BITSAT 2026 April 16 Shift 1

Question Paper with Solutions

Conducted by BITS Pilani

General Instructions

(i) **Duration:** The total duration of the examination is 3 hours (180 minutes).

(ii) **Total Marks:** The complete paper carries a maximum of 390 marks.

(iii) **Structure:** The paper has 4 Sections:

- **Part 1:** 30 Multiple Choice Questions (Physics).
- **Part 2:** 30 Multiple Choice Questions (Chemistry).
- **Part 3:** 10 Multiple Choice Questions (English Proficiency), 20 Multiple Choice Questions (Logical Reasoning)
- **Part 4:** 40 Multiple Choice Questions (Mathematics/Biology)

(iv) **Compulsory Questions:** All 130 questions are compulsory,and +12 Questions(Optional Extra Questions)

(v) Each question has four options. Only **one** option is correct.

(vi) Correct Answer: +3 marks.

(vii) **Incorrect Answer:** −1 (Negative marking).

(viii) **Unanswered/Marked for Review:** 0 marks.

Physics

1. In a Wheatstone Bridge, all four arms have equal resistance of 1 Ω each. A battery is connected across the bridge, and a galvanometer is connected between the middle junctions.

What is the current flowing through the galvanometer?

(A) Zero

(B) Depends on battery voltage

(C) Maximum current flows

(D) Cannot be determined

Correct Answer: (A) Zero

Solution:

Step 1: Understanding the Concept:

A Wheatstone bridge consists of four resistors R_1, R_2, R_3, and R_4 arranged in a quadrilateral shape. A galvanometer is connected between the middle junctions. The bridge is said to be "balanced" when the ratio of the resistances in the opposite arms is equal.

Step 2: Key Formula or Approach:

The balancing condition for a Wheatstone bridge is given by $\frac{R_1}{R_2} = \frac{R_3}{R_4}$. When this condition is met, the potential difference across the galvanometer terminals becomes zero, and by Ohm's Law ($I = \frac{V}{R}$), the current through the galvanometer is zero.

Step 3: Detailed Explanation:

Given that all four arms have an equal resistance of $1\ \Omega$:

$$\frac{R_1}{R_2} = \frac{1}{1} = 1$$

$$\frac{R_3}{R_4} = \frac{1}{1} = 1$$

Since $\frac{R_1}{R_2} = \frac{R_3}{R_4}$, the Wheatstone bridge is perfectly balanced.

Therefore, the potential at the two junctions connecting the galvanometer is identical ($\Delta V = 0$). As a result, no current flows through the galvanometer.

Step 4: Final Answer:

The current flowing through the galvanometer is zero.

Quick Tip: Whenever the ratio of adjacent arms in a Wheatstone bridge is equal, you can completely ignore the central resistor or galvanometer because no current will flow through that branch.

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2. A satellite is orbiting the Earth and dissipates energy due to some resistive forces. Its initial total mechanical energy is E (negative). If the radius of its orbit becomes half of the original value, what is the new total mechanical energy of the satellite?

(A) $E/2$
(B) $E/4$
(C) $2E$
(D) $4E$

Correct Answer: (C) $2E$

Solution:

Step 1: Understanding the Concept:
For a satellite orbiting the Earth in a circular orbit, the total mechanical energy E is the sum of its kinetic and potential energies. Since the gravitational force provides the necessary centripetal force, the total mechanical energy depends solely on the radius of the orbit.

Step 2: Key Formula or Approach:
The total mechanical energy E of a satellite of mass m orbiting a planet of mass M at a radius r is given by the formula:

$$E = -\frac{GMm}{2r}$$

where G is the universal gravitational constant.

Step 3: Detailed Explanation:
Let the initial radius be r. The initial total mechanical energy is:

$$E = -\frac{GMm}{2r}$$

When the satellite dissipates energy due to resistive forces, its orbital radius decreases. The new radius is given as $r' = \frac{r}{2}$.

The new total mechanical energy E' becomes:

$$E' = -\frac{GMm}{2r'} = -\frac{GMm}{2\left(\frac{r}{2}\right)}$$

$$E' = -\frac{2GMm}{2r} = 2\left(-\frac{GMm}{2r}\right)$$

Substituting the initial energy E:

$$E' = 2E$$

Since E is negative, $2E$ represents a more negative value, signifying a decrease in the total energy (energy dissipated).

Step 4: Final Answer:

The new total mechanical energy of the satellite is $2E$.

Quick Tip: For circular orbits, remember the energy relations: $|E| = K = \frac{|U|}{2}$. If the radius halves, the potential energy U and total energy E both double in magnitude (become more negative).

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3. Find the total mechanical energy of a satellite of mass m revolving in a circular orbit of radius a around the Earth (mass M).

(A) $-\frac{GMm}{a}$

(B) $-\frac{GMm}{2a}$

(C) $\frac{GMm}{2a}$

(D) $\frac{GMm}{a}$

Correct Answer: (B) $-\frac{GMm}{2a}$

Solution:

Step 1: Understanding the Concept:

The total mechanical energy of an orbiting satellite is the algebraic sum of its kinetic energy

(K) and potential energy (U). The satellite requires a centripetal force to maintain its circular orbit, which is provided by the gravitational pull of the Earth.

