BITSAT 2026 April 15 Shift 2

Question Paper with Solutions

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General Instructions

(i) **Duration:** The total duration of the examination is 3 hours (180 minutes).

(ii) **Total Marks:** The complete paper carries a maximum of 390 marks.

(iii) **Structure:** The paper has 4 Sections:

- **Part 1:** 30 Multiple Choice Questions (Physics).
- **Part 2:** 30 Multiple Choice Questions (Chemistry).
- **Part 3:** 10 Multiple Choice Questions (English Proficiency),
  20 Multiple Choice Questions (Logical Reasoning)
- **Part 4:** 40 Multiple Choice Questions (Mathematics/Biology)

(iv) **Compulsory Questions:** All 130 questions are compulsory,and +12 Questions(Optional Extra Questions)

(v) Each question has four options. Only **one** option is correct.

(vi) Correct Answer: +3 marks.

(vii) **Incorrect Answer:** −1 (Negative marking).

(viii) **Unanswered/Marked for Review:** 0 marks.

Physics

1. An ideal spring with spring-constant k is hung from the ceiling and a mass M is attached to its lower end. The mass is released with the spring initially unstretched. Then the maximum

extension in the spring is

(A) $4Mg/k$

(B) $2Mg/k$

(C) Mg/k

(D) $Mg/2k$

Correct Answer: (B) $2Mg/k$

Solution:

Step 1: Understanding the Concept:

When the mass is released, it falls under gravity, and the spring stretches. The mass will momentarily come to rest at the maximum extension $x_{\max}$. At this point, the loss in gravitational potential energy is completely converted into the elastic potential energy of the spring.

Step 2: Key Formula or Approach:

Using the Work-Energy Theorem or Conservation of Mechanical Energy:

$$\Delta K + \Delta U_g + \Delta U_s = 0$$

Since the initial and final velocities are zero, $\Delta K = 0$.

Step 3: Detailed Explanation:

Let the maximum extension be $x_{\max}$. The loss in gravitational potential energy of the mass is:

$$\Delta U_g = -Mgx_{\max}$$

The gain in elastic potential energy of the spring is:

$$\Delta U_s = \frac{1}{2}kx_{\max}^2$$

Equating the magnitudes (or setting the sum to zero):

$$Mgx_{\max} = \frac{1}{2}kx_{\max}^2$$

Since $x_{\max} \neq 0$:

$$Mg = \frac{1}{2}kx_{\max}$$

$$x_{\max} = \frac{2Mg}{k}$$

Step 4: Final Answer:

The maximum extension in the spring is $2Mg/k$.

Quick Tip: Do not confuse the maximum extension with the equilibrium extension! At equilibrium, the forces balance ($Mg = kx \implies x = Mg/k$), but at maximum extension, the velocity is zero, and the extension is twice the equilibrium extension.

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2. In a mixture of gases, the average number of degrees of freedom per molecule is 6. The rms speed of the molecule of the gas is c, then the velocity of sound in the gas is

(A) $\frac{c}{\sqrt{3}}$
(B) $\frac{c}{\sqrt{2}}$
(C) $\frac{2c}{3}$
(D) $\frac{3c}{3}$

Correct Answer: (C) $\frac{2c}{3}$

Solution:

Step 1: Understanding the Concept:

The root mean square (rms) speed of gas molecules and the velocity of sound in the gas are both related to the temperature and the molar mass of the gas. The velocity of sound also depends on the adiabatic index (γ).

Step 2: Key Formula or Approach:

The formula for rms speed is:

$$v_{\text{rms}} = c = \sqrt{\frac{3RT}{M}}$$

The formula for the velocity of sound in a gas is:

$$v_s = \sqrt{\frac{\gamma RT}{M}}$$

The adiabatic index γ is related to the degree of freedom (f) by:

$$\gamma = 1 + \frac{2}{f}$$

Step 3: Detailed Explanation:

Given the average degrees of freedom $f = 6$:

$$\gamma = 1 + \frac{2}{6} = 1 + \frac{1}{3} = \frac{4}{3}$$

From the rms speed equation, we can express $\sqrt{\frac{RT}{M}}$ in terms of c:

$$\sqrt{\frac{RT}{M}} = \frac{c}{\sqrt{3}}$$

Substitute this into the velocity of sound equation:

$$v_s = \sqrt{\gamma} \cdot \sqrt{\frac{RT}{M}} = \sqrt{\frac{4}{3}} \cdot \left(\frac{c}{\sqrt{3}}\right)$$

$$v_s = \frac{2}{\sqrt{3}} \cdot \frac{c}{\sqrt{3}} = \frac{2c}{3}$$

Step 4: Final Answer:

The velocity of sound in the gas is $\frac{2c}{3}$.

