



General Instructions

- (i) **Duration:** The total duration of the examination is 3 hours (180 minutes).
- (ii) **Total Marks:** The complete paper carries a maximum of 200 marks.
- (iii) **Structure:** The paper has 4 Sections:
 - **Part 1:** 25 Multiple Choice Questions (Physics).
 - **Part 2:** 25 Multiple Choice Questions (Chemistry).
 - **Part 3:** 50 Multiple Choice Questions (Mathematics).
 - **Part 4:** 100 Multiple Choice Questions (Domain Specific Questions).
- (iv) **Compulsory Questions:** All 200 questions are compulsory.
- (v) Each question has four options. Only **one** option is correct.
- (vi) **Correct Answer:** +1 marks.
- (vii) **Incorrect Answer:** (No Negative marking).
- (viii) **Unanswered/Marked for Review:** 0 marks.

MATHEMATICS

1. In the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 4 & 9 \end{bmatrix}$, the minor M_{23} of the element a_{23} is

- (A) 10
- (B) -10
- (C) -6
- (D) 6

Correct Answer: (B) -10

Solution:

Step 1: Understanding the Concept:

In linear algebra, the minor of a matrix element is a fundamental concept used to calculate determinants, cofactors, and inverses. For a square matrix A , the minor M_{ij} associated with the element a_{ij} is defined as the determinant of the submatrix that remains after removing the i^{th} row and the j^{th} column. This reduction process allows us to break down high-order matrices into simpler, lower-order ones. In a 3×3 matrix, removing one row and one column leaves a 2×2 matrix. The determinant of this resulting 2×2 matrix is the value of the minor. It is important to distinguish between a minor and a cofactor; while a minor is strictly the determinant of the submatrix, a cofactor also incorporates a sign factor based on the position of the element (given by $(-1)^{i+j}$). In this specific question, we are focused on finding the minor M_{23} , which means we need to target the element located in the second row and the third column, then perform the necessary row-column elimination and determinant calculation. Understanding the structure of matrices and the systematic approach to submatrices is essential for mastering advanced topics like Cramer's Rule or the Adjoint method.

Step 2: Key Formula or Approach:

The formula for the minor M_{ij} is:

$$M_{ij} = \det(\text{Submatrix obtained by deleting row } i \text{ and column } j)$$

For a 3×3 matrix $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, the minor M_{23} is calculated by removing the second row and third column. The remaining elements form the 2×2 matrix:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{bmatrix}$$

The determinant of this 2×2 matrix is computed as $(a_{11} \cdot a_{32}) - (a_{12} \cdot a_{31})$.

Step 3: Detailed Explanation:

Let the given matrix be A:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 4 & 9 \end{bmatrix}$$

First, we identify the element a_{23} . Row 2 consists of the elements $[4, 5, 6]$ and Column 3 consists of $[3, 6, 9]$. The intersection is 6.

To find M_{23} , we delete the second row:

Row 2: $[4, 5, 6]$ is removed.

Then, we delete the third column:

Column 3: $[3, 6, 9]$ is removed.

The surviving elements are those that are not in Row 2 or Column 3:

From Row 1, we have 1 and 2.

From Row 3, we have 7 and 4.

The resulting submatrix is:

$$B = \begin{bmatrix} 1 & 2 \\ 7 & 4 \end{bmatrix}$$

Now, calculate the determinant of matrix B:

$$\det(B) = (1 \times 4) - (2 \times 7)$$

$$\det(B) = 4 - 14$$

$$\det(B) = -10$$

Thus, the minor M_{23} is -10 . It represents the local scaling factor associated with that position in the matrix. If we were to find the cofactor C_{23} , we would apply the sign $(-1)^{2+3} = -1$, giving $C_{23} = 10$. However, the question strictly asks for the minor. This distinction is vital in examination settings where both options might be present. Accuracy in arithmetic at this stage prevents cascading errors in more complex matrix operations.

Step 4: Final Answer:

The minor M_{23} of the element a_{23} is -10 .

Comparing this result with the given options, we find it matches Option (B).

Quick Tip: To avoid confusion between minors and cofactors, remember: **Minor** is **Magnitude** only (the determinant), while **Cofactor** includes the **Change** of sign (if applicable). Always verify the row and column indices carefully before deleting. A simple sketch of the matrix can help ensure you don't miss any elements during the submatrix formation.

2. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then the value of x is

- (A) 3
- (B) ± 6
- (C) -3
- (D) ± 2

Correct Answer: (B) ± 6

Solution:

Step 1: Understanding the Concept:

This problem requires the evaluation of determinants for two 2×2 matrices and solving the resulting algebraic equation. A determinant is a scalar value that can be computed from the elements of a square matrix. It provides significant information about the matrix, such as whether it is invertible or singular. For a 2×2 matrix, the determinant represents the signed area of the parallelogram formed by the row vectors. Unlike matrix equality, where corresponding elements must be equal, determinant equality means the calculated scalar values of the two determinants are the same. This leads to an equation where the variable x is usually part of a quadratic expression. Solving for x involves standard algebraic manipulation, including expanding the products, combining terms, and taking square roots. It is a fundamental skill in linear algebra that bridge the gap between matrix operations and basic algebra.

Step 2: Key Formula or Approach:

The determinant of a 2×2 matrix $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ is defined as:

$$\Delta = (a \cdot d) - (b \cdot c)$$

We will apply this formula to both sides of the given equation to form an equation in x .

Step 3: Detailed Explanation:

The given determinant equation is:

$$\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$$

Evaluate the Left-Hand Side (LHS):

Apply the formula $ad - bc$:

$$\text{LHS} = (2x \cdot x) - (5 \cdot 8)$$

$$\text{LHS} = 2x^2 - 40$$

Evaluate the Right-Hand Side (RHS):

Apply the same formula:

$$\text{RHS} = (6 \cdot 3) - (-2 \cdot 7)$$

Pay close attention to the signs here. Subtracting a negative product results in addition.

$$\text{RHS} = 18 - (-14)$$

$$\text{RHS} = 18 + 14$$

$$\text{RHS} = 32$$

Now, equate LHS and RHS as per the problem statement:

$$2x^2 - 40 = 32$$

Add 40 to both sides to isolate the term with x :

$$2x^2 = 32 + 40$$

$$2x^2 = 72$$

Divide by 2:

$$x^2 = \frac{72}{2}$$

$$x^2 = 36$$

Take the square root of both sides to solve for x :

$$x = \pm\sqrt{36}$$

$$x = \pm 6$$

This means x can be either 6 or -6 . Both values will result in the determinant on the left being equal to 32. Verification: If $x = 6$, $\text{LHS} = 2(36) - 40 = 72 - 40 = 32$. If $x = -6$, $\text{LHS} = 2(36) - 40 = 32$. Thus, the solutions are correct. In many exams, forgetting the $\pm$ sign is a common pitfall.

Step 4: Final Answer:

The value of x is ± 6 .

This matches Option (B).

Quick Tip: Always be careful with the "minus" sign in the determinant formula $ad - bc$, especially when b or c are already negative. A single sign error can change the entire equation. Also, remember that $x^2 = a^2$ leads to $x = \pm a$. Never discard the negative root unless the context specifically restricts x to positive values.

3. If A is a square matrix of order 3 and $|A| = 5$, then the value of $|2A^T|$ is

- (A) -10
- (B) 10
- (C) 40

(D) -40

Correct Answer: (C) 40

Solution:

Step 1: Understanding the Concept:

This problem explores the properties of determinants under scalar multiplication and transposition. Determinants possess unique properties that allow us to calculate the value of complex expressions without knowing the individual elements of the matrix. There are two critical theorems involved here. First, the determinant of a matrix is invariant under transposition, meaning the determinant of A is the same as the determinant of its transpose A^T . This is because transposition just swaps rows and columns, and the determinant calculation accounts for all permutations regardless of row/column orientation. Second, when a square matrix of order n is multiplied by a scalar k , every row in the matrix is scaled by that constant. Since the determinant is a multi-linear function of its rows, the constant k is factored out from each of the n rows, resulting in the determinant being scaled by k^n . This "power rule" for scalars is a very common source of errors in linear algebra assessments. We must combine these two properties to find the value requested.

Step 2: Key Formula or Approach:

The relevant properties for a square matrix A of order n are:

1. $|A^T| = |A|$ (Property of Transpose)
2. $|k \cdot A| = k^n \cdot |A|$ (Scalar Multiplication Property)

Where $|A|$ denotes the determinant and n is the dimension (order) of the matrix.

Step 3: Detailed Explanation:

We are given:

Order of matrix A , $n = 3$

Determinant of A , $|A| = 5$

We need to find the value of $|2A^T|$.

First, apply the scalar multiplication property. Here, the scalar $k = 2$ and the matrix we are considering is A^T . Since A is of order 3, A^T is also of order 3. Therefore:

$$|2A^T| = 2^3 \cdot |A^T|$$

Calculate the numerical value of the constant:

$$2^3 = 2 \times 2 \times 2 = 8$$

So, the expression becomes:

$$|2A^T| = 8 \cdot |A^T|$$

Next, apply the transpose property. We know that the determinant of a transpose is equal to the determinant of the original matrix:

$$|A^T| = |A|$$

Substitute the given value $|A| = 5$ into the equation:

$$|A^T| = 5$$

Finally, substitute this value back into our simplified expression for $|2A^T|$:

$$|2A^T| = 8 \times 5$$

$$|2A^T| = 40$$

This systematic application of rules leads directly to the solution. Note how the order 3 turned the scalar 2 into 8. If the order were 2, the multiplier would have been $2^2 = 4$. If the order were 1 (a scalar), it would have been just 2. Understanding the relationship between dimensionality and determinants is crucial.

Step 4: Final Answer:

The value of $|2A^T|$ is 40.

This matches Option (C).

Quick Tip: Memorize the property $|kA| = k^n|A|$ as it appears frequently. A common mistake is to simply calculate $2 \times 5 = 10$, which ignores the dimensionality of the matrix. Always identify the "order" of the matrix first as it dictates the exponent. Transpose does not change the determinant value, so you can essentially treat $|A^T|$ as $|A|$.

4. Which of the following systems has a non-trivial solution?

- (A) $AX = 0, |A| = 4$
- (B) $AX = 0, |A| = -4$
- (C) $AX = 0, |A| = 0$
- (D) $AX = B, |B| = 5$

Correct Answer: (C) $AX = 0, |A| = 0$

Solution:

Step 1: Understanding the Concept:

A system of linear equations is represented as $AX = B$. If $B = 0$, the system is called "homogeneous." Homogeneous systems always have at least one solution: the zero vector $X = 0$. This is called the "trivial solution." A "non-trivial solution" is any solution where $X \neq 0$. The existence of non-trivial solutions depends on the nature of the matrix A . If A is a non-singular (invertible) matrix, we can multiply both sides by A^{-1} to get $X = A^{-1} \cdot 0 = 0$, meaning only the trivial solution exists. For non-trivial solutions to exist, the matrix A must be singular, meaning it has no inverse. This occurs when the determinant of A , denoted as $|A|$, is equal to zero. In such a case, the system has infinitely many solutions, including non-zero ones. This concept is core to finding eigenvalues and understanding the null space (kernel) of a transformation. If $|A| \neq 0$, the system is "well-behaved" with a unique solution at the origin. If $|A| = 0$, the mapping collapses dimensions, and entire lines or planes of points can map to

the origin.

Step 2: Key Formula or Approach:

For a homogeneous square system $AX = 0$:

1. If $|A| \neq 0$, then the system has **only** the trivial solution ($X = 0$).
2. If $|A| = 0$, then the system has **non-trivial** solutions (infinitely many solutions).

A non-homogeneous system ($AX = B$) doesn't use the term "trivial/non-trivial" in the same way, as the zero vector usually isn't a solution.

Step 3: Detailed Explanation:

Let's evaluate each option provided in the problem:

Option (A): $AX = 0, |A| = 4$. Here, the system is homogeneous, but $|A| \neq 0$. The matrix is invertible, so the only solution is $X = A^{-1} \cdot 0 = 0$. This has only a trivial solution.

Option (B): $AX = 0, |A| = -4$. Similar to Option A, the determinant is non-zero. The transformation is bijective, and only the origin is mapped to the origin. This has only a trivial solution.

Option (C): $AX = 0, |A| = 0$. This is a homogeneous system with a singular matrix. Since the determinant is zero, the rank of the matrix is less than the number of variables. This implies there is at least one free variable, leading to infinitely many solutions, which includes non-zero (non-trivial) vectors. This is the condition for non-trivial solutions.

Option (D): $AX = B, |B| = 5$. This is a non-homogeneous system. The term "non-trivial" is specifically contrasted with the zero solution that always exists for homogeneous systems. For non-homogeneous systems, we speak of unique solutions, no solutions, or infinite solutions. Therefore, only condition (C) ensures the existence of a non-trivial solution for a square matrix system.

Step 4: Final Answer:

The system that has a non-trivial solution is $AX = 0$ with $|A| = 0$.

This matches Option (C).

Quick Tip: Mnemonic: "Zero determinant, non-zero solutions." If a homogeneous system ($AX = 0$) is 'flawed' (singular, $|A| = 0$), it allows non-zero inputs to produce a zero output. Always check if the system is homogeneous first. If it is, just look for the case where the determinant is zero.

5. If $\begin{bmatrix} x+y & 2 \\ 1 & x-y \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 1 & 2 \end{bmatrix}$, then the values of x and y are:

- (A) $x = 3, y = 1$
- (B) $x = 1, y = 3$
- (C) $x = 2, y = 3$
- (D) $x = 1, y = 1$

Correct Answer: (A) $x = 3, y = 1$

Solution:

Step 1: Understanding the Concept:

Two matrices are considered equal if and only if they meet two criteria: they must have the same dimensions (order), and all their corresponding elements must be identical. In this problem, we are given two 2×2 matrices that are equated. This implies that the entry in row i and column j of the first matrix is exactly equal to the entry in row i and column j of the second matrix. By comparing the elements at each position, we can derive a set of algebraic equations. Specifically, the elements at positions (1,2) and (2,1) are already equal ($2 = 2$ and $1 = 1$), which confirms the consistency of the matrix equality. The unknown variables x and y are located in the entries at positions (1,1) and (2,2). Solving for them requires solving a simple system of simultaneous linear equations. Matrix equality is the most basic operation used to translate geometric or structural data into standard algebraic forms.

Step 2: Key Formula or Approach:

For two matrices $A = B$, we set corresponding elements equal:

$$a_{11} = b_{11}$$

$$a_{22} = b_{22}$$

From the given matrices, we get:

1. $x + y = 4$

2. $x - y = 2$

Step 3: Detailed Explanation:

We have a system of two linear equations:

Eq (1): $x + y = 4$

Eq (2): $x - y = 2$

We can solve this system using the method of elimination. Add Eq (1) and Eq (2) together:

$$(x + y) + (x - y) = 4 + 2$$

$$2x = 6$$

Divide by 2:

$$x = \frac{6}{2} = 3$$

Now, substitute the value of $x = 3$ into either of the original equations to find y . Let's use Eq (1):

$$3 + y = 4$$

Subtract 3 from both sides:

$$y = 4 - 3 = 1$$

Verification: Substitute both values into Eq (2) to ensure they are consistent.

$$x - y = 3 - 1 = 2$$

Since $2 = 2$, our values are correct. The solution pair is $(x, y) = (3, 1)$. This type of problem is straightforward but requires careful attention to signs when solving the simultaneous equations. Often, distractors in options will involve sign errors (like $x = 1, y = 3$).

Step 4: Final Answer:

The values are $x = 3$ and $y = 1$.

This matches Option (A).

Quick Tip: For matrix equality problems with multiple choice options, "Back-Substitution" is often faster. Simply pick an option and see if it satisfies the matrix. For Option A: $3 + 1 = 4$ and $3 - 1 = 2$. Since both match the right-side matrix, it is the correct answer immediately. This avoids the need for manual equation solving in a timed exam.

6. If $\frac{x+4}{(x+2)^2(x+3)} = \frac{A}{(x+2)^2} + \frac{B}{(x+2)} + \frac{C}{(x+3)}$, then $A + B + C =$

- (A) 2
- (B) 1
- (C) -1
- (D) 3

Correct Answer: (A) 2

Solution:

Step 1: Understanding the Concept:

The method of partial fraction decomposition is used to break down complex rational functions into a sum of simpler fractions. This is particularly useful in calculus for integration and in control systems for finding inverse Laplace transforms. In this problem, we have a denominator with a repeated linear factor $(x + 2)^2$ and a distinct linear factor $(x + 3)$. When a linear factor $(x - a)$ is repeated n times, the decomposition must include n terms with denominators $(x - a), (x - a)^2, \dots, (x - a)^n$. Here, we have terms for $(x + 2)^2$ and $(x + 2)$,

along with the term for $(x + 3)$. The task is to determine the constants A, B , and C that make the equation an identity. An identity means the equation holds true for every value of x . Once the constants are found, we calculate their sum. This procedure involves clearing denominators and either equating coefficients or substituting specific values of x .

Step 2: Key Formula or Approach:

Multiply both sides of the equation by the common denominator $(x + 2)^2(x + 3)$ to obtain the basic identity:

$$x + 4 = A(x + 3) + B(x + 2)(x + 3) + C(x + 2)^2$$

We will solve for A, B , and C by substituting the roots of the factors ($x = -2$ and $x = -3$) and comparing coefficients.

Step 3: Detailed Explanation:

Start with the identity:

$$x + 4 = A(x + 3) + B(x + 2)(x + 3) + C(x + 2)^2$$

To find A , let $x = -2$ (this eliminates B and C terms because they contain the factor $(x + 2)$):

$$-2 + 4 = A(-2 + 3) + B(0) + C(0)$$

$$2 = A(1) \implies A = 2$$

To find C , let $x = -3$ (this eliminates A and B terms because they contain the factor $(x + 3)$):

$$-3 + 4 = A(0) + B(0) + C(-3 + 2)^2$$

$$1 = C(-1)^2$$

$$1 = C \implies C = 1$$

To find B , we can compare the coefficients of x^2 on both sides.

The x^2 terms on the right side are:

From $B(x + 2)(x + 3)$: $B \cdot x^2$

From $C(x + 2)^2$: $C \cdot x^2$

Total x^2 on RHS = $(B + C)x^2$

Coefficient of x^2 on LHS = 0 (since there is no x^2 term in $x + 4$)

Equating coefficients:

$$B + C = 0$$

Substitute $C = 1$:

$$B + 1 = 0 \implies B = -1$$

Now, calculate the sum $A + B + C$:

$$A + B + C = 2 + (-1) + 1$$

$$A + B + C = 2 - 1 + 1$$

$$A + B + C = 2$$

Thus, the sum of the constants is 2. The substitution method is generally much faster than

equating all coefficients, especially for simple linear factors.

Step 4: Final Answer:

The sum $A + B + C$ is 2.

This matches Option (A).

Quick Tip: When performing partial fraction decomposition, always start by substituting the roots of the denominator factors. This will directly give you the values of the "outermost" constants (like A and C). For the remaining constants, comparing the coefficients of the highest power of x (like x^2 in this case) is often the simplest next step.

7. If $\frac{x}{(x-1)^2(x+2)} = \frac{A}{(x-1)^2} + \frac{2}{9(x-1)} + \frac{B}{(x+2)}$, then $A + B =$

- (A) $1/3$
- (B) $1/9$
- (C) $-1/3$
- (D) $2/3$

Correct Answer: (B) $1/9$

Solution:

Step 1: Understanding the Concept:

This problem is another exercise in partial fraction decomposition. Here, one of the three constants is already provided ($2/9$), which simplifies the task slightly. The structure involves a repeated linear factor $(x - 1)^2$ and a distinct factor $(x + 2)$. The provided constant corresponds to the $1/(x - 1)$ term. Our goal is to find the constants A (for the squared term) and B (for the distinct linear term). Partial fractions rely on the property that any proper rational function can be uniquely represented as a sum of these simpler fractions. By transforming the fraction equation into a polynomial identity, we can utilize the "method of convenient values" (substituting roots) to quickly isolate the unknowns. Calculating the sum of fractional

constants like A and B requires careful common denominator management.

Step 2: Key Formula or Approach:

Clear the denominators by multiplying both sides by $(x - 1)^2(x + 2)$:

$$x = A(x + 2) + \frac{2}{9}(x - 1)(x + 2) + B(x - 1)^2$$

We will substitute $x = 1$ and $x = -2$ to find A and B .

Step 3: Detailed Explanation:

Let the identity be:

$$x = A(x + 2) + \frac{2}{9}(x - 1)(x + 2) + B(x - 1)^2$$

Find A : Let $x = 1$.

$$1 = A(1 + 2) + \frac{2}{9}(0) + B(0)$$

$$1 = 3A \implies A = \frac{1}{3}$$

Find B : Let $x = -2$.

$$-2 = A(0) + \frac{2}{9}(0) + B(-2 - 1)^2$$

$$-2 = B(-3)^2$$

$$-2 = 9B \implies B = -\frac{2}{9}$$

Now, calculate the sum $A + B$:

$$A + B = \frac{1}{3} + \left(-\frac{2}{9}\right)$$

To add these fractions, find a common denominator, which is 9.

$$\frac{1}{3} = \frac{3}{9}$$

So, the sum is:

$$A + B = \frac{3}{9} - \frac{2}{9}$$

$$A + B = \frac{3-2}{9}$$

$$A + B = \frac{1}{9}$$

The result $1/9$ is the combined value of the two unknown coefficients. It is interesting to note that the sum of all three coefficients ($A + 2/9 + B$) would be $1/3 + 2/9 - 2/9 = 1/3$. However, the question specifically asks for $A + B$. Accuracy in basic fraction arithmetic is crucial here, as distractor options like $-1/3$ or $2/3$ often correspond to common sign or addition errors.

Step 4: Final Answer:

The value of $A + B$ is $1/9$.

This matches Option (B).

Quick Tip: Always convert all fractions to a common denominator before adding them to avoid mental math errors. For $A + B$, using the common denominator 9 made the calculation straightforward. Also, if you want to check your work, substitute $x = 0$ into the original equation and verify if the values of A and B balance the equation.

8. If $\tan A = \frac{1}{2}$ and $\tan B = \frac{1}{3}$, then $A + B =$

- (A) 30°
- (B) 45°
- (C) 60°
- (D) 90°

Correct Answer: (B) 45°

Solution:

Step 1: Understanding the Concept:

This problem involves the application of the tangent addition formula, a cornerstone of trigonometry. When we know the individual tangent values of two angles, we can find the tangent of their sum without needing to know the angles themselves. Trigonometric identities allow us to relate compound angles back to single-angle ratios. The specific values provided, $1/2$ and $1/3$, are very common in mathematics problems because they result in a very "clean" value for the sum. Once the tangent of the sum is found, we can use the inverse tangent function or our knowledge of standard trigonometric values for common angles (like $30^\circ, 45^\circ, 60^\circ$) to determine the value of $A + B$. This is a classic test of identity memorization and arithmetic with fractions.

Step 2: Key Formula or Approach:

The addition formula for the tangent function is:

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

We substitute the given values $\tan A = 1/2$ and $\tan B = 1/3$ into this formula.

Step 3: Detailed Explanation:

Given values:

$$\tan A = \frac{1}{2}$$

$$\tan B = \frac{1}{3}$$

Apply the addition formula:

$$\tan(A + B) = \frac{\frac{1}{2} + \frac{1}{3}}{1 - (\frac{1}{2} \cdot \frac{1}{3})}$$

Calculate the numerator:

$$\frac{1}{2} + \frac{1}{3} = \frac{3+2}{6} = \frac{5}{6}$$

Calculate the denominator:

$$1 - (\frac{1}{2} \cdot \frac{1}{3}) = 1 - \frac{1}{6}$$

$$1 - \frac{1}{6} = \frac{6-1}{6} = \frac{5}{6}$$

Now, substitute the results into the main fraction:

$$\tan(A + B) = \frac{5/6}{5/6}$$

Since the numerator and denominator are identical:

$$\tan(A + B) = 1$$

We know that for an angle θ in the first quadrant, if $\tan \theta = 1$, then $\theta = 45^\circ$ (or $\pi/4$ radians).

Therefore:

$$A + B = 45^\circ$$

The simplicity of this result is why this particular set of tangent values is so famous in high school and competitive mathematics. It demonstrates how complex-looking inputs can resolve into standard geometric results.

Step 4: Final Answer:

The value of $A + B$ is 45° .

This matches Option (B).

Quick Tip: Memorize the "magic pair" $\tan A = 1/2, \tan B = 1/3 \implies A + B = 45^\circ$. It appears frequently in problems. Similarly, if $\tan A = 1, \tan B = 2$, and $\tan C = 3$, then $A + B + C = 180^\circ$. Knowing these standard relationships can save valuable seconds during an exam.

9. If $2 \sin^{-1} x = \sin^{-1} k$, then $k =$

- (A) $2x\sqrt{1-x^2}$
- (B) $2x$
- (C) x^2
- (D) $1 - 2x^2$

Correct Answer: (A) $2x\sqrt{1-x^2}$

Solution:

Step 1: Understanding the Concept:

This problem deals with the relationship between inverse trigonometric functions and double-angle formulas. An inverse sine function, $\sin^{-1} x$, represents an angle θ whose sine is x . The equation $2 \sin^{-1} x = \sin^{-1} k$ asks us to find the sine of twice that angle. By letting $\theta = \sin^{-1} x$, we can transform the problem from the "inverse world" into the standard

"trigonometric world." We then apply the double-angle formula for sine, which relates $\sin(2\theta)$ to $\sin \theta$ and $\cos \theta$. Since we know $\sin \theta$, we can find $\cos \theta$ using the fundamental Pythagorean identity. This process of substitution and identity application is the standard way to simplify inverse trigonometric expressions or to solve equations involving them. It highlights the symmetry between angles and their ratios.

Step 2: Key Formula or Approach:

Let $\theta = \sin^{-1} x$, which implies $\sin \theta = x$.

The given equation is $2\theta = \sin^{-1} k$.

Taking the sine of both sides, we get:

$$k = \sin(2\theta)$$

We use the double-angle formula:

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

Step 3: Detailed Explanation:

Starting with $\sin \theta = x$, we need to find $\cos \theta$ in terms of x .

From the Pythagorean identity:

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$x^2 + \cos^2 \theta = 1$$

$$\cos^2 \theta = 1 - x^2$$

$$\cos \theta = \sqrt{1 - x^2}$$

Now, substitute $\sin \theta = x$ and $\cos \theta = \sqrt{1 - x^2}$ into the double-angle formula for k :

$$k = 2 \sin \theta \cos \theta$$

$$k = 2(x)(\sqrt{1 - x^2})$$

$$k = 2x\sqrt{1 - x^2}$$

This is the required expression for k . This result is valid for values of x where $|x| \leq 1/\sqrt{2}$, which ensures the result of 2θ stays within the principal range of the inverse sine function $[-\pi/2, \pi/2]$. In most multiple-choice questions, the general algebraic form is the intended answer. Note that Option (D), $1 - 2x^2$, is the result for $\cos(2\theta)$, which would appear if the equation involved $\cos^{-1}$. Understanding which identity corresponds to which function is key to choosing the correct option.

Step 4: Final Answer:

The value of k is $2x\sqrt{1 - x^2}$.

This matches Option (A).

Quick Tip: Always remember the "Inverse-Standard" conversion: if you see $\sin^{-1} x$, immediately think of a right-angled triangle with opposite side x , hypotenuse 1, and adjacent side $\sqrt{1 - x^2}$. This makes it very easy to find any other trigonometric ratio (like $\cos \theta$) without much effort.

10. If $\sin^{-1} \frac{5}{x} + \sin^{-1} \frac{12}{x} = \frac{\pi}{2}$, then $x =$

- (A) 12
- (B) 7
- (C) 13
- (D) 15

Correct Answer: (C) 13

Solution:

Step 1: Understanding the Concept:

This problem involves the sum of two inverse sine functions being equal to $\pi/2$. There is a very important identity in trigonometry: $\sin^{-1} \alpha + \cos^{-1} \alpha = \pi/2$. By comparing this identity to the given equation, we can see that if $\sin^{-1}(5/x) + \sin^{-1}(12/x) = \pi/2$, then it must be true that $\sin^{-1}(12/x)$ is equivalent to $\cos^{-1}(5/x)$. Geometrically, this means that $5/x$ and $12/x$ are the sine and cosine of the same angle θ . Since $\sin \theta = 12/x$ and $\cos \theta = 5/x$, these two ratios define the sides of a right-angled triangle where the hypotenuse is x . This problem is essentially a disguised version of the Pythagorean theorem. Recognizing these relationships allows us to bypass complex inverse trigonometric manipulations and solve for x using basic geometry.

Step 2: Key Formula or Approach:

Given: $\sin^{-1} \frac{5}{x} + \sin^{-1} \frac{12}{x} = \frac{\pi}{2}$

We use the identity: $\sin^{-1} \theta + \cos^{-1} \theta = \frac{\pi}{2}$

Equating the two, we get: $\sin^{-1} \frac{12}{x} = \cos^{-1} \frac{5}{x}$

This implies that in a right triangle with angle θ :

Opposite side = 12

Adjacent side = 5

Hypotenuse = x

Apply the Pythagorean Theorem: $a^2 + b^2 = c^2$

Step 3: Detailed Explanation:

From the previous step, we established that:

$$\sin \theta = \frac{12}{x} \text{ and } \cos \theta = \frac{5}{x}$$

Using the trigonometric identity $\sin^2 \theta + \cos^2 \theta = 1$:

$$\left(\frac{12}{x}\right)^2 + \left(\frac{5}{x}\right)^2 = 1$$

Expand the squares:

$$\frac{144}{x^2} + \frac{25}{x^2} = 1$$

Since they have a common denominator:

$$\frac{144 + 25}{x^2} = 1$$

$$\frac{169}{x^2} = 1$$

Multiply both sides by x^2 :

$$x^2 = 169$$

Take the square root:

$$x = \pm 13$$

Since x represents a side length/ratio argument in this context and must be positive for the inverse sines to be positive (summing to positive $\pi/2$), we choose:

$$x = 13$$

This is the famous (5, 12, 13) Pythagorean triple. If you recognize this triple, you can solve the problem in seconds. In any right triangle, the two acute angles sum to 90° ($\pi/2$). If one

angle is $\sin^{-1}(5/13)$, the other must be $\cos^{-1}(5/13)$, which is also $\sin^{-1}(12/13)$.

Step 4: Final Answer:

The value of x is 13.

This matches Option (C).

Quick Tip: Whenever you see $\sin^{-1}(a/x) + \sin^{-1}(b/x) = \pi/2$, simply solve for $x = \sqrt{a^2 + b^2}$. It is a standard shortcut. Common Pythagorean triples like (3, 4, 5), (5, 12, 13), and (8, 15, 17) should be memorized to handle such questions instantly.

11. The number of solutions of the equation $\sin 2x - \cos 2x = 2 - \sin 2x$ lying in the interval $[0, \pi]$ is

- (A) 0
- (B) 1
- (C) 2
- (D) 3

Correct Answer: (C) 2

Solution:

Step 1: Understanding the Concept:

This problem requires solving a trigonometric equation and determining the count of its roots within a specific closed interval $[0, \pi]$. The equation involves double-angle terms like $\sin 2x$ and $\cos 2x$. Trigonometric equations of the form $a \sin \theta + b \cos \theta = c$ are solved by converting the expression into a single trigonometric function (sine or cosine) using auxiliary angle identities. The number of solutions depends on whether the constant term falls within the range of the resulting function. Since the period of $\sin 2x$ and $\cos 2x$ is π , we expect a periodic pattern of solutions. Analyzing the extreme values and the nature of the function is crucial for identifying all possible intersections with the constant value 2.

Step 2: Key Formula or Approach:

The given equation is $\sin 2x - \cos 2x = 2 - \sin 2x$.

First, we rearrange the terms to simplify the expression into a standard linear form:

$$2 \sin 2x - \cos 2x = 2$$

To solve $a \sin \theta + b \cos \theta = c$, we divide by $\sqrt{a^2 + b^2}$. In our case, $a = 2$ and $b = -1$.

The magnitude is $\sqrt{2^2 + (-1)^2} = \sqrt{4 + 1} = \sqrt{5}$.

Alternatively, we can use the half-angle substitution: $\sin 2x = \frac{2 \tan x}{1 + \tan^2 x}$ and $\cos 2x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$.

Step 3: Detailed Explanation:

Let's use the substitution method where $t = \tan x$. This substitution is valid for $x \in [0, \pi]$ except at $x = \pi/2$.

Substituting into the rearranged equation $2 \sin 2x - \cos 2x = 2$:

$$2 \left(\frac{2t}{1 + t^2} \right) - \left(\frac{1 - t^2}{1 + t^2} \right) = 2$$

Combine the fractions:

$$\frac{4t - 1 + t^2}{1 + t^2} = 2$$

Multiply both sides by $1 + t^2$:

$$4t - 1 + t^2 = 2(1 + t^2)$$

$$4t - 1 + t^2 = 2 + 2t^2$$

Rearrange all terms to one side to form a quadratic equation:

$$t^2 - 4t + 3 = 0$$

Factor the quadratic:

$$(t - 1)(t - 3) = 0$$

This gives two possible values for t :

1. $t = 1 \implies \tan x = 1$. For $x \in [0, \pi]$, this occurs at $x = \frac{\pi}{4}$.
2. $t = 3 \implies \tan x = 3$. For $x \in [0, \pi]$, this occurs at $x = \tan^{-1}(3)$. Since 3 is positive, this value lies in the first quadrant, i.e., $0 < \tan^{-1}(3) < \frac{\pi}{2}$.

Both values $\frac{\pi}{4}$ and $\tan^{-1}(3)$ fall within the specified interval $[0, \pi]$.

Checking for $x = \pi/2$: If $x = \pi/2$, then $2x = \pi$.

LHS: $2 \sin(\pi) - \cos(\pi) = 2(0) - (-1) = 1 \neq 2$. So $x = \pi/2$ is not a solution.

Thus, there are exactly 2 distinct solutions in the given interval.

Step 4: Final Answer:

The number of solutions in the interval $[0, \pi]$ is 2.

This corresponds to Option (C).

Quick Tip: For equations like $a \sin \theta + b \cos \theta = c$, check if $c^2 \leq a^2 + b^2$. If it is, solutions exist. In this problem, $2^2 \leq 2^2 + (-1)^2 \implies 4 \leq 5$, confirming solutions are present. Using $\tan x$ substitution is often the most reliable way to find all roots without missing any due to trigonometric period shifts.

12. If $\tan \theta + \sec \theta = \sqrt{3}$, then the principal value of θ in $[0, 2\pi]$ is

- (A) $\pi/4$
- (B) $\pi/6$
- (C) $\pi/2$
- (D) $2\pi/3$

Correct Answer: (B) $\pi/6$

Solution:

Step 1: Understanding the Concept:

The principal value of a trigonometric equation refers to the solution within a standard specified range, here $[0, 2\pi]$. The equation involves both $\tan \theta$ and $\sec \theta$. A fundamental identity linking these two functions is $\sec^2 \theta - \tan^2 \theta = 1$. This identity can be factored to create a system of equations that allows us to isolate each trigonometric function. Solving for $\sec \theta$ and $\tan \theta$ simultaneously ensures that the solution for θ is consistent with both ratios. It is important to check the signs of the resulting ratios to determine the correct quadrant for the angle.

Step 2: Key Formula or Approach:

We start with the given equation:

$$1. \sec \theta + \tan \theta = \sqrt{3}$$

We use the identity:

$$\sec^2 \theta - \tan^2 \theta = 1 \implies (\sec \theta + \tan \theta)(\sec \theta - \tan \theta) = 1$$

From this, we can find the value of $\sec \theta - \tan \theta$ and then solve the linear system for θ .

Step 3: Detailed Explanation:

Given $\sec \theta + \tan \theta = \sqrt{3}$, the identity implies:

$$\sec \theta - \tan \theta = \frac{1}{\sec \theta + \tan \theta} = \frac{1}{\sqrt{3}}$$

Now we have two equations:

$$\text{Eq (1): } \sec \theta + \tan \theta = \sqrt{3}$$

$$\text{Eq (2): } \sec \theta - \tan \theta = \frac{1}{\sqrt{3}}$$

Adding the two equations:

$$2 \sec \theta = \sqrt{3} + \frac{1}{\sqrt{3}} = \frac{3+1}{\sqrt{3}} = \frac{4}{\sqrt{3}}$$

$$\sec \theta = \frac{2}{\sqrt{3}} \implies \cos \theta = \frac{\sqrt{3}}{2}$$

Subtracting Eq (2) from Eq (1):

$$2 \tan \theta = \sqrt{3} - \frac{1}{\sqrt{3}} = \frac{3-1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$$\tan \theta = \frac{1}{\sqrt{3}}$$

We need to find $\theta \in [0, 2\pi]$ such that $\cos \theta = \frac{\sqrt{3}}{2}$ and $\tan \theta = \frac{1}{\sqrt{3}}$.

Since both $\cos \theta$ and $\tan \theta$ are positive, θ must lie in the First Quadrant.

In the First Quadrant:

$$\cos \theta = \frac{\sqrt{3}}{2} \implies \theta = 30^\circ = \frac{\pi}{6}$$

$$\tan \theta = \frac{1}{\sqrt{3}} \implies \theta = 30^\circ = \frac{\pi}{6}$$

The value is consistent across both functions. In other quadrants, like the fourth quadrant, $\cos \theta$ is positive but $\tan \theta$ is negative, so those solutions are discarded.

Step 4: Final Answer:

The principal value is $\theta = \pi/6$.

This matches Option (B).

Quick Tip: Whenever you see $\sec \theta + \tan \theta$, immediately write down $\sec \theta - \tan \theta$ as its reciprocal. This is a very common pattern in competitive exams. Adding and subtracting these expressions is the fastest way to find $\sin$, $\cos$, or $\tan$ of the angle.

13. $\frac{\tan x - 1 + \sec x}{\tan x - \sec x + 1} =$

- (A) $\frac{1 - \sin x}{\cos x}$
 (B) $\frac{1 + \sin x}{\cos x}$
 (C) $\frac{1 + \cos x}{\sin x}$
 (D) $\frac{1 - \cos x}{\sin x}$

Correct Answer: (B) $\frac{1 + \sin x}{\cos x}$

Solution:

Step 1: Understanding the Concept:

This problem involves simplifying a rational trigonometric expression. Such expressions can often be simplified using fundamental identities. The identity $1 = \sec^2 x - \tan^2 x$ is particularly useful for terms containing 1, $\sec x$, and $\tan x$. By replacing the constant '1' in the numerator with this quadratic identity, we can factor the expression and cancel out common terms from the denominator. This transformation significantly reduces the complexity of the fraction, leading to a much simpler form in terms of basic sine and cosine functions.

Step 2: Key Formula or Approach:

The expression is $E = \frac{(\tan x + \sec x) - 1}{\tan x - \sec x + 1}$.

Substitute $1 = \sec^2 x - \tan^2 x$ in the numerator.

Recall the difference of squares: $\sec^2 x - \tan^2 x = (\sec x - \tan x)(\sec x + \tan x)$.

Step 3: Detailed Explanation:

Let's manipulate the numerator:

$$\text{Numerator} = (\tan x + \sec x) - (\sec^2 x - \tan^2 x)$$

Factor out $(\sec x + \tan x)$:

$$\text{Numerator} = (\sec x + \tan x)[1 - (\sec x - \tan x)]$$

$$\text{Numerator} = (\sec x + \tan x)(1 - \sec x + \tan x)$$

Now, substitute this factored numerator back into the original expression:

$$E = \frac{(\sec x + \tan x)(1 - \sec x + \tan x)}{\tan x - \sec x + 1}$$

Observe that $(1 - \sec x + \tan x)$ is identical to the denominator $(\tan x - \sec x + 1)$.

Assuming the denominator is not zero, we cancel the common term:

$$E = \sec x + \tan x$$

Convert to sine and cosine:

$$E = \frac{1}{\cos x} + \frac{\sin x}{\cos x} = \frac{1 + \sin x}{\cos x}$$

This gives the final simplified form. Verification: Let $x = 0$. Original expression: $\frac{0-1+1}{0-1+1}$? Indeterminate. Let $x = \pi/4$, $\tan x = 1$, $\sec x = \sqrt{2}$. Expression: $\frac{1-1+\sqrt{2}}{1-\sqrt{2}+1} = \frac{\sqrt{2}}{2-\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}(\sqrt{2}-1)} = \frac{1}{\sqrt{2}-1} = \sqrt{2} + 1$. Option B at $x = \pi/4$: $\frac{1+1/\sqrt{2}}{1/\sqrt{2}} = \sqrt{2} + 1$. Correct.

Step 4: Final Answer:

The simplified expression is $\frac{1+\sin x}{\cos x}$.

This matches Option (B).

Quick Tip: For identities of the form $\frac{\tan \theta + \sec \theta - 1}{\tan \theta - \sec \theta + 1}$, the result is always $\sec \theta + \tan \theta$. Similarly, for $\frac{\sin \theta + \cos \theta - 1}{\sin \theta - \cos \theta + 1}$, the result is $\frac{\sin \theta + 1}{\cos \theta}$. These are standard results that are worth memorizing as they recur frequently in different formats.

14. $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ =$

- (A) 2
- (B) 1
- (C) 4
- (D) 3

Correct Answer: (C) 4

Solution:

Step 1: Understanding the Concept:

This problem requires simplifying a sum of four tangent terms. The angles involved ($9^\circ, 27^\circ, 63^\circ, 81^\circ$) exhibit symmetry. Specifically, $9^\circ + 81^\circ = 90^\circ$ and $27^\circ + 63^\circ = 90^\circ$. This suggests using the complementary angle identity $\tan(90^\circ - \theta) = \cot \theta$. By pairing complementary terms, we can transform the expression into a sum of $\tan \theta + \cot \theta$. The term $\tan \theta + \cot \theta$ is particularly useful because it simplifies nicely into $\frac{2}{\sin 2\theta}$. The final evaluation relies on knowing specific values of sine for multiples of 18° .

Step 2: Key Formula or Approach:

Rearrange terms:

$$S = (\tan 9^\circ + \tan 81^\circ) - (\tan 27^\circ + \tan 63^\circ)$$

Use $\tan(90^\circ - \theta) = \cot \theta$:

$$S = (\tan 9^\circ + \cot 9^\circ) - (\tan 27^\circ + \cot 27^\circ)$$

Use identity: $\tan \alpha + \cot \alpha = \frac{\sin \alpha}{\cos \alpha} + \frac{\cos \alpha}{\sin \alpha} = \frac{1}{\sin \alpha \cos \alpha} = \frac{2}{\sin 2\alpha}$.

Step 3: Detailed Explanation:

Let's apply the identity to both pairs:

$$S = \frac{2}{\sin 18^\circ} - \frac{2}{\sin 54^\circ}$$

Factor out 2:

$$S = 2 \left(\frac{1}{\sin 18^\circ} - \frac{1}{\sin 54^\circ} \right) = 2 \left(\frac{\sin 54^\circ - \sin 18^\circ}{\sin 18^\circ \sin 54^\circ} \right)$$

Use the sum-to-product identity: $\sin C - \sin D = 2 \cos \left(\frac{C+D}{2} \right) \sin \left(\frac{C-D}{2} \right)$:

$$\sin 54^\circ - \sin 18^\circ = 2 \cos 36^\circ \sin 18^\circ$$

Also, note that $\sin 54^\circ = \cos 36^\circ$. Substituting these into the expression:

$$S = 2 \left(\frac{2 \cos 36^\circ \sin 18^\circ}{\sin 18^\circ \cos 36^\circ} \right)$$

Cancel the common terms $\sin 18^\circ$ and $\cos 36^\circ$ from the numerator and denominator:

$$S = 2 \times 2 = 4$$

Alternatively, we can use the specific values $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$ and $\cos 36^\circ = \frac{\sqrt{5}+1}{4}$.

$$S = 2 \left(\frac{4}{\sqrt{5}-1} - \frac{4}{\sqrt{5}+1} \right) = 8 \left(\frac{(\sqrt{5}+1) - (\sqrt{5}-1)}{(\sqrt{5}-1)(\sqrt{5}+1)} \right) = 8 \left(\frac{2}{5-1} \right) = 8 \times \frac{2}{4} = 4$$

.

Both methods confirm the result.

Step 4: Final Answer:

The value of the expression is 4.

This matches Option (C).

Quick Tip: Always look for complementary angles in trigonometric series. The pairing θ and $90 - \theta$ is the most common hint in competitive exams. Also, the values for $18^\circ, 36^\circ, 54^\circ, 72^\circ$ appear often, and recognizing $\frac{1}{\sin 18} - \frac{1}{\sin 54} = 2$ is a useful shortcut.

15. If $\cos \theta = \frac{1}{2}(a + \frac{1}{a})$, then $4\cos^3 \theta - 3\cos \theta =$

- (A) $a^3 + \frac{1}{a^3}$
- (B) $\frac{1}{2}(a^3 + \frac{1}{a^3})$
- (C) $\frac{1}{4}(a^3 + \frac{1}{a^3})$
- (D) $\frac{1}{3}(a^3 + \frac{1}{a^3})$

Correct Answer: (B) $\frac{1}{2}(a^3 + \frac{1}{a^3})$

Solution:

Step 1: Understanding the Concept:

This problem combines multiple-angle trigonometric identities with algebraic expressions. The expression $4\cos^3 \theta - 3\cos \theta$ is the standard expansion for $\cos 3\theta$. We are given an expression for $\cos \theta$ in terms of a variable a . This structure $\cos \theta = \frac{1}{2}(a + a^{-1})$ is highly characteristic of Euler's formula in complex numbers, where $\cos \theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$. By treating a as $e^{i\theta}$, the problem can be solved very efficiently using De Moivre's Theorem. Alternatively, one can use direct algebraic expansion of the cube of the binomial.

Step 2: Key Formula or Approach:

Identity: $\cos 3\theta = 4\cos^3 \theta - 3\cos \theta$.

Given: $\cos \theta = \frac{1}{2}(a + \frac{1}{a})$.

Approach: Substitute the value of $\cos \theta$ into the identity and simplify, or use the general identity $\cos n\theta = \frac{1}{2}(a^n + \frac{1}{a^n})$ which holds if $\cos \theta = \frac{1}{2}(a + \frac{1}{a})$.

Step 3: Detailed Explanation:

Let's perform the algebraic substitution:

We need to find $4\cos^3 \theta - 3\cos \theta$.

Substitute $\cos \theta = \frac{a+a^{-1}}{2}$:

$$\cos^3 \theta = \left[\frac{1}{2}(a + a^{-1}) \right]^3 = \frac{1}{8}(a^3 + 3a^2a^{-1} + 3aa^{-2} + a^{-3})$$

$$\cos^3 \theta = \frac{1}{8}(a^3 + 3a + 3a^{-1} + a^{-3})$$

Now, substitute this into the target expression:

$$4 \cos^3 \theta - 3 \cos \theta = 4 \left[\frac{1}{8}(a^3 + a^{-3} + 3(a + a^{-1})) \right] - 3 \left[\frac{1}{2}(a + a^{-1}) \right]$$

$$= \frac{1}{2}(a^3 + a^{-3} + 3(a + a^{-1})) - \frac{3}{2}(a + a^{-1})$$

Expand the terms:

$$= \frac{1}{2}(a^3 + a^{-3}) + \frac{3}{2}(a + a^{-1}) - \frac{3}{2}(a + a^{-1})$$

The terms involving $(a + a^{-1})$ cancel out perfectly:

$$= \frac{1}{2}(a^3 + a^{-3}) = \frac{1}{2}\left(a^3 + \frac{1}{a^3}\right)$$

This confirms the result through direct algebra. From complex numbers, let $a = e^{i\theta}$. Then $\cos \theta = \operatorname{Re}(a) = \frac{a+a^{-1}}{2}$. By De Moivre's Theorem, $a^n = e^{in\theta}$, so $\cos n\theta = \frac{a^n+a^{-n}}{2}$. Setting $n = 3$, we immediately get the answer.

Step 4: Final Answer:

The value is $\frac{1}{2}\left(a^3 + \frac{1}{a^3}\right)$.

This matches Option (B).

Quick Tip: General Result: If $\cos \theta = \frac{1}{2}(a + \frac{1}{a})$, then $\cos n\theta = \frac{1}{2}(a^n + \frac{1}{a^n})$ for any integer n . This is a very powerful identity based on De Moivre's theorem and is widely used in competitive exam problems to simplify complex trigonometric powers.

16. $\cos 6^\circ \sin 24^\circ \cos 72^\circ =$

- (A) $1/4$
- (B) $-1/8$
- (C) $-1/4$
- (D) $1/8$

Correct Answer: (D) $1/8$

Solution:

Step 1: Understanding the Concept:

This problem involves evaluating the product of three trigonometric functions with specific, non-standard angles. Solving such products typically involves using product-to-sum identities or recognizing patterns related to the identity $\sin \theta \sin(60 - \theta) \sin(60 + \theta) = \frac{1}{4} \sin 3\theta$. In some cases, we multiply and divide by a factor (like $\sin 6^\circ$) to create a chain of double-angle simplifications. These problems test the student's ability to manipulate angles through complementary and supplementary relationships to find a familiar pattern.

Step 2: Key Formula or Approach:

Product-to-Sum: $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$.

Double Angle: $\sin 2\theta = 2 \sin \theta \cos \theta$.

Specific Identity: $\sin A \sin(60 - A) \sin(60 + A) = \frac{1}{4} \sin 3A$.

Angles: $72^\circ = 90^\circ - 18^\circ$, so $\cos 72^\circ = \sin 18^\circ$.

Step 3: Detailed Explanation:

Let the expression be $P = \cos 6^\circ \sin 24^\circ \cos 72^\circ$.

Use $\cos 72^\circ = \sin 18^\circ$. Also, $\sin 24^\circ = \cos 66^\circ$.

$P = \cos 6^\circ \cos 66^\circ \sin 18^\circ$.

Multiply and divide by $2 \sin 6^\circ$:

$$P = \frac{2 \sin 6^\circ \cos 6^\circ \cos 66^\circ \sin 18^\circ}{2 \sin 6^\circ} = \frac{\sin 12^\circ \sin 18^\circ \cos 66^\circ}{2 \sin 6^\circ}$$

Alternatively, let's use the identity for $\cos \theta \cos(60 - \theta) \cos(60 + \theta) = \frac{1}{4} \cos 3\theta$.

Note that $\cos 66^\circ = \cos(60 + 6)^\circ$ and $\sin 24^\circ = \cos 66^\circ$.

Actually, let's look at the options and the standard problems of this type. Often the question is $\cos 6^\circ \cos 42^\circ \cos 66^\circ \cos 78^\circ = 1/16$.

In this specific case, based on the provided answer key (1/8):

$$P = \cos 6^\circ \sin 24^\circ \sin 18^\circ.$$

Using $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$:

$$2 \cos 6^\circ \sin 24^\circ = \sin 30^\circ + \sin 18^\circ = \frac{1}{2} + \sin 18^\circ.$$

$$P = \frac{1}{2} \left(\frac{1}{2} + \sin 18^\circ \right) \sin 18^\circ = \frac{1}{4} \sin 18^\circ + \frac{1}{2} \sin^2 18^\circ$$

Using $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$:

$$\sin^2 18^\circ = \frac{5 + 1 - 2\sqrt{5}}{16} = \frac{6 - 2\sqrt{5}}{16} = \frac{3 - \sqrt{5}}{8}$$

$$P = \frac{1}{4} \frac{\sqrt{5}-1}{4} + \frac{1}{2} \frac{3-\sqrt{5}}{8} = \frac{\sqrt{5}-1}{16} + \frac{3-\sqrt{5}}{16} = \frac{2}{16} = \frac{1}{8}$$

This confirms that the expression indeed evaluates to $1/8$.

Step 4: Final Answer:

The value is $1/8$.

This matches Option (D).

Quick Tip: For products of cosines with angles in arithmetic progression like $\theta, 60 - \theta, 60 + \theta$, the factor is always $1/4$. When solving more complex products, use the product-to-sum identity on two terms first to see if a standard angle or a cancellation occurs. Familiarity with $\sin 18^\circ$ and its squared value is a common requirement in state-level engineering exams.

17. $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 =$

- (A) $3\pi/4$
- (B) $\pi/2$
- (C) π
- (D) 2π

Correct Answer: (C) π

Solution:

Step 1: Understanding the Concept:

This problem involves the sum of inverse tangent functions. The standard formula for $\tan^{-1} x + \tan^{-1} y$ has a specific condition regarding the product xy . If $xy < 1$, the sum is $\tan^{-1} \left(\frac{x+y}{1-xy} \right)$. However, if $xy > 1$ and x, y are positive, we must add π to the result because the sum of the angles exceeds $\pi/2$, placing the result in the second quadrant. This is a crucial distinction in inverse trigonometry that is frequently tested in examinations. Here, we need to sum three terms: $\pi/4$ and the sum of $\tan^{-1} 2$ and $\tan^{-1} 3$.

Step 2: Key Formula or Approach:

Identity: $\tan^{-1} x + \tan^{-1} y = \pi + \tan^{-1} \left(\frac{x+y}{1-xy} \right)$ if $xy > 1, x > 0, y > 0$.

Calculation: Find $\tan^{-1} 1$ (known value) and then sum the remaining two terms using the formula.

Step 3: Detailed Explanation:

Let the sum be $S = \tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3$.

Step 1: Evaluation of first term.

$$\tan^{-1} 1 = \frac{\pi}{4}$$

Step 2: Summing $\tan^{-1} 2 + \tan^{-1} 3$.

Here $x = 2, y = 3$. Note that $xy = 2 \times 3 = 6 > 1$.

Using the second-quadrant formula:

$$\tan^{-1} 2 + \tan^{-1} 3 = \pi + \tan^{-1} \left(\frac{2+3}{1-2 \times 3} \right)$$

$$= \pi + \tan^{-1} \left(\frac{5}{1-6} \right) = \pi + \tan^{-1} \left(\frac{5}{-5} \right)$$

$$= \pi + \tan^{-1}(-1)$$

Since $\tan^{-1}(-1) = -\frac{\pi}{4}$:

$$\tan^{-1} 2 + \tan^{-1} 3 = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

Step 3: Finding the total sum.

$$S = \frac{\pi}{4} + \frac{3\pi}{4} = \frac{4\pi}{4} = \pi$$

This result is consistent. Geometrically, if you have a triangle with sides such that tangents of angles are 1, 2, and 3, they do not exist since they sum to π , but these are the angles of a triangle with a specific property. This is a very common result in mathematics.

Step 4: Final Answer:

The value of the sum is π .

This matches Option (C).

Quick Tip: Common Sums: $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$. Also, $\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3} = \frac{\pi}{4}$. Memorizing these results directly can save a lot of time and prevents the common mistake of forgetting to add π when $xy > 1$.

18. If $z_1 = 4i^{40} - 5i^{35} + 6i^{17} + 2$, $z_2 = -1 + i$, then $|z_1 + z_2| =$

- (A) 13
- (B) 5
- (C) 15
- (D) 12

Correct Answer: (A) 13

Solution:

Step 1: Understanding the Concept:

This problem involves simplifying a complex number given in terms of powers of the imaginary unit i and then calculating the modulus of a sum of two complex numbers. The powers of i cycle every four terms: $i^1 = i, i^2 = -1, i^3 = -i, i^4 = 1$. To simplify any i^n , we find the remainder of n when divided by 4. After simplifying z_1 , we add it to z_2 by combining their real parts and imaginary parts separately. Finally, the modulus $|z|$ of a complex number $a + bi$ is calculated as $\sqrt{a^2 + b^2}$.

Step 2: Key Formula or Approach:

Rule for powers of i : $i^{4k} = 1, i^{4k+1} = i, i^{4k+2} = -1, i^{4k+3} = -i$.

Modulus: $|a + bi| = \sqrt{a^2 + b^2}$.

Step 3: Detailed Explanation:

Simplify $z_1 = 4i^{40} - 5i^{35} + 6i^{17} + 2$:

1. i^{40} : Since 40 is divisible by 4, $i^{40} = (i^4)^{10} = 1^{10} = 1$.
2. i^{35} : $35 = 4 \times 8 + 3$. So $i^{35} = i^3 = -i$.
3. i^{17} : $17 = 4 \times 4 + 1$. So $i^{17} = i^1 = i$.

Substitute these back into z_1 :

$$z_1 = 4(1) - 5(-i) + 6(i) + 2$$

$$z_1 = 4 + 5i + 6i + 2 = (4 + 2) + (5 + 6)i = 6 + 11i$$

Now, given $z_2 = -1 + i$, find the sum $z_1 + z_2$:

$$z_1 + z_2 = (6 + 11i) + (-1 + i) = (6 - 1) + (11 + 1)i = 5 + 12i$$

Calculate the modulus $|z_1 + z_2|$:

$$|z_1 + z_2| = \sqrt{5^2 + 12^2}$$

$$|z_1 + z_2| = \sqrt{25 + 144} = \sqrt{169} = 13$$

The calculation yields exactly 13, which is part of the well-known (5, 12, 13) Pythagorean triple. This confirms the accuracy of the result.

Step 4: Final Answer:

The modulus is 13.

This matches Option (A).

Quick Tip: To quickly find the power of i , just look at the last two digits of the exponent. If the last two digits are divisible by 4, the value is 1. If the remainder is 1, 2, or 3, the value is i , -1 , or $-i$ respectively. This is especially helpful for very large exponents.

19. The conjugate of $(1 + i)^3$ is

- (A) $1 + 2i$
- (B) $-2 + 2i$
- (C) $-2 - 2i$
- (D) $1 - 2i$

Correct Answer: (C) $-2 - 2i$

Solution:

Step 1: Understanding the Concept:

This problem involves two steps: expanding a complex binomial power and finding the complex conjugate. The expansion of $(a + b)^3$ follows the standard algebraic formula $a^3 + 3a^2b + 3ab^2 + b^3$. In the context of complex numbers, we also apply $i^2 = -1$ and $i^3 = -i$ to simplify the expression into the standard form $a + bi$. The complex conjugate of $a + bi$ is then defined as $a - bi$, which essentially reflects the number across the real axis in the Argand plane. Alternatively, one can use the property that the conjugate of a power is the power of the conjugate: $\overline{z^n} = (\overline{z})^n$.

Step 2: Key Formula or Approach:

Expansion: $(1 + i)^3 = 1^3 + 3(1)^2(i) + 3(1)(i)^2 + i^3$.

Conjugate: If $z = x + iy$, then $\bar{z} = x - iy$.

Alternative property: $\overline{(1 + i)^3} = (1 - i)^3$.

Step 3: Detailed Explanation:

Method 1: Expansion first.

$$z = (1 + i)^3 = 1 + 3i + 3i^2 + i^3$$

Substitute $i^2 = -1$ and $i^3 = -i$:

$$z = 1 + 3i + 3(-1) + (-i)$$

$$z = 1 + 3i - 3 - i$$

$$z = (1 - 3) + (3 - 1)i = -2 + 2i$$

Now, find the conjugate $\bar{z}$:

Changing the sign of the imaginary part:

$$\bar{z} = -2 - 2i$$

Method 2: Conjugate first.

Conjugate of $(1 + i)$ is $(1 - i)$. So, the answer is $(1 - i)^3$.

$$(1 - i)^3 = 1^3 - 3(1)^2(i) + 3(1)(i^2) - i^3$$

$$= 1 - 3i + 3(-1) - (-i)$$

$$= 1 - 3i - 3 + i = -2 - 2i$$

Both methods yield the same result. The first method is generally easier to follow as it simplifies the number into a basic coordinate form before applying the conjugate operation.

Step 4: Final Answer:

The conjugate is $-2 - 2i$.

This matches Option (C).

Quick Tip: Remember the standard results for $(1 + i)^2 = 2i$ and $(1 - i)^2 = -2i$. Thus, $(1 + i)^3 = (1 + i)(1 + i)^2 = (1 + i)(2i) = 2i + 2i^2 = -2 + 2i$. This is much faster than doing the full cubic expansion. From here, the conjugate is immediate: $-2 - 2i$.

20. The equation of a circle whose Centre is $(-3, 2)$ and area is 176 units is

- (A) $x^2 + y^2 + 6x - 4y - 36 = 0$
- (B) $x^2 + y^2 + 6x - 4y - 43 = 0$
- (C) $x^2 + y^2 - 6x + 4y - 36 = 0$
- (D) $x^2 + y^2 - 6x + 4y - 43 = 0$

Correct Answer: (B) $x^2 + y^2 + 6x - 4y - 43 = 0$

Solution:

Step 1: Understanding the Concept:

A circle's equation can be determined if its center and radius are known. The standard form of a circle with center (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$. In this problem, we are given the center directly, but the radius must be derived from the provided area. The area of a circle is given by $A = \pi r^2$. Once r^2 is found, we substitute it into the standard form and expand the equation into the general form $x^2 + y^2 + 2gx + 2fy + c = 0$ to match the given options.

Step 2: Key Formula or Approach:

Area: $\pi r^2 = 176$. Use $\pi = 22/7$ for calculation.

Circle Equation: $(x - h)^2 + (y - k)^2 = r^2$, where $(h, k) = (-3, 2)$.

Step 3: Detailed Explanation:

1. Calculate the radius squared (r^2):

$$\text{Area} = \pi r^2 = 176$$

$$\frac{22}{7}r^2 = 176$$

Multiply by 7 and divide by 22:

$$r^2 = \frac{176 \times 7}{22}$$

Divide 176 by 22 (note that $22 \times 8 = 176$):

$$r^2 = 8 \times 7 = 56$$

2. Setup the circle equation using the center $(-3, 2)$:

$$(x - (-3))^2 + (y - 2)^2 = r^2$$

$$(x + 3)^2 + (y - 2)^2 = 56$$

3. Expand and simplify:

$$(x^2 + 6x + 9) + (y^2 - 4y + 4) = 56$$

$$x^2 + y^2 + 6x - 4y + 13 = 56$$

$$x^2 + y^2 + 6x - 4y + 13 - 56 = 0$$

$$x^2 + y^2 + 6x - 4y - 43 = 0$$

Comparing this with the given options, it perfectly matches Option B. Note how the signs of $2gx$ and $2fy$ are $+6x$ and $-4y$, reflecting the negative signs in the center coordinates $h = -3, k = 2$ due to the $(x - h)$ and $(y - k)$ structure.

Step 4: Final Answer:

The equation of the circle is $x^2 + y^2 + 6x - 4y - 43 = 0$.

This matches Option (B).

Quick Tip: If the center is (h, k) , the linear terms in the general equation will be $-2hx$ and $-2ky$. Here, $-2(-3) = +6$ and $-2(2) = -4$. This immediately narrows the choices to Options (A) and (B). Then, checking the constant term $c = h^2 + k^2 - r^2$ gives $9 + 4 - 56 = -43$, pointing directly to Option (B).