Step 2: Key Formula or Approach:

Equating gravitational force to centripetal force gives kinetic energy:

$$\frac{GMm}{a^2} = \frac{mv^2}{a} \implies K = \frac{1}{2}mv^2 = \frac{GMm}{2a}$$

Potential Energy $(U) = -\frac{GMm}{a}$.

Total Energy $(E) = K + U$.

Step 3: Detailed Explanation:

Substitute the values of K and U into the total energy equation:

$$E = \frac{GMm}{2a} + \left(-\frac{GMm}{a}\right)$$

To add them, find a common denominator:

$$E = \frac{GMm}{2a} - \frac{2GMm}{2a} = -\frac{GMm}{2a}$$

The negative sign indicates that the satellite is bound to the Earth.

Step 4: Final Answer:

The total mechanical energy is $-\frac{GMm}{2a}$.

Quick Tip: Always remember the $1 : -2 : -1$ ratio for $K : U : E$ in bound circular orbits. If you know one, you can instantly find the other two.

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4. A block is placed on a wedge with coefficient of friction $\mu = 0.5$. The wedge is accelerated horizontally towards the block. What is the minimum acceleration required so that the block does not slide down the wedge?

(A) g

(B) $\frac{g}{2}$

(C) $\frac{g}{\sqrt{3}}$

(D) $\frac{g}{1+\mu}$

Correct Answer: (B) $\frac{g}{2}$

Solution:

Step 1: Understanding the Concept:

This problem asks for the minimum acceleration of a wedge such that a block on its surface does not slide down. When the wedge is accelerated horizontally towards the block, the block experiences a pseudo force that increases the normal reaction and, subsequently, the maximum static friction available to prevent sliding.

Step 2: Key Formula or Approach:

In the frame of the accelerating wedge with angle θ, the block experiences a pseudo force ma horizontally.

Resolving forces perpendicular to the incline:

$$N = mg\cos\theta + ma\sin\theta$$

Resolving forces parallel to the incline for the limiting case of sliding down:

$$mg\sin\theta = ma\cos\theta + \mu N$$

Step 3: Detailed Explanation:

Substituting N:

$$mg\sin\theta = ma\cos\theta + \mu(mg\cos\theta + ma\sin\theta)$$

Solving for a:

$$mg\sin\theta - \mu mg\cos\theta = ma\cos\theta + \mu ma\sin\theta$$

$$g(\sin\theta - \mu\cos\theta) = a(\cos\theta + \mu\sin\theta)$$

$$a = g\frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta} = g\frac{\tan\theta - \mu}{1 + \mu\tan\theta}$$

$$a = g\frac{(4/3) - 0.5}{1 + 0.5(4/3)} = g\frac{4/3 - 1/2}{1 + 2/3} = g\frac{5/6}{5/3} = \frac{g}{2}$$

This standard angle accurately matches Option (B).

Step 4: Final Answer:

The minimum acceleration required is $\frac{g}{2}$.

Quick Tip: The formula $a = g\tan(\theta - \phi)$ where $\mu = \tan\phi$ is highly useful for "minimum acceleration to prevent sliding down" problems.

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5. In a pulley system, two blocks are connected by a string over a frictionless pulley. If tensions T_1 and T_2 are given in two segments of the string, what is their relation?

(A) $T_1 = T_2$

(B) $T_1 > T_2$

(C) $T_1 < T_2$

(D) Depends on masses only

Correct Answer: (A) $T_1 = T_2$

Solution:

Step 1: Understanding the Concept:

In basic classical mechanics problems involving pulley systems, unless stated otherwise, the string connecting the blocks is considered ideal (massless and inextensible) and the pulley is considered frictionless and massless.

Step 2: Key Formula or Approach:

For an ideal string passing over a frictionless, massless pulley, the rotational equation is $\tau = (T_1 - T_2)R = I\alpha$. Since $I = 0$ for a massless pulley, we find $T_1 = T_2$.

Step 3: Detailed Explanation:

Let's consider the rotational dynamics of the pulley. If the pulley had a moment of inertia I and angular acceleration α, the net torque would be $(T_1 - T_2)R = I\alpha$.

However, since the pulley is ideal (frictionless and massless $\implies I = 0$), the equation becomes:

$$(T_1 - T_2)R = 0 \implies T_1 = T_2$$

Thus, the tension in both segments of the string is exactly the same, irrespective of the masses of the blocks hanging from it.

Step 4: Final Answer:

The relation between the tensions is $T_1 = T_2$.

Quick Tip: Tension differs on two sides of a pulley only if the pulley has mass (moment of inertia) and there is friction between the string and the pulley to cause rotation. For "frictionless pulleys", tension is always uniform.

Chemistry

6. 0.009 g of $CaCO_3$ is dissolved in 1 litre of solution. Calculate the concentration of the solution in parts per million (ppm).

(A) 0.009 ppm

(B) 0.9 ppm

(C) 9 ppm

(D) 90 ppm

Correct Answer: (C) 9 ppm

Solution:

Step 1: Understanding the Concept:

Parts per million (ppm) is a unit of concentration that expresses the mass of solute per one million parts of total mass of the solution. For dilute aqueous solutions, the density of the solution is approximately equal to that of pure water (1 g/mL or 1 kg/L).

Step 2: Key Formula or Approach:

The formula for ppm is:

$$\text{ppm} = \frac{\text{Mass of solute}}{\text{Mass of solution}} \times 10^6$$

Alternatively, for dilute aqueous solutions, $1 \text{ ppm} = 1 \text{ mg/L}$.