Quick Tip: A useful shortcut relation between the speed of sound and rms speed is $v_s = v_{\text{rms}}\sqrt{\frac{\gamma}{3}}$. If you memorize this, you can directly plug in $\gamma = \frac{4}{3}$ to get the answer instantly!

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3. Five identical springs are used in the three configurations as shown in figure. The time periods of vertical oscillations in configurations (a), (b) and (c) are in the ratio.

(A) $1 : \sqrt{2} : \frac{1}{\sqrt{2}}$
(B) $2 : \sqrt{2} : \frac{1}{\sqrt{2}}$
(C) $\frac{1}{\sqrt{2}} : 2 : 1$
(D) $2 : \frac{1}{\sqrt{2}} : 1$

Correct Answer: (A) $1 : \sqrt{2} : \frac{1}{\sqrt{2}}$

Solution:

Step 1: Understanding the Concept:

The time period of a mass-spring system is determined by the equivalent spring constant k_{eq} of the given configuration. Series and parallel combinations of springs alter the equivalent stiffness.

Step 2: Key Formula or Approach:

The time period of oscillation is given by:

$$T = 2\pi\sqrt{\frac{m}{k_{\text{eq}}}}$$

For springs in series: $\frac{1}{k_{\text{eq}}} = \frac{1}{k_1} + \frac{1}{k_2}$

For springs in parallel: $k_{\text{eq}} = k_1 + k_2$

Step 3: Detailed Explanation:

Let the spring constant of each identical spring be k.

Configuration (a): A single spring is attached to the mass.

$$k_a = k$$

$$T_a = 2\pi\sqrt{\frac{m}{k}}$$

Configuration (b): Two springs are connected in series.

$$\frac{1}{k_b} = \frac{1}{k} + \frac{1}{k} = \frac{2}{k} \implies k_b = \frac{k}{2}$$

$$T_b = 2\pi\sqrt{\frac{m}{k/2}} = \sqrt{2}\left(2\pi\sqrt{\frac{m}{k}}\right) = \sqrt{2}T_a$$

Configuration (c): Two springs are connected in parallel.

$$k_c = k + k = 2k$$

$$T_c = 2\pi\sqrt{\frac{m}{2k}} = \frac{1}{\sqrt{2}}\left(2\pi\sqrt{\frac{m}{k}}\right) = \frac{1}{\sqrt{2}}T_a$$

Taking the ratio of their time periods:

$$T_a : T_b : T_c = T_a : \sqrt{2}T_a : \frac{1}{\sqrt{2}}T_a = 1 : \sqrt{2} : \frac{1}{\sqrt{2}}$$

Step 4: Final Answer:

The ratio of the time periods is $1 : \sqrt{2} : \frac{1}{\sqrt{2}}$.

Quick Tip: Remember that springs in series become "looser" (lower k, higher T), while springs in parallel become "stiffer" (higher k, lower T). This intuitive check ensures your computed ratios make logical sense.

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4. A man of mass m starts falling towards a planet of mass M and radius R. As he reaches near to the surface, he realizes that he will pass through a small hole in the planet. As he enters the hole, he sees that the planet is really made of two pieces: a spherical shell of negligible thickness of mass $3M/4$ and a point mass $M/4$ at the centre. Change in the force of gravity experienced by the man is

(A) $\frac{3}{4}\frac{GMm}{R^2}$

(B) 0

(C) $\frac{1}{3}\frac{GMm}{R^2}$

(D) $\frac{4}{3}\frac{GMm}{R^2}$

Correct Answer: (A) $\frac{3}{4}\frac{GMm}{R^2}$

Solution:

Step 1: Understanding the Concept:

The gravitational force outside a spherically symmetric mass distribution behaves as if the entire mass is concentrated at the center. However, inside a uniform spherical shell, the net gravitational force exerted by the shell is zero (Shell Theorem).

