Solution:

Topic of the Question:

The topic of this question is relations and functions, specifically focusing on classifying a relation on a finite set based on reflexivity, symmetry, and transitivity.

Step 1 : Understanding the Question:

We are given a set $A = \{1,2,3,4,5\}$ and a relation R defined on A by the condition $R = \{(x,y) : x, y \in A, x+y=5\}$. We need to determine whether this relation is reflexive, symmetric, and/or transitive.

Step 2 : Key Formulas and Approach:

- Reflexive: A relation R on set A is reflexive if $(a,a) \in R$ for every element $a \in A$.
- Symmetric: A relation R on set A is symmetric if $(a,b) \in R \implies (b,a) \in R$ for all $a, b \in A$.
- Transitive: A relation R on set A is transitive if $(a,b) \in R$ and $(b,c) \in R \implies (a,c) \in R$ for all $a, b, c \in A$.

Step 3 : Detailed Explanation:

- We start by writing the relation R in roster form by finding all pairs (x, y) of elements in A that satisfy $x + y = 5$:
- If $x = 1$, then $y = 4$ (since $1 + 4 = 5$ and $4 \in A$). This gives the pair $(1, 4)$.
- If $x = 2$, then $y = 3$ (since $2 + 3 = 5$ and $3 \in A$). This gives the pair $(2, 3)$.
- If $x = 3$, then $y = 2$ (since $3 + 2 = 5$ and $2 \in A$). This gives the pair $(3, 2)$.
- If $x = 4$, then $y = 1$ (since $4 + 1 = 5$ and $1 \in A$). This gives the pair $(4, 1)$.
- If $x = 5$, then $y = 0$, but since $0 \notin A$, this does not yield a valid pair.
- Thus, in roster form, the relation is: $R = \{(1, 4), (2, 3), (3, 2), (4, 1)\}$.
- We test for reflexivity: For R to be reflexive, it must contain $(1, 1), (2, 2), (3, 3), (4, 4)$, and $(5, 5)$. Since $(1, 1) \notin R$, the relation is not reflexive.
- We test for symmetry: We check if the reverse of each pair is also in the relation. We have $(1, 4) \in R$ and $(4, 1) \in R$, and we also have $(2, 3) \in R$ and $(3, 2) \in R$. Since $(x, y) \in R \implies (y, x) \in R$ holds true for all pairs, the relation is symmetric.
- We test for transitivity: For R to be transitive, if $(a, b) \in R$ and $(b, c) \in R$, then (a, c) must be in R. Let us choose the pairs $(1, 4) \in R$ and $(4, 1) \in R$. Here, $a = 1, b = 4$, and $c = 1$. For transitivity to hold, $(a, c) = (1, 1)$ must be in R. However, $(1, 1) \notin R$. Therefore, the relation is not transitive.

Step 4 : Final Answer:

The relation is symmetric, but it is neither reflexive nor transitive. This matches the statement in Option (C).

Quick Tip: To quickly test for transitivity in relations with symmetric elements, look for cyclic pairs like (a, b) and (b, a). For the relation to remain transitive, the diagonal element (a, a) must be present in the set. If it is missing, you can immediately conclude that the relation is not transitive.

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12. If A and B are symmetric matrices of same order such that $AB + BA = X$ and $AB - BA = Y$, then $(XY)^T =$

(A) XY

(B) X^TY^T

(C) $-YX$

(D) $-Y^TX^T$

Correct Answer: (C) $-YX$

Solution:

Topic of the Question:

The topic of this question is linear algebra, focusing on the properties of symmetric and skew-symmetric matrices and the rules governing matrix transposes.

Step 1 : Understanding the Question:

We are given two symmetric matrices A and B of the same order. Two new matrices are defined as $X = AB + BA$ and $Y = AB - BA$. We need to find the simplified expression for the transpose of their product, $(XY)^T$.

Step 2 : Key Formulas and Approach:

- Definition of a symmetric matrix: A matrix M is symmetric if $M^T = M$.

- Definition of a skew-symmetric matrix: A matrix M is skew-symmetric if $M^{\mathrm{T}} = -M$.
- Reversal law for the transpose of a product of matrices: $(M_1 M_2)^{\mathrm{T}} = M_2^{\mathrm{T}} M_1^{\mathrm{T}}$.
- Distributive property of transposes over addition and subtraction: $(M_1 \pm M_2)^{\mathrm{T}} = M_1^{\mathrm{T}} \pm M_2^{\mathrm{T}}$.

Step 3 : Detailed Explanation:

- Since A and B are given as symmetric matrices, we have the relations: $A^{\mathrm{T}} = A$ and $B^{\mathrm{T}} = B$.
- First, we analyze matrix $X = AB + BA$ by taking its transpose: $X^{\mathrm{T}} = (AB + BA)^{\mathrm{T}}$.
- Applying the transpose properties: $X^{\mathrm{T}} = (AB)^{\mathrm{T}} + (BA)^{\mathrm{T}} = B^{\mathrm{T}}A^{\mathrm{T}} + A^{\mathrm{T}}B^{\mathrm{T}}$.
- Substituting $A^{\mathrm{T}} = A$ and $B^{\mathrm{T}} = B$, we get: $X^{\mathrm{T}} = BA + AB = AB + BA = X$. Thus, X is a symmetric matrix.
- Next, we analyze matrix $Y = AB - BA$ by taking its transpose: $Y^{\mathrm{T}} = (AB - BA)^{\mathrm{T}}$.
- Applying the transpose properties: $Y^{\mathrm{T}} = (AB)^{\mathrm{T}} - (BA)^{\mathrm{T}} = B^{\mathrm{T}}A^{\mathrm{T}} - A^{\mathrm{T}}B^{\mathrm{T}}$.
- Substituting $A^{\mathrm{T}} = A$ and $B^{\mathrm{T}} = B$, we get: $Y^{\mathrm{T}} = BA - AB = -(AB - BA) = -Y$. Thus, Y is a skew-symmetric matrix.
- Now, we find the transpose of the product XY using the reversal law: $(XY)^{\mathrm{T}} = Y^{\mathrm{T}}X^{\mathrm{T}}$.
- Substituting our results for X^{T} and Y^{T} into this expression: $(XY)^{\mathrm{T}} = (-Y)(X) = -YX$.

Step 4 : Final Answer:

The expression $(XY)^T$ simplifies to $-YX$, which corresponds to Option (C).

Quick Tip: For any two symmetric matrices A and B of the same order, the sum of their products $AB + BA$ is always a symmetric matrix, and their difference $AB - BA$ is always a skew-symmetric matrix. Recognizing these standard relations can save you valuable time during exams.
