

BITSAT 2026 April 15 Shift 1

Question Paper with Solutions

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General Instructions

- (i) **Duration:** The total duration of the examination is 3 hours (180 minutes).
- (ii) **Total Marks:** The complete paper carries a maximum of 390 marks.
- (iii) **Structure:** The paper has 4 Sections:
 - **Part 1:** 30 Multiple Choice Questions (Physics).
 - **Part 2:** 30 Multiple Choice Questions (Chemistry).
 - **Part 3:** 10 Multiple Choice Questions (English Proficiency),
20 Multiple Choice Questions (Logical Reasoning)
 - **Part 4:** 40 Multiple Choice Questions (Mathematics/Biology)
- (iv) **Compulsory Questions:** All 130 questions are compulsory, and +12 Questions (Optional Extra Questions)
- (v) Each question has four options. Only **one** option is correct.
- (vi) **Correct Answer:** +3 marks.
- (vii) **Incorrect Answer:** -1 (Negative marking).
- (viii) **Unanswered/Marked for Review:** 0 marks.

Physics

1. A car is moving on a horizontal curved road with radius 50 m. The approximate maximum speed of car will be, if friction between tyres and road is 0.34. [Take $g = 10 \text{ ms}^{-2}$]

- (a) 3.4 ms^{-1}
- (b) 22.4 ms^{-1}
- (C) 13 ms^{-1}
- (d) 17 ms^{-1}

Correct Answer: (C) 13 ms^{-1}

Solution:

Step 1: Understanding the Concept:

When a car moves on a level, horizontal circular road, the frictional force between the tyres and the road provides the necessary centripetal force to keep the car in its circular path.

The car reaches its maximum safe speed when the required centripetal force is exactly equal to the maximum available static friction.

Step 2: Key Formula or Approach:

The maximum speed v is determined by the balance between centripetal force and limiting friction:

$$\frac{mv^2}{r} = \mu mg$$

Solving for v :

$$v = \sqrt{\mu rg}$$

Where μ is the coefficient of friction, r is the radius, and g is the acceleration due to gravity.

Step 3: Detailed Explanation:

From the question, we have the following data:

Radius of the curved road, $r = 50 \text{ m}$

Coefficient of friction, $\mu = 0.34$

Acceleration due to gravity, $g = 10 \text{ ms}^{-2}$

Substituting these values into the formula:

$$v = \sqrt{0.34 \times 50 \times 10}$$

$$v = \sqrt{170}$$

To find the approximate value, we know that $13^2 = 169$.

Thus, $\sqrt{170} \approx 13.03 \text{ ms}^{-1}$.

Comparing with the given options, 13 ms^{-1} is the closest approximate value.

Step 4: Final Answer:

The approximate maximum speed of the car is 13 ms^{-1} .

Quick Tip: For flat roads, remember the direct relation $v \propto \sqrt{\mu}$. If the friction coefficient is quadrupled, the safe speed only doubles. Always look for nearby perfect squares to estimate roots quickly in exams.

2. An electron moving along the x-axis has a position given by $x = 20te^{-t}$ m, where t is in second. How far is the electron from the origin when it momentarily stop?

- (a) 20 m
- (b) 20 em
- (c) $\frac{20}{e}$ m
- (d) zero

Correct Answer: (c) $\frac{20}{e}$ m

Solution:

Step 1: Understanding the Concept:

The velocity of a particle is the first derivative of its position with respect to time.

The term "momentarily stop" indicates that the instantaneous velocity of the electron becomes zero at a specific instant of time.

Step 2: Key Formula or Approach:

Calculate velocity v by differentiating x with respect to t :

$$v = \frac{dx}{dt}$$

Set $v = 0$ to find the time t of stopping, then substitute this t back into the original x equation.