Step 2: Key Formula or Approach:

Newton's law of universal gravitation:

$$F = \frac{GMm}{r^2}$$

Step 3: Detailed Explanation:

Just outside the surface of the planet:

The planet's total mass is $M = \frac{3M}{4}(\text{shell}) + \frac{M}{4}(\text{point mass})$. At a distance R (just outside), the entire mass behaves as a point mass at the center. The force experienced by the man is:

$$F_{\text{out}} = \frac{GMm}{R^2}$$

Just inside the hole (inside the spherical shell):

Once the man crosses the spherical shell of mass $\frac{3M}{4}$, this shell no longer exerts any net gravitational force on him (by Gauss's Law for gravity or Shell Theorem). The only force acting on him is due to the point mass $\frac{M}{4}$ situated at the center, which is still at a distance R away. The force experienced by the man inside is:

$$F_{\text{in}} = \frac{G(\frac{M}{4})m}{R^2} = \frac{1}{4}\frac{GMm}{R^2}$$

Change in force:

The sudden change in the force as he crosses the shell is:

$$\Delta F = F_{\text{out}} - F_{\text{in}} = \frac{GMm}{R^2} - \frac{1}{4}\frac{GMm}{R^2} = \frac{3}{4}\frac{GMm}{R^2}$$

Step 4: Final Answer:

The change in the force of gravity experienced is $\frac{3}{4}\frac{GMm}{R^2}$.

Quick Tip: The Shell Theorem dictates that for any point inside a uniform spherical shell, the gravitational field contributed by the shell is exactly zero. Always separate the planet's mass into internal and external components relative to the object's position.

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5. A steel rod of diameter 1.0 cm is clamped firmly at each end when its temperature is 25°C so that it cannot contract on cooling. The tension in the rod at 0°C is ($\alpha = 1 \times 10^{-5}$/°C, $Y = 2 \times 10^{11}$ N/m^2)

(A) 3925 N

(B) 7000 N

(C) 7400 N

(D) 4700 N

Correct Answer: (A) 3925 N

Solution:

Step 1: Understanding the Concept:

When a rod is clamped at both ends, a change in temperature attempts to alter its length. Since its length is constrained, a thermal stress is developed within the rod. Cooling the rod creates tension because it wants to contract but is prevented from doing so.

Step 2: Key Formula or Approach:

The thermal strain is given by: $\frac{\Delta L}{L} = \alpha \Delta T$

The thermal stress is: Stress $= Y \times$ Strain $= Y\alpha\Delta T$

The tension (force) developed is: $F = \text{Stress} \times A = YA\alpha\Delta T$

Step 3: Detailed Explanation:

Given values: Diameter $d = 1.0 \text{ cm} = 10^{-2}$ m

Young's modulus $Y = 2 \times 10^{11}$ N/m^2

Coefficient of linear expansion $\alpha = 1 \times 10^{-5}$ /°C

Change in temperature $\Delta T = 25°\text{C} - 0°\text{C} = 25°\text{C}$

Calculate the cross-sectional area A:

$$A = \frac{\pi d^2}{4} = \frac{3.14 \times (10^{-2})^2}{4} = \frac{3.14 \times 10^{-4}}{4} = 0.785 \times 10^{-4} \text{ m}^2$$

Now compute the tension F:

$$F = YA\alpha\Delta T$$

$$F = (2 \times 10^{11}) \times (0.785 \times 10^{-4}) \times (1 \times 10^{-5}) \times 25$$

$$F = (2 \times 25) \times 0.785 \times 10^{11} \times 10^{-9}$$

$$F = 50 \times 0.785 \times 10^2$$

$$F = 5000 \times 0.785 = 3925 \text{ N}$$

Step 4: Final Answer:

The tension in the rod is 3925 N.

Quick Tip: Always convert all given quantities (like diameter from cm to meters) into standard SI units before applying formulas to avoid massive powers-of-10 errors in your final result.

Chemistry

6. Half-life of zero order reaction $A \rightarrow$ product is 1 hour, when initial concentration of reactant is 2.0 mol L^{-1}. The time required to decrease concentration of A from 0.50 to 0.25 mol L^{-1} is:

(a) 0.5 hour

(b) 4 hour

(c) 15 min

(d) 60 min

Correct Answer: (c) 15 min

Solution:

Step 1: Understanding the Concept:

For a zero-order reaction, the rate of reaction is constant and independent of the reactant concentration. The half-life of a zero-order reaction is directly proportional to its initial concentration.