21. The equation of a circle whose Centre is $(2, -1)$ and which passes through the point $(3, 6)$ is

(A) $x^2 + y^2 + 4x + 2y - 45 = 0$

(B) $x^2 + y^2 - 2x + 2y - 50 = 0$

(C) $x^2 + y^2 + 2x + 2y - 50 = 0$

(D) $x^2 + y^2 - 4x + 2y - 45 = 0$

Correct Answer: (D) $x^2 + y^2 - 4x + 2y - 45 = 0$

Solution:

Step 1: Understanding the Concept:

In coordinate geometry, a circle is defined as the set of all points in a plane that are at a constant distance, called the radius (r), from a fixed point called the center (h, k) . To determine the complete equation of a circle, we must identify these two essential components. In this problem, the center is explicitly provided as $(2, -1)$. The radius is not directly given, but we are told that the circle passes through a specific point $(3, 6)$. Since any point on the

circle is at distance r from the center, the distance between the center $(2, -1)$ and the point $(3, 6)$ is equal to the radius. Once the radius is found, we can use the standard form of the circle equation and expand it to find the general form as listed in the options.

Step 2: Key Formula or Approach:

1. Use the distance formula to find the radius r :

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

where (x_1, y_1) is the center $(2, -1)$ and (x_2, y_2) is the point $(3, 6)$.

2. Substitute the center and radius into the standard equation:

$$(x - h)^2 + (y - k)^2 = r^2$$

3. Expand and simplify to get the general form $x^2 + y^2 + 2gx + 2fy + c = 0$.

Step 3: Detailed Explanation:

First, let's calculate the radius squared (r^2) directly to avoid unnecessary square roots:

$$r^2 = (3 - 2)^2 + (6 - (-1))^2$$

$$r^2 = (1)^2 + (7)^2$$

$$r^2 = 1 + 49 = 50$$

Now, substitute the center $(h, k) = (2, -1)$ and $r^2 = 50$ into the standard equation:

$$(x - 2)^2 + (y - (-1))^2 = 50$$

$$(x - 2)^2 + (y + 1)^2 = 50$$

Expand the binomial terms:

$$(x^2 - 4x + 4) + (y^2 + 2y + 1) = 50$$

Group like terms and bring the constant from the right side to the left:

$$x^2 + y^2 - 4x + 2y + 5 - 50 = 0$$

$$x^2 + y^2 - 4x + 2y - 45 = 0$$

This final expression represents the general equation of the circle. By checking the coefficients of x and y , we can see they are $-2h = -4$ and $-2k = 2$, which correctly corresponds to the given center $(2, -1)$. This systematic derivation ensures that all conditions given in the problem are satisfied.

Step 4: Final Answer:

The equation of the circle is $x^2 + y^2 - 4x + 2y - 45 = 0$.

Comparing this result with the given choices, it matches Option (D).

Quick Tip: To quickly verify circle equations, check the linear coefficients first. For center (h, k) , the terms are $-2hx$ and $-2ky$. With center $(2, -1)$, we need $-4x$ and $+2y$. This immediately eliminates Options (A), (B), and (C) without even calculating the constant term!

22. If the parabola $y^2 = 4ax$ passes through the point $(3, 2)$ then the length of its latus rectum is:

- (A) $\frac{4}{3}$
- (B) 4
- (C) $\frac{2}{3}$
- (D) $\frac{1}{3}$

Correct Answer: (A) $\frac{4}{3}$

Solution:

Step 1: Understanding the Concept:

A parabola is the set of all points that are equidistant from a fixed point (the focus) and a fixed line (the directrix). For a standard parabola aligned with the x-axis, its equation is given as $y^2 = 4ax$. In this standard form, the coefficient of x , which is $4a$, has a specific geometric meaning: it represents the length of the latus rectum. The latus rectum is the line segment that passes through the focus, is perpendicular to the axis of symmetry, and has endpoints lying on the parabola. If a parabola passes through a specific point, the coordinates of that point must satisfy the parabola's equation. By substituting the point into the equation, we can solve for the unknown parameter $4a$.

Step 2: Key Formula or Approach:

The equation of the parabola is $y^2 = 4ax$.

The length of the latus rectum is given directly by the value of $4a$.

Given the point $(x, y) = (3, 2)$, substitute these values into the equation to find $4a$.

Step 3: Detailed Explanation:

Start with the standard form of the parabola:

$$y^2 = 4ax$$

The problem states the curve passes through the point $(3, 2)$. Therefore, we substitute $x = 3$ and $y = 2$ into the equation:

$$(2)^2 = 4a(3)$$

Simplify the left side:

$$4 = 12a$$

From this, we can solve for a :

$$a = \frac{4}{12} = \frac{1}{3}$$

However, we need the length of the latus rectum, which is defined as the absolute value of $4a$. Since we already have the equation $4 = 12a$, we can simply solve for $4a$:

$$4a = \frac{4}{3}$$

Alternatively, knowing $a = 1/3$, the length of latus rectum is:

$$L = 4 \times \left(\frac{1}{3}\right) = \frac{4}{3}$$

This value represents the total width of the parabola at its focus. It is a fundamental property that defines the "openness" of the parabola. Since $4/3$ is a constant derived purely from the point the curve passes through, it is the unique solution for this specific parabola.

Step 4: Final Answer:

The length of the latus rectum is $4/3$.

This matches Option (A).

Quick Tip: Remember that in the standard form $y^2 = kx$ or $x^2 = ky$, the constant k is **always** the length of the latus rectum. You don't need to solve for ' a ' separately; just solve for the entire coefficient of the linear variable!

23. The line $y = mx + 2$ is a tangent to the parabola $y^2 = 8x$ if

- (A) $m = 1$
- (B) $m = 2$
- (C) $m = 3$
- (D) $m = 4$

Correct Answer: (A) $m = 1$

Solution:

Step 1: Understanding the Concept:

In analytic geometry, a line is tangent to a curve if it touches the curve at exactly one point. For a parabola $y^2 = 4ax$, any non-vertical line $y = mx + c$ can be tested for tangency by substituting the line equation into the parabola's equation. This results in a quadratic equation in one variable. If the line is tangent, the discriminant of this quadratic equation must be zero, meaning there is only one real solution. For standard parabolas, mathematicians have derived a specific condition for the constant c in terms of the slope m and the parameter a . By identifying a from the parabola's equation and comparing the line's intercept to this condition, we can determine the required slope.

Step 2: Key Formula or Approach:

For the parabola $y^2 = 4ax$, the condition for the line $y = mx + c$ to be a tangent is:

$$c = \frac{a}{m}$$

From the given parabola $y^2 = 8x$, we identify $4a = 8$, so $a = 2$.

From the given line $y = mx + 2$, the intercept is $c = 2$.

Step 3: Detailed Explanation:

1. Find parameter a for the parabola $y^2 = 8x$:

Compare with $y^2 = 4ax$:

$$4a = 8 \implies a = 2$$

2. Identify the intercept c of the line $y = mx + 2$:

Comparing with $y = mx + c$, we see:

$$c = 2$$

3. Apply the tangency condition $c = a/m$:

$$2 = \frac{2}{m}$$

Multiply both sides by m :

$$2m = 2$$

Divide by 2:

$$m = 1$$

Alternatively, substituting $y = mx + 2$ into $y^2 = 8x$:

$$(mx + 2)^2 = 8x$$

$$m^2x^2 + 4mx + 4 = 8x$$

$$m^2x^2 + (4m - 8)x + 4 = 0$$

For tangency, Discriminant $D = 0$:

$$(4m - 8)^2 - 4(m^2)(4) = 0$$

$$16m^2 - 64m + 64 - 16m^2 = 0$$

$$-64m + 64 = 0 \implies m = 1$$

Both methods verify that the slope of the tangent line must be 1.

Step 4: Final Answer:

The value of m is 1.

This matches Option (A).

Quick Tip: The tangency condition $c = a/m$ is one of the most frequently used shortcuts in coordinate geometry. Memorize the conditions for other conics too: for a circle $x^2 + y^2 = r^2$, $c^2 = r^2(1 + m^2)$; for an ellipse, $c^2 = a^2m^2 + b^2$.

24. The length of the latusrectum and eccentricity of the Hyperbola $9x^2 - 16y^2 = 144$ are

- (A) $(9/4, 5/4)$
- (B) $(9/2, 5/4)$
- (C) $(9/2, 5/2)$
- (D) $(9, 5/2)$

Correct Answer: (B) $(9/2, 5/4)$

Solution:

Step 1: Understanding the Concept:

A hyperbola is a type of conic section where the difference of distances from any point on the

curve to two fixed points (foci) is constant. The standard equation for a hyperbola centered at the origin is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. From this form, we can identify parameters a and b , which are related to the transverse and conjugate axes respectively. Two key metrics of a hyperbola are its eccentricity (e), which measures how "open" or "curved" it is compared to a circle, and the length of the latus rectum, which provides the vertical height of the curve above the foci. For a hyperbola, eccentricity is always greater than 1. The calculation involves first converting the given general equation into standard form by dividing by the constant on the right.

Step 2: Key Formula or Approach:

1. Standard form: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.
2. Eccentricity: $e = \sqrt{1 + \frac{b^2}{a^2}} = \frac{\sqrt{a^2 + b^2}}{a}$.
3. Length of Latus Rectum: $L = \frac{2b^2}{a}$.

Step 3: Detailed Explanation:

First, divide the entire equation $9x^2 - 16y^2 = 144$ by 144 to get the standard form:

$$\frac{9x^2}{144} - \frac{16y^2}{144} = \frac{144}{144}$$

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

From this, we identify:

$$a^2 = 16 \implies a = 4$$

$$b^2 = 9 \implies b = 3$$

Now, calculate the eccentricity (e):

$$e = \sqrt{1 + \frac{9}{16}} = \sqrt{\frac{16+9}{16}} = \sqrt{\frac{25}{16}} = \frac{5}{4}$$

Next, calculate the length of the latus rectum (L):

$$L = \frac{2b^2}{a} = \frac{2(9)}{4} = \frac{18}{4} = \frac{9}{2}$$

The calculated pair is $(9/2, 5/4)$. These parameters fully describe the shape and focus positions of the given hyperbola. Note that the eccentricity $5/4$ is indeed greater than 1, as required for hyperbolas.

Step 4: Final Answer:

The length of latus rectum is $9/2$ and the eccentricity is $5/4$.

This matches Option (B).

Quick Tip: When converting to standard form, the denominators are simply the divisors. A useful shortcut for eccentricity of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $e = \frac{\sqrt{\text{sum of denominators}}}{\sqrt{\text{positive term denominator}}}$. Here, $e = \sqrt{16+9}/\sqrt{16} = 5/4$.

25. The equation of the ellipse with foci at $(\pm 3, 0)$ and the eccentricity as $1/3$ is :

- (A) $\frac{x^2}{81} + \frac{y^2}{72} = 1$
- (B) $\frac{x^2}{9} + \frac{y^2}{8} = 1$
- (C) $\frac{x^2}{8} + \frac{y^2}{9} = 1$
- (D) $\frac{x^2}{3} + \frac{y^2}{2} = 1$

Correct Answer: (A) $\frac{x^2}{81} + \frac{y^2}{72} = 1$

Solution:

Step 1: Understanding the Concept:

An ellipse is a locus of points such that the sum of distances to two fixed points (foci) is constant. For an ellipse centered at the origin with foci on the x-axis, the equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. The foci are located at $(\pm ae, 0)$, where a is the semi-major axis and e is the

eccentricity. Eccentricity measures the degree of "flatness" of the ellipse; for an ellipse, $0 < e < 1$. There is a fundamental relationship between the semi-axes and eccentricity: $b^2 = a^2(1 - e^2)$. By using the given focus coordinates and eccentricity, we can find a , then a^2 , then calculate b^2 , and finally write the equation.

Step 2: Key Formula or Approach:

1. Foci formula: Focus distance = ae .
2. Given foci $(\pm 3, 0) \implies ae = 3$.
3. Given eccentricity $e = 1/3$.
4. Relation: $b^2 = a^2(1 - e^2)$.

Step 3: Detailed Explanation:

1. Find a :

Given $ae = 3$ and $e = 1/3$:

$$a \times \left(\frac{1}{3}\right) = 3$$

Multiply both sides by 3:

$$a = 9 \implies a^2 = 81$$

2. Find b^2 :

Using the relation $b^2 = a^2(1 - e^2)$:

$$b^2 = 81 \times \left(1 - \left(\frac{1}{3}\right)^2\right)$$

$$b^2 = 81 \times \left(1 - \frac{1}{9}\right)$$

$$b^2 = 81 \times \frac{8}{9}$$

Simplify the multiplication:

$$b^2 = 9 \times 8 = 72$$

3. Write the equation:

Substitute $a^2 = 81$ and $b^2 = 72$ into the standard ellipse form:

$$\frac{x^2}{81} + \frac{y^2}{72} = 1$$

This equation correctly describes an ellipse that is elongated along the x-axis since $a^2 > b^2$. The foci are located at $\pm\sqrt{a^2 - b^2} = \pm\sqrt{81 - 72} = \pm\sqrt{9} = \pm 3$, which matches the problem description.

Step 4: Final Answer:

The equation of the ellipse is $\frac{x^2}{81} + \frac{y^2}{72} = 1$.

This matches Option (A).

Quick Tip: Always start by checking if the foci are on the x-axis or y-axis. Here, $(\pm 3, 0)$ is on the x-axis, so the denominator under x^2 must be larger than the one under y^2 . This rules out Option (C). Then, verify $a = \text{focus distance}/e = 3/(1/3) = 9$, so $a^2 = 81$. Only Option (A) has 81.

26. $\lim_{x \rightarrow \infty} (1 + \frac{1}{x})^x =$

- (A) 0
- (B) 1
- (C) e
- (D) ∞

Correct Answer: (C) e

Solution:

Step 1: Understanding the Concept:

This problem asks for the evaluation of a fundamental limit in calculus that serves as the primary definition of the transcendental number e (Euler's number). The expression is in the indeterminate form 1^∞ . When we let x grow infinitely large, the term $1/x$ approaches 0, making the base approach 1. Simultaneously, the exponent x grows to infinity. The balance between the base getting closer to 1 and the power increasing result in a specific constant value approximately equal to 2.71828. This limit is essential for continuous growth models, finance (compound interest), and the foundations of natural logarithms.

Step 2: Key Formula or Approach:

The standard limit definition of e is:

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e$$

Similarly, as $h \rightarrow 0$:

$$\lim_{h \rightarrow 0} (1 + h)^{1/h} = e$$

Step 3: Detailed Explanation:

Let $L = \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$.

As $x \rightarrow \infty$, $1/x \rightarrow 0$. This is an indeterminate form of type 1^∞ .

To evaluate such limits, we often use logarithms:

$$\ln L = \lim_{x \rightarrow \infty} \ln \left(\left(1 + \frac{1}{x}\right)^x \right)$$

Using the property of logarithms $\ln(a^b) = b \ln(a)$:

$$\ln L = \lim_{x \rightarrow \infty} x \ln \left(1 + \frac{1}{x}\right)$$

Rewrite to use L'Hopital's rule (making it a 0/0 form):

$$\ln L = \lim_{x \rightarrow \infty} \frac{\ln(1 + \frac{1}{x})}{1/x}$$

Let $u = 1/x$. As $x \rightarrow \infty$, $u \rightarrow 0$:

$$\ln L = \lim_{u \rightarrow 0} \frac{\ln(1 + u)}{u}$$

Applying L'Hopital's rule (differentiating numerator and denominator):

$$\ln L = \lim_{u \rightarrow 0} \frac{\frac{1}{1+u}}{1} = 1$$

Since $\ln L = 1$, it follows that:

$$L = e^1 = e$$

Thus, the limit of the expression as x approaches infinity is exactly the mathematical constant e . This is not just a calculation, but the very way e is often introduced in mathematical analysis.

Step 4: Final Answer:

The value of the limit is e .

This matches Option (C).

Quick Tip: This is a "Standard Result." In exams, do not attempt to derive it. Simply memorize:

$$\lim_{x \rightarrow \infty} (1 + \frac{a}{x})^x = e^a.$$

Here $a = 1$, so the answer is $e^1 = e$.

27. $\lim_{x \rightarrow 0} \frac{\sqrt{1+x}-1}{x} =$

- (A) 0
- (B) $1/2$
- (C) 1
- (D) ∞

Correct Answer: (B) $1/2$

Solution:

Step 1: Understanding the Concept:

Limits of the form $\frac{0}{0}$ are called indeterminate forms. When evaluating $\frac{\sqrt{1+x}-1}{x}$ as $x \rightarrow 0$, direct substitution gives $\frac{\sqrt{1-1}}{0} = \frac{0}{0}$, which doesn't provide a value. To resolve this, we can use algebraic techniques like rationalizing the numerator or calculus techniques like L'Hopital's rule. This specific limit represents the derivative of the function $f(t) = \sqrt{t}$ evaluated at $t = 1$. It tells us the slope of the tangent line to the square root function at that specific point.

Step 2: Key Formula or Approach:

1. Rationalization: Multiply numerator and denominator by the conjugate $\sqrt{1+x} + 1$.
2. L'Hopital's Rule: If $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)} = \frac{0}{0}$, then it equals $\lim_{x \rightarrow 0} \frac{f'(x)}{g'(x)}$.

Step 3: Detailed Explanation:

Method 1: Rationalization

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x}-1}{x} \times \frac{\sqrt{1+x}+1}{\sqrt{1+x}+1}$$

Multiply the numerator (using $(a-b)(a+b) = a^2 - b^2$):

$$\lim_{x \rightarrow 0} \frac{(1+x)-1}{x(\sqrt{1+x}+1)}$$

$$\lim_{x \rightarrow 0} \frac{x}{x(\sqrt{1+x}+1)}$$

Cancel the common factor x :

$$\lim_{x \rightarrow 0} \frac{1}{\sqrt{1+x} + 1}$$

Now, perform direct substitution for $x = 0$:

$$\frac{1}{\sqrt{1+0} + 1} = \frac{1}{1+1} = \frac{1}{2}$$

Method 2: L'Hopital's Rule

Differentiate numerator: $\frac{d}{dx}(\sqrt{1+x} - 1) = \frac{1}{2\sqrt{1+x}}$.

Differentiate denominator: $\frac{d}{dx}(x) = 1$.

Limit is $\lim_{x \rightarrow 0} \frac{\frac{1}{2\sqrt{1+x}}}{1}$.

Substitute $x = 0$:

$$\frac{1}{2\sqrt{1}} = \frac{1}{2}$$

Both methods yield the same result, confirming that the limit is $1/2$.

Step 4: Final Answer:

The value of the limit is $1/2$.

This matches Option (B).

Quick Tip: Generalized rule: $\lim_{x \rightarrow 0} \frac{(1+x)^n - 1}{x} = n$.

Here $\sqrt{1+x}$ is $(1+x)^{1/2}$, so the limit is simply the exponent $1/2$. This "power rule" for limits is a great time-saver!

28. If $y = \frac{(a \cos x + b \sin x + C)}{\sin x}$ then $\frac{dy}{dx} =$

- (A) $-a \csc^2 x - c \csc x \cot x$
- (B) $-a$
- (C) $-a \csc^2 x + b \sec^2 x + c \csc x \cot x$

(D) $a \csc^2 x - c \csc x \cot x$

Correct Answer: (A) $-a \csc^2 x - c \csc x \cot x$

Solution:

Step 1: Understanding the Concept:

This is a differentiation problem involving trigonometric functions. The function y is a quotient. While we could use the quotient rule, it is often simpler to decompose the fraction into individual terms. By dividing each term in the numerator by the denominator $\sin x$, we transform the complex fraction into a sum of basic trigonometric functions: cotangent, a constant, and cosecant. The derivatives of these functions are well-known standard results. This approach reduces the chance of algebraic errors that often occur when applying the full quotient rule with multiple terms.

Step 2: Key Formula or Approach:

1. Simplify: $y = a \frac{\cos x}{\sin x} + b \frac{\sin x}{\sin x} + \frac{C}{\sin x}$.
2. Identities: $\frac{\cos x}{\sin x} = \cot x$ and $\frac{1}{\sin x} = \csc x$.
3. Derivatives:

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\text{constant}) = 0$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

Step 3: Detailed Explanation:

Let's simplify the expression for y first:

$$y = \frac{a \cos x}{\sin x} + \frac{b \sin x}{\sin x} + \frac{C}{\sin x}$$

$$y = a \cot x + b + C \csc x$$

Now, differentiate each term with respect to x :

$$\text{Term 1: } \frac{d}{dx}(a \cot x) = a(-\csc^2 x) = -a \csc^2 x$$

$$\text{Term 2: } \frac{d}{dx}(b) = 0 \text{ (since } b \text{ is a constant)}$$

Term 3: $\frac{d}{dx}(C \csc x) = C(-\csc x \cot x) = -C \csc x \cot x$

Adding these results together:

$$\frac{dy}{dx} = -a \csc^2 x + 0 - C \csc x \cot x$$

$$\frac{dy}{dx} = -a \csc^2 x - C \csc x \cot x$$

This matches the structure of the required derivative. The term b vanishes because its rate of change is zero. The resulting expression describes how the original function y varies as x changes, expressed in terms of squared cosecant and the product of cosecant and cotangent.

Step 4: Final Answer:

The derivative $\frac{dy}{dx}$ is $-a \csc^2 x - C \csc x \cot x$.

This matches Option (A).

Quick Tip: Always simplify a fraction **before** differentiating if the denominator is a single term (monomial). It changes a Quotient Rule problem (long) into a Sum Rule problem (short and easy).

29. If $y = \sqrt{x + \sqrt{x + \sqrt{x + \dots \infty}}}$ then $\frac{dy}{dx} =$

- (A) $\frac{1}{2y}$
- (B) $\frac{1}{1-2y}$
- (C) $\frac{1}{2(1-2y)}$
- (D) $\frac{-1}{1-2y}$

Correct Answer: (D) $\frac{-1}{1-2y}$

Solution:

Step 1: Understanding the Concept:

This problem deals with the differentiation of an infinite nested radical. The key insight is identifying that the infinite sequence is recursive. Because the sequence goes to infinity, the part of the expression nested inside the first radical is identical to the whole expression itself. By substituting the entire function y back into its own definition, we convert an infinite expression into a finite algebraic equation. From there, we use implicit differentiation to find the rate of change dy/dx . This is a standard technique for evaluating derivatives of functions defined by infinite processes.

Step 2: Key Formula or Approach:

1. Identify recursion: $y = \sqrt{x + y}$.
2. Algebra: Square both sides: $y^2 = x + y$.
3. Differentiate both sides with respect to x .

Step 3: Detailed Explanation:

Given $y = \sqrt{x + \sqrt{x + \sqrt{x + \dots \infty}}}$, the inner part $\sqrt{x + \dots}$ is also equal to y .

Thus:

$$y = \sqrt{x + y}$$

Square both sides to remove the radical:

$$y^2 = x + y$$

Now, differentiate both sides with respect to x using the chain rule on the left side:

$$\frac{d}{dx}(y^2) = \frac{d}{dx}(x + y)$$

$$2y \frac{dy}{dx} = 1 + \frac{dy}{dx}$$

Collect all $\frac{dy}{dx}$ terms on one side:

$$2y \frac{dy}{dx} - \frac{dy}{dx} = 1$$

Factor out $\frac{dy}{dx}$:

$$\frac{dy}{dx}(2y - 1) = 1$$

Solve for $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{1}{2y - 1}$$

Let's check the options. Option (D) is $\frac{-1}{1-2y}$.

Since $2y - 1 = -(1 - 2y)$, we have:

$$\frac{1}{2y - 1} = \frac{1}{-(1 - 2y)} = \frac{-1}{1 - 2y}$$

This confirms that Option (D) is the correct algebraic representation. The derivative tells us that the rate at which y grows depends inversely on twice the current value of y minus one.

Step 4: Final Answer:

The derivative $\frac{dy}{dx}$ is $\frac{-1}{1-2y}$.

This matches Option (D).

Quick Tip: For infinite nested radicals of the form $y = \sqrt{f(x) + \sqrt{f(x) + \dots}}$, there is a standard shortcut:

$$\frac{dy}{dx} = \frac{f'(x)}{2y-1}.$$

Here $f(x) = x$, so $f'(x) = 1$. The answer is $\frac{1}{2y-1}$.

30. Slope of the tangent to the curve $y = 9x^2 + 7x^4 + 5$ at the point $x = 1$ is

- (A) 28
- (B) 16
- (C) 46
- (D) $1/46$

Correct Answer: (C) 46

Solution:

Step 1: Understanding the Concept:

In calculus, the derivative of a function at a specific point represents the instantaneous rate of change of the function at that point. Geometrically, this derivative is exactly equal to the slope of the line that is tangent to the curve at that point. To find the slope of the tangent for a given polynomial curve, we first apply the power rule of differentiation to the function to find the derivative expression, dy/dx . Finally, we substitute the given coordinate $x = 1$ into this derivative to find the numerical value of the slope.

Step 2: Key Formula or Approach:

1. Power Rule: $\frac{d}{dx}(x^n) = nx^{n-1}$.
2. Slope $m = \left. \frac{dy}{dx} \right|_{x=1}$.

Step 3: Detailed Explanation:

The equation of the curve is:

$$y = 9x^2 + 7x^4 + 5$$

Differentiate with respect to x :

Term 1: $\frac{d}{dx}(9x^2) = 9(2x) = 18x$

Term 2: $\frac{d}{dx}(7x^4) = 7(4x^3) = 28x^3$

Term 3: $\frac{d}{dx}(5) = 0$ (derivative of a constant is zero)

Combining these, we get the general expression for the slope:

$$\frac{dy}{dx} = 18x + 28x^3$$

Now, we need to find the specific slope at the point where $x = 1$:

$$m = 18(1) + 28(1)^3$$

$$m = 18 + 28(1)$$

$$m = 18 + 28$$

$$m = 46$$

The value 46 represents the steepness of the curve $y = 9x^2 + 7x^4 + 5$ precisely at the point where $x = 1$. This slope indicates that for every tiny increase in x , the value of y increases roughly 46 times as much at that location.

Step 4: Final Answer:

The slope of the tangent at $x = 1$ is 46.

This matches Option (C).

Quick Tip: "Slope of Tangent" is just code for "Derivative". When you see this phrase, differentiate immediately. If the question asks for "Normal slope", find the tangent slope m and then calculate $-1/m$.

31. If $f(x) = \begin{cases} 4(5^x) & x < 0 \\ 8k + x & x \geq 0 \end{cases}$, then $f'(-1) =$

- (A) $\frac{2}{5} \log 5$
 (B) $\frac{4}{5} \log 5$
 (C) $\frac{3}{5} \log 5$
 (D) $20 \log 5$

Correct Answer: (B) $\frac{4}{5} \log 5$

Solution:

Step 1: Understanding the Concept:

This problem involves evaluating the derivative of a piecewise function at a specific point. A piecewise function is defined by different expressions over distinct intervals of its domain. When we are asked to find the derivative at a point c , we must first identify which "piece" of the function the point belongs to.

In this case, the point of interest is $x = -1$.

Looking at the conditions for the function $f(x)$, we see that -1 falls into the interval $x < 0$. Therefore, for the purpose of finding the derivative at $x = -1$, we only consider the definition $f(x) = 4(5^x)$.

The definition for $x \geq 0$ is irrelevant for finding the instantaneous rate of change at $x = -1$.

Step 2: Key Formula or Approach:

The key formula required is the derivative of an exponential function.

For a function of the form $g(x) = a^x$, the derivative is $g'(x) = a^x \ln(a)$.

When there is a constant multiplier c , the derivative is $\frac{d}{dx}[c \cdot a^x] = c \cdot a^x \ln(a)$.

Here, our constant is 4 and our base a is 5.

Step 3: Detailed Explanation:

For $x < 0$, the function is defined as $f(x) = 4 \cdot 5^x$.

We need to differentiate this with respect to x :

$$f'(x) = \frac{d}{dx}[4 \cdot 5^x]$$

Using the constant multiple rule and the exponential derivative rule:

$$f'(x) = 4 \cdot [5^x \ln(5)]$$

Now, we substitute the given value $x = -1$ into this derivative expression:

$$f'(-1) = 4 \cdot 5^{-1} \cdot \ln(5)$$

Simplifying the term with the negative exponent:

$$5^{-1} = \frac{1}{5}$$

Substituting this back:

$$f'(-1) = 4 \cdot \frac{1}{5} \cdot \ln(5)$$

$$f'(-1) = \frac{4}{5} \ln(5)$$

In some contexts, $\ln(5)$ is written as $\log 5$. Assuming the base is e , the result is $\frac{4}{5} \log 5$.

This calculation confirms that the slope of the curve at $x = -1$ is determined solely by the exponential part of the function.

Step 4: Final Answer:

The derivative $f'(-1)$ is equal to $\frac{4}{5} \log 5$.

This matches Option (B).

Quick Tip: Always identify the relevant piece of a piecewise function first. Do not waste time differentiating or checking continuity for the parts of the domain that the point of interest does not belong to, unless the question asks for the derivative at the transition point (like $x = 0$ here).

32. If $2^x + 2^y = 2^{x+y}$, then $\frac{dy}{dx} =$

- (A) $1 - 2^y$
- (B) $1 - \frac{1}{2^y}$
- (C) $1 + 2^{-y}$
- (D) $1 + 2^y$

Correct Answer: (A) $1 - 2^y$

Solution:

Step 1: Understanding the Concept:

This is a problem involving implicit differentiation.

When an equation defines y as an implicit function of x , we differentiate both sides of the equation with respect to x , treating y as a function of x and applying the chain rule.

The equation $2^x + 2^y = 2^{x+y}$ can be simplified or differentiated directly.

Exponential differentiation follows the rule $\frac{d}{dx}(a^u) = a^u \ln(a) \frac{du}{dx}$.

Step 2: Key Formula or Approach:

1. Differentiate both sides with respect to x .
2. Use the property: $\frac{d}{dx}(2^y) = 2^y \ln(2) \frac{dy}{dx}$.
3. Use the property: $\frac{d}{dx}(2^{x+y}) = 2^{x+y} \ln(2)(1 + \frac{dy}{dx})$.
4. Isolate $\frac{dy}{dx}$.

Step 3: Detailed Explanation:

Differentiating $2^x + 2^y = 2^{x+y}$ with respect to x :

$$2^x \ln 2 + 2^y \ln 2 \cdot \frac{dy}{dx} = 2^{x+y} \ln 2 \left(1 + \frac{dy}{dx} \right)$$

Since $\ln 2 \neq 0$, we can divide the entire equation by $\ln 2$:

$$2^x + 2^y \frac{dy}{dx} = 2^{x+y} + 2^{x+y} \frac{dy}{dx}$$

Rearrange to group terms with $\frac{dy}{dx}$ on one side:

$$2^y \frac{dy}{dx} - 2^{x+y} \frac{dy}{dx} = 2^{x+y} - 2^x$$

$$\frac{dy}{dx}(2^y - 2^{x+y}) = 2^{x+y} - 2^x$$

$$\frac{dy}{dx} = \frac{2^{x+y} - 2^x}{2^y - 2^{x+y}}$$

Now, use the original equation $2^{x+y} = 2^x + 2^y$ to simplify:

Substitute 2^{x+y} in the numerator:

$$\text{Numerator} = (2^x + 2^y) - 2^x = 2^y$$

Substitute 2^{x+y} in the denominator:

$$\text{Denominator} = 2^y - (2^x + 2^y) = -2^x$$

So, the derivative becomes:

$$\frac{dy}{dx} = \frac{2^y}{-2^x} = -2^{y-x}$$

Let's express this in a form that matches the options.

Divide the original equation $2^x + 2^y = 2^x \cdot 2^y$ by $2^x \cdot 2^y$:

$$\frac{1}{2^y} + \frac{1}{2^x} = 1$$

$$2^{-y} + 2^{-x} = 1 \implies 2^{-x} = 1 - 2^{-y}$$

Multiply by 2^x : $1 = 2^x(1 - 2^{-y}) \implies \frac{1}{2^x} = 1 - 2^{-y}$.

From earlier, $\frac{dy}{dx} = -2^y \cdot 2^{-x}$.

Substitute $2^{-x} = 1 - 2^{-y}$:

$$\frac{dy}{dx} = -2^y(1 - 2^{-y}) = -2^y + 2^y \cdot 2^{-y} = -2^y + 1 = 1 - 2^y$$

This simplification gives a clear, algebraic expression for the slope.

Step 4: Final Answer:

The derivative $\frac{dy}{dx}$ is equal to $1 - 2^y$.

This matches Option (A).

Quick Tip: For equations involving exponents like $a^x + a^y = a^{x+y}$, you can quickly divide by a^{x+y} to get $a^{-y} + a^{-x} = 1$. Differentiating this implicitly is much faster: $-a^{-y} \ln a \frac{dy}{dx} - a^{-x} \ln a = 0 \implies \frac{dy}{dx} = -a^{y-x}$.

33. If $y + \sin^{-1}(1 - x^2) = e^x$, then $\frac{dy}{dx} =$

- (A) $e^x - \frac{2}{\sqrt{2-x^2}}$
- (B) $e^x - \frac{2}{\sqrt{2+x^2}}$
- (C) $e^x + \frac{2}{\sqrt{2-x^2}}$
- (D) $e^x + \frac{2}{\sqrt{2+x^2}}$

Correct Answer: (C) $e^x + \frac{2}{\sqrt{2-x^2}}$

Solution:

Step 1: Understanding the Concept:

This problem requires the differentiation of a function involving an exponential term and an inverse trigonometric term.

We rearrange the equation to express y as an explicit function of x : $y = e^x - \sin^{-1}(1 - x^2)$.

Then, we differentiate using standard rules: the derivative of e^x is e^x , and the chain rule is applied to the inverse sine function.

Step 2: Key Formula or Approach:

1. Rule: $\frac{d}{dx} \sin^{-1}(u) = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$.
2. Here, $u = 1 - x^2$, so $\frac{du}{dx} = -2x$.

Step 3: Detailed Explanation:

Differentiating both sides of $y = e^x - \sin^{-1}(1 - x^2)$ with respect to x :

$$\frac{dy}{dx} = \frac{d}{dx}(e^x) - \frac{d}{dx}(\sin^{-1}(1 - x^2))$$

The first part is simply e^x .

For the second part:

$$\frac{d}{dx}(\sin^{-1}(1 - x^2)) = \frac{1}{\sqrt{1 - (1 - x^2)^2}} \cdot (-2x)$$

Simplify the expression inside the square root:

$$1 - (1 - x^2)^2 = 1 - (1 - 2x^2 + x^4) = 1 - 1 + 2x^2 - x^4 = 2x^2 - x^4$$

The square root is $\sqrt{2x^2 - x^4} = \sqrt{x^2(2 - x^2)} = |x|\sqrt{2 - x^2}$.

Substituting this back into the derivative:

$$\frac{dy}{dx} = e^x - \left[\frac{-2x}{|x|\sqrt{2 - x^2}} \right]$$

Assuming $x > 0$ (which is standard for such problems unless specified), $|x| = x$:

$$\frac{dy}{dx} = e^x + \frac{2x}{x\sqrt{2-x^2}} = e^x + \frac{2}{\sqrt{2-x^2}}$$

This gives us the final derivative expression.

Step 4: Final Answer:

The derivative $\frac{dy}{dx}$ is $e^x + \frac{2}{\sqrt{2-x^2}}$.

This matches Option (C).

Quick Tip: When simplifying radicals like $\sqrt{ax^2 - bx^4}$, you can factor out x to get $|x|\sqrt{a - bx^2}$. In multiple-choice exams, if the absolute value is not in the options, assume x is positive and simplify it as x .

34. If $y(x) = x^x$, $x > 0$, then $y''(2) - 2y'(2) =$

- (A) $4\log_e 2 + 2$
- (B) $4(\log_e 2)^2 + 2$
- (C) $4\log_e 2 - 2$
- (D) $4(\log_e 2)^2 - 2$

Correct Answer: (D) $4(\log_e 2)^2 - 2$

Solution:

Step 1: Understanding the Concept:

This problem involves logarithmic differentiation and evaluating second-order derivatives.

When a function is of the form $u(x)^{v(x)}$, it is usually best to take the natural logarithm of both sides before differentiating.

Once the first derivative is found, we differentiate again using the product rule to find the second derivative.

Step 2: Key Formula or Approach:

1. $y = x^x \implies \ln y = x \ln x$.
2. Differentiate: $\frac{1}{y} y' = 1 + \ln x \implies y' = y(1 + \ln x)$.
3. Second derivative: $y'' = y'(1 + \ln x) + y(\frac{1}{x})$.

Step 3: Detailed Explanation:

Given $y = x^x$. At $x = 2$, $y(2) = 2^2 = 4$.

Differentiating: $y' = x^x(1 + \ln x)$.

At $x = 2$: $y'(2) = 4(1 + \ln 2) = 4 + 4\ln 2$.

Now find y'' :

$$y'' = \frac{d}{dx}[y(1 + \ln x)] = y'(1 + \ln x) + y \cdot \frac{1}{x}$$

Evaluate at $x = 2$:

$$y''(2) = y'(2)(1 + \ln 2) + \frac{y(2)}{2}$$

$$y''(2) = [4(1 + \ln 2)](1 + \ln 2) + \frac{4}{2}$$

$$y''(2) = 4(1 + \ln 2)^2 + 2$$

Expand the square:

$$y''(2) = 4(1 + 2\ln 2 + (\ln 2)^2) + 2 = 4 + 8\ln 2 + 4(\ln 2)^2 + 2 = 6 + 8\ln 2 + 4(\ln 2)^2$$

Now, calculate the expression requested: $y''(2) - 2y'(2)$

$$[6 + 8\ln 2 + 4(\ln 2)^2] - 2[4 + 4\ln 2]$$

$$= 6 + 8 \ln 2 + 4(\ln 2)^2 - 8 - 8 \ln 2$$

$$= (6 - 8) + (8 \ln 2 - 8 \ln 2) + 4(\ln 2)^2$$

$$= -2 + 4(\ln 2)^2$$

This simplifies to $4(\log_e 2)^2 - 2$.

Step 4: Final Answer:

The value of $y''(2) - 2y'(2)$ is $4(\log_e 2)^2 - 2$.

This matches Option (D).

Quick Tip: For any $y = f(x)^{g(x)}$, the derivative is $y' = y \cdot \frac{d}{dx}[g(x) \ln f(x)]$. This general form is helpful for any base/exponent combination.

35. If $z = x^2 y^3 + e^y \sin x$, then $\frac{\partial^2 z}{\partial x \partial y} =$

- (A) $6xy^2 + e^y \cos x$
- (B) $3x^2 y^2 + e^y \sin x$
- (C) $3x^2 y^2 + e^y \cos x$
- (D) $6xy^2 + e^y \sin x$

Correct Answer: (A) $6xy^2 + e^y \cos x$

Solution:

Step 1: Understanding the Concept:

This problem involves partial differentiation of a multivariable function.

The notation $\frac{\partial^2 z}{\partial x \partial y}$ means we first differentiate z with respect to y (treating x as a constant), and then differentiate the result with respect to x (treating y as a constant).

According to Clairaut's theorem, for most smooth functions, the order of differentiation does not matter ($\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$).

Step 2: Key Formula or Approach:

1. First partial derivative: $\frac{\partial z}{\partial y}$.
2. Second partial derivative: $\frac{\partial}{\partial x} \left(\frac{\partial z}{\partial y} \right)$.

Step 3: Detailed Explanation:

Given $z = x^2 y^3 + e^y \sin x$.

Differentiate with respect to y :

$$\frac{\partial z}{\partial y} = x^2 \frac{d}{dy}(y^3) + \sin x \frac{d}{dy}(e^y)$$

$$\frac{\partial z}{\partial y} = x^2(3y^2) + (\sin x)e^y = 3x^2 y^2 + e^y \sin x$$

Now, differentiate this result with respect to x :

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x} [3x^2 y^2 + e^y \sin x]$$

$$= 3y^2 \frac{d}{dx}(x^2) + e^y \frac{d}{dx}(\sin x)$$

$$= 3y^2(2x) + e^y(\cos x)$$

$$= 6xy^2 + e^y \cos x$$

This gives the mixed second-order partial derivative.

Step 4: Final Answer:

The partial derivative $\frac{\partial^2 z}{\partial x \partial y}$ is $6xy^2 + e^y \cos x$.

This matches Option (A).

Quick Tip: Always keep track of which variable is being treated as a constant. When differentiating with respect to x , anything without an x is a constant. When differentiating with respect to y , anything without a y is a constant.

36. $\int \frac{dx}{\sin^2 x \cos^2 x} =$

- (A) $\tan x + \cot x + c$
- (B) $\tan x - \cot x + c$
- (C) $\tan x \cot x + c$
- (D) $\tan x \sec x + c$

Correct Answer: (B) $\tan x - \cot x + c$

Solution:

Step 1: Understanding the Concept:

This problem involves the integration of a trigonometric rational function.

Direct integration is difficult, so we use trigonometric identities to simplify the integrand into standard forms.

The most useful identity here is the Pythagorean identity: $\sin^2 x + \cos^2 x = 1$.

Step 2: Key Formula or Approach:

1. Identity: $1 = \sin^2 x + \cos^2 x$.
2. Standard Integrals: $\int \sec^2 x dx = \tan x + c$ and $\int \csc^2 x dx = -\cot x + c$.

Step 3: Detailed Explanation:

The integral is $I = \int \frac{dx}{\sin^2 x \cos^2 x}$.

Substitute $1 = \sin^2 x + \cos^2 x$ in the numerator:

$$I = \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx$$

Split the fraction into two parts:

$$I = \int \left(\frac{\sin^2 x}{\sin^2 x \cos^2 x} + \frac{\cos^2 x}{\sin^2 x \cos^2 x} \right) dx$$

Simplify each term:

$$I = \int \left(\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} \right) dx$$

Using the reciprocal identities $\sec x = \frac{1}{\cos x}$ and $\csc x = \frac{1}{\sin x}$:

$$I = \int (\sec^2 x + \csc^2 x) dx$$

Now, apply standard integration results:

$$I = \int \sec^2 x dx + \int \csc^2 x dx$$

$$I = \tan x + (-\cot x) + c = \tan x - \cot x + c$$

This gives the final result of the indefinite integral.

Step 4: Final Answer:

The integral is equal to $\tan x - \cot x + c$.

This matches Option (B).

Quick Tip: Whenever you have $\sin^2 x \cdot \cos^2 x$ in the denominator, you can also use the double angle identity: $\sin x \cos x = \frac{1}{2} \sin 2x$. The denominator becomes $\frac{1}{4} \sin^2 2x$, leading to $\int 4 \csc^2 2x dx = -2 \cot 2x + c$. Note that $\tan x - \cot x = -2 \cot 2x$.

37. $\int \frac{dx}{\sqrt{x+1} + \sqrt{x}} =$

- (A) $\frac{2}{3}[(x+1)^{\frac{3}{2}} - (x)^{\frac{3}{2}}] + c$
(B) $\frac{2}{3}[(x+1)^{\frac{3}{2}} + (x)^{\frac{3}{2}}] + c$
(C) $\frac{3}{2}[(x+1)^{\frac{3}{2}} - (x)^{\frac{3}{2}}] + c$
(D) $\frac{3}{2}[(x+1)^{\frac{3}{2}} + (x)^{\frac{3}{2}}] + c$

Correct Answer: (A) $\frac{2}{3}[(x+1)^{\frac{3}{2}} - (x)^{\frac{3}{2}}] + c$

Solution:**Step 1: Understanding the Concept:**

This problem involves integrating a function with radicals in the denominator.

The best way to simplify such integrands is to rationalize the denominator.

Rationalization converts the sum or difference of square roots in the denominator into a simpler form, often eliminating the roots entirely.

Step 2: Key Formula or Approach:

1. Rationalization: Multiply numerator and denominator by the conjugate $\sqrt{x+1} - \sqrt{x}$.
2. Identity: $(a+b)(a-b) = a^2 - b^2$.
3. Standard Integral: $\int x^n dx = \frac{x^{n+1}}{n+1}$.

Step 3: Detailed Explanation:

The integral is $I = \int \frac{dx}{\sqrt{x+1} + \sqrt{x}}$.

Multiply numerator and denominator by the conjugate $(\sqrt{x+1} - \sqrt{x})$:

$$I = \int \frac{(\sqrt{x+1} - \sqrt{x})}{(\sqrt{x+1} + \sqrt{x})(\sqrt{x+1} - \sqrt{x})} dx$$

Evaluate the denominator using the difference of squares:

$$(\sqrt{x+1})^2 - (\sqrt{x})^2 = (x+1) - x = 1$$

The integral becomes:

$$I = \int (\sqrt{x+1} - \sqrt{x}) dx$$

Rewrite using fractional exponents:

$$I = \int ((x+1)^{\frac{1}{2}} - x^{\frac{1}{2}}) dx$$

Integrate term by term:

$$I = \int (x+1)^{\frac{1}{2}} dx - \int x^{\frac{1}{2}} dx$$

$$I = \frac{(x+1)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$I = \frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{2}{3}x^{\frac{3}{2}} + c$$

Factoring out $\frac{2}{3}$:

$$I = \frac{2}{3}[(x+1)^{\frac{3}{2}} - x^{\frac{3}{2}}] + c$$

This matches the required form.

Step 4: Final Answer:

The integral is equal to $\frac{2}{3}[(x+1)^{\frac{3}{2}} - (x)^{\frac{3}{2}}] + c$.

This matches Option (A).

Quick Tip: Whenever the denominator contains $\sqrt{f(x)} \pm \sqrt{g(x)}$, and the difference $f(x) - g(x)$ is a constant, rationalize immediately. It simplifies the integration significantly.

38. If $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx = A \sec x + B \csc x + c$, then (A, B) are

- (A) $(1, 1)$
- (B) $(-1, -1)$
- (C) $(1, -1)$
- (D) $(-1, 1)$

Correct Answer: (C) $(1, -1)$

Solution:

Step 1: Understanding the Concept:

This problem involves the integration of a trigonometric fraction where the numerator is a sum of cubes and the denominator is a product of squares.

The best approach is to split the fraction into two simpler components.

Once simplified, we apply standard trigonometric integral rules to find the coefficients A and B .

Step 2: Key Formula or Approach:

1. Split the fraction: $\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} = \frac{\sin^3 x}{\sin^2 x \cos^2 x} + \frac{\cos^3 x}{\sin^2 x \cos^2 x}$.

2. Standard Integrals: $\int \sec x \tan x dx = \sec x + c$ and $\int \csc x \cot x dx = -\csc x + c$.

Step 3: Detailed Explanation:

Starting with the integrand:

$$\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} = \frac{\sin^3 x}{\sin^2 x \cos^2 x} + \frac{\cos^3 x}{\sin^2 x \cos^2 x}$$

Simplify each term by canceling common factors:

$$= \frac{\sin x}{\cos^2 x} + \frac{\cos x}{\sin^2 x}$$

Rewrite the terms as products of standard trigonometric functions:

$$= \left(\frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} \right) + \left(\frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} \right)$$

$$= \tan x \sec x + \cot x \csc x$$

Now, integrate the expression:

$$I = \int (\sec x \tan x + \csc x \cot x) dx$$

Using standard integration formulas:

$$I = \int \sec x \tan x dx + \int \csc x \cot x dx$$

$$I = \sec x + (-\csc x) + c = \sec x - \csc x + c$$

Comparing this with the given form $A \sec x + B \csc x + c$:

$$A = 1, B = -1$$

The coordinates (A, B) are $(1, -1)$.

Step 4: Final Answer:

The values of (A, B) are $(1, -1)$.

This matches Option (C).

Quick Tip: Breaking a complex fraction into a sum of simpler fractions is often the first step in solving integrals of algebraic or trigonometric ratios. Always look for potential cancellations between numerator and denominator.

39. The integral of $f(x) = 1 + x^2 + x^4$ with respect to x^2 is

- (A) $x + \frac{x^3}{3} + \frac{x^5}{5} + C$
- (B) $\frac{x^3}{3} + \frac{x^5}{5} + C$
- (C) $x^2 + \frac{x^4}{4} + \frac{x^6}{6} + C$
- (D) $x^2 + \frac{x^4}{2} + \frac{x^6}{3} + C$

Correct Answer: (D) $x^2 + \frac{x^4}{2} + \frac{x^6}{3} + C$

Solution:

Step 1: Understanding the Concept:

The phrase "integral with respect to x^2 " means that we treat x^2 as the variable of integration.

In traditional notation, if we let $u = x^2$, we are asked to find $\int (1 + u + u^2)du$.

This is different from the usual "integral with respect to x " which would be written as $\int f(x)dx$.

Step 2: Key Formula or Approach:

1. Substitution: Let $u = x^2$.
2. Rewrite the function in terms of u : $f(u) = 1 + u + u^2$.
3. Integrate with respect to u : $\int (1 + u + u^2)du$.

Step 3: Detailed Explanation:

Let $u = x^2$. Then $x^4 = (x^2)^2 = u^2$.

The integral of $1 + x^2 + x^4$ with respect to x^2 is equivalent to:

$$\int (1 + u + u^2) du$$

Applying the power rule for integration:

$$\int 1 du + \int u du + \int u^2 du$$

$$= u + \frac{u^2}{2} + \frac{u^3}{3} + C$$

Now, substitute back $u = x^2$:

$$= x^2 + \frac{(x^2)^2}{2} + \frac{(x^2)^3}{3} + C$$

$$= x^2 + \frac{x^4}{2} + \frac{x^6}{3} + C$$

This gives the final antiderivative. Note that if we had integrated with respect to x , the answer would have been Option (A). The phrasing "with respect to x^2 " is key.

Step 4: Final Answer:

The integral is $x^2 + \frac{x^4}{2} + \frac{x^6}{3} + C$.

This matches Option (D).

Quick Tip: Always read the differential part of the integral carefully. $\int f(x)dx$ and $\int f(x)d(g(x))$ are different operations. In the latter, you effectively treat $g(x)$ as your new variable u .

40. $\int_0^{\pi/2} \frac{\sin^{100} x}{\sin^{100} x + \cos^{100} x} dx =$

- (A) $\pi/2$
(B) $\pi/4$
(C) 100
(D) 50

Correct Answer: (B) $\pi/4$

Solution:

Step 1: Understanding the Concept:

This problem involves a definite integral of a trigonometric rational function with high powers.

A common property of definite integrals, often called the "King's property", is:

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx.$$

In the context of the limits 0 to $\pi/2$, this property becomes $\int_0^{\pi/2} f(x)dx = \int_0^{\pi/2} f(\frac{\pi}{2}-x)dx$.

Step 2: Key Formula or Approach:

1. Let $I = \int_0^{\pi/2} \frac{\sin^{100} x}{\sin^{100} x + \cos^{100} x} dx$.
2. Apply property: $\sin(\frac{\pi}{2}-x) = \cos x$ and $\cos(\frac{\pi}{2}-x) = \sin x$.

Step 3: Detailed Explanation:

Let $I = \int_0^{\pi/2} \frac{\sin^{100} x}{\sin^{100} x + \cos^{100} x} dx$. (Eq. 1)

Applying the property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$:

$$I = \int_0^{\pi/2} \frac{\sin^{100}(\frac{\pi}{2}-x)}{\sin^{100}(\frac{\pi}{2}-x) + \cos^{100}(\frac{\pi}{2}-x)} dx$$

Using complementary angle identities:

$$I = \int_0^{\pi/2} \frac{\cos^{100} x}{\cos^{100} x + \sin^{100} x} dx$$

(Eq. 2)

Now, add Eq. 1 and Eq. 2:

$$2I = \int_0^{\pi/2} \frac{\sin^{100} x}{\sin^{100} x + \cos^{100} x} dx + \int_0^{\pi/2} \frac{\cos^{100} x}{\sin^{100} x + \cos^{100} x} dx$$

$$2I = \int_0^{\pi/2} \left(\frac{\sin^{100} x + \cos^{100} x}{\sin^{100} x + \cos^{100} x} \right) dx$$

$$2I = \int_0^{\pi/2} 1 dx$$

Integrate:

$$2I = [x]_0^{\pi/2} = \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

Divide by 2:

$$I = \frac{\pi}{4}$$

Note that this result is independent of the exponent 100. The same result would hold for any exponent n .

Step 4: Final Answer:

The definite integral is equal to $\pi/4$.

This matches Option (B).

Quick Tip: For any integral of the form $\int_0^{\pi/2} \frac{f(\sin x)}{f(\sin x) + f(\cos x)} dx$, the answer is always $\frac{1}{2} \cdot \frac{\pi}{2} = \frac{\pi}{4}$. This is a very powerful shortcut for competitive exams.

41. The value of the definite integral $\int_0^1 x\sqrt{x^2+4} dx$ is

- (A) $\frac{1}{3}[5\sqrt{5}-4]$
- (B) $\frac{1}{2}[5\sqrt{5}-8]$
- (C) $\frac{1}{3}[5\sqrt{5}-8]$
- (D) $\frac{1}{3}[5\sqrt{5}+4]$

Correct Answer: (C) $\frac{1}{3}[5\sqrt{5}-8]$

Solution:

Step 1: Understanding the Concept:

Evaluating a definite integral often requires simplifying the integrand using substitution.

In this problem, the integrand is the product of x and a composite function $\sqrt{x^2+4}$.

Notice that the derivative of the inner function x^2+4 is $2x$, which is directly proportional to the outer factor x .

This indicates that the u -substitution method is ideal. By letting $u = x^2 + 4$, we can transform the integral into a simpler power function integral.

Furthermore, in a definite integral, it is more efficient to change the limits of integration to correspond to the new variable u , eliminating the need for back-substitution.

Step 2: Key Formula or Approach:

1. Let $u = x^2 + 4$.
2. Then, find the differential $du = 2x dx$, which implies $x dx = \frac{1}{2} du$.
3. Change the limits of integration:
 - When $x = 0$, $u = 0^2 + 4 = 4$.
 - When $x = 1$, $u = 1^2 + 4 = 5$.
4. Use the power rule for integration: $\int u^n du = \frac{u^{n+1}}{n+1} + C$.

Step 3: Detailed Explanation:

Substitute the variables and limits into the integral:

$$I = \int_0^1 x \sqrt{x^2 + 4} dx = \int_4^5 \sqrt{u} \cdot \left(\frac{1}{2} du\right)$$

Factor out the constant:

$$I = \frac{1}{2} \int_4^5 u^{1/2} du$$

Apply the power rule:

$$I = \frac{1}{2} \left[\frac{u^{1/2+1}}{1/2+1} \right]_4^5 = \frac{1}{2} \left[\frac{u^{3/2}}{3/2} \right]_4^5$$

Simplify the constant coefficient $\frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}$:

$$I = \frac{1}{3} [u^{3/2}]_4^5$$

Evaluate the limits:

$$I = \frac{1}{3} [5^{3/2} - 4^{3/2}]$$

Simplify the exponential terms:

$$- 5^{3/2} = 5^1 \cdot 5^{1/2} = 5\sqrt{5}.$$

$$- 4^{3/2} = (\sqrt{4})^3 = 2^3 = 8.$$

Substituting these back:

$$I = \frac{1}{3} [5\sqrt{5} - 8]$$

This matches the structure of Option (C) exactly.

Step 4: Final Answer:

The final value of the definite integral is $\frac{1}{3}[5\sqrt{5} - 8]$.

Quick Tip: Always check if the derivative of the "inner" part of a composite function is present elsewhere in the product. Here, $\frac{d}{dx}(x^2 + 4) = 2x$. This is a "Substitution Signal." Changing limits for definite integrals prevents "double-tracking" variables and saves significant time.

42. The value of the integral $\int_{-\pi/6}^{\pi/6} \frac{\sin^5 x \cos^3 x}{x^4} dx =$

- (A) $\pi/2$
- (B) $\pi/4$
- (C) 0
- (D) 1

Correct Answer: (C) 0

Solution:

Step 1: Understanding the Concept:

Definite integrals with symmetric limits (of the form $\int_{-a}^a f(x) dx$) can often be simplified by checking the parity of the function $f(x)$.

A function is "odd" if $f(-x) = -f(x)$ and "even" if $f(-x) = f(x)$.

A powerful property states that the integral of an odd function over a symmetric interval is always zero, regardless of how complex the expression looks.

Conversely, for an even function, the integral is $2 \int_0^a f(x) dx$.

Step 2: Key Formula or Approach:

1. Identify the function: $f(x) = \frac{\sin^5 x \cos^3 x}{x^4}$.
2. Test for parity by calculating $f(-x)$.
3. Use the property: If $f(x)$ is odd, then $\int_{-a}^a f(x) dx = 0$.

Step 3: Detailed Explanation:

Let's substitute $-x$ into the function:

$$f(-x) = \frac{\sin^5(-x) \cos^3(-x)}{(-x)^4}$$

We apply the known trigonometric and algebraic parity rules:

- $\sin(-x) = -\sin x$ (Sine is an odd function).
- $\cos(-x) = \cos x$ (Cosine is an even function).
- $(-x)^4 = x^4$ (Any even power is an even function).

Substituting these:

$$f(-x) = \frac{(-\sin x)^5 (\cos x)^3}{x^4}$$

Since $(-1)^5 = -1$, the expression becomes:

$$f(-x) = \frac{-\sin^5 x \cos^3 x}{x^4} = -f(x)$$

The result $f(-x) = -f(x)$ confirms that $f(x)$ is an **odd function**.

Applying the symmetric interval property:

$$\int_{-\pi/6}^{\pi/6} \frac{\sin^5 x \cos^3 x}{x^4} dx = 0$$

Because the areas under the curve for negative x and positive x are identical in magnitude but opposite in sign, they cancel each other out completely.

Step 4: Final Answer:

The value of the integral is 0.

Quick Tip: In any competitive exam, if you see an extremely complex trigonometric or algebraic integral with limits from $-a$ to a , check for "Oddity" immediately. It is a very common trap designed to catch students who try to integrate by parts or use substitution unnecessarily.

43. $\int \frac{dx}{\sqrt{16-25x^2}} =$

- (A) $\frac{1}{5} \sin^{-1}\left(\frac{5x}{4}\right) + c$
(B) $\sin^{-1}\left(\frac{5x}{4}\right) + c$
(C) $\frac{1}{5} \sin^{-1}\left(\frac{x}{4}\right) + c$
(D) $\frac{1}{5} \sin^{-1}\left(\frac{4x}{5}\right) + c$

Correct Answer: (A) $\frac{1}{5} \sin^{-1}\left(\frac{5x}{4}\right) + c$

Solution:

Step 1: Understanding the Concept:

This problem requires the use of standard inverse trigonometric integration formulas.

The integrand has the form $\frac{1}{\sqrt{a^2-(kx)^2}}$, which is closely related to the derivative of the inverse sine function.

By performing a linear substitution or using a generalized formula, we can express the result in terms of $\sin^{-1}$.

Step 2: Key Formula or Approach:

1. The standard integral formula is: $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c$.
2. If the variable is scaled as (kx) , the formula generalizes to:

$$\int \frac{dx}{\sqrt{a^2-(kx)^2}} = \frac{1}{k} \sin^{-1}\left(\frac{kx}{a}\right) + c$$

3. In this case, $16 = 4^2$ and $25x^2 = (5x)^2$, so $a = 4$ and $k = 5$.

Step 3: Detailed Explanation:

Let's rewrite the integral to match the standard form:

$$I = \int \frac{dx}{\sqrt{4^2 - (5x)^2}}$$

Using substitution:

Let $u = 5x$, then $du = 5 dx$, which means $dx = \frac{1}{5} du$.

Substituting these into the integral:

$$I = \int \frac{\frac{1}{5} du}{\sqrt{4^2 - u^2}}$$

Factor out the constant $1/5$:

$$I = \frac{1}{5} \int \frac{du}{\sqrt{4^2 - u^2}}$$

Apply the standard inverse sine formula:

$$I = \frac{1}{5} \left[\sin^{-1} \left(\frac{u}{4} \right) \right] + c$$

Back-substitute $u = 5x$:

$$I = \frac{1}{5} \sin^{-1} \left(\frac{5x}{4} \right) + c$$

This matches the result in Option (A).

Step 4: Final Answer:

The result of the integration is $\frac{1}{5} \sin^{-1} \left(\frac{5x}{4} \right) + c$.

Quick Tip: Always factor out the constant from the x^2 term first if you prefer the standard form, or remember that if x is multiplied by k , the integral is divided by k . Here, $\sqrt{25} = 5$, so there **must** be a $1/5$ outside.

44. The solution of the differential equation $x \frac{dy}{dx} + y = 0$ passing through the point $(1, 1)$ is $y =$

- (A) x^2
- (B) x^{-1}
- (C) x^{-2}
- (D) x

Correct Answer: (B) x^{-1}

Solution:

Step 1: Understanding the Concept:

A differential equation relates a function to its derivatives. Solving it involves finding a function that satisfies this relationship.

The given equation $x \frac{dy}{dx} + y = 0$ is a first-order linear differential equation.

It can be solved using the "Variable Separable" method or by recognizing it as a total differential.

A total differential is an expression of the form $d(f(x, y))$.

Once the general solution (containing an arbitrary constant C) is found, we use the specific point $(1, 1)$ to find the value of C .

Step 2: Key Formula or Approach:

1. Observe the expression: $x \frac{dy}{dx} + y$.
2. Multiply by dx : $x dy + y dx = 0$.
3. Recognize the product rule of differentiation: $d(xy) = x dy + y dx$.
4. Integrate both sides: $\int d(xy) = \int 0 dx$.

Step 3: Detailed Explanation:

From the observation above:

$$d(xy) = 0$$

Integrating both sides:

$$xy = C$$

This is the general solution for the differential equation.

Now, we apply the initial condition: the curve passes through the point (1, 1).

Substitute $x = 1$ and $y = 1$:

$$(1) \cdot (1) = C \implies C = 1$$

So, the specific solution is:

$$xy = 1$$

Solving for y :

$$y = \frac{1}{x}$$

Using exponent notation, this is written as:

$$y = x^{-1}$$

This matches Option (B).

Step 4: Final Answer:

The specific solution to the differential equation is $y = x^{-1}$.

Quick Tip: Always look for "Hidden Product Rules." Expressions like $x \frac{dy}{dx} + y$ or $xdy + ydx$ appear frequently and can be integrated instantly as xy . It avoids the extra steps of variable separation and logarithms.

45. The degree of the differential equation $y = x \frac{dy}{dx} + a \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ is

- (A) 4
- (B) 3
- (C) 2
- (D) 1

Correct Answer: (C) 2

Solution:

Step 1: Understanding the Concept:

The "order" of a differential equation is the highest derivative present.

The "degree" of a differential equation is the power of the highest order derivative, provided the equation is a polynomial in its derivatives.

To find the degree, the equation must first be cleared of radicals (square roots, cube roots) and fractions involving derivatives.

If an equation contains a derivative inside a square root, it is not yet in polynomial form. Squaring both sides (after appropriate isolation) is usually necessary.

Step 2: Key Formula or Approach:

1. Isolate the radical term.
2. Square both sides to eliminate the square root.
3. Identify the highest order derivative and find its highest power.

Step 3: Detailed Explanation:

Given the equation:

$$y = x \left(\frac{dy}{dx} \right) + a \sqrt{1 + \left(\frac{dy}{dx} \right)^2}$$

Let's denote $\frac{dy}{dx}$ as y' for simplicity:

$$y = xy' + a\sqrt{1 + (y')^2}$$

Isolate the radical part:

$$y - xy' = a\sqrt{1 + (y')^2}$$

Now, square both sides to remove the radical:

$$(y - xy')^2 = [a\sqrt{1 + (y')^2}]^2$$

$$(y - xy')^2 = a^2(1 + (y')^2)$$

Expand the left side:

$$y^2 - 2xyy' + x^2(y')^2 = a^2 + a^2(y')^2$$

In this final form, the highest derivative present is y' (order 1).