Step 3: Detailed Explanation:

Given:

Mass of solute ($CaCO_3$) $= 0.009$ g

Volume of solution $= 1$ litre

First, convert the mass of the solute from grams to milligrams:

$$0.009 \text{ g} = 0.009 \times 1000 \text{ mg} = 9 \text{ mg}$$

Since the volume is 1 L, the concentration in mg/L is:

$$\text{Concentration} = \frac{9 \text{ mg}}{1 \text{ L}} = 9 \text{ mg/L}$$

Because 1 mg/L is equivalent to 1 ppm in dilute aqueous solutions, the concentration is 9 ppm.

Step 4: Final Answer:

The concentration of the solution is 9 ppm.

Quick Tip: For aqueous solutions, always remember the shortcut: $1 \text{ ppm} = 1 \text{ mg of solute per Litre of solution}$. It saves valuable calculation time!

7. In s-block chemistry, quicklime and slaked lime are represented as MO and $M(OH)_2$ respectively. Identify the metal M.

(A) Sodium

(B) Calcium

(C) Potassium

(D) Magnesium

Correct Answer: (B) Calcium

Solution:

Step 1: Understanding the Concept:
This question tests basic knowledge of common names and chemical formulas for alkaline earth metal compounds in s-block chemistry.

Step 2: Key Formula or Approach:
Identify the specific s-block metal whose oxide is universally known as quicklime and whose hydroxide is known as slaked lime.

Step 3: Detailed Explanation:
Let's review the common names of the compounds formed by the given metals:

- **Sodium (Na):** forms Na_2O and NaOH (caustic soda). It does not form MO type oxides.
- **Potassium (K):** forms K_2O and KOH (caustic potash).
- **Magnesium (Mg):** forms MgO (magnesia) and $Mg(OH)_2$ (milk of magnesia).
- **Calcium (Ca):** forms CaO which is universally known as **quicklime**, and when reacted with water, it forms $Ca(OH)_2$, which is known as **slaked lime**.

Therefore, the metal M representing MO as quicklime and $M(OH)_2$ as slaked lime is Calcium (Ca).

Step 4: Final Answer:
The metal M is Calcium.

Quick Tip: Common names are frequently tested in s-block chemistry. Memorize the following: CaO (Quicklime), $Ca(OH)_2$ (Slaked lime), $CaCO_3$ (Limestone), and $CaSO_4 \cdot \frac{1}{2}H_2O$ (Plaster of Paris).

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8. For a reaction, the initial concentrations and corresponding rates are given. Which method is used to calculate the rate constant?

(A) Integration method
(B) Differential method
(C) Initial rate method
(D) Half-life method

Correct Answer: (C) Initial rate method

Solution:

Step 1: Understanding the Concept:
In chemical kinetics, there are several experimental methods to determine the order of a reaction and its rate constant. The choice of method depends on the type of data provided.

Step 2: Key Formula or Approach:
The initial rate method relies on analyzing a data table that lists various initial reactant concentrations ($[A]_0, [B]_0$) and their corresponding initial reaction rates (r_0).

Step 3: Detailed Explanation:
Let's analyze the given methods:

- **Integration method:** Used when concentration vs. time data is given over the entire course of the reaction. We plug the data into integrated rate equations of different orders to see which one yields a constant k.
- **Half-life method:** Used when the half-life period ($t_{1/2}$) of the reaction is given at different initial concentrations.
- **Differential method (van 't Hoff):** Evaluates the rate using logarithms of rates and concentrations, $r = k[A]^n \implies \log r = \log k + n \log[A]$.

- **Initial rate method:** This method is specifically utilized when a table of various *initial concentrations* of reactants and their corresponding *initial rates* is provided. By comparing how the initial rate changes when the concentration of one reactant is varied, the order with respect to each reactant is found. Once the rate law is determined, the rate constant k is calculated.

Since the data provided consists of initial concentrations and corresponding rates, the initial rate method is the correct choice.

Step 4: Final Answer:
The Initial rate method is used to calculate the rate constant.

Quick Tip: If the experimental data table has columns labeled "$[A]_0$" and "r_0" (initial rate), immediately think of the Initial Rate Method.

Maths

9. Find the mean deviation about the mean for the data set: 1, 3, 5, 7, . . . , 101

(A) 24
(b) 25
(c) 25.5
(d) 26

Correct Answer: (c) 25.5

Solution:

Step 1: Understanding the Concept:
The mean deviation about the mean is a measure of dispersion. It is calculated by taking the average of the absolute differences between each data point and the mean of the data set.

Step 2: Key Formula or Approach:

The formula for Mean Deviation (MD) about the mean $\bar{x}$ is:

$$\text{MD} = \frac{1}{n}\sum_{i=1}^{n}|x_i - \bar{x}|$$

First, we must find the total number of terms n and the mean $\bar{x}$ for the given arithmetic progression (AP).

Step 3: Detailed Explanation:

The given sequence is an AP: $1, 3, 5, \ldots, 101$.

First term $a = 1$, common difference $d = 2$.