Step 2: Key Formula or Approach:

The half-life ($t_{1/2}$) for a zero-order reaction is given by:

$$t_{1/2} = \frac{[A]_0}{2k}$$

The integrated rate law for a zero-order reaction is:

$$[A]_t = [A]_0 - kt$$

where k is the rate constant.

Step 3: Detailed Explanation:

First, calculate the rate constant k using the given half-life information:

$$1 \text{ hour} = \frac{2.0 \text{ mol L}^{-1}}{2k}$$

$$k = \frac{2.0}{2 \times 1} = 1.0 \text{ mol L}^{-1} \text{ h}^{-1}$$

Next, find the time required to decrease the concentration from 0.50 mol L^{-1} to 0.25 mol L^{-1}.

Here, the starting concentration for this interval is $[A]_0 = 0.50$ mol L^{-1} and the final concentration is $[A]_t = 0.25$ mol L^{-1}. Using the integrated rate law:

$$0.25 = 0.50 - (1.0) \times t$$

$$1.0 \times t = 0.50 - 0.25 = 0.25 \text{ hours}$$

Convert the time into minutes:

$$t = 0.25 \times 60 \text{ min} = 15 \text{ min}$$

Step 4: Final Answer:

The time required is 15 minutes.

Quick Tip: For a zero-order reaction, the time required to reduce the concentration by a specific amount $\Delta[A]$ is simply $\Delta[A]/k$. It does not depend on the absolute starting concentration, only on the difference!

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7. The absolute configuration of:

(Image represents a Fischer projection with C2 and C3 chiral centers. Top group: CO_2H, Bottom group: CH_3. At C2: H on left, OH on right. At C3: H on left, Cl on right.)

(a) (2S,3S)

(b) (2R,3R)

(c) (2R,3S)

(d) (2S,3R)

Correct Answer: (d) (2S,3R)

Solution:

Step 1: Understanding the Concept:

The absolute configuration (R/S) of chiral centers in a Fischer projection is determined using the Cahn-Ingold-Prelog (CIP) priority rules. If the lowest priority group is on a horizontal bond, the actual configuration is opposite to the apparent direction (clockwise vs. counter-clockwise).

Step 2: Key Formula or Approach:

1. Assign priority numbers (1 to 4) to the four groups attached to the chiral center based on atomic number.
2. Trace the path from $1 \to 2 \to 3$.
3. If the path is clockwise, the apparent configuration is R. If counter-clockwise, it is S.
4. Since the lowest priority group (-H) is on the horizontal bond in both centers, reverse the result: $R \to S$ and $S \to R$.

Step 3: Detailed Explanation:

For C2 (Top chiral center):

Groups attached to C2: -OH, -CH(Cl)CH_3 (C3 group), -CO_2H, -H.

Priorities at C2: 1. -OH (Oxygen has the highest atomic number, 8) 2. -CH(Cl)CH_3 (Carbon is attached to Cl, C, H. Max atomic number is 17) 3. -CO_2H (Carbon is attached to O, O, O. Max atomic number is 8) 4. -H (Lowest priority) Note: Group 2 wins over Group 3 because the first point of difference yields Cl (17) vs O (8). Positions on Fischer projection: Top(3), Bottom(2), Right(1), Left(4). Path $1 \to 2 \to 3$ goes from Right to Bottom to Top, which forms a **clockwise** circle. Apparent = R.

Since -H is horizontal (Left), we reverse the result. Therefore, actual configuration = **2S**.

For C3 (Bottom chiral center):

Groups attached to C3: -Cl, -CH(OH)CO_2H (C2 group), -CH_3, -H.

Priorities at C3: 1. -Cl (Highest atomic number, 17) 2. -CH(OH)CO_2H (Carbon attached to O, C, H) 3. -CH_3 (Carbon attached to H, H, H) 4. -H Positions on Fischer projection: Right(1), Top(2), Bottom(3), Left(4). Path $1 \to 2 \to 3$ goes from Right to Top to Bottom, which forms a **counter-clockwise** circle. Apparent = S.

Since -H is horizontal (Left), we reverse the result. Therefore, actual configuration = **3R**.