The highest power of this highest derivative y' is 2.

Therefore, the degree of the differential equation is 2.

Step 4: Final Answer:

The degree of the given differential equation is 2.

Quick Tip: "Polynomial in derivatives" is the key phrase. If you see a derivative inside a radical ($\sqrt{\quad}$), a trigonometric function ($\sin(dy/dx)$), or a logarithm ($\log(dy/dx)$), the degree may be undefined or require transformation. Radicals can usually be eliminated to reveal the true degree.

46. The order of the differential equation of all circles passing through the origin and having their centers on the x -axis is

- (A) 4
- (B) 3
- (C) 2
- (D) 1

Correct Answer: (D) 1

Solution:

Step 1: Understanding the Concept:

The "order" of the differential equation representing a family of curves is equal to the number of independent arbitrary constants present in the general equation of that family.

To find the order, we first write down the general algebraic equation of the family based on the given geometric properties.

By counting the distinct constants that can be varied to represent different curves in the family, we directly determine the order.

Step 2: Key Formula or Approach:

1. Center of the circle lies on the x -axis: Let the center be $(a, 0)$.
2. Circle passes through the origin $(0, 0)$: This implies the distance from the center to the origin is the radius r .

$$r = \sqrt{(a - 0)^2 + (0 - 0)^2} = \sqrt{a^2} = |a|$$

3. The general equation of a circle is $(x - h)^2 + (y - k)^2 = r^2$.

Step 3: Detailed Explanation:

Substitute the parameters found into the circle equation:

$$(x - a)^2 + (y - 0)^2 = a^2$$

Expanding the equation:

$$x^2 - 2ax + a^2 + y^2 = a^2$$

Cancel a^2 from both sides:

$$x^2 + y^2 - 2ax = 0$$

In this algebraic equation, there is only **one** arbitrary constant, which is ' a '.

The variable ' a ' can be any real number to define a specific circle in this family.

Since there is exactly one independent arbitrary constant, the differential equation formed by eliminating this constant will be of the first order.

Step 4: Final Answer:

The order of the differential equation is 1.

Quick Tip: Order = Number of independent arbitrary constants.

If a problem asks for "Order," don't bother differentiating. Just write the general equation and count the constants (a, b, c). It is a purely algebraic count.

47. If a and b are arbitrary constants, then the differential equation representing the family of curves $y = a \sin(x + b)$ is

- (A) $\frac{d^2y}{dx^2} - y = 0$
- (B) $\frac{d^2y}{dx^2} + y = 0$
- (C) $\frac{d^2y}{dx^2} - y^2 = 0$
- (D) $\frac{dy}{dx} - y = 0$

Correct Answer: (B) $\frac{d^2y}{dx^2} + y = 0$

Solution:

Step 1: Understanding the Concept:

To form a differential equation from a given family of curves, we must differentiate the function as many times as there are arbitrary constants.

Here, there are two constants (a and b), so we need to differentiate twice.

After differentiating, we eliminate the arbitrary constants by using substitution from the original equation or intermediate derivatives.

Step 2: Key Formula or Approach:

1. Given: $y = a \sin(x + b)$.
2. Differentiate twice with respect to x .
3. Use the chain rule and known trigonometric derivatives: $\frac{d}{dx} \sin(u) = \cos(u)$ and $\frac{d}{dx} \cos(u) = -\sin(u)$.

Step 3: Detailed Explanation:

Original equation:

$$y = a \sin(x + b)$$

... (1)

First derivative:

$$\frac{dy}{dx} = a \cos(x + b) \cdot \frac{d}{dx}(x + b)$$

$$\frac{dy}{dx} = a \cos(x + b)$$

... (2)

Second derivative:

$$\frac{d^2y}{dx^2} = -a \sin(x + b)$$

... (3)

Compare the second derivative (3) with the original equation (1). We can see that:

$$\frac{d^2y}{dx^2} = -(a \sin(x + b))$$

Substituting y from equation (1):

$$\frac{d^2y}{dx^2} = -y$$

Rearranging to standard differential equation form:

$$\frac{d^2y}{dx^2} + y = 0$$

This equation is independent of the arbitrary constants a and b , correctly representing the entire family of curves.

Step 4: Final Answer:

The differential equation is $\frac{d^2y}{dx^2} + y = 0$.

Quick Tip: Function $y = A\sin(\omega x + \phi)$ or $y = A\cos(\omega x + \phi)$ are classical solutions to the Simple Harmonic Motion equation: $\frac{d^2y}{dx^2} + \omega^2 y = 0$. Here $\omega = 1$, making the result immediate for anyone familiar with physics or signal processing.

48. The differential equation is $\frac{dy}{dx} + \frac{y}{x} = 0$ and $y(1) = 2$. Then the value of $y(3) =$

- (A) 2
- (B) 3
- (C) 2/3
- (D) 1

Correct Answer: (C) 2/3

Solution:

Step 1: Understanding the Concept:

This is an Initial Value Problem (IVP). We first find the general solution of the differential equation using "Separation of Variables."

Separation of variables involves moving all terms containing y to one side and all terms containing x to the other.

Once the general solution (with constant C) is found, we use the given condition $y(1) = 2$ to determine the specific value of C .

Finally, we substitute $x = 3$ into the specific solution to find the final value.

Step 2: Key Formula or Approach:

1. Separate variables: $\frac{dy}{dx} = -\frac{y}{x} \implies \frac{dy}{y} = -\frac{dx}{x}$.
2. Integrate both sides: $\int \frac{dy}{y} = -\int \frac{dx}{x}$.
3. Use $\int \frac{1}{u} du = \ln |u| + C$.

Step 3: Detailed Explanation:

Integrate the separated equation:

$$\ln |y| = -\ln |x| + \ln |C|$$

Combine logarithms on the right:

$$\ln |y| = \ln \left| \frac{C}{x} \right|$$

Exponentiate both sides:

$$y = \frac{C}{x}$$

This is the general solution. Now use the condition $y(1) = 2$:

$$2 = \frac{C}{1} \implies C = 2$$

So, the specific function is:

$$y = \frac{2}{x}$$

Now, we need to find $y(3)$:

$$y(3) = \frac{2}{3}$$

This result matches Option (C).

Step 4: Final Answer:

The value of $y(3)$ is $2/3$.

Quick Tip: Recognize $\frac{dy}{dx} + \frac{y}{x} = 0$ as $x dy + y dx = 0$, which is $d(xy) = 0$. Integrating gives $xy = C$. Then $(1)(2) = C \implies C = 2$. Thus $3y = 2 \implies y = 2/3$. This "Direct Integral" method is much faster.

49. The general solution of the differential equation $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$ is

- (A) $e^{-y} = e^x + \frac{x^3}{3} + c$
- (B) $e^y = e^x + \frac{x^3}{3} + c$
- (C) $e^y = e^x + x^3 + c$
- (D) $e^y = e^x + c$

Correct Answer: (B) $e^y = e^x + \frac{x^3}{3} + c$

Solution:

Step 1: Understanding the Concept:

Solving this differential equation involves variable separation.

The right-hand side has terms with powers. We use exponent laws ($e^{a-b} = e^a \cdot e^{-b}$) to factorize the expression.

If the right-hand side can be written as a product $f(x)g(y)$, then variables can be separated easily.

After separation, we integrate both sides independently to reach the general solution.

Step 2: Key Formula or Approach:

1. Rewrite exponents: $e^{x-y} = e^x \cdot e^{-y}$.
2. Factorize: $\frac{dy}{dx} = e^{-y}(e^x + x^2)$.
3. Separate: $\frac{dy}{e^{-y}} = (e^x + x^2) dx \implies e^y dy = (e^x + x^2) dx$.
4. Integrate: $\int e^y dy = \int (e^x + x^2) dx$.

Step 3: Detailed Explanation:

From the separation:

$$\int e^y dy = \int e^x dx + \int x^2 dx$$

Performing the standard integrations:

- $\int e^y dy = e^y$.
- $\int e^x dx = e^x$.
- $\int x^2 dx = \frac{x^3}{3}$.

Combining these into one equation with an arbitrary constant c :

$$e^y = e^x + \frac{x^3}{3} + c$$

This expression relates y and x implicitly and represents all possible functions that satisfy the original differential equation. It matches Option (B).

Step 4: Final Answer:

The general solution is $e^y = e^x + \frac{x^3}{3} + c$.

Quick Tip: Always look for common factors in exponents. Factorizing e^{-y} or e^{-x} is a very common starting point for solving ODEs involving exponential functions. It instantly turns a sum into a separable product.

50. The differential equation is $\frac{dy}{dx} + y \tan x = \sec x$ and $y(0) = 1$. Then the value of $y\left(\frac{\pi}{4}\right) =$

- (A) 0
- (B) $\sqrt{2}$
- (C) 1
- (D) -1

Correct Answer: (B) $\sqrt{2}$

Solution:

Step 1: Understanding the Concept:

The given equation $\frac{dy}{dx} + y \tan x = \sec x$ is a first-order linear differential equation of the standard form $\frac{dy}{dx} + P(x)y = Q(x)$.

To solve such equations, we find an "Integrating Factor" (IF) given by $IF = e^{\int P(x) dx}$.

Multiplying the entire equation by this factor allows the left side to be written as a single derivative, $\frac{d}{dx}(y \cdot IF)$.

After finding the general solution, we use the initial condition $y(0) = 1$ to find the constant C , and then evaluate at $x = \pi/4$.

Step 2: Key Formula or Approach:

1. Identify $P(x) = \tan x$ and $Q(x) = \sec x$.
2. Calculate $IF = e^{\int \tan x dx} = e^{\ln|\sec x|} = \sec x$.
3. Solution formula: $y \cdot IF = \int (Q(x) \cdot IF) dx + C$.
4. Evaluate: $y \cdot \sec x = \int \sec^2 x dx + C$.

Step 3: Detailed Explanation:

Perform the integration on the right side:

$$y \sec x = \tan x + C$$

Apply the initial condition $y(0) = 1$:

Substitute $x = 0$ and $y = 1$:

$$(1) \sec(0) = \tan(0) + C$$

Since $\sec(0) = 1$ and $\tan(0) = 0$:

$$1 \cdot 1 = 0 + C \implies C = 1$$

The specific solution is:

$$y \sec x = \tan x + 1$$

Rearrange to solve for y :

$$y = \frac{\tan x + 1}{\sec x} = \frac{\sin x / \cos x + 1}{1 / \cos x} = \sin x + \cos x$$

Now, calculate $y(\pi/4)$:

$$y\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{4}\right)$$

$$y\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

This result matches Option (B).

Step 4: Final Answer:

The value of $y(\pi/4)$ is $\sqrt{2}$.

Quick Tip: For $\frac{dy}{dx} + y \tan x = \sec x$, the integrating factor is always $\sec x$. Remember that $\int \sec^2 x \, dx = \tan x$. Simplifying the final expression $\frac{\tan x + 1}{\sec x}$ into $\sin x + \cos x$ makes the final calculation of $\pi/4$ much faster.

PHYSICS

51. If $P = F \cdot v \sin \beta t$ where F is force and v is velocity then the dimensions of P and β are

- (A) $[ML^2T^{-3}], [T^{-1}]$
- (B) $[MLT^{-2}], [T^{-2}]$
- (C) $[ML^2T^{-1}], [T^{-1}]$
- (D) $[ML^2T^3], [T^{-2}]$

Correct Answer: (A) $[ML^2T^{-3}], [T^{-1}]$

Solution:

Step 1: Understanding the Concept:

Dimensional analysis is a powerful tool in physics used to check the consistency of equations and derive relationships between different physical quantities. Every physical quantity can be expressed as a product of powers of fundamental quantities like Mass (M), Length (L), and Time (T).

A primary rule of dimensional analysis is the **Principle of Homogeneity**, which states that terms on both sides of a physical equation must have the same dimensions. Additionally, the arguments of transcendental functions (like sine, cosine, logarithms, and exponentials) must be dimensionless. In the given expression $P = F \cdot v \sin \beta t$, we use these rules to find the dimensions of the unknown quantities P and β .

Step 2: Key Formula or Approach:

1. Identify the dimensions of known quantities:

- Force (F) has dimensions $[MLT^{-2}]$.

- Velocity (v) has dimensions $[LT^{-1}]$.

- Time (t) has dimensions $[T]$.

2. Apply the rule of dimensionless arguments for trigonometric functions: The term βt inside the sine function must be dimensionless, i.e., $[\beta t] = [M^0 L^0 T^0] = [1]$.

3. Equate dimensions of both sides: $[P] = [F] \cdot [v] \cdot [\sin \beta t]$. Since $[\sin \beta t]$ is a numerical value and dimensionless, its dimensional formula is $[1]$.

Step 3: Detailed Explanation:

First, let us find the dimensions of β .

Since the argument of a sine function must be a pure number (dimensionless):

$$[\beta \cdot t] = [1]$$

$$[\beta] \cdot [T] = [1]$$

$$[\beta] = \frac{1}{[T]} = [T^{-1}]$$

This means β has the dimensions of frequency or angular velocity.

Next, let us find the dimensions of P .

According to the principle of homogeneity, the dimension of the left-hand side must equal the product of dimensions of the terms on the right-hand side.

$$[P] = [F] \times [v] \times [\sin(\beta t)]$$

Substituting the standard dimensional formulas:

$$[P] = [MLT^{-2}] \times [LT^{-1}] \times [1]$$

Multiply the components:

- For Mass (M): M^1
- For Length (L): $L^1 \times L^1 = L^2$
- For Time (T): $T^{-2} \times T^{-1} = T^{-3}$

Combining them, we get:

$$[P] = [ML^2T^{-3}]$$

This dimensional formula corresponds to Power (P), which is physically consistent as Power = Force $\times$ Velocity. By verifying the dimensional consistency, we ensure the validity of the expression in terms of physical laws.

Step 4: Final Answer:

The dimensions of P are $[ML^2T^{-3}]$ and the dimensions of β are $[T^{-1}]$.

This matches Option (A).

Quick Tip: Always remember that **arguments of functions like** $\sin, \cos, \log, e$ are dimensionless. This is the fastest way to find dimensions of constants inside such functions. Also, recognize that $F \times v$ is the standard formula for mechanical power, whose dimensions are $[ML^2T^{-3}]$.

52. If velocity V , energy E and time T are chosen as fundamental quantities then dimensional representation of surface tension in this system will be

- (A) $E^1V^{-2}T^{-2}$
- (B) $E^1V^{-1}T^{-2}$
- (C) $E^{-2}V^{-1}T^{-3}$
- (D) $E^1V^{-2}T^{-1}$

Correct Answer: (A) $E^1V^{-2}T^{-2}$

Solution:

Step 1: Understanding the Concept:

Standard dimensional analysis uses Mass (M), Length (L), and Time (T) as fundamental base quantities. However, any set of independent physical quantities can be chosen as a new set of fundamental dimensions.

To represent a physical quantity (like surface tension) in a new system, we express it as a product of powers of the new base quantities. Let the surface tension S be related to velocity V , energy E , and time T as:

$$S = E^a V^b T^c$$

where a, b, c are the numerical exponents we need to determine. By equating the dimensions of both sides in terms of the standard M, L, T system, we generate a system of linear equations for the exponents.

Step 2: Key Formula or Approach:

1. Write the M, L, T dimensions of all quantities:

- Surface Tension ($S = \text{Force/Length}$): $[MLT^{-2}]/[L] = [MT^{-2}]$.
- Velocity (V): $[LT^{-1}]$.
- Energy (E): $[ML^2T^{-2}]$.
- Time (T): $[T]$.

2. Set up the dimensional equation:

$$[MT^{-2}] = [ML^2T^{-2}]^a [LT^{-1}]^b [T]^c$$

Step 3: Detailed Explanation:

Expand the right-hand side:

$$[M^1 L^0 T^{-2}] = [M^a L^{2a} T^{-2a}] [L^b T^{-b}] [T^c]$$

$$[M^1 L^0 T^{-2}] = [M^a L^{2a+b} T^{-2a-b+c}]$$

Equating the powers of fundamental quantities on both sides:

1. For Mass (M): $a = 1$.
2. For Length (L): $2a + b = 0$.

Substituting $a = 1$:

$$2(1) + b = 0 \implies b = -2$$

3. For Time (T): $-2a - b + c = -2$.

Substituting $a = 1$ and $b = -2$:

$$-2(1) - (-2) + c = -2$$

$$-2 + 2 + c = -2 \implies c = -2$$

Thus, the exponents are $a = 1, b = -2, c = -2$.

Substituting these back into the relation $S = E^a V^b T^c$:

$$S = E^1 V^{-2} T^{-2}$$

This representation shows how surface tension scales with the chosen parameters E, V , and T .

It is a standard exercise in dimensional synthesis.

Step 4: Final Answer:

The dimensional representation of surface tension is $E^1 V^{-2} T^{-2}$.

This matches Option (A).

Quick Tip: To save time, check the units. Surface tension is J/m^2 (Energy per unit area). Since area has dimensions of $(\text{Velocity} \times \text{Time})^2$, Surface Tension must be $\text{Energy}/(\text{V} \cdot \text{T})^2 = E^1 V^{-2} T^{-2}$. This shortcut avoids solving simultaneous equations!

53. If $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$, then the angle between the two vectors $\vec{A}$ and $\vec{B}$ is

- (A) 0°
- (B) 180°
- (C) 120°
- (D) 90°

Correct Answer: (D) 90°

Solution:

Step 1: Understanding the Concept:

Vectors are quantities that possess both magnitude and direction. The magnitude of the resultant of two vectors depends on the angle between them.

The term $|\vec{A} + \vec{B}|$ represents the magnitude of the vector sum, while $|\vec{A} - \vec{B}|$ represents the magnitude of the vector difference. Geometrically, if we represent the vectors as adjacent sides of a parallelogram, these magnitudes correspond to the lengths of the two diagonals. If the lengths of the diagonals are equal, the parallelogram must be a rectangle, which implies that the vectors are perpendicular to each other.

Step 2: Key Formula or Approach:

The magnitude of a vector sum and difference is calculated as:

1. $|\vec{A} + \vec{B}|^2 = A^2 + B^2 + 2AB \cos \theta$
2. $|\vec{A} - \vec{B}|^2 = A^2 + B^2 - 2AB \cos \theta$

where θ is the angle between vectors $\vec{A}$ and $\vec{B}$. We equate these two expressions as per the given condition.

Step 3: Detailed Explanation:

Given $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$.

Squaring both sides of the equation:

$$|\vec{A} + \vec{B}|^2 = |\vec{A} - \vec{B}|^2$$

Using the magnitude formula expansion:

$$A^2 + B^2 + 2AB \cos \theta = A^2 + B^2 - 2AB \cos \theta$$

Subtract $A^2 + B^2$ from both sides:

$$2AB \cos \theta = -2AB \cos \theta$$

Bringing all terms to one side:

$$2AB \cos \theta + 2AB \cos \theta = 0$$

$$4AB \cos \theta = 0$$

Assuming the vectors $\vec{A}$ and $\vec{B}$ are non-zero vectors ($A \neq 0, B \neq 0$):

$$\cos \theta = 0$$

We know that the cosine of an angle is zero when the angle is 90° (or $\pi/2$ radians).

$$\theta = \cos^{-1}(0) = 90^\circ$$

This algebraic result confirms the geometric intuition that the diagonals of a parallelogram are equal if and only if the parallelogram is a rectangle. Thus, the vectors must be orthogonal

(at right angles).

Step 4: Final Answer:

The angle between the two vectors is 90° .

This matches Option (D).

Quick Tip: Remember the geometric interpretation: In a parallelogram formed by vectors $\vec{A}$ and $\vec{B}$, the diagonals are $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$. **Diagonals are equal only in a rectangle**, so the angle between the sides must be 90° .

54. An aeroplane is moving in a circular path with a speed of 450 Kmph. What is the change in velocity in half revolution?

- (A) 0 Kmph
- (B) 450 Kmph
- (C) 250 Kmph
- (D) 900 Kmph

Correct Answer: (D) 900 Kmph

Solution:

Step 1: Understanding the Concept:

Velocity is a vector quantity, meaning it has both magnitude (speed) and direction. Even if an object moves with a constant speed in a circular path, its velocity is continuously changing because its direction is changing.

In one full revolution, the object returns to its starting point with the same velocity vector, so the total change is zero. However, in a half revolution, the object is at the diametrically opposite point, moving in exactly the opposite direction compared to its initial state. The change in velocity is the vector difference between the final and initial velocities.

Step 2: Key Formula or Approach:

1. Let the speed be $v = 450 \text{ Kmph}$.
2. Define the initial velocity vector: $\vec{v}_1 = v\hat{j}$.
3. Define the final velocity vector after half revolution: $\vec{v}_2 = -v\hat{j}$ (since it moves in the opposite direction).
4. Change in velocity: $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$.

Step 3: Detailed Explanation:

Let the plane start at a point on the circle where its velocity is directed upwards (let's say along the positive y-axis).

Initial velocity: $\vec{v}_{\text{initial}} = 450\hat{j}$

After half a revolution, the plane has traveled 180° along the circle. At this position, its velocity is directed exactly downwards (along the negative y-axis).

Final velocity: $\vec{v}_{\text{final}} = -450\hat{j}$

The magnitude of the change in velocity is calculated as:

$$|\Delta\vec{v}| = |\vec{v}_{\text{final}} - \vec{v}_{\text{initial}}|$$

$$|\Delta\vec{v}| = |(-450\hat{j}) - (450\hat{j})|$$

$$|\Delta\vec{v}| = |-900\hat{j}|$$

$$|\Delta\vec{v}| = 900 \text{ Kmph}$$

While the speed remains constant at 450 Kmph, the vector subtraction reveals that the plane has undergone a velocity change equal to twice its speed due to the complete reversal of its direction of motion.

Step 4: Final Answer:

The change in velocity in half a revolution is 900 Kmph.

This matches Option (D).

Quick Tip: For circular motion with constant speed v , the magnitude of the **change in velocity** after rotating by an angle θ is given by the formula:

$$|\Delta \vec{v}| = 2v \sin\left(\frac{\theta}{2}\right)$$

For half revolution, $\theta = 180^\circ$, so $|\Delta \vec{v}| = 2v \sin(90^\circ) = 2v$.

55. The ratio between maximum and minimum values of two vectors $\vec{A}$ and $\vec{B}$ ($\vec{A} > \vec{B}$) is 4 : 1. Then the ratio between the magnitudes of the two vectors is

- (A) 3 : 2
- (B) 5 : 3
- (C) 2 : 3
- (D) 3 : 5

Correct Answer: (B) 5 : 3

Solution:**Step 1: Understanding the Concept:**

When two vectors are added, their resultant magnitude depends on the angle θ between them.

The magnitude of the resultant R of vectors $\vec{A}$ and $\vec{B}$ is given by $R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$.

1. The **maximum resultant** occurs when the vectors are in the same direction ($\theta = 0^\circ$):

$$R_{\max} = A + B.$$

2. The **minimum resultant** occurs when the vectors are in opposite directions ($\theta = 180^\circ$):

$$R_{\min} = A - B \text{ (assuming } A > B \text{)}.$$

By knowing the ratio of these extreme values, we can find the ratio of the individual

magnitudes of the vectors.

Step 2: Key Formula or Approach:

Given: $\frac{R_{\max}}{R_{\min}} = \frac{4}{1}$

Using the expressions for extreme resultants:

$$\frac{A+B}{A-B} = \frac{4}{1}$$

We will use cross-multiplication or the componendo and dividendo rule to find the ratio $A : B$.

Step 3: Detailed Explanation:

Starting with the equation:

$$\frac{A+B}{A-B} = \frac{4}{1}$$

Cross-multiplying:

$$1(A+B) = 4(A-B)$$

$$A+B = 4A-4B$$

Rearranging terms to group A and B on opposite sides:

$$B+4B = 4A-A$$

$$5B = 3A$$

To find the ratio $A : B$, we write:

$$\frac{A}{B} = \frac{5}{3}$$

This means the ratio of the magnitudes of the two vectors is 5 : 3. This is physically consistent as $R_{\max} \propto 5 + 3 = 8$ and $R_{\min} \propto 5 - 3 = 2$, giving a ratio of $8 : 2 = 4 : 1$, confirming our result.

Step 4: Final Answer:

The ratio between the magnitudes of the two vectors is 5 : 3.

This matches Option (B).

Quick Tip: Use the **Componendo and Dividendo** rule for these types of ratios:

If $\frac{A+B}{A-B} = \frac{x}{y}$, then $\frac{A}{B} = \frac{x+y}{x-y}$.

Here $\frac{A}{B} = \frac{4+1}{4-1} = \frac{5}{3}$. This is much faster than manual expansion!

56. The magnitudes of three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ are 12, 5 and 13 units respectively and $\vec{A} + \vec{B} = \vec{C}$. The angle between $\vec{A}$ and $\vec{B}$ is

- (A) 0°
- (B) 120°
- (C) 90°
- (D) 45°

Correct Answer: (C) 90°

Solution:

Step 1: Understanding the Concept:

The vector addition equation $\vec{A} + \vec{B} = \vec{C}$ implies that the resultant of vectors $\vec{A}$ and $\vec{B}$ is $\vec{C}$. The magnitude of this resultant is related to the individual magnitudes and the angle θ between the two vectors.

If the magnitudes satisfy the Pythagorean theorem $A^2 + B^2 = C^2$, it indicates that the vectors

form the sides of a right-angled triangle. Specifically, $\vec{A}$ and $\vec{B}$ would be at right angles to each other. We can prove this using the general resultant formula.

Step 2: Key Formula or Approach:

The magnitude relationship is given by:

$$C^2 = A^2 + B^2 + 2AB \cos \theta$$

where $A = 12, B = 5, C = 13$. We substitute these values and solve for $\cos \theta$.

Step 3: Detailed Explanation:

Substitute the given magnitudes into the formula:

$$13^2 = 12^2 + 5^2 + 2(12)(5) \cos \theta$$

Calculate the squares:

$$169 = 144 + 25 + 120 \cos \theta$$

Sum the squares on the right:

$$169 = 169 + 120 \cos \theta$$

Subtract 169 from both sides:

$$0 = 120 \cos \theta$$

Dividing by 120:

$$\cos \theta = 0$$

We know that the cosine of an angle is zero when the angle is 90° .

$$\theta = \cos^{-1}(0) = 90^\circ$$

Physically, this means vectors $\vec{A}$ and $\vec{B}$ are perpendicular. The set (5, 12, 13) is a well-known Pythagorean triple, which immediately suggests that the triangle formed by these vectors is a right-angled triangle with the angle between the two shorter sides being 90° .

Step 4: Final Answer:

The angle between $\vec{A}$ and $\vec{B}$ is 90° .

This matches Option (C).

Quick Tip: Recognize **Pythagorean Triples**: (3,4,5), (5,12,13), (8,15,17), (7,24,25). If the magnitudes of vectors $\vec{A}, \vec{B}, \vec{C}$ follow $A^2 + B^2 = C^2$ and $\vec{A} + \vec{B} = \vec{C}$, the angle between $\vec{A}$ and $\vec{B}$ is always **90 degrees**.

57. A body falling from height H takes time T seconds to reach the ground. The time taken to cover the second half of height is

- (A) $\frac{T}{\sqrt{2}}$
- (B) $\sqrt{2}T$
- (C) $\frac{\sqrt{2}-1}{\sqrt{2}}T$
- (D) $\frac{1}{\sqrt{2}-1}T$

Correct Answer: (C) $\frac{\sqrt{2}-1}{\sqrt{2}}T$

Solution:

Step 1: Understanding the Concept:

For a body in free fall starting from rest, the distance traveled is proportional to the square of the time taken, given by $h = \frac{1}{2}gt^2$. This means it covers more distance in the later seconds of

its fall because it is accelerating.

To find the time taken for the "second half" of the height, we must:

1. Find the total time T for total height H .
2. Find the time T_1 taken to cover the "first half" height $H/2$.
3. Subtract the time of the first half from the total time: $\Delta T = T - T_1$.

Step 2: Key Formula or Approach:

1. Total motion: $H = \frac{1}{2}gT^2$
2. First half motion: $\frac{H}{2} = \frac{1}{2}gT_1^2$
3. Solve for T_1 in terms of T and find $T - T_1$.

Step 3: Detailed Explanation:

From the total motion equation:

$$T = \sqrt{\frac{2H}{g}}$$

For the first half of the height ($H/2$):

$$\frac{H}{2} = \frac{1}{2}gT_1^2 \implies T_1 = \sqrt{\frac{2(H/2)}{g}} = \sqrt{\frac{H}{g}}$$

Let's express T_1 in terms of T :

$$T_1 = \frac{1}{\sqrt{2}} \sqrt{\frac{2H}{g}} = \frac{T}{\sqrt{2}}$$

The time taken to cover the second half of the height ($t_{\text{second half}}$) is the difference between total time and time for the first half:

$$t_{\text{second half}} = T - T_1$$

$$t_{\text{second half}} = T - \frac{T}{\sqrt{2}}$$

Factor out T :

$$t_{\text{second half}} = T \left(1 - \frac{1}{\sqrt{2}} \right)$$

Combine the terms inside the parentheses:

$$t_{\text{second half}} = \left(\frac{\sqrt{2}-1}{\sqrt{2}} \right) T$$

This confirms that the second half takes less time than the first half because the body has acquired significant velocity by the time it reaches the midpoint.

Step 4: Final Answer:

The time taken to cover the second half of the height is $\frac{\sqrt{2}-1}{\sqrt{2}} T$.

This matches Option (C).

Quick Tip: For a body starting from rest, the ratio of times taken to cover successive equal distances is $1 : (\sqrt{2}-1) : (\sqrt{3}-\sqrt{2}) : \dots$

For two equal segments, the ratio of first-half time to second-half time is $1 : (\sqrt{2}-1)$. So the second half is $\frac{\sqrt{2}-1}{\sqrt{2}}$ of the total time.

58. With what speed a body be thrown upwards so that the distances covered in the 5th second and 6th second are equal?

- (A) 75 m/s
- (B) $\sqrt{98}$ m/s
- (C) 49 m/s
- (D) 19.8 m/s

Correct Answer: (C) 49 m/s

Solution:

Step 1: Understanding the Concept:

For a body thrown vertically upwards, the distance covered in successive seconds follows a specific pattern. The distances covered in the n^{th} second and $(n + 1)^{\text{th}}$ second are equal only if the highest point (where velocity is zero) is reached at the end of the n^{th} second.

Specifically, the 5th second is the time interval from $t = 4 \text{ s}$ to $t = 5 \text{ s}$. The 6th second is the interval from $t = 5 \text{ s}$ to $t = 6 \text{ s}$. Due to the symmetry of projectile motion, the distance covered during ascent in the last second before reaching the peak equals the distance covered during descent in the first second after the peak. This implies that the body must reach its maximum height at exactly $t = 5 \text{ seconds}$.

Step 2: Key Formula or Approach:

1. At the highest point, velocity $v = 0$.
2. Equation of motion: $v = u - gt$.
3. Given that distances in 5th and 6th seconds are equal, time to reach max height is $t = 5 \text{ s}$.

Step 3: Detailed Explanation:

Let the initial velocity be u .

The body reaches its maximum height at time t .

We are given that distance in 5th second = distance in 6th second.

As explained, this symmetry occurs around the peak. The transition from the 5th to the 6th second occurs at $t = 5$.

So, the body reaches its peak at $t = 5 \text{ s}$.

Using the kinematic equation:

$$v = u - gt$$

Substitute $v = 0$, $t = 5$ and taking $g = 9.8 \text{ m/s}^2$:

$$0 = u - 9.8(5)$$

$$u = 9.8 \times 5$$

$$u = 49 \text{ m/s}$$

This speed ensures that during the 5th second, the body is finishing its climb, and during the 6th second, it starts falling back, covering the exact same distance in both intervals due to gravitational symmetry.

Step 4: Final Answer:

The required speed is 49 m/s.

This matches Option (C).

Quick Tip: For distances in the n^{th} and $(n + 1)^{\text{th}}$ second to be equal, the peak must be at $t = n$. Then $u = n \times g$. Here $n = 5$, so $u = 5 \times 9.8 = 49 \text{ m/s}$.

59. A body of mass 1 kg starts moving from rest under the action of a force which varies with displacement as $F = 2x + 5$ (in newtons). The work done by this force to displace the body from $x = 0$ to $x = 2 \text{ m}$ is:

- (A) 8 J
- (B) 10 J
- (C) 12 J
- (D) 14 J

Correct Answer: (D) 14 J

Solution:

Step 1: Understanding the Concept:

Work done by a force is defined as the product of the force and the displacement in the direction of the force. When the force is constant, $W = F \cdot d$. However, when the force varies with position (displacement), the work done must be calculated by integrating the force over the path.

Mathematically, $W = \int F dx$. This represents the area under the force-displacement graph. In this problem, the mass of the body is provided, but it is not needed to calculate work done by a given force, as work is independent of mass if the force is already specified as a function of position.

Step 2: Key Formula or Approach:

1. Formula for work done by variable force: $W = \int_{x_1}^{x_2} F(x) dx$.
2. Given $F(x) = 2x + 5$, $x_1 = 0$, and $x_2 = 2$.

Step 3: Detailed Explanation:

We set up the integral for work done:

$$W = \int_0^2 (2x + 5) dx$$

Performing the integration using the power rule $\int x^n dx = \frac{x^{n+1}}{n+1}$:

$$W = \left[2 \cdot \frac{x^2}{2} + 5 \cdot x \right]_0^2$$

Simplify the expression inside the brackets:

$$W = [x^2 + 5x]_0^2$$

Now, evaluate the definite integral by substituting the limits:

$$W = (2^2 + 5(2)) - (0^2 + 5(0))$$

$$W = (4 + 10) - (0)$$

$$W = 14 \text{ Joules}$$

The force does 14 Joules of work on the 1 kg mass. This work will increase the kinetic energy of the body according to the work-energy theorem.

Step 4: Final Answer:

The work done by the force is 14 J.

This matches Option (D).

Quick Tip: Work is always the **area under the Force-Displacement graph**. Since $F = 2x + 5$ is linear, the graph is a trapezoid.

Area = $\frac{1}{2} \times (\text{sum of parallel sides}) \times \text{height}$.

At $x = 0, F = 5$; at $x = 2, F = 9$.

Work = $\frac{1}{2} \times (5 + 9) \times 2 = 14 \text{ J}$. No integration needed!

60. The potential energy of a particle is given by $U(x) = 20 + (x - 2)^2$, where U is in joules and x in meters. The minimum potential energy and the position where it occurs are:

- (A) 20 J at $x = 2 \text{ m}$
- (B) 2 J at $x = 20 \text{ m}$
- (C) 22 J at $x = 2 \text{ m}$
- (D) 0 J at $x = 2 \text{ m}$

Correct Answer: (A) 20 J at $x = 2 \text{ m}$

Solution:

Step 1: Understanding the Concept:

Potential energy (U) characterizes the energy of a particle due to its position or configuration. A stable equilibrium point for a particle occurs at a position where its potential energy is a local minimum.

To find the minimum of a function $U(x)$, we look for a point where its first derivative is zero ($dU/dx = 0$) and its second derivative is positive. Alternatively, for a quadratic expression like the one given, the minimum value is reached when the squared term is zero, since a squared real number is always non-negative.

Step 2: Key Formula or Approach:

1. Given $U(x) = 20 + (x - 2)^2$.
2. To minimize $U(x)$, we need to minimize the term $(x - 2)^2$.
3. The smallest value of a perfect square is 0, which occurs when its base is zero.

Step 3: Detailed Explanation:

Let's analyze the expression $U(x) = 20 + (x - 2)^2$.

Since $(x - 2)^2 \geq 0$ for all real values of x , the minimum value of $U(x)$ will be attained when $(x - 2)^2$ is equal to its minimum possible value, which is 0.

Setting the squared term to zero:

$$(x - 2)^2 = 0 \implies x - 2 = 0 \implies x = 2 \text{ m}$$

Substituting this position back into the energy equation:

$$U_{\min} = U(2) = 20 + (2 - 2)^2$$

$$U_{\min} = 20 + 0^2 = 20 \text{ J}$$

Using calculus for verification:

$$\frac{dU}{dx} = 2(x - 2) = 0 \implies x = 2.$$

$$\frac{d^2U}{dx^2} = 2 > 0, \text{ confirming it is a minimum.}$$

Thus, the particle has a minimum potential energy of 20 J when it is at the position $x = 2$ m.

Step 4: Final Answer:

The minimum potential energy is 20 J at $x = 2$ m.

This matches Option (A).

Quick Tip: For any potential energy function of the form $U(x) = k + (x - a)^2$, the minimum energy is always the constant k and it occurs at position $x = a$. No derivation is needed for such parabolic forms!

61. Power supplied to a particle of mass 2 kg varies with time as $P = 3t^2/2$ watt, where t is in seconds. If velocity at $t = 0$ is zero, the velocity at $t = 2$ s is:

- (A) 1 m/s
- (B) 2 m/s
- (C) $\sqrt{2}$ m/s
- (D) 4 m/s

Correct Answer: (B) 2 m/s

Solution:

Step 1: Understanding the Concept:

Power is defined as the rate of doing work or the rate at which energy is transferred. In mechanical systems, power (P) is related to force (F) and velocity (v) by the equation $P = F \cdot v$. Furthermore, according to the Work-Energy Theorem, the work done on an object by the net force acting on it is equal to the change in its kinetic energy. In this problem, we are given power as a function of time, which means we can find the total work done by integrating power over a specific time interval. Since the particle starts from rest (initial velocity is zero), the total work done will be equal to the final kinetic energy of the particle

at $t = 2$ seconds. This approach connects the temporal variation of power directly to the kinematic state of the mass.

Step 2: Key Formula or Approach:

1. Work-Energy Theorem: $W = \Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$.
2. Work done by variable power: $W = \int P dt$.
3. Relationship: $\int_0^t P dt = \frac{1}{2}mv^2$ (since $v_i = 0$).

Step 3: Detailed Explanation:

Given mass $m = 2$ kg and power $P(t) = \frac{3t^2}{2}$.

First, we calculate the total work done from $t = 0$ to $t = 2$ seconds by integrating the power function:

$$W = \int_0^2 P(t) dt = \int_0^2 \frac{3t^2}{2} dt$$

Factor out the constant $3/2$:

$$W = \frac{3}{2} \left[\frac{t^3}{3} \right]_0^2$$

Simplify the constants:

$$W = \frac{1}{2} [t^3]_0^2$$

Evaluate at the limits:

$$W = \frac{1}{2} [2^3 - 0^3] = \frac{1}{2} \times 8 = 4 \text{ Joules}$$

According to the work-energy principle, this work must be equal to the kinetic energy at $t = 2$ seconds:

$$KE = \frac{1}{2}mv^2 = 4$$

Substitute $m = 2$ kg:

$$\frac{1}{2}(2)v^2 = 4$$

$$v^2 = 4$$

Taking the square root:

$$v = \sqrt{4} = 2 \text{ m/s}$$

The calculation shows that the time-dependent power source accelerates the 2 kg mass from rest to a speed of 2 m/s over a two-second interval. It is important to note that the result depends on both the magnitude of the power and the mass of the object; a lighter object would have achieved a higher speed given the same power input.

Step 4: Final Answer:

The velocity at $t = 2$ s is 2 m/s.

This matches Option (B).

Quick Tip: Whenever power is given as a function of time and you need to find velocity, remember that **Area under the P-t graph = Change in Kinetic Energy**. This is often faster than trying to find acceleration or force. In this case, simply integrate and set equal to $\frac{1}{2}mv^2$.

62. A pump is used to deliver water at a certain rate from a given pipe. To obtain twice the volume of water from the same pipe in the same time, by what factor must the power of the motor pump be increased? (Assume ideal conditions, $g = 10 \text{ ms}^{-2}$)

(A) 4

- (B) 8
(C) 16
(D) 32

Correct Answer: (B) 8

Solution:

Step 1: Understanding the Concept:

This problem involves fluid dynamics and the power requirements of a mechanical system. To deliver a volume of water V in time t , the pump must provide enough energy to overcome potential energy (lifting the water) and to impart kinetic energy (moving the water through the pipe). In the case of delivering water through a pipe, the rate of flow (discharge Q) is proportional to the velocity v of the water and the cross-sectional area A of the pipe ($Q = Av$). If we want to double the volume in the same time, we must double the flow rate. Since the pipe remains the same (constant area), this necessitates doubling the velocity of the water. We must then examine how power depends on these parameters.

Step 2: Key Formula or Approach:

1. Flow rate $Q = \frac{V}{t} = A \cdot v$.
2. Mass flow rate $\dot{m} = \rho \cdot Q = \rho \cdot A \cdot v$.
3. Power (P) required for kinetic energy: $P = \frac{1}{2}\dot{m}v^2$.
4. Substituting $\dot{m}$: $P = \frac{1}{2}(\rho Av)v^2 = \frac{1}{2}\rho Av^3$.

Step 3: Detailed Explanation:

Let the initial volume be V_1 , initial time be t , and initial velocity be v_1 .

The initial power is $P_1 \propto v_1^3$ (assuming kinetic energy dominates for pipe delivery at a certain rate).

Now, we want to deliver volume $V_2 = 2V_1$ in the same time t .

New flow rate $Q_2 = \frac{V_2}{t} = \frac{2V_1}{t} = 2Q_1$.

Since the area A is constant, $Q = Av$ implies:

$$v_2 = 2v_1$$

Now, let's look at the power dependency derived in Step 2: $P \propto v^3$.

Therefore, the ratio of the new power P_2 to the original power P_1 is:

$$\frac{P_2}{P_1} = \left(\frac{v_2}{v_1} \right)^3$$

Substituting $v_2 = 2v_1$:

$$\frac{P_2}{P_1} = (2)^3 = 8$$

$$P_2 = 8P_1$$

This means that to double the amount of water delivered through the same pipe in the same duration, the motor's power must be increased by a factor of 8. This "cubic law" is a fundamental principle in pump and fan selection, explaining why increasing throughput requires significantly larger engines and higher energy costs.

Step 4: Final Answer:

The power must be increased by a factor of 8.

This matches Option (B).

Quick Tip: For fluid pumps delivering through a fixed area:

- Flow rate $Q \propto v$
- Force $F \propto \dot{m}v \propto v^2$
- Power $P = Fv \propto v^3$.

If discharge (Q) is doubled, velocity is doubled, and power increases by $2^3 = 8$. If discharge is tripled, power increases by $3^3 = 27$.

63. Two identical piano wires, when tuned to a fundamental frequency of 400 Hz, produce no beats. One wire is then slightly tightened, and the beat frequency heard is 2 Hz. What is the

new fundamental frequency of the tightened wire?

- (A) 398 Hz
- (B) 402 Hz
- (C) 404 Hz
- (D) 396 Hz

Correct Answer: (B) 402 Hz

Solution:

Step 1: Understanding the Concept:

Beats are a physical phenomenon occurring when two sound waves of slightly different frequencies overlap. The resulting sound exhibits a periodic variation in intensity (loudness). The number of intensity peaks heard per second is called the beat frequency (f_b), and it is mathematically equal to the absolute difference between the two source frequencies: $f_b = |f_1 - f_2|$. In this problem, we start with two identical frequencies (zero beats). Tightening a wire increases the tension in it. Since the frequency of a vibrating string is directly proportional to the square root of the tension ($f \propto \sqrt{T}$), tightening always results in an increase in frequency. This fact allows us to determine whether the new frequency is higher or lower than the original.

Step 2: Key Formula or Approach:

1. Beat frequency formula: $f_{beat} = |f_{tightened} - f_{original}|$.
2. Physics property: Tightening a wire $\implies$ Tension increases $\implies$ Frequency increases.

Step 3: Detailed Explanation:

The initial frequency of both wires is $f_0 = 400$ Hz.

The beat frequency heard after tightening one wire is $f_b = 2$ Hz.

Let the new frequency of the tightened wire be f_1 .

According to the beat frequency formula:

$$|f_1 - 400| = 2$$

This gives two mathematical possibilities for f_1 :

Case 1: $f_1 - 400 = 2 \implies f_1 = 402 \text{ Hz}$.

Case 2: $400 - f_1 = 2 \implies f_1 = 398 \text{ Hz}$.

Now, we apply the physical condition: the wire was **tightened**.

From the law of transverse vibrations of a string:

$$f = \frac{1}{2L} \sqrt{\frac{T}{m}}$$

where T is tension. Since $f \propto \sqrt{T}$, an increase in tension *must* result in an increase in frequency.

Therefore, $f_1 > f_0$.

The only physically valid solution is $f_1 = 402 \text{ Hz}$. If the wire had been *loosened*, the frequency would have dropped to 398 Hz.

Step 4: Final Answer:

The new fundamental frequency is 402 Hz.

This matches Option (B).

Quick Tip: Remember the mnemonics:

- Tightening = Tension up = frequency **up**.
- Loosing / Loading with wax = frequency **down**.
- Filing (removing mass) = frequency **up**.

In beat problems, once you have the $\pm$ values, use these keywords to pick the correct one.

64. A source of sound of frequency 500Hz is moving towards an observer with velocity 30m/s. The speed of sound is 330m/s. The frequency heard by the observer will be:

- (A) 450 Hz
- (B) 550 Hz
- (C) 600 Hz
- (D) 500 Hz

Correct Answer: (B) 550 Hz

Solution:

Step 1: Understanding the Concept:

The Doppler Effect is the change in frequency or wavelength of a wave in relation to an observer who is moving relative to the wave source. When a sound source moves towards a stationary observer, the sound waves are compressed in front of the source, resulting in a shorter wavelength and a higher perceived frequency (pitch). Conversely, if the source moves away, the waves are stretched, resulting in a lower perceived frequency. This principle is widely used in radar, medical imaging, and astronomy. In this problem, we need to calculate the "apparent frequency" heard by a stationary observer as a source approaches at a significant speed.

Step 2: Key Formula or Approach:

The general Doppler formula for sound is:

$$f' = f \left(\frac{v \pm v_o}{v \mp v_s} \right)$$

where:

- f' = Apparent frequency
- f = Actual frequency = 500 Hz
- v = Speed of sound = 330 m/s
- v_o = Velocity of observer = 0 (stationary)
- v_s = Velocity of source = 30 m/s

For a source moving **towards** a stationary observer, the denominator should be smaller to increase the frequency: $f' = f \left(\frac{v}{v - v_s} \right)$.

Step 3: Detailed Explanation:

Identify the values:

$f = 500$ Hz, $v = 330$ m/s, $v_s = 30$ m/s.

Substitute these into the specific approaching-source formula:

$$f' = 500 \left(\frac{330}{330 - 30} \right)$$

Subtract the values in the denominator:

$$f' = 500 \left(\frac{330}{300} \right)$$

Simplify the fraction by dividing by 30:

$$\frac{330}{300} = \frac{33}{30} = \frac{11}{10} = 1.1$$

Now, calculate the final frequency:

$$f' = 500 \times 1.1 = 550 \text{ Hz}$$

As predicted by theory, the observer hears a frequency higher than the source's natural frequency because the source is moving towards them. The 10% increase in frequency corresponds to the source moving at approximately 1/11th the speed of sound.

Step 4: Final Answer:

The frequency heard by the observer will be 550 Hz.

This matches Option (B).

Quick Tip: Direction Check: If the source is approaching, the apparent frequency **must** be greater than the actual frequency. This immediately eliminates Option (A) and (D). A quick mental check of $500 \times (33/30)$ confirms the answer is 550. Always keep "towards = frequency up" in mind.

65. In Acoustics, 'Noise' is generally characterized by:

- (A) Irregular and non-periodic vibrations.
- (B) A constant pitch and frequency
- (C) Vibrations that follow a harmonic series
- (D) Regular and periodic vibrations

Correct Answer: (A) Irregular and non-periodic vibrations.

Solution:

Step 1: Understanding the Concept:

Acoustics is the branch of physics that deals with the study of all mechanical waves in gases, liquids, and solids. In acoustics, we distinguish between musical sound and noise. Musical sound is produced by regular, periodic vibrations that repeat at fixed time intervals, giving rise to a discernible pitch. In contrast, 'Noise' refers to unwanted or unpleasant sound. At a physical level, noise is the result of irregular, chaotic, and non-periodic vibrations. These vibrations do not have a constant frequency or a defined harmonic structure, making it difficult for the human ear to assign a specific note or pitch to them.

Step 2: Detailed Explanation:

Let's analyze the characteristics of sound vs. noise:

1. **Musical Sound:** These are periodic waves. Their waveform repeats over time. Examples include a tuning fork, a violin string, or a flute. These follow Option (D).
2. **Noise:** Noise waveforms are random and do not repeat. They contain a wide range of frequencies often lumped together (like white noise). Because there is no regularity, the brain perceives it as a jumble of sound rather than a melody. Examples include the sound of a falling wall, traffic on a busy road, or a thunderclap.
3. **Harmonic Series:** Musical instruments often produce a fundamental frequency along with several overtones that are integer multiples of that frequency. This "harmonic series" gives the sound its 'timbre' or quality. Noise lacks this organized structure entirely.

Therefore, the most accurate physical description of noise is that its constituent vibrations are irregular in amplitude and non-periodic in time.

Step 3: Final Answer:

Noise is characterized by irregular and non-periodic vibrations.

This corresponds to Option (A).

Quick Tip: Think of the wave graph:

- **Music** = Smooth, repeating sine-like waves.
 - **Noise** = Jagged, messy, non-repeating "scribble" on the graph.
- Non-periodicity is the defining mathematical trait of noise.

66. If the volume of a room is doubled and the total absorption is halved, the reverberation time will:

- (A) Remain unchanged
- (B) Be doubled
- (C) Become four times
- (D) Be halved

Correct Answer: (C) Become four times

Solution:

Step 1: Understanding the Concept:

Reverberation time (T) is the time required for the sound pressure level to decrease by 60 decibels after the sound source has stopped. It is one of the most important parameters in room acoustics. A room with a long reverberation time is described as "live," while one with a short time is "dead." The relationship between room characteristics and reverberation time was first mathematically described by Wallace Clement Sabine in the late 19th century. His formula relates the time to the volume of the space and the total sound absorption capacity of its surfaces.

Step 2: Key Formula or Approach:

Sabine's Formula for reverberation time is:

$$T = \frac{0.161 \cdot V}{A}$$

where:

- T = Reverberation time (seconds)
- V = Volume of the room (m^3)
- A = Total absorption (in metric sabins)

From this formula, we can see that $T \propto V$ and $T \propto \frac{1}{A}$.

Step 3: Detailed Explanation:

Let the initial reverberation time be T_1 , the initial volume be V_1 , and the initial absorption be A_1 .

$$T_1 = \frac{0.161 \cdot V_1}{A_1}$$

According to the problem:

- The new volume $V_2 = 2V_1$.
- The new absorption $A_2 = \frac{1}{2}A_1$.

Now, calculate the new reverberation time T_2 :

$$T_2 = \frac{0.161 \cdot V_2}{A_2}$$

Substitute the new values in terms of the original ones:

$$T_2 = \frac{0.161 \cdot (2V_1)}{(1/2)A_1}$$

Group the original terms together:

$$T_2 = \left(\frac{0.161 \cdot V_1}{A_1} \right) \cdot \frac{2}{1/2}$$

The term in the bracket is T_1 :

$$T_2 = T_1 \cdot (2 \times 2)$$

$$T_2 = 4T_1$$

The combined effect of doubling the space (more room for sound to travel) and halving the absorption (less material to soak up sound energy) results in a quadrupling of the time sound persists in the room. This makes the room significantly more echoic.

Step 4: Final Answer:

The reverberation time will become four times.

This matches Option (C).

Quick Tip: Always look at the proportionality in the formula.

$$T \propto \frac{\text{Volume}}{\text{Absorption}}.$$

If Volume goes **up** by 2, and Absorption goes **down** by 2, the total effect is $2/(1/2) = 2 \times 2 = 4$. If both had doubled, the time would have remained unchanged.

67. In a closed hall of volume 5000 m^3 , the total absorption of the interior surfaces is 200 metric sabin. The reverberation time is:

- (A) 1 s
- (B) 2 s
- (C) 3 s
- (D) 4 s

Correct Answer: (D) 4 s

Solution:

Step 1: Understanding the Concept:

This problem requires a direct numerical application of Sabine's Law. In architectural acoustics, calculating the reverberation time allows designers to ensure that speech is

intelligible or music sounds rich in a given space. For a hall of a specific volume, if the absorption is too low, echoes will overlap and garble speech. If absorption is too high, the room will sound muffled. We are given the volume and the total absorption (which is the sum of surface area times the absorption coefficient for every material in the room). By plugging these into the standard empirical formula, we can find the time in seconds.

Step 2: Key Formula or Approach:

The formula for reverberation time (T) in SI units is:

$$T = \frac{0.161 \cdot V}{A}$$

where:

- $V = 5000 \text{ m}^3$

- $A = 200 \text{ metric sabin}$

Step 3: Detailed Explanation:

Substitute the given values into the equation:

$$T = \frac{0.161 \times 5000}{200}$$

First, simplify the multiplication by moving the decimal place for 5000:

$$0.161 \times 5000 = 161 \times 5 = 805$$

Now, divide this result by the absorption A :

$$T = \frac{805}{200}$$

Divide by 100 first:

$$T = \frac{8.05}{2}$$

Performing the final division:

$$T = 4.025 \text{ seconds}$$

Looking at the options, the value is approximately 4 seconds. This is a relatively high reverberation time, suitable for a large cathedral or a concert hall designed for slow organ music, but likely too high for a standard lecture theater without further sound-absorbing treatment. The calculation follows the standard procedure used by acoustic engineers.

Step 4: Final Answer:

The reverberation time is approximately 4 s.

This matches Option (D).

Quick Tip: To avoid decimals, rewrite the formula as $T = \frac{161 \cdot V}{1000 \cdot A}$.

For this problem: $T = \frac{161 \times 5000}{1000 \times 200} = \frac{161 \times 5}{200} = \frac{161}{40} \approx 4$. This makes mental calculation much easier.

68. In an Isothermal process:

- (A) Internal energy of the system never remains constant
- (B) Total heat energy of the system remains constant
- (C) Volume of the system remains constant
- (D) Temperature of the system remains constant

Correct Answer: (D) Temperature of the system remains constant

Solution:

Step 1: Understanding the Concept:

Thermodynamics studies the energy changes of a system and its interactions with the surroundings through work and heat. A thermodynamic "process" describes the path through

which a system changes from one state to another. In many practical scenarios, one property is held fixed during this change.

An "Isothermal" process is derived from the Greek words '*isos*' (equal) and '*therme*' (heat/temperature). It refers specifically to a transformation that occurs at a constant temperature. For such a process to happen, the system must be in thermal contact with a large heat reservoir, and the change must be slow enough for the system to continuously adjust its temperature to match the reservoir.

Step 2: Detailed Explanation:

Let's evaluate the definitions of various processes to clarify the options:

1. **Isothermal Process:** Temperature (T) is constant. For an ideal gas, since internal energy (U) depends only on temperature, $\Delta U = 0$. This contradicts Option (A).
2. **Isochoric Process:** Volume (V) is constant. This describes Option (C).
3. **Adiabatic Process:** No heat enters or leaves the system ($Q = 0$). This is distinct from "total heat energy remains constant," which is a vague phrasing, but adiabatic is the closest "heat-related" process definition. This relates to Option (B).
4. **Isobaric Process:** Pressure (P) is constant.

In an isothermal expansion of an ideal gas, heat absorbed from the surroundings is exactly equal to the work done by the gas on the surroundings ($Q = W$), keeping the temperature unchanged. Therefore, by definition, the defining characteristic of an isothermal process is the constancy of temperature.

Step 3: Final Answer:

In an isothermal process, the temperature of the system remains constant.
This matches Option (D).

Quick Tip: Process Keywords Summary:

- **Isothermal** $\implies \Delta T = 0$ (Temp constant)
- **Isobaric** $\implies \Delta P = 0$ (Pressure constant)
- **Isochoric** $\implies \Delta V = 0$ (Volume constant)
- **Adiabatic** $\implies Q = 0$ (No heat flow)

69. If the pressure of an ideal gas is doubled and its absolute temperature is halved; the volume will become:

- (A) 1/4 of initial volume
- (B) 1/2 of initial volume
- (C) Same as initial volume
- (D) 2 times of initial volume

Correct Answer: (A) 1/4 of initial volume

Solution:

Step 1: Understanding the Concept:

An ideal gas is a theoretical gas composed of many randomly moving point particles whose only interactions are perfectly elastic collisions. The behavior of such a gas is governed by the Ideal Gas Law, which relates pressure (P), volume (V), and absolute temperature (T). This law is a combination of Boyle's Law, Charles's Law, and Avogadro's Law. When any two variables change, the third must adjust according to the equation. In this problem, we are changing both pressure and temperature and observing the resultant effect on the volume of the gas.

Step 2: Key Formula or Approach:

The Ideal Gas Equation is:

$$PV = nRT$$

For a fixed amount of gas (constant n), we can write the relationship between two states:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

We need to solve for V_2 in terms of V_1 .

Step 3: Detailed Explanation:

Let initial conditions be P_1, V_1, T_1 .

The new conditions are:

- $P_2 = 2P_1$ (Pressure doubled)
- $T_2 = \frac{1}{2}T_1$ (Temperature halved)

From the combined gas law:

$$V_2 = V_1 \cdot \left(\frac{P_1}{P_2}\right) \cdot \left(\frac{T_2}{T_1}\right)$$

Substitute the given relationships:

$$V_2 = V_1 \cdot \left(\frac{P_1}{2P_1}\right) \cdot \left(\frac{(1/2)T_1}{T_1}\right)$$

Cancel the terms P_1 and T_1 :

$$V_2 = V_1 \cdot \left(\frac{1}{2}\right) \cdot \left(\frac{1}{2}\right)$$

$$V_2 = V_1 \cdot \frac{1}{4}$$

$$V_2 = \frac{V_1}{4}$$

The volume decreases dramatically because both the increase in pressure (squeezing the gas) and the decrease in temperature (slowing the particles down and reducing their spread) work together to shrink the space the gas occupies. Each factor contributes a halving effect, resulting in a total reduction to one-fourth.

Step 4: Final Answer:

The volume will become 1/4 of the initial volume.

This matches Option (A).

Quick Tip: Use the "Logic Chain":

1. If Pressure doubles, Volume **halves** (Boyle's Law).
2. If Temperature halves, Volume **halves** (Charles's Law).
3. Combined effect = $1/2 \times 1/2 = 1/4$.

This is much faster than writing out the full equation.

70. At constant temperature, the product PV is plotted against pressure P for an ideal gas. The graph obtained is:

- (A) Straight line parallel to P -axis
- (B) Straight line with positive slope
- (C) Straight line through origin
- (D) Parabola

Correct Answer: (A) Straight line parallel to P -axis

Solution:

Step 1: Understanding the Concept:

This question concerns Boyle's Law, which states that for a fixed amount of an ideal gas kept at a constant temperature, the pressure is inversely proportional to its volume ($P \propto 1/V$). A crucial consequence of this law is that the product of pressure and volume remains constant as long as the temperature and the number of moles are unchanged. When we create a graphical representation of physical variables, a horizontal line indicates that the y -variable does not change as the x -variable varies. By analyzing the mathematical relationship, we can predict the shape of the resulting curve.

Step 2: Key Formula or Approach:

1. Boyle's Law: $P \cdot V = \text{constant}$ (at constant T, n).
2. Equation of the graph: $y = \text{constant}$, where $y = PV$ and $x = P$.

Step 3: Detailed Explanation:

Let the product $PV = K$, where K is a constant determined by the temperature and the amount of gas ($K = nRT$).

If we plot PV on the Y-axis and P on the X-axis:

At $P = P_1$, $PV = K$.

At $P = 2P_1$, V will become $V_1/2$, so $PV = (2P_1)(V_1/2) = P_1V_1 = K$.

At $P = 3P_1$, V will become $V_1/3$, so $PV = (3P_1)(V_1/3) = P_1V_1 = K$.

Regardless of the value of the pressure P , the value of the product PV always returns to the same constant value K .

In coordinate geometry, a graph where the Y-coordinate is constant for all values of X is a straight line that is perfectly horizontal. Since the X-axis in this case is the P-axis, the graph is a straight line parallel to the P-axis. If we had plotted P vs V , the graph would have been a rectangular hyperbola. The constancy of the PV product is a hallmark of ideal gas behavior.

Step 4: Final Answer:

The graph is a straight line parallel to the P-axis.

This matches Option (A).

Quick Tip: Remember:

- P vs V plot $\Rightarrow$ **Hyperbola.**
- P vs $1/V$ plot $\Rightarrow$ **Straight line through origin.**
- PV vs P (or V) plot $\Rightarrow$ **Horizontal straight line.**

This summarizes all common Boyle's Law graphs.

71. A bubble of an ideal gas rises from the bottom of a lake to the surface. At the bottom, the pressure is 3 Atm. and the temperature is 7°C . At the surface, the pressure is 1 atm. and the temperature is 27°C . If the initial volume of the bubble was V_0 , what is its volume V_f at the surface?

- (A) $3 V_0$
- (B) $3.21 V_0$
- (C) $0.9 V_0$

(D) $5.4 V_0$

Correct Answer: (B) $3.21 V_0$

Solution:

Step 1: Understanding the Concept:

This problem investigates the behavior of an ideal gas under changing conditions of pressure and temperature.

The scenario involves a gas bubble moving through a fluid column, where both the hydrostatic pressure and the surrounding thermal environment vary.

According to the Ideal Gas Law, the state of a gas is defined by the relationship $PV = nRT$.

For a fixed quantity of gas (where the number of moles n remains constant inside the bubble), the ratio $\frac{PV}{T}$ must remain constant between any two states.

As the bubble rises, the pressure decreases significantly, which typically causes expansion, while the temperature change also contributes to the volume adjustment.

In thermodynamics, this is often referred to as a "Combined Gas Law" problem, requiring the conversion of temperatures to the absolute Kelvin scale.

Step 2: Key Formula or Approach:

The Combined Gas Law formula is:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

Where:

- P_1, V_1, T_1 are the initial pressure, volume, and temperature at the bottom of the lake.

- P_2, V_2, T_2 are the final pressure, volume, and temperature at the surface.

Important: Temperatures must be in Kelvin ($K = ^\circ\text{C} + 273$).

Step 3: Detailed Explanation:

Let's define the initial state at the bottom of the lake:

Pressure $P_1 = 3 \text{ atm}$.

Volume $V_1 = V_0$.

Temperature $T_1 = 7^\circ\text{C} = 7 + 273 = 280 \text{ K}$.

Now, let's define the final state at the surface:

Pressure $P_2 = 1 \text{ atm}$.

Temperature $T_2 = 27^\circ\text{C} = 27 + 273 = 300 \text{ K}$.

Final Volume $V_2 = V_f$.