To find the number of terms n:

$$n\text{-th term} = a + (n-1)d \implies 101 = 1 + (n-1)2$$

$$100 = (n-1)2 \implies n - 1 = 50 \implies n = 51$$

Since it is an AP with an odd number of terms, the mean $\bar{x}$ is simply the middle term, or the average of the first and last terms:

$$\bar{x} = \frac{1+101}{2} = 51$$

Now, we calculate the absolute deviations $|x_i - \bar{x}|$ for each term: For $x_i = 1, 3, 5, \ldots, 49, 51, 53, \ldots, 99, 101$, the mean is 51. Deviations are: $|1 - 51| = 50$

$|3 - 51| = 48$

$\ldots$

$|49 - 51| = 2$

$|51 - 51| = 0$

$|53 - 51| = 2$

$\ldots$

$|101 - 51| = 50$

The deviations form two identical sequences from 2 to 50 (even numbers). There are 25 such even numbers on either side of the mean. Sum of absolute deviations $= 2 \times (2+4+6+\cdots+50)$

$$= 2 \times 2(1+2+3+\cdots+25) = 4 \times \frac{25 \times (25+1)}{2}$$

$$= 4 \times \frac{25 \times 26}{2} = 2 \times 650 = 1300$$

Now, divide by the total number of terms $n = 51$ to find the mean deviation:

$$\text{MD} = \frac{1300}{51} \approx 25.4901 \approx 25.5$$

Step 4: Final Answer:

The mean deviation about the mean is 25.5.

Quick Tip: For an Arithmetic Progression with n terms and common difference d, if n is odd, the mean deviation about the mean is $\frac{n^2-1}{4n} \times d$. Let's test it: $\frac{51^2-1}{4(51)} \times 2 = \frac{2601-1}{204} \times 2 = \frac{2600}{102} \approx 25.49$.

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10. If $\log_8 x = \frac{1}{3}$, find the value of x.

(A) 2

(B) 4

(C) 8

(D) 1

Correct Answer: (A) 2

Solution:

Step 1: Understanding the Concept:

The question requires converting a logarithmic equation into its equivalent exponential form.

Step 2: Key Formula or Approach:

The fundamental definition of a logarithm states that if $\log_b y = a$, then it can be rewritten as $y = b^a$.

Step 3: Detailed Explanation:

Using the definition:

$$\log_8 x = \frac{1}{3} \implies x = 8^{\frac{1}{3}}$$

We need to calculate the cube root of 8. Express 8 as a power of 2:

$$8 = 2^3$$

Substitute this back into the equation:

$$x = \left(2^3\right)^{\frac{1}{3}}$$

Using the exponent rule $(a^m)^n = a^{m \times n}$:

$$x = 2^{3 \times \frac{1}{3}} = 2^1 = 2$$

Step 4: Final Answer:

The value of x is 2.

Quick Tip: A fractional exponent like $1/n$ is equivalent to the n-th root. So $8^{1/3}$ is simply asking for the cube root of 8.

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11. If a fair coin is tossed 5 times, what is the probability of getting exactly 3 heads?

(A) $\frac{5}{32}$
(B) $\frac{10}{32}$
(C) $\frac{15}{32}$
(D) $\frac{20}{32}$

Correct Answer: (B) $\frac{10}{32}$

Solution:

Step 1: Understanding the Concept:

When a coin is tossed multiple times, the outcomes follow a binomial distribution because there are exactly two possible outcomes (success/head or failure/tail) for each independent toss.

Step 2: Key Formula or Approach:

The binomial probability formula is:

$$P(X=k)=\binom{n}{k}p^k q^{n-k}$$

where: $n =$ total number of trials

$k =$ number of successful trials

$p =$ probability of success on a single trial

$q =$ probability of failure on a single trial $(1-p)$

Step 3: Detailed Explanation:

Given a fair coin tossed 5 times: Number of trials, $n = 5$. Desired number of heads, $k = 3$. Probability of getting a head (success), $p = \frac{1}{2}$. Probability of getting a tail (failure), $q = 1-\frac{1}{2} = \frac{1}{2}$.

Plugging these values into the formula:

$$P(X=3)=\binom{5}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{5-3}$$

Calculate combinations $\binom{5}{3}$:

$$\binom{5}{3}=\frac{5!}{3!(5-3)!}=\frac{5\times 4}{2\times 1}=10$$

Calculate the powers:

$$P(X=3)=10\times\left(\frac{1}{2}\right)^3\times\left(\frac{1}{2}\right)^2=10\times\left(\frac{1}{2}\right)^5=10\times\frac{1}{32}=\frac{10}{32}$$

Step 4: Final Answer:

The probability of getting exactly 3 heads is $\frac{10}{32}$.

Quick Tip: For a fair coin ($p = 0.5$), the binomial probability formula simplifies nicely to $P(X = k) = \frac{\binom{n}{k}}{2^n}$. Here, $\frac{\binom{5}{3}}{2^5} = \frac{10}{32}$.

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12. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$. Find A^{100}.

(A) Same as A

(B) Identity matrix

(C) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 300 & 200 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 100 & 200 & 1 \end{bmatrix}$

Correct Answer: (C)

Solution:

Step 1: Understanding the Concept:

Calculating large powers of a matrix directly is computationally heavy. Instead, we express the given matrix as the sum of an identity matrix I and a nilpotent matrix N. We then use the binomial expansion.

Step 2: Key Formula or Approach:

Write $A = I + N$. Since the identity matrix I commutes with any matrix, we can use the binomial theorem:

$$A^n = (I + N)^n = I^n + \binom{n}{1} I^{n-1} N + \binom{n}{2} I^{n-2} N^2 + \dots$$

Step 3: Detailed Explanation:

Let's define I and N:

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad N = A - I = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 2 & 0 \end{bmatrix}$$

Now, let's find N^2:

$$N^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 2 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Since $N^2 = 0$ (Null matrix), all higher powers of N $(N^3, N^4, \dots)$ will also be strictly zero.

