

AP ECET 2026 Civil Engineering

Question Paper with Solutions PDF

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General Instructions

- (i) **Duration:** The total duration of the examination is 3 hours (180 minutes).
- (ii) **Total Marks:** The complete paper carries a maximum of 200 marks.
- (iii) **Structure:** The paper has 4 Sections:
 - **Section A:** 50 Multiple Choice Questions (Mathematics).
 - **Section B:** 25 Multiple Choice Questions (Physics).
 - **Section C:** 25 Multiple Choice Questions (Chemistry).
 - **Section D:** 100 Multiple Choice Questions (Civil Engineering).
- (iv) **Compulsory Questions:** All 200 questions are compulsory.
- (v) Each question has four options. Only **one** option is correct.
- (vi) **Correct Answer:** +1 marks.
- (vii) **Incorrect Answer:** (No Negative marking).
- (viii) **Unanswered/Marked for Review:** 0 marks.

MATHEMATICS

1. In the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 4 & 9 \end{bmatrix}$, the minor M_{23} of the a_{23} is ____.

(1) 10

- (2) -10
- (3) -6
- (4) 6

Correct Answer: (2) -10

Solution:

Step 1: Understanding the Concept:

The minor M_{ij} of an element a_{ij} in a matrix is the determinant of the submatrix left after deleting the i^{th} row and j^{th} column.

Step 2: Key Formula or Approach:

For an element a_{23} , we delete the 2nd row and 3rd column of matrix A and calculate the determinant of the remaining 2×2 matrix.

Step 3: Detailed Explanation:

$$\text{Matrix } A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 4 & 9 \end{bmatrix}.$$

The element a_{23} is in the 2nd row and 3rd column (which is 6).

Deleting the 2nd row $[4, 5, 6]$ and the 3rd column $[3, 6, 9]^T$, we get:

$$M_{23} = \begin{vmatrix} 1 & 2 \\ 7 & 4 \end{vmatrix}$$

Calculating the determinant:

$$M_{23} = (1 \times 4) - (2 \times 7)$$

$$M_{23} = 4 - 14 = -10$$

Step 4: Final Answer:

The minor M_{23} is -10.

Quick Tip: To find a minor, simply "hide" the row and column of the element and find the determinant of what remains. Don't confuse it with a cofactor, which also requires a sign change based on position.

2. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$ then the value of x is ____.

- (1) 3
- (2) ± 6
- (3) -3
- (4) 6

Correct Answer: (2) ± 6

Solution:

Step 1: Understanding the Concept:

Unlike matrices, where corresponding elements are equal, in the case of determinants, we must evaluate the numerical value of the determinants on both sides to solve for the variable.

Step 2: Key Formula or Approach:

For a 2×2 determinant $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$, the value is $ad - bc$.

Step 3: Detailed Explanation:

Evaluate the left-hand side (LHS):

$$\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = (2x \times x) - (5 \times 8) = 2x^2 - 40$$

Evaluate the right-hand side (RHS):

$$\begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix} = (6 \times 3) - (-2 \times 7) = 18 - (-14) = 18 + 14 = 32$$

Set LHS = RHS:

$$2x^2 - 40 = 32$$

$$2x^2 = 32 + 40$$

$$2x^2 = 72$$

$$x^2 = 36$$

$$x = \pm 6$$

Step 4: Final Answer:

The value of x is ± 6 .

Quick Tip: Common Error: Do not equate elements like $2x = 6$. This is only allowed in matrix equality, not determinant equality.

3. If A is a square matrix of order 3 and $|A| = 5$, then the value of $|2A^T|$ is ____.

- (1) -10
- (2) 10
- (3) 40
- (4) -40

Correct Answer: (3) 40

Solution:**Step 1: Understanding the Concept:**

This problem uses properties of determinants regarding scalar multiplication and transposes. Specifically, for a matrix of order n , the scalar comes out raised to the power n .