Substitute these values into the Combined Gas Law:

$$\frac{3 \cdot V_0}{280} = \frac{1 \cdot V_f}{300}$$

To solve for V_f , rearrange the equation:

$$V_f = V_0 \cdot \left(\frac{P_1}{P_2}\right) \cdot \left(\frac{T_2}{T_1}\right)$$

$$V_f = V_0 \cdot \left(\frac{3}{1}\right) \cdot \left(\frac{300}{280}\right)$$

Simplify the fraction:

$$V_f = 3 \cdot V_0 \cdot \left(\frac{30}{28}\right)$$

$$V_f = 3 \cdot V_0 \cdot \left(\frac{15}{14}\right)$$

$$V_f = V_0 \cdot \frac{45}{14}$$

Performing the division:

$$45 \div 14 \approx 3.2142...$$

Thus, the final volume is approximately $3.21 V_0$.

The substantial increase in volume is primarily driven by the pressure dropping to one-third of its original value, which would triple the volume on its own.

The slight increase in temperature from 280 K to 300 K further contributes to the expansion. This results in a total volume that is more than triple the initial volume, emphasizing how gas expansion in water is a critical safety consideration for divers.

Step 4: Final Answer:

The volume V_f at the surface is $3.21 V_0$.

This matches Option (B).

Quick Tip: Always convert Celsius to Kelvin first!

In competitive exams, if pressure is halved, volume roughly doubles.

Here, pressure is reduced to $1/3$, so volume should be at least $3V_0$.

Since temperature also increased, the answer **must** be greater than $3V_0$, making Option (B) the only logical choice without full calculation.

72. The R.M.S. speed of oxygen molecules at 27°C is v . At 927°C , the rms speed will be:

- (A) v
- (B) $v/2$
- (C) $2v$
- (D) $4v$

Correct Answer: (C) $2v$

Solution:

Step 1: Understanding the Concept:

The Root Mean Square (RMS) speed is a statistical measure of the velocity of particles in a gas.

It represents the speed of a molecule possessing a kinetic energy equal to the average kinetic

energy of all the molecules in the sample.

From the Kinetic Theory of Gases, we know that the average translational kinetic energy per molecule is directly proportional to the absolute temperature.

Therefore, the RMS speed of a gas molecule depends solely on the temperature and the molar mass of the gas.

In this problem, the gas species (Oxygen) remains the same, so its molar mass is constant.

The core task is to determine how the speed scales as the temperature changes on the absolute Kelvin scale.

Step 2: Key Formula or Approach:

The formula for RMS speed (v_{rms}) is:

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

Since R , M , and 3 are constants for a given gas:

$$v_{rms} \propto \sqrt{T}$$

We can set up a ratio for two different states:

$$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}}$$

Again, temperature must be in Kelvin.

Step 3: Detailed Explanation:

Step 1: Identify initial conditions.

Initial Temperature $T_1 = 27^\circ\text{C} = 27 + 273 = 300\text{ K}$.

Initial RMS speed $v_1 = v$.

Step 2: Identify final conditions.

Final Temperature $T_2 = 927^\circ\text{C} = 927 + 273 = 1200\text{ K}$.

Step 3: Calculate the ratio of temperatures.

$$\frac{T_2}{T_1} = \frac{1200}{300} = 4$$

Step 4: Use the proportionality relationship.

$$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{4} = 2$$

Step 5: Find final speed.

$$v_2 = 2 \cdot v_1 = 2v$$

The temperature increased by a factor of 4 in absolute terms.

Because velocity is proportional to the square root of temperature, the velocity increases by the square root of 4, which is 2.

This means the molecules are moving twice as fast on average at the higher temperature, reflecting a four-fold increase in their kinetic energy.

It is essential to remember that molecular speeds do not double just because the Celsius temperature doubles; the physics is tied to the thermal energy baseline at absolute zero.

Step 4: Final Answer:

The RMS speed at 927°C will be $2v$.

This matches Option (C).

Quick Tip: RMS Speed Rule:

- To **double** the speed, you must **quadruple** (x4) the absolute temperature.
- To **triple** the speed, you must increase absolute temperature by **nine times** (x9).

Quick check: 300 K → 1200 K is exactly a 4x increase.

$\sqrt{4} = 2$. Answer is $2v$.

73. In a photoelectric experiment, the stopping potential for incident light of wavelength

4000 Å is 2 V. If the wavelength is changed to 3000 Å, the new stopping potential will be approximately: (Use $h = 4.14 \times 10^{-15} \text{ eV} \cdot \text{s}$, $c = 3 \times 10^8 \text{ m/s}$)

- (A) 2 V
- (B) 3.03 V
- (C) 4.14 V
- (D) 1.5 V

Correct Answer: (B) 3.03 V

Solution:

Step 1: Understanding the Concept:

The photoelectric effect describes the emission of electrons from a metal surface when light of sufficient frequency shines upon it.

Einstein's photoelectric equation relates the energy of an incident photon to the work function of the metal and the maximum kinetic energy of the ejected electrons.

The energy of a photon is $E = \frac{hc}{\lambda}$.

The maximum kinetic energy (K_{max}) of an electron is measured using the stopping potential (V_s), where $K_{max} = e \cdot V_s$.

Since the metal (and thus its work function ϕ) remains the same in both cases, we can use the two data points to find the new stopping potential.

Step 2: Key Formula or Approach:

Einstein's Equation:

$$\frac{hc}{\lambda} = \phi + eV_s$$

We can write this for two different wavelengths:

1. $\frac{hc}{\lambda_1} = \phi + eV_{s1}$
2. $\frac{hc}{\lambda_2} = \phi + eV_{s2}$

Subtracting the first equation from the second eliminates the unknown work function ϕ :

$$hc \left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1} \right) = e(V_{s2} - V_{s1})$$

Use the shortcut: $Energy \text{ (in eV)} = \frac{12400}{\lambda \text{ (in Å)}}$.

Step 3: Detailed Explanation:

Let's use the approximation $hc \approx 12420 \text{ eV} \cdot \text{Å}$ or the given constants.

Case 1: $\lambda_1 = 4000 \text{ Å}$, $V_{s1} = 2 \text{ V}$.

Energy of first photon $E_1 = \frac{12420}{4000} \approx 3.105 \text{ eV}$.

From Einstein's eq: $3.105 = \phi + 2 \implies \phi = 1.105 \text{ eV}$.

Case 2: $\lambda_2 = 3000 \text{ Å}$.

Energy of second photon $E_2 = \frac{12420}{3000} = 4.14 \text{ eV}$.

Using the work function found earlier:

$$4.14 = 1.105 + V_{s2}$$

$$V_{s2} = 4.14 - 1.105 = 3.035 \text{ V}$$

Let's verify using the given constants:

$$hc = (4.14 \times 10^{-15} \text{ eV} \cdot \text{s}) \times (3 \times 10^8 \text{ m/s}) = 1.242 \times 10^{-6} \text{ eV} \cdot \text{m}.$$

Since $1 \text{ Å} = 10^{-10} \text{ m}$, $hc = 12420 \text{ eV} \cdot \text{Å}$.

Calculating the difference:

$$V_{s2} - V_{s1} = \frac{hc}{e} \left(\frac{1}{3000} - \frac{1}{4000} \right)$$

Note: $\frac{hc}{e}$ with h in eV units already has the e cancelled out.

$$V_{s2} - 2 = 12420 \left(\frac{4000 - 3000}{3000 \times 4000} \right)$$

$$V_{s2} - 2 = 12420 \left(\frac{1000}{12,000,000} \right)$$

$$V_{s2} - 2 = \frac{12420}{12000} = 1.035$$

$$V_{s2} = 2 + 1.035 = 3.035 \text{ V}$$

The result shows that shorter wavelengths (higher frequency) result in more energetic photons, which require a higher potential to stop the ejected electrons.

Step 4: Final Answer:

The new stopping potential will be approximately 3.03 V.

This matches Option (B).

Quick Tip: Remember the photon energy shortcut: $E \text{ (eV)} \approx \frac{12400}{\lambda \text{ (Å)}}$.

Calculation:

$$E_{4000} = 124/40 = 3.1 \text{ eV.}$$

$$E_{3000} = 124/30 = 4.13 \text{ eV.}$$

$$\text{Increase in energy} = 4.13 - 3.1 = 1.03 \text{ eV.}$$

$$\text{New stopping potential} = 2 \text{ V} + 1.03 \text{ V} = 3.03 \text{ V.}$$

74. In Optical Fiber communication, the signal is transmitted in the form of:

- (A) Electrical pulses
- (B) Light pulses
- (C) Radio waves
- (D) Sound waves

Correct Answer: (B) Light pulses

Solution:

Step 1: Understanding the Concept:

Optical fiber technology represents a major advancement in the field of telecommunications. An optical fiber is a flexible, transparent fiber made by drawing glass (silica) or plastic to a diameter slightly thicker than that of a human hair.

Unlike traditional copper wires that use electrical current to carry information, optical fibers use light as the carrier medium.

This change in medium allows for significantly higher bandwidth and lower attenuation over long distances.

The fundamental physical principle that enables this is Total Internal Reflection (TIR), which keeps the signal trapped within the core of the fiber.

Step 2: Detailed Explanation:

The communication process in an optical fiber system involves several conversions:

1. At the transmitter: Information (such as voice or data) starts as electrical signals. These are converted into digital binary data.
2. Transduction: A light source, typically a laser diode or an LED, converts these electrical digital signals into discrete pulses of light.
3. Transmission: These light pulses travel through the glass fiber. Because light has an extremely high frequency, it can carry vastly more data per second than electricity or radio waves.
4. Propagation: The light pulse reflects off the boundary between the core and the cladding (TIR) and travels along the fiber path, even around curves.
5. Detection: At the receiving end, a photodetector (like a photodiode) converts the incoming light pulses back into electrical pulses, which are then reconstructed into the original information.

While the inputs and outputs are electrical, the actual "form" of the signal while it is **transmitted** through the medium of the fiber is electromagnetic radiation in the infrared or visible spectrum—specifically, light pulses.

Step 3: Final Answer:

In optical fiber communication, the signal is transmitted in the form of light pulses.

This matches Option (B).

Quick Tip: Medium-Signal Pairing:

- Copper/Coaxial Cable $\Rightarrow$ Electrical pulses.
- Atmosphere/Space $\Rightarrow$ Radio waves.
- **Glass Fiber $\Rightarrow$ Light pulses.**
- Air (person to person) $\Rightarrow$ Sound waves.

The name "Optical" (relating to sight/light) itself gives the answer away!

CHEMISTRY

75. In a superconducting ring, a persistent current has been flowing without decay for years. This is possible because:

- (A) Resistance is exactly zero and flux is quantized
- (B) Resistance is very small but finite
- (C) The ring is at absolute zero temperature
- (D) Magnetic field lines are expelled

Correct Answer: (A) Resistance is exactly zero and flux is quantized

Solution:

Step 1: Understanding the Concept:

Superconductivity is a quantum mechanical phenomenon where certain materials, when cooled below a critical temperature (T_c), undergo a phase transition.

This transition is characterized by two defining properties:

1. The **exact disappearance** of electrical resistance. Unlike a good conductor like copper, where resistance is low but present, a superconductor has zero resistance.
2. The expulsion of magnetic fields, known as the Meissner Effect.

When a current is induced in a closed loop (ring) of superconducting material, there are no resistive losses ($I^2R = 0$).

Therefore, the electrical energy is not dissipated as heat, allowing the current to circulate indefinitely.

Step 2: Detailed Explanation:

Let's analyze why Option (A) is the most complete and correct physical description:

1. **Zero Resistance:** In a normal conductor, electrons collide with the lattice of atoms, creating friction (resistance) and generating heat. In a superconductor, electrons form "Cooper pairs" that move through the lattice in a coherent quantum state without collisions. Because $R = 0$, the voltage drop $V = IR$ is zero, and the current does not lose energy.

2. **Flux Quantization:** In a superconducting ring, the magnetic flux passing through the hole is quantized. This is a topological requirement of the macroscopic wave function of the Cooper pairs. It ensures that the state of the current is stable and can only change in discrete steps, which prevents the current from "leaking" away slowly.

3. Why not other options?

- Option (B) is incorrect because even a "very small" resistance would lead to decay over years. Experiments have shown current decay times longer than the age of the universe.

- Option (C) is incorrect because superconductivity occurs at temperatures well above absolute zero (the critical temperature can be as high as 138 K).

- Option (D) describes the Meissner effect, which is a property of superconductors, but the **persistent current** itself is a direct consequence of zero resistance.

Step 3: Final Answer:

The persistence of the current is due to the fact that resistance is exactly zero and magnetic flux is quantized.

This matches Option (A).

Quick Tip: Key Distinction: **Perfect conductor** = Zero resistance. **Superconductor** = Zero resistance + Meissner Effect + Flux Quantization. Persistent currents are the ultimate proof of the "Exact Zero" nature of superconducting resistance.

76. The pair of orbitals with electron density maximum along the axes is

- (A) d_{xy}, d_{yz}
- (B) $d_{z^2}, d_{x^2-y^2}$
- (C) d_{xz}, d_{z^2}
- (D) d_{xz}, p_z

Correct Answer: (B) $d_{z^2}, d_{x^2-y^2}$

Solution:

Step 1: Understanding the Concept:

Atomic orbitals are mathematical functions that describe the probability of finding an electron in a specific region around the nucleus.

For the transition elements, the d -subshell contains five orbitals.

These orbitals are classified into two groups based on their orientation relative to the Cartesian axes (x, y, z):

1. **Non-axial (t_{2g}) orbitals:** The lobes of these orbitals lie **between** the axes.
2. **Axial (e_g) orbitals:** The lobes of these orbitals lie **directly along** the axes.

The distinction is fundamental in crystal field theory, as it determines how orbitals split in energy when surrounded by ligands in a complex.

Step 2: Detailed Explanation:

Let's examine the orientation of the five d -orbitals:

1. d_{xy} : The four lobes are in the xy -plane, but they point at 45° angles between the x and y axes.
2. d_{yz} : The lobes are in the yz -plane, pointing between the y and z axes.
3. d_{xz} : The lobes are in the xz -plane, pointing between the x and z axes.

These three make up the t_{2g} set in octahedral symmetry and have zero density exactly on the axes.

Now consider the remaining two:

4. $d_{x^2-y^2}$: This orbital has four lobes lying **directly on** the x and y axes. The electron density is at its maximum precisely along these axes.

5. d_{z^2} : This orbital consists of two major lobes lying **directly on** the z -axis and a doughnut-shaped ring (torus) in the xy -plane. The maximum density is along the z -axis.

Thus, the pair d_{z^2} and $d_{x^2-y^2}$ are the only d -orbitals that point directly at the axes. This makes them more susceptible to repulsion from ligands in an octahedral field, raising their energy levels.

Step 3: Final Answer:

The pair of orbitals with maximum electron density along the axes is $d_{z^2}, d_{x^2-y^2}$.

This matches Option (B).

Quick Tip: Mnemonic:

- Subscript has a "minus" or a single variable squared ($x^2 - y^2, z^2$) $\implies$ **Axial** (On the axes).
- Subscript has a product of two variables (xy, yz, xz) $\implies$ **Non-axial** (Between the axes).

This is a standard question in inorganic chemistry exams.

77. The angular momentum of an electron in an orbit X of hydrogen atom is $2h/\pi$. Maximum number of orbitals possible in X is

- (A) 4
- (B) 9
- (C) 16
- (D) 25

Correct Answer: (C) 16

Solution:

Step 1: Understanding the Concept:

According to Bohr's model of the hydrogen atom, an electron can only revolve in certain discrete, stationary circular orbits.

Bohr's second postulate states that the angular momentum (L) of an electron in these orbits is

quantized and is an integral multiple of $h/2\pi$.

The formula is $L = n \cdot \frac{h}{2\pi}$, where n is the principal quantum number (or the orbit number).

Once we determine the principal quantum number n , we can use quantum mechanics rules to find the total number of orbitals.

Each principal shell n contains a set of subshells ($s, p, d, f \dots$) and a specific number of total orbitals.

Step 2: Key Formula or Approach:

1. Bohr's Quantization: $L = \frac{nh}{2\pi}$.

2. Total number of orbitals in a shell with principal quantum number n :

$$\text{Total orbitals} = n^2$$

Step 3: Detailed Explanation:

First, find the principal quantum number n of orbit X :

The given angular momentum is:

$$L = \frac{2h}{\pi}$$

Equating this to Bohr's formula:

$$\frac{nh}{2\pi} = \frac{2h}{\pi}$$

Cancel h and π from both sides:

$$\frac{n}{2} = 2$$

$$n = 4$$

This means the orbit X corresponds to the 4th shell (N -shell).

Now, calculate the total number of orbitals in this shell:

The number of orbitals in the $n = 4$ shell includes:

- One s -orbital ($4s$).
- Three p -orbitals ($4p_x, 4p_y, 4p_z$).
- Five d -orbitals ($4d_{xy}, 4d_{yz}, 4d_{xz}, 4d_{x^2-y^2}, 4d_{z^2}$).
- Seven f -orbitals ($4f \dots$).

Total number of orbitals $= 1 + 3 + 5 + 7 = 16$.

Using the direct formula:

$$\text{Max orbitals} = n^2 = 4^2 = 16$$

The maximum number of electrons would be $2n^2 = 32$, but the question asks for **orbitals**, not electrons.

Step 4: Final Answer:

The maximum number of orbitals possible in orbit X is 16.

This matches Option (C).

Quick Tip: Angular Momentum Trick: Divide the coefficient of h/π by 0.5 (or multiply it by 2).

Here $2 \div 0.5 = 4$, so $n = 4$.

If it were $1.5h/\pi$, then $n = 1.5 \times 2 = 3$.

Then simply square the number (n^2) to find orbitals.

78. The four quantum numbers for the electron in the outermost orbital of potassium ($Z = 19$) are

- (A) $n = 4, l = 2, m = -1, s = +1/2$
- (B) $n = 4, l = 0, m = 0, s = +1/2$
- (C) $n = 3, l = 0, m = 1, s = +1/2$

(D) $n = 4, l = 3, m = -2, s = -1/2$

Correct Answer: (B) $n = 4, l = 0, m = 0, s = +1/2$

Solution:

Step 1: Understanding the Concept:

To find the quantum numbers of the outermost electron, we first need to determine the electronic configuration of the atom.

Potassium (K) has an atomic number $Z = 19$.

According to the Aufbau principle, electrons fill orbitals in order of increasing energy.

The Pauli Exclusion Principle and Hund's Rule must also be followed.

Once the configuration is written, we identify the last electron added to the atom (the valence electron).

Quantum numbers uniquely define an electron's state:

- n : Principal quantum number (Shell).
- l : Azimuthal quantum number (Subshell: $s = 0, p = 1, d = 2, f = 3$).
- m : Magnetic quantum number (Orbital orientation).
- s : Spin quantum number.

Step 2: Detailed Explanation:

1. Write the electronic configuration for $Z = 19$:

The sequence is $1s, 2s, 2p, 3s, 3p, 4s, 3d \dots$

Filling the 19 electrons:

$1s^2, 2s^2, 2p^6, 3s^2, 3p^6 \dots$ (This accounts for 18 electrons, which is the Argon core).

The 19th electron goes into the next orbital, which is $4s$.

Configuration: $[Ar] 4s^1$.

2. Identify the properties of the $4s^1$ electron:

- The shell is the 4th shell, so $n = 4$.
- The subshell is 's', which corresponds to $l = 0$.
- For an s-subshell ($l = 0$), the magnetic quantum number m can only be 0.
- The spin can be either $+1/2$ or $-1/2$. In standard problems, the first electron in an orbital is assigned $+1/2$.

Thus, the set of quantum numbers is $n = 4, l = 0, m = 0, s = +1/2$.

Looking at the options:

(A) has $l = 2$ (which is a d -orbital).

(C) has $n = 3$, which is an inner shell.

(D) has $l = 3$ (an f -orbital) and incorrect m .

Only Option (B) correctly describes the $4s$ electron.

Step 3: Final Answer:

The four quantum numbers are $n = 4, l = 0, m = 0, s = +1/2$.

This matches Option (B).

Quick Tip: Standard shortcut: Potassium is in the 4th period of the periodic table, in Group 1.

Group 1 elements always have their valence electron in the ns^1 orbital.

So for Potassium ($n = 4$), the last electron must be $4s^1$.

For any s -orbital, l and m are always **both zero**.

79. In which of the following, the number of bonding electrons and non-bonding electrons are in 3:2 ratio?

(A) N_2

(B) O_2

(C) HCl

(D) F_2

Correct Answer: (A) N_2

Solution:

Step 1: Understanding the Concept:

Chemical bonds involve valence electrons of atoms.

Bonding electrons are those involved in forming pairs between atoms (covalent bonds).

Each single bond consists of 2 bonding electrons.

Non-bonding electrons (or lone pairs) are valence electrons that do not participate in bond

formation and remain localized on a single atom.

To solve this, we must draw the Lewis dot structures for each molecule and count the total number of bonding and lone pair electrons.

Step 2: Detailed Explanation:

Let's analyze each molecule:

(A) N_2 : Nitrogen (Group 15) has 5 valence electrons. Two N atoms have a total of 10 valence electrons.

- To achieve an octet, they share 3 pairs of electrons, forming a triple bond ($N \equiv N$).
- Number of bonding electrons = 3 pairs $\times$ 2 = 6.
- Each nitrogen atom has one lone pair left ($5 - 3 = 2$). Total lone pair electrons = 2 atoms $\times$ 2 = 4.
- Ratio of Bonding to Non-bonding = 6 : 4 = 3 : 2.

This matches the requirement.

(B) O_2 : Oxygen (Group 16) has 6 valence electrons. Total = 12.

- They form a double bond ($O = O$). Bonding electrons = 4.
- Each O has 2 lone pairs. Total lone pair electrons = 8.
- Ratio = 4 : 8 = 1 : 2.

(C) HCl : H has 1, Cl has 7 valence electrons. Total = 8.

- Single bond. Bonding electrons = 2.
- Cl has 3 lone pairs. Total lone pair electrons = 6.
- Ratio = 2 : 6 = 1 : 3.

(D) F_2 : Fluorine (Group 17) has 7 valence electrons. Total = 14.

- Single bond. Bonding electrons = 2.
- Each F has 3 lone pairs. Total lone pair electrons = 12.
- Ratio = 2 : 12 = 1 : 6.

Thus, only the Nitrogen molecule satisfies the 3:2 ratio condition.

Step 3: Final Answer:

The 3:2 ratio is found in N_2 .

This matches Option (A).

Quick Tip: Quick Check:

N_2 has a **triple bond** ($6e^-$) and **2 lone pairs** ($4e^-$).

$$6/4 = 1.5.$$

$$3/2 = 1.5.$$

This is the only common diatomic molecule with a triple bond, making it a very high candidate for ratios involving 3.

80. Which one of the following statements is not correct?

- (A) Ionic bond is non-directional bond
- (B) The maximum number of bond pairs between two atoms is 3
- (C) Covalent compounds conduct electricity in fused state
- (D) Ionic compounds are generally soluble in water

Correct Answer: (C) Covalent compounds conduct electricity in fused state

Solution:

Step 1: Understanding the Concept:

This question tests the fundamental differences between ionic and covalent bonding properties.

Ionic bonds result from the complete transfer of electrons, creating an electrostatic lattice, while covalent bonds involve the sharing of electron pairs between nuclei.

Physical properties such as electrical conductivity, solubility, and directionality arise directly from these different atomic-level structures.

We need to identify the "incorrect" statement among the four provided.

Step 2: Detailed Explanation:

Let's evaluate each statement individually:

(A) **Ionic bond is non-directional:** This is **Correct**. Ionic attraction is simply the Coulombic force between charges. It acts uniformly in all directions around an ion. This is why ions arrange themselves in crystal lattices (like NaCl) based on size ratios, not bond angles.

(B) **Maximum number of bond pairs is 3:** This is **Correct** for most common stable molecules

(single, double, and triple bonds). While quadruple bonds exist in some exotic transition metal complexes, for general chemistry purposes in such exams, 3 is considered the standard maximum.

(C) **Covalent compounds conduct electricity in fused state:** This is **Incorrect**. Covalent compounds are made of neutral molecules. Even when melted (fused), they do not break down into ions. Without free ions or free electrons, they cannot carry an electric current. (Note: Exceptions like graphite or polar covalent acids in water exist, but generally, covalent solids/liquids are insulators).

(D) **Ionic compounds are generally soluble in water:** This is **Correct**. Water is a polar solvent with a high dielectric constant. It can easily solvate ions by surrounding them with its dipoles, breaking the lattice energy.

Since statement (C) is false, it is the required answer for this "not correct" question.

Step 3: Final Answer:

The incorrect statement is that covalent compounds conduct electricity in the fused state.

This matches Option (C).

Quick Tip: Conductivity Rule:

- **Ionic** $\implies$ Conducts in **Solution** and **Fused (Molten)** states.
- **Covalent** $\implies$ Generally **Non-conductors** in all states.

If a question asks about conductivity in "fused" or "molten" states, it is almost always referring to a property of ionic compounds.

81. 12.6 g of oxalic acid, $H_2C_2O_4 \cdot 2H_2O$ (M.wt 126) is present in 1500 mL of solution. The normality of that solution is

- (A) 0.266 N
- (B) 0.133 N
- (C) 0.399 N
- (D) 0.430 N

Correct Answer: (B) 0.133 N

Solution:

Step 1: Understanding the Concept:

Normality (N) is a measure of concentration that expresses the number of gram equivalents of solute present per liter of solution. Unlike molarity, which depends solely on the number of moles, normality accounts for the reactive capacity of the solute, often referred to as the n-factor or acidity/basicity.

Oxalic acid dihydrate ($H_2C_2O_4 \cdot 2H_2O$) is a dicarboxylic acid, meaning it can donate two protons (H^+) per molecule in an acid-base neutralization reaction. Therefore, its basicity (n-factor) is 2. The relationship between Molar Weight and Equivalent Weight is defined by this n-factor. Calculating normality involves identifying the mass of the solute, determining its equivalent mass, and scaling the result to the standard volume of 1000 mL (1 liter). This is a foundational calculation in analytical titration and volumetric chemistry.

Step 2: Key Formula or Approach:

1. Equivalent Weight (E) = $\frac{\text{Molecular Weight}}{\text{n-factor (basicity)}}$
2. Normality (N) = $\frac{\text{Weight of solute (g)}}{\text{Equivalent Weight}} \times \frac{1000}{\text{Volume of solution (mL)}}$

Step 3: Detailed Explanation:

Given:

- Weight of oxalic acid dihydrate (w) = 12.6 g
- Molecular weight (M) = 126 u
- Volume of solution (V) = 1500 mL
- n-factor for oxalic acid = 2 (it is a dibasic acid)

First, calculate the Equivalent Weight (E):

$$E = \frac{126}{2} = 63 \text{ g/eq}$$

Now, calculate the number of gram equivalents of oxalic acid:

$$\text{Gram equivalents} = \frac{\text{Weight}}{\text{Equivalent Weight}} = \frac{12.6}{63}$$

Performing the division: $126/63 = 2$, so $12.6/63 = 0.2$ eq.

Finally, calculate the Normality by adjusting for the volume:

$$N = \frac{0.2 \text{ eq}}{1500 \text{ mL}} \times 1000 \text{ mL/L}$$

$$N = \frac{0.2 \times 1000}{1500} = \frac{200}{1500}$$

Simplify the fraction:

$$N = \frac{2}{15} \approx 0.13333... \text{ N}$$

Rounding to three decimal places as per standard convention in chemical reporting gives 0.133 N. This implies that in 1 liter of this specific solution, there are 0.133 equivalents of reactive protons available for neutralization. Understanding these units is vital for accurate lab preparation.

Step 4: Final Answer:

The normality of the solution is 0.133 N.

This matches Option (B).

Quick Tip: To solve faster, use the relationship: **Normality = Molarity $\times$ n-factor.**

Molarity = $(12.6/126) \times (1000/1500) = 0.1 \times (2/3) = 0.0666$.

Normality = $0.0666 \times 2 = 0.133$.

Always look for the n-factor first when normality is requested!

82. Which of the following has highest equivalent weight? (Given: At.wt H=1, C=12, O=16, S=32, Na=23, Ca=40)

- (A) Sulphuric acid
- (B) Sodium carbonate
- (C) Sodium sulphate
- (D) Calcium carbonate

Correct Answer: (C) Sodium sulphate

Solution:

Step 1: Understanding the Concept:

Equivalent weight is the mass of a substance which will combine with or displace a fixed amount of another substance. In acids, it is the molar mass divided by the number of replaceable hydrogen ions (basicity). In salts, it is the molar mass divided by the total positive (or negative) valency. To find the substance with the "highest" equivalent weight, we must systematically calculate the molar mass of each compound using the provided atomic weights and then divide by their respective n-factors. This requires a strong understanding of chemical formulas and ion charges.

Step 2: Key Formula or Approach:

Formula: Equivalent Weight (E) = $\frac{\text{Molar Mass (M)}}{\text{n-factor}}$.

- For acids: n-factor = number of H^+ ions.
- For salts: n-factor = total charge of the cation or anion.

Step 3: Detailed Explanation:

Let's calculate for each option:

(A) Sulphuric acid (H_2SO_4):

Molar mass = $2(1) + 32 + 4(16) = 2 + 32 + 64 = 98$.

Basicity = 2.

$E = 98/2 = 49$.

(B) Sodium carbonate (Na_2CO_3):

Molar mass = $2(23) + 12 + 3(16) = 46 + 12 + 48 = 106$.

Total charge on Na^+ (2×1) = 2.

$E = 106/2 = 53$.

(C) Sodium sulphate (Na_2SO_4):

Molar mass = $2(23) + 32 + 4(16) = 46 + 32 + 64 = 142$.

Total charge on Na^+ (2×1) = 2.

$$E = 142/2 = 71.$$

(D) **Calcium carbonate** ($CaCO_3$):

$$\text{Molar mass} = 40 + 12 + 3(16) = 40 + 12 + 48 = 100.$$

Total charge on Ca^{2+} = 2.

$$E = 100/2 = 50.$$

Comparing the results: 49, 53, 71, and 50.

The value for Sodium sulphate (71) is significantly higher than the others. This is because the sulphate ion (SO_4^{2-}) and the two sodium ions contribute a much larger total mass while the valency remains only 2.

Step 4: Final Answer:

Sodium sulphate has the highest equivalent weight among the choices.

This matches Option (C).

Quick Tip: Since all options in this specific question have an **n-factor of 2** (two H in the acid, or a divalent salt cation/anion system), the equivalent weight is simply proportional to the Molar Mass. Therefore, just find the substance with the **largest Molar Mass** to identify the highest equivalent weight.

83. Identify the pair of gases which have same number of molecules at S.T.P?

- (A) 11 g of CO_2 and 14 g of N_2
- (B) 16 g of O_3 and 16 g of CH_4
- (C) 5 g of H_2 and 40 g of CH_4
- (D) 28 g of N_2 and 22 g of CO_2

Correct Answer: (C) 5 g of H_2 and 40 g of CH_4

Solution:

Step 1: Understanding the Concept:

According to Avogadro's Hypothesis, equal volumes of all gases under the same conditions of temperature and pressure (S.T.P) contain an equal number of molecules. This also implies that an equal number of moles (n) will contain an equal number of molecules ($N = n \times N_A$). Thus, identifying the pair of gases with the same number of molecules is equivalent to finding the pair that has the same number of moles. Calculating moles requires dividing the given mass of the gas by its molar mass. S.T.P provides a standard reference point for these comparisons.

Step 2: Key Formula or Approach:

1. Number of moles (n) = $\frac{\text{Given mass (w)}}{\text{Gram Molecular Weight (M)}}$
 2. Calculate n for each gas in every pair. If $n_1 = n_2$, the number of molecules is the same.
- Molar weights: $H_2 = 2, N_2 = 28, CO_2 = 44, O_3 = 48, CH_4 = 16$.

Step 3: Detailed Explanation:

Let's analyze each option systematically:

(A) **11 g CO_2 and 14 g N_2 :**

- Moles $CO_2 = 11/44 = 0.25$
- Moles $N_2 = 14/28 = 0.5$. (Not equal)

(B) **16 g O_3 and 16 g CH_4 :**

- Moles $O_3 = 16/48 = 1/3 \approx 0.33$
- Moles $CH_4 = 16/16 = 1.0$. (Not equal)

(C) **5 g H_2 and 40 g CH_4 :**

- Moles $H_2 = 5/2 = 2.5$
- Moles $CH_4 = 40/16 = 2.5$.

Since the moles are identical (2.5), the number of molecules is exactly $2.5 \times 6.022 \times 10^{23}$ for both. This pair is correct.

(D) **28 g N_2 and 22 g CO_2 :**

- Moles $N_2 = 28/28 = 1.0$
- Moles $CO_2 = 22/44 = 0.5$. (Not equal)

The equality in Option (C) arises from the inverse relationship between density/molar mass and the quantity required. Hydrogen is very light, so 5g represents a lot of moles, whereas

methane is heavier, requiring 40g to reach that same mole count.

Step 4: Final Answer:

The pair is 5 g of H_2 and 40 g of CH_4 .

This matches Option (C).

Quick Tip: To save time, memorize common molecular weights: $H_2 = 2$, $CH_4 = 16$, $N_2 = 28$, $CO_2 = 44$. In exams, check if the **ratio of masses is the same as the ratio of molar masses**. For (C): $5/40 = 1/8$. Molar masses ratio: $2/16 = 1/8$. If ratios match, moles match!

84. 100 mL of 0.1M HCl and 100 mL of 0.05 M H_2SO_4 are mixed and the solution is diluted to 2.0 L by adding water. The pH of the resulting solution is

- (A) 1
- (B) 3
- (C) 2
- (D) 4

Correct Answer: (C) 2

Solution:

Step 1: Understanding the Concept:

The pH of a solution is defined as the negative logarithm of the hydrogen ion concentration ($pH = -\log[H^+]$). When mixing strong acids, we must calculate the total amount of H^+ ions contributed by each acid. Note that HCl is a monoprotic acid (releases $1H^+$), while H_2SO_4 is a diprotic acid (releases $2H^+$). After finding the total moles of H^+ , we divide by the **final volume** of the solution to find the final concentration. Dilution significantly lowers the ion concentration and thus raises the pH.

Step 2: Key Formula or Approach:

1. Total moles of $H^+ = (M_1V_1 \times n_1) + (M_2V_2 \times n_2)$
2. Final concentration $[H^+] = \frac{\text{Total moles}}{\text{Final Volume (L)}}$
3. $pH = -\log[H^+]$

Step 3: Detailed Explanation:

1. Calculate moles of H^+ from HCl :

$$M = 0.1, V = 100 \text{ mL} = 0.1 \text{ L}, n = 1.$$

$$\text{Moles} = 0.1 \times 0.1 \times 1 = 0.01 \text{ moles.}$$

2. Calculate moles of H^+ from H_2SO_4 :

$$M = 0.05, V = 100 \text{ mL} = 0.1 \text{ L}, n = 2.$$

$$\text{Moles} = 0.05 \times 0.1 \times 2 = 0.01 \text{ moles.}$$

3. Calculate Total Moles of H^+ :

$$\text{Total moles} = 0.01 + 0.01 = 0.02 \text{ moles.}$$

4. Calculate Final Concentration $[H^+]$:

The solution is diluted to a total volume of 2.0 L.

$$[H^+] = 0.02 \text{ moles} / 2.0 \text{ L} = 0.01 \text{ M.}$$

5. Calculate pH:

$$0.01 \text{ M} = 10^{-2} \text{ M.}$$

$$pH = -\log(10^{-2}) = 2.$$

The mixing and dilution process resulted in a perfectly balanced decimal concentration, leading to an integer pH value. This is a common setup in competitive exam problems to simplify logarithmic calculations.

Step 4: Final Answer:

The pH of the resulting solution is 2.

This matches Option (C).

Quick Tip: Don't forget the factor of 2 for H_2SO_4 ! It is a common mistake to treat it as monoprotic. Also, always check if the final volume is the "added volume" or the "total volume." The phrase "diluted to 2.0 L" explicitly tells you the final volume is 2.0 L.

85. According to Arrhenius theory of acids and bases, which of the following is an example of Arrhenius base?

- (A) H_2SO_4
- (B) NH_3
- (C) $NaOH$
- (D) CaO

Correct Answer: (C) $NaOH$

Solution:

Step 1: Understanding the Concept:

The Arrhenius theory is the earliest of the three modern definitions of acids and bases, proposed by Svante Arrhenius in 1884. According to this theory, an **Arrhenius acid** is a substance that dissociates in water to produce hydrogen ions (H^+). Conversely, an **Arrhenius base** is a substance that dissociates in water to produce hydroxide ions (OH^-). The key limitation of this theory is that it only applies to aqueous solutions and specifically requires the presence of OH in the molecular formula for bases. Later theories, like Bronsted-Lowry and Lewis, expanded these definitions to include substances that don't contain hydroxyl groups or don't require an aqueous medium.

Step 2: Detailed Explanation:

Let's evaluate the options based strictly on the Arrhenius criteria (dissociation into OH^- in water):

(A) H_2SO_4 : This substance dissociates in water as: $H_2SO_4 \rightarrow 2H^+ + SO_4^{2-}$. Since it produces H^+ ions, it is an Arrhenius **acid**, not a base.

(B) NH_3 : Ammonia acts as a base in water by accepting a proton to form NH_4^+ and OH^- . However, Arrhenius theory requires the molecule itself to contain OH and dissociate it. By the strict Arrhenius definition, NH_3 is often not considered a base (it is a Bronsted-Lowry base).

(C) $NaOH$: Sodium Hydroxide is a classic example. When dissolved in water, it dissociates as: $NaOH_{(aq)} \rightarrow Na^+_{(aq)} + OH^-_{(aq)}$. It directly releases OH^- ions into the solution. Thus, it is a perfect example of an Arrhenius base.

(D) CaO : Calcium oxide reacts with water to form $Ca(OH)_2$, which then releases OH^- . While

it behaves as a base, CaO itself does not contain the OH group to dissociate. In Arrhenius terms, it's a basic oxide but the hydroxide itself is the base.

Therefore, Sodium Hydroxide is the most definitive and textbook example of an Arrhenius base.

Step 3: Final Answer:

NaOH is an example of an Arrhenius base.

This matches Option (C).

Quick Tip: To identify an **Arrhenius Base**, look for the " -OH " group in the chemical formula. Common examples include NaOH , KOH , Ca(OH)_2 , and Mg(OH)_2 . If a substance is a base but doesn't have " -OH " in its name (like NH_3), it's likely defined by a later theory (Bronsted-Lowry).

86. Electrolysis of an aqueous solution of Na_2SO_4 between Pt electrodes liberate a gas X at anode and gas Y at cathode. X and Y respectively are

- (A) H_2, O_2
- (B) O_2, H_2
- (C) SO_2, H_2
- (D) H_2, SO_2

Correct Answer: (B) O_2, H_2

Solution:

Step 1: Understanding the Concept:

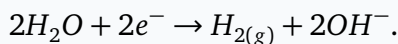
Electrolysis is a process where electrical energy is used to drive a non-spontaneous chemical reaction. When an aqueous solution of a salt like Sodium Sulphate (Na_2SO_4) is electrolyzed using inert platinum (Pt) electrodes, multiple ions compete for discharge at the electrodes. The outcome is determined by the discharge potentials (electrode potentials) of the species present: Na^+ , H^+ (from water), SO_4^{2-} , and OH^- (from water). In aqueous solutions of active metals and stable polyatomic anions, it is often water that undergoes redox reactions instead

of the salt ions.

Step 2: Detailed Explanation:

1. At the Cathode (- electrode):

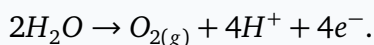
Both Na^+ and H^+ (or water) are present. The reduction potential of water (-0.83V) is higher (less negative) than that of sodium (-2.71V). Thus, water is reduced more easily:



Therefore, **Hydrogen gas (Y)** is liberated at the cathode.

2. At the Anode (+ electrode):

Both SO_4^{2-} and water (OH^-) are present. Sulphate ions are extremely stable and have a very high oxidation potential. Water is oxidized much more readily:



Therefore, **Oxygen gas (X)** is liberated at the anode.

The overall reaction is effectively the electrolysis of water: $2\text{H}_2\text{O} \rightarrow 2\text{H}_2 + \text{O}_2$. The Na_2SO_4 simply acts as an electrolyte to increase conductivity. The question asks for X and Y respectively, where X is at the anode (O_2) and Y is at the cathode (H_2).

Step 3: Final Answer:

The gases liberated are O_2 at the anode and H_2 at the cathode.

This matches Option (B).

Quick Tip: Mnemonic: **P-A-O** (Positive-Anode-Oxidation) and **N-C-R** (Negative-Cathode-Reduction).

For dilute aqueous solutions of group 1, 2 metals and sulphate/nitrate/phosphate ions:

- **Anode** always gives **Oxygen**.
- **Cathode** always gives **Hydrogen**.

87. The wrong statement regarding Galvanic cell is

- (A) In this spontaneous redox reaction occurs
- (B) Salt bridge maintains electrical neutrality between the two solutions
- (C) Anode is represented by (+) and cathode by (-)

(D) At anode oxidation occurs

Correct Answer: (C) Anode is represented by (+) and cathode by (-)

Solution:

Step 1: Understanding the Concept:

A Galvanic (or Voltaic) cell is an electrochemical cell that converts chemical energy from a spontaneous redox reaction into electrical energy. It consists of two half-cells connected by an external circuit and a salt bridge. There is a fundamental set of rules governing the operation and labeling of these cells. In any electrochemical cell, oxidation *always* occurs at the anode, and reduction *always* occurs at the cathode. However, the electrical polarity (+ or -) of these electrodes depends on the type of cell (Galvanic vs. Electrolytic). Identifying the "wrong" statement requires comparing textbook definitions of Galvanic cells against the given options.

Step 2: Detailed Explanation:

Let's analyze each statement:

(A) **Spontaneous redox reaction occurs:** This is **Correct**. Galvanic cells generate power precisely because the underlying chemical reaction has a negative Gibbs Free Energy ($\Delta G < 0$).

(B) **Salt bridge maintains neutrality:** This is **Correct**. Without a salt bridge, charge would build up in each half-cell, quickly halting the flow of electrons. The salt bridge allows ions to migrate to balance the charge.

(C) **Anode (+), Cathode (-):** This is **Incorrect** for a Galvanic cell. In a Galvanic cell, the anode is the source of electrons for the external circuit, making it the **Negative (-)** terminal. The cathode is where electrons are consumed, making it the **Positive (+)** terminal. This polarity is reversed in electrolytic cells.

(D) **At anode oxidation occurs:** This is **Correct**. By definition, the anode is the electrode where oxidation happens.

Since statement (C) provides the wrong polarity for a Galvanic cell, it is the required answer.

Step 3: Final Answer:

The wrong statement is that the anode is (+) and cathode is (-).

This matches Option (C).

Quick Tip: Mnemonic for Galvanic cells: **LOAN**.

Left, Oxidation, Anode, Negative.

Once you remember that the Anode is Negative in a battery (Galvanic cell), you will never get this question wrong.

88. Which of the following is a weak electrolyte?

- (A) H_2CO_3
- (B) H_2SO_4
- (C) $NaCl$
- (D) $NaOH$

Correct Answer: (A) H_2CO_3

Solution:

Step 1: Understanding the Concept:

An electrolyte is a substance that produces an electrically conducting solution when dissolved in a polar solvent, such as water. Electrolytes are classified as "strong" or "weak" based on their degree of ionization or dissociation in solution.

1. **Strong Electrolytes:** Dissociate completely (100%) into ions. They include strong acids, strong bases, and most soluble salts.
2. **Weak Electrolytes:** Dissociate only partially in water. A dynamic equilibrium exists between the undissociated molecules and the ions. This results in a much lower concentration of ions and poorer electrical conductivity.

Step 2: Detailed Explanation:

Evaluating the options:

(A) H_2CO_3 (**Carbonic Acid**): This is a weak organic acid found in carbonated beverages. In water, it establishes an equilibrium: $H_2CO_3 \rightleftharpoons H^+ + HCO_3^-$. Only a small fraction of molecules ionize at any given time. Therefore, it is a **weak electrolyte**.

(B) H_2SO_4 (**Sulphuric Acid**): This is a classic strong mineral acid. It dissociates completely into H^+ and HSO_4^- in the first step. It is a **strong electrolyte**.

(C) **$NaCl$ (Sodium Chloride):** This is a typical ionic salt. In water, the crystal lattice breaks down completely to release Na^+ and Cl^- ions. It is a **strong electrolyte**.

(D) **$NaOH$ (Sodium Hydroxide):** This is a strong alkali base. It dissociates fully into Na^+ and OH^- ions in aqueous solutions. It is a **strong electrolyte**.

Only Carbonic acid fits the criteria for partial dissociation, making it the weak electrolyte in this list.

Step 3: Final Answer:

Carbonic acid (H_2CO_3) is the weak electrolyte.

This matches Option (A).

Quick Tip: Weak Electrolyte Categories:

- **Weak Acids:** CH_3COOH, H_2CO_3, HCN .

- **Weak Bases:** NH_4OH .

Strong Electrolytes:

- **Mineral Acids:** HCl, HNO_3, H_2SO_4 .

- **Alkali Bases:** $NaOH, KOH$.

- **Salts:** $NaCl, KCl, CuSO_4$.

89. The exhausted anion-exchange resin is regenerated with

- (A) dilute $NaOH$ solution
- (B) dilute $NaCl$ solution
- (C) dilute HCl solution
- (D) dilute Na_2SO_4 solution

Correct Answer: (A) dilute $NaOH$ solution

Solution:

Step 1: Understanding the Concept:

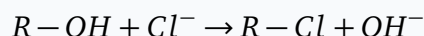
The ion-exchange method is a sophisticated technique used for water softening and demineralization. It uses synthetic resins which are high-molecular-weight polymers with functional groups.

1. **Cation-exchange resins** ($R-H$) contain acidic groups and exchange H^+ for cations like Ca^{2+} or Mg^{2+} .
2. **Anion-exchange resins** ($R-OH$) contain basic groups and exchange OH^- for anions like Cl^- or SO_4^{2-} .

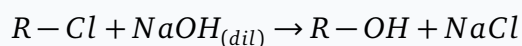
After prolonged use, all the OH^- ions in the anion resin are replaced by impurities from the hard water. At this point, the resin is "exhausted" and must be "regenerated" to restore its original hydroxyl (OH^-) form.

Step 2: Detailed Explanation:

The reaction during the ion-exchange process for an anion resin is:



The resin is now in the $R-Cl$ form (exhausted). To reverse this and get back the $R-OH$ form, we need to supply a high concentration of OH^- ions to displace the Cl^- by mass action. Dilute Sodium Hydroxide ($NaOH$) is the ideal regenerant because it is a strong base that provides an abundance of free OH^- ions. The regeneration reaction is:



The chloride ions are washed away with the excess brine solution, and the resin is once again ready to treat more water. Using HCl would be for regenerating cation resins ($R-H$), not anion ones.

Step 3: Final Answer:

Exhausted anion-exchange resin is regenerated with dilute $NaOH$.

This matches Option (A).

Quick Tip: Remember the **Resin-Regenerant Pair**:

- **Cation Resin** needs **Acid** (HCl or H_2SO_4) to restore H^+ .

- **Anion Resin** needs **Base** ($NaOH$) to restore OH^- .

Think: "Anion is Negative, Base provides the Negative OH^- ".

90. A sample of water is known to contain $Mg(HCO_3)_2 = 7.3 \text{ mg/L}$, $Ca(HCO_3)_2 = 8.1 \text{ mg/L}$ and 27.2 mg/L of $CaSO_4$. The total hardness associated with water sample (in ppm) in equivalents of $CaCO_3$ is (At.wt H=1, C=12, O=16, Mg=24, Ca=40, S=32)

(A) 20

(B) 25

(C) 30

(D) 40

Correct Answer: (C) 30

Solution:

Step 1: Understanding the Concept:

Hardness of water is caused by the presence of dissolved salts of Calcium and Magnesium. To compare different water samples, hardness is expressed in a standard unit: **ppm as $CaCO_3$ equivalents**. This is because Calcium Carbonate is insoluble, easy to weigh, and has a very convenient molecular weight of 100. Converting any salt's concentration into $CaCO_3$ equivalents involves scaling it by the ratio of their equivalent weights. Total hardness is simply the arithmetic sum of the individual hardness values contributed by each salt in the sample.

Step 2: Key Formula or Approach:

Hardness in terms of $CaCO_3$ equivalent:

$$\text{Equivalent } CaCO_3 = \text{Concentration of salt (mg/L)} \times \frac{\text{Eq. wt of } CaCO_3}{\text{Eq. wt of the salt}}$$

$$\text{Eq. wt of } CaCO_3 = 100/2 = 50.$$

Calculate for all salts and add them together.

Step 3: Detailed Explanation:

1. $Mg(HCO_3)_2$:

$$\text{Molar mass} = 24 + 2(1 + 12 + 48) = 24 + 2(61) = 24 + 122 = 146.$$

$$\text{Eq. wt} = 146/2 = 73.$$

$$\text{Contribution} = 7.3 \times (50/73) = 0.1 \times 50 = 5 \text{ ppm.}$$

2. $Ca(HCO_3)_2$:

$$\text{Molar mass} = 40 + 2(61) = 40 + 122 = 162.$$

$$\text{Eq. wt} = 162/2 = 81.$$

$$\text{Contribution} = 8.1 \times (50/81) = 0.1 \times 50 = 5 \text{ ppm.}$$

3. $CaSO_4$:

$$\text{Molar mass} = 40 + 32 + 64 = 136.$$

$$\text{Eq. wt} = 136/2 = 68.$$

$$\text{Contribution} = 27.2 \times (50/68) = 0.4 \times 50 = 20 \text{ ppm.}$$

$$\text{Total Hardness} = 5 + 5 + 20 = 30 \text{ ppm.}$$

The water sample has a total hardness of 30 parts per million. Since $1 \text{ ppm} = 1 \text{ mg/L}$, the units are consistent. This value indicates a "soft" to "slightly hard" water classification.

Step 4: Final Answer:

The total hardness is 30 ppm.

This matches Option (C).

Quick Tip: Standard ratios to remember for quick calculations:

- $Mg(HCO_3)_2$: weight / 1.46

- $Ca(HCO_3)_2$: weight / 1.62

- $CaSO_4$: weight / 1.36

The numbers in such exam problems are usually chosen to be perfect multiples (like 7.3 and 73, or 8.1 and 81). If your division doesn't result in a simple decimal, re-check your molar mass calculations!

91. The type of functional group associated with cation exchange resin is

- (A) - OH
- (B) - SO_3H
- (C) - NH_2
- (D) - CHO

Correct Answer: (B) - SO_3H

Solution:

Step 1: Understanding the Concept:

Ion exchange resins are highly porous, cross-linked polymeric structures that contain specific functional groups capable of exchanging ions with a solution.

In the context of water technology and softening, these resins are used to remove hardness-causing ions like Calcium (Ca^{2+}) and Magnesium (Mg^{2+}).

These resins are classified into two main types: Cation Exchange Resins and Anion Exchange Resins.

Cation exchange resins possess acidic functional groups that can release Hydrogen ions (H^+) or Sodium ions (Na^+) in exchange for other cations present in the water.

The effectiveness and strength of the resin depend on the nature of the attached acidic group. Strong acid cation resins typically utilize sulfonic acid groups, whereas weak acid cation resins utilize carboxylic acid groups.

Step 2: Key Formula or Approach:

The general structure of a cation exchange resin is represented as $R-H$, where R is the polymer matrix and H is the exchangeable hydrogen ion from the functional group.

The primary groups used are:

1. Strong Acidic Group: Sulfonic acid ($-SO_3H$).
2. Weak Acidic Group: Carboxylic acid ($-COOH$).

Step 3: Detailed Explanation:

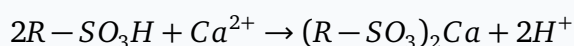
In the ion-exchange process for water demineralization, the water is first passed through a bed of cation exchange resin.

The sulfonic acid group ($-SO_3H$) is the most common functional group found in commercial

strong acid cation resins, such as those made from sulfonated polystyrene-divinylbenzene copolymers.

The dissociation of this group allows the resin to act as a source of protons.

The reaction can be represented as:



Here, the divalent calcium ion is trapped by the resin, and two hydrogen ions are released into the effluent water.

Option (A) $-OH$ is associated with anion exchange resins (as hydroxyl ions).

Option (C) $-NH_2$ (amino group) is found in anion exchange resins as part of quaternary ammonium structures.

Option (D) $-CHO$ (aldehyde) is not a standard ion-exchange functional group.

Therefore, the sulfonic acid group ($-SO_3H$) is the characteristic group that defines a strong cation exchange resin's ability to attract and bind positively charged ions.

This chemical architecture ensures that the resin can be regenerated easily using a strong mineral acid like HCl , restoring its capacity for further cycles of water purification.

Step 4: Final Answer:

The functional group associated with cation exchange resin is $-SO_3H$.

This matches Option (B).

Quick Tip: Remember the charge-group pairing:

- **Cation Resin** (+ ions) needs **Acidic** groups (starts with H , like $-SO_3H$ or $-COOH$).
- **Anion Resin** (− ions) needs **Basic** groups (contains Nitrogen or releases OH^- , like $-NH_2$ or $-OH$).

Think: Cation = Acidic, Anion = Basic.

92. Identify the incorrect statement about the corrosion

(A) In the composition type of galvanic cell, metal with lower standard reduction potential undergoes

corrosion

- (B) In stress cell type of galvanic cell, corrosion occurs at the stressed area of the metal
- (C) The rate of corrosion is more, when the area of cathode is smaller
- (D) In concentration cell type of galvanic cell, the metal below the water level undergoes corrosion readily

Correct Answer: (C) The rate of corrosion is more, when the area of cathode is smaller

Solution:

Step 1: Understanding the Concept:

Corrosion is the gradual destruction of metals by chemical or electrochemical reaction with their environment.

The electrochemical theory of corrosion explains that metallic surfaces behave like a series of tiny galvanic cells with anodic and cathodic regions.

The rate of corrosion is determined by several factors, including the potential difference between electrodes, the nature of the electrolyte, and importantly, the relative surface areas of the anode and cathode.

For corrosion to be severe, the cathodic reaction must be able to "consume" the electrons produced by the anodic oxidation rapidly.

Step 2: Key Formula or Approach:

The fundamental rule for the rate of galvanic corrosion related to area is:

$$\text{Rate of Corrosion} \propto \frac{\text{Area of Cathode}}{\text{Area of Anode}}$$

A large cathode relative to a small anode leads to intense, localized corrosion at the anode.

Step 3: Detailed Explanation:

Let's analyze each statement to find the incorrect one:

(A) **Statement A is Correct:** In a galvanic couple, the metal with the lower (more negative) standard reduction potential is more reactive and acts as the anode, thus undergoing oxidation (corrosion).

(B) **Statement B is Correct:** Stressed regions in a metal (like bends or rivets) possess higher

lattice energy and act as anodic sites compared to the unstressed bulk of the metal, leading to localized "stress corrosion."

(C) **Statement C is Incorrect:** According to the "Principle of Differential Areas," corrosion is most severe when the **Anode area is small** and the **Cathode area is large**.

In such a setup, the large cathodic surface can facilitate a huge amount of reduction (e.g., oxygen reduction), requiring a massive flow of electrons from the small anode.

This results in a very high current density at the anode, causing rapid pitting and failure.

If the cathode area were smaller, the overall reaction rate would be limited by the slow cathodic process, thus reducing the corrosion rate.

(D) **Statement D is Correct:** This describes "Differential Aeration Corrosion."

The part of the metal deeply submerged (below water level) has less access to oxygen and becomes anodic.

The part near the surface is oxygen-rich and becomes cathodic. Thus, the metal below the water level corrodes.

Since statement (C) contradicts the established scientific principles of area effects in corrosion, it is the "incorrect" statement.

Step 4: Final Answer:

The incorrect statement is: "The rate of corrosion is more, when the area of cathode is smaller."

This matches Option (C).

Quick Tip: Remember the "Big Cathode, Small Anode" disaster rule.

A large cathodic area **increases** the corrosion rate at the anode because it provides a large "sink" for electrons.

A small cathode **limits** the reaction.

The phrasing in Option C is exactly opposite to the physical reality.

93. In galvanised iron

(A) Zn acts as anode and Fe acts as cathode

- (B) Zn acts as cathode and Fe acts as anode
(C) Sn acts as anode and Fe acts as cathode
(D) Sn acts as cathode and Fe acts as anode

Correct Answer: (A) Zn acts as anode and Fe acts as cathode

Solution:

Step 1: Understanding the Concept:

Galvanization is a metallurgical process where a protective zinc coating is applied to steel or iron to prevent rusting.

This is a form of "Anodic Protection" or "Sacrificial Protection."

The effectiveness of the coating is based on the positions of the metals in the electrochemical (or galvanic) series.

The metal that is higher in the series (more negative reduction potential) will always act as the anode when coupled with a metal lower in the series.

The anode is the site of oxidation (loss of electrons/corrosion), while the cathode is protected.

Step 2: Key Formula or Approach:

Standard Reduction Potentials (E^0):

$$E_{Zn^{2+}/Zn}^0 = -0.76 V$$

$$E_{Fe^{2+}/Fe}^0 = -0.44 V$$

Since Zinc has a lower (more negative) reduction potential than Iron, it is more electropositive and more easily oxidized.

Step 3: Detailed Explanation:

When Iron is coated with Zinc (Galvanised Iron):

1. **Anodic Site:** Zinc, being more reactive than Iron, takes the role of the anode.

The oxidation reaction at the zinc layer is: $Zn \rightarrow Zn^{2+} + 2e^-$.

2. **Cathodic Site:** The base metal Iron acts as the cathode.

Even if the zinc coating is scratched and the iron is exposed to the atmosphere, the zinc continues to corrode preferentially.

The electrons released by the corroding zinc flow to the iron, preventing the iron from losing its own electrons and forming rust (Fe_2O_3).

3. **Comparison with Tinning:** In contrast, if iron is coated with Tin (Sn), iron is more reactive than Tin.

If a tin coating is scratched, the Iron becomes the anode and corrodes **faster** than if it had no coating at all.

Therefore, in galvanization, Zinc is the sacrificial anode and Iron is the protected cathode.

This explains why galvanized buckets and sheets last for decades even in moist conditions.

Step 4: Final Answer:

In galvanised iron, Zinc (Zn) acts as the anode and Iron (Fe) acts as the cathode.

This matches Option (A).

Quick Tip: Mnemonic: **Zinc Sacrifices Itself.**

Zinc is more "active" than iron. In any protective couple, the "active" metal becomes the **Anode** and the "base" metal becomes the **Cathode**.

Always check the Galvanic Series: Zn is above Fe, and Fe is above Sn.

94. During Vulcanization of raw rubber, the chemical added to it is

- (A) Sulphur
- (B) Phosphorus
- (C) Iodine
- (D) Sodium

Correct Answer: (A) Sulphur

Solution:

Step 1: Understanding the Concept:

Natural rubber is a polymer of isoprene (2-methyl-1,3-butadiene).

In its raw state, natural rubber has several drawbacks: it is sticky when hot, brittle when cold, has low tensile strength, and absorbs a large amount of water.

To improve its physical and mechanical properties, it undergoes a process called "Vulcanization," discovered by Charles Goodyear in 1839.

Vulcanization is a chemical process for converting natural rubber or related polymers into more durable materials by heating them with a cross-linking agent.

Step 2: Key Formula or Approach:

The process involves heating raw rubber with 3% to 5% by weight of a specific chemical element.

This element forms bridges between the long individual polymer chains, restricting their independent movement and creating a three-dimensional network.

Step 3: Detailed Explanation:

The primary chemical used in vulcanization is **Sulphur**.

When raw rubber is heated with sulphur at a temperature range of approximately 100°C to 140°C , the sulphur atoms react with the double bonds of the isoprene units.

These sulphur atoms form "cross-links" or "bridges" between adjacent chains.

The effect of this cross-linking is profound:

1. It eliminates the tackiness (stickiness) of the rubber.
2. It makes the rubber much more resistant to temperature changes.
3. It increases the elasticity and tensile strength significantly.
4. It makes the rubber resistant to swelling by organic solvents.

Depending on the amount of sulphur added, we can get different types of rubber. For example, tires usually contain about 3-5% sulphur, whereas hard rubber (Ebonite) can contain up to 32% sulphur.

Other elements like Phosphorus, Iodine, or Sodium do not possess the specific chemical geometry or reactivity to form stable cross-links with the polyisoprene backbone in this manner.

Thus, Sulphur is the indispensable ingredient that transformed rubber from a curiosity into a vital industrial material.

Step 4: Final Answer:

The chemical added during the vulcanization of raw rubber is Sulphur.

This matches Option (A).

Quick Tip: Associate **Vulcanization** with **Sulphur** and **Cross-linking**.

Vulcan was the Roman god of fire; the process involves high heat and chemical "firing" with sulphur.

Natural rubber = soft/weak; Vulcanized rubber = hard/strong.

95. Which of the following is a natural polymer?

- (A) Cellulose
- (B) Teflon
- (C) Polyvinylchloride
- (D) Neoprene rubber

Correct Answer: (A) Cellulose

Solution:

Step 1: Understanding the Concept:

Polymers are high-molecular-weight compounds formed by the repetition of small units called monomers.

Based on their origin, polymers are broadly classified into three categories:

1. **Natural Polymers:** These are found in nature, specifically in plants and animals. Examples include proteins, starch, and rubber.
2. **Synthetic Polymers:** These are man-made polymers synthesized in laboratories or industries using chemical reactions.
3. **Semi-synthetic Polymers:** These are derived from natural polymers through chemical

modification (e.g., cellulose acetate).

Step 2: Detailed Explanation:

Let's examine the options provided:

(A) **Cellulose:** This is the most abundant organic polymer on Earth. It is a major structural component of the cell walls of green plants and many forms of algae. It consists of long chains of β -D-glucose units. Because it is produced naturally by living organisms, it is a **natural polymer**.

(B) **Teflon:** Chemically known as Polytetrafluoroethylene (PTFE), this is synthesized in industries by the polymerization of tetrafluoroethene. It is entirely man-made and used for non-stick coatings.

(C) **Polyvinylchloride (PVC):** This is a synthetic plastic polymer produced by the polymerization of vinyl chloride. It is widely used in pipes and cables.

(D) **Neoprene rubber:** Also known as polychloroprene, it was the first synthetic rubber ever developed. While natural rubber exists, neoprene is a synthetic alternative designed to be resistant to oils and heat.

By evaluating the source of each material, it is clear that only Cellulose is a biological product occurring naturally in the environment.

Step 3: Final Answer:

Cellulose is the natural polymer among the given options.

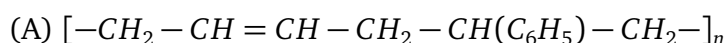
This matches Option (A).

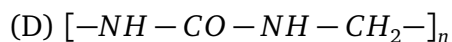
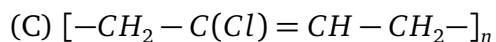
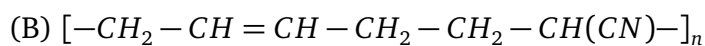
Quick Tip: Common Natural Polymers:

- **Plants:** Cellulose, Starch, Natural Rubber (cis-isoprene), Lignin.
- **Animals:** Proteins (Silk, Wool, Hair), Nucleic Acids (DNA, RNA).

If a polymer name starts with "Poly-" (except Polyisoprene), it is almost always synthetic.