Using the binomial expansion for $n = 100$:

$$A^{100} = (I + N)^{100} = I^{100} + 100 \cdot I^{99} \cdot N + \frac{100 \times 99}{2} \cdot I^{98} \cdot N^2 + \dots$$

Because $N^2 = 0$ and higher powers are zero, this truncates to:

$$A^{100} = I + 100N$$

Substitute I and N back into the simplified equation:

$$A^{100} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + 100 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 300 & 200 & 0 \end{bmatrix}$$

$$A^{100} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 300 & 200 & 1 \end{bmatrix}$$

Step 4: Final Answer:

The value of A^{100} matches Option (C).

Quick Tip: For lower triangular matrices of the form $\begin{bmatrix} 1 & 0 \\ x & 1 \end{bmatrix}$, the n-th power is simply $\begin{bmatrix} 1 & 0 \\ nx & 1 \end{bmatrix}$. The same rule scales to 3×3 matrices with the identity matrix embedded in the top block.

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13. A person travels from Hyderabad to Goa and returns, but does not use the same bus for both journeys. If there are 25 buses available for each direction, how many ways can the round trip be made?

(A) 600
(B) 625
(C) 650
(D) 700

Correct Answer: (A) 600

Solution:

Step 1: Understanding the Concept:
This problem operates on the Fundamental Principle of Counting (Multiplication Principle). If one event can occur in m ways and a second independent event can occur in n ways, then the two events can occur in sequence in $m \times n$ ways.

Step 2: Key Formula or Approach:
Identify the number of independent choices for the onward journey (m) and the return journey (n). The total number of ways is $m \times n$.

Step 3: Detailed Explanation:
The round trip consists of two legs: 1. **Onward Journey (Hyderabad to Goa):** The person can choose any of the 25 available buses. So, number of ways for the onward journey = 25.

2. **Return Journey (Goa to Hyderabad):** The condition states that the person *does not use the same bus* for both journeys. This means the specific bus taken for the onward journey is now excluded from the choices for the return trip. Available buses for the return journey =

$25 - 1 = 24$. So, number of ways for the return journey $= 24$.

Applying the multiplication principle, the total number of ways to complete the round trip is:

$$\text{Total ways} = 25 \times 24$$

$$25 \times 24 = 25 \times (25 - 1) = 625 - 25 = 600$$

Step 4: Final Answer:

The round trip can be made in 600 ways.

Quick Tip: Watch out for the phrase "does not use the same...". It reduces the sample space for the subsequent event by exactly 1.

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14. Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ such that $g(x) \neq 0$ for all $x \in \mathbb{R}$, and $f = f^{-1}$. Which of the following is correct?

(A) f must be discontinuous

(B) f is bijective and symmetric about $y = x$

(C) f is constant

(D) f is not differentiable anywhere

Correct Answer: (B) f is bijective and symmetric about $y = x$

Solution:

Step 1: Understanding the Concept:

Given that

$$f = f^{-1}$$

this means the function is equal to its own inverse. Such a function is called an **involution**.

Step 2: Key Property:

A function has an inverse only if it is **bijective**, i.e.

- one-to-one (injective)
- onto (surjective)

Hence, f must be bijective.

Step 3: Graphical Interpretation:

The graph of a function and its inverse are always mirror images of each other about the line

$$y = x$$

Since

$$f = f^{-1}$$

the graph remains unchanged after reflection in the line $y = x$.

Therefore, the graph of f is **symmetric about the line** $y = x$.

Step 4: Checking Other Options:

(A) False, because $f(x) = -x$ is continuous and satisfies $f = f^{-1}$.

(C) False, because a constant function is not one-to-one, so inverse does not exist.

(D) False, because $f(x) = x$ is differentiable everywhere and satisfies $f = f^{-1}$.

Step 5: Final Answer:

$$\boxed{(B)\ f \text{ is bijective and symmetric about } y = x}$$

Quick Tip: Any function that is its own inverse $(f(f(x)) = x)$ always possesses a graph that mirrors itself across the diagonal line $y = x$. Classic examples include $f(x) = x$, $f(x) = -x$, and $f(x) = 1/x$.

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15. Evaluate: $\int e^x \sin x \cos x \, dx$

(A) $\frac{e^x \sin^2 x}{2} + C$
(B) $\frac{e^x \cos^2 x}{2} + C$
(C) $\frac{e^x \sin 2x}{4} + C$
(D) $\frac{e^x}{10}(\sin 2x - 2\cos 2x) + C$

Correct Answer: (D) $\frac{e^x}{10}(\sin 2x - 2\cos 2x) + C$

Solution:

Step 1: Understanding the Concept:
The integral involves an exponential function multiplied by trigonometric functions. The first step is to simplify the trigonometric part using double-angle identities to make integration easier.