Step 3: Detailed Explanation:

Given position function: $x = 20te^{-t}$

Differentiating using the product rule $\frac{d}{dt}(uv) = u\frac{dv}{dt} + v\frac{du}{dt}$:

$$v = \frac{d}{dt}(20te^{-t}) = 20 \left[t \cdot \frac{d}{dt}(e^{-t}) + e^{-t} \cdot \frac{d}{dt}(t) \right]$$

$$v = 20[t \cdot (-e^{-t}) + e^{-t} \cdot 1]$$

$$v = 20e^{-t}(1 - t)$$

Setting $v = 0$ to find the time of stopping:

$$20e^{-t}(1 - t) = 0$$

Since $20e^{-t}$ cannot be zero for finite time, we have $1 - t = 0$, which gives $t = 1$ second.

Now, calculate the position at $t = 1$ s:

$$x = 20(1)e^{-1} = \frac{20}{e} \text{ m}$$

Step 4: Final Answer:

The electron is at a distance of $\frac{20}{e}$ m from the origin.

Quick Tip: For any function of form $t^n e^{-at}$, the maximum value or stationary point always occurs at $t = n/a$. Here $n = 1$ and $a = 1$, so $t = 1$ immediately without full differentiation steps.

3. The ranges and heights for two projectiles projected with the same initial velocity at angles 42° and 48° with the horizontal are R_1 , R_2 and H_1 , H_2 respectively. Choose the correct option :

- (a) $R_1 > R_2$ and $H_1 = H_2$
- (b) $R_1 = R_2$ and $H_1 < H_2$
- (c) $R_1 < R_2$ and $H_1 < H_2$
- (d) $R_1 = R_2$ and $H_1 = H_2$

Correct Answer: (b) $R_1 = R_2$ and $H_1 < H_2$

Solution:**Step 1: Understanding the Concept:**

Two projectiles launched with the same speed have identical horizontal ranges if their angles of projection are complementary (sum up to 90°).

Maximum height depends solely on the vertical component of initial velocity, which increases as the angle of projection increases towards 90° .

Step 2: Key Formula or Approach:

Horizontal Range: $R = \frac{u^2 \sin(2\theta)}{g}$

Maximum Height: $H = \frac{u^2 \sin^2 \theta}{2g}$

Step 3: Detailed Explanation:

1. Comparing Ranges:

Check if the angles are complementary: $42^\circ + 48^\circ = 90^\circ$.

Since the sum is 90° , $\sin(2 \times 42^\circ) = \sin 84^\circ$ and $\sin(2 \times 48^\circ) = \sin 96^\circ = \sin(180^\circ - 96^\circ) = \sin 84^\circ$.

Therefore, $R_1 = R_2$.

2. Comparing Heights:

Height is proportional to $\sin^2 \theta$.

Since $48^\circ > 42^\circ$, it follows that $\sin 48^\circ > \sin 42^\circ$.

Squaring both sides preserves the inequality: $\sin^2 48^\circ > \sin^2 42^\circ$.

Thus, $H_2 > H_1$, or $H_1 < H_2$.

Step 4: Final Answer:

The correct condition is $R_1 = R_2$ and $H_1 < H_2$.

Quick Tip: To remember this quickly: Range is symmetrical about 45° (complementary angles work), but Height always increases monotonically with the angle between 0° and 90° .

4. A particle moves under the effect of a force $F = Cx$ from $x = 0$ to $x = x_1$. The work done in the process is

- (a) Cx_1^2
- (b) $\frac{1}{2}Cx_1^2$
- (c) Cx_1
- (d) Zero

Correct Answer: (b) $\frac{1}{2}Cx_1^2$

Solution:

Step 1: Understanding the Concept:

Work done by a variable force acting in the direction of displacement is calculated by integrating the force over the path of motion.

Step 2: Key Formula or Approach:

Work done formula:

$$W = \int F dx$$

With limits from $x = 0$ to $x = x_1$.

Step 3: Detailed Explanation:

Substitute the given force expression $F = Cx$ into the integral:

$$W = \int_0^{x_1} Cx dx$$

Factor out the constant C :

$$W = C \int_0^{x_1} x dx$$

Integrate x using the power rule $\int x^n dx = \frac{x^{n+1}}{n+1}$:

$$W = C \left[\frac{x^2}{2} \right]_0^{x_1}$$

Evaluate at limits:

$$W = C \left[\frac{x_1^2}{2} - \frac{0^2}{2} \right]$$

$$W = \frac{1}{2} C x_1^2$$

Step 4: Final Answer:

The work done in the process is $\frac{1}{2} C x_1^2$.