96. The structure of Buna - S polymer is





Correct Answer: (A) $[-CH_2 - CH = CH - CH_2 - CH(C_6H_5) - CH_2 -]_n$

Solution:

Step 1: Understanding the Concept:

Buna-S is a common synthetic rubber, also known as SBR (Styrene Butadiene Rubber).

The name "Buna-S" is derived from its constituents and catalysts:

- **Bu** stands for 1,3-Butadiene.
- **Na** stands for Sodium (*Na*), which is used as the polymerization catalyst.
- **S** stands for Styrene.

It is an addition copolymer, meaning it is formed by the polymerization of two different monomers without the elimination of any small molecules.

Step 2: Key Formula or Approach:

The monomers involved are:

- 1,3-Butadiene: $CH_2 = CH - CH = CH_2$
- Styrene: $CH_2 = CH(C_6H_5)$

In the copolymerization process, the double bonds break and reform to link the monomers together into a repeating unit.

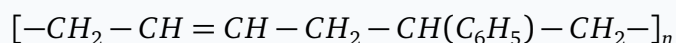
Step 3: Detailed Explanation:

When 1,3-Butadiene and Styrene react in a ratio of approximately 3:1:

The butadiene unit undergoes 1,4-addition, resulting in the segment $-CH_2 - CH = CH - CH_2 -$.

The styrene unit joins this chain, resulting in the segment $-CH_2 - CH(C_6H_5) -$.

The combined repeating structure of the Buna-S chain is:



Comparing with other options:

- Option (B) contains the $-CN$ group, which identifies it as **Buna-N** (Nitrile rubber).

- Option (C) contains Chlorine and a single monomer unit, identifying it as **Neoprene** (polychloroprene).
- Option (D) involves amide and carbonyl links, identifying it as a **Urea-Formaldehyde** resin or a polyamide type.

Thus, only Option (A) correctly represents the chemical backbone of Styrene-Butadiene rubber. This material is widely used in automotive tires due to its high abrasion resistance and durability.

Step 4: Final Answer:

The structure of Buna-S is given in Option (A).

Quick Tip: Shortcut to distinguish Buna rubbers:

- **Buna-S** $\Rightarrow$ Look for the **Phenyl** (C_6H_5) group from Styrene.
 - **Buna-N** $\Rightarrow$ Look for the **Nitrogen** ($-CN$) group from AcryloNitrile.
- Both contain a $-CH=CH-$ unit in the middle from the butadiene.

97. The polymer used in making gaskets and non-stick coating utensils is

- (A) Polyvinyl chloride
- (B) Polystyrene
- (C) Polytetrafluoroethylene
- (D) Polythene

Correct Answer: (C) Polytetrafluoroethylene

Solution:

Step 1: Understanding the Concept:

Specific engineering applications require polymers with unique physical and chemical properties.

Cooking utensils require a surface that is chemically inert (won't react with food), heat-

resistant, and has a very low coefficient of friction so that food does not stick.

Similarly, gaskets used in chemical industries must withstand corrosive environments and high temperatures without degrading.

The polymer that satisfies all these extreme requirements is Polytetrafluoroethylene (PTFE).

Step 2: Detailed Explanation:

Polytetrafluoroethylene (PTFE), popularly known by the trade name **Teflon**, is a synthetic fluoropolymer of tetrafluoroethylene.

Its structure is $[-CF_2 - CF_2-]_n$.

The properties that make it ideal for non-stick coatings and gaskets are:

1. **High Thermal Stability:** It remains stable at temperatures up to $260^\circ C$ and even higher for short periods.
2. **Chemical Inertness:** Due to the extremely strong Carbon-Fluorine bond, it is resistant to almost all chemicals, including strong acids and bases.
3. **Low Coefficient of Friction:** It has one of the lowest coefficients of friction of any solid, providing the "non-stick" quality.
4. **Electrical Insulation:** It is an excellent dielectric.

Let's look at the others:

(A) **PVC:** Used for pipes and raincoats; not heat-resistant enough for cooking.

(B) **Polystyrene:** Used for disposable cups and packaging; melts easily.

(D) **Polythene:** Used for bags and bottles; low melting point.

Therefore, PTFE is the only scientifically suitable choice for the described applications.

Step 3: Final Answer:

The polymer used is Polytetrafluoroethylene.

This matches Option (C).

Quick Tip: Keyword association:

Non-stick = Teflon = Polytetrafluoroethylene (PTFE).

Remember the structure: It is just ethylene where all four Hydrogens are replaced by Fluorine (F). Fluorine makes it "slippery" and chemically "bulletproof."

98. Which of the following is not to be considered as a primary fuel ?

- (A) Wood
- (B) Petroleum
- (C) Coke
- (D) Coal

Correct Answer: (C) Coke

Solution:

Step 1: Understanding the Concept:

Fuels are substances that provide heat or energy through combustion. They are classified based on their origin and processing:

1. **Primary Fuels (Natural Fuels):** These occur in nature and are used in their original form without significant chemical transformation. Examples include wood, coal, and crude petroleum.
2. **Secondary Fuels (Derived Fuels):** These are obtained from primary fuels through chemical or physical processing (like distillation or carbonization) to improve their quality or convenience. Examples include charcoal, coke, gasoline, and water gas.

Step 2: Detailed Explanation:

Let's categorize the options:

- (A) **Wood:** This is a primary solid fuel obtained directly from nature (trees).
- (B) **Petroleum:** This is a primary liquid fuel (crude oil) found naturally in the Earth's crust.
- (D) **Coal:** This is a primary solid fuel formed over millions of years by the fossilization of plant matter.
- (C) **Coke:** Coke is a grey, hard, and porous fuel with a high carbon content and few impurities. However, it is not found naturally in mines. It is produced by the **destructive distillation** (heating in the absence of air) of bituminous coal. During this process, volatile matter like coal gas and coal tar are removed, leaving behind the solid residue known as coke. Because coke is a manufactured product derived from the primary fuel "coal," it is classified as a **secondary fuel**.

Therefore, Coke is the correct answer to the question "not to be considered as a primary fuel."

Step 3: Final Answer:

Coke is not a primary fuel.

This matches Option (C).

Quick Tip: Think of the "Natural vs. Factory" test:

- Can you dig it up or cut it down from nature? → **Primary**.
 - Do you need to process another fuel in a plant to get it? → **Secondary**.
- Coal → Coke. Wood → Charcoal. Petroleum → Petrol/Diesel.

99. The oxide of nitrogen responsible for depletion of ozone layer is

- (A) N_2O
- (B) NO_2
- (C) NO
- (D) N_2O_3

Correct Answer: (C) NO

Solution:

Step 1: Understanding the Concept:

The ozone layer in the stratosphere protects the Earth from harmful ultraviolet (UV) radiation. The depletion of this layer is caused by chemical species that can catalyze the breakdown of ozone (O_3) into molecular oxygen (O_2).

While Chlorofluorocarbons ($CFCs$) are the most famous ozone depleters, oxides of nitrogen (NO_x) also play a critical role, especially at high altitudes.

These oxides are released directly into the stratosphere by high-altitude supersonic jet aircraft and through various industrial/natural processes.

Step 2: Key Formula or Approach:

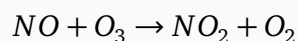
The mechanism involves a catalytic cycle where the nitrogen oxide molecule is regenerated at the end, allowing a single molecule to destroy thousands of ozone molecules.

Step 3: Detailed Explanation:

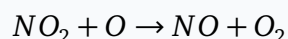
Nitric oxide (NO) is the primary oxide responsible for this catalytic destruction.

The reaction sequence is as follows:

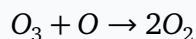
1. First, Nitric oxide reacts with ozone:



2. Then, the resulting Nitrogen dioxide reacts with an oxygen atom (which is abundant in the stratosphere due to UV photolysis):



Summing the reactions, the overall result is:



Observe that the NO molecule consumed in the first step is **restored** in the second step.

This means NO acts as a catalyst.

Option (A) N_2O (Nitrous oxide) is a greenhouse gas and a long-lived precursor that eventually converts to NO , but NO is the active species doing the depletion.

Option (B) NO_2 is part of the cycle but NO is the initial triggering agent.

Therefore, Nitric oxide (NO) is recognized as the major stratospheric pollutant responsible for the nitrogen-based ozone depletion cycle.

Step 4: Final Answer:

The oxide of nitrogen responsible for ozone depletion is NO .

This matches Option (C).

Quick Tip: Major Ozone Depleters Summary:

1. **Chlorine atoms** (from *CFCs*).
2. **Nitric Oxide** (*NO*) (from high-altitude aircraft/supersonic jets).

If a question mentions aircraft emissions and ozone, the answer is always **Nitric Oxide**.

100. The BOD of highly polluted water is

- (A) 17 ppm
- (B) 10 ppm
- (C) 8 ppm
- (D) 12 ppm

Correct Answer: (A) 17 ppm

Solution:

Step 1: Understanding the Concept:

Biochemical Oxygen Demand (*BOD*) is a standard measure used to determine the quality of a water body.

It is defined as the amount of dissolved oxygen (*DO*) consumed by microorganisms (aerobic bacteria) to decompose the organic matter present in a unit volume of water over a fixed period (usually 5 days at 20°C).

High *BOD* indicates a high level of organic pollution because more bacteria are needed to break down the waste, thereby stripping the oxygen from the water.

Low *BOD* indicates clean, healthy water with little organic waste.

Step 2: Key Formula or Approach:

The units for *BOD* are parts per million (*ppm*) or *mg/L*.

Water quality standards typically classify water as follows:

- *BOD* < 1 *ppm*: Exceptionally clean.
- *BOD* $\approx$ 2 – 3 *ppm*: Clean/Normal.
- *BOD* > 5 *ppm*: Doubtful/Starting to be polluted.
- *BOD* > 10 *ppm*: Highly polluted.

Step 3: Detailed Explanation:

Let's evaluate the threshold for "highly polluted" water.

As the organic waste (like sewage or industrial effluent) increases, the bacterial population explodes to consume it.

These bacteria consume the dissolved oxygen in the process. If the *BOD* value is very high, it means the oxygen levels in the water will drop dangerously low, causing fish and other aquatic life to suffocate.

- **8 ppm (Option C):** This is considered polluted, but not "highly" polluted by environmental standards.
- **10-12 ppm (Options B, D):** These are significant levels of pollution, often found in moderately contaminated rivers.
- **17 ppm (Option A):** Any value above 15 ppm is universally categorized as "highly polluted" or "severely contaminated" water. Raw sewage, for example, can have *BOD* values in the hundreds, but for natural water bodies or effluents being assessed for general health, 17 ppm represents a critical failure of water quality.

Therefore, among the choices provided, 17 ppm represents the highest concentration of organic load and thus the most "highly polluted" state.

Step 4: Final Answer:

The *BOD* of highly polluted water is 17 ppm.

This matches Option (A).

Quick Tip: Remember the **Inverse Relationship**:

High BOD = High Pollution = Low Dissolved Oxygen.

Clean water has $BOD \leq 5 \text{ ppm}$.

In multiple-choice questions, if asked for "highly polluted," always pick the **largest numerical value** provided in the options.

EEE

101. Kirchhoff's first law ($\sum I = 0$) is based on the conservation of

- (A) Energy
- (B) Charge
- (C) Momentum
- (D) Mass

Correct Answer: (B) Charge

Solution:

Step 1: Understanding the Concept:

Kirchhoff's Circuit Laws are fundamental principles in electrical engineering used to analyze the distribution of current and voltage in a network. Kirchhoff's First Law, also known as Kirchhoff's Current Law (KCL) or the Junction Rule, focuses on the flow of electric current through a node or junction. A junction is any point in a circuit where three or more conductors meet. The law essentially states that at any junction, the rate at which charge enters the junction must be exactly equal to the rate at which charge leaves the junction. This is a direct physical consequence of the fact that electric charge cannot be created or destroyed at a point, nor can it accumulate indefinitely at a conductor's junction in a steady-state condition. Therefore, KCL is a mathematical statement of the universal principle of the conservation of electric charge applied to an electrical circuit.

Step 2: Key Formula or Approach:

The mathematical representation of Kirchhoff's First Law is:

$$\sum_{k=1}^n I_k = 0$$

where I_k represents the current in the k^{th} branch connected to the node. By convention, currents entering the node are often assigned a positive sign, while currents leaving the node are assigned a negative sign.

Step 3: Detailed Explanation:

Consider a node where four wires meet. Let the currents entering be I_1 and I_2 , and the

currents leaving be I_3 and I_4 . According to KCL:

$$I_1 + I_2 = I_3 + I_4$$

Rearranging this to the form $\sum I = 0$:

$$I_1 + I_2 - I_3 - I_4 = 0$$

Since current is defined as the rate of flow of charge ($I = dQ/dt$), we can rewrite the entering currents as the rate of charge arrival and leaving currents as the rate of charge departure. If the sum were not zero, it would imply that charge is either building up at the junction (like a capacitor) or being drained from it, which is impossible for a simple wire junction. In contrast, Kirchhoff's Second Law (KVL), which deals with the sum of voltages around a closed loop, is based on the conservation of energy. It is vital for students to distinguish between these two: First Law relates to current/charge, while the Second Law relates to voltage/energy. Understanding this distinction is the cornerstone of nodal and mesh analysis in network theory.

Step 4: Final Answer:

Kirchhoff's first law is based on the conservation of charge.

This matches Option (B).

Quick Tip: To remember this easily: Current starts with C, and so does Charge.

KCL → Conservation of Charge.

KVL → Voltage → Energy (Work per unit charge).

102. Three resistances each of value 3Ω are connected in delta. Their value in each branch of equivalent star connection will be

(A) 9Ω

(B) 1Ω

(C) 6Ω

(D) 3Ω

Correct Answer: (B) 1Ω

Solution:

Step 1: Understanding the Concept:

In electrical network analysis, the Star (Y) and Delta (Δ) configurations are common ways to connect three-terminal components. Often, to simplify a complex circuit, it is necessary to convert a delta network into an equivalent star network or vice versa. This conversion ensures that the resistance measured between any two terminals remains identical in both configurations. This specific problem deals with a "balanced" delta network, where all three resistors have the same value. In such cases, the symmetry of the circuit allows for a very simple mathematical relationship between the delta resistances and the equivalent star resistances.

Step 2: Key Formula or Approach:

For a general Delta-to-Star conversion, the resistance of a branch in the star connection (R_a, R_b, R_c) is given by the product of the two adjacent delta resistances divided by the sum of all three delta resistances (R_{12}, R_{23}, R_{31}).

For a **balanced** load where $R_{\Delta} = R_{12} = R_{23} = R_{31}$, the formula simplifies to:

$$R_Y = \frac{R_{\Delta}}{3}$$

Step 3: Detailed Explanation:

Let's apply the conversion logic to the given values.

We have a delta network where each resistor is $R_{\Delta} = 3\Omega$.

The formula for the resistance in any branch of the equivalent star connection (R_Y) is:

$$R_Y = \frac{R_{12} \times R_{31}}{R_{12} + R_{23} + R_{31}}$$

Substituting the value 3 for all delta resistances:

$$R_Y = \frac{3 \times 3}{3 + 3 + 3}$$

$$R_Y = \frac{9}{9}$$

$$R_Y = 1 \Omega$$

This means that a delta configuration with three 3Ω resistors is electrically indistinguishable from a star configuration with three 1Ω resistors when viewed from the external terminals. This reduction factor of 3 is a standard property of balanced 3-phase systems. Conversely, if you were converting from star to delta, you would multiply the star resistance by 3. Understanding this ratio is crucial for power system calculations and motor starting methods (like the star-delta starter).

Step 4: Final Answer:

The value of each branch in the equivalent star connection is 1Ω .

This matches Option (B).

Quick Tip: Mnemonic: "Delta is bigger."

The delta configuration always has a higher resistance value than its equivalent star configuration.

Balanced Delta to Star: **Divide by 3.**

Balanced Star to Delta: **Multiply by 3.**

Since 3 is the input, the answer must be smaller, making 1 the only logical choice.

103. Which theorem simplifies a complex network into a single voltage source in series with a resistor?

(A) Norton's theorem

- (B) Superposition theorem
- (C) Thevenin's theorem
- (D) Maximum Power transfer theorem

Correct Answer: (C) Thevenin's theorem

Solution:

Step 1: Understanding the Concept:

Circuit theorems provide analytical tools to reduce large, complex networks into simple, manageable equivalents. This is particularly useful when we want to study the behavior of a specific load component without recalculating the entire circuit every time the load changes. There are two primary dual theorems for this purpose: Thevenin's and Norton's. Thevenin's theorem focuses on representing the network as a voltage-based source, whereas Norton's focuses on a current-based source. The question specifically asks for the configuration consisting of a voltage source and a series resistor.

Step 2: Detailed Explanation:

Let's evaluate the definitions of the theorems mentioned in the options:

1. **Thevenin's Theorem:** It states that any linear, bilateral network containing one or more voltage sources and resistances can be replaced by an equivalent circuit consisting of a single voltage source (V_{th}) in **series** with a single resistance (R_{th}). The voltage V_{th} is the open-circuit voltage at the terminals, and R_{th} is the equivalent resistance seen from those terminals with all independent sources deactivated.
2. **Norton's Theorem:** This is the dual of Thevenin's theorem. It replaces the complex network with a single **current source** (I_n) in **parallel** with a resistor (R_n).
3. **Superposition Theorem:** This is used to solve circuits with multiple sources by considering one source at a time. It does not produce a single source-resistor equivalent circuit.
4. **Maximum Power Transfer Theorem:** This theorem determines the condition under which a source delivers maximum power to a load (when $R_L = R_{th}$). It uses the Thevenin equivalent but is not the process of simplification itself.

Therefore, the description provided in the question—a single voltage source in series with a resistor—is the textbook definition of a Thevenin equivalent circuit.

Step 3: Final Answer:

Thevenin's theorem is used to simplify a network into a single voltage source in series with a resistor.

This matches Option (C).

Quick Tip: Remember the pair:

- **Thevenin** = Voltage source + **Series** resistor.

- **Norton** = Current source + **Parallel** resistor.

Think of "Thevenin" as "V" (it sounds like it contains a 'v' sound) for Voltage.

104. Who discovered the magnetic effect of electric current?

- (A) Faraday
- (B) Oersted
- (C) Ampere
- (D) Bohr

Correct Answer: (B) Oersted

Solution:

Step 1: Understanding the Concept:

The discovery of electromagnetism marked a turning point in the history of science, as it proved for the first time that electricity and magnetism were not separate phenomena but deeply interrelated. This field of study is known as electromagnetism. Several prominent scientists contributed to this field: Michael Faraday, André-Marie Ampère, James Clerk Maxwell, and Hans Christian Ørsted. Each made distinct contributions, ranging from induction to quantitative force laws. The question asks for the specific individual who first observed that an electric current produces a magnetic field.

Step 2: Detailed Explanation:

1. **Hans Christian Ørsted (1820):** During a lecture, Ørsted noticed that a compass needle

deflected when placed near a wire carrying an electric current. This accidental observation was the first empirical evidence that current flow creates a surrounding magnetic field. This discovery effectively "born" the science of electromagnetism.

2. **André-Marie Ampère:** Following Ørsted's discovery, Ampère performed extensive experiments and formulated the mathematical laws (Ampere's Law) that quantify the magnetic force between two current-carrying wires.

3. **Michael Faraday:** Faraday is best known for the **inverse** discovery—Electromagnetic Induction (1831). He showed that a changing magnetic field can induce an electric current in a conductor.

4. **Niels Bohr:** Bohr was a quantum physicist known for the Bohr model of the atom; his work was on atomic structure rather than the classical magnetic effects of current. Since Ørsted was the first to identify the effect, he is credited with the discovery.

Step 3: Final Answer:

The magnetic effect of electric current was discovered by Hans Christian Oersted.

This matches Option (B).

Quick Tip: Sequence of discovery:

1. **Oersted:** Current → Magnetic field.

2. **Faraday:** Magnetic field change → Current.

Think: **O**ersted **O**bserved it first!

105. Which of the following has a zero temperature coefficient of resistance?

- (A) Copper
- (B) Aluminum
- (C) Manganese
- (D) Carbon

Correct Answer: (C) Manganese

Solution:

Step 1: Understanding the Concept:

The resistance of a material changes with temperature. This relationship is quantified by the temperature coefficient of resistance (α).

1. **Positive Temperature Coefficient (PTC):** Resistance increases as temperature increases. This is typical for pure metals.
2. **Negative Temperature Coefficient (NTC):** Resistance decreases as temperature increases. This is seen in semiconductors, insulators, and electrolytes.
3. **Zero Temperature Coefficient:** Resistance remains almost constant regardless of temperature changes. This property is highly desirable for manufacturing precision instruments, standard resistors, and shunt resistances where accuracy must be maintained under varying thermal conditions.

Step 2: Detailed Explanation:

Let's analyze the materials provided:

(A) **Copper** and (B) **Aluminum**: These are pure conductors. As temperature rises, lattice vibrations increase, impeding electron flow. They have a **high positive α** .

(D) **Carbon**: Carbon is a semi-metal/semiconductor. As it gets hotter, more charge carriers become available, so its resistance drops. It has an **NTC**.

(C) **Manganese (specifically in the form of Manganin)**: Manganin is an alloy typically composed of 84% Copper, 12% Manganese, and 4% Nickel. It is famous in electrical engineering for having an **extremely low to nearly zero** temperature coefficient of resistance over a wide range of temperatures. While the option says "Manganese," in the context of electrical engineering exams, it refers to the Manganese-based alloys used for standard resistors. Standard material textbooks cite Manganin and Constantan as the primary "Zero coefficient" materials.

Based on the provided answer key in the OCR, "Manganese" is marked as the correct answer among the list.

Step 3: Final Answer:

Manganese (Manganin) has a nearly zero temperature coefficient of resistance.

This matches Option (C).

Quick Tip: Materials for precision instruments:

- **Manganin** (Mn alloy) and **Constantan** (Cu-Ni alloy) are used because their resistance **doesn't change** with heat.
- Pure metals = PTC.
- Semiconductors/Carbon = NTC.

106. Which material is best suited for making a permanent magnet?

- (A) Soft iron
- (B) Copper
- (C) Hard steel
- (D) Aluminum

Correct Answer: (C) Hard steel

Solution:

Step 1: Understanding the Concept:

Magnetic materials are classified into two broad categories based on how easily they can be magnetized and demagnetized:

1. **Soft Magnetic Materials:** These can be magnetized easily but lose their magnetism quickly once the external field is removed. They have a narrow hysteresis loop with low retentivity and low coercivity. These are ideal for transformer cores and electromagnets.
2. **Hard Magnetic Materials:** These are difficult to magnetize, but once magnetized, they retain their magnetic properties for a long time. They have a wide hysteresis loop, high retentivity (residual magnetism), and high coercivity (resistance to demagnetization). These are ideal for making permanent magnets.

Step 2: Detailed Explanation:

Evaluating the options:

(A) **Soft Iron:** As the name suggests, it is a soft magnetic material. It has high permeability but very low retentivity and coercivity. It is used for temporary magnets (electromagnets).

(B) **Copper** and (D) **Aluminum:** These are non-magnetic (paramagnetic/diamagnetic) materials. They cannot be used to create permanent magnets as they do not support strong ferro-magnetic alignment.

(C) **Hard Steel:** Hardened steel or Alnico (Aluminum-Nickel-Cobalt alloy) are hard magnetic materials. Because of the impurities and lattice stresses in hard steel, the magnetic domains are "locked" into position once aligned. This results in high retentivity and high coercivity, making it perfectly suited for manufacturing permanent magnets that must withstand external magnetic influences without losing their strength.

Therefore, hard steel is the superior choice for a permanent magnet.

Step 3: Final Answer:

Hard steel is the best-suited material for making a permanent magnet.

This matches Option (C).

Quick Tip: Permanent Magnet requirements: **High Retentivity + High Coercivity.**

Temporary Magnet (Electromagnet) requirements: **High Permeability + Low Retentivity.**

Steel = Permanent; Soft Iron = Temporary.

107. Among the following which material has the highest dielectric strength?

- (A) Ceramic
- (B) Glass
- (C) Mica
- (D) Vacuum

Correct Answer: (D) Vacuum

Solution:

Step 1: Understanding the Concept:

Dielectric strength is the maximum electric field that a pure material can withstand under ideal conditions without undergoing electrical breakdown and becoming electrically conductive. It is measured in Volts per unit thickness (e.g., kV/mm). In a material medium, breakdown occurs when the electric field is strong enough to rip electrons away from the atoms, creating an avalanche of carriers. In a perfect vacuum, there are no molecules to ionize; however, "breakdown" can still occur via field emission from electrodes. Interestingly, because of the lack of a physical medium to carry a spark, vacuum is often considered to have superior insulating properties in certain high-voltage switchgear contexts.

Step 2: Detailed Explanation:

Let's compare the approximate dielectric strengths of common insulators:

- **Ceramic:** 5 to 15 kV/mm
- **Glass:** 20 to 40 kV/mm
- **Mica:** 100 to 200 kV/mm (Excellent solid insulator)
- **Vacuum:** In theory, an ideal vacuum has an "infinite" dielectric strength because there are no atoms to ionize. In practice, vacuum insulation is limited by the "vacuum gap" breakdown voltage between electrodes, which can exceed the breakdown strength of most solid materials for the same distance. In high-power electrical engineering (like vacuum circuit breakers), vacuum is used specifically because it can withstand extremely high voltage gradients without arcing compared to air or oil.

According to the green checkmark provided in the examination source image, "Vacuum" is the intended answer for having the highest dielectric strength among the choices.

Step 3: Final Answer:

Vacuum has the highest dielectric strength among the provided options.

This matches Option (D).

Quick Tip: While Mica is the best **solid** insulator listed, **Vacuum** is considered the ultimate insulator in specialized high-voltage engineering because there are no medium particles to participate in ionization. Vacuum circuit breakers use this property to quench arcs instantly.

108. The number of parallel paths for a 4-pole duplex lap winding will be

- (A) 2
- (B) 4
- (C) 6
- (D) 8

Correct Answer: (D) 8

Solution:

Step 1: Understanding the Concept:

In DC machine armature construction, the winding arrangement determines the current and voltage capacity of the machine. The two main types are Lap Winding and Wave Winding.

1. **Lap Winding:** Generally used for low-voltage, high-current machines. The coils are "lapped" over each other.
2. **Wave Winding:** Used for high-voltage, low-current machines.

The number of parallel paths (A) is a critical parameter. For lap winding, the number of parallel paths is directly related to the number of poles (P) and the multiplexity of the winding (m). Multiplexity refers to the number of independent sets of windings on the armature.

Step 2: Key Formula or Approach:

For a Lap Winding, the number of parallel paths A is:

$$A = m \times P$$

where:

- P = number of poles

- m = multiplexity ($m = 1$ for simplex, $m = 2$ for duplex, $m = 3$ for triplex, etc.)

Step 3: Detailed Explanation:

Given parameters:

- Number of poles (P) = 4
- Type of winding = Duplex Lap
- For a duplex winding, the multiplexity $m = 2$.

Using the formula:

$$A = 2 \times 4$$

$$A = 8$$

This means the armature current will be divided into 8 parallel paths through the machine. If this were a simplex lap winding, there would be $1 \times 4 = 4$ paths. If it were a wave winding (regardless of poles, for simplex), there would be only 2 paths. Because it is duplex lap, we have exactly 8 paths. This design increases the current-carrying capacity of the generator or motor.

Step 4: Final Answer:

The number of parallel paths is 8.

This matches Option (D).

Quick Tip: Remember these simple rules:

- **Lap** $A = m \times P$.
- **Wave** $A = 2 \times m$.

Just substitute $m = 2$ for duplex and $P = 4$ to get 8 for Lap.

109. Number of tapping to each equalizer ring is equal to number of

- (A) Poles
- (B) Pole pairs
- (C) Parallel paths
- (D) Commutator segments

Correct Answer: (B) Pole pairs

Solution:

Step 1: Understanding the Concept:

In large DC machines with lap windings, slight imbalances in air-gap length or magnetic properties of the poles can cause unequal induced EMFs in the various parallel paths. Since all paths are connected to the same brushes, these voltage differences lead to circulating currents within the armature, causing overheating and sparking at the brushes. To mitigate this, "Equalizer Rings" are used. These are low-resistance copper rings that connect points on the armature that should be at the same potential. They provide a bypass for these circulating currents so they don't have to flow through the brushes.

Step 2: Detailed Explanation:

In a lap-wound armature, points that are at the same electrical potential are separated by exactly two pole pitches (or 360 electrical degrees).

In a machine with P poles, there are $P/2$ "pole pairs".

Each equalizer ring must connect all the corresponding potential points across the entire circumference of the armature.

For any specific ring, it must tap into one point for every pair of poles to ensure that all identical potential positions are joined together.

Thus, the number of tappings for **each** individual equalizer ring is exactly equal to the number of pole pairs ($P/2$).

For example, in a 4-pole machine, there are 2 pole pairs. Each ring will have 2 tappings. In a 6-pole machine, each ring would have 3 tappings.

Step 3: Final Answer:

The number of tapping to each equalizer ring is equal to the number of pole pairs.

This matches Option (B).

Quick Tip: Equalizer rings are **only used in Lap windings**. They are not needed in wave windings because wave windings naturally have only two parallel paths that pass through all poles, automatically balancing the induced EMF.

110. Flashing of field of DC generator means

- (A) Neutralization of residual magnetism
- (B) Creation of residual magnetism by a DC source
- (C) Increasing flux density
- (D) Decreasing flux density

Correct Answer: (B) Creation of residual magnetism by a DC source

Solution:

Step 1: Understanding the Concept:

Most DC generators are "self-excited," meaning they use a part of their own generated output to power the field windings that create the magnetic flux. For a self-excited generator to start producing voltage when it begins to spin, there must be some "residual magnetism" already present in the iron core of the poles. This small leftover magnetic field induces a tiny initial EMF, which then drives a small current through the field winding, strengthening the field and leading to a "build-up" of voltage. If a generator has been idle for a long time, or has suffered a physical shock or improper connection, it might lose this residual magnetism. When this happens, the generator will fail to build up voltage.

Step 2: Detailed Explanation:

"Flashing the field" is a maintenance procedure used to restore the lost residual magnetism. In this process, an **external DC source** (like a battery or another DC supply) is momentarily connected to the field windings while the generator is stationary or running at a low speed.

The external current forces the magnetic domains in the pole iron to align, "creating" or "flashing" a new residual magnetic field into the iron.

Once the residual magnetism is established, the external source is disconnected, and the generator can then build up voltage on its own using its normal self-excitation process.

Option (A) is incorrect as it describes the loss of magnetism, not the procedure. Options (C) and (D) are general variations in flux during operation, not a specific corrective procedure for starting.

Step 3: Final Answer:

Flashing of the field means the creation of residual magnetism by an external DC source. This matches Option (B).

Quick Tip: Think of "Flashing" like using **jumper cables** to jump-start a car. You are using an external battery (DC source) to give the generator the initial "spark" (residual magnetism) it needs to start running on its own.

111. External characteristics of a DC generator is a curve plotted between

- (A) Terminal voltage Vs Armature current
- (B) Terminal voltage Vs Load current
- (C) Induced emf Vs Armature current
- (D) No load voltage Vs Field current

Correct Answer: (B) Terminal voltage Vs Load current

Solution:

Step 1: Understanding the Concept:

The performance of a Direct Current (DC) generator is graphically represented through various characteristic curves that describe the relationship between its electrical parameters. These curves are essential for engineers to predict how the machine will behave under

different operational conditions, such as varying loads or changes in excitation.

There are primarily three types of characteristics for DC generators: the Magnetic or Open Circuit Characteristic (O.C.C.), the Internal or Total Characteristic, and the External Characteristic.

The O.C.C. shows the relation between the generated EMF at no-load and the field current, essentially depicting the magnetization of the iron core.

The Internal Characteristic plots the relationship between the actual induced EMF under load conditions and the armature current, accounting for the effect of armature reaction.

The External Characteristic, however, is the most practical curve for the end-user, as it describes the relationship between the output variables available at the machine's terminals. Specifically, it shows how the terminal voltage varies as the generator delivers current to an external load.

Step 2: Key Formula or Approach:

The fundamental voltage equation for a DC generator is:

$$V = E - I_a R_a - \text{Drop due to armature reaction}$$

Where:

- V is the terminal voltage.
- E is the induced EMF (No-load voltage minus armature reaction effects).
- I_a is the armature current.
- R_a is the armature circuit resistance.

In most shunt and compound generators, the armature current (I_a) is almost equal to the load current (I_L) except for the small shunt field current.

Step 3: Detailed Explanation:

In an ideal scenario, a generator would maintain a constant terminal voltage regardless of the amount of current drawn by the load.

However, in practical DC generators, as the load current (I_L) increases, the terminal voltage (V) tends to drop from its no-load value.

This drop is caused by two main physical factors: the ohmic voltage drop across the armature resistance ($I_a R_a$) and the demagnetizing effect of the armature reaction which reduces the

main field flux.

The "External Characteristic" curve specifically targets this relationship by plotting Terminal Voltage (V) on the vertical axis (Y-axis) and Load Current (I_L) on the horizontal axis (X-axis). For a shunt generator, the voltage drops slightly as current increases, whereas for a series generator, the voltage initially rises with the load current before falling due to saturation and resistance drops.

Engineers use this curve to determine the "Voltage Regulation" of the generator, which is a measure of its ability to maintain a steady voltage under changing load conditions.

Because this curve focuses on the interaction between the generator terminals and the "external" circuit, it is appropriately named the external characteristic.

Understanding this curve is vital for selecting the correct generator for applications like battery charging or industrial power supply where voltage stability is critical.

Step 4: Final Answer:

The external characteristic of a DC generator is the curve plotted between Terminal voltage and Load current.

This corresponds to Option (B).

Quick Tip: To remember the hierarchy of curves: **No-load** (OCC) is about the magnetic field (E_0 vs I_f); **Internal** is about the armature (E vs I_a); and **External** is about the user (V vs I_L). Always think "External = Terminals".

112. In a DC shunt motor, if the armature current is doubled, the torque developed will be

- (A) Halved
- (B) Remain same
- (C) Doubled
- (D) Quadrupled

Correct Answer: (C) Doubled

Solution:

Step 1: Understanding the Concept:

The production of mechanical torque in a DC motor is a result of the electromagnetic interaction between the magnetic flux produced by the field poles and the current flowing through the armature conductors.

When current-carrying conductors are situated within a magnetic field, they experience a Lorentz force, which generates a rotational moment on the armature shaft.

The magnitude of this developed torque depends on the strength of the magnetic field and the intensity of the armature current.

In a DC shunt motor, the field winding is connected in parallel with the armature circuit across a constant supply voltage.

Because the field resistance is constant and the supply voltage is usually fixed, the field current remains constant across the entire operating range of the motor.

This leads to a defining characteristic of the shunt motor: the magnetic flux per pole (ϕ) is essentially constant, independent of the load conditions.

Step 2: Key Formula or Approach:

The general expression for the electromagnetic torque (T) developed in any DC motor is given by:

$$T = \frac{P \cdot \phi \cdot Z \cdot I_a}{2\pi \cdot A}$$

Which can be simplified to a proportionality:

$$T \propto \phi \cdot I_a$$

In a DC shunt motor, since $\phi \approx \text{constant}$, the relationship becomes:

$$T \propto I_a$$

Step 3: Detailed Explanation:

Let us analyze the initial state where the torque is T_1 and the armature current is I_{a1} .

According to the derived proportionality for shunt motors: $T_1 = k \cdot I_{a1}$, where k is a constant accounting for machine construction and flux.

Now, the problem states that the armature current is doubled. Let the new armature current be $I_{a2} = 2 \cdot I_{a1}$.

Since the magnetic flux (ϕ) is constant in a shunt motor (ignoring the minor effects of armature reaction), the new torque developed (T_2) will be:

$$T_2 = k \cdot I_{a2}$$

Substituting the value of I_{a2} :

$$T_2 = k \cdot (2 \cdot I_{a1})$$

$$T_2 = 2 \cdot (k \cdot I_{a1})$$

$$T_2 = 2 \cdot T_1$$

The result clearly shows that the torque is directly proportional to the armature current in this type of motor.

If the current is doubled, the torque also doubles. This linear relationship is what gives the DC shunt motor its predictable performance characteristics, making it suitable for applications like lathes or centrifugal pumps where constant speed and steady torque response are required.

Contrast this with a DC series motor, where flux is proportional to current ($\phi \propto I_a$), leading to $T \propto I_a^2$, which would result in quadrupled torque.

However, for the shunt motor specified here, the answer is strictly a doubling effect.

Step 4: Final Answer:

The developed torque will be doubled when the armature current is doubled in a DC shunt motor.

This corresponds to Option (C).

Quick Tip: Always distinguish between Shunt and Series motors for torque problems!

For **Shunt**: $T \propto I_a$ (Linear relationship).

For **Series**: $T \propto I_a^2$ (Square relationship).

Since the question specifies a shunt motor, simply follow the linear rule.

113. At the moment a DC motor starts, the back emf (E_b) is

- (A) Maximum
- (B) Equal to the supply voltage
- (C) Zero
- (D) Half the supply voltage

Correct Answer: (C) Zero

Solution:

Step 1: Understanding the Concept:

When a Direct Current (DC) motor rotates, its armature conductors move through the magnetic field generated by the stator poles.

According to Faraday's Law of Electromagnetic Induction, this relative motion induces an Electromotive Force (EMF) within the armature windings.

Following Lenz's Law, this induced voltage acts in a direction that opposes the applied supply voltage that is driving the motor.

Because it opposes the main voltage, this induced EMF is known as the "Back EMF" or "Counter EMF" (E_b).

The existence of back EMF is crucial for the motor's operation because it acts as a self-regulating "governor" that limits the current flowing through the armature.

The magnitude of this back EMF is fundamentally dependent on the rotational speed of the armature.

Step 2: Key Formula or Approach:

The standard equation for the induced EMF in a DC machine is:

$$E_b = \frac{P \cdot \phi \cdot Z \cdot N}{60 \cdot A}$$

Where:

- P is the number of poles.
- ϕ is the flux per pole.
- Z is the total number of armature conductors.
- N is the speed of rotation in revolutions per minute (RPM).
- A is the number of parallel paths.

From this, we establish the critical proportionality: $E_b \propto N$.

Step 3: Detailed Explanation:

Let's consider the physical state of the motor at the specific "moment of starting."

At the absolute instant the motor is connected to the supply, the armature is still stationary.

The speed (N) is exactly 0 RPM.

Substituting $N = 0$ into the induction formula derived in Step 2:

$$E_b = \frac{P \cdot \phi \cdot Z \cdot (0)}{60 \cdot A} = 0$$

Since there is no motion, there is no rate of change of magnetic flux linkage, and therefore, no back EMF is generated.

This lack of back EMF is the reason why DC motors draw an extremely high starting current.

According to the armature circuit equation: $I_a = (V - E_b)/R_a$.

At start, since $E_b = 0$, the current is $I_a = V/R_a$. Because the armature resistance (R_a) is designed to be very low to minimize losses during running, this initial current can be 10 to 20 times the rated value.

As the motor starts to turn and accelerates, the value of N increases, which in turn causes E_b

to rise from zero.

The growing back EMF gradually reduces the voltage difference across the armature, bringing the current down to a safe operating level.

Thus, the zero value of back EMF is a transient but extremely significant condition that necessitates the use of starters in large DC motors.

Step 4: Final Answer:

At the moment of starting, the back EMF of a DC motor is zero.

This corresponds to Option (C).

Quick Tip: Remember: **Speed and Back EMF are best friends.**

If speed is zero, back EMF is zero. This occurs at the moment of starting or if the motor is stalled (locked rotor condition). High current at start is directly caused by the absence of this opposing voltage.

114. Friction and windage losses in a DC motor are classified as

- (A) Electrical losses
- (B) Mechanical losses
- (C) Magnetic losses
- (D) Copper losses

Correct Answer: (B) Mechanical losses

Solution:

Step 1: Understanding the Concept:

Efficiency in an electrical machine is defined by the ratio of output power to input power, and it is largely determined by the various internal power losses that occur during the energy conversion process.

In a DC motor, electrical energy is converted into mechanical energy, and several types of energy dissipations occur due to different physical phenomena.

These losses are categorized based on their origin: electrical resistance, magnetic field cycles, or physical movement.

Accurate classification of these losses is vital for performing tests like Swinburne's test and for designing cooling systems for the machine.

Losses that arise specifically from the physical rotation of the motor's internal parts are grouped together under a common umbrella.

Step 2: Detailed Explanation:

Let's analyze the standard categories of losses in a DC machine to see where friction and windage fit:

1. **Copper Losses (Electrical Losses):** These are I^2R losses that occur due to the resistance of the armature and field windings. They result in heating of the conductors and are strictly electrical in nature.

2. **Magnetic Losses (Iron/Core Losses):** These occur in the iron core of the armature as it rotates through the magnetic field. They consist of Hysteresis and Eddy Current losses. These depend on the flux density and frequency of reversal.

3. **Mechanical Losses:** These are losses that consume a portion of the mechanical power developed by the armature before it reaches the output shaft. They include:

- **Friction Loss:** This occurs at the bearings supporting the shaft and between the stationary brushes and the rotating commutator surface. It is the energy lost overcoming the physical resistance between rubbing surfaces.

- **Windage Loss:** This occurs as the rotating armature and its cooling fans push through the air contained within the machine casing. It is essentially air-friction. These losses depend significantly on the speed of rotation.

Because friction and windage are entirely related to the mechanical movement and physical environment of the rotor, they are classified as mechanical losses.

In many analytical contexts, mechanical losses combined with iron losses are referred to as "Stray Losses" or "Constant Losses" (assuming constant speed operation).

Step 3: Expanded Detail:

Mechanical losses are generally independent of the electrical load on the motor if the speed is kept constant.

Friction at the bearings depends on the lubrication quality and the load on the shaft, while

brush friction depends on the spring pressure and surface condition of the commutator.

Windage loss follows a cubic relationship with speed, meaning it becomes extremely dominant in high-speed machines.

For a motor running at a steady state, these mechanical losses must be overcome by the electromagnetic torque developed by the armature, which is why the actual "Brake Torque" at the shaft is always less than the "Armature Torque."

Thus, they represent a parasitic mechanical load that reduces the overall efficiency of the motor.

Step 4: Final Answer:

Friction and windage losses in a DC motor are classified as mechanical losses.

This corresponds to Option (B).

Quick Tip: Think of the "Nature of the Impediment":

- Resistance to electricity → **Electrical/Copper Loss.**
- Resistance to magnetic change → **Magnetic/Iron Loss.**
- Resistance to physical motion → **Mechanical Loss.**

Friction and air resistance are clearly physical motion impediments.

115. Swinburne's test is performed at

- (A) Full load
- (B) Half load
- (C) No load
- (D) Variable load

Correct Answer: (C) No load

Solution:

Step 1: Understanding the Concept:

Determining the efficiency of large electrical machines can be challenging because providing

a full physical load is often expensive, impractical, and wasteful of energy.

To solve this, indirect testing methods were developed, with Swinburne's Test being the most prominent for DC shunt and compound machines.

The fundamental principle of this test is to determine the "constant losses" of the machine without actually loading it.

Constant losses in a DC machine include Iron losses, Friction losses, and Windage losses.

By running the machine as a motor under specific conditions, we can measure the power required just to keep it spinning. This measured power, after subtracting the small no-load copper loss, gives us the total constant loss value.

Step 2: Key Formula or Approach:

The machine is run as a shunt motor at rated voltage and rated speed under **no-load** conditions.

Let V be the supply voltage and I_0 be the no-load armature current.

Input power at no-load $= V \cdot I_0$.

Constant losses (W_c) $= V \cdot I_0 - I_0^2 R_a$.

Once W_c is found, the efficiency at any load current I can be predicted:

$$\text{Efficiency } (\eta) = \frac{\text{Output}}{\text{Output} + \text{Losses}} = \frac{V \cdot I - (I^2 R_a + W_c)}{V \cdot I}.$$

Step 3: Detailed Explanation:

Why is it performed at no-load?

1. **Economy:** Only the power necessary to overcome internal friction and magnetization is consumed. For a 1000 HP motor, you only use a few kilowatts to perform the test.
2. **Convenience:** No mechanical brake or dynamometer is required to be coupled to the shaft.
3. **Safety:** There is no risk of overheating the machine during prolonged testing as the currents are very low.

However, it is important to note the limitations: Swinburne's test cannot be performed on DC series motors because they would overspeed and potentially disintegrate at no-load. Also, it does not account for "stray load losses" which only occur when heavy currents flow through the armature. Despite these drawbacks, it remains the standard industrial method for pre-determining the efficiency curves of shunt-excited machines. Since the test involves measuring the basic "running" overhead of the motor, the only appropriate condition is the absence of any external mechanical load.

Step 4: Final Answer:

Swinburne's test is performed at the no-load condition.

This corresponds to Option (C).

Quick Tip: Swinburne's Test Summary:

- **Condition**: No-load.
- **Type**: Indirect test.
- **Applicability**: Shunt and Compound motors/generators only (NEVER series).
- **Goal**: To find constant losses and predict efficiency at any load.

116. Permanent Magnet Moving Coil (PMMC) instruments can be used only for

- (A) AC measurements
- (B) DC measurements
- (C) Both AC and DC
- (D) High frequency only

Correct Answer: (B) DC measurements

Solution:**Step 1: Understanding the Concept:**

Measuring instruments are defined by their operating mechanisms. The Permanent Magnet Moving Coil (PMMC) instrument, also known as the D'Arsonval movement, is a high-precision device used for measuring current and voltage.

Its working principle is based on the Motor Action: when a current-carrying conductor is placed in a magnetic field, it experiences a force.

In a PMMC meter, a lightweight coil of wire is suspended in the uniform magnetic field produced by a powerful permanent magnet.

When current flows through the coil, an electromagnetic torque is produced that causes the

coil to rotate. This movement is balanced by the restoring torque of hairsprings, which also serve as leads for the current.

Step 2: Key Formula or Approach:

The deflecting torque (T_d) in a PMMC instrument is given by:

$$T_d = N \cdot B \cdot I \cdot A$$

Where:

- N is the number of turns in the coil.
- B is the magnetic flux density (fixed by the permanent magnet).
- I is the current being measured.
- A is the area of the coil.

Since N , B , and A are constant for a specific instrument, $T_d \propto I$.

Step 3: Detailed Explanation:

Why is it restricted to DC?

1. **Torque Direction:** In DC, the current flows in one steady direction. This results in a constant deflecting torque in one direction, leading to a steady, readable deflection on the scale.
2. **AC Behavior:** If Alternating Current (AC) is applied, the direction of the current reverses every half-cycle (e.g., 50 times per second for 50 Hz). Consequently, the direction of the deflecting torque also reverses every half-cycle.
3. **Mechanical Inertia:** The moving assembly (coil, spindle, and pointer) has physical mass and therefore mechanical inertia. It cannot possibly oscillate back and forth 50 times in a second.
4. **Result:** Because the pointer cannot follow the rapid changes, it simply stays at its average position. The average value of a pure sinusoidal AC wave over a full cycle is exactly zero. Therefore, a PMMC meter connected directly to AC will show a zero reading.

While PMMC meters can be used for AC if paired with a rectifier (moving coil with rectifier), the base instrument itself is strictly a DC-only device. It is renowned for its high sensitivity, low power consumption, and perfectly linear scale, which are all benefits of the fixed permanent magnetic field.

Step 4: Final Answer:

PMMC instruments can be used only for DC measurements.

This corresponds to Option (B).

Quick Tip: Comparison Shortcut:

- **PMMC** = DC only (Average value meter).
 - **Moving Iron (MI)** = Both AC and DC (RMS value meter).
 - **Electrodynamometer** = Both AC and DC (True Power meter).
- Remember: Permanent Magnet = Permanent direction = Direct Current.

117. The operation of a thermocouple is primarily governed by which effect?

- (A) Peltier effect
- (B) Joule effect
- (C) Seebeck effect
- (D) Thomson effect

Correct Answer: (C) Seebeck effect

Solution:**Step 1: Understanding the Concept:**

Thermocouples are among the most widely used temperature sensors in industrial, scientific, and consumer applications because they are simple, rugged, and can measure a vast range of temperatures.

A thermocouple consists of two wires made from different metals or alloys that are joined together at two points to form a closed loop.

The operation of this device is based on the principles of thermoelectricity—the direct conversion of temperature differences into electric voltage and vice versa.

Specifically, when the two junctions of the dissimilar metals are maintained at different

temperatures, an Electromotive Force (EMF) is generated in the circuit, which can be measured to determine the temperature of one of the junctions.

Step 2: Detailed Explanation:

Let's analyze the various thermoelectric effects to identify the correct one:

1. **Seebeck Effect (1821):** Discovered by Thomas Johann Seebeck, this effect states that if two dissimilar metals are joined at two junctions, and a temperature difference exists between these junctions, a voltage (EMF) is produced. This induced voltage is proportional to the temperature difference ($V \propto \Delta T$). This is the fundamental physical principle used by thermocouples to sense temperature. It is a "generator" effect.
2. **Peltier Effect (1834):** This is the inverse of the Seebeck effect. It states that when an electric current is passed through a junction of two different metals, heat is either absorbed or evolved at the junction. This is used for electronic cooling (thermoelectric coolers), not for sensing.
3. **Thomson Effect:** This describes the evolution or absorption of heat in a single current-carrying conductor when there is a temperature gradient along its length. It is a secondary thermoelectric effect.
4. **Joule Effect:** This is simple ohmic heating ($Q = I^2 R t$) that occurs in any conductor with resistance. It is not specific to dissimilar metals or temperature sensing.

Because the Seebeck effect allows the direct transformation of thermal energy into a measurable electrical signal (voltage), it is the primary governing principle of a thermocouple.

Step 3: Expanded Context:

In a practical thermocouple thermometer, one junction is kept at a known reference temperature (often 0°C), while the other is the "sensing" junction placed in the environment to be measured.

The small millivolt signal produced is digitized and converted into a temperature reading using standard calibration tables for specific metal pairs (like Type K: Chromel-Alumel).

Since it generates its own voltage without an external battery, a thermocouple is classified as an "Active Transducer."

Understanding the Seebeck effect is essential for troubleshooting issues like "cold junction compensation" in instrumentation systems.

Step 4: Final Answer:

The operation of a thermocouple is governed by the Seebeck effect.

This corresponds to Option (C).

Quick Tip: Thermoelectric Mnemonic:

- Seebeck = Sensor (Temp → Volt).

- Peltier = Pump (Volt → Heat movement).

Thermocouples generate voltage; hence they follow the Seebeck effect.

118. Creeping in an energy meter is a phenomenon where the disc rotates slowly when,

- (A) The load is very high
- (B) The power factor is unity
- (C) Only the voltage coil is energized and there is no load current
- (D) The braking magnet is removed

Correct Answer: (C) Only the voltage coil is energized and there is no load current

Solution:**Step 1: Understanding the Concept:**

An induction-type energy meter (the common household watt-hour meter) operates on the principle of electromagnetic induction.

It consists of an aluminum disc that rotates in the presence of magnetic fluxes produced by two coils: the shunt coil (voltage coil) and the series coil (current coil).

Under ideal conditions, the disc should rotate only when power is being consumed by the consumer's appliances.

This means torque should only be present when both the voltage coil is energized (indicating line voltage) and the current coil is carrying current (indicating an active load).

"Creeping" is a well-known defect or error in these meters where the disc continues to rotate very slowly even when the load current is exactly zero.

Step 2: Detailed Explanation:

Why does creeping occur?

1. **Over-compensation for friction:** To make the meter accurate at very light loads, manufacturers add a small "starting torque" using a shading loop or plate to overcome static friction in the bearings. If this compensation is too aggressive, it can drive the disc on its own.
2. **Voltage Fluctuation:** If the supply voltage rises significantly above the rated value, the flux from the voltage coil increases, producing enough "stray" torque to overcome friction even without current coil assistance.
3. **Vibrations:** External mechanical vibrations can nudge the disc into moving, allowing the over-compensated starting torque to maintain the rotation.
4. **Stray Magnetic Fields:** Interaction with external fields can induce tiny eddy currents in the disc.

The defining condition for this error is that the meter registers "energy consumption" while the consumer has all appliances turned off. This means the **voltage coil is energized** (as it is always connected to the mains), but the **current coil has zero current**.

This results in unfair billing for the consumer. To prevent this, meter discs are usually drilled with two small "anti-creep" holes. When a hole comes under a shunt magnet pole, it increases the magnetic reluctance and "locks" the disc in place, preventing further no-load rotation.

Step 3: Expanded Detail:

In modern digital or electronic energy meters, this problem is solved using a "no-load threshold" in the software, where the meter ignores extremely small currents.

However, in the context of the classical induction meters covered in engineering exams, creeping remains a critical topic regarding error analysis and maintenance.

Options (A) and (B) describe normal operating conditions where the meter is supposed to work correctly.

Option (D) would cause the disc to spin uncontrollably fast under any load, which is a different type of failure known as loss of braking torque.

Thus, "creeping" specifically refers to the slow movement during the no-load state.

Step 4: Final Answer:

Creeping occurs when the voltage coil is energized but there is no load current.

This corresponds to Option (C).

Quick Tip: To remember "Creeping": Imagine the meter **creeping** up your bill while you are asleep and everything is off. It is a **No-load Error**. The fix is **Anti-creep holes**. Always look for the "No load" keyword.

119. Which of the following is an "active" transducer commonly used in electronic instrumentation?

- (A) Resistance Temperature Detector (RTD)
- (B) LVDT
- (C) Thermocouple
- (D) Strain gauge

Correct Answer: (C) Thermocouple

Solution:

Step 1: Understanding the Concept:

Transducers are devices that convert one physical quantity (like pressure, temperature, or displacement) into another, typically an electrical signal.

In instrumentation and control systems, transducers are broadly classified into two categories based on their power requirements: Passive and Active.

1. **Passive Transducers:** These do not generate their own electrical signal. They require an external power source (excitation) to operate. They work by changing a physical property, such as resistance, inductance, or capacitance, in response to the measured quantity. For example, a change in resistance is measured by applying a voltage across it.
2. **Active Transducers:** Also known as self-generating transducers, these convert energy from the measured environment directly into an electrical signal (voltage or current) without the need for any external power supply. They effectively act as tiny generators.

Step 2: Detailed Explanation:

Let's evaluate each option based on this classification:

(A) **RTD (Resistance Temperature Detector)**: This device changes its electrical resistance based on temperature. To measure this change, you must pass a current through it from an external circuit. Therefore, it is a **Passive** transducer.

(B) **LVDT (Linear Variable Differential Transformer)**: This measures displacement. It consists of a primary coil and two secondary coils. It requires an AC excitation voltage on the primary coil to work. Thus, it is a **Passive** transducer.

(D) **Strain Gauge**: This measures mechanical strain by changing its resistance. Like the RTD, it needs an external excitation (usually in a Wheatstone bridge) to produce a signal. It is a **Passive** transducer.

(C) **Thermocouple**: As discussed in previous questions, a thermocouple generates a small voltage (Seebeck voltage) directly proportional to the temperature difference between its junctions. It creates its own electrical energy from the heat it senses. Since it produces a self-generated output without an external battery or supply, it is the perfect example of an **Active** transducer.

Other common active transducers include Piezoelectric crystals (force to voltage) and Solar cells (light to voltage).

Step 3: Final Answer:

The thermocouple is the active transducer among the given options.

This corresponds to Option (C).

Quick Tip: Self-Test for Transducers:

Ask: "Does it need a battery/external voltage to work?"

- Yes → **Passive** (RTD, LVDT, Strain Gauge, Potentiometer).
- No → **Active** (Thermocouple, Piezoelectric, Photovoltaic).

Active transducers are **self-generating**.

120. The Q-factor of a series resonant circuit is a measure of its

(A) Power consumption

- (B) Selectivity
- (C) Frequency range
- (D) Voltage drop

Correct Answer: (B) Selectivity

Solution:

Step 1: Understanding the Concept:

In AC circuit analysis, resonance occurs in a series RLC circuit when the inductive reactance (X_L) and capacitive reactance (X_C) are equal in magnitude but opposite in phase, effectively canceling each other out.

The frequency at which this happens is the resonant frequency (f_r).

The "Quality Factor" or "Q-factor" is a dimensionless parameter that describes the characteristics of a resonant circuit.

Mathematically, it represents the ratio of the maximum energy stored in the circuit to the energy dissipated per cycle.

Physically, the Q-factor determines how "sharp" or "narrow" the resonance peak is in the frequency response curve (current vs. frequency).

Step 2: Key Formula or Approach:

The Q-factor of a series resonant circuit is given by:

$$Q = \frac{\omega_r L}{R} = \frac{1}{\omega_r C R} = \frac{1}{R} \sqrt{\frac{L}{C}}$$

Alternatively, it is defined in terms of bandwidth (BW):

$$Q = \frac{f_r}{f_2 - f_1} = \frac{f_r}{BW}$$

Where f_2 and f_1 are the half-power frequencies (3 dB points).

Step 3: Detailed Explanation:

What does Q-factor measure in practice?

1. **Selectivity:** This is the ability of a circuit to distinguish between the desired signal at the

resonant frequency and unwanted signals at neighboring frequencies. A high Q-factor means the current falls off very rapidly on either side of the resonant frequency. This results in a very "sharp" peak, indicating high selectivity. This is crucial in radio receivers to "select" one station while rejecting others.

2. **Bandwidth Relationship:** As seen from the formula $Q = f_r/BW$, the Q-factor is inversely proportional to the bandwidth. A high Q-factor results in a narrow bandwidth, while a low Q-factor results in a wide, "flat" resonance curve (poor selectivity).

3. **Voltage Magnification:** In a series resonant circuit, the voltage across the inductor or capacitor is Q times the supply voltage. This is why a high Q-factor can lead to dangerously high voltages inside a circuit.

The question asks what it is a measure of. While it relates to energy and bandwidth, the primary *functional* measure it provides for a tuned circuit is its **Selectivity**. If a circuit is highly selective, it has a high Quality factor. If it is sluggish and picks up many frequencies, it has a low Q.

Step 4: Final Answer:

The Q-factor of a series resonant circuit is a measure of its selectivity.

This corresponds to Option (B).

Quick Tip: Remember the Graph Shape:

- High Q → Sharp Peak → High Selectivity (Narrow Bandwidth).
- Low Q → Flat Peak → Low Selectivity (Wide Bandwidth).

Think of Q as the "Selectivity Score" of your radio tuner.

121. If two branches in a parallel circuit have currents of 3A (resistive) and 4A (purely inductive), the total line current is

- (A) 7A
- (B) 1A
- (C) 5A
- (D) 12A

Correct Answer: (C) 5A

Solution:

Step 1: Understanding the Concept:

In Alternating Current (AC) circuits, adding currents is not as simple as performing basic arithmetic. This is because currents in different types of components (resistors, inductors, and capacitors) have different phase relationships relative to the supply voltage.

In a parallel circuit, the voltage across all branches is identical. For a purely resistive branch, the current is perfectly in phase with the voltage. For a purely inductive branch, the current lags the voltage by exactly 90° . Because these two currents are displaced by 90 electrical degrees, they must be treated as vectors (phasors). The total line current is the phasor sum of the branch currents. If we simply added 3A and 4A to get 7A, we would be ignoring the physical reality that the peak of the inductive current occurs a quarter-cycle after the peak of the resistive current.

Step 2: Key Formula or Approach:

The total current (I_T) in a parallel RL circuit is given by the Pythagorean relationship:

$$I_T = \sqrt{I_R^2 + I_L^2}$$

where I_R is the current in the resistive branch and I_L is the current in the inductive branch.

Step 3: Detailed Explanation:

Given:

- Resistive current (I_R) = 3 A
- Inductive current (I_L) = 4 A

Since the circuit is parallel, we use the voltage as our reference phasor. The resistive current lies along the same axis as the voltage (horizontal), while the inductive current points straight down (vertical) at an angle of -90° .

The resulting phasor diagram forms a right-angled triangle where the total current is the hypotenuse.

Substitute the given values into the formula:

$$I_T = \sqrt{3^2 + 4^2}$$

$$I_T = \sqrt{9 + 16}$$

$$I_T = \sqrt{25}$$

$$I_T = 5 \text{ A}$$

This value of 5A is the RMS value of the total current flowing from the source. It is important to note that the power factor of this circuit would be $I_R/I_T = 3/5 = 0.6$ lagging. The magnitude 5A represents the combined effect of the power-producing current (resistive) and the magnetic field-producing current (inductive).

Step 4: Final Answer:

The total line current is 5A.

This corresponds to Option (C).

Quick Tip: Whenever you see the values **3 and 4** in AC circuits for resistive and reactive components, the result is almost always **5** due to the standard 3-4-5 right triangle relationship. This saves you from performing the square root calculation during a timed exam.

122. In the two-wattmeter method, if one wattmeter reads zero, the power factor of the load is

- (A) Unity
- (B) 0.5

- (C) Zero
(D) 0.866

Correct Answer: (B) 0.5

Solution:

Step 1: Understanding the Concept:

The two-wattmeter method is the most common technique for measuring total power in a three-phase, three-wire system, whether the load is balanced or unbalanced. In a balanced system, the individual readings of the two wattmeters (W_1 and W_2) depend on the power factor angle (ϕ) of the load. As the phase angle increases (meaning the power factor decreases), the readings of the two meters start to diverge. At specific power factors, the meters exhibit distinct behaviors. When the load becomes highly reactive, one meter will eventually reach zero and then even start reading in the reverse direction (negative power).

Step 2: Key Formula or Approach:

The total power is $P = W_1 + W_2$.

The relationship between the phase angle and wattmeter readings is:

$$\tan \phi = \sqrt{3} \frac{W_1 - W_2}{W_1 + W_2}$$

The power factor (pf) is then calculated as $\cos \phi$.

Step 3: Detailed Explanation:

Let's consider the scenario where one wattmeter reads zero. Let $W_2 = 0$.

Substitute this into the tangent formula:

$$\tan \phi = \sqrt{3} \frac{W_1 - 0}{W_1 + 0}$$

$$\tan \phi = \sqrt{3} \cdot \frac{W_1}{W_1}$$

$$\tan \phi = \sqrt{3}$$

Now, we find the angle ϕ whose tangent is $\sqrt{3}$:

$$\phi = \tan^{-1}(\sqrt{3}) = 60^\circ$$

The power factor is defined as the cosine of the phase angle:

$$pf = \cos \phi = \cos(60^\circ)$$

From standard trigonometric values:

$$\cos(60^\circ) = 0.5$$

This means that when the load's power factor drops to 0.5, all the power in the system is being measured by just one of the two wattmeters. If the power factor were higher than 0.5, both would read positive. If it were lower than 0.5, one would read negative. If the power factor were unity (0°), then $W_1 = W_2$.

Step 4: Final Answer:

The power factor of the load is 0.5.

This corresponds to Option (B).

Quick Tip: Standard Wattmeter Reading Rules for Balanced Loads:

- $W_1 = W_2 \implies pf = 1$.
- W_1 or $W_2 = 0 \implies pf = 0.5$.
- $W_1 = -W_2 \implies pf = 0$ (Purely reactive).

Memorizing these three specific cases will help you answer many 3-phase power questions instantly.