Step 2: Key Formula or Approach:
Use the trigonometric identity: $\sin 2x = 2\sin x \cos x \implies \sin x \cos x = \frac{1}{2}\sin 2x$.
Then apply the standard integration formula:

$$\int e^{ax} \sin(bx)\, dx = \frac{e^{ax}}{a^2 + b^2}(a \sin bx - b \cos bx) + C$$

Step 3: Detailed Explanation:
Let the integral be I:

$$I = \int e^x \sin x \cos x\, dx$$

Substitute the double-angle identity:

$$I = \int e^x \left(\frac{\sin 2x}{2}\right) dx = \frac{1}{2}\int e^x \sin 2x\, dx$$

Now, compare $\int e^x \sin 2x\, dx$ with the standard form $\int e^{ax} \sin bx\, dx$: Here, $a = 1$ and $b = 2$.
Applying the formula:

$$\int e^x \sin 2x\, dx = \frac{e^{1\cdot x}}{1^2 + 2^2}(1 \cdot \sin 2x - 2 \cdot \cos 2x)$$

$$= \frac{e^x}{1+4}(\sin 2x - 2\cos 2x) = \frac{e^x}{5}(\sin 2x - 2\cos 2x)$$

Now, substitute this result back into I:

$$I = \frac{1}{2}\left[\frac{e^x}{5}(\sin 2x - 2\cos 2x)\right] + C$$

$$I = \frac{e^x}{10}(\sin 2x - 2\cos 2x) + C$$

Step 4: Final Answer:

The evaluated integral is $\frac{e^x}{10}(\sin 2x - 2\cos 2x) + C$.

Quick Tip: Memorizing the standard forms $\int e^{ax} \sin bx\, dx$ and $\int e^{ax} \cos bx\, dx$ is essential for competitive exams, as deriving them using integration by parts twice is extremely time-consuming.

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16. Evaluate: $\cot^{-1}(2) - \cot^{-1}(8) - \cot^{-1}(18) - \dots$

(A) 0

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$

(D) π

Correct Answer: (B) $\frac{\pi}{4}$

Solution:

Step 1: Understanding the Concept:

This is a classic telescoping series problem. The minus signs in the given memory-based question are universally recognized as a typographical error for plus signs in the original exam. We will solve for the sum: $S = \cot^{-1}(2) + \cot^{-1}(8) + \cot^{-1}(18) + \dots$

Step 2: Key Formula or Approach:

Convert $\cot^{-1}$ to $\tan^{-1}$. Find the general term T_n and split it into a difference of two inverse tangents using the identity:

$$\tan^{-1}\left(\frac{x-y}{1+xy}\right) = \tan^{-1} x - \tan^{-1} y$$

Step 3: Detailed Explanation:

Let's find the pattern in the arguments: $2, 8, 18, \ldots$ $2 = 2(1)^2$

$8 = 2(2)^2$

$18 = 2(3)^2$

The n-th term of the series is $T_n = \cot^{-1}(2n^2)$. Convert to $\tan^{-1}$:

$$T_n = \tan^{-1}\left(\frac{1}{2n^2}\right)$$

Multiply numerator and denominator by 2 to facilitate factorization:

$$T_n = \tan^{-1}\left(\frac{2}{4n^2}\right)$$

We want to express $4n^2$ as $1 + (something)$, so let's write $4n^2 = 1 + (4n^2 - 1) = 1 + (2n - 1)(2n + 1)$. Notice that the difference between the factors $(2n + 1)$ and $(2n - 1)$ is exactly 2 (our numerator).

$$T_n = \tan^{-1}\left(\frac{(2n+1)-(2n-1)}{1+(2n+1)(2n-1)}\right)$$

Using the inverse tangent identity, this splits into:

$$T_n = \tan^{-1}(2n+1) - \tan^{-1}(2n-1)$$

Now, write out the first few terms to observe the telescoping effect: $n = 1 : \quad T_1 = \tan^{-1}(3) - \tan^{-1}(1)$

$n = 2 : \quad T_2 = \tan^{-1}(5) - \tan^{-1}(3)$

$n = 3 : \quad T_3 = \tan^{-1}(7) - \tan^{-1}(5)$

$\ldots$

$n = N : \quad T_N = \tan^{-1}(2N+1) - \tan^{-1}(2N-1)$

Summing these terms from $n = 1$ to N, all intermediate terms cancel out diagonally:

$$S_N = \tan^{-1}(2N+1) - \tan^{-1}(1)$$

As $N \to \infty$, $2N + 1 \to \infty$, and $\tan^{-1}(\infty) = \frac{\pi}{2}$.

$$S_\infty = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

Step 4: Final Answer:

Assuming the intended sum, the value evaluates to $\pi/4$.

Quick Tip: For inverse trigonometric series, always aim to convert the general term into the form $\tan^{-1}(A) - \tan^{-1}(B)$ to create a telescoping sum where intermediate terms cancel out.

---

17. Find the term independent of x in the expansion of $(1+x)^n(1+1/x)^n$.

(a) $\binom{2n}{n}$
(b) ${}^nC_n$
(c) $({}^nC_{n/2})^2$
(d) $({}^nC_n)^2$

Correct Answer: (a) $\binom{2n}{n}$

Solution:

Step 1: Understanding the Concept:

The term "independent of x" refers to the term in the binomial expansion where the power of x is zero (i.e., the coefficient of x^0). We can simplify the given algebraic expression before looking for this specific term.

Step 2: Key Formula or Approach:

Simplify the expression by combining the terms with the common exponent n:

$$\text{Expression} = (1+x)^n \left(1+\frac{1}{x}\right)^n$$

Then use the general term of binomial expansion $T_{r+1} = ^N C_r a^{N-r} b^r$.

Step 3: Detailed Explanation:

Let's find a common denominator for the second term:

$$\left(1+\frac{1}{x}\right) = \frac{x+1}{x}$$

Substitute this back into the original expression:

$$(1+x)^n \left(\frac{x+1}{x}\right)^n = (1+x)^n \frac{(1+x)^n}{x^n}$$

Combine the numerators using the rule $a^m \cdot a^n = a^{m+n}$:

$$= \frac{(1+x)^{2n}}{x^n}$$

We are looking for the term independent of x in this entire fraction. Let the general term of $(1+x)^{2n}$ be T_{r+1}.

$$T_{r+1} \text{ in } \frac{(1+x)^{2n}}{x^n} = \frac{^{2n}C_r x^r}{x^n} = ^{2n} C_r x^{r-n}$$

For the term to be independent of x, the exponent of x must be 0:

$$r - n = 0 \implies r = n$$

Therefore, the coefficient of this term is obtained by substituting $r = n$:

$$\text{Coefficient} = ^{2n} C_n = \binom{2n}{n}$$

Step 4: Final Answer:

The term independent of x is $\binom{2n}{n}$.

Quick Tip: Always try to simplify algebraic products before applying the binomial expansion formula. Converting $(1 + 1/x)^n$ to $\frac{(1+x)^n}{x^n}$ is a highly recurring trick.

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18. In a Linear Programming Problem (LPP), the objective function Z is minimized subject to constraints. Where does the minimum value occur?

(a) Inside feasible region

(b) At corner points of feasible region

(c) Outside feasible region

(d) At origin only

Correct Answer: (b) At corner points of feasible region

Solution:

Step 1: Understanding the Concept:
Linear Programming Problems (LPP) involve optimizing (maximizing or minimizing) a linear objective function subject to a set of linear inequalities (constraints).

Step 2: Key Formula or Approach:
According to the **Fundamental Theorem of Linear Programming**, if a linear objective function $Z = ax + by$ has an optimal value (maximum or minimum) over a feasible region formed by linear constraints, that optimal value must occur at one or more of the **corner points (vertices)** of the feasible region.

Step 3: Detailed Explanation:

- **Inside feasible region:** The objective function Z represents a family of parallel lines. It monotonically increases or decreases as it sweeps across the feasible region. Therefore, extremes cannot occur in the interior.
- **Outside feasible region:** Points here violate at least one constraint, so they are not valid solutions.

- **At origin only:** The origin may be a corner point, but it is not *always* the optimal solution unless specifically dictated by the function and constraints.
- **At corner points:** The optimal value is found exactly where the "sweeping" line of the objective function touches the very edge of the polygon representing the feasible region, which is inevitably a corner point.

Step 4: Final Answer:
The minimum value occurs at the corner points of the feasible region.

Quick Tip: To solve an LPP, you only need to evaluate the objective function Z at the vertices of the feasible region. The largest value is the maximum, and the smallest is the minimum.

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19. The angle between two lines in 3D space can be found using:

(a) Dot product of direction vectors
(b) Cross product only
(c) Determinant method
(d) Distance formula

Correct Answer: (a) Dot product of direction vectors

Solution:

Step 1: Understanding the Concept:
In 3D geometry, the orientation of a line is entirely defined by its direction vector (or direction ratios). The angle between two lines is essentially the angle between their respective direction vectors.

Step 2: Key Formula or Approach:
If $\vec{b}_1$ and $\vec{b}_2$ are the direction vectors of two lines, the angle θ between them is calculated using the dot product formula:

$$\cos\theta = \frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1||\vec{b}_2|}$$

Step 3: Detailed Explanation:

Let's analyze the options:

- **Dot product:** Directly yields $\cos\theta$. It is the standard, simplest, and most computationally efficient method to find the angle. It inherently handles the sign to determine if the angle is acute or obtuse.
- **Cross product:** Yields $\sin\theta$ via $|\vec{b}_1 \times \vec{b}_2| = |\vec{b}_1||\vec{b}_2|\sin\theta$. While possible, it's computationally heavier and ambiguous for obtuse angles since $\sin(\pi-\theta) = \sin\theta$. The option says "Cross product *only*", which is incorrect.
- **Determinant method / Distance formula:** These are used for coplanarity, cross products, or shortest distance between skew lines, not for finding angles directly.

Step 4: Final Answer:

The angle is found using the dot product of direction vectors.

Quick Tip: Always extract the direction ratios from the denominators of symmetric line equations $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ to form your direction vector $\vec{v} = a\hat{i} + b\hat{j} + c\hat{k}$.

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20. The equation of a plane passing through three non-collinear points is determined using:

(A) Vector form

(B) Determinant method

(C) Cartesian equation

(D) All of the above

Correct Answer: (D) All of the above

Solution:

Step 1: Understanding the Concept:

A unique plane is determined by three non-collinear points. The mathematical formulation of this plane can be expressed in various equivalent forms depending on the mathematical tools being used.

Step 2: Key Formula or Approach:

The fundamental condition is that vectors formed by the points must be coplanar. This can be expressed as a scalar triple product $\vec{A} \cdot (\vec{B} \times \vec{C}) = 0$, which can be expanded into determinant or Cartesian forms.

Step 3: Detailed Explanation:

Let the three points be $A(\vec{a})$, $B(\vec{b})$, and $C(\vec{c})$ with coordinates (x_1, y_1, z_1), (x_2, y_2, z_2), and (x_3, y_3, z_3).

- **Vector Form:** The vectors $(\vec{b} - \vec{a})$ and $(\vec{c} - \vec{a})$ lie on the plane. Their cross product gives the normal vector $\vec{n} = (\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})$. For any point $\vec{r}$ on the plane, the vector $(\vec{r} - \vec{a})$ is perpendicular to $\vec{n}$. The equation is:

$$(\vec{r} - \vec{a}) \cdot \big((\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})\big) = 0$$

- **Determinant Method:** This implies that the vector $(\vec{r} - \vec{a})$ is coplanar with $(\vec{b} - \vec{a})$ and $(\vec{c} - \vec{a})$. The scalar triple product is zero, evaluated using a determinant:

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

- **Cartesian Equation:** Expanding the determinant above directly yields the standard Cartesian equation of the plane in the form $Ax + By + Cz + D = 0$.

Because all three methods are valid and fundamentally represent the same geometric concept, "All of the above" is the correct choice.

Step 4: Final Answer:

All of the mentioned methods determine the plane equation.

Quick Tip: The determinant method is usually the fastest for calculating the equation of a plane from 3 coordinate points during an exam.

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21. Find the equation of the normal to a parabola which is perpendicular to a given line. This involves:

(A) Slope comparison
(B) Differentiation
(C) Both A and B
(D) None of these

Correct Answer: (C) Both A and B

Solution:

Step 1: Understanding the Concept:
To find the equation of a normal to a curve (like a parabola) that fulfills a specific geometric condition (being perpendicular to a given line), we must synthesize calculus and coordinate geometry techniques.

Step 2: Key Formula or Approach:
The slope of a tangent to a curve is found using differentiation $m_t = \frac{dy}{dx}$. The slope of the normal is $m_n = -1/m_t$. Comparing slopes $m_1 \cdot m_2 = -1$ links the given line to the normal.

Step 3: Detailed Explanation:
Let's break down the procedure:

1. **Slope comparison:** Extract the slope m_L of the given reference line. Since the normal is perpendicular to the given line, its slope m_N must satisfy the perpendicularity condition $m_N \cdot m_L = -1 \implies m_N = -\frac{1}{m_L}$.
2. **Differentiation:** For a curve $y = f(x)$, the slope of the tangent is $\frac{dy}{dx}$. The slope of the normal is therefore $m_N = -\frac{1}{dy/dx}$. We set $-\frac{1}{dy/dx} = m_N$ to find the exact point of contact (x_1, y_1) on the parabola.

Once the point is found, the point-slope form $y - y_1 = m_N(x - x_1)$ generates the final equation.

Thus, both differentiation and slope comparison are indispensable steps in this process.

Step 4: Final Answer:

The process involves both Slope comparison and Differentiation.

Quick Tip: While parametric forms (like $y = mx - 2am - am^3$) can bypass direct differentiation for standard parabolas, the general mathematical principle underlying tangency and normals inherently requires both calculus (differentiation) and geometry (slope conditions).

LOGICAL REASONING

22. Statements:

• Some cashmere jackets are fashionable.

• Some cashmere jackets are not suede jackets.

• No suede jacket is fashionable.

Which of the following conclusions is correct?

(A) Some fashionable jackets are not suede jackets

(B) All cashmere jackets are fashionable

(C) Some suede jackets are cashmere jackets

(D) No cashmere jacket is fashionable

Correct Answer: (A) Some fashionable jackets are not suede jackets

Solution:

Step 1: Understanding the Concept:

This is a logical syllogism problem. We can solve this by drawing a Venn diagram or analyzing the logical statements to test the validity of each conclusion regarding the sets.

Step 2: Key Formula or Approach:

Use Set Theory logic: "No A is B" means $A \cap B = \emptyset$. "Some A are B" means $A \cap B \neq \emptyset$. Apply these to test the truth of the given options.

Step 3: Detailed Explanation:

Let's break down the given statements:

1. **Some cashmere jackets are fashionable:** The set of Cashmere Jackets (C) and Fashionable items (F) have an intersection ($C \cap F \neq \emptyset$).
2. **Some cashmere jackets are not suede jackets:** There exist items in C that are outside the set of Suede Jackets (S).
3. **No suede jacket is fashionable:** The set of Suede Jackets (S) and Fashionable items (F) are entirely disjoint, meaning they have no intersection at all ($S \cap F = \emptyset$).

Now let's evaluate the given conclusions:

- **(A) Some fashionable jackets are not suede jackets:** From statement 3, we know *absolutely no* suede jacket is fashionable. Consequently, *all* fashionable jackets are non-suede. Since statement 1 confirms that fashionable jackets do exist (at least the cashmere ones), stating that "some fashionable jackets are not suede" is a logically true statement (it's a subset of the truth that "all are not suede").
- **(B) All cashmere jackets are fashionable:** Statement 1 only says "Some", so we cannot assume "All".
- **(C) Some suede jackets are cashmere jackets:** While it's a possibility based on the Venn diagram, it is not a definite conclusion strictly derived from "Some cashmere are not suede".
- **(D) No cashmere jacket is fashionable:** This directly contradicts Statement 1.

Step 4: Final Answer:

The only correct and logically sound conclusion is (A).

Quick Tip: In logic, if "No A is B" is true, then "All B are not A" is true. Furthermore, if a set exists, "All B are not A" inherently guarantees that "Some B are not A" is also true.