Quick Tip: Notice that the force is linear ($F \propto x$). The area under a Force-Displacement graph for a linear force is a triangle with base x_1 and height Cx_1 . Area = $\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times x_1 \times Cx_1 = \frac{1}{2}Cx_1^2$.

5. Three-point particles of masses 1.0 kg, 1.5 kg and 2.5 kg are placed at three corners of a right-angle triangle of sides 4.0 cm, 3.0 cm and 5.0 cm as shown in the figure. The center of mass of the system is at a point:

- (a) 0.6 cm right and 2.0 cm above 1 kg mass
- (b) 1.5 cm right and 1.2 cm above 1 kg mass
- (c) 2.0 cm right and 0.9 cm above 1 kg mass
- (d) 0.9 cm right and 2.0 cm above 1 kg mass

Correct Answer: (d) 0.9 cm right and 2.0 cm above 1 kg mass

Solution:

Step 1: Understanding the Concept:

The Center of Mass (COM) coordinates for a system of point masses are calculated as the mass-weighted average of their positions.

Step 2: Key Formula or Approach:

$$X_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$Y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

Let 1 kg mass be at origin (0,0). Then 1.5 kg is at (3,0) and 2.5 kg is at (0,4).

Step 3: Detailed Explanation:

1. Assign coordinates based on the diagram:

- Mass $m_1 = 1.0$ kg at $(x_1, y_1) = (0, 0)$.

- Mass $m_2 = 1.5$ kg at $(x_2, y_2) = (3, 0)$.

- Mass $m_3 = 2.5$ kg at $(x_3, y_3) = (0, 4)$.

Total mass $M = 1.0 + 1.5 + 2.5 = 5.0$ kg.

2. Calculate X_{cm} :

$$X_{cm} = \frac{(1.0 \times 0) + (1.5 \times 3) + (2.5 \times 0)}{5.0} = \frac{4.5}{5.0} = 0.9 \text{ cm}$$

3. Calculate Y_{cm} :

$$Y_{cm} = \frac{(1.0 \times 0) + (1.5 \times 0) + (2.5 \times 4)}{5.0} = \frac{10}{5.0} = 2.0 \text{ cm}$$

Step 4: Final Answer:

The center of mass is 0.9 cm to the right and 2.0 cm above the 1 kg mass.

Quick Tip: Always choose the origin at the location of one of the masses. This sets its contributions to zero, significantly simplifying the arithmetic and reducing chances of calculation errors.

Chemistry

1. The number of valence electrons present in the metal among Cr, Co, Fe and Ni which has the lowest enthalpy of atomisation is

(a) 8

(b) 9

- (c) 6
(d) 10

Correct Answer: (c) 6

Solution:

Step 1: Understanding the Concept:

Enthalpy of atomisation ($\Delta_a H$) is the enthalpy change that occurs when one mole of a substance is completely dissociated into atoms in the gaseous state.

For transition metals, it depends on the strength of metallic bonds, which is determined by the number of valence electrons (unpaired electrons in $(n-1)d$ and ns orbitals) available for bonding.

Step 2: Key Formula or Approach:

1. Compare the experimental enthalpy of atomisation values for the given 3d series transition metals.
2. Identify the metal with the lowest value.
3. Determine its electronic configuration and count the total valence electrons $((n-1)d + ns)$.

Step 3: Detailed Explanation:

The standard enthalpies of atomisation ($\Delta_a H^\circ$) for the given metals in kJ/mol are:

- Chromium (Cr): 397 kJ/mol
- Iron (Fe): 416 kJ/mol
- Cobalt (Co): 425 kJ/mol
- Nickel (Ni): 430 kJ/mol

Among these, Chromium (Cr) has the lowest enthalpy of atomisation.

The electronic configuration of Chromium ($Z = 24$) is $[Ar]3d^5 4s^1$.

Number of valence electrons = Electrons in $3d$ + Electrons in $4s$

Number of valence electrons = $5 + 1 = 6$.