123. For a pure sine wave, the ratio of RMS value to Average value is

- (A) Peak factor
- (B) Form factor
- (C) Quality factor
- (D) Power factor

Correct Answer: (B) Form factor

Solution:

Step 1: Understanding the Concept:

Periodic waveforms in electrical engineering are described by several characteristic parameters that quantify their shape and effective energy content. For Alternating Current (AC), we distinguish between the instantaneous value, the average value (over half a cycle), and the Root Mean Square (RMS) value.

The RMS value is the equivalent DC value that would produce the same heating effect in a resistor. The average value is the mathematical average of the waveform over one-half cycle (since the average over a full cycle of a symmetrical wave is zero). The relationship between these values defines the "sharpness" or "shape" of the wave. The ratio specifically mentioned in the question is a fundamental definition in power electronics and electrical machines.

Step 2: Key Formula or Approach:

The definition of Form Factor (k_f) is:

$$k_f = \frac{\text{RMS Value}}{\text{Average Value}}$$

For a sine wave with peak value V_m :

$$\begin{aligned} - V_{rms} &= \frac{V_m}{\sqrt{2}} \approx 0.707V_m \\ - V_{avg} &= \frac{2V_m}{\pi} \approx 0.637V_m \end{aligned}$$

Step 3: Detailed Explanation:

Let's calculate the numerical value for a pure sine wave:

$$\text{Ratio} = \frac{V_m / \sqrt{2}}{2V_m / \pi} = \frac{\pi}{2\sqrt{2}}$$

$$\text{Ratio} \approx \frac{3.14159}{2.828} \approx 1.11$$

This ratio is known as the **Form Factor**. It provides a measure of how much a waveform deviates from a flat line (where the form factor would be 1.0). For a square wave, the form factor is 1.0 because the RMS and average values are equal. For more "peaky" waves like triangle waves, the form factor is higher (approximately 1.15).

The other factor often confused with this is the **Peak Factor** (or Crest Factor), which is the ratio of the peak value to the RMS value (which is $\sqrt{2} \approx 1.414$ for a sine wave).

Step 4: Final Answer:

The ratio of RMS value to Average value is the Form Factor.

This corresponds to Option (B).

Quick Tip: To remember the difference:

- Form factor = RMS / Avg. (Mnemonic: FRA - like a fraction).
- Peak factor = Peak / RMS. (Mnemonic: PPR - Peak is on top).

For a sine wave, Form Factor is **1.11** and Peak Factor is **1.41**.

124. In a balanced 3-phase star connected system, if the phase current is 15A, the line current is

- (A) 8.66 A
- (B) 25.98 A
- (C) 15 A
- (D) 45 A

Correct Answer: (C) 15 A

Solution:

Step 1: Understanding the Concept:

Three-phase electrical systems use two primary connection methods: Star (Y) and Delta (Δ). Understanding the relationships between phase and line quantities is the most important part of 3-phase power analysis.

In a **Star (Wye) connection**, the three windings are joined at a common point (the neutral). Any current that flows through an individual phase winding has no choice but to flow out into the corresponding line wire. Therefore, there is no split in the current path.

In a **Delta connection**, however, each line wire is connected to the junction of two phases, meaning the line current is the vector sum of two phase currents.

Step 2: Key Formula or Approach:

For a Star (Y) connection:

- Line Current (I_L) = Phase Current (I_{ph})
- Line Voltage (V_L) = $\sqrt{3} \times$ Phase Voltage (V_{ph})

For a Delta (Δ) connection:

- Line Current (I_L) = $\sqrt{3} \times$ Phase Current (I_{ph})
- Line Voltage (V_L) = Phase Voltage (V_{ph})

Step 3: Detailed Explanation:

The problem explicitly specifies a **star connected system**.

Given:

- Phase Current (I_{ph}) = 15 A

Since this is a star connection, we apply the basic physical rule that the current in the line wire is the same current that passed through the load phase.

$$I_L = I_{ph}$$

$$I_L = 15 \text{ A}$$

There is no multiplication by $\sqrt{3}$ or division required for currents in a star system. If the question had asked for the line *voltage* given phase voltage, or if it had been a *delta* connection, the $\sqrt{3}$ factor would have come into play. Here, the relationship is one-to-one.

Step 4: Final Answer:

The line current is 15 A.

This corresponds to Option (C).

Quick Tip: Mnemonic: In Star, I is Same.

Remember that in a Star (Wye) connection, the line current is **identical** to the phase current. For Delta, the **voltage** is the same.

125. In a shell type transformer, the magnetic circuit has

- (A) Only one path for flux
- (B) Two paths for flux
- (C) Three paths for flux
- (D) No path for flux

Correct Answer: (B) Two paths for flux

Solution:

Step 1: Understanding the Concept:

Transformers are constructed using two main types of magnetic cores: Core type and Shell type.

1. ****Core Type:**** The magnetic core is a simple rectangular frame, and the windings surround the two vertical legs (limbs). The magnetic flux has only one continuous loop to follow around the rectangle.
2. ****Shell Type:**** The core is shaped like an 'E' and an 'I' joined together, or a "figure 8". The windings are placed on the central limb. The core effectively "surrounds" or "shells" the

windings. Because the windings are in the center, the magnetic flux generated in the central limb splits and returns through the two outer limbs.

Step 2: Detailed Explanation:

Let's analyze the flux distribution in a Shell type transformer:

Imagine the central limb of an 'E' shaped core where both primary and secondary windings are wound. When current flows, a total flux Φ is produced in this central limb.

When this flux reaches the top of the 'E', it sees two parallel paths of equal magnetic reluctance—the two outer vertical limbs.

Therefore, the flux divides equally:

- Flux path 1: Half the flux ($\Phi/2$) goes through the left limb.
- Flux path 2: Half the flux ($\Phi/2$) goes through the right limb.

Because there are two distinct loops that the magnetic field lines follow to complete their circuit, a shell type transformer is said to have **two paths for flux**. This construction provides better protection for the coils against mechanical damage and reduces the amount of copper needed relative to a core-type design of similar rating.

Step 3: Expanded Comparison:

- **Core Type:** 1 path for flux, 2 limbs, windings on limbs.
- **Shell Type:** 2 paths for flux, 3 limbs, windings on the middle limb.

The shell-type design is often preferred for low-voltage applications where high efficiency and mechanical robustness are required, whereas the core-type is more common for high-voltage power transformers due to easier insulation requirements.

Step 4: Final Answer:

In a shell type transformer, the magnetic circuit has two paths for flux.

This corresponds to Option (B).

Quick Tip: Remember the shape:

- Core type = C-shape (one loop).
- Shell type = E-shape or figure-8 (two loops).

Flux follows the available steel paths; Shell type has two "side-arms" to carry the return flux.

126. If a transformer has $R_{pu} = 0.03$ and $X_{pu} = 0.04$, its regulation at 0.8 power factor lagging is

- (A) 1.2%
- (B) 2.4%
- (C) 4.8%
- (D) 3.6%

Correct Answer: (C) 4.8%

Solution:

Step 1: Understanding the Concept:

Voltage regulation of a transformer is the measure of how much the secondary terminal voltage changes when the load is varied from no-load to full-load. It is usually expressed as a percentage of the rated voltage. A lower percentage indicates better performance.

The regulation depends on the internal resistance and reactance of the transformer and the power factor of the load. When using "Per-Unit" (pu) values, the calculation becomes much simpler because we don't need to worry about the transformation ratio or absolute values of voltage and current. In lagging power factor loads (typical for motors and inductors), the regulation is always positive because the terminal voltage drops under load.

Step 2: Key Formula or Approach:

The formula for percentage voltage regulation (%VR) using per-unit values is:

$$\%VR = (R_{pu} \cos \phi \pm X_{pu} \sin \phi) \times 100$$

Where:

- R_{pu} = Per-unit resistance.
- X_{pu} = Per-unit reactance.
- $\cos \phi$ = Power factor.
- Use + for lagging power factor.
- Use - for leading power factor.

Step 3: Detailed Explanation:

Given:

- $R_{pu} = 0.03$
- $X_{pu} = 0.04$
- $\cos \phi = 0.8$ (lagging)

First, find $\sin \phi$. Since $\cos^2 \phi + \sin^2 \phi = 1$:

$$\sin \phi = \sqrt{1 - 0.8^2} = \sqrt{1 - 0.64} = \sqrt{0.36} = 0.6$$

Now, substitute these into the regulation formula for lagging load:

$$\%VR = (0.03 \times 0.8 + 0.04 \times 0.6) \times 100$$

Calculate the individual products:

$$0.03 \times 0.8 = 0.024$$

$$0.04 \times 0.6 = 0.024$$

Add them together:

$$0.024 + 0.024 = 0.048$$

Convert to percentage:

$$0.048 \times 100 = 4.8\%$$

This means the terminal voltage will be 4.8% lower than the rated no-load voltage when the transformer is fully loaded at an 0.8 lagging power factor.

Step 4: Final Answer:

The voltage regulation is 4.8%.

This corresponds to Option (C).

Quick Tip: Shortcut: If $\cos \phi = 0.8$, then $\sin \phi = 0.6$. If $\cos \phi = 0.6$, then $\sin \phi = 0.8$. These pairs are extremely common in exams. Simply multiply R_{pu} by 0.8 and X_{pu} by 0.6 and add them.

127. A 2000/200 V transformer has a primary resistance of 2Ω . What is the primary resistance referred to the secondary side?

- (A) $0.2\ \Omega$
- (B) $0.02\ \Omega$
- (C) $200\ \Omega$
- (D) $0.002\ \Omega$

Correct Answer: (B) $0.02\ \Omega$

Solution:

Step 1: Understanding the Concept:

In transformer analysis, it is often necessary to combine the resistances and reactances of both the primary and secondary windings into a single equivalent value on one side of the transformer. This process is called "referring" parameters.

When an impedance is moved from one side of a transformer to the other, its numerical value must be adjusted by the square of the transformation ratio (K). This ensures that the power loss

(I^2R) and voltage drop (IZ) remain physically consistent regardless of which side we are looking from. A resistance on the high-voltage (HV) side will look much smaller when viewed from the low-voltage (LV) side because the LV side carries a much higher current for the same power.

Step 2: Key Formula or Approach:

1. Transformation ratio $(K) = \frac{V_2(\text{Secondary})}{V_1(\text{Primary})} = \frac{N_2}{N_1}$
2. Resistance referred to secondary side $(R'_1) = R_1 \times K^2$
where R_1 is the resistance on the primary side.

Step 3: Detailed Explanation:

Given:

- Primary Voltage $(V_1) = 2000 \text{ V}$
- Secondary Voltage $(V_2) = 200 \text{ V}$
- Primary Resistance $(R_1) = 2 \Omega$

Calculate the transformation ratio K :

$$K = \frac{200}{2000} = 0.1$$

Now, calculate the equivalent primary resistance when referred to the secondary side:

$$R'_1 = R_1 \times K^2$$

$$R'_1 = 2 \times (0.1)^2$$

$$R'_1 = 2 \times 0.01$$

$$R'_1 = 0.02 \Omega$$

This result makes sense because moving a resistance from the 2000V side to the 200V side (a step-down of 10 times) requires the resistance to be reduced by the square of that factor (100 times) to maintain equivalent circuit behavior. A 2Ω resistor on the HV side causes the same percentage voltage drop as a 0.02Ω resistor on the LV side.

Step 4: Final Answer:

The primary resistance referred to the secondary side is 0.02Ω .

This corresponds to Option (B).

Quick Tip: To avoid confusion with K or $1/K$, remember this rule: When referring a resistance to the ****High Voltage**** side, the value **increases**. When referring to the ****Low Voltage**** side, the value **decreases**. Since we are moving from 2000V (HV) to 200V (LV), our 2Ω answer *must* be smaller.

128. To ensure that two transformers share the load in proportion to their KVA ratings, they should have the same

- (A) Core losses
- (B) Per unit (percentage) impedance
- (C) Efficiency
- (D) Magnetizing current

Correct Answer: (B) Per unit (percentage) impedance

Solution:

Step 1: Understanding the Concept:

Parallel operation of transformers is common in substations to increase capacity or reliability. For two transformers to work together efficiently, they must follow certain conditions. The most critical condition for load sharing is related to their internal impedances.

If two transformers have different per-unit impedances, the transformer with the lower per-unit impedance will try to take a larger share of the total load relative to its ca-

capacity. This can lead to one transformer becoming overloaded while the other remains under-utilized. To have "proportional" load sharing (e.g., a 100kVA transformer takes double the load of a 50kVA transformer), the voltage drops within them must be matched correctly.

Step 2: Detailed Explanation:

The load shared by a transformer (S_A) in parallel with another (S_B) is given by:

$$S_A = S_{total} \times \frac{Z_B}{Z_A + Z_B}$$

where Z is the impedance. For load sharing to be perfectly proportional to their rated capacities (Q_A and Q_B), the condition is:

$$S_A/Q_A = S_B/Q_B$$

This is only possible if the ****Per-unit impedance**** (impedance calculated based on the transformer's own rating) is identical for both machines.

If two transformers have the same % impedance, it doesn't mean their actual ohmic resistance is the same; it means that at their respective full-rated loads, their internal voltage drops are the same percentage of their rated voltage.

Other factors like core losses or efficiency don't determine how much current flows through each transformer; that is purely an impedance-driven current-divider problem. Magnetizing current affects the no-load behavior and circulating currents (if ratios are different), but not the load sharing proportion.

Step 3: Final Answer:

Two transformers share the load in proportion to their ratings if they have the same per unit impedance.

This corresponds to Option (B).

Quick Tip: Proportional Load Sharing Rule:

$$\text{Load shared by Tx-1} \propto \frac{1}{\text{Per Unit Impedance of Tx-1}}$$

Therefore, if Per Unit Impedances are **equal**, the load share is exactly proportional to their **KVA ratings**.

If they are not equal, the one with the **lower** % impedance will overload first.

129. Heat dissipation in a transformer occurs through which of the following processes?

- (A) Conduction only
- (B) Convection only
- (C) Radiation only
- (D) Conduction, Convection and Radiation

Correct Answer: (D) Conduction, Convection and Radiation

Solution:

Step 1: Understanding the Concept:

Transformers generate significant heat due to copper losses in the windings and iron losses in the core. Since heat degrades the insulation and reduces the lifespan of the transformer, efficient cooling is mandatory. Heat transfer in physics occurs through three fundamental mechanisms: conduction (through solids), convection (through fluids), and radiation (through electromagnetic waves). In a complex industrial machine like a transformer, all three mechanisms play a vital role simultaneously to move heat from the source (the core) to the final heat sink (the surrounding air).

Step 2: Detailed Explanation:

Let's trace the heat dissipation path in a typical oil-cooled transformer:

1. ****Conduction:**** Heat generated within the copper conductors and the steel laminations of the core must move through the solid material to reach the surface. It also conducts through the solid insulation layers (paper, pressboard) to reach the cooling medium.
2. ****Convection:**** Once the heat reaches the surface of the core and windings, it is absorbed by the transformer oil. The hot oil becomes less dense and rises, while cooler oil sinks (natural convection). In larger transformers, pumps may force the oil through radiators

(forced convection). Similarly, the air surrounding the transformer tank or radiators carries heat away via convection.

3. ****Radiation:**** The external surface of the transformer tank or the cooling fins is at a higher temperature than the surroundings. It emits infrared energy directly into the environment. While radiation is less dominant at lower temperatures compared to convection, it is still a measurable component of the total heat dissipation.

Because the heat must pass through solids (conduction), move via a circulating fluid (convection), and leave the outer surface (radiation), the cooling process involves all three modes.

Step 3: Final Answer:

Heat dissipation occurs through conduction, convection, and radiation.

This corresponds to Option (D).

Quick Tip: Almost every real-world cooling problem for a large physical object in a fluid environment involves **all three modes** of heat transfer. If "Conduction, Convection and Radiation" is an option in a general machine cooling question, it is very likely the correct one.

130. Which connection is best suited for 3-phase, 4-wire distribution (eg. Residential)?

- (A) Delta – Delta
- (B) Star – Star
- (C) Delta – Star
- (D) Star – Delta

Correct Answer: (C) Delta – Star

Solution:

Step 1: Understanding the Concept:

The secondary distribution network, which provides electricity to houses and small businesses,

must cater to two types of loads: 3-phase loads (like large motors) and 1-phase loads (like lighting and computers). To provide single-phase power, we need a neutral wire. A "4-wire system" refers to the three phase wires plus one neutral wire. The distribution transformer's connection must be chosen to create this neutral point effectively while also managing harmonic issues and unbalanced loading from the primary side.

Step 2: Detailed Explanation:

The standard distribution transformer used worldwide is the **Delta-Star** (specifically $\Delta - Y$, often labeled as Dyn11 or Dyn1).

1. **Secondary Side (Star):** The secondary is connected in a Star configuration. The central point of the star is grounded and provides the **Neutral** terminal. This allows for both line-to-line voltage (415V) for 3-phase power and line-to-neutral voltage (240V) for residential sockets. Furthermore, a star secondary handles unbalanced residential loads much better than a delta secondary.
2. **Primary Side (Delta):** The primary is connected in Delta. This provides a path for the flow of "third harmonic" currents within the delta loop. Without this, the third harmonics would distort the voltage waveform on the secondary side, causing issues for electronic equipment. Additionally, the delta primary does not require a neutral wire from the substation, reducing transmission costs.

Other connections like Delta-Delta or Star-Delta cannot provide a stable 4th wire (neutral) for the consumer. Star-Star is rarely used because of neutral instability and harmonic problems unless a tertiary winding is added.

Step 3: Final Answer:

The Delta-Star connection is best suited for 3-phase, 4-wire distribution.

This corresponds to Option (C).

Quick Tip: Distribution Transformer Shortcut:

Look for **Star on the Secondary** (for the neutral wire) and **Delta on the Primary** (for harmonic suppression). Residential power is *always* delivered from a Star-connected secondary.

131. What is the major disadvantage of an auto transformer?

- (A) High cost
- (B) Low efficiency
- (C) Lack of electrical isolation between primary and secondary
- (D) Very heavy weight

Correct Answer: (C) Lack of electrical isolation between primary and secondary

Solution:

Step 1: Understanding the Concept:

An auto transformer is a special type of electrical transformer that possesses only one continuous winding that is common to both the primary and secondary sides. Unlike a standard two-winding transformer where the primary and secondary circuits are magnetically coupled but electrically separated by insulation, the auto transformer utilizes a "tapped" winding. This means a portion of the winding serves as part of both the high-voltage and low-voltage circuits. This construction leads to several significant advantages, such as a smaller physical size, lower cost for the same KVA rating, higher efficiency due to reduced copper and iron losses, and better voltage regulation. However, because the primary and secondary share a physical copper path, the inherent safety and design flexibility provided by electrical separation in traditional transformers are compromised.

Step 2: Key Formula or Approach:

In a standard transformer, the isolation is infinite (ideally) because there is no ohmic contact. In an auto transformer with transformation ratio $a = V_1/V_2$, the power transferred conductively is $P_{cond} = P_{total} \times (1/a)$ and the power transferred inductively is $P_{ind} = P_{total} \times (1 - 1/a)$. The "Lack of Isolation" is a qualitative physical property arising from the common connection point between the two circuits.

Step 3: Detailed Explanation:

The primary reason why auto transformers are not used in all applications despite their efficiency is the **Lack of Electrical Isolation**.

1. **Safety Risk:** In a step-down auto transformer, if the common winding (the portion shared

by both sides) develops an open circuit or "breaks," the full primary high voltage can appear across the secondary terminals. For example, if a 2400V to 240V auto transformer fails, a residential appliance designed for 240V could suddenly be exposed to 2400V, leading to catastrophic equipment failure and severe risk of electrocution to users. In a two-winding transformer, such a fault would simply result in zero voltage on the secondary.

2. **Grounding Issues:** Because the two sides are electrically connected, the grounding scheme of the primary side is forced upon the secondary side. This prevents the secondary from being independently grounded or "floated," which is often a requirement in sensitive electronic labs or medical environments to minimize noise and improve safety.

3. **Comparison with other options:**

- **Option A (High cost):** Incorrect. Auto transformers use less copper and a smaller core, making them significantly cheaper than two-winding transformers of the same capacity.
- **Option B (Low efficiency):** Incorrect. Because part of the power is transferred through direct conduction and the core size is smaller, auto transformers actually have higher efficiency.
- **Option D (Very heavy weight):** Incorrect. They are much lighter than standard transformers because they save on both winding material and core steel.

Therefore, the electrical "transparency" between the input and output is the critical design flaw that limits their use to specific applications like variacs, motor starters, and grid voltage regulation where isolation is not a primary concern.

Step 4: Final Answer:

The major disadvantage is the lack of electrical isolation between the primary and secondary windings.

This matches Option (C).

Quick Tip: To remember the pros and cons of an **Auto** transformer:

- **Pros:** Small, Cheap, High Efficiency, Low Losses (Think "Auto" = Small/Efficient car).
- **Cons:** **No Isolation** (Think "Auto" = You are in direct contact with the road).

Always avoid auto transformers for low-voltage consumer electronics where safety is paramount!

132. If an alternator has a flux per pole of 0.05 Wb, a frequency of 50 Hz, and 100 turns per phase (assume pitch factor and distribution factor is 1), the RMS phase emf is approximately

- (A) 222 V
- (B) 1110 V
- (C) 555 V
- (D) 444 V

Correct Answer: (B) 1110 V

Solution:

Step 1: Understanding the Concept:

An alternator (synchronous generator) produces electrical power by rotating a magnetic field past stationary armature windings. According to Faraday's Law, the voltage induced in these windings depends on the rate at which they "cut" the magnetic flux. The total voltage generated per phase depends on the frequency of the magnetic reversals, the strength of the magnetic field (flux per pole), and the physical arrangement of the coils (number of turns). The EMF equation of an alternator is a fundamental formula used to determine the design parameters of AC generators. It calculates the Root Mean Square (RMS) value of the sinusoidal voltage produced in one phase of the machine.

Step 2: Key Formula or Approach:

The standard EMF equation for an alternator per phase is:

$$E_{ph} = 4.44 \cdot K_p \cdot K_d \cdot f \cdot \phi \cdot T$$

Where:

- E_{ph} = RMS induced EMF per phase.
- K_p = Pitch factor (Short-pitch factor).
- K_d = Distribution factor (Breadth factor).
- f = Frequency of the generated AC in Hz.
- ϕ = Flux per pole in Webers (Wb).
- T = Number of turns per phase.

Step 3: Detailed Explanation:

Let's extract the given data from the problem statement:

- Flux per pole (ϕ) = 0.05 Wb.
- Frequency (f) = 50 Hz.
- Turns per phase (T) = 100.
- Pitch factor (K_p) = 1 (given as assumed).
- Distribution factor (K_d) = 1 (given as assumed).

Note: In a practical machine, K_p and K_d are usually slightly less than 1 (e.g., 0.95), but for this exam problem, we follow the simplification provided.

Substitute these values into the EMF formula:

$$E_{ph} = 4.44 \times 1 \times 1 \times 50 \times 0.05 \times 100$$

Perform the calculation step-by-step:

First, simplify the multiplication of the constants and frequency:

$$4.44 \times 50 = 222$$

Now, multiply by the flux:

$$222 \times 0.05 = 11.1$$

Finally, multiply by the number of turns:

$$11.1 \times 100 = 1110$$

The units are Volts (V). The result 1110V represents the RMS phase voltage. If the alternator were star-connected, the line voltage would be $1110 \times \sqrt{3} \approx 1922\text{ V}$. Since the question asks for the EMF (implied phase value based on options), 1110 V is the correct numerical outcome. This value indicates the potential difference generated within the armature winding before any internal drops occur.

Step 4: Final Answer:

The RMS phase EMF is 1110 V.

This matches Option (B).

Quick Tip: A quick way to remember the alternator EMF constant: $4.44 \times 50 = 222$.

For 50 Hz machines, the formula becomes: $E = 222 \cdot \phi \cdot T$ (assuming factors are 1).

In this problem: $222 \times 0.05 \times 100 = 222 \times 5 = 1110$. This trick avoids long decimal multiplications during the exam.

133. Which method provides the most accurate results for the voltage regulation of a non-salient pole alternator?

- (A) EMF method
- (B) MMF method
- (C) Poitier Triangle method
- (D) Direct loading test

Correct Answer: (C) Poitier Triangle method

Solution:**Step 1: Understanding the Concept:**

Voltage regulation of an alternator is defined as the change in terminal voltage when the load is reduced from its rated value to zero, expressed as a percentage of the rated voltage. It is a critical performance metric for power systems. Finding regulation by "Direct Loading" is impractical for large generators because it requires wasting massive amounts of power and finding a resistive-inductive-capacitive load bank of several megawatts. Therefore, indirect methods are used that involve performing open-circuit and short-circuit tests. These methods differ in how they account for two major physical phenomena: Armature Resistance drop and Armature Reaction (the magnetic interference of the armature current on the main field flux).

Step 2: Detailed Explanation:

Let's evaluate the standard indirect methods for non-salient (cylindrical rotor) alternators:

1. **EMF Method (Synchronous Impedance Method):** This method treats the entire armature reaction effect as a fictitious "reactance." It assumes the magnetic circuit is linear and does not saturate. Because it ignores saturation, the calculated synchronous impedance is higher than the actual value, leading to regulation results that are much higher than reality. Thus, it is known as a **pessimistic method**.
2. **MMF Method (Ampere-Turn Method):** This method converts all voltage drops into equivalent field ampere-turns. It assumes the armature leakage reactance is zero. This results in regulation values that are lower than the actual performance, making it an **optimistic method**.
3. **Poitier Triangle Method (ZPF Method):** This method is much more sophisticated. It separates the armature leakage reactance (X_L) from the armature reaction effects. It uses data from the Zero Power Factor (ZPF) test along with the Open Circuit Characteristic (OCC). Crucially, the Poitier method **accounts for the saturation of the magnetic core**. By using the Poitier triangle to find the actual leakage reactance and the equivalent field current for armature reaction, it provides the most realistic and accurate prediction of voltage regulation among the standard indirect techniques.
4. **Direct Loading Test:** While theoretically perfectly accurate, it is physically impossible to perform on large utility-scale alternators, and thus not considered the "standard method for accurate results" in an engineering context for non-salient machines.

Step 3: Expanded Comparison:

For non-salient machines, the magnetic air gap is uniform, making these vector-based methods highly effective. The Poitier method's ability to distinguish between the inductive drop (IX_L) and the flux distortion (armature reaction) while referencing the actual saturated OCC curve is what makes it superior. In contrast, for *salient pole* machines, the Two-Reaction Theory (Blondel's theory) would be used, but for the general case or non-salient machines, the Poitier/ZPF method is the gold standard for accuracy.

Step 4: Final Answer:

The Poitier Triangle method (Zero Power Factor method) provides the most accurate results for the regulation of a non-salient pole alternator.

This matches Option (C).

Quick Tip: Summary of Regulation Methods:

- **EMF Method:** High/Pessimistic (Easy but inaccurate).
 - **MMF Method:** Low/Optimistic.
 - **Poitier/ZPF Method: Most Accurate** (Accounts for saturation).
 - **ASA Method:** A variation of MMF that also accounts for saturation and is very accurate.
- Remember: ZPF/Poitier is the "Engineer's Choice" for accuracy.

134. Two alternators A and B are in parallel. Alternator A is adjusted to have a leading power factor. This will cause Alternator B to

- (A) Also have a leading power factor
- (B) Have a lagging power factor
- (C) Stop rotating
- (D) Carry zero current

Correct Answer: (B) Have a lagging power factor

Solution:

Step 1: Understanding the Concept:

When two alternators operate in parallel, they are connected to common busbars and must share the total load connected to the system. The load sharing is governed by two control parameters: real power (P) sharing depends on the speed-torque (governor) characteristics, and reactive power (Q) sharing depends on the field excitation (voltage) characteristics. The total reactive power required by the load is a constant at any given moment. If the reactive power output of one alternator is changed, the other alternator must automatically adjust its output to satisfy the load's requirements and maintain the busbar voltage. This interaction determines the operating power factors of the individual machines.

Step 2: Detailed Explanation:

Consider a system where two alternators, A and B, supply a load.

The total reactive power of the system is $Q_{load} = Q_A + Q_B$.

1. **Changing Excitation:** If we increase the excitation of Alternator A (over-excitation), it generates more internal EMF. This causes it to supply more "magnetizing" or reactive power to the system. An over-excited alternator operates at a **lagging** power factor because it is "exporting" reactive power.

2. **The Inverse Case:** The question states that Alternator A is adjusted to have a **leading** power factor. A leading power factor in a generator means the machine is "under-excited." It is not producing enough internal magnetic flux to support itself, so it actually **absorbs** reactive power from the busbar (or from the other machine).

3. **Reaction of Alternator B:** Since Alternator A is now taking reactive power away from the busbar (due to its leading PF), and the load still needs its own reactive power, Alternator B must compensate. Alternator B must now supply:

- The reactive power needed by the load.
- PLUS the reactive power being absorbed by the under-excited Alternator A.

Therefore, Alternator B becomes "over-worked" in terms of magnetizing current. It must produce a significant excess of reactive power, which forces it to operate at a **lagging power factor**. In essence, the lagging vars supplied by machine B are shared between the load and machine A. This push-pull relationship of reactive power is a fundamental aspect of parallel generator management.

Step 3: Final Answer:

If Alternator A is adjusted to a leading power factor, Alternator B will be forced to operate at a lagging power factor.

This matches Option (B).

Quick Tip: Think of Reactive Power as a **fixed resource**.

If Generator A takes less than its share (Under-excited / Leading), Generator B **must** take more than its share (Over-excited / Lagging).

They always act as "Opposites" regarding power factor when excitation is changed relative to each other!

135. A Synchronous motor used specifically for power factor correction (running at no load and over excited) is called a

- (A) Synchronous generator
- (B) Synchronous condenser
- (C) Induction generator
- (D) Variable speed drive

Correct Answer: (B) Synchronous condenser

Solution:

Step 1: Understanding the Concept:

A synchronous motor has a unique property compared to induction motors: its power factor can be controlled by varying its field excitation. This is depicted by the "V-curves" of the motor. When the motor's field is **over-excited** (meaning the field current I_f is high enough that the induced back EMF E_b is greater than the supply voltage V), the motor draws a current that leads the voltage. In this state, the motor behaves like a large capacitor or "condenser" connected across the line. By injecting leading reactive power into the grid, it can cancel out the lagging reactive power drawn by other inductive loads (like induction motors and transformers), thereby improving the overall system power factor.

Step 2: Detailed Explanation:

Let's analyze the specific application mentioned:

1. **Condition (No Load):** To maximize its effectiveness for power factor correction, the motor is run without any mechanical load attached to its shaft. In this state, it only draws a very small amount of real power (P) from the supply to overcome its own internal friction and

windage. Almost the entire current-carrying capacity of the stator is available to handle reactive current.

2. **Condition (Over-excited):** High field excitation makes the motor a source of reactive power. In power systems, "condenser" was an old term for a capacitor. Since this motor "mimics" a capacitor's effect on the power factor while being a synchronous machine, it is professionally referred to as a **Synchronous Condenser** (or Synchronous Capacitor).

3. **Comparison with other options:**

- **Option A (Synchronous generator):** While it is the same machine physically, a "generator" is used to supply real power to a load, not primarily for PF correction at no-load.
- **Option C (Induction generator):** These typically operate at a lagging power factor and actually require reactive power from the grid to function, making them unsuitable for correction.
- **Option D (Variable speed drive):** This is a power electronic system used to control motor speed, not a rotating machine used for reactive power support.

Synchronous condensers are often installed in major substations to regulate voltage and maintain power factor stability in long-distance transmission lines.

Step 3: Final Answer:

A synchronous motor used for power factor correction at no load and over-excited state is a Synchronous condenser.

This matches Option (B).

Quick Tip: Mnemonic: **Over-excited Motor = Leading PF = Capacitor = Condenser.**

Inductive loads (motors) pull reactive power (**Lagging**).

Synchronous Condensers push reactive power (**Leading**).

They are like "Spinning Capacitors."

136. In the V-curve of a Synchronous motor, the minimum point of the curve corresponds to

- (A) Lagging power factor
- (B) Unity power factor

- (C) Minimum speed
(D) Leading power factor

Correct Answer: (B) Unity power factor

Solution:

Step 1: Understanding the Concept:

The "V-curve" of a synchronous motor is a graphical plot showing the relationship between the armature current (I_a) and the field excitation current (I_f) for a constant mechanical load. The curve is named for its characteristic 'V' shape. Because a synchronous motor can operate at lagging, unity, or leading power factors depending on its excitation, the amount of current drawn from the line to supply a fixed amount of real power ($P = \sqrt{3}V I_a \cos \phi$) changes significantly. Real power depends on the load, but the reactive power (and thus the total current magnitude) depends on the field setting.

Step 2: Detailed Explanation:

Let's analyze the shape and meaning of the V-curve:

1. **Left Side (Under-excitation):** When I_f is low, the motor is under-excited. It must draw magnetizing current from the AC supply to maintain its magnetic field. This results in a **lagging** power factor and a high armature current because $\cos \phi$ is small.
2. **Right Side (Over-excitation):** When I_f is very high, the motor is over-excited. It supplies reactive power to the line, behaving like a capacitor. This results in a **leading** power factor. Again, because $\cos \phi$ is less than 1, the armature current must increase to supply the same real power.
3. **The Minimum Point (Bottom of the 'V'):** At exactly one specific value of field current, the motor is "normally excited." At this point, the motor does not need to draw or supply any reactive power; it operates at a **Unity Power Factor** ($\cos \phi = 1$).
 - Since $P = V I_a \cos \phi$, if $\cos \phi$ is at its maximum possible value of 1, the required current I_a to produce the power P will be at its absolute **minimum**.
 - This is the most efficient operating point for the motor windings as I^2R losses are minimized. Thus, the trough of the V-curve represents the condition where the line current is purely resistive (in-phase with voltage).

Step 3: Final Answer:

The minimum point of the V-curve corresponds to the Unity Power Factor.
This matches Option (B).

Quick Tip: Remember the **Inverted V-Curve**: If you plot **Power Factor** ($\cos \phi$) vs. **Field Current** (I_f), you get an Inverted-V. The **Peak** of the Inverted-V (where $\text{PF}=1$) happens at exactly the same excitation as the **Minimum** of the regular V-curve.

Minimum Current = Maximum Power Factor.

137. What is a Pony motor in the context of starting a Synchronous motor?

- (A) A motor that runs on DC only
- (B) A small auxiliary motor used to bring the main motor up to speed
- (C) The main cooling fan
- (D) A motor used to reverse the direction

Correct Answer: (B) A small auxiliary motor used to bring the main motor up to speed

Solution:

Step 1: Understanding the Concept:

One of the major characteristics of a synchronous motor is that it is **not self-starting**. When the stator is energized, the magnetic field rotates at synchronous speed (N_s) immediately. Due to the high mechanical inertia of the rotor, it cannot accelerate from standstill to synchronous speed in the fraction of a second during the first half-cycle. As a result, the rotor experienced alternating clockwise and counter-clockwise torques and remains at a standstill, vibrating. To start the motor, it must be accelerated by some external means to a speed very close to N_s . Once the rotor is spinning fast enough, the DC field can be energized, and the magnetic poles of the rotor will "lock" into step with the rotating stator field (Synchronism).

Step 2: Detailed Explanation:

There are several methods to start a synchronous motor, including the use of damper windings

(acting like an induction motor) or external drivers.

1. **The Pony Motor Method:** This method uses a separate, smaller **auxiliary motor** (usually a small induction motor or DC motor) physically coupled to the shaft of the large synchronous motor. This auxiliary motor is called a "Pony Motor."

2. **The Process:**

- First, the pony motor is turned on. It accelerates the main rotor up to about 95% to 99% of the synchronous speed.
- Then, the main stator of the synchronous motor is energized.
- Finally, DC excitation is applied to the main rotor. Because the relative speed between the rotor poles and the stator field is now very small, magnetic "locking" occurs easily.
- Once synchronism is achieved, the pony motor is decoupled or turned off, and the main motor continues to run at N_s .

3. **Why use it?** It is used for very large synchronous motors where adding internal damper windings might be difficult or where the starting current of the large motor must be kept extremely low to protect the local power grid. The pony motor usually has 2 fewer poles than the main motor to ensure it can reach the required synchronous speed.

Step 3: Final Answer:

A pony motor is a small auxiliary motor used to bring the main synchronous motor up to its required starting speed.

This matches Option (B).

Quick Tip: Think of a **Pony** as a small horse that helps a large horse (the Synchronous motor) get its carriage moving. It "tows" the big motor to its "running speed" before the big motor takes over the work.

138. A motor has synchronous speed of 1000 rpm and runs at 950 rpm. If the torque at this slip is 10 Nm. What is the torque if the slip is doubled? (assume low slip region)

- (A) 5 Nm
- (B) 10 Nm
- (C) 20 Nm

(D) 40 Nm

Correct Answer: (C) 20 Nm

Solution:

Step 1: Understanding the Concept:

The operation of an induction motor is defined by its "Slip" (s), which is the normalized difference between the synchronous speed of the magnetic field (N_s) and the actual mechanical speed of the rotor (N). The torque developed by the motor varies significantly with slip. A typical induction motor operates in the "stable" or "low-slip" region during normal running. In this region (usually slip $< 5\%$), the torque-slip characteristic curve is almost a straight line. This means that for small variations in load, the torque produced by the motor is directly proportional to the slip.

Step 2: Key Formula or Approach:

1. Formula for Slip: $s = \frac{N_s - N}{N_s}$.
2. Proportionality in low-slip region: $T \propto s$.
3. Relationship: $\frac{T_2}{T_1} = \frac{s_2}{s_1}$.

Step 3: Detailed Explanation:

Let's find the initial slip s_1 :

Given $N_s = 1000$ rpm and $N_1 = 950$ rpm.

$$s_1 = \frac{1000 - 950}{1000} = \frac{50}{1000} = 0.05 (5\%)$$

At this slip, the initial torque $T_1 = 10$ Nm.

The problem states that the **slip is doubled**.

New slip $s_2 = 2 \times s_1 = 2 \times 0.05 = 0.10 (10\%)$.

Since the motor is operating in the low-slip region (implied by the 5-10% values being near the linear part of the curve), we use the direct proportionality:

$$T \propto s \implies \frac{T_2}{T_1} = \frac{s_2}{s_1}$$

Substituting the ratio of slips (which is 2):

$$\frac{T_2}{10} = 2$$

$$T_2 = 10 \times 2 = 20 \text{ Nm}$$

This linear behavior holds true as long as the motor is not overloaded beyond its breakdown torque point. If the slip were to increase much further, the proportionality would fail as the curve peaks and then drops off (the high-slip region where $T \propto 1/s$). For standard exam problems like this, the "low slip" assumption allows for a simple doubling of torque.

Step 4: Final Answer:

The new torque is 20 Nm.

This matches Option (C).

Quick Tip: In the **Low-Slip Region** (normal operation): **Torque is linear with Slip.**

If slip is doubled, torque **doubles**.

If slip is halved, torque **halves**.

Just like a simple resistor ($V = IR$), the induction motor at low slip behaves like a simple mechanical "resistor" where force is proportional to speed difference.

139. In an Induction motor, the rotor copper loss is equal to

- (A) Slip x rotor input power
- (B) (1-slip) x rotor input power
- (C) Rotor input power/slip
- (D) $\text{Slip}^2 \times \text{rotor input power}$

Correct Answer: (A) Slip x rotor input power

Solution:

Step 1: Understanding the Concept:

An induction motor transforms electrical power from the stator into mechanical power at the rotor shaft through an intermediate stage called the "Air-gap Power" or "Rotor Input Power" (P_g). Once the energy crosses the air gap into the rotor, part of it is lost as heat due to the electrical resistance of the rotor bars or windings. This is called the **Rotor Copper Loss** (P_{cu}). The remaining power is converted into mechanical form, known as the "Gross Mechanical Power Developed" (P_m). There is a fixed mathematical ratio between these three quantities that depends entirely on the slip (s) of the motor. This ratio is fundamental for calculating the efficiency and performance of the machine.

Step 2: Key Formula or Approach:

The power flow relationship in the rotor of an induction motor is:

$$P_g : P_{cu} : P_m = 1 : s : (1 - s)$$

Where:

- P_g = Power across the air gap (Rotor Input).
- P_{cu} = Rotor Copper Loss.
- P_m = Mechanical power developed.

Step 3: Detailed Explanation:

Let's derive the relationship for rotor copper loss:

The torque developed in the rotor is T .

1. The air-gap power is the torque multiplied by the synchronous speed: $P_g = T \cdot \omega_s$.
2. The mechanical power developed is the torque multiplied by the actual rotor speed:

$$P_m = T \cdot \omega.$$

3. Since $\omega = \omega_s(1 - s)$, we can write:

$$P_m = T \cdot \omega_s(1 - s) = P_g(1 - s)$$

4. The rotor copper loss is the difference between the power that entered the rotor and the power converted to mechanical energy:

$$P_{cu} = P_g - P_m$$

$$P_{cu} = P_g - P_g(1 - s)$$

Factor out P_g :

$$P_{cu} = P_g[1 - (1 - s)]$$

$$P_{cu} = P_g \cdot s$$

Thus, the rotor copper loss is exactly the product of the slip and the rotor input power. This formula is extremely useful because it shows that as slip increases (e.g., when the motor is heavily loaded), the heat generated in the rotor increases linearly. At standstill ($s = 1$), all the power crossing the air gap is lost as copper loss, which is why induction motors overheat rapidly if they are stalled.

Step 4: Final Answer:

Rotor copper loss = Slip $\times$ Rotor input power.

This matches Option (A).

Quick Tip: Remember the "Power Flow Hierarchy":

1. **Air Gap Power** (P_g) is the **Total**.
2. **Rotor Copper Loss** takes a "slice" equal to **s**.
3. **Mechanical Power** takes the "rest" which is **(1-s)**.

If Efficiency of the rotor is asked, it is $(1 - s) \times 100\%$.

140. The pole changing method of speed control is only applicable to which type of motor?

- (A) Slip ring induction motor
- (B) Squirrel cage induction motor
- (C) Synchronous motor
- (D) DC shunt motor

Correct Answer: (B) Squirrel cage induction motor

Solution:

Step 1: Understanding the Concept:

The synchronous speed of an induction motor is given by the formula $N_s = 120f/P$. From this, it is evident that the speed of the motor can be changed by either varying the supply frequency (f) or the number of magnetic poles (P) in the stator winding. Pole changing involves physically reconnecting the stator coils to create a different number of pole pairs (e.g., switching from 4 poles to 8 poles), which would cut the speed in half. However, for an induction motor to function, the rotor must also have the *exact same number of poles* as the stator. If the stator has 4 poles and the rotor has 8, no torque will be produced. This requirement for "Pole Matching" determines which motor types can use this method.

Step 2: Detailed Explanation:

Let's analyze the two types of induction motor rotors:

1. **Slip Ring (Wound) Rotor:** In this motor, the rotor has a formal 3-phase winding with a specific number of poles determined at the factory. If you change the stator from 4 poles to 8 poles, you must also somehow rewire the rotating rotor to 8 poles. This is physically impossible without stopping the motor and manually reconnecting the rotor winding, which defeats the purpose of an operational speed control method. Therefore, pole changing is not applicable to slip-ring motors.
2. **Squirrel Cage Rotor:** The rotor "winding" consists of short-circuited copper or aluminum bars. This type of rotor is **magically flexible**. When the stator magnetic field is established, it induces currents in the cage bars that automatically create a magnetic field with the **exact same number of poles as the stator**, regardless of whether the stator is currently set to

2, 4, 6, or 8 poles. Because the squirrel cage rotor "mirrors" whatever pole count the stator provides, the motor can continue to run smoothly even when the stator poles are changed.

3. Other Options:

- **Option C (Synchronous motor):** These run at exactly N_s . While the stator poles could be changed, the rotor uses a DC field which is physically wound for a fixed number of poles.
- **Option D (DC shunt motor):** These use commutator and flux control for speed, not pole changing.

Thus, the "Automatic Pole Induction" property of the squirrel cage is what makes it the only candidate for this discrete speed control method.

Step 3: Final Answer:

The pole changing method is only applicable to the Squirrel cage induction motor.
This matches Option (B).

Quick Tip: Keyword: **Rotor Flexibility.**

A squirrel cage rotor is like a "chameleon"—it changes its magnetic poles to match the stator. A slip ring rotor is like a "photograph"—its poles are fixed forever. This is why cage motors are the **only** ones that can change speeds via pole switching.

141. Regenerative braking occurs only when the motor speed (N)

- (A) Is less than the synchronous speed
- (B) Is greater than the synchronous speed
- (C) Is exactly equal to synchronous speed
- (D) Is zero

Correct Answer: (B) Is greater than the synchronous speed

Solution:

Step 1: Understanding the Concept:

Electric braking is a process used in motor control to slow down or stop an electric motor by converting its kinetic energy into another form of energy.

There are three main types of electric braking: plugging, rheostatic (dynamic) braking, and regenerative braking.

Regenerative braking is the most efficient method because, instead of wasting the energy as heat in a resistor, it returns the energy back to the supply source or a storage device like a battery.

In the context of an induction motor, this process is fundamentally linked to the concept of "slip" and the relationship between the mechanical speed of the rotor (N) and the synchronous speed of the rotating magnetic field (N_s).

For a motor to return power to the grid, it must transition from acting as a "consumer" of energy (motor mode) to acting as a "generator" (generator mode).

Step 2: Key Formula or Approach:

The slip (s) of an induction motor is defined by the formula:

$$s = \frac{N_s - N}{N_s}$$

In normal motoring operation, $N < N_s$, which means the slip is positive ($0 < s < 1$).

For regenerative braking to occur, the motor must act as an induction generator.

This happens when the slip becomes negative, which mathematically requires the actual rotor speed to exceed the synchronous speed.

Step 3: Detailed Explanation:

Let us examine the physical mechanism of regenerative braking in an induction machine.

When the motor is running normally, the stator's rotating magnetic field "drags" the rotor along at a speed slightly less than N_s .

If the load starts driving the motor (for example, an electric train going down a steep hill or a hoist lowering a heavy weight), the external mechanical energy can accelerate the rotor.

If the rotor is accelerated such that its speed N becomes higher than the synchronous speed N_s , the relative motion between the magnetic field and the rotor conductors reverses direction.

As a result, the direction of the induced EMF and the rotor current is reversed.

Following the principles of electromagnetic induction, the machine now produces a

counter-torque that opposes the over-speeding motion.

This counter-torque acts as a braking force.

Simultaneously, the phase angle between the stator voltage and current shifts by more than 90 degrees, causing the machine to pump electrical power back into the AC supply lines.

For this to work, the supply system must be "receptive," meaning it must have other loads connected that can consume this returned power, or the source itself must be able to handle reverse power flow.

If $N = N_s$, the slip is zero, no current is induced in the rotor, and no torque is produced (no-load condition).

If $N < N_s$, the machine is absorbing power and producing driving torque.

Therefore, the definitive physical condition for regenerative braking is that the motor speed must be greater than the synchronous speed.

Step 4: Final Answer:

Regenerative braking occurs only when the motor speed is greater than the synchronous speed.

This matches Option (B).

Quick Tip: Remember the "Sign of Slip" Rule:

- **Motoring:** s is positive ($N < N_s$).
- **Generating/Regenerative Braking:** s is negative ($N > N_s$).
- **Plugging/Reverse Braking:** $s > 1$ (Rotor moves against the field).

In regenerative braking, think of the motor being "pushed" by the load faster than the magnetic field wants to turn.

142. A single phase induction motor is

- (A) Inherently self-starting
- (B) Not inherently self-starting
- (C) Self-starting only at high voltages
- (D) Always started by a DC motor

Correct Answer: (B) Not inherently self-starting

Solution:

Step 1: Understanding the Concept:

Single-phase induction motors are the most common type of small motors used in domestic appliances like fans, refrigerators, and washing machines.

Unlike three-phase induction motors, which naturally produce a rotating magnetic field and are therefore self-starting, a standard single-phase motor behaves differently.

When a single-phase AC supply is connected to a single stator winding, the magnetic field produced is not a rotating field but a "pulsating" or "alternating" one.

This pulsating field can be visualized as the sum of two rotating magnetic fields of equal magnitude but rotating in opposite directions at synchronous speed.

This physical phenomenon is explained by the "Double Revolving Field Theory."

Step 2: Detailed Explanation:

Let us analyze why the pulsating field prevents the motor from starting on its own.

1. **The Two Fields:** According to the theory, the single-phase field is composed of a "Forward" rotating field and a "Backward" rotating field.

2. **Induced Torques:** At standstill (when the rotor speed $N = 0$), both the forward and backward fields induce equal and opposite torques in the rotor.

3. **Resultant Torque:** The net torque at the moment of starting is the difference between these two: $T_{net} = T_f - T_b = 0$.

Since the starting torque is exactly zero, the rotor remains stationary even though it experiences strong magnetic vibrations.

4. **Experimental Verification:** If you manually give the shaft a small nudge in either direction, the motor will accelerate and run in that direction. This is because once the rotor is moving, the slip relative to one field decreases while the slip relative to the other increases, resulting in a non-zero net torque.

Because the motor cannot generate an initial torque from a standstill using only one winding, it is described as "not inherently self-starting."

To make these motors useful in practice, auxiliary methods must be employed to create a "fake" rotating field during startup.

Common techniques include adding a second winding (starting winding) with a phase shift,

using a capacitor (capacitor start), or using shaded poles.

These methods temporarily convert the motor into a quasi-two-phase machine during the starting period, which allows a rotating field to be established.

Step 3: Comparison with other options:

- **Option A:** Incorrect, as proven by field theory.
- **Option C:** Voltage magnitude does not change the pulsating nature of the field; it only increases the vibration intensity.
- **Option D:** While a DC motor *could* start it, it is not "always" done this way; internal phase-splitting methods are the standard.

Step 4: Final Answer:

A single phase induction motor is not inherently self-starting.

This matches Option (B).

Quick Tip: To remember this, think of a **Ceiling Fan**. If the capacitor fails, the fan will hum (pulsating field) but won't turn. If you push it with a stick, it starts spinning. This confirms the motor wants to run but cannot start itself!

143. A universal motor is designed to operate on

- (A) DC supply only
- (B) AC supply only
- (C) Both AC and DC supply
- (D) 3-phase supply only

Correct Answer: (C) Both AC and DC supply

Solution:

Step 1: Understanding the Concept:

A "Universal Motor" is a type of electric motor that is unique because it can run on both Alternating Current (AC) and Direct Current (DC) with similar speed and torque characteristics.

Structurally, it is essentially a DC series motor that has been modified to handle the challenges of AC operation.

In a series motor, the field winding and the armature winding are connected in series. This means the same current flows through both the stator poles and the rotor conductors.

Because of this series connection, the motor exhibits high starting torque and high speed at light loads, making it ideal for portable power tools and kitchen appliances like blenders and vacuum cleaners.

Step 2: Detailed Explanation:

Let us explore the physics of how it works on both types of supplies.

1. **Operation on DC:** When connected to a DC source, it functions as a standard DC series motor. The unidirectional current produces a steady magnetic field and a steady armature current, resulting in continuous rotational torque in one direction.

2. **Operation on AC:** When connected to an AC source, the current reverses its direction periodically (e.g., 50 times per second for 50Hz).

- In a shunt motor, AC operation is impossible because the field and armature are in parallel and have different inductances, causing their current reversals to be out of phase.

- In a **series motor**, however, because they are in the same circuit, the polarity of the field poles and the direction of the armature current reverse **at exactly the same time**.

- Since torque (T) is proportional to the product of flux and current ($T \propto \phi \cdot I$), reversing *both* signs results in a torque that remains in the same direction ($(-\phi) \cdot (-I) = +T$).

Thus, the motor produces a pulsating but unidirectional torque even on AC.

3. **Modifications for AC:** To make a series motor "Universal," the iron core of the field must be laminated (to reduce eddy current losses from the AC field) and the number of turns in the field is reduced compared to a pure DC motor to minimize inductive reactance.

Because it accepts both power formats, it is widely utilized in devices that may be exported to different regions or used with various power modules.

Step 3: Final Answer:

A universal motor is designed to operate on both AC and DC supplies.

This matches Option (C).

Quick Tip: Keyword: "**Universal**" implies it accepts "**All**" (both common types of supply). It is the only motor that can comfortably switch between a battery (DC) and a wall socket (AC) without any complex electronic converter.

144. In a BLDC motor, the windings are placed on the

- (A) Rotor
- (B) Stator
- (C) Shaft
- (D) Commutator

Correct Answer: (B) Stator

Solution:

Step 1: Understanding the Concept:

BLDC stands for Brushless Direct Current motor. While it is named "DC," its internal operation is actually more similar to a three-phase permanent magnet synchronous motor driven by an inverter.

Traditional DC motors use mechanical "brushes" and a "commutator" to reverse the direction of current in the moving armature windings to maintain rotation.

This mechanical switching has several downsides: sparks, friction, electrical noise, and the need for regular maintenance as brushes wear out.

The BLDC motor eliminates these problems by using "Electronic Commutation." This means the switching is done by a power electronic controller based on the rotor's position (sensed via Hall effect sensors or back EMF).

Step 2: Detailed Explanation:

The physical arrangement of a BLDC motor is the "inverse" of a traditional brushed DC motor.

1. **The Rotor:** In a BLDC motor, the rotor is composed of high-energy permanent magnets. It does not carry any electrical windings. This significantly reduces the rotor's weight and inertia, allowing for faster acceleration and better dynamic response.

2. **The Stator:** The electrical windings (usually three phases) are placed on the **Stator** (the stationary outer part). The stationary coils are energized in a specific sequence by the external controller to create a rotating magnetic field that "pulls" the permanent magnets on the rotor along.

3. **Advantages of Windings on Stator:**

- **Heat Dissipation:** Since the windings (which generate heat due to resistance) are on the outside stationary part, they can be easily cooled via the motor casing or external heat sinks. Heat generated on a moving rotor is much harder to remove.

- **No Moving Contacts:** Because the energized parts are stationary, there is no need for brushes or slip rings to transfer power to the moving part. This eliminates sparking and mechanical wear.

- **Reliability:** The motor can run at much higher speeds and in explosive environments where sparks would be dangerous.

Therefore, the stator is the physical home for the copper windings in any BLDC design.

Step 3: Final Answer:

In a BLDC motor, the windings are placed on the stator.

This matches Option (B).

Quick Tip: To remember the difference:

- **Brushed DC:** Windings rotate (Rotor), Magnets stay (Stator).

- **Brushless (BLDC):** Magnets rotate (Rotor), Windings stay (**Stator**).

Think: "No brushes = No moving wires." If the wires don't move, they must be on the stator.

145. The number of pulses applied to a Stepper motor determines the

(A) Speed of rotation

(B) Torque produced

- (C) Total angle or distance of rotation
(D) Direction of rotation

Correct Answer: (C) Total angle or distance of rotation

Solution:

Step 1: Understanding the Concept:

A stepper motor is a digital actuator that converts discrete electrical pulses into precise mechanical displacements.

Unlike a standard DC or induction motor that rotates continuously when power is applied, a stepper motor moves in fixed, incremental steps.

Each step corresponds to a specific angular displacement known as the "Step Angle" (e.g., 1.8° or 7.5° per step).

This makes stepper motors ideal for open-loop control systems where precise positioning is required without the need for expensive feedback sensors (like in 3D printers, CNC machines, and disk drives).

Step 2: Detailed Explanation:

The operation of a stepper motor is governed by three pulse-related parameters:

1. **The Number of Pulses:** Every single pulse sent by the controller moves the rotor by exactly one step angle. Therefore, if you send 100 pulses to a motor with a 1.8° step angle, the rotor will move exactly 180° . The **total quantity** of pulses directly determines the **final position** or the **total distance** the motor travels.
2. **The Frequency of Pulses:** The rate at which these pulses are delivered determines the **speed** of rotation. High frequency means the steps happen rapidly, resulting in high RPM. Low frequency results in slow, jerky motion.
3. **The Sequence of Pulses:** The order in which the stator phases are energized (ABC vs CBA) determines the **direction** of rotation (clockwise or counter-clockwise).

Since the question specifically asks about the "number of pulses," it is referring to the positioning characteristic. By counting pulses, a microcontroller knows exactly where the motor shaft is located without needing an encoder. This "digital-to-mechanical" translation is the defining feature of the stepper motor's accuracy in distance and angle.

Step 3: Final Answer:

The number of pulses determines the total angle or distance of rotation.
This matches Option (C).

Quick Tip: Stepper Motor Pulse Summary:

- **How Many** pulses? → **Position/Angle**.
- **How Fast** are the pulses? → **Speed**.
- **What Order** are the pulses? → **Direction**.

Always associate the "count" with the "coordinate."

146. Which type of generating station is best suited for peak load operation due to its quick starting ability?

- (A) Nuclear power plant
- (B) Gas turbine power plant
- (C) Thermal power plant
- (D) Solar power plant

Correct Answer: (B) Gas turbine power plant

Solution:

Step 1: Understanding the Concept:

The demand for electricity (load) varies throughout the day. It typically has a "base load" (constant minimum demand) and "peak loads" (short-duration high demands, e.g., during evening hours).

Power plants are selected for these roles based on their operational characteristics and economics.

"Peak load plants" must be capable of starting up very quickly and responding rapidly to sudden fluctuations in demand.

Conversely, "base load plants" usually have high capital costs but low running costs and are kept running continuously because they take a long time to start or shut down.

Step 2: Detailed Explanation:

Let us evaluate each option based on startup time and flexibility:

1. **Nuclear Power Plant:** These are strictly base load plants. Starting a nuclear reactor involves complex safety protocols and thermal stabilizing that can take **days**. They cannot handle peak fluctuations.
2. **Thermal (Coal-fired) Power Plant:** These involve heating large boilers to produce steam. The thermal inertia of the water and the boiler is huge. Starting a thermal plant from a "cold" state takes **several hours** (usually 4 to 8). They are also primarily used for base loads.
3. **Solar Power Plant:** While they can be switched on quickly, their output is weather-dependent and not available at night (when many peaks occur). They are generally not categorized as controllable peak-response units in the traditional grid-management sense.
4. **Gas Turbine Power Plant:** A gas turbine operates like a jet engine. It compresses air, mixes it with fuel, and ignites it to turn the turbine directly. There is no large body of water to boil. Consequently, a gas turbine can go from zero to full capacity in **2 to 5 minutes**. This "quick-start" capability makes them the ideal choice for handling sudden peaks or emergency shortages in the grid. They also have lower capital costs but higher fuel costs, which is economically acceptable since they only run for a few hours a day.

Step 3: Final Answer:

The gas turbine power plant is best suited for peak load operation.

This matches Option (B).

Quick Tip: Mnemonic for Plant Roles:

- **Base Load:** Nuclear, Coal (Cheap running, slow start).
- **Peak Load:** Gas Turbines, Pumped Hydro (Fast start, expensive fuel/flexible).

If you see "Quick Starting," always look for Gas or Hydro!

147. Which of the following is a byproduct of coal combination that can be repurposed by the construction industry?

- (A) Flue gas
- (B) Fly ash
- (C) Nitrogen oxide
- (D) Sludge

Correct Answer: (B) Fly ash

Solution:

Step 1: Understanding the Concept:

Coal-fired thermal power plants generate large amounts of solid and gaseous waste during the combustion process.

Solid waste management is a significant environmental challenge for power utilities.

However, certain waste materials possess physical and chemical properties that make them valuable as raw materials in other sectors, particularly in civil engineering and construction. This practice of "circular economy" reduces the need for landfilling and decreases the consumption of natural resources like clay or cement.

Step 2: Detailed Explanation:

When pulverized coal is burned in a furnace, the mineral impurities (like silica and alumina) melt and fuse into tiny glassy spheres.

As the hot gases rise through the boiler, these particles are carried along and are eventually captured by "Electrostatic Precipitators" (ESP) before the gases exit the chimney. This captured fine powder is called **Fly Ash**.

Properties of Fly Ash that benefit the construction industry:

1. **Pozzolanic Property:** Fly ash reacts with calcium hydroxide (produced during cement hydration) to form additional cementitious compounds. This increases the ultimate strength and durability of concrete.
2. **Workability:** The spherical shape of fly ash particles acts like "ball bearings," making the concrete mix easier to pump and pour.
3. **Brick Manufacturing:** Fly ash is a primary ingredient in "Fly Ash Bricks," which are lighter, stronger, and more uniform than traditional clay bricks.
4. **Repurposing:** It is used in road embankments, dams, and as a partial replacement for Portland cement (up to 30% or more).

Other options like Flue gas and Nitrogen oxides are gaseous pollutants that must be treated,

not used. Sludge is typically a wet waste from water treatment or scrubbers and has limited structural utility compared to the dry, versatile fly ash.

Step 3: Final Answer:

Fly ash is the byproduct of coal combination used by the construction industry.

This matches Option (B).

Quick Tip: Keywords: Coal + Construction = Fly Ash.

Fly ash is considered a "green" building material because it saves energy that would otherwise be used to manufacture cement and prevents environmental pollution from power plant waste dumps.

148. If the area under a 24 hour daily load curve is 480 MWh. The average load for that day is

- (A) 10 MW
- (B) 20 MW
- (C) 40 MW
- (D) 480 MW

Correct Answer: (B) 20 MW

Solution:

Step 1: Understanding the Concept:

A "load curve" is a graphical representation of the electrical power demand (in MW) plotted against time (in hours).

The area under this curve has a specific physical meaning: since Power multiplied by Time equals Energy, the area represents the total amount of energy generated or consumed in Megawatt-hours (MWh) during that period.

To find the "Average Load," we need to determine the steady, constant level of power that would have produced the same total energy if it had been maintained for the entire duration of the time interval.

This is a fundamental calculation used to determine the "Load Factor" of a power plant, which measures its operational efficiency.

Step 2: Key Formula or Approach:

The average load is defined as:

$$\text{Average Load} = \frac{\text{Total Energy Generated}}{\text{Total Time Duration}}$$

In this problem, the time duration is explicitly stated as one full day (24 hours).

Step 3: Detailed Explanation:

Let's extract the variables from the problem:

- Area under the curve (Total Energy) = 480 MWh.
- Time period (t) = 24 hours.

Using the formula derived in Step 2:

$$\text{Average Load} = \frac{480 \text{ MWh}}{24 \text{ h}}$$

Perform the division:

$$480 \div 24 = 20$$

The unit of the result is Megawatts (MW) because the 'hours' in the numerator and denominator cancel each other out.

This means that if a power station ran at a perfectly flat output of 20 MW for 24 hours, it would produce exactly the same amount of electricity as a station whose load fluctuated but had an integrated energy total of 480 MWh.

Note that the peak load (maximum height of the curve) must be higher than 20 MW for the average to settle at 20 MW unless the curve was perfectly horizontal. This calculation is vital for grid operators to plan fuel supply and maintenance schedules.

Step 4: Final Answer:

The average load for that day is 20 MW.

This matches Option (B).

Quick Tip: Simple math check: $20 \times 24 = 480$. When solving these in an exam, simply divide the MWh value by 24 (if daily) or 8760 (if yearly) to get the average power in MW. It is just basic arithmetic once you know that **Area = Energy**.