Step 2: Key Formula or Approach:

1. $|kA| = k^n|A|$, where n is the order of the matrix.
2. $|A^T| = |A|$.

Step 3: Detailed Explanation:

Given $|A| = 5$ and order $n = 3$.

We need to find $|2A^T|$.

First, applying the property $|kA| = k^n|A|$:

$$|2A^T| = 2^3|A^T|$$

Since $2^3 = 8$:

$$|2A^T| = 8|A^T|$$

Using the property $|A^T| = |A|$:

$$|2A^T| = 8|A|$$

Substitute $|A| = 5$:

$$|2A^T| = 8 \times 5 = 40$$

Step 4: Final Answer:

The value of $|2A^T|$ is 40.

Quick Tip: Always identify the order n of the matrix first. If the order was 2, the answer would be $2^2 \times 5 = 20$.

4. Which of the following systems has non trivial solution?

- (1) $AX = 0, |A| = 4$
- (2) $AX = 0, |A| = -4$
- (3) $AX = 0, |A| = 0$
- (4) $AX = B, |B| = 5$

Correct Answer: (3) $AX = 0, |A| = 0$

Solution:

Step 1: Understanding the Concept:

A homogeneous system of linear equations $AX = 0$ always has the trivial solution $X = 0$. For it

to have a non-trivial solution (infinitely many solutions), the matrix A must be singular.

Step 2: Key Formula or Approach:

For a homogeneous system $AX = 0$:

1. If $|A| \neq 0$, the system has only a trivial solution ($X = 0$).
2. If $|A| = 0$, the system has non-trivial solutions.

Step 3: Detailed Explanation:

In options (1) and (2), $|A| \neq 0$, so these systems have only the unique trivial solution.

In option (4), the system is non-homogeneous ($AX = B$), which typically refers to consistency rather than the concept of "non-trivial" solutions as applied to homogeneous systems.

In option (3), since $|A| = 0$, the matrix is singular, which implies that the rows/columns are linearly dependent, leading to at least one free variable and thus non-trivial solutions.

Step 4: Final Answer:

The system with a non-trivial solution is $AX = 0$ where $|A| = 0$.

Quick Tip: Remember: "Singular matrix ($|A| = 0$) = Non-trivial solutions" for homogeneous equations. If the determinant is anything other than zero, you only get the zero solution.

5. If $\begin{bmatrix} x+y & 2 \\ 1 & x-y \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 1 & 2 \end{bmatrix}$, then the values of x and y are:

- (1) $x = 3, y = 1$
- (2) $x = 1, y = 3$
- (3) $x = 2, y = 3$
- (4) $x = 1, y = 1$

Correct Answer: (1) $x = 3, y = 1$

Solution:

Step 1: Understanding the Concept:

Two matrices are equal if and only if their corresponding elements are identical. We can set

up a system of linear equations by equating the corresponding entries.

Step 2: Key Formula or Approach:

Equate a_{11} with b_{11} and a_{22} with b_{22} .

Step 3: Detailed Explanation:

From the given matrices:

$$x + y = 4 \quad \text{---(i)}$$

$$x - y = 2 \quad \text{---(ii)}$$

Adding equations (i) and (ii):

$$(x + y) + (x - y) = 4 + 2$$

$$2x = 6 \implies x = 3$$

Substituting $x = 3$ into equation (i):

$$3 + y = 4 \implies y = 1$$

Step 4: Final Answer:

The values are $x = 3$ and $y = 1$.

Quick Tip: When dealing with matrix equality, always double-check the constant elements (like 2 and 1 in this problem) to ensure the matrices are actually comparable.

6. If $\frac{x+4}{(x+2)^2(x+3)} = \frac{A}{(x+2)^2} + \frac{B}{x+2} + \frac{C}{x+3}$ then $A + B + C =$

- (1) 2
- (2) 1
- (3) -1
- (4) 3

Correct Answer: (2) 1

Solution:

Step 1: Understanding the Concept:

This problem involves partial fraction decomposition. We need to find the constants A , B , and C by clearing the denominators and equating the numerators.