Step 4: Final Answer:

The metal with the lowest enthalpy of atomisation is Cr, which has 6 valence electrons.

Quick Tip: In the 3d series, enthalpy of atomisation generally increases up to the middle and then decreases, but Cr and Mn show local minima because their stable half-filled d^5 configurations result in weaker metallic bonding compared to their neighbors.

2. The Zeta potential is related to which property of colloids"

- (a) Colour
- (b) Tyndall effect
- (c) Charge on the surface of colloidal particles
- (d) Brownian movement

Correct Answer: (c) Charge on the surface of colloidal particles

Solution:

Step 1: Understanding the Concept:

Colloidal particles carry an electric charge which is responsible for the stability of the sol. When a colloidal particle is placed in a medium, it adsorbs certain ions to form a fixed layer. This attracts oppositely charged ions in the solution, forming a mobile or diffused layer.

Step 2: Key Formula or Approach:

Analyze the definition of Zeta potential (electrokinetic potential) in the context of the Helmholtz electrical double layer.

Step 3: Detailed Explanation:

The combination of the two layers of opposite charges around the colloidal particle is called the Helmholtz electrical double layer.

The potential difference between the fixed layer and the diffused layer of opposite charges is called the electrokinetic potential or Zeta potential.

This potential arises due to the presence of charge on the surface of colloidal particles.

It is a crucial factor in determining the stability of colloidal systems; high zeta potential values indicate stable colloids due to electrostatic repulsion.

Step 4: Final Answer:

Zeta potential is fundamentally related to the charge on the surface of colloidal particles.

Quick Tip: Remember that Zeta potential values outside the range of -30 mV to $+30$ mV usually indicate a stable colloidal suspension that will not aggregate easily.

3. Based on the data given below:

$$E_{Cr_2O_7^{2-}/Cr^{3+}}^0 = 1.33 \text{ V} \quad E_{Cl_2/Cl^-}^0 = 1.36 \text{ V}$$
$$E_{MnO_4^-/Mn^{2+}}^0 = 1.51 \text{ V} \quad E_{Cr^{3+}/Cr}^0 = -0.74 \text{ V}$$

the strongest reducing agent is:

- (a) Mn^{2+}
- (b) Cr
- (c) MnO_4^-
- (d) Cl^-

Correct Answer: (b) Cr

Solution:**Step 1: Understanding the Concept:**

A reducing agent is a species that loses electrons (undergoes oxidation).

The tendency of a species to be reduced is measured by its standard reduction potential (E^0).

The species with the lowest (most negative) E^0 value has the greatest tendency to undergo oxidation and therefore acts as the strongest reducing agent.

Step 2: Key Formula or Approach:

1. List all given standard reduction potentials.
2. Identify the lowest E^0 value.
3. The reduced form (the species on the right side of the reduction half-reaction) corresponding to the lowest E^0 is the strongest reducing agent.

Step 3: Detailed Explanation:

Given potentials:

1. $E^0(\text{MnO}_4^-/\text{Mn}^{2+}) = 1.51 \text{ V}$

2. $E^0(\text{Cl}_2/\text{Cl}^-) = 1.36 \text{ V}$

3. $E^0(\text{Cr}_2\text{O}_7^{2-}/\text{Cr}^{3+}) = 1.33 \text{ V}$

4. $E^0(\text{Cr}^{3+}/\text{Cr}) = -0.74 \text{ V}$

The lowest value is -0.74 V for the couple Cr^{3+}/Cr .

This means Cr metal has the highest tendency to oxidize to Cr^{3+} among the given species.

Hence, Cr is the strongest reducing agent.

Step 4: Final Answer:

Based on the minimum reduction potential, Cr is the strongest reducing agent.

Quick Tip: Reduction Potential $\propto$ Oxidising Power.

Oxidation Potential $\propto$ Reducing Power.

Since Oxidation Potential = $-\text{Reduction Potential}$, the most negative E_{red}^0 gives the highest reducing power.