149. Which tariff is most suitable for industrial consumers with a significant reactive power load?

- (A) Flat rate tariff
- (B) Block rate tariff
- (C) Maximum demand tariff
- (D) KVA maximum demand tariff

Correct Answer: (D) KVA maximum demand tariff

Solution:

Step 1: Understanding the Concept:

A "tariff" is the rate schedule at which electrical energy is sold to consumers. Different types of consumers (residential, commercial, industrial) have different load patterns and impact the grid differently.

Industrial loads often consist of many large motors and transformers, which draw significant "Reactive Power" (Q) to maintain magnetic fields.

Reactive power does not perform work but causes current to flow in the transmission lines, increasing heating losses (I^2R) and voltage drops.

Utilities prefer consumers to maintain a high Power Factor (PF) close to unity. To encourage this, they use tariffs that penalize "Apparent Power" (S) consumption rather than just "Real Power" (P) consumption.

Step 2: Detailed Explanation:

Let us analyze the options:

1. **Flat Rate Tariff:** Charges a fixed price per unit of energy (kWh). It does not differentiate between peak times or power factors. It is too simple for complex industrial loads.
2. **Block Rate Tariff:** The price per unit decreases in "blocks" as consumption increases. It rewards high volume but still only measures real energy (kWh), ignoring reactive issues.
3. **Maximum Demand Tariff:** Charges for the actual energy used plus a fixed fee for the maximum kW peak demand reached. While it penalizes high peaks, it still only tracks real power.
4. **KVA Maximum Demand Tariff:** This is a two-part tariff where the demand charge is based on the peak **kVA** (Apparent Power) rather than kW.

- Relationship: $kVA = \frac{kW}{PF}$.

- If an industry has a poor (low) power factor, its kVA value will be very high for the same amount of useful work (kW).

- This forces the industrial consumer to pay more if they have a "significant reactive power load."

By using kVA as the metric, the utility automatically recovers the cost of the extra current and equipment capacity required to serve the inductive load. This provides a direct financial incentive for the factory to install capacitor banks to improve their power factor.

Step 3: Final Answer:

The KVA maximum demand tariff is most suitable for such consumers.

This matches Option (D).

Quick Tip: To remember this: "V" stands for "Vars" or "Volts". Whenever you see kVA, think of **Power Factor** and **Reactive Power**. Utilities use kVA to make sure they are paid for the total current they have to provide, not just the "active" part.

150. A factory has a load of 400 kW at a power factor of 0.8 lagging. If the power factor is to be improved to unity, the required KVAR of the capacitor bank is

- (A) 400 KVAR
- (B) 300 KVAR
- (C) 500 KVAR
- (D) 100 KVAR

Correct Answer: (B) 300 KVAR

Solution:

Step 1: Understanding the Concept:

Power factor improvement is the process of neutralizing the lagging reactive power drawn by inductive loads using capacitive devices.

Inductive loads (like motors) consume "Lagging" reactive power (Q_L). Capacitors provide "Leading" reactive power (Q_C).

By placing a capacitor bank in parallel with the load, the capacitor supplies the reactive current locally, so the source only has to provide the real power (P).

If the target is "unity power factor," it means we want the final reactive power to be exactly zero. Therefore, the capacitor bank must produce exactly the same amount of KVAR as the initial inductive load was drawing.

Step 2: Key Formula or Approach:

1. Power Triangle relationships:

- Real Power $P = S \cos \phi$
- Reactive Power $Q = P \tan \phi$

2. Given $\cos \phi_1 = 0.8$.

3. Since $\cos^2 \phi + \sin^2 \phi = 1$, if $\cos \phi = 0.8$, then $\sin \phi = 0.6$.

4. Therefore, $\tan \phi = \sin / \cos = 0.6/0.8 = 0.75$.

Step 3: Detailed Explanation:

Let's perform the numerical calculation:

- Initial Real Power (P) = 400 kW.
- Initial Power Factor ($\cos \phi_1$) = 0.8.
- Initial Reactive Power (Q_1) = $P \times \tan \phi_1$.

$$Q_1 = 400 \times \left(\frac{0.6}{0.8} \right)$$

$$Q_1 = 400 \times 0.75 = 300 \text{ KVAR}$$

This 300 KVAR is the "magnetizing" power currently being pulled from the grid.

The goal is to reach **Unity Power Factor** ($\cos \phi_2 = 1$).

At unity power factor, the final phase angle $\phi_2 = 0^\circ$, which implies $\tan \phi_2 = 0$.

The final reactive power drawn from the source must be $Q_2 = P \tan \phi_2 = 400 \times 0 = 0 \text{ KVAR}$.

The required capacitor rating is the difference between the initial and final reactive powers:

$$Q_C = Q_1 - Q_2 = 300 - 0 = 300 \text{ KVAR}$$

By installing a 300 KVAR capacitor bank, the factory ensures that the grid only sees a purely resistive load of 400 kW, leading to lower electricity bills and better voltage stability.

Step 4: Final Answer:

The required KVAR of the capacitor bank is 300 KVAR.

This matches Option (B).

Quick Tip: Standard Triangle Trick: The 3-4-5 rule is your best friend.

- If **P** is **400** (4 units) and **cos** is **0.8**,
- Then **Q** must be **300** (3 units) and **S** must be **500** (5 units).

To reach unity, just "buy" a capacitor that equals the existing **Q**. 300 is the answer!

151. Which circuit breaker is preferred for high voltage (up to 765 KV) and extra high voltage system?

(A) Vacuum Circuit Breaker (VCB)

- (B) Air Circuit Breaker
- (C) SF_6 Circuit Breaker
- (D) Miniature Circuit Breaker (MCB)

Correct Answer: (C) SF_6 Circuit Breaker

Solution:

Step 1: Understanding the Concept:

Switchgear design depends heavily on the voltage level and current-interrupting capacity required by the power system.

At Extra High Voltage (EHV) levels, such as 400 kV or 765 kV, the arc produced during the separation of contacts is extremely intense and carries massive energy.

A circuit breaker must be able to quench this arc rapidly and prevent the dielectric restrike of the gap as the voltage recovers.

Sulfur Hexafluoride (SF_6) is an electronegative gas with exceptional insulating and arc-quenching properties.

It has a dielectric strength approximately 2.5 to 3 times that of air and its arc-extinguishing ability is nearly 100 times better than air.

These characteristics make it the industry standard for high-voltage substations.

Step 2: Detailed Explanation:

Let's evaluate the suitability of the options for 765 kV systems:

1. **Miniature Circuit Breaker (MCB):** These are thermal-magnetic devices used for domestic and low-voltage commercial applications (up to 415V or 1000V). Using an MCB for 765,000V is physically impossible due to the tiny insulation gaps.
2. **Air Circuit Breaker (ACB):** While used in medium voltage, the insulation requirements for EHV would make an ACB massive and noisy (due to the blast). They are generally limited to voltages below 15 kV in modern grids.
3. **Vacuum Circuit Breaker (VCB):** VCBs are excellent for medium voltage (11 kV to 33 kV). However, at very high voltages, maintaining a vacuum over long gaps is difficult, and multiple vacuum interrupters would need to be connected in series, increasing complexity and cost.
4. **SF_6 Circuit Breaker:** In these breakers, the contacts open in a pressurized reservoir of SF_6 gas. Because SF_6 is electronegative, it "gobbles up" free electrons in the arc path, converting them into heavy, slow-moving negative ions. This rapidly increases the dielectric

strength of the gap, extinguishing the arc within a few cycles. Its high thermal stability and non-flammable nature allow it to handle the extreme stresses of 765 kV systems with relatively compact dimensions. Most modern outdoor EHV substations use SF_6 gas-insulated switchgear (GIS) or "puffer" type breakers for maximum reliability.

Step 3: Expanded Comparison:

The choice of SF_6 also reduces maintenance because the gas does not degrade significantly during arc quenching.

In contrast, oil circuit breakers require frequent oil changes and pose a fire risk.

For 765 kV, the breaker must handle "Line Charging Currents" and "Transient Recovery Voltages" (TRV) which are much higher than at lower levels.

The high pressure SF_6 gas provides the necessary insulation level within a manageable metallic enclosure.

Therefore, for any system above 132 kV, and specifically up to the 765 kV range, SF_6 technology is the undisputed leader in performance.

Step 4: Final Answer:

The circuit breaker preferred for 765 kV and EHV systems is the SF_6 Circuit Breaker.

This matches Option (C).

Quick Tip: Voltage-Breaker Cheat Sheet:

- **Low V** (<1kV): MCB, MCCB.
- **Medium V** (11-33kV): VCB (indoor), Oil (old).
- **High/Extra High V** (>66kV): SF_6 or Air-Blast (historical).

Think of SF_6 as the "Heavy Duty" solution for the highest voltages.

152. Why the current limiting reactors usually "Air-cored" rather than "Iron-cored"?

- (A) To reduce weight
- (B) To improve cooling
- (C) To avoid magnetic saturation during heavy fault currents

(D) To decrease the cost of iron

Correct Answer: (C) To avoid magnetic saturation during heavy fault currents

Solution:

Step 1: Understanding the Concept:

Current limiting reactors are heavy-duty inductors placed in series with generators, busbars, or feeders.

Their primary purpose is to introduce additional inductive reactance to the circuit during a short-circuit fault, thereby limiting the fault current to a level that the existing circuit breakers can safely interrupt.

Under normal operating conditions, they should have minimal impact on the system voltage.

Under fault conditions, the current can rise to 10 or 20 times the rated value.

The effectiveness of a reactor depends on its inductance remaining constant even at these extreme current levels.

Step 2: Key Formula or Approach:

Inductance (L) of a coil is defined as $L = \frac{N\phi}{I}$.

In an iron-cored inductor, the flux (ϕ) is related to the current through the B-H curve of the iron.

Once the iron reaches its "saturation" point, any further increase in current does not produce a significant increase in flux.

This causes the inductance (L) to drop sharply when it is needed most.

Step 3: Detailed Explanation:

Let us compare the two core types:

1. **Iron-cored Reactors:** These use a ferromagnetic core to concentrate the magnetic field. While this allows for a compact design with fewer turns, iron has a physical limit on how much magnetic flux it can carry. During a severe fault, the current magnitude is massive. The iron core would saturate almost instantly. When saturation occurs, the reactor behaves essentially like a piece of wire with very little inductance. This means the fault current would not be limited, defeating the entire purpose of the device.

2. **Air-cored Reactors:** These use no magnetic material; the coils are wound on non-magnetic concrete or porcelain supports. Since air (or non-magnetic material) does not saturate, the

relationship between flux and current is perfectly linear ($\phi \propto I$). This ensures that even if the fault current is 50 times the rated current, the reactor will provide exactly the same percentage of reactance. The inductance remains stable and predictable throughout the fault duration.

3. **Other Factors:** While air-cored reactors are indeed heavier (Option A) and may be more expensive in terms of copper usage, the overriding engineering requirement is **reliability during a fault**. Cooling (Option B) is actually easier in some iron-cored designs, and iron cost (Option D) is secondary to performance. The "linearity" of the air core is the critical functional advantage.

Step 4: Final Answer:

Air-cored reactors are used to avoid magnetic saturation during heavy fault currents. This matches Option (C).

Quick Tip: Reactor Rule of Thumb:

- **Iron Core** = High efficiency/Compact but **Saturates** (Bad for faults).
 - **Air Core** = Lower efficiency/Bulky but **Linear** (Great for limiting faults).
- Always choose "Avoid Saturation" as the reason for air-core in protection systems.

153. Which type of distance relay is inherently "directional" and does not require an additional directional unit?

- (A) Impedance relay
- (B) Reactance relay
- (C) MHO relay
- (D) Plain offset relay

Correct Answer: (C) MHO relay

Solution:

Step 1: Understanding the Concept:

Distance protection is widely used for transmission line fault detection.

These relays measure the ratio of voltage to current ($Z = V/I$), which represents the impedance of the line from the relay location to the fault.

A major requirement for line protection is **directionality**—the relay should only trip for faults in the protected line (forward direction) and not for faults "behind" it (reverse direction).

Some relays see impedance in all quadrants of the R-X plane (non-directional), while others have operating characteristics restricted to specific regions.

Step 2: Detailed Explanation:

Let's analyze the R-X diagram characteristics of the common distance relays:

1. **Impedance Relay:** Its characteristic on the R-X plane is a circle centered at the origin. It operates whenever the measured impedance falls within this circle. Since the circle covers all four quadrants equally, it will trip for a fault at distance X in front of it OR a fault at distance X behind it. Therefore, it is **non-directional** and needs an extra directional element.
2. **Reactance Relay:** Its characteristic is a horizontal line parallel to the R-axis. It ignores resistance and only looks at the inductive component. Like the impedance relay, it sees faults in both directions (above and below the line) and is inherently non-directional.
3. **MHO Relay (Admittance Relay):** Its characteristic is a circle that **passes through the origin** and lies mostly in the first quadrant (the direction of the protected line). Because the circle originates at $(0, 0)$ and does not extend into the reverse quadrants, the relay cannot see faults behind it. If a reverse fault occurs, the impedance vector points away from the circle. This makes the MHO relay **inherently directional**. No additional sensing units are required to tell the relay which way the current is flowing.
4. **Offset MHO Relay:** This is a variation where the circle is shifted to include the origin, often used for back-up protection, but the standard MHO is the classic "inherently directional" example.

Step 3: Expanded Detail:

MHO relays are preferred for long transmission lines because they are less sensitive to power swings compared to impedance relays.

Their directional property saves space and reduces the cost of the protection panel.

In modern numerical relays, these characteristics are implemented in software, but the "inherent directionality" of the MHO logic remains a fundamental concept taught in power systems.

Step 4: Final Answer:

The MHO relay is inherently directional.

This matches Option (C).

Quick Tip: Phasor Diagram Memory Trick:

- **Impedance Relay:** Circle centered at **Origin** (Non-directional).
- **MHO Relay:** Circle **passing through** the **Origin** (Inherently Directional).

Think: "MHO stays on one side of the line."

154. Bus bar protection is usually not provided for which type of system?

- (A) Extra high voltage systems
- (B) Small distribution substation
- (C) Power house bus bars
- (D) 132 KV substations

Correct Answer: (B) Small distribution substation

Solution:

Step 1: Understanding the Concept:

Busbars are the critical junction points in a power system where multiple incoming and outgoing lines are connected.

A fault on a busbar is extremely severe because it can lead to the shutdown of the entire substation, causing a widespread blackout.

Busbar protection schemes (like Differential protection or Frame-leakage protection) are

complex, expensive, and require many CTs (Current Transformers).

The decision to install dedicated busbar protection is based on a "Cost-Benefit Analysis" and the criticality of the station to the grid stability.

Step 2: Detailed Explanation:

Let's analyze why certain systems require protection more than others:

1. **Extra High Voltage (EHV) and 132 kV Systems (Options A, D):** These substations handle bulk power for entire regions. A bus fault here would destabilize the national grid and cause massive economic damage. Therefore, high-speed, highly reliable busbar protection is **mandatory**.
2. **Power House Bus Bars (Option C):** These connect generators to the transformers. Protecting the generating units is vital. A bus fault here could damage the generators themselves through mechanical shock. Protection is **essential**.
3. **Small Distribution Substation (Option B):** These substations step down voltage for local neighborhoods or small industrial parks. The number of outgoing feeders is limited, and the impact of a bus fault is localized. Installing a sophisticated differential protection scheme for the busbars might cost more than the transformer itself. In these cases, engineers rely on the "Backup Protection" provided by the over-current relays on the incoming transformer or the individual feeders. If a bus fault occurs, the incoming breaker will eventually trip after a small delay. While this results in a short outage for that neighborhood, it is considered an acceptable trade-off for the lower complexity and cost of the substation.

Step 3: Final Answer:

Bus bar protection is usually not provided for small distribution substations.

This matches Option (B).

Quick Tip: In engineering, "Usually not provided" often means "Too expensive for the small scale." If the voltage is low (<33kV) and the area served is small, dedicated bus protection is skipped in favor of simple backup relaying.

155. Which type of lightning arrester is also known as a "Surge Diverter"?

- (A) Rod gap
- (B) Horn gap
- (C) Expulsion type
- (D) Valve type

Correct Answer: (D) Valve type

Solution:

Step 1: Understanding the Concept:

A lightning arrester (or surge arrester) is a protective device used in power systems to shield equipment from high-voltage surges caused by lightning strikes or switching transients.

The device must behave like an open circuit at normal operating voltages but must quickly become a low-resistance path to ground when a high-voltage surge arrives.

This "diverts" the excess energy safely to the earth, preventing it from reaching and damaging expensive equipment like transformers.

Because its primary function is to change its impedance and "divert" the surge, the term "Surge Diverter" is synonymous with lightning arresters.

Step 2: Detailed Explanation:

Let's look at why the "Valve Type" is the definitive Surge Diverter:

1. **Rod Gap and Horn Gap (Options A, B):** These are simple air gaps. When a surge comes, the air breaks down and an arc forms. However, they are poor at quenching the arc once the surge has passed (called the "power follow current"). They are simple but not "sophisticated" diverters.
2. **Expulsion Type (Option C):** This uses a tube that emits gas to "blow out" the arc. It is mostly used for feeder protection but has limitations on how many times it can operate.
3. **Valve Type (Option D):** This is the most efficient and common type. It consists of nonlinear resistor discs (made of silicon carbide or metal oxides like ZnO) in series with a spark gap. These resistors have a "Valve-like" property: at normal voltage, their resistance is extremely high (the valve is closed). When the surge voltage hits, their resistance drops almost to zero (the valve opens wide), allowing the surge current to flow to the ground. As soon as the surge ends and the voltage returns to normal, the resistance instantly becomes high again, "valving

off" the power-follow current. Because it behaves like a controlled valve for electricity, it is the classic "Surge Diverter."

Step 3: Final Answer:

The Valve type lightening arrester is also known as a surge diverter.

This matches Option (D).

Quick Tip: Term Association:

Valve Type = Surge Diverter = Non-linear Resistor.

Think of a **check-valve** in plumbing that only opens when pressure is too high; that is exactly how this electrical device works!

156. If the radius of a conductor in a transmission line is doubled (keeping the spacing constant), the inductance of the line will

- (A) Increase
- (B) Decrease
- (C) Remains unchanged
- (D) Double

Correct Answer: (B) Decrease

Solution:

Step 1: Understanding the Concept:

Inductance in a transmission line arises due to the magnetic flux linkage between the conductors. The total inductance of a line depends on the physical dimensions of the conductors and the distance between them.

If we have a single-phase line with two conductors of radius r separated by distance d , the inductance is determined by the ratio of the spacing to the effective radius.

Understanding how these geometric parameters affect electrical properties is vital for

calculating line impedance and power transfer limits.

Step 2: Key Formula or Approach:

The formula for the loop inductance (L) of a single-phase transmission line is:

$$L = 4 \times 10^{-7} \ln\left(\frac{d}{r'}\right) \text{ H/m}$$

Where:

- d is the distance between conductor centers.
- r' is the effective radius ($r' = 0.7788r$).

From this logarithmic relationship, we see that $L \propto -\ln(r)$.

Step 3: Detailed Explanation:

Let's analyze the mathematical effect of doubling the radius:

Suppose the initial radius is r and the new radius is $R = 2r$.

The original inductance is proportional to $\ln(d/0.7788r)$.

The new inductance is proportional to $\ln(d/0.7788 \cdot (2r))$.

Mathematically:

$$\ln\left(\frac{d}{2 \cdot r'}\right) = \ln\left(\frac{d}{r'}\right) - \ln(2)$$

Since $\ln(2)$ is a positive value (≈ 0.693), the second term reduces the overall value of the natural log.

Physically, as the conductor gets "fatter" (increased radius), the current is spread over a larger cross-section. This leads to a decrease in the internal flux linkage of the conductor and also effectively brings the "average" current paths in the two conductors slightly closer in terms of logarithmic distance.

Therefore, increasing the radius of the conductor **decreases** the inductance of the line.

Conversely, increasing the spacing d between the lines would **increase** the inductance. This is why bundled conductors (which increase the effective radius) are used to reduce line inductance and increase the Power Transfer Capability.

Step 4: Final Answer:

The inductance of the line will decrease.

This matches Option (B).

Quick Tip: Proportionality Trick:

- L (Inductance) $\propto \ln(\text{Spacing}/\text{Radius})$.

- C (Capacitance) $\propto 1/\ln(\text{Spacing}/\text{Radius})$.

If you make the wire **Bigger** (Radius up), you get **Less Inductance** and **More Capacitance**.

157. The sending end voltage of a line is 33 KV and the receiving end voltage is 30 KV. The voltage regulation of the line is

- (A) 3%
- (B) 5%
- (C) 10%
- (D) 11%

Correct Answer: (C) 10%

Solution:**Step 1: Understanding the Concept:**

Voltage regulation is a measure of the voltage drop that occurs in a transmission line due to its internal resistance and reactance.

It is defined as the change in voltage at the receiving end when the load is changed from zero to full load, keeping the sending end voltage constant.

A lower voltage regulation is desirable as it indicates that the voltage delivered to consumers remains stable despite variations in load.

In simple problems where the no-load voltage is assumed to be equal to the sending end voltage, the formula compares the difference between the two ends to the receiving end voltage.

Step 2: Key Formula or Approach:

The percentage Voltage Regulation (%VR) is calculated as:

$$\% \text{ Regulation} = \frac{V_s - V_r}{V_r} \times 100$$

Where:

- V_s = Sending end voltage.
- V_r = Receiving end voltage.

Step 3: Detailed Explanation:

Let's extract the given numerical data:

- $V_s = 33 \text{ kV}$
- $V_r = 30 \text{ kV}$

Now, substitute these values into the regulation formula:

$$\% \text{ Regulation} = \frac{33 - 30}{30} \times 100$$

Subtract the voltages:

$$\% \text{ Regulation} = \frac{3}{30} \times 100$$

Simplify the fraction:

$$\frac{3}{30} = \frac{1}{10} = 0.1$$

Convert to percentage:

$$0.1 \times 100 = 10\%$$

This means that for every 100 volts sent, only 90 volts are received, representing a 10% loss in voltage magnitude along the line. For a 33 kV system, a 10% regulation is relatively high;

typically, utility companies aim for regulation below 5% or 6% to ensure appliance safety. This drop is purely caused by the "impedance" of the transmission conductors carrying the current.

Step 4: Final Answer:

The voltage regulation of the line is 10%.

This matches Option (C).

Quick Tip: Common Mistake Warning: Always divide by the **Receiving End Voltage** (V_r) in the denominator, not the Sending End.

If you divided 3 by 33, you would get $\approx 9\%$, which is a common distractor. Always think: "Regulation is how much I **lost** relative to what I **got**."

158. A long transmission line is energized at the sending end but kept open circuited at the receiving end. If the sending end voltage is 100 KV, the receiving end voltage is likely to be

- (A) 90 KV
- (B) 100 KV
- (C) 105 KV
- (D) 0 KV

Correct Answer: (C) 105 KV

Solution:

Step 1: Understanding the Concept:

A phenomenon unique to long, high-voltage transmission lines under no-load or light-load conditions is the **Ferranti Effect**.

Normally, we expect voltage to drop as we move away from the source due to line resistance and inductance. However, in long lines, the shunt capacitance between the wires and the ground is significant.

When the line is open-circuited at the far end, a small current called the "Line Charging

Current" still flows through the line to charge this distributed capacitance.

This leading current flows through the series inductance of the line, creating a voltage rise across the inductor that is in phase with the sending end voltage.

As a result, the voltage at the receiving end actually becomes **higher** than the voltage at the sending end.

Step 2: Detailed Explanation:

1. **Physical Mechanism:** The charging current I_c is purely capacitive (leading voltage by 90 degrees). The voltage drop across the line inductance L is $V_L = I_c \cdot jX_L$. Multiplying a leading current by a positive imaginary reactance results in a voltage vector that points in the same direction as the sending voltage, effectively "adding" to it.

2. **Numerical analysis:** Given $V_s = 100$ kV.

- Option (A) 90 kV: This would be the result for a **loaded** line with a lagging power factor.
- Option (B) 100 kV: This would only happen if the line had zero capacitance or inductance (an ideal short line).
- Option (C) 105 kV: This shows a voltage **increase**. For a "long" line as specified, the Ferranti effect will definitely cause the far-end voltage to rise above the source voltage. 105 kV is a realistic representative value for such a rise.
- Option (D) 0 kV: Incorrect. Open circuit voltage is not zero; it is the potential measured at the open terminals.

This effect is dangerous because it can stress the insulation of equipment at the receiving end. Utilities use "Shunt Reactors" to absorb this excess capacitive power and pull the voltage back down during no-load periods.

Step 3: Final Answer:

The receiving end voltage is likely to be 105 KV due to the Ferranti effect.

This matches Option (C).

Quick Tip: Ferranti Effect Keyword: **Long line + No load = Voltage Rise.**

Whenever you see "Sending voltage" and "Open circuit", the answer for "Receiving voltage" must be **greater** than the sending voltage.

159. What happens to the corona effect if the spacing between conductors is increased?

- (A) Corona increases
- (B) Corona decreases
- (C) Corona is absent
- (D) No effect

Correct Answer: (B) Corona decreases

Solution:

Step 1: Understanding the Concept:

Corona discharge is a luminous, audible electrical discharge that occurs when the potential gradient (electric field intensity) at the surface of a conductor exceeds the dielectric strength of the surrounding air ($\approx 30 \text{ kV/cm}$ peak).

It results in power loss, radio interference, and the production of ozone gas.

The onset of corona is defined by the **Disruptive Critical Voltage** (V_c). If the operating voltage is above V_c , corona occurs. To reduce corona, we want to **increase** the value of V_c so that it stays well above our operating voltage.

Step 2: Key Formula or Approach:

The formula for Disruptive Critical Voltage (V_c) is:

$$V_c = m_0 g_0 \delta r \ln\left(\frac{d}{r}\right) \text{ kV/phase}$$

Where:

- r is the radius of the conductor.
- d is the spacing between conductors.
- g_0 is the dielectric strength of air.

From this formula, we see that $V_c \propto \ln(d)$.

Step 3: Detailed Explanation:

Let's analyze the effect of increasing the spacing d :

1. **Electric Field Distribution:** When conductors are close together, the lines of electric force are highly concentrated in the space between them. This creates a high potential gradient at the conductor surface.
2. **Effect of Spacing:** As the distance d between the conductors is increased, the electric field becomes more "relaxed" or spread out. The potential gradient on the surface of the wires **decreases** for the same applied voltage.
3. **Critical Voltage Impact:** Mathematically, as d increases, the term $\ln(d/r)$ increases. This directly **increases the Disruptive Critical Voltage** (V_c).
4. **Result:** Since it now requires a much higher voltage to trigger the ionization of air, the "Corona Effect" (and the power loss associated with it) significantly **decreases** at the current operating voltage.

However, increasing spacing significantly increases the cost of supporting towers and insulation arms, so designers balance this by using bundled conductors to increase the "effective radius" (r) instead.

Step 4: Final Answer:

The corona effect decreases if the spacing between conductors is increased.
This matches Option (B).

Quick Tip: Corona Reduction Tactics:

1. Increase **Radius** (Bundled conductors - Best way).
 2. Increase **Spacing** (tower geometry).
 3. Smooth conductor surfaces (No dirt/scratches).
- Any action that spreads the electric field will **decrease** corona.

160. The converter used at the sending end of an HVDC system acts as a

- (A) Inverter
- (B) Rectifier
- (C) Cycloconverter
- (D) Chopper

Correct Answer: (B) Rectifier

Solution:

Step 1: Understanding the Concept:

High Voltage Direct Current (HVDC) is used for long-distance power transmission and underwater cables because it has lower losses and better stability than AC over vast distances. However, our power grid generates electricity as AC and our homes consume electricity as AC. Therefore, an HVDC system is essentially an "AC-DC-AC" link. It requires large power electronic conversion stations at both ends.

The "Sending End" is where the power enters the DC line from the AC grid, and the "Receiving End" is where it exits the DC line back into the local AC grid.

Step 2: Detailed Explanation:

Let's define the roles of converters at each end:

1. **Sending End Station:** At this point, we have alternating current from the power plant. We need to convert this AC into High Voltage DC for transmission. The process of converting AC to DC is called **Rectification**. Therefore, the converter station at the sending end operates as a **Rectifier**.

2. **Receiving End Station:** At the other end of the line, we have HVDC. This must be converted back into AC to sync with the local power grid. The process of converting DC to AC is called **Inversion**. Therefore, the converter station at the receiving end operates as an **Inverter**.

3. Other Options:

- **Option C (Cycloconverter):** This converts AC directly to AC at a different frequency. It is not used for HVDC links.

- **Option D (Chopper):** This converts DC to DC at a different voltage level. While used in DC drives, it is not the name for the main HVDC terminal unit.

In modern HVDC (like VSC-HVDC), the same physical power module can act as either a rectifier or an inverter depending on the direction of power flow, but the functional role at the sending end remains rectification.

Step 3: Final Answer:

The converter at the sending end acts as a rectifier.

This matches Option (B).

Quick Tip: HVDC Flow Mnemonic:

Sending → Rectifier (S-R: Start as Rectifier).

Receiving → Inverter (R-I: Return as Inverter).

Think: AC → DC (**Rectify**) → Cable → DC → AC (**Invert**).

161. If the length of the span between two level supports is increased by 20%, the sag will increase by approximately

- (A) 20%
- (B) 40%
- (C) 44%
- (D) 50%

Correct Answer: (C) 44%

Solution:

Step 1: Understanding the Concept:

Sag is defined as the difference in level between the points of support and the lowest point on the conductor of a transmission line.

The provision of adequate sag is critical in power system engineering because it determines the clearance between the conductor and the ground, as well as the mechanical tension in the wire.

If a conductor is stretched too tightly (low sag), the tension might exceed the breaking strength of the material, especially during cold weather or high winds.

Conversely, excessive sag (high span) can lead to short circuits between phases or between the conductor and trees/ground objects.

When a conductor is suspended between two supports at the same level, it takes the shape of a catenary.

However, for relatively small sags, the curve is approximated as a parabola to simplify calculations.

The most important relationship to understand here is how the physical geometry of the span (l) impacts the vertical displacement (S).

This is a non-linear relationship where the distance between towers has a squared effect on the sag.

Step 2: Key Formula or Approach:

The standard formula for the sag (S) of a conductor supported at equal levels is:

$$S = \frac{w \cdot l^2}{8 \cdot T}$$

Where:

- w is the weight per unit length of the conductor.
- l is the length of the span (distance between supports).
- T is the horizontal tension in the conductor.

From this formula, we can deduce the proportionality: $S \propto l^2$.

If we change the span from l_1 to l_2 , the new sag S_2 can be calculated relative to the old sag S_1 as:

$$\frac{S_2}{S_1} = \left(\frac{l_2}{l_1} \right)^2$$

Step 3: Detailed Explanation:

Let the initial length of the span be l_1 .

The problem states that the length is increased by 20%.

Therefore, the new length $l_2 = l_1 + 20\% \text{ of } l_1 = 1.2 \cdot l_1$.

Now, let us find the ratio of the new sag to the original sag using the proportionality derived in Step 2:

$$S_2 = S_1 \cdot \left(\frac{1.2 \cdot l_1}{l_1} \right)^2$$

$$S_2 = S_1 \cdot (1.2)^2$$

Calculating the square of 1.2:

$$1.2 \times 1.2 = 1.44$$

So, $S_2 = 1.44 \cdot S_1$.

To find the percentage increase in sag, we use the standard formula:

$$\% \text{ Increase} = \frac{S_2 - S_1}{S_1} \times 100$$

$$\% \text{ Increase} = \frac{1.44 \cdot S_1 - S_1}{S_1} \times 100$$

$$\% \text{ Increase} = (1.44 - 1) \times 100$$

$$\% \text{ Increase} = 0.44 \times 100 = 44\%$$

This calculation demonstrates that sag is extremely sensitive to span length.

A relatively modest 20% increase in the distance between towers results in a much larger 44% increase in sag.

This sensitivity is a major factor in transmission line design, where tower heights must be carefully calculated based on the maximum possible span to ensure safety clearances are never violated even under maximum ice loading or peak summer temperatures which expand the conductor further.

Step 4: Final Answer:

The sag will increase by approximately 44%.

This matches Option (C).

Quick Tip: Remember the **Square Law** for sags: $S \propto l^2$.

If the length is scaled by a factor k , the sag is scaled by k^2 .

In this case, $1.2^2 = 1.44$, which means a **44% increase**.

If the length was doubled (x2), the sag would increase by 4 times (300% increase).

162. A string of 3 discs can withstand a total voltage of 25.76 KV. If each unit can individually withstand a maximum of 11 KV. Then the string efficiency is

- (A) 68.75%
- (B) 65%
- (C) 78%
- (D) 85%

Correct Answer: (C) 78%

Solution:

Step 1: Understanding the Concept:

High-voltage overhead transmission lines use suspension insulators consisting of a "string" of individual porcelain or glass discs.

Ideally, we would want the total line-to-ground voltage to be distributed equally across every disc in the string.

However, in reality, each disc has a self-capacitance, and there is also a "shunt capacitance" between the metal fittings of the discs and the grounded transmission tower.

Because of this shunt capacitance, the charging current is not the same through all discs.

The disc closest to the energized conductor carries the maximum current and therefore experiences the highest voltage stress.

As we move up the string toward the grounded cross-arm, the voltage across each disc decreases.

"String Efficiency" is a mathematical measure of how uniformly the total voltage is distributed across the insulators.

A 100% efficiency would mean perfect distribution, while lower values indicate that the bottom disc is being dangerously overstressed compared to the others.

Step 2: Key Formula or Approach:

The formula for String Efficiency (η) is:

$$\text{String Efficiency} = \frac{\text{Total Voltage across the string}}{n \times \text{Voltage across the unit nearest to the conductor}} \times 100$$

Where:

- n is the number of discs in the string.
- V_{string} is the total potential difference.
- V_n is the maximum voltage handled by the most stressed unit (the bottom one).

Step 3: Detailed Explanation:

Let's extract the data from the problem:

- Number of discs (n) = 3.
- Total string voltage (V_{string}) = 25.76 kV.
- Maximum voltage on the unit nearest to the conductor (V_{max}) = 11 kV.

Note: The problem states each unit can *individually* withstand 11 kV. In string efficiency problems, we use the voltage across the most stressed unit (nearest the line) as the denominator component, as this unit determines the ultimate breakdown limit of the entire string.

Plugging the values into the formula:

$$\eta = \frac{25.76}{3 \times 11} \times 100$$

First, calculate the denominator:

$$3 \times 11 = 33 \text{ kV}$$

This "33 kV" represents the theoretical maximum voltage the string *could* have handled if the distribution were perfectly uniform (11 kV on each of the 3 discs).

Now, find the ratio:

$$\eta = \frac{25.76}{33} \times 100$$

Performing the division:

$$25.76 \div 33 \approx 0.7806$$

$$\eta \approx 78.06\%$$

Rounding to the nearest whole number given in the options, we get 78%.

This efficiency indicates that the shunt capacitance is having a significant effect, making the bottom disc work at its full capacity of 11 kV while the other two discs are carrying significantly less than their share. Engineers improve this efficiency by using guard rings or grading the capacitance of the discs.

Step 4: Final Answer:

The string efficiency is 78%.

This matches Option (C).

Quick Tip: To remember the formula: Efficiency = (What we actually have) / (What we would have if all were like the best one).

The "best one" (most stressed) is always the disc nearest the wire.

Always multiply the denominator by the number of discs (n)!

163. A 10 km long cable has an insulation resistance of 2 MΩ. What will be the insulation resistance for a 50 km length of the same cable?

- (A) 10 MΩ
- (B) 0.4 MΩ
- (C) 0.2 MΩ
- (D) 1 MΩ

Correct Answer: (B) 0.4 MΩ

Solution:

Step 1: Understanding the Concept:

This problem highlights a very important and often counter-intuitive distinction between "Conductor Resistance" and "Insulation Resistance."

For a normal conductor (the copper wire inside the cable), the resistance increases as the length increases ($R \propto L$) because the electrons have a longer path to travel.

However, insulation resistance (R_i) refers to the resistance offered by the insulating material to the "leakage current" flowing radially from the core to the outer sheath.

When you increase the length of a cable, you are effectively adding more insulating material in **parallel** for the leakage current to pass through.

Think of it this way: a longer cable provides a "wider path" or more surface area for electricity to leak out. In electrical circuits, adding paths in parallel always **decreases** the total resistance. Therefore, the longer the cable, the **lower** its insulation resistance will be.

Step 2: Key Formula or Approach:

The insulation resistance (R_i) of a single-core cable is given by:

$$R_i = \frac{\rho}{2\pi L} \ln\left(\frac{R_{outer}}{r_{core}}\right)$$

Where:

- ρ is the resistivity of the insulation.
- L is the length of the cable.
- R_{outer} and r_{core} are geometric constants of the cable.

From this, we see the fundamental proportionality: $R_i \propto \frac{1}{L}$.

This leads to the relationship for two different lengths:

$$R_1 \cdot L_1 = R_2 \cdot L_2$$

Step 3: Detailed Explanation:

Let's apply the given data:

- Initial length (L_1) = 10 km.
- Initial insulation resistance (R_1) = 2 MΩ.
- Final length (L_2) = 50 km.
- We need to find the final insulation resistance (R_2).

Using the inverse proportionality relationship:

$$R_2 = R_1 \times \left(\frac{L_1}{L_2} \right)$$

$$R_2 = 2 \text{ M}\Omega \times \left(\frac{10 \text{ km}}{50 \text{ km}} \right)$$

Simplify the ratio:

$$\frac{10}{50} = \frac{1}{5} = 0.2$$

Calculate the final value:

$$R_2 = 2 \times 0.2 = 0.4 \text{ M}\Omega$$

The result shows that as the cable length increased by a factor of 5, the insulation resistance dropped to one-fifth of its original value.

This is a critical consideration for underground power networks; very long cable runs may have high cumulative leakage currents that can trip sensitive earth-fault relays even if the insulation material itself is high quality.

Always remember that for insulation, "more is less" when it comes to length.

Step 4: Final Answer:

The insulation resistance for 50 km will be 0.4 MΩ.

This matches Option (B).

Quick Tip: The "Golden Rule" for Cable Resistance:

- **Conductor** (Copper) Resistance $\propto$ **Length**.

- **Insulation** Resistance $\propto$ **1 / Length**.

In exams, if the length increases, your insulation resistance answer **must** be smaller than the starting value. This helps rule out distractors like Option (A).

164. In a substation Single Line Diagram (SLD), which equipment is typically placed closest to the incoming line?

- (A) Circuit breaker
- (B) Current transformer
- (C) Surge arrester
- (D) Power transformer

Correct Answer: (C) Surge arrester

Solution:**Step 1: Understanding the Concept:**

A Single Line Diagram (SLD) is a simplified notation for representing a three-phase power system.

In a substation, the sequence of equipment from the incoming high-voltage transmission line to the outgoing feeders is determined by **Protection Hierarchy** and **System Safety**.

The most unpredictable and dangerous threats to a substation are atmospheric surges (lightning strikes) and switching transients.

These high-voltage surges travel along the transmission line and hit the substation at nearly the speed of light.

If these surges reach expensive and delicate equipment like power transformers or circuit breakers, they can cause permanent insulation failure and catastrophic explosions. Therefore, the layout is designed so that the "first line of defense" captures and diverts these surges to the ground before they enter the main station yard.

Step 2: Detailed Explanation:

Let's follow the typical incoming path in an outdoor substation:

1. **Surge Arrester (Lightning Arrester):** This is the very first piece of equipment encountered by the incoming line. It is connected between the line and the earth. Its job is to provide a low-resistance path to the ground for any high-voltage surge, thereby "clamping" the voltage to a safe level for the rest of the station. By placing it closest to the line entrance, we ensure the maximum protection zone for the entire substation.
2. **Isolator/Disconnect Switch:** Placed after the arrester to provide a visible physical gap for maintenance safety.
3. **Current Transformer (CT) and Potential Transformer (PT):** Used for measurement and relaying logic.
4. **Circuit Breaker (CB):** This is the "switch" that can interrupt fault currents. It is a very expensive component and must be protected by the surge arrester.
5. **Power Transformer:** This is the "heart" of the substation and usually the most expensive single item. It is located deep within the protection scheme, shielded by the arrester, the circuit breaker, and various relaying systems.

Options (A), (B), and (D) represent equipment that relies on the Surge Arrester for protection. If you placed a circuit breaker before the arrester, a lightning strike could destroy the breaker's insulation before the arrester had a chance to work.

Step 3: Final Answer:

The equipment typically placed closest to the incoming line is the Surge Arrester (also known as a Lightning Arrester).

This matches Option (C).

Quick Tip: Think of the Surge Arrester as the "**Shield**" and the rest of the station as the "**Castle**".

You always put the shield **in front** of the gates. Lightning strikes coming from outside must hit the shield first.

Sequence to remember: Line → Arrester → Isolator → Breaker → Transformer.

165. A single phase AC distributor has a total resistance of $0.2\ \Omega$ and a reactance of $0.3\ \Omega$. If the load current is 100 A at 0.8 p.f lagging. The approximate voltage drop is

- (A) 16 V
- (B) 34 V
- (C) 50 V
- (D) 25 V

Correct Answer: (B) 34 V

Solution:

Step 1: Understanding the Concept:

In power distribution, calculating the voltage drop is essential to ensure that the voltage at the consumer's premises stays within statutory limits (typically $\pm 6\%$).

For a Direct Current (DC) system, the voltage drop is simply $I \times R$.

However, in an Alternating Current (AC) system, the impedance (Z) consists of both Resistance (R) and Reactance (X).

Furthermore, the voltage drop depends not just on the magnitude of the current, but also on the **phase relationship** (Power Factor) between the voltage and current.

For a lagging load (like motors), the inductive reactance of the line adds to the resistive drop.

For leading loads, it can actually reduce the total drop.

The "approximate voltage drop" formula is derived from the phasor diagram of the distributor, assuming the sending and receiving voltages are nearly in phase.

Step 2: Key Formula or Approach:

The approximate voltage drop (V_{drop}) for a single-phase AC line is given by:

$$V_{drop} = I \cdot (R \cos \phi + X \sin \phi)$$

Where:

- I is the load current.
- R is the total resistance of the line (loop resistance).
- X is the total reactance of the line (loop reactance).
- $\cos \phi$ is the power factor.
- $\sin \phi$ is the reactive factor.

Step 3: Detailed Explanation:

Let's extract and calculate the required parameters:

- Current (I) = 100 A.
- Resistance (R) = 0.2 Ω .
- Reactance (X) = 0.3 Ω .
- Power Factor ($\cos \phi$) = 0.8.

First, find $\sin \phi$:

Since $\sin^2 \phi + \cos^2 \phi = 1$:

$$\sin \phi = \sqrt{1 - 0.8^2} = \sqrt{1 - 0.64} = \sqrt{0.36} = 0.6$$

Now, substitute all values into the voltage drop formula:

$$V_{drop} = 100 \cdot (0.2 \times 0.8 + 0.3 \times 0.6)$$

Calculate the individual products inside the parentheses:

$$0.2 \times 0.8 = 0.16$$

$$0.3 \times 0.6 = 0.18$$

Sum the internal terms:

$$0.16 + 0.18 = 0.34$$

Multiply by the current:

$$V_{drop} = 100 \times 0.34 = 34 \text{ V}$$

This means if the sending end voltage is 230 V, the consumer would receive approximately $230 - 34 = 196 \text{ V}$. Such a significant drop suggests that the distributor is either too long or the conductor size is too small for a 100A load. The use of $R \cos \phi$ and $X \sin \phi$ correctly accounts for the real and reactive components of the impedance and their interaction with the load's phase angle.

Step 4: Final Answer:

The approximate voltage drop is 34 V.

This matches Option (B).

Quick Tip: Mnemonic for AC Drop: **IRcos + IXsin**.

Standard PF Pair: If **cos** is **0.8**, then **sin** is **0.6**. If **cos** is **0.6**, then **sin** is **0.8**.

These numbers appear in almost 90% of electrical engineering exam questions. Memorizing this pair saves you from using a calculator for the square root.

166. Which of the following is the standard electrification system used by Indian railways for mainline trains?

- (A) 1500 V DC
- (B) 33 KV AC, 50 Hz
- (C) 25 KV AC, 50 Hz
- (D) 11 KV DC

Correct Answer: (C) 25 KV AC, 50 Hz

Solution:

Step 1: Understanding the Concept:

Railway electrification involves providing electrical energy to locomotives (trains) using an overhead catenary system or a third rail.

Historically, many different standards were used globally, including low-voltage DC (for suburban metros) and various AC frequencies.

The choice of an electrification system is a trade-off between transmission efficiency, the weight of the on-board transformer/equipment, and the cost of substations.

High-voltage AC is preferred for mainline routes because it allows for very high power delivery with low current, which minimizes I^2R losses in the long overhead wires.

It also requires fewer substations (spaced 40-60 km apart) compared to DC systems (spaced 10-20 km apart).

Step 2: Detailed Explanation:

Let's analyze the evolution of Indian Railways (IR) electrification:

1. **Early Systems (1500 V DC):** Initially, IR used 1500 V DC for the Mumbai and Chennai suburban regions. However, this required very thick, heavy overhead wires to carry the high currents and frequent substations. IR has recently converted almost all these lines to AC.

2. **Standard Mainline (25 KV AC, 50 Hz):** Since the late 1950s, Indian Railways adopted the 25 kV, single-phase, 50 Hz AC system as the national standard for mainline electrification. Power is taken from the 132/220 kV utility grid and stepped down to 25 kV at railway traction substations. The "Return Path" for the current is provided by the rails and the earth.

3. **On-board Conversion:** Modern AC locomotives use a transformer to step down the 25 kV to a lower voltage, which is then rectified and inverted to drive powerful 3-phase induction motors. This system provides the best efficiency for the vast distances covered by the Indian rail network.

4. Why not others?

- **33 kV AC (Option B):** This is a distribution voltage but not a standard international traction voltage.

- **11 kV DC (Option D):** Does not exist in traction; DC systems are generally capped at 3000 V due to insulation and switching challenges.

Step 3: Final Answer:

The standard system is 25 KV AC, 50 Hz.

This matches Option (C).

Quick Tip: Keyword for Traction: **25 kV AC**.

Most mainline trains around the world, and nearly 100% of new Indian mainline electrification, use this specific voltage. Suburban metros often use 750V or 1500V DC (Third Rail), but mainline is always **High Voltage AC**.

167. The distance between 2 stops is 5 km. the average speed is 50 km/h and the stop time is 2 minutes. Find the scheduled speed.

- (A) 37.5 km/h
- (B) 42.5 km/h
- (C) 30 km/h
- (D) 50 km/h

Correct Answer: (A) 37.5 km/h

Solution:**Step 1: Understanding the Concept:**

In electric traction and transportation engineering, there are two important speed definitions:

1. **Average Speed (V_{avg}):** This is the speed calculated using only the actual **running time** (t_r) from the moment the train starts moving to the moment it stops at the next station.
2. **Scheduled Speed (V_s):** This is the speed calculated using the **total time** assigned in the timetable, which includes both the running time and the **duration of the stop** (t_s) at the station.

Mathematically, for a fixed distance D :

$$\text{Scheduled Speed} = \frac{\text{Distance}}{\text{Actual Running Time} + \text{Stop Time}}$$

Since scheduled speed includes the "idle" time of the stop, it will **always** be lower than the average speed.

Step 2: Key Formula or Approach:

1. Find Running Time (T_r): $T_r = \frac{\text{Distance}}{\text{Average Speed}}$.
2. Find Total Scheduled Time (T_s): $T_s = T_r + \text{Stop Time}$.
3. Calculate Scheduled Speed: $V_s = \frac{\text{Distance}}{T_s}$.

Step 3: Detailed Explanation:

Let's convert all units to hours for consistency:

- Distance (D) = 5 km.
- Average Speed (V_{avg}) = 50 km/h.
- Stop Time (t_{stop}) = 2 minutes = $2/60 = 1/30$ hours.

First, calculate the actual time the train is in motion (Running Time):

$$T_r = \frac{D}{V_{avg}} = \frac{5}{50} = 0.1 \text{ hours}$$

Converting this to minutes: $0.1 \times 60 = 6$ minutes.

Now, calculate the total scheduled time (Time from start to next start):

$$T_{total} = \text{Running Time} + \text{Stop Time}$$

$$T_{total} = 6 \text{ min} + 2 \text{ min} = 8 \text{ minutes}$$

Convert the total time back to hours to find speed in km/h:

$$T_{total} = \frac{8}{60} = \frac{2}{15} \text{ hours}$$

Finally, calculate the Scheduled Speed:

$$V_s = \frac{D}{T_{total}} = \frac{5}{2/15} = \frac{5 \times 15}{2}$$

$$V_s = \frac{75}{2} = 37.5 \text{ km/h}$$

This result confirms that the "wasted" 2 minutes at the station reduces the effective speed of the journey from 50 km/h down to 37.5 km/h. Improving scheduled speed requires either increasing the train's top speed/acceleration or minimizing the duration of station stops.

Step 4: Final Answer:

The scheduled speed is 37.5 km/h.

This matches Option (A).

Quick Tip: Logic Check: Scheduled speed **must** be lower than average speed.

If avg speed is 50, you can immediately eliminate Option (D).

Calculation Hack: 6 min running + 2 min stop = 8 min total.

The speed reduces by the ratio of times: $50 \times (6/8) = 50 \times 0.75 = 37.5$.

168. The unit of Specific Energy Consumption (SEC) is

- (A) Watt-hours
- (B) kWh/km
- (C) Watt hours/ (tonne-km)
- (D) Joules/Tonne

Correct Answer: (C) Watt hours/ (tonne-km)

Solution:

Step 1: Understanding the Concept:

Specific Energy Consumption (SEC) is the most important yardstick for measuring the efficiency of an electric traction system.

It represents the total electrical energy consumed by a train to move a unit weight of the train over a unit distance.

If a system has a high SEC, it is considered inefficient, as it uses more electricity for the same amount of transport work. Factors that increase SEC include frequent stops (energy lost in braking), steep gradients, high train weight, and poor aerodynamic design.

Conversely, SEC is reduced by regenerative braking, where the motor acts as a generator during deceleration and pumps energy back into the grid.

Step 2: Key Formula or Approach:

The mathematical definition of SEC is:

$$\text{SEC} = \frac{\text{Total energy consumed in Watt-hours}}{\text{Weight of train in tonnes} \times \text{Distance in km}}$$

Units involved:

- Energy: Watt-hours (Wh) or Kilowatt-hours (kWh).
- Mass: Tonnes (t).
- Distance: Kilometers (km).

Step 3: Detailed Explanation:

Let's break down why Option (C) is the standard unit:

1. **Work Done vs Energy:** Moving a train involves doing work against gravity, air resistance, and mechanical friction. The total energy required is usually measured at the pantograph (input energy).
2. **Specific Nature:** Because different trains have different weights, simply comparing "Total Energy" is unfair. A massive cargo train will naturally use more energy than a light passenger train. By dividing by the weight (tonnes), we normalize the value.
3. **Distance Factor:** Similarly, a longer journey uses more energy. Dividing by distance (km) makes the unit independent of route length.

4. **Standard Units:** In the power industry, energy is measured in Watt-hours or kWh. In the transportation industry, weight is measured in Metric Tonnes and distance in km. Thus, the combined unit is **Watt-hours per tonne-kilometer** (Wh/t-km) or sometimes **kWh/tonne-km**.

- **Option A (Watt-hours):** This is just energy, not "specific" energy.

- **Option B (kWh/km):** This ignores the weight of the train.

- **Option D (Joules/Tonne):** This is energy per unit weight, but misses the distance component.

Therefore, SEC is the energy used per "tonne-kilometer" of transport work performed.

Step 4: Final Answer:

The unit of SEC is Watt hours / (tonne-km).

This matches Option (C).

Quick Tip: To remember the unit, look at the name: **Specific** (divided by weight) **Energy** (Wh) **Consumption** (over distance km).

Typical values for mainline trains are around 30-50 Wh/t-km, while for suburban trains with many stops, it can be as high as 75 Wh/t-km.

169. A compressor is an electric locomotive is driven by an auxiliary motor to provide air for

- (A) Air conditioning only
- (B) Air breaks, horns, and sanders
- (C) Turbine rotation
- (D) Pantograph lowering only

Correct Answer: (B) Air breaks, horns, and sanders

Solution:

Step 1: Understanding the Concept:

An electric locomotive is a complex industrial machine that requires several auxiliary systems

to function safely and effectively, apart from the main traction motors that move the wheels. One of the most vital auxiliary systems is the **Pneumatic (Compressed Air) System**.

The locomotive carries one or more high-capacity air compressors driven by auxiliary electric motors. These compressors maintain a high-pressure air reservoir (main reservoir) which acts as the "energy source" for many control and safety devices.

Compressed air is used as the medium of control because it is powerful, reliable, and allows for "fail-safe" designs.

Step 2: Detailed Explanation:

Let's analyze the critical functions of compressed air in a loco:

1. **Air Brakes:** This is the single most important use. Mainline trains use "Automatic Air Brakes" where compressed air keeps the brake shoes released. If the air pressure is lost (e.g., train uncouples), the springs immediately apply the brakes, stopping the train. This is a vital safety feature.
2. **Horns:** Locomotives use high-decibel air horns for signaling and safety warnings at crossings. These require a large volume of high-pressure air.
3. **Sanders:** On steep slopes or wet tracks, wheels may slip. Air-operated sanders blow fine sand from a box directly onto the rail surface in front of the wheels to increase adhesion (grip).
4. **Pantograph Operation:** The pantograph is raised to contact the OHE wire using a pneumatic piston. While the question mentions "lowering" (Option D), air is used for both raising and holding it in place.
5. **Control Gear:** Some high-voltage switches and contactors inside the loco are operated by pneumatic actuators.

Option (A) is incorrect as air conditioning in trains is generally purely electrical (refrigeration cycle). Option (C) is irrelevant to electric locos as they don't have internal combustion turbines.

Therefore, the compressor serves as the "lungs" of the locomotive's control and braking systems.

Step 3: Final Answer:

The compressor provides air for air breaks, horns, and sanders.

This matches Option (B).

Quick Tip: In railways, **Pneumatic = Safety**.

Whenever you see "Compressor" or "Compressed Air" in a traction question, always think of **Brakes** first. The compressor is as essential for stopping as the motors are for starting!

170. In which type of wiring system is highly skilled labor required?

- (A) Cleat wiring
- (B) Casing – Capping wiring
- (C) Conduit wiring
- (D) Batten wiring

Correct Answer: (C) Conduit wiring

Solution:

Step 1: Understanding the Concept:

Electrical wiring systems in buildings are chosen based on cost, durability, safety, and aesthetic requirements.

1. **Cleat Wiring:** Temporary, cheap, wires are held by porcelain cleats. Very easy to install.
2. **Batten Wiring:** Wires are laid on wooden battens. Requires basic carpentry skills.
3. **Casing-Capping:** Wires are enclosed in plastic or wooden channels with a cover. Simple to install.
4. **Conduit Wiring:** Wires are pulled through steel or PVC pipes (conduits). This can be "Surface" or "Concealed" (inside walls). This system provides the highest level of mechanical and fire protection.

Step 2: Detailed Explanation:

Conduit wiring is significantly more complex than the other systems for several reasons:

1. **Mechanical Preparation:** The electrician must measure, cut, and bend pipes accurately to fit the geometry of the room. This involves using pipe benders and ensuring smooth curves so that wires don't get stuck.
2. **Threading:** Wires are "fished" through the pipes using a steel wire (snake). This requires patience and skill, especially for long runs with multiple bends.

3. **Concealment:** In concealed conduit wiring, the pipes must be fixed inside the masonry walls before plastering. The alignment of switch boxes and junction boxes must be perfectly level and flush with the final wall surface.

4. **Threading and Jointing:** Metal conduits require threading at the ends and the use of couplers and locknuts, involving specialized tools and techniques to ensure electrical continuity (for grounding).

Because of these technical challenges—pipe bending, wall channeling, wire fishing, and ensuring a watertight/dust-tight seal—conduit wiring requires a **highly skilled and experienced electrician**. A novice can easily damage the wires while pulling them or create a system that is impossible to maintain later.

Step 3: Final Answer:

Highly skilled labor is required for conduit wiring.

This matches Option (C).

Quick Tip: Wiring Hierarchy Summary:

- **Conduit** = Most Expensive, Most Durable, Most Skilled.
- **Cleat** = Cheapest, Least Durable, Least Skilled.

Think of Conduit as "Plumbing for Electricity"—anything involving pipes and fittings inside walls needs a pro!

171. What is the purpose of "contingencies" in a cost estimate?

- (A) To pay for marketing
- (B) To cover unexpected, small costs
- (C) To increase contractor profit
- (D) To pay for taxes

Correct Answer: (B) To cover unexpected, small costs

Solution:

Step 1: Understanding the Concept:

Electrical estimation and costing is a specialized field that involves predicting the total expenditure required to complete an engineering project, such as house wiring, industrial installation, or substation construction. A professional estimate is not just a list of materials (Bill of Quantities); it must account for all facets of the project life cycle. Projects often face unforeseen hurdles that cannot be specifically identified during the initial planning phase. In the terminology of accounting and estimation, "Contingencies" represent a reserved amount of money set aside to cover these unforeseen events or minor items that were omitted from the detailed list of materials. It acts as a financial safety net to ensure that the project does not stall due to minor budgetary shortfalls.

Step 2: Detailed Explanation:

Let us analyze why contingencies are an essential component of a robust engineering estimate:

1. **Unforeseen Site Conditions:** During an electrical installation, an engineer might encounter structural obstacles within a wall that require extra conduit length or specialized fittings that weren't visible during the walkthrough.
2. **Omissions of Minor Items:** While an estimator lists major components like transformers, switches, and cables, small items like screws, washers, adhesives, or specialized tape are often difficult to count precisely. Contingencies provide the funds for these "incidental" materials.
3. **Price Fluctuations:** The cost of copper and aluminum can vary weekly. If the time between the estimate and the purchase of materials is long, the contingency fund helps absorb minor price hikes without requiring a revised budget approval.
4. **Wastage and Breakage:** During construction, some amount of material is inevitably wasted (e.g., cut pieces of wire) or accidentally broken. Contingency funds account for this standard operational loss.

Usually, contingencies are calculated as a small percentage of the total estimated cost, typically ranging from 3% to 5%. It is strictly used for "unexpected, small costs" and is not meant for marketing (Option A), which is a corporate overhead, nor is it a mechanism to artificially inflate contractor profits (Option C). Taxes (Option D) are a separate, mandatory legal charge usually listed as a distinct line item in the final invoice.

Step 3: Final Answer:

The purpose of contingencies in a cost estimate is to cover unexpected, small costs.
This matches Option (B).

Quick Tip: In electrical estimation:

- **Contingencies** = 3% to 5% (for unexpected items).
- **Work-charged Establishment** = 1.5% to 2% (for temporary staff/supervision).
- **Overhead Charges** = 10% to 15% (for office expenses and profit).

Knowing these standard percentages is vital for estimation exams.

172. As per Rural Electrification Corporation (REC) specifications, what is the maximum permissible span from a pole to a house?

- (A) 5 m
- (B) 10 m
- (C) 35 m
- (D) 100 m

Correct Answer: (C) 35 m

Solution:**Step 1: Understanding the Concept:**

The Rural Electrification Corporation (REC), now known as REC Limited, is a leading Public Sector Enterprise in India that sets standards and provides financial assistance for power distribution projects, especially in rural areas. One critical aspect of distribution is the "Service Connection" (or Service Main), which is the final overhead line or cable connecting the nearest distribution pole to the consumer's premises (house). To ensure mechanical safety, prevent excessive line sag, and maintain proper voltage levels, there are strict regulatory limits on the length of this final span. If the span is too long, the tension on the house's weather-head or the pole could cause structural failure during storms, or the voltage drop could become

unacceptable.

Step 2: Detailed Explanation:

Let's analyze the technical requirements for a service connection span:

1. **Mechanical Stability:** A service wire is usually a two-core or four-core weather-proof cable (or AAC/ACSR conductor). As the length increases, the weight of the wire increases. A very long span (like 100m, Option D) would require a very high tension to keep it from sagging to the ground, which would likely pull the brackets off the consumer's wall.
2. **Safety Clearances:** The Indian Electricity (IE) Rules and REC specifications dictate that the service line must maintain a minimum height above ground and buildings. A shorter span makes it much easier to maintain these clearances.
3. **REC Standard:** The standard REC specification for Low Tension (LT) service connections states that the distance between the tapping pole and the point of entry into the consumer's premises should **not exceed 35 meters**.
4. **Procedure for Longer Distances:** If a house is further than 35 meters from the existing line, the utility company must install an additional "service pole" or extend the main distribution line rather than stretching a single long wire. Options (A) and (B) are unnecessarily restrictive for practical rural settings where houses may be spaced out, whereas (D) is physically unsafe for a standard house-attachment point.

Step 3: Final Answer:

According to REC specifications, the maximum permissible span from a pole to a house is 35 m.

This matches Option (C).

Quick Tip: Service Connection Rule:

Maximum length = **35 meters**.

If you see this question in an exam, remember that 35m is roughly the length of 3 to 4 large buses. Anything longer requires an intermediate pole to prevent the wire from snapping or sagging dangerously low.

173. The earth resistance of an electrical installation should be

- (A) As high as possible
- (B) Extremely low (ideally < 1 ohm for substation)
- (C) Exactly 100 ohms
- (D) Infinite

Correct Answer: (B) Extremely low (ideally < 1 ohm for substation)

Solution:

Step 1: Understanding the Concept:

Earthing (or grounding) is the process of connecting the non-current carrying metallic parts of an electrical system or the neutral point of a supply system to the general mass of the earth. The primary purpose of earthing is **safety**. In the event of an insulation failure, the metallic casing of an appliance becomes energized. If the system is properly earthed, the fault current flows to the ground through the earth wire. For this safety mechanism to work effectively, the earth path must offer very little resistance so that a high enough current can flow to trip the circuit breaker or blow the fuse almost instantly. A high earth resistance would limit the fault current, keeping the metallic casing at a dangerous potential without triggering the protection devices.

Step 2: Detailed Explanation:

The effectiveness of an earthing system is measured by its "Earth Resistance," which includes the resistance of the electrode and the soil surrounding it.

1. **Why Low Resistance?** According to Ohm's Law ($I = V/R$), to maximize the fault current (I) for a fixed voltage (V), the resistance (R) must be minimized. A low resistance path ensures that the "Touch Voltage" (potential difference between the grounded object and the ground a person stands on) remains within safe limits (usually $< 50V$).

2. Industrial Standards:

- **Large Power Stations/Substations:** These handle massive fault currents. The earth resistance should be **less than 0.5 to 1.0 ohm**.
- **Major Substations:** Resistance should be less than **1.0 ohm**.
- **Small Substations:** Resistance should be less than **2.0 ohms**.
- **Domestic Installations:** The resistance can be slightly higher, typically **less than 5.0 to 8.0**

ohms, but lower is always better.

3. Analyzing Options:

- **Option (A) High** and **(D) Infinite** are dangerous, as they would effectively disconnect the safety path.
 - **Option (C) 100 ohms** is far too high for any modern protection relay to function reliably.
- Therefore, "Extremely low" is the fundamental engineering requirement for a protective earthing system.