Step 4: Final Answer:

The absolute configuration is (2S, 3R).

Quick Tip: When assigning priorities, evaluate atoms one bond further away only if there is a tie at the first point of attachment. Here, C3's attached Chlorine (17) easily beats the Oxygens (8) on the carboxylic acid group!

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8. The one giving maximum number of isomeric alkenes on dehydrohalogenation reaction is (excluding rearrangement):

(a) 1-Bromo-2-methylbutane

(b) 2-Bromopropane

(c) 2-Bromopentane

(d) 2-Bromo-3,3-dimethylpentane

Correct Answer: (c) 2-Bromopentane

Solution:

Step 1: Understanding the Concept:

Dehydrohalogenation (an E2 elimination reaction) removes a halogen atom and a hydrogen atom from an adjacent (β) carbon, creating a carbon-carbon double bond. The number of isomeric alkenes depends on the number of different types of β-hydrogens and the possibility of geometrical (cis/trans or E/Z) isomerism in the resulting alkenes.

Step 2: Detailed Explanation:

Let's analyze each option independently:

(a) 1-Bromo-2-methylbutane: Br-CH_2-CH(CH_3)-CH_2-CH_3

The halogen is at C1. The only adjacent carbon is C2.

Removal of H from C2 gives: CH_2=C(CH_3)-CH_2-CH_3 (2-methylbut-1-ene).

This terminal alkene does not exhibit geometrical isomerism. Total isomers = 1.

(b) 2-Bromopropane: CH_3-CH(Br)-CH_3

The β-carbons are identical (both are methyl groups).

Elimination gives: CH_2=CH-CH_3 (Propene).

Propene has no geometrical isomerism. Total isomers = 1.

(c) 2-Bromopentane: CH_3-CH(Br)-CH_2-CH_2-CH_3

There are two different adjacent carbons with hydrogens: C1 and C3.

Elimination with C1 gives: CH_2=CH-CH_2-CH_2-CH_3 (Pent-1-ene) $\rightarrow$ No geometrical isomerism (1 isomer).

Elimination with C3 gives: CH_3-CH=CH-CH_2-CH_3 (Pent-2-ene).

Pent-2-ene can exist as both *cis*- and *trans*- isomers (2 isomers).

Total isomers = 1 + 2 = 3.

(d) 2-Bromo-3,3-dimethylpentane: CH_3-CH(Br)-C(CH_3)$_2$-CH_2-CH_3

The β-carbons are C1 and C3. However, C3 is a quaternary carbon (it has no hydrogens attached to it!).

Therefore, elimination can only occur by removing a hydrogen from C1.

Product: CH_2=CH-C(CH_3)$_2$-CH_2-CH_3 (3,3-dimethylpent-1-ene).

No geometrical isomerism possible. Total isomers = 1.

Step 3: Final Answer:

2-Bromopentane gives the maximum number of isomeric alkenes (3 isomers).

Quick Tip: Always check the internal alkenes formed during elimination for cis/trans (E/Z) isomerism. A single structural product can often represent two separate stereoisomers!

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9. Pick the correct statement about electron and photon:

(a) both electron and photons are fermions

(b) electron is a fermion and photons are bosons

(c) electron is boson and photons are fermions

(d) both electron and photons are bosons

Correct Answer: (b) electron is a fermion and photons are bosons

Solution:

Step 1: Understanding the Concept:

In quantum mechanics, all identical particles are classified into two broad categories based on their intrinsic spin: fermions and bosons.

- **Fermions** have half-integer spins (e.g., $1/2, 3/2$). They follow Fermi-Dirac statistics and obey the Pauli Exclusion Principle.
- **Bosons** have integer spins (e.g., $0, 1, 2$). They follow Bose-Einstein statistics and do not obey the Pauli Exclusion Principle (multiple bosons can occupy the same quantum state).

Step 2: Detailed Explanation:

An electron is a fundamental particle with a spin of $1/2$. Therefore, it is classified as a fermion. A photon is a gauge boson that mediates the electromagnetic force and has a spin of 1. Therefore, it is classified as a boson.

Thus, the statement "electron is a fermion and photons are bosons" is the physically correct categorization.

Step 3: Final Answer:

Option (b) is correct.