4. An evacuated glass vessel weighs 40.0 g when empty, 135.0 g when filled with a liquid of density 0.95 g mL^{-1} and 40.5 g when filled with an ideal gas at 0.82 atm at 250 K. The molar mass of the gas in g mol^{-1} is:

(Given: $R = 0.082 \text{ L atm K}^{-1} \text{ mol}^{-1}$)

- (a) 35
- (b) 50
- (c) 75
- (d) 125

Correct Answer: (d) 125

Solution:

Step 1: Understanding the Concept:

To find the molar mass (M) of a gas using the ideal gas law, we need the mass of the gas (m), the pressure (P), the volume (V), and the temperature (T).

The volume of the vessel can be found by using a liquid of known density.

Step 2: Key Formula or Approach:

1. Volume of vessel (V) = $\frac{\text{Mass of liquid}}{\text{Density of liquid}}$.
2. Mass of gas (m) = (Mass of vessel + gas) - (Mass of empty vessel).
3. Ideal gas equation: $PV = nRT = \frac{m}{M}RT \implies M = \frac{mRT}{PV}$.

Step 3: Detailed Explanation:

- Mass of empty vessel = 40.0 g
- Mass of vessel + liquid = 135.0 g
- Mass of liquid = 135.0 - 40.0 = 95.0 g
- Volume of vessel (V) = $\frac{95.0 \text{ g}}{0.95 \text{ g/mL}} = 100 \text{ mL} = 0.1 \text{ L}$
- Mass of vessel + gas = 40.5 g
- Mass of gas (m) = 40.5 - 40.0 = 0.5 g
- Given: $P = 0.82 \text{ atm}$, $T = 250 \text{ K}$, $R = 0.082 \text{ L atm K}^{-1} \text{ mol}^{-1}$
- Calculating molar mass (M):

$$M = \frac{0.5 \times 0.082 \times 250}{0.82 \times 0.1}$$

$$M = \frac{0.5 \times 250}{1} \times \frac{0.082}{0.82 \times 0.1}$$

$$M = \frac{125 \times 0.082}{0.082} = 125 \text{ g/mol}$$

Step 4: Final Answer:

The molar mass of the gas is 125 g mol^{-1} .

Quick Tip: Always convert volume to Liters (L) when using $R = 0.082 \text{ L atm K}^{-1} \text{ mol}^{-1}$ to ensure unit consistency.

5. Which one of the following equations does not correctly represent the first law of thermodynamics for the given processes involving an ideal gas? (Assume non-expansion work is zero)

- (a) Cyclic process: $q = -w$
- (b) Adiabatic process: $\Delta U = -w$
- (c) Isochoric process: $\Delta U = q$
- (d) Isothermal process: $q = -w$

Correct Answer: (b) Adiabatic process: $\Delta U = -w$

Solution:

Step 1: Understanding the Concept:

The First Law of Thermodynamics is stated as $\Delta U = q + w$, where ΔU is the change in internal energy, q is the heat added to the system, and w is the work done on the system (IUPAC convention).

Step 2: Key Formula or Approach:

Apply the conditions for each thermodynamic process to the first law equation $\Delta U = q + w$.

Step 3: Detailed Explanation:

(a) **Cyclic process:** Since internal energy is a state function, for a complete cycle, $\Delta U = 0$. Thus, $0 = q + w \implies q = -w$. (Correct)

(b) **Adiabatic process:** By definition, there is no heat exchange with the surroundings, so $q = 0$.

The equation becomes $\Delta U = 0 + w \implies \Delta U = w$.

The option states $\Delta U = -w$, which is incorrect under the standard IUPAC convention. (Incorrect)

(c) **Isochoric process:** The volume is constant, so expansion work $w = -P\Delta V = 0$.

The equation becomes $\Delta U = q + 0 \implies \Delta U = q$. (Correct)

(d) **Isothermal process:** For an ideal gas, internal energy depends only on temperature. Since temperature is constant, $\Delta U = 0$.

Thus, $0 = q + w \implies q = -w$. (Correct)

Step 4: Final Answer:

Equation (b) does not correctly represent the first law for an adiabatic process.

Quick Tip: In Chemistry competitive exams, always use the IUPAC sign convention ($\Delta U = q + w$) unless specified otherwise. In this convention, work done **on** the system is positive.