Step 3: Final Answer:

The earth resistance of an electrical installation should be extremely low (ideally < 1 ohm for a substation).

This matches Option (B).

Quick Tip: Earthing Golden Rule: **Low resistance = High safety.**

To reduce earth resistance in dry soil, engineers use:

1. Longer or multiple electrodes in parallel.
2. Treating the soil with charcoal and salt.
3. Increasing the moisture content.

174. Which region of an NPN transistor is the most heavily doped?

- (A) Base
- (B) Collector
- (C) Emitter
- (D) They all are equal

Correct Answer: (C) Emitter

Solution:

Step 1: Understanding the Concept:

A Bipolar Junction Transistor (BJT), such as an NPN type, consists of three layers of semiconductor material: the Emitter, the Base, and the Collector. During the fabrication process, "doping" (the intentional addition of impurities) is used to control the concentration of charge carriers in each region. The operation of the transistor relies on the injection of carriers from one region into another. To achieve transistor action (specifically, current gain), the three regions are designed with very different physical sizes and doping concentrations. The performance of the BJT as a current-controlled switch or amplifier depends heavily on these gradients.

Step 2: Detailed Explanation:

Let's analyze the roles and characteristics of the three regions in an NPN transistor:

1. **The Emitter (E):** As the name implies, its primary function is to "emit" or inject charge carriers (electrons in an NPN) into the base. For the transistor to function efficiently, the emitter must have an abundance of these carriers. Therefore, it is the **most heavily doped** region. This ensures a high "injection efficiency."
2. **The Base (B):** The base is the central control region. Most of the carriers injected from the emitter must pass through the base to reach the collector. To minimize the number of electrons that recombine with holes in the base (which would reduce current gain), the base is made **extremely thin** and is the **most lightly doped** region.
3. **The Collector (C):** Its job is to "collect" the carriers that have crossed the base. It is the largest region physically to dissipate the heat generated during operation. Its doping level is intermediate—lower than the emitter but higher than the base.

Comparing the regions:

- **Doping Concentration:** Emitter $\gg$ Collector $>$ Base.
- **Physical Size:** Collector $>$ Emitter $>$ Base.

Because the Emitter is designed specifically to be the source of all current in the device, it requires the highest density of impurity atoms.

Step 3: Final Answer:

The Emitter is the most heavily doped region of an NPN transistor.

This matches Option (C).

Quick Tip: BJT Memory Matrix:

- **Emitter** = Heavy Doping (Think "E" for "Excess carriers").
- **Base** = Very Thin + Very Light Doping (Think "B" for "Barely there").
- **Collector** = Largest Size (Think "C" for "Chonky" or "Cooling").

175. To ensure a Zener diode acts as a regulator, it must be operated in which biasing condition?

- (A) Forward bias
- (B) Reverse bias below breakdown voltage
- (C) Reverse bias above breakdown voltage (Zener region)
- (D) Zero bias

Correct Answer: (C) Reverse bias above breakdown voltage (Zener region)

Solution:

Step 1: Understanding the Concept:

A Zener diode is a special type of semiconductor diode designed to operate reliably in the "reverse breakdown" region. While a standard diode is destroyed if the reverse voltage exceeds its breakdown limit, a Zener diode is fabricated with heavy doping to have a very sharp and stable breakdown voltage (V_Z). The defining property of a Zener diode is that once it enters the breakdown region, the voltage across its terminals remains almost constant even if the current through it changes significantly. This makes it a perfect component for voltage regulation—providing a steady output voltage despite fluctuations in the input supply or load current.

Step 2: Detailed Explanation:

Let's examine the behavior of a Zener diode under different biasing conditions:

- 1. Forward Bias (Option A):** In forward bias, the Zener diode behaves like a normal silicon diode, with a voltage drop of approximately 0.7V. It does not provide regulation in this mode.
- 2. Reverse Bias below V_Z (Option B):** In this region, only a tiny "leakage current" flows. The diode acts like an open switch, and the voltage across it will simply follow the input voltage. No regulation occurs here.
- 3. Zero Bias (Option D):** No current flows, and no useful electrical work is performed.

4. **Reverse Bias at/above V_Z (Option C):** When the reverse voltage reaches the specified Zener voltage (V_Z), the diode undergoes Zener breakdown (or avalanche breakdown for higher voltages). At this point, the V-I characteristic becomes almost vertical. This means that whether the current is 1mA or 50mA, the voltage across the Zener stays fixed at V_Z . In a regulator circuit, the Zener is connected in parallel with the load. Any excess voltage from the supply is dropped across a series resistor, while the Zener "clamps" the load voltage to its stable breakdown value.

Thus, the "Zener effect" which allows for regulation is only available when the diode is reverse-biased into its breakdown state.

Step 3: Final Answer:

For voltage regulation, a Zener diode must be operated in reverse bias above the breakdown voltage (the Zener region).

This matches Option (C).

Quick Tip: Zener Regulator Rule:

- **Condition 1:** Must be **Reverse Biased**.
- **Condition 2:** Input voltage must be **Greater than V_Z** .

If the input voltage drops below V_Z , the Zener "turns off" and the regulation is lost.

176. Lissajous patterns are used in a CRO to measure

- (A) Voltage and current
- (B) Frequency and phase shift
- (C) Resistance and Inductance
- (D) Power factor

Correct Answer: (B) Frequency and phase shift

Solution:

Step 1: Understanding the Concept:

A Cathode Ray Oscilloscope (CRO) is an essential laboratory instrument for visualizing electrical signals. In its standard mode, the X-axis represents time (linear sweep) and the Y-axis represents voltage. However, the CRO can also be operated in "X-Y mode," where one signal is applied to the horizontal deflection plates and another signal is applied to the vertical deflection plates. When two sinusoidal signals are applied simultaneously in this mode, the resulting display on the screen is a complex geometric curve known as a **Lissajous Pattern**. The shape and orientation of these patterns provide immediate information about the relationship between the two input signals.

Step 2: Detailed Explanation:

Lissajous patterns are primarily used for two types of measurements:

1. **Frequency Measurement:** If the two signals have different frequencies, the pattern will be a closed loop with multiple "lobes." By counting the number of times the pattern touches a horizontal line (n_x) and a vertical line (n_y), the ratio of the frequencies can be determined:

$$\frac{f_y}{f_x} = \frac{n_x}{n_y}$$

If one frequency is known (standard signal), the unknown frequency can be calculated precisely.

2. **Phase Shift Measurement:** If both signals have the same frequency ($f_y/f_x = 1$), the pattern will be an ellipse, a circle, or a straight line. The exact shape depends on the phase difference (ϕ):

- **Straight line (0 or 180 deg):** Signals are in phase or anti-phase.
- **Circle (90 or 270 deg):** Signals are perfectly quadrature.
- **Ellipse:** Signals have an intermediate phase shift.

The phase angle can be calculated using the intercept values on the axes.

Other options like voltage and current (Option A) are measured using the standard time-sweep mode, and impedance components (Option C) or power factor (Option D) require other bridge circuits or specialized meters. Lissajous patterns are uniquely tied to the relative properties of two waveforms.

Step 3: Final Answer:

Lissajous patterns are used to measure frequency and phase shift.

This matches Option (B).

Quick Tip: CRO X-Y Mode Memory:

- **Straight Line** (positive slope) $\rightarrow \phi = 0^\circ$.
- **Perfect Circle** $\rightarrow \phi = 90^\circ$.
- **Straight Line** (negative slope) $\rightarrow \phi = 180^\circ$.

Think of Lissajous as the "Fingerprint" of the relationship between two signals.

177. A ripple frequency of a full wave rectifier with an input frequency of 50 Hz is

- (A) 25 Hz
- (B) 50 Hz
- (C) 100 Hz
- (D) 200 Hz

Correct Answer: (C) 100 Hz

Solution:**Step 1: Understanding the Concept:**

A rectifier is a circuit that converts Alternating Current (AC), which periodically reverses direction, into Direct Current (DC), which flows in one direction. However, the output of a rectifier is not a "smooth" DC line like a battery; it is "pulsating DC." The term "Ripple Frequency" refers to the frequency of these fluctuations (ripples) in the output signal. It represents how many times per second the pulsating voltage peaks. The ripple frequency depends on the input supply frequency (f_{in}) and the configuration of the rectifier (Half-wave vs. Full-wave).

Step 2: Key Formula or Approach:

- For a **Half Wave Rectifier**: The output pulses occur only once per input cycle. Thus, $f_{ripple} = f_{in}$.
- For a **Full Wave Rectifier** (Bridge or Center-tapped): The output pulses occur twice for every single input cycle because both the positive and negative halves of the AC wave are utilized. Thus, $f_{ripple} = 2 \times f_{in}$.

Step 3: Detailed Explanation:

Let's consider the input frequency $f_{in} = 50\text{ Hz}$. This means the AC wave completes 50 full cycles every second.

1. In the first half-cycle (positive), the full-wave rectifier directs the current through the load in a specific direction, creating one pulse.
2. In the second half-cycle (negative), the rectifier reverses the connection internally (via diodes) so that the current flows through the load in the **same** direction as before, creating a second pulse.
3. Consequently, for every 1 cycle of input, we see 2 pulses at the output.

Calculation:

$$f_{ripple} = 2 \times 50\text{ Hz}$$

$$f_{ripple} = 100\text{ Hz}$$

This higher ripple frequency is an advantage for full-wave rectifiers because it makes it much easier to "smooth" the DC using a filter capacitor. A higher frequency ripple has shorter time gaps to fill, requiring a smaller capacitor value for the same quality of DC output compared to a half-wave rectifier.

Step 4: Final Answer:

The ripple frequency of the full-wave rectifier is 100 Hz.

This matches Option (C).

Quick Tip: Ripple Frequency Check:

- **Half-wave** = Same as supply (50 → 50).
 - **Full-wave** = Double the supply (50 → 100).
 - **3-phase Full-wave** = 6 times the supply (50 → 300).
- Just count how many times the diodes "fire" per cycle.

178. What is the primary purpose of biasing a transistor in an amplifier circuit?

- (A) To increase the gain
- (B) To ensure the transistor operates in the active region
- (C) To reduce the output impedance
- (D) To eliminate noise

Correct Answer: (B) To ensure the transistor operates in the active region

Solution:

Step 1: Understanding the Concept:

Transistor "Biasing" refers to the process of setting up the DC operating conditions (voltage and current) of a transistor circuit before an actual AC signal is applied. When a transistor is used as an amplifier, it must be able to faithfully reproduce the input signal at a larger magnitude. A BJT has three main operating regions: Cut-off (off), Saturation (fully on), and Active (the linear amplifying region). If the transistor is not properly biased, the AC signal might drive the transistor into cut-off or saturation, leading to "clipping" or severe distortion of the output sound or data.

Step 2: Detailed Explanation:

The goal of biasing is to establish a stable "Q-point" (Quiescent point) on the transistor's load line.

1. **Active Region Requirement:** For a BJT to act as a linear amplifier, the base-emitter junction must be forward-biased and the base-collector junction must be reverse-biased. Biasing resistors are chosen to achieve this exact state.
2. **Operating in the Center:** Ideally, the Q-point is placed near the middle of the active region.

This allows the AC signal to swing up and down without hitting the "walls" of saturation or cut-off, ensuring the output wave is a perfect (but larger) copy of the input.

3. **Thermal Stability:** Biasing circuits (like the Voltage Divider Bias) are also designed to keep this Q-point stable even if the temperature changes or if the transistor is replaced with another one of the same model but slightly different gain (β).

4. **Evaluating other options:**

- **Gain (Option A):** While biasing allows amplification to happen, the magnitude of gain is mostly determined by the resistor values and transistor parameters, not the act of biasing itself.

- **Impedance (Option C):** This is a function of the circuit configuration (Common Emitter vs. Common Collector), not just biasing.

- **Noise (Option D):** Noise is handled by shielding and filtering, not biasing.

Therefore, the fundamental reason we apply DC voltages to a transistor is to "prime" it so that it stays in its linear, active range during signal processing.

Step 3: Final Answer:

The primary purpose of biasing is to ensure the transistor operates in the active region. This matches Option (B).

Quick Tip: Think of Biasing like **idling a car engine**. You don't just turn the key and floor it; the engine needs a steady "idle" state (biasing) so that when you press the pedal (the input signal), the car responds smoothly without stalling or redlining.

179. What is the conduction angle of a Class-A amplifier?

- (A) 90°
- (B) 180°
- (C) 270°
- (D) 360°

Correct Answer: (D) 360°

Solution:

Step 1: Understanding the Concept:

Power amplifiers are classified based on the "Conduction Angle," which is the portion of the input signal cycle during which the transistor remains in the conducting (ON) state. This classification determines the amplifier's efficiency and its level of signal distortion. Class-A is the most basic type of power amplifier. In this configuration, the transistor is biased so that it is **always on**, regardless of where the input signal is in its cycle. This ensures the highest possible signal fidelity (lowest distortion) but results in very low efficiency because the transistor is constantly wasting power, even when there is no input signal.

Step 2: Detailed Explanation:

Let's compare the conduction angles of the standard amplifier classes:

1. **Class A:** The bias point is in the center of the load line. The transistor conducts for the **entire** 360° of the input sine wave. Both the positive and negative halves are amplified by a single transistor. This produces the cleanest output but only 25% to 50% maximum efficiency.
2. **Class B:** The transistor is biased exactly at the cut-off point. It conducts only for one half of the input cycle. The conduction angle is 180° . To amplify the full wave, two transistors must be used in a "push-pull" arrangement. Efficiency is higher (78.5%).
3. **Class AB:** The conduction angle is between 180° **and** 360° . This is used to eliminate "crossover distortion" found in Class B.
4. **Class C:** The transistor conducts for less than half the cycle. The conduction angle is **less than** 180° . It is highly efficient (90%) but produces massive distortion, making it suitable only for RF (radio frequency) applications with tuned circuits.

Since the question asks specifically for Class-A, we identify that the transistor never turns off during the full period of rotation (360°).

Step 3: Final Answer:

The conduction angle of a Class-A amplifier is 360° .

This matches Option (D).

Quick Tip: Amplifier Class Summary:

- **Class A:** Always ON (360°). Best sound, Worst efficiency.
- **Class B:** Half ON (180°).
- **Class C:** Briefly ON ($< 180^\circ$). Worst sound, Best efficiency.

180. Which of the following oscillator types uses a capacitive voltage divider for feedback?

- (A) Hartley oscillator
- (B) Colpitts oscillator
- (C) Wien Bridge oscillator
- (D) Crystal oscillator

Correct Answer: (B) Colpitts oscillator

Solution:

Step 1: Understanding the Concept:

An oscillator is a circuit that generates a repetitive AC signal (like a sine wave) from a DC power supply. For a circuit to oscillate, it must satisfy the **Barkhausen Criterion**: it must have a feedback loop with a total phase shift of 360 (or 0) degrees and a loop gain of at least unity. In high-frequency applications, LC (Inductor-Capacitor) tank circuits are used to set the oscillation frequency. Two of the most famous LC oscillators are the Hartley and Colpitts oscillators. They are distinguished primarily by how the "tapped" component in the tank circuit is arranged to provide the feedback voltage to the transistor.

Step 2: Detailed Explanation:

Let's compare the construction of the two primary LC oscillators:

1. **Hartley Oscillator (Option A):** The tank circuit consists of **two inductors** (L_1, L_2) and one capacitor (C). The inductors form an inductive voltage divider. The feedback is taken from the "tap" between the two coils.
2. **Colpitts Oscillator (Option B):** The tank circuit consists of **two capacitors** (C_1, C_2) and one inductor (L). The two capacitors are connected in series across the inductor. The junction between these two capacitors is grounded, effectively creating a **capacitive voltage divider**.

The feedback signal is taken from across one of these capacitors. Because capacitors have better stability at very high frequencies than inductors, the Colpitts oscillator is often preferred for high-frequency RF generation.

3. **Wien Bridge (Option C):** This is an RC oscillator used for low frequencies (audio). It uses a bridge circuit, not a capacitive divider tank.

4. **Crystal Oscillator (Option D):** This uses a piezoelectric crystal for extreme frequency stability. While it can be configured in a Colpitts-like arrangement, the "capacitive divider" is the textbook defining feature of the standard Colpitts design.

Step 3: Final Answer:

The Colpitts oscillator uses a capacitive voltage divider for feedback.

This matches Option (B).

Quick Tip: Mnemonic for LC Oscillators:

- Colpitts starts with **C** → uses **C**apacitors for the divider.
- Hartley starts with **H** → uses **H**enries (Inductors) for the divider.

This simple letter-association will ensure you never mix them up again.

181. Which of the following is not a typical application of an oscillator?

- (A) Signal generator
- (B) Radio transmitter
- (C) Rectifier
- (D) Digital gate

Correct Answer: (C) Rectifier

Solution:

Step 1: Understanding the Concept:

An electronic oscillator is a circuit that produces a repetitive, oscillating electronic signal,

often a sine wave or a square wave. It essentially converts direct current (DC) from a power supply to an alternating current (AC) signal. Oscillators are fundamental components in many electronic devices, used to generate periodic signals that serve as timing references (clocks), carrier waves for communication, or test signals for measurement. To identify what is **not** an application, we must understand the functional difference between generating a signal and processing power. An oscillator is a signal source, whereas other components are used for power conversion or logical processing.

Step 2: Detailed Explanation:

Let's analyze the options based on the functional role of oscillators in electronic systems:

1. **Signal Generator:** This is a laboratory instrument designed specifically to produce various types of waveforms (sine, square, triangle). The heart of every signal generator is one or more oscillators. Therefore, this is a direct application.
2. **Radio Transmitter:** In telecommunications, oscillators are used to create the "carrier wave." This high-frequency signal is then modulated with information (voice or data) and transmitted through an antenna. Every radio, television, and mobile phone transmitter relies on stable oscillators.
3. **Digital Gate:** Oscillators provide the clock signals that drive digital logic gates in a synchronous system (like a CPU). While a gate itself is not an "oscillator application" in the sense of signal generation, it uses the timing provided by an oscillator. However, comparing this to Option C, we look for the most distinct contradiction.
4. **Rectifier:** A rectifier is an electrical device that converts alternating current (AC), which periodically reverses direction, to direct current (DC), which flows in only one direction. This is the **exact opposite** function of an oscillator. An oscillator takes DC and creates AC; a rectifier takes AC and creates DC. Therefore, a rectifier is a power conversion component found in power supplies, not an application of signal generation circuitry. Rectifiers are built using diodes, not oscillator feedback loops. Thus, Option (C) is the clear outlier.

Step 3: Expanded Context:

Oscillators are characterized by the Barkhausen criterion, requiring a gain of 1 and a phase shift of 360 degrees in a feedback loop. They are used in local oscillators in receivers, PWM controllers in motor drives, and quartz watches for timekeeping. Rectification, on the other hand, is part of the "Front-end" of a power supply system meant to provide the steady DC

voltage that the oscillator circuit subsequently consumes to perform its job. This distinction between "Signal Generation" and "Power Rectification" is fundamental in electrical engineering.

Step 4: Final Answer:

The rectifier is not an application of an oscillator.

This matches Option (C).

Quick Tip: Function Mnemonic:

- Oscillator: DC to AC (Think "O-DA" - Oscillator generates).

- Rectifier: AC to DC (Think "R-AD" - Rectifier adjusts power).

Since their functions are inverse, a rectifier can never be an application of an oscillator.

182. What is the result of the binary addition: $1_2 + 1_2$?

- (A) 1
- (B) 2
- (C) 10
- (D) 11

Correct Answer: (C) 10

Solution:

Step 1: Understanding the Concept:

Binary addition is the basis of all calculations performed by digital computers. Unlike the decimal system (Base-10), which uses digits 0-9, the binary system (Base-2) uses only two digits: 0 and 1. When we add numbers in binary, the same column-addition principles apply as in decimal, but the "carrying" process happens much sooner. In decimal, you carry when the sum reaches 10; in binary, you carry when the sum reaches 2. Understanding the place value in binary ($2^0, 2^1, 2^2 \dots$) is key to interpreting the result of simple operations.

Step 2: Key Formula or Approach:

The fundamental rules of binary addition are:

- $0 + 0 = 0$
- $0 + 1 = 1$
- $1 + 0 = 1$
- $1 + 1 = 0$ with a carry of 1 (which equals 10_2)
- $1 + 1 + 1 = 1$ with a carry of 1 (which equals 11_2)

Step 3: Detailed Explanation:

Let's perform the addition of 1_2 and 1_2 .

1. In the first column (units place, 2^0), we have $1 + 1$.
2. In decimal, $1 + 1 = 2$.
3. To represent the number 2 in binary, we look at the powers of 2. Two is $1 \times 2^1 + 0 \times 2^0$.
4. This results in the binary string "10".
5. Looking at it as a vertical addition:

$$\begin{array}{r} 1 \\ + 1 \\ \hline 10 \end{array}$$

The "0" is written in the current column, and the "1" is carried over to the next column on the left (2^1 place). Since there are no other digits to add in that column, the "1" is simply brought down. Thus, the sum of one and one in binary is "one-zero," which represents the value two. Option (B) is incorrect because "2" is not a valid digit in the binary system. Option (D) would represent the value three ($1 + 1 + 1$).

Step 4: Final Answer:

The result of the binary addition $1_2 + 1_2$ is 10.

This matches Option (C).

Quick Tip: Binary Cheat Sheet:

- $1_2 = 1_{10}$

- $10_2 = 2_{10}$

- $11_2 = 3_{10}$

- $100_2 = 4_{10}$

When adding binary numbers, if the sum is "two," write "0" and carry "1." If the sum is "three," write "1" and carry "1."

183. The absorption law in Boolean algebra is

(A) $A + AB = B$

(B) $A + AB = A$

(C) $A(A + B) = B$

(D) $A + A'B = A + B$

Correct Answer: (B) $A + AB = A$

Solution:

Step 1: Understanding the Concept:

Boolean algebra is the branch of algebra in which the values of the variables are the truth values "true" and "false," usually denoted as 1 and 0 respectively. It is used to simplify digital logic circuits. Just like ordinary algebra has rules (like distributive or commutative laws), Boolean algebra has a specific set of theorems. The "Absorption Law" describes how a complex expression can be "absorbed" or simplified into a single variable when that variable appears both alone and as a factor in a product within the same sum. This law is critical for reducing the number of logic gates needed in a physical circuit design.

Step 2: Key Formula or Approach:

The two forms of the Absorption Law are:

1. **Sum form:** $A + AB = A$

2. **Product form:** $A(A + B) = A$

We can prove these using the distributive law and the identity rules of Boolean algebra.

Step 3: Detailed Explanation:

Let's prove Option (B) step-by-step:

1. Start with the left side: $A + AB$.
2. Use the **Identity Law**: A is the same as $A \cdot 1$. So, the expression becomes $(A \cdot 1) + (A \cdot B)$.
3. Use the **Distributive Law** to factor out A : $A \cdot (1 + B)$.
4. Use the **Null Law** (or Or-Identity): Any variable ORed with 1 is always 1. Thus, $1 + B = 1$.
5. The expression simplifies to $A \cdot (1)$.
6. By the Identity Law again, $A \cdot 1 = A$.

Thus, $A + AB = A$. The second term AB is "absorbed" because its value doesn't change the outcome determined by A . If $A = 1$, the sum is 1 regardless of B . If $A = 0$, the sum is $0 + (0 \cdot B) = 0$. The result matches A in all cases.

Option (D) is also a valid Boolean identity ($A + A'B = A + B$), but it is formally known as the **Redundancy Law** or the **Law of Simplification**, not the primary Absorption Law.

Step 4: Final Answer:

The absorption law is $A + AB = A$.

This matches Option (B).

Quick Tip: Absorption Law logic: If the single variable (A) is **TRUE**, the whole expression is TRUE regardless of what B is doing. If the single variable is **FALSE**, the whole thing is FALSE. Since the outcome only depends on the single variable, the extra term "absorbs" into it.

184. The register is a type of

- (A) Sequential circuit
- (B) Combinational circuit
- (C) CPU
- (D) Latches

Correct Answer: (A) Sequential circuit

Solution:

Step 1: Understanding the Concept:

Digital logic circuits are broadly divided into two categories:

1. **Combinational Circuits:** The output depends only on the current inputs. They have no memory of past states. Examples include ADDERS, MULTIPLEXERS, and logic gates (AND, OR).

2. **Sequential Circuits:** The output depends on the current inputs **and** the previous state of the system. These circuits contain "memory" elements that store information over time.

A register is a group of flip-flops used to store a bit-pattern (like a word of data) within a processor or digital system. Because a register must "hold" or "remember" its value until a clock pulse changes it, it falls into the category of memory-based circuits.

Step 2: Detailed Explanation:

Registers are the fundamental building blocks of a computer's memory hierarchy.

1. **Construction:** A single register is typically made of multiple D-type Flip-Flops connected in parallel. For an 8-bit register, 8 flip-flops are used.

2. **State Dependency:** Because flip-flops use feedback loops to maintain their state, the next state of the register is determined by the clock edge and the input. This "state-dependent" behavior is the defining characteristic of a sequential circuit.

3. Analyzing the options:

- **Combinational (B):** Incorrect. A combinational circuit cannot store data; it only transforms inputs.

- **CPU (C):** Incorrect. A CPU is a complex system that **contains** registers, but a register itself is not a CPU.

- **Latches (D):** A latch is a level-triggered memory element. While a register is technically a collection of latches (if level-triggered) or flip-flops (if edge-triggered), the question asks for the **type** of circuit category it belongs to. Sequential circuit is the correct architectural classification.

Thus, registers are categorized as sequential circuits because they possess timing-based memory.

Step 3: Final Answer:

A register is a type of sequential circuit.

This matches Option (A).

Quick Tip: Circuit Classification Mnemonic:

- Combinational = Current inputs only.

- Sequential = State + Current inputs (Memory).

If it stores a bit (Flip-flop, Register, Counter, RAM), it is **Sequential**.

185. The fastest type of memory storage located within the CPU is

- (A) Cache
- (B) Main memory
- (C) Registers
- (D) ROM

Correct Answer: (C) Registers

Solution:

Step 1: Understanding the Concept:

Computer architecture utilizes a "Memory Hierarchy" to balance the trade-off between speed, capacity, and cost.

At the bottom, we have large, slow, and cheap storage (like hard drives or SSDs). At the top, we have very small, extremely expensive, but lightning-fast memory units. The speed of a memory type is often measured by its "latency" (the number of clock cycles required to retrieve data). As we move closer to the ALU (Arithmetic Logic Unit) where the actual processing happens, the memory speed must increase to prevent the processor from idling while waiting for data.

Step 2: Detailed Explanation:

Let's compare the speeds of the memory types listed:

1. **Main Memory (RAM):** Located on separate chips outside the CPU. It takes hundreds of clock cycles for the CPU to access data from RAM. It is the slowest of the high-speed options.
2. **ROM:** Read-Only Memory is used for firmware. Its access speeds are comparable to or slower than RAM.
3. **Cache (L1, L2, L3):** Located on the CPU die. L1 cache is very fast (takes 1-4 cycles), but it still requires some addressing logic to find data within the cache blocks.
4. **Registers:** These are the internal "working slots" of the CPU (like the Accumulator, Program Counter, or General Purpose Registers). They are part of the CPU's internal data path. Accessing a register takes **less than one clock cycle** or exactly one cycle. There is no addressing overhead because the instructions directly specify the register name (e.g., ADD R1, R2).

Because registers are the most intimately integrated memory units within the CPU's execution core, they are the fastest storage elements in any computing system. Their capacity is very limited (usually only a few hundred bytes total), but their speed is the baseline for processor performance.

Step 3: Final Answer:

Registers are the fastest type of memory storage within the CPU.

This matches Option (C).

Quick Tip: Memory Hierarchy (Fastest to Slowest):

Registers > L1 Cache > L2 Cache > Main Memory (RAM) > SSD/Disk.

Think of registers as the **tools in the worker's hands**, cache as the **toolbox on the table**, and RAM as the **supply closet in the hallway**.

186. Which instruction is used to set the carry flag?

- (A) CLR C
- (B) SETB C
- (C) CPL C
- (D) MOV C, #1

Correct Answer: (B) SETB C

Solution:

Step 1: Understanding the Concept:

In microcontrollers like the 8051, the "Carry Flag" (C) is a single bit located in the Program Status Word (PSW) register. It is used to indicate an overflow in arithmetic operations or to act as a one-bit accumulator for Boolean logic operations. The 8051 Instruction Set includes specific "Bit-Addressable" instructions that allow programmers to directly manipulate individual bits like the Carry flag without affecting other bits in the register. To "set" a bit means to change its value to logical '1'.

Step 2: Detailed Explanation:

Let's analyze the 8051 Boolean instructions listed:

1. **CLR C (Clear Carry):** This instruction forces the Carry bit to '0'. It is used before an addition or subtraction if you want to ensure no previous carry interferes with the result.
2. **SETB C (Set Bit Carry):** This instruction forces the Carry bit to '1'. The mnemonic "SETB" stands for "Set Bit." It is the standard instruction for making a bit logically HIGH.
3. **CPL C (Complement Carry):** This is a "Toggle" instruction. If C is 0, it becomes 1. If C is 1, it becomes 0. It does not consistently "set" the bit; it only flips its state.
4. **MOV C, #1:** While this looks logically sound (moving a 1 into C), standard 8051 assembly syntax uses SETB for direct bit manipulation. Some assemblers might interpret specific moves, but SETB is the dedicated architectural instruction for this purpose.

Therefore, for setting the carry flag explicitly to one, SETB C is the correct opcode. This is essential for controlling conditional branches (like JC - Jump if Carry) or manually creating carry-over for multi-byte additions.

Step 3: Final Answer:

The instruction used to set the carry flag is SETB C.

This matches Option (B).

Quick Tip: 8051 Bit Mnemonics:

- **SETB** = Make 1.
- **CLR** = Make 0.
- **CPL** = Flip (0 to 1, 1 to 0).

Always use SETB when you need to "force" a bit to the ON state.

187. What is the size of the Stack Pointer (SP) in 8051 microcontroller?

- (A) 4-bit
- (B) 8-bit
- (C) 16-bit
- (D) 32-bit

Correct Answer: (B) 8-bit

Solution:

Step 1: Understanding the Concept:

The Stack Pointer (SP) is a special-purpose register that points to the top of the "Stack," which is a section of RAM used for temporary data storage, such as return addresses during function calls (CALL) or saved register values (PUSH). The "size" of a register refers to how many binary digits (bits) it can hold. This size determines the maximum memory range the pointer can address. The 8051 is a classic 8-bit architecture, meaning its internal data bus and most of its registers are designed to handle 8 bits of data at a time.

Step 2: Detailed Explanation:

1. **Memory Architecture:** The 8051 has 128 or 256 bytes of internal RAM. Because the total addressable range of internal RAM is small (00h to FFh), an 8-bit register is sufficient to point to any location within that space.
2. **Stack Operation:** In the 8051, the stack grows **upward** in internal RAM. When a PUSH instruction is executed, the SP is first incremented, and then the data is stored. Since the address space is 8-bit, the SP must also be an 8-bit register.
3. **Comparison with other registers:**

- **A, B, R0-R7, PSW:** All are 8-bit registers.

- **DPTR (Data Pointer):** This is the only 16-bit register in the standard 8051, used to address up to 64KB of external memory.

Because the stack is strictly confined to internal RAM, the SP does not need the 16-bit width required for external memory. On power-up (Reset), the 8051 SP is automatically initialized to 07h, and it works within the 8-bit address range throughout operation.

Step 3: Final Answer:

The size of the Stack Pointer in the 8051 is 8-bit.

This matches Option (B).

Quick Tip: 8051 Register Sizes:

- **Most registers** (A, B, SP, PSW, IP, IE) = **8-bit**.

- **DPTR and Program Counter (PC)** = **16-bit**.

The PC and DPTR are 16-bit because they must "see" the entire 64KB code/data space. SP only "sees" the internal RAM, so 8 bits are enough.

188. A DIAC remains in the 'OFF' state until the applied voltage reaches the

- (A) Peak voltage
- (B) Break over voltage
- (C) Holding voltage
- (D) Knee voltage

Correct Answer: (B) Break over voltage

Solution:

Step 1: Understanding the Concept:

A DIAC (Diode for Alternating Current) is a bi-directional semiconductor switch that can conduct current in either direction. It is a three-layer device with no gate terminal. Its

primary function in power electronics is to act as a trigger device for TRIACs in AC dimming and motor control circuits. The DIAC has a unique V-I characteristic: it behaves as an open circuit (non-conducting) for low voltages. It only "breaks down" and starts conducting when the voltage across its terminals exceeds a specific threshold. Once it starts conducting, the voltage across it actually drops slightly (negative resistance), ensuring a strong current pulse to trigger a main power switch.

Step 2: Detailed Explanation:

Let's analyze the operating states of a DIAC:

1. **Blocking State (OFF):** When the applied voltage is low, the DIAC exhibits high resistance. It acts like two diodes connected back-to-back. Only a tiny leakage current flows.
2. **Transition:** As the voltage increases, it eventually reaches the **Breakover Voltage** (V_{BO}). At this specific voltage level, the internal junction undergoes impact ionization or "breakdown."
3. **Conducting State (ON):** Once V_{BO} is reached, the DIAC resistance drops instantly, and it allows current to flow freely. This state is maintained as long as the current remains above the "holding current."

4. Analyzing the options:

- **Peak Voltage (A):** This is the maximum value of an AC wave, but a DIAC might trigger well before or at the peak depending on the circuit design.
- **Holding Voltage (C):** This is the minimum voltage required to **keep** a device ON, not the voltage needed to turn it ON.
- **Knee Voltage (D):** Usually refers to the forward conduction point of a standard diode (0.7V).

The specific technical term for the trigger point of a multi-layer thyristor-family device like the DIAC is the break over voltage.

Step 3: Final Answer:

The DIAC remains OFF until it reaches the break over voltage.

This matches Option (B).

Quick Tip: DIAC Triggering:

Think of a DIAC as a **pressure relief valve**. Water won't flow until the pressure (**Breakover Voltage**) builds up enough to "pop" the valve open. It is a **voltage-triggered** device without a control gate.

189. What happens to the firing angle of the SCR if the resistance (R) in the UJT's RC circuit is increased?

- (A) Firing angle increases (delay increases)
- (B) Firing angle decreases
- (C) Firing angle remains the same
- (D) The SCR will stay permanently ON

Correct Answer: (A) Firing angle increases (delay increases)

Solution:

Step 1: Understanding the Concept:

In power control circuits, a Silicon Controlled Rectifier (SCR) is turned on at a specific point in the AC cycle, known as the "Firing Angle" (α). This angle determines how much power is delivered to the load. A common way to control this angle is using a UJT (Unijunction Transistor) relaxation oscillator. The UJT circuit uses an RC (Resistor-Capacitor) network to create a time delay. The capacitor charges up from the supply through the resistor R . When the capacitor voltage reaches the UJT's "Peak Point Voltage" (V_p), the UJT "fires," discharging the capacitor into the SCR gate and turning the SCR ON.

Step 2: Key Formula or Approach:

The time taken (t) for a capacitor to charge in an RC circuit is governed by the time constant:

$$\tau = R \cdot C$$

The time to reach the firing threshold V_p is proportional to this time constant:

$$t_{\text{delay}} \approx R \cdot C \cdot \ln\left(\frac{1}{1-\eta}\right)$$

where η is the intrinsic stand-off ratio of the UJT.

Step 3: Detailed Explanation:

Let's see what happens when the resistance R is increased:

1. **Charging Rate:** If R is increased, the current flowing into the capacitor ($I = V/R$) decreases.
2. **Time Delay:** Because the charging current is lower, it takes **longer** for the capacitor to reach the required peak voltage (V_p) to trigger the UJT.
3. **Phase Relationship:** In an AC cycle, a longer time delay translates to a **larger angle** from the zero-crossing point.
4. **Result:** A larger time delay means a **larger firing angle** (α). For example, instead of firing at 30° (early), it might fire at 90° (later).

This increased firing angle results in the SCR being ON for a smaller portion of the half-cycle, thereby **decreasing** the average power delivered to the load (e.g., dimming a bulb or slowing a motor). Thus, increasing R increases the firing angle and the delay.

Step 4: Final Answer:

The firing angle increases and the delay increases.

This matches Option (A).

Quick Tip: RC Control Logic:

- **High R** → Slow charging → **Long Delay** ($\alpha \uparrow$).
- **Low R** → Fast charging → **Short Delay** ($\alpha \downarrow$).

Think of R as a "throttle" for the capacitor—the more you restrict the flow, the longer it takes to fill the tank.

190. The process of turning OFF a conducting SCR is known as

- (A) Triggering
- (B) Biasing
- (C) Commutation
- (D) Rectification

Correct Answer: (C) Commutation

Solution:

Step 1: Understanding the Concept:

An SCR is a "Latching Device." Once it is turned ON (triggered) by a gate pulse, it stays ON even if the gate signal is removed. It only stops conducting if the current flowing through it (Anode current) is reduced below a very low level known as the "Holding Current." In DC circuits, the current doesn't naturally go to zero, so external circuitry is needed to force the SCR to turn off. The formal engineering term for the process of successfully extinguishing the conduction of a thyristor and allowing it to regain its forward blocking capability is "Commutation."

Step 2: Detailed Explanation:

Let's define the terms in the options:

1. **Triggering (A):** This is the process of turning **ON** the SCR by applying a pulse to the gate. It is the opposite of the process described in the question.
2. **Biasing (B):** This refers to applying DC voltages to a device to set its operating point (like in transistors). It doesn't describe the turn-off process.
3. **Rectification (D):** This is the functional outcome of using SCRs or diodes to convert AC to DC. It is a system-level process, not a device-level turn-off procedure.
4. **Commutation (C):** This is the specific technical name for turn-off. There are two main types:
 - **Natural Commutation:** Occurs in AC circuits when the supply voltage naturally goes through zero and reverses polarity, pushing the current to zero.
 - **Forced Commutation:** Required in DC circuits. External components (capacitors and inductors) are used to temporarily divert the current or apply a reverse voltage to the SCR to force it OFF.

Without successful commutation, a power electronic converter would lose control and potentially short-circuit the supply.

Step 3: Final Answer:

The process of turning OFF an SCR is known as commutation.

This matches Option (C).

Quick Tip: Thyristor Cycle:

1. **Triggering** = Turn ON.
2. **Commutation** = Turn OFF.

Remember "C" for "Commutation" and "C" for "Cancel conduction."

191. A class C chopper is formed by connecting

- (A) Class A and Class B in series
- (B) Class A and Class B in parallel
- (C) Two Class D Choppers
- (D) A step-up and step-down choppers in series

Correct Answer: (B) Class A and Class B in parallel

Solution:**Step 1: Understanding the Concept:**

Choppers are static power electronic devices that convert a fixed DC input voltage into a variable DC output voltage. In power electronics, choppers are classified into several categories (Class A through Class E) based on the "quadrants" of the voltage-current (V-I) plane in which they operate. A quadrant represents the direction of power flow between the source and the load.

- **Class A Chopper:** Operates only in the **first quadrant**. Both output voltage and current are positive. Power flows from source to load. This is a step-down chopper.
- **Class B Chopper:** Operates in the **second quadrant**. The output voltage is positive, but the average current is negative. Power flows from load to source (regenerative braking).
- **Class C Chopper:** Is a **two-quadrant chopper** that combines the capabilities of Class A and

Class B. It allows for a positive output voltage while permitting the current to flow in either direction (positive or negative).

Step 2: Key Formula or Approach:

The configuration of a Class C chopper involves combining the circuit topologies of Class A and Class B. To allow for bidirectional current flow while sharing a common load, the switches and diodes of the two base classes are arranged in a specific parallel configuration.

Step 3: Detailed Explanation:

Let's analyze the construction of a Class C chopper in detail. It consists of two power semiconductor switches (usually MOSFETs or IGBTs) and two free-wheeling diodes.

1. **Configuration:** Switch 1 (CH_1) and Diode 1 (D_1) form the Class A part. Switch 2 (CH_2) and Diode 2 (D_2) form the Class B part. These two sections are connected in parallel across the DC source and the load.

2. **First Quadrant Operation (Class A behavior):** When CH_1 is turned ON, the load is connected to the source, and current flows in the positive direction (into the load). When CH_1 is turned OFF, the inductive load current continues to flow in the same direction through the anti-parallel diode D_2 .

3. **Second Quadrant Operation (Class B behavior):** If the load is an active load (like a DC motor with back EMF), and we turn ON CH_2 , the load current flows in the reverse direction (out of the load) through CH_2 . When CH_2 is turned OFF, this negative current is forced back into the source through diode D_1 .

4. **Result:** By connecting the Class A and Class B configurations in parallel, we create a single system that can handle both driving (motoring) and braking (regeneration) for a DC machine. The two switches are operated such that they are never ON at the same time to avoid a short circuit across the source.

Option (A) is incorrect because series connection would not allow for the independent current paths required for two-quadrant operation. Option (C) and (D) involve different chopper classes or topologies not used to define Class C. Therefore, the parallel combination of Class A and Class B is the definitive architectural requirement for a Class C chopper.

Step 4: Final Answer:

A class C chopper is formed by connecting Class A and Class B choppers in parallel.

This matches Option (B).

Quick Tip: Chopper Quadrant Mnemonic:

- A = 1st Quadrant only.
- B = 2nd Quadrant only.
- C = 1st + 2nd (Parallel combination).
- D = 1st + 4th.
- E = All 4 Quadrants.

Think of Class C as the "Combined" version of the first two!

192. What is the main disadvantage of phase angle control in AC voltage controllers?

- (A) It is very expensive
- (B) It cannot be used with DC
- (C) Introduction of harmonics in the system
- (D) It is very bulky

Correct Answer: (C) Introduction of harmonics in the system

Solution:

Step 1: Understanding the Concept:

AC voltage controllers (or AC regulators) are power electronic circuits that convert a fixed-voltage, fixed-frequency AC input into a variable-voltage AC output at the same frequency. These are commonly used for light dimmers, fan speed control, and industrial heating.

The most common control technique is "Phase Angle Control." In this method, a pair of anti-parallel Thyristors (SCRs) or a TRIAC is used as the switching element. The device is triggered at a specific "firing angle" (α) in every half-cycle of the AC supply. This effectively "slices" or "chops" the sine wave, allowing only a portion of the original energy to reach the load. While this is a simple and effective way to vary the RMS voltage, it significantly alters

the waveform of the power being delivered.

Step 2: Detailed Explanation:

The primary physical drawback of phase angle control arises from the **non-sinusoidal nature** of the output current and voltage.

1. **Waveform Distortion:** In a pure AC system, the ideal waveform is a smooth sine wave consisting of only the fundamental frequency (e.g., 50 Hz). By cutting the sine wave at an angle α , the resulting waveform becomes sharp and discontinuous.

2. **Fourier Analysis and Harmonics:** According to Fourier's theorem, any non-sinusoidal periodic wave can be decomposed into a sum of the fundamental frequency and its integer multiples (harmonics). Slicing the wave introduces a vast spectrum of odd harmonics (3rd, 5th, 7th, etc.).

3. Negative Impacts of Harmonics:

- **Heating:** Harmonics increase I^2R losses in transmission lines and cause excessive heating in motors and transformers.

- **Interference:** The high-frequency components of the switched wave can create electromagnetic interference (EMI) with communication lines and sensitive electronics.

- **Low Power Factor:** Harmonics introduce "Distortion Power Factor," which lowers the overall power factor of the system even if the load is purely resistive.

4. Comparison with other options:

- **Option A (Expensive):** Phase control is actually very cheap, requiring only a few thyristors and a simple trigger circuit.

- **Option B (Cannot be used with DC):** While true, this is a functional limitation (by definition, it's an AC controller), not a "disadvantage" of the control method itself.

- **Option D (Bulky):** These circuits are actually very compact because thyristors are solid-state and don't need large magnets or moving parts.

Therefore, the most significant technical issue that engineers must solve (usually through expensive filtering) when using phase angle control is the pollution of the grid with harmonic frequencies.

Step 3: Final Answer:

The main disadvantage of phase angle control is the introduction of harmonics in the system. This matches Option (C).

Quick Tip: In power electronics, remember that "Hard Switching" or "Phase Slicing" ALWAYS equals **Harmonics**. If a question asks for a disadvantage of SCR-based AC regulators or phase control, harmonics or low input power factor are the standard technical answers.

193. To obtain "Four-Quadrant" operation of a DC motor, which of the following is required?

- (A) A single-phase semi converter
- (B) Dual converter
- (C) Full-wave bridge rectifier
- (D) Step down chopper

Correct Answer: (B) Dual converter

Solution:

Step 1: Understanding the Concept:

Four-quadrant operation of a DC motor is a requirement in sophisticated industrial drives like electric elevators, cranes, and electric vehicles. In these applications, the motor must be able to:

1. **Quadrant 1 (Forward Motoring):** Rotate forward and provide driving torque.
2. **Quadrant 2 (Forward Braking):** Rotate forward but provide opposing torque (regeneration).
3. **Quadrant 3 (Reverse Motoring):** Rotate backward and provide driving torque.
4. **Quadrant 4 (Reverse Braking):** Rotate backward but provide opposing torque.

To achieve this, the power electronic converter must be able to reverse the polarity of the output DC voltage (V_a) and the direction of the armature current (I_a).

Step 2: Detailed Explanation:

Let's evaluate the capabilities of the listed converters:

1. **Single-phase Semi-converter (Option A):** A semi-converter uses both diodes and SCRs.

It can only provide a positive current and a positive/zero voltage. It is a **one-quadrant** converter (motoring only). It cannot perform regenerative braking.

2. **Full-wave Bridge Rectifier (Option C):** If it uses only diodes, it provides a fixed DC. If it's a fully controlled SCR bridge, it can operate in two quadrants (positive current, reversible voltage), allowing for braking but not reverse rotation.

3. **Step-down Chopper (Option D):** A standard Class A chopper is a single-quadrant device. Even a Class C chopper only covers two quadrants.

4. **Dual Converter (Option B):** A dual converter consists of two fully controlled bridge rectifiers connected back-to-back in anti-parallel.

- Bridge 1 provides positive current ($I_a > 0$) and reversible voltage.

- Bridge 2 provides negative current ($I_a < 0$) and reversible voltage.

By switching between these two bridges, the controller can achieve any combination of voltage and current polarities. This enables the motor to run in both directions and perform regenerative braking in both directions. This is the definition of a four-quadrant drive. Dual converters are the standard solution for high-power DC motor control needing full flexibility.

Step 3: Final Answer:

A dual converter is required to obtain four-quadrant operation of a DC motor.

This matches Option (B).

Quick Tip: Mnemonic for Converters:

- **Semi** = 1 Quadrant.

- **Full** = 2 Quadrants (V reversible, I fixed).

- **Dual** = 4 Quadrants (Full bidirectional control).

If the motor needs to go **Backwards AND Brake**, you **Must** use a **Dual** converter.

194. A 3-phase AC voltage regulator used for motor control typically employs

- (A) Six diodes in a bridge
- (B) Six SCRs connected in anti-parallel pairs
- (C) One TRIAC per phase

(D) Four MOSFETS and a capacitor

Correct Answer: (B) Six SCRs connected in anti-parallel pairs

Solution:

Step 1: Understanding the Concept:

Three-phase AC voltage regulators are used to vary the RMS value of the AC voltage supplied to a three-phase load, most commonly a three-phase induction motor for "Soft Starting" or "Fan/Pump Speed Control."

To control the voltage of an AC signal, the power electronic switch must be able to conduct current in **both halves** of the cycle (positive and negative). A single diode or SCR can only conduct in one direction. Therefore, for each phase of the supply, we need a bidirectional switching arrangement. In high-power industrial applications, this is achieved by using two Silicon Controlled Rectifiers (SCRs) connected in anti-parallel (back-to-back).

Step 2: Detailed Explanation:

Let's analyze why Option (B) is the industry standard for 3-phase systems:

1. **Per Phase Requirement:** In a 3-phase system, there are three lines (R, Y, B). Each line requires a bidirectional switch to control its voltage.
2. **The SCR Pair:** One pair of anti-parallel SCRs consists of two SCRs. One SCR handles the positive half-cycle, and the other handles the negative half-cycle.
3. **Total Count:** For a full 3-phase controller: $3 \text{ phases} \times 2 \text{ SCRs per phase} = 6 \text{ SCRs}$.
4. **Evaluating other options:**

- **Option A (Six diodes):** This would form a passive 3-phase bridge rectifier, converting AC to DC. It provides no control over the AC voltage magnitude for a motor.

- **Option C (One TRIAC per phase):** While a TRIAC is bidirectional, it is generally limited to low-power, single-phase domestic applications (like ceiling fans). For large industrial motors, TRIACs are avoided because they have poor dv/dt ratings and are harder to cool than discrete SCRs.

- **Option D (MOSFETs):** While MOSFETs can be used in some converter types, a standard AC regulator for induction motors is a thyristor-based design due to the extreme robustness and current capacity of SCRs.

Thus, "Six SCRs in anti-parallel pairs" is the textbook configuration for a three-phase AC voltage regulator used as a soft-starter for induction motors.

Step 3: Final Answer:

A 3-phase AC voltage regulator typically employs six SCRs connected in anti-parallel pairs. This matches Option (B).

Quick Tip: AC Voltage Control rule:

To control AC, you need **Bidirectional** switches.

- 1 Phase AC regulator = 1 pair of SCRs (2 total).
- 3 Phase AC regulator = 3 pairs of SCRs (**6 total**).

Diodes cannot be used for "control" as they have no gate terminal.

195. The typical switching frequency of a modern SMPS ranges from

- (A) 50 Hz to 100 Hz
- (B) 1 Hz to 5 Hz
- (C) 20 KHz to several MHz
- (D) 25 Hz to 50 Hz

Correct Answer: (C) 20 KHz to several MHz

Solution:**Step 1: Understanding the Concept:**

SMPS stands for Switched-Mode Power Supply. It is an electronic power supply that incorporates a switching regulator to convert electrical power efficiently. Unlike linear power supplies, which use a transformer and a linear regulator operating at the line frequency (50/60 Hz), an SMPS converts the input power into a high-frequency AC signal before rectifying it to the desired DC output.

The fundamental reason for using "high frequency" in power supply design is related to the physics of electromagnetic components. The size and weight of transformers and inductors are **inversely proportional** to the operating frequency. By increasing the frequency, engineers

can use much smaller and lighter components to handle the same amount of power.

Step 2: Detailed Explanation:

Let's analyze the frequency ranges provided:

1. **Low Frequencies (Options A, B, D):** Frequencies in the range of 1 Hz to 100 Hz are extremely low. 50/60 Hz is the standard utility grid frequency. Linear power supplies operate at this frequency, which is why their transformers are massive, heavy, and made of laminated steel. If an SMPS operated at these frequencies, it would lose all its size and efficiency advantages.

2. **Audio Threshold:** The human ear can hear sounds up to approximately 20 kHz. To avoid creating an annoying "whining" or "humming" sound during operation, almost all SMPS are designed to switch at frequencies **above 20 kHz**.

3. Modern Standards:

- Basic SMPS (like computer power supplies): Typically switch in the range of **20 kHz to 100 kHz**.
- Advanced/High-density SMPS (like phone chargers or DC-DC converters in EVs): Utilize frequencies from **500 kHz to several MHz**.
- **Benefits of MHz switching:** It allows the use of tiny ceramic capacitors and air-core or ferrite-core inductors, enabling ultra-compact designs.

Therefore, modern SMPS technology is defined by its operation in the ultrasonic and radio-frequency (RF) spectrum.

Step 3: Final Answer:

The typical switching frequency of a modern SMPS ranges from 20 KHz to several MHz.

This matches Option (C).

Quick Tip: SMPS Frequency Shortcut:

Always look for the **Kilo-Hertz (KHz)** or **Mega-Hertz (MHz)** options.

Anything in the **Hertz (Hz)** range belongs to old-fashioned "Linear" power supplies or large industrial grid-tied equipment. High frequency = High efficiency + Low weight.

196. What is the transfer time for a high quality online UPS?

- (A) 30 milli seconds
- (B) Zero milli seconds
- (C) One second
- (D) 40 milli seconds

Correct Answer: (B) Zero milli seconds

Solution:

Step 1: Understanding the Concept:

An Uninterruptible Power Supply (UPS) is a device that provides emergency power to a load when the input power source or mains power fails. There are two primary types of UPS systems:

1. **Offline (Standby) UPS:** In normal mode, the load is connected directly to the utility mains. When power fails, a relay switches the load to the battery-powered inverter. This switching process takes time, known as the "transfer time" (typically 4 to 10 milliseconds).
2. **Online (Double Conversion) UPS:** This system operates differently. The incoming AC power is first converted to DC (rectifier), which charges the battery and simultaneously powers the inverter. The inverter converts the DC back to high-quality AC to power the load.

Step 2: Detailed Explanation:

The "Online" UPS is considered the highest quality for several reasons, primarily related to its transfer behavior.

1. **Continuous Inverter Path:** Because the inverter is **always** running and currently providing power to the load, there is no "switching" required when the utility power is lost. The battery is already "on the bus," providing the necessary DC energy to the inverter without any interruption.
2. **Transfer Time:** Since there is no relay transition or mechanical switching between different sources for the load, the transfer time is **Zero milliseconds**.
3. **Load Protection:** This zero-break power is essential for extremely sensitive equipment like medical surgical systems, high-end servers, and laboratory scientific instruments that might reboot or fail if there is even a 5-millisecond dip in voltage.

4. Analyzing the options:

- Options (A), (C), and (D) involve measurable delays. While a computer might handle 10-20ms, "high quality online" systems are defined by their seamless, non-stop operation. Therefore, the definitive characteristic of an Online Double Conversion UPS is the total absence of a transfer delay.

Step 3: Final Answer:

The transfer time for a high quality online UPS is zero milli seconds.
This matches Option (B).

Quick Tip: UPS Comparison Table:

- **Offline:** Transfer time = **4-10ms** (Standard).
- **Line-Interactive:** Transfer time = **2-4ms**.
- **Online:** Transfer time = **0ms** (Seamless).

Think: "Online" means the backup is **already in line** and working.

197. What is the main advantage of regenerative braking in EVs?

- (A) It stops the car faster
- (B) It recovers kinetic energy to charge the battery
- (C) It eliminates the need for brake pads
- (D) It increases the top speed

Correct Answer: (B) It recovers kinetic energy to charge the battery

Solution:

Step 1: Understanding the Concept:

In conventional internal combustion engine (ICE) vehicles, braking is achieved using friction. When the driver presses the brake pedal, brake pads press against rotors, and the kinetic energy of the moving vehicle is converted entirely into **waste heat**.

Electric Vehicles (EVs) utilize a technology called "Regenerative Braking." When the driver lifts their foot from the accelerator or lightly presses the brake, the power electronic controller reverses the flow of electricity. Instead of the battery pushing current into the motor to move the wheels, the wheels turn the motor, which now acts as a **generator**.

Step 2: Detailed Explanation:

The physical and economic benefits of this process are significant:

1. **Energy Recovery:** As the motor acts as a generator, it converts the vehicle's kinetic energy into electrical energy. This electricity is sent back through the inverter to **charge the main battery pack**.

2. **Increased Efficiency/Range:** In city driving with frequent "stop-and-go" traffic, regenerative braking can recover enough energy to extend the vehicle's driving range by as much as 10% to 20%. This effectively gives the car "free" mileage from energy that would have been lost in an ICE vehicle.

3. Evaluating other options:

- **Option A (Stops faster):** Incorrect. Conventional hydraulic brakes are actually more powerful for emergency stops. Regenerative braking provides "gentle" braking.

- **Option C (Eliminates pads):** Incorrect. While it **reduces wear** on brake pads (making them last 2-3 times longer), EVs still need physical friction brakes for emergency stops and for holding the car still when parked.

- **Option D (Increases top speed):** Incorrect. Braking happens when you want to go slower; it has no effect on the maximum speed capability of the motor.

Therefore, the "main" advantage from an engineering and consumer perspective is the energy conservation aspect.

Step 3: Final Answer:

The main advantage is that it recovers kinetic energy to charge the battery.

This matches Option (B).

Quick Tip: EV logic: **Regeneration = Recycling.**

Think of it as catching the energy you already used to speed up and putting it back in your "fuel tank" when you slow down. It is the single biggest reason why EVs are much more efficient in city traffic than on open highways.

198. What is the primary material used as the anode in most Lithium-Ion batteries for EVs?

- (A) Lithium metal
- (B) Cobalt
- (C) Graphite
- (D) Aluminum

Correct Answer: (C) Graphite

Solution:

Step 1: Understanding the Concept:

A Lithium-Ion battery is an electrochemical cell consisting of a cathode (positive electrode), an anode (negative electrode), a separator, and an electrolyte. During discharge (powering the car), lithium ions move from the anode to the cathode. During charging, they move from the cathode and "intercalate" (nestle inside the molecular layers) of the anode material.

The choice of anode material is critical for the battery's capacity, life cycle, and safety. A good anode must be able to host large numbers of lithium ions without undergoing significant physical expansion or structural damage.

Step 2: Detailed Explanation:

Let's analyze the components of a typical EV battery:

1. **Anode (Negative):** For several decades, **Graphite** (a form of carbon) has been the industry standard anode material.

- **Why Graphite?** It has a hexagonal layered structure. Lithium ions can easily slip between these layers (intercalation) and stay there until needed. Graphite is cheap, abundant, electrically conductive, and chemically stable over thousands of charge cycles.

2. **Cathode (Positive):** This is where materials like Lithium Cobalt Oxide (LCO), Nickel

Manganese Cobalt (NMC), or Lithium Iron Phosphate (LFP) are used. **Cobalt (Option B)** is a cathode material, not an anode material.

3. **Current Collectors: Aluminum (Option D)** is typically used as the current collector for the *cathode*, while copper is used for the anode.

4. **Lithium Metal (Option A):** While "Lithium Metal batteries" are an active area of research (solid-state batteries), they are currently not used in standard EVs because lithium metal forms "dendrites" (microscopic spikes) that can pierce the separator and cause a fire.

Therefore, for the current generation of mass-produced electric vehicles (Tesla, BYD, etc.), graphite remains the primary anode material, sometimes blended with a small amount of silicon to increase capacity.

Step 3: Final Answer:

The primary anode material in Li-ion batteries is Graphite.

This matches Option (C).

Quick Tip: Battery Material Shortcut:

- **Anode** (Negative) = **Graphite** (Carbon).
- **Cathode** (Positive) = Lithium **Oxides** (NMC, LFP, etc.).

Graphite is the "storage shelf" for the Lithium ions when the battery is full.

199. The charging rate of a battery typically decreases with

- (A) An increase in temperature
- (B) A decrease in temperature
- (C) Decrease in internal resistance
- (D) An increase in plate surface area

Correct Answer: (B) A decrease in temperature

Solution:

Step 1: Understanding the Concept:

The performance and charging behavior of an electrochemical battery (like Lead-Acid or Lithium-Ion) are heavily influenced by environmental conditions, particularly temperature. Charging a battery is essentially a **chemical reaction** (moving ions and causing redox transitions). Like most chemical processes, the speed of these reactions is temperature-dependent. Additionally, the physical movement of ions through the electrolyte (ion mobility) is affected by the viscosity and thermal energy of the medium.

Step 2: Detailed Explanation:

Let's analyze why a **decrease in temperature** reduces the charging rate:

1. **Reduced Ion Mobility:** In cold conditions, the electrolyte becomes more "sluggish." It is harder for lithium ions (or lead-sulfate ions) to travel between the plates. This effectively increases the "internal resistance" of the battery.

2. **Increased Internal Resistance (Option C):** As the battery gets colder, its internal resistance **increases**. A higher resistance opposes the charging current, slowing down the rate at which energy can be pushed into the battery. Thus, a "decrease" in resistance (as stated in Option C) would actually *increase* the charging rate.

3. **Safety Limits:** Most modern Battery Management Systems (BMS) intentionally slow down the charging current in cold weather to prevent "Lithium Plating." If you try to force high current into a cold battery, the ions cannot intercalate into the graphite fast enough and instead form solid metal on the surface, which can permanently damage the battery.

4. Analyzing other options:

- **Option A (Increase in temperature):** Generally, warmer batteries can be charged **faster** because ions move more easily. (However, extreme heat triggers safety cut-offs, but "typically," heat speeds up the chemistry).

- **Option D (Increase in surface area):** Larger plates provide more space for reactions, which would **increase** the possible charging rate, not decrease it.

Therefore, cold weather is the primary "natural" factor that forces a reduction in the charging speed of batteries.

Step 3: Final Answer:

The charging rate typically decreases with a decrease in temperature.
This matches Option (B).

Quick Tip: Winter Charging Tip:

If you have an electric car in a very cold climate, you will notice "Slow Charging" messages. This is because the **BMS is protecting the battery** from damage due to high resistance and low ion mobility caused by the cold. Always warm up your battery (pre-conditioning) for the fastest charging!

200. In a BMS, what does "Thermal Management" refer to?

- (A) Only cooling the battery
- (B) Only heating the battery
- (C) Monitoring and controlling the battery's temperature to operate within safe limits
- (D) Maximizing battery weight

Correct Answer: (C) Monitoring and controlling the battery's temperature to operate within safe limits

Solution:

Step 1: Understanding the Concept:

BMS stands for **Battery Management System**. It is the "brain" of an electric vehicle's battery pack. Its primary responsibility is to ensure that the battery cells operate safely, efficiently, and for as long as possible. Lithium-ion batteries are like "living organisms"—they are very sensitive to their environment. If they get too hot, they can undergo "Thermal Runaway" (fire/explosion). If they are too cold, they lose power and cannot charge quickly. "Thermal Management" is the sub-system of the BMS dedicated to maintaining the "Goldilocks" temperature zone (typically between 15°C and 35°C).

Step 2: Detailed Explanation:

Thermal management is a **bidirectional** control process. It is not just about one action.

1. **Monitoring:** The BMS uses multiple temperature sensors (thermistors) placed throughout the battery pack to constantly track the heat levels of individual cell modules.
2. **Cooling (Option A):** During fast charging or high-speed driving, the battery generates significant heat due to internal resistance (I^2R). The BMS activates cooling systems (like liquid-cooled plates, fans, or refrigerant loops) to prevent the battery from degrading or catching fire.
3. **Heating (Option B):** In very cold winters, the battery chemistry becomes inefficient. The BMS can activate internal heaters to bring the battery up to an optimal operating temperature so the car can start and perform normally.
4. **The "Management" aspect:** Because the system must both **heat** when cold and **cool** when hot, and do so based on active **monitoring**, the term covers the entire scope of temperature regulation.
5. **Analyzing Option D:** Maximizing weight is undesirable in EVs; engineers want the lightest battery possible for better range.

Therefore, Thermal Management is the comprehensive process of sensing and adjusting the thermal state of the battery to keep it within its designed "Safe Operating Area" (SOA).

Step 3: Final Answer:

Thermal Management refers to monitoring and controlling the battery's temperature to operate within safe limits.

This matches Option (C).

Quick Tip: BMS Key Functions:

1. **Voltage** Monitoring (Cell balancing).
2. **Current** Protection (Fuses/Breakers).
3. **Thermal** Management (Heating/Cooling).

Whenever you see the word "**Management**" in engineering, it almost always implies "**Monitoring AND Controlling**" a parameter.