Step 2: Key Formula or Approach:

Multiply the entire equation by the common denominator $(x + 2)^2(x + 3)$:

$$x + 4 = A(x + 3) + B(x + 2)(x + 3) + C(x + 2)^2$$

Step 3: Detailed Explanation:

To find the constants:

1. Let $x = -2$:

$$-2 + 4 = A(-2 + 3) + B(0) + C(0) \implies 2 = A(1) \implies A = 2$$

2. Let $x = -3$:

$$-3 + 4 = A(0) + B(0) + C(-3 + 2)^2 \implies 1 = C(-1)^2 \implies C = 1$$

3. To find B , compare the coefficients of x^2 on both sides:

$$0 = B + C$$

$$0 = B + 1 \implies B = -1$$

Calculating $A + B + C$:

$$2 + (-1) + 1 = 2$$

Wait, let's re-verify the calculation: $2 - 1 + 1 = 2$. Let's re-check the sum. $A = 2, B = -1, C = 1$. Sum is 2. (Note: Re-checking options, if the question meant specific individual values, but the sum is requested).

Step 4: Final Answer:

The value of $A + B + C$ is 2.

Quick Tip: To quickly find the sum $A + B + C$, you can sometimes substitute a convenient value for x . However, comparing coefficients of the highest power (x^2) is often the fastest way to relate B and C .

7. If $\frac{x}{(x-1)^2(x+2)} = \frac{A}{(x-1)^2} + \frac{2}{9(x-1)} + \frac{B}{x+2}$ then $A + B = \underline{\hspace{1cm}}$.

- (1) $1/3$
- (2) $1/9$
- (3) $-1/3$
- (4) $2/3$

Correct Answer: (2) $1/9$

Solution:**Step 1: Understanding the Concept:**

This problem involves partial fraction decomposition for a repeated linear factor. We need to find the constants A and B by equating the numerators after finding a common denominator.

Step 2: Key Formula or Approach:

Multiply the entire equation by the common denominator $(x-1)^2(x+2)$:

$$x = A(x+2) + \frac{2}{9}(x-1)(x+2) + B(x-1)^2$$

Step 3: Detailed Explanation:

To find A , substitute $x = 1$:

$$1 = A(1+2) + 0 + 0 \implies 1 = 3A \implies A = 1/3$$

To find B , substitute $x = -2$:

$$-2 = 0 + 0 + B(-2 - 1)^2 \implies -2 = 9B \implies B = -2/9$$

Now, calculate $A + B$:

$$A + B = \frac{1}{3} + \left(-\frac{2}{9}\right)$$

$$A + B = \frac{3}{9} - \frac{2}{9} = \frac{1}{9}$$

Step 4: Final Answer:

The value of $A + B$ is $1/9$.

Quick Tip: To solve for a specific constant quickly, substitute the roots of the denominator. For A , use $x = 1$; for B , use $x = -2$.

8. If $\tan A = 1/2$ and $\tan B = 1/3$, then $A + B = \underline{\hspace{1cm}}$.

- (1) 30°
- (2) 45°
- (3) 60°
- (4) 90°

Correct Answer: (2) 45°

Solution:

Step 1: Understanding the Concept:

We can determine the sum of two angles by using the tangent addition formula if the tangent values of the individual angles are known.

Step 2: Key Formula or Approach:

Use the identity:

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

Step 3: Detailed Explanation:

Substitute the given values $\tan A = 1/2$ and $\tan B = 1/3$:

$$\tan(A + B) = \frac{\frac{1}{2} + \frac{1}{3}}{1 - (\frac{1}{2} \times \frac{1}{3})}$$

$$\tan(A + B) = \frac{\frac{3+2}{6}}{1 - \frac{1}{6}} = \frac{\frac{5}{6}}{\frac{5}{6}}$$

$$\tan(A + B) = 1$$

Since $\tan(45^\circ) = 1$:

$$A + B = 45^\circ$$

Step 4: Final Answer:

The sum $A + B$ is 45° .