Maths

1. If $|z_1| = 2$, $|z_2| = 3$, $|z_3| = 4$ and $|2z_1 + 3z_2 + 4z_3| = 4$, then absolute value of $8z_2z_3 + 27z_3z_1 + 64z_1z_2$ equals

- (A) 24
- (B) 48
- (C) 72
- (D) 96

Correct Answer: (D) 96

Solution:

Step 1: Understanding the Concept:

The problem utilizes fundamental properties of complex numbers relating the modulus and

the conjugate.

Specifically, we use the fact that for any complex number z , $|z|^2 = z\bar{z}$.

The modulus of a product is the product of the moduli: $|z_1 z_2| = |z_1| |z_2|$.

Also, the property of conjugates in addition is used: $\bar{z}_1 + \bar{z}_2 = \overline{z_1 + z_2}$.

Step 2: Key Formula or Approach:

From the given moduli, we can express the conjugates in terms of the original variables:

$$|z_1| = 2 \implies |z_1|^2 = 4 \implies z_1 \bar{z}_1 = 4 \implies \bar{z}_1 = \frac{4}{z_1}$$

$$|z_2| = 3 \implies |z_2|^2 = 9 \implies z_2 \bar{z}_2 = 9 \implies \bar{z}_2 = \frac{9}{z_2}$$

$$|z_3| = 4 \implies |z_3|^2 = 16 \implies z_3 \bar{z}_3 = 16 \implies \bar{z}_3 = \frac{16}{z_3}$$

Step 3: Detailed Explanation:

We are asked to find $|8z_2 z_3 + 27z_3 z_1 + 64z_1 z_2|$.

First, factor out $z_1 z_2 z_3$ from the entire expression:

$$= |z_1 z_2 z_3| \cdot \left| \frac{8}{z_1} + \frac{27}{z_2} + \frac{64}{z_3} \right|$$

Separate the modulus of the product:

$$= |z_1| |z_2| |z_3| \cdot \left| \frac{8}{z_1} + \frac{27}{z_2} + \frac{64}{z_3} \right|$$

Substitute the values $|z_1| = 2$, $|z_2| = 3$, $|z_3| = 4$:

$$= (2 \cdot 3 \cdot 4) \cdot \left| 2 \cdot \left(\frac{4}{z_1} \right) + 3 \cdot \left(\frac{9}{z_2} \right) + 4 \cdot \left(\frac{16}{z_3} \right) \right|$$

Using the conjugate relations from Step 2:

$$= 24 \cdot |2\bar{z}_1 + 3\bar{z}_2 + 4\bar{z}_3|$$

By conjugate properties, $|2\bar{z}_1 + 3\bar{z}_2 + 4\bar{z}_3| = |\overline{2z_1 + 3z_2 + 4z_3}| = |2z_1 + 3z_2 + 4z_3|$.

Given $|2z_1 + 3z_2 + 4z_3| = 4$, the final result is:

$$= 24 \cdot 4 = 96$$

Step 4: Final Answer:

The absolute value of the expression is 96.

Quick Tip: When you see products of multiple complex numbers like $z_2 z_3$, factoring out the common product $z_1 z_2 z_3$ often transforms the expression into reciprocal terms $\frac{1}{z_1}$, which can be replaced by conjugates when moduli are given.

2. If $a > 0, b > 0, c > 0$ and a, b, c are distinct, then $(a + b)(b + c)(c + a)$ is greater than

- (A) $2(a + b + c)$
- (B) $3(a + b + c)$
- (C) $6abc$
- (D) $8abc$

Correct Answer: (D) $8abc$

Solution:

Step 1: Understanding the Concept:

This problem uses the Arithmetic Mean-Geometric Mean (AM-GM) Inequality.

For positive real numbers, the AM is always greater than or equal to the GM.

Since the numbers a, b, c are distinct, the sum of any two will be strictly greater than twice their geometric mean.

Step 2: Key Formula or Approach:

The AM-GM inequality for two positive numbers x and y is:

$$\frac{x+y}{2} \geq \sqrt{xy} \implies x+y \geq 2\sqrt{xy}$$

We apply this individually to each binomial factor in the expression.