Quick Tip: Matter particles (like electrons, protons, quarks) are usually fermions, while force-carrying particles (like photons, gluons, W/Z bosons) are bosons.

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10. Which hydride among the following is less stable?

(a) BeH_2

(b) NH_3

(c) HF

(d) LiH

Correct Answer: (a) BeH_2

Solution:

Step 1: Understanding the Concept:

The stability of hydrides depends heavily on their bonding characteristics. Elements in the second period form different types of hydrides: ionic, covalent, and electron-deficient.

Step 2: Detailed Explanation:

Let's look at the nature of each given hydride:

- **HF (Hydrogen Fluoride):** A highly stable covalent compound due to the extremely strong and polar H-F bond.
- **NH_3 (Ammonia):** A very stable, electron-precise covalent hydride with strong N-H bonds.
- **LiH (Lithium Hydride):** A stable, saline (ionic) hydride featuring strong electrostatic attractions between Li^+ and H^- ions in its crystal lattice.
- **BeH_2 (Beryllium Hydride):** Beryllium is a small atom with high ionization energy, leading it to form covalent rather than ionic bonds. However, it lacks enough electrons

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to form a complete octet, making BeH_2 an *electron-deficient* compound. It exists as a complex polymeric structure with 3-center 2-electron (3c-2e) hydrogen bridges.

Because of its electron-deficient and polymeric nature, BeH_2 is thermodynamically the least stable of the group. It is notoriously difficult to synthesize directly from its elements and decomposes relatively easily compared to the stable octets/lattices in the other choices.

Step 3: Final Answer:

BeH_2 is the least stable hydride.

Quick Tip: Electron-deficient molecules (Group 2 and Group 13 hydrides like BeH_2, B_2H_6) are generally highly reactive and less stable than their electron-precise (Group 14-17) or strongly ionic (Group 1) counterparts.

Maths

11. Let $f(x) = \sin x, g(x) = \cos x, h(x) = x^2$ **then**

$$\lim_{x\to 1} \frac{f(g(h(x))) - f(g(h(1)))}{x-1} =$$

(A) 0

(B) $-2\sin 1 \cos(\cos 1)$

(C) ∞

(D) $-2\sin 1 \cos 1$

Correct Answer: (B) $-2\sin 1 \cos(\cos 1)$

Solution:

Step 1: Understanding the Concept:

The given limit expression matches the precise limit definition of a derivative at a specific point.

$$\lim_{x \to a} \frac{F(x) - F(a)}{x - a} = F'(a)$$

Step 2: Key Formula or Approach:

Let $F(x) = f(g(h(x)))$. We need to find the derivative $F'(x)$ using the Chain Rule and evaluate it at $x = 1$.

Step 3: Detailed Explanation:

First, construct the composite function $F(x)$:

$$F(x) = f(g(x^2)) = f(\cos(x^2)) = \sin(\cos(x^2))$$

Now, differentiate $F(x)$ with respect to x:

$$F'(x) = \frac{d}{dx}\left[\sin(\cos(x^2))\right]$$

Apply the Chain Rule sequentially (outside to inside):

$$F'(x) = \cos(\cos(x^2)) \cdot \frac{d}{dx}[\cos(x^2)]$$

$$F'(x) = \cos(\cos(x^2)) \cdot [-\sin(x^2)] \cdot \frac{d}{dx}[x^2]$$

$$F'(x) = \cos(\cos(x^2)) \cdot [-\sin(x^2)] \cdot (2x)$$

$$F'(x) = -2x \sin(x^2) \cos(\cos(x^2))$$

Finally, evaluate the derivative at $x = 1$:

$$F'(1) = -2(1) \sin(1^2) \cos(\cos(1^2))$$

$$F'(1) = -2 \sin(1) \cos(\cos 1)$$

Step 4: Final Answer:

The limit is $-2 \sin 1 \cos(\cos 1)$.

Quick Tip: Recognizing the limit definition of a derivative saves you from attempting complicated L'Hôpital's rule expansions or Taylor series on heavily nested trigonometric functions.

12. The variance of 20 observations is 5. If each observation is multiplied by 2, then the new variance of the resulting observation is

(A) $2^3 \times 5$
(B) $2^2 \times 5$
(C) 2×5
(D) $2^4 \times 5$

Correct Answer: (B) $2^2 \times 5$

Solution:

Step 1: Understanding the Concept:

Variance measures the spread of a set of data. If every observation in a dataset is scaled (multiplied) by a constant factor, the variance is scaled by the square of that constant.