Quick Tip: This is a very common result in trigonometry. Whenever $\tan A$ and $\tan B$ are reciprocals of consecutive integers (like 2 and 3), their sum often results in 45° .

9. If $2 \sin^{-1} x = \sin^{-1} k$ then $k = \underline{\hspace{2cm}}$.

(1) $2x\sqrt{1-x^2}$

(2) x

(3) x^2

(4) $1 - 2x^2$

Correct Answer: (1) $2x\sqrt{1-x^2}$

Solution:

Step 1: Understanding the Concept:

This problem requires the use of inverse trigonometric identities, specifically the double angle formula for sine expressed in inverse form.

Step 2: Key Formula or Approach:

Let $\sin^{-1} x = \theta$, then $x = \sin \theta$. The expression becomes $2\theta = \sin^{-1} k$, or $\sin(2\theta) = k$.

Step 3: Detailed Explanation:

We know the double angle identity:

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

Substitute $\sin \theta = x$. Since $\cos \theta = \sqrt{1 - \sin^2 \theta}$, we have $\cos \theta = \sqrt{1 - x^2}$.

$$k = 2x \sqrt{1 - x^2}$$

Thus:

$$2 \sin^{-1} x = \sin^{-1}(2x \sqrt{1 - x^2})$$

Step 4: Final Answer:

The value of k is $2x \sqrt{1 - x^2}$.

Quick Tip: To remember this, think of the standard sine double-angle formula $\sin 2A = 2 \sin A \cos A$. The square root part $\sqrt{1 - x^2}$ simply represents the cosine term.

10. If $\sin^{-1} \frac{5}{x} + \sin^{-1} \frac{12}{x} = \frac{\pi}{2}$, then $x = \underline{\hspace{1cm}}$.

- (1) 12
- (2) 7
- (3) 13
- (4) 15

Correct Answer: (3) 13

Solution:

Step 1: Understanding the Concept:

We use the identity $\sin^{-1} \theta + \cos^{-1} \theta = \frac{\pi}{2}$. By comparing this identity to the given equation, we can relate sine and cosine functions.

Step 2: Key Formula or Approach:

If $\sin^{-1} \alpha + \sin^{-1} \beta = \frac{\pi}{2}$, then $\sin^{-1} \alpha = \frac{\pi}{2} - \sin^{-1} \beta$. Since $\frac{\pi}{2} - \sin^{-1} \beta = \cos^{-1} \beta$, we have $\sin^{-1} \alpha = \cos^{-1} \beta$.

Step 3: Detailed Explanation:

Given:

$$\sin^{-1} \frac{5}{x} = \cos^{-1} \frac{12}{x}$$

Let $\cos^{-1} \frac{12}{x} = \phi$, then $\cos \phi = \frac{12}{x}$. From the identity $\sin^2 \phi + \cos^2 \phi = 1$:

$$\sin \phi = \sqrt{1 - \left(\frac{12}{x}\right)^2} = \sqrt{\frac{x^2 - 144}{x^2}}$$

So, $\sin^{-1} \frac{5}{x} = \sin^{-1} \frac{\sqrt{x^2 - 144}}{x}$. Equating the arguments:

$$\frac{5}{x} = \frac{\sqrt{x^2 - 144}}{x}$$

$$5 = \sqrt{x^2 - 144}$$

$$25 = x^2 - 144 \implies x^2 = 169 \implies x = 13$$

Step 4: Final Answer:

The value of x is 13.

Quick Tip: Recognize the Pythagorean triplet (5, 12, 13). In equations of the form $\sin^{-1}(a/x) + \sin^{-1}(b/x) = \pi/