Step 3: Detailed Explanation:

Applying AM-GM to the pairs:

1) For a, b : $a+b > 2\sqrt{ab}$ (strict inequality because a, b are distinct)

2) For b, c : $b+c > 2\sqrt{bc}$ (strict inequality because b, c are distinct)

3) For c, a : $c+a > 2\sqrt{ca}$ (strict inequality because c, a are distinct)

Now, multiply these three inequalities together:

$$(a+b)(b+c)(c+a) > (2\sqrt{ab}) \cdot (2\sqrt{bc}) \cdot (2\sqrt{ca})$$

Combine the terms on the right-hand side:

$$(a+b)(b+c)(c+a) > 2 \cdot 2 \cdot 2 \cdot \sqrt{a \cdot b \cdot b \cdot c \cdot c \cdot a}$$

$$(a+b)(b+c)(c+a) > 8\sqrt{a^2b^2c^2}$$

Since $a, b, c > 0$, $\sqrt{a^2b^2c^2} = abc$.

Thus, $(a+b)(b+c)(c+a) > 8abc$.

Step 4: Final Answer:

The given expression is strictly greater than $8abc$.

Quick Tip: For products of sums involving positive variables, the AM-GM inequality is usually the most efficient tool.

Remember that if variables are distinct, the inequality sign is strict ($>$).

3. If $A = 1 + r^a + r^{2a} + r^{3a} + \dots \infty$ and $B = 1 + r^b + r^{2b} + r^{3b} + \dots \infty$, then a/b is equal to

- (A) $\log_B(A)$
- (B) $\log_{1-B}(1-A)$
- (C) $\log_{\frac{B-1}{B}}\left(\frac{A-1}{A}\right)$
- (D) None of these

Correct Answer: (C) $\log_{\frac{B-1}{B}}\left(\frac{A-1}{A}\right)$

Solution:

Step 1: Understanding the Concept:

The values A and B represent the sums of infinite geometric series.

For a geometric series $a_1 + a_1R + a_1R^2 + \dots$ to converge, the common ratio $|R|$ must be less than 1.

Step 2: Key Formula or Approach:

The sum of an infinite GP is given by:

$$S_{\infty} = \frac{a_1}{1-R}$$

Where a_1 is the first term and R is the common ratio.

Step 3: Detailed Explanation:

For series A : first term $a_1 = 1$, common ratio $R = r^a$.

$$A = \frac{1}{1-r^a} \Rightarrow 1-r^a = \frac{1}{A} \Rightarrow r^a = 1 - \frac{1}{A} = \frac{A-1}{A}$$

Taking natural log: $a \ln(r) = \ln\left(\frac{A-1}{A}\right) \dots (i)$

For series B : first term $a_1 = 1$, common ratio $R = r^b$.

$$B = \frac{1}{1-r^b} \Rightarrow 1-r^b = \frac{1}{B} \Rightarrow r^b = 1 - \frac{1}{B} = \frac{B-1}{B}$$

Taking natural log: $b \ln(r) = \ln\left(\frac{B-1}{B}\right) \dots (ii)$

Divide equation (i) by equation (ii):

$$\frac{a \ln(r)}{b \ln(r)} = \frac{\ln\left(\frac{A-1}{A}\right)}{\ln\left(\frac{B-1}{B}\right)}$$

Using the change of base formula for logarithms $\frac{\ln x}{\ln y} = \log_y(x)$:

$$\frac{a}{b} = \log_{\frac{B-1}{B}}\left(\frac{A-1}{A}\right)$$

Step 4: Final Answer:

The ratio a/b is equal to $\log_{\frac{B-1}{B}}\left(\frac{A-1}{A}\right)$.

Quick Tip: To solve problems with variables in exponents, isolate the exponential base term first, then use the logarithmic property: $\frac{\ln x}{\ln y} = \log_y x$.

4. If the 17^{th} and the 18^{th} terms in the expansion of $(2+a)^{50}$ are equal, then the coefficient of x^{35} in the expansion of $(a+x)^{-2}$ is

(A) -35

(B) 3

(C) 36

(D) -36

Correct Answer: (D) -36

Solution:

Step 1: Understanding the Concept:

The general term in the expansion of $(x + y)^n$ is given by $T_{r+1} = \binom{n}{r} x^{n-r} y^r$.