Step 2: Key Formula or Approach:

For a random variable X or a set of observations, the property of variance under a linear transformation is:

$$\text{Var}(aX) = a^2\text{Var}(X)$$

where a is a constant multiplier.

Step 3: Detailed Explanation:

Let the original variance of the observations be $\text{Var}(X) = 5$.

Each observation is multiplied by the constant $a = 2$.

The new variance will be:

$$\text{Var}(2X) = 2^2 \times \text{Var}(X)$$

$$\text{Var}(2X) = 2^2 \times 5 = 4 \times 5 = 20$$

Matching this with the given options, the result is explicitly written as $2^2 \times 5$. Note that the number of observations ($n = 20$) is irrelevant to the scaling property of the variance itself.

Step 4: Final Answer:

The new variance is $2^2 \times 5$.

Quick Tip: Standard Deviation scales linearly ($\text{SD}_{\text{new}} = |a| \times \text{SD}_{\text{old}}$), but Variance scales quadratically ($\text{Var}_{\text{new}} = a^2 \times \text{Var}_{\text{old}}$). Do not let the "number of observations" distract you!

13. If $A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, P = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ **and** $X = APA^T$**, then** $A^T X^{50} A =$

(A) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(B) $\begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix}$

(C) $\begin{pmatrix} 25 & 1 \\ 1 & -25 \end{pmatrix}$

(D) $\begin{pmatrix} 1 & 50 \\ 0 & 1 \end{pmatrix}$

Correct Answer: (D) $\begin{pmatrix} 1 & 50 \\ 0 & 1 \end{pmatrix}$

Solution:

Step 1: Understanding the Concept:

To compute high powers of a composite matrix expression efficiently, we look for properties like symmetry, inverse relationships, or nilpotency that will cause terms to cancel out during multiplication.

Step 2: Key Formula or Approach:

Notice the properties of matrix A: $A^T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = A$.

$A^2 = A \cdot A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$.

Therefore, $A = A^T = A^{-1}$.

Step 3: Detailed Explanation:

Given $X = APA^T$. Since $A^T = A$, we can write $X = APA$.

Let's find X^2:

$$X^2 = (APA)(APA) = AP(A^2)PA$$

Since $A^2 = I$:

$$X^2 = AP(I)PA = AP^2A$$

Extending this logic to the 50th power via induction:

$$X^{50} = AP^{50}A$$

Now, substitute this back into the target expression $A^T X^{50} A$:

$$A^T X^{50} A = A(AP^{50}A)A = (A^2)P^{50}(A^2)$$

Again, since $A^2 = I$:

$$IP^{50}I = P^{50}$$

So, the entire problem simplifies to finding P^{50}. Let's calculate the first few powers of P:

$$P = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$P^2 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$

$$P^3 = P^2P = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$$

By induction, the general formula is $P^n = \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$.

Therefore, for $n = 50$:

$$P^{50} = \begin{pmatrix} 1 & 50 \\ 0 & 1 \end{pmatrix}$$

Step 4: Final Answer:

The result matrix is $\begin{pmatrix} 1 & 50 \\ 0 & 1 \end{pmatrix}$.

Quick Tip: Whenever you see an expression like $X = PAP^{-1}$ or $X = APA$ (where $A^2 = I$), immediately recognize it as a similarity transformation. Computing X^n will always collapse the inner terms into the identity matrix, leaving you to only compute the power of the central matrix!

14. The locus of the mid-point of a chord of the circle $x^2 + y^2 = 4$, which subtends a right angle at the origin is

(A) $x + y = 2$

(B) $x^2 + y^2 = 1$

(C) $x^2 + y^2 = 2$

(D) $x + y = 1$

Correct Answer: (C) $x^2 + y^2 = 2$

Solution:

Step 1: Understanding the Concept:

We can solve this using either simple coordinate geometry properties (distance from center) or the homogenization method. Using the geometric properties of the circle is the most straightforward and fastest method.

Step 2: Key Formula or Approach:

For a circle centered at the origin, a chord that subtends a specific angle 2θ at the center creates an isosceles triangle with the two radii. The perpendicular distance d from the center to the chord (which is also the midpoint) is given by:

$$d = r \cos\theta$$

Step 3: Detailed Explanation:

The circle is $x^2 + y^2 = 4$. Its center is $(0, 0)$ and its radius is $r = 2$.

Let the midpoint of the chord be (h, k).

The distance from the center $(0, 0)$ to the midpoint (h, k) is $d = \sqrt{h^2 + k^2}$.

The chord subtends an angle of 90° at the origin (center). The line joining the center to the midpoint bisects this angle.

Therefore, the angle θ between the radius and the perpendicular bisector is $90^\circ/2 = 45^\circ$.

Using the relation from the right-angled triangle formed:

$$d = r\cos(45^\circ)$$

Substitute d and r:

$$\sqrt{h^2 + k^2} = 2\left(\frac{1}{\sqrt{2}}\right) = \sqrt{2}$$

Squaring both sides to find the locus:

$$h^2 + k^2 = 2$$

Replacing (h, k) with general coordinates (x, y), the equation of the locus is:

$$x^2 + y^2 = 2$$

Step 4: Final Answer:

The locus of the midpoint is $x^2 + y^2 = 2$.

Quick Tip: Drawing a quick diagram for locus problems involving circles often reveals simple geometric relationships (like right or isosceles triangles) that bypass tedious algebra completely!

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15. If the system of linear equations $2x + y - z = 7$, $x - 3y + 2z = 1$, $x + 4y + \delta z = k$ (where $\delta, k \in \mathbb{R}$) has infinitely many solutions, then $\delta + k$ is equal to:

(A) -3

(B) 3

(C) 6

(D) 9

Correct Answer: (B) 3

Solution:

Step 1: Understanding the Concept:

For a system of three linear equations to have infinitely many solutions, the determinant of the coefficient matrix (Δ) must be zero, and the system must be consistent (which implies that the third equation is a linear combination of the first two).

Step 2: Key Formula or Approach:

1. Set determinant $\Delta = 0$ to find δ.

2. Determine the scalar coefficients to express the left hand side of Eq 3 as a linear combination of Eq 1 and Eq 2.

3. Apply the same scalar combination to the constant terms on the right hand side to find k.

Step 3: Detailed Explanation:

The system is: 1) $2x + y - z = 7$ 2) $x - 3y + 2z = 1$ 3) $x + 4y + \delta z = k$

Finding δ:

Set the determinant of the coefficient matrix to 0:

$$\Delta = \begin{vmatrix} 2 & 1 & -1 \\ 1 & -3 & 2 \\ 1 & 4 & \delta \end{vmatrix} = 0$$

Expand along the first row:

$$2(-3\delta - 8) - 1(\delta - 2) - 1(4 - (-3)) = 0$$

$$-6\delta - 16 - \delta + 2 - 7 = 0$$

$$-7\delta - 21 = 0 \implies 7\delta = -21 \implies \delta = -3$$

Finding k:

Since there are infinitely many solutions, the third equation must be a linear combination of the first two. Let:

$$\text{Eq}_3 = a \cdot \text{Eq}_1 + b \cdot \text{Eq}_2$$

Matching the coefficients for x and y: For x: $2a + b = 1$ For y: $a - 3b = 4$ Solving these simultaneous equations:

From the second equation, $a = 4 + 3b$. Substitute into the first:

$$2(4 + 3b) + b = 1$$

$$8 + 6b + b = 1 \implies 7b = -7 \implies b = -1$$

Then, $a = 4 + 3(-1) = 1$. Let's verify with the z coefficient (which is $\delta = -3$): $a(-1) + b(2) = 1(-1) + (-1)(2) = -1 - 2 = -3$. This perfectly matches our δ. Now, apply the same linear combination to the constant terms to find k:

$$k = a(7) + b(1) = 1(7) + (-1)(1) = 7 - 1 = 6$$

Thus, $k = 6$.

Calculating the final required value:

$$\delta + k = -3 + 6 = 3$$

Step 4: Final Answer:

The value of $\delta + k$ is 3.

Quick Tip: To bypass Cramer's Rule determinants (like $\Delta_x = \Delta_y = 0$), identifying the linear dependence directly via $a \cdot \text{Eq}_1 + b \cdot \text{Eq}_2$ is often much faster and less prone to algebraic sign errors.

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