After finding the value of a , we use the general term formula for negative integer exponents: the coefficient of x^k in $(1 + x)^{-n}$ is $(-1)^k \binom{n+k-1}{k}$.

Step 2: Key Formula or Approach:

For $(2 + a)^{50}$:

$$T_{17} = \binom{50}{16} 2^{34} a^{16} \text{ and } T_{18} = \binom{50}{17} 2^{33} a^{17}.$$

Set them equal: $T_{17} = T_{18}$.

Also, for $(1 + x)^{-2}$, the coefficient of x^r is $(-1)^r (r + 1)$.

Step 3: Detailed Explanation:

Equating terms:

$$\binom{50}{16} 2^{34} a^{16} = \binom{50}{17} 2^{33} a^{17}$$

Divide both sides by $2^{33} a^{16}$:

$$\binom{50}{16} \cdot 2 = \binom{50}{17} \cdot a$$

$$a = 2 \cdot \frac{\binom{50}{16}}{\binom{50}{17}}$$

Using the property $\frac{\binom{n}{r}}{\binom{n}{r-1}} = \frac{n-r+1}{r}$, we have $\frac{\binom{50}{16}}{\binom{50}{17}} = \frac{17}{50-17+1} = \frac{17}{34} = \frac{1}{2}$.

Thus, $a = 2 \cdot \frac{1}{2} = 1$.

Now, we need the coefficient of x^{35} in $(1 + x)^{-2}$.

Expansion: $(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \dots + (-1)^r(r+1)x^r + \dots$

For $r = 35$, the coefficient is $(-1)^{35}(35+1) = -1 \cdot 36 = -36$.

Step 4: Final Answer:

The coefficient is -36.

Quick Tip: For $(1+x)^{-2}$, the coefficient of x^r is simply $(-1)^r(r+1)$. This special case is worth memorizing to save time in binomial problems.

5. Number of solutions of equations $\sin 9\theta = \sin \theta$ in the interval $[0, 2\pi]$ is

- (A) 16
- (B) 17
- (C) 18
- (D) 15

Correct Answer: (B) 17

Solution:

Step 1: Understanding the Concept:

To find the number of solutions for $\sin A = \sin B$, we use the general solution form.

The interval $[0, 2\pi]$ represents one full circle.

Step 2: Key Formula or Approach:

The equation $\sin 9\theta = \sin \theta$ can be solved using:

$$9\theta = n\pi + (-1)^n\theta.$$

Equivalently, it splits into:

$$1) 9\theta = 2k\pi + \theta \implies 8\theta = 2k\pi \implies \theta = \frac{k\pi}{4}$$

$$2) 9\theta = (2k+1)\pi - \theta \implies 10\theta = (2k+1)\pi \implies \theta = \frac{(2k+1)\pi}{10}$$

Step 3: Detailed Explanation:

Set 1: $\theta = \frac{k\pi}{4}$ in $[0, 2\pi]$.

Values of k are 0, 1, 2, 3, 4, 5, 6, 7, 8.

Solutions: $\{0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}, 2\pi\}$ (9 solutions).

Set 2: $\theta = \frac{(2k+1)\pi}{10}$ in $[0, 2\pi]$.

Values of k are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

Solutions: $\{\frac{\pi}{10}, \frac{3\pi}{10}, \frac{\pi}{2}, \frac{7\pi}{10}, \frac{9\pi}{10}, \frac{11\pi}{10}, \frac{13\pi}{10}, \frac{3\pi}{2}, \frac{17\pi}{10}, \frac{19\pi}{10}\}$ (10 solutions).

Checking for intersection: $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ are present in both sets.

Total unique solutions = $9 + 10 - 2 = 17$.

Step 4: Final Answer:

There are 17 unique solutions in the given interval.

Quick Tip: When counting solutions for $\sin p\theta = \sin q\theta$, ensure you subtract the common solutions from the two sets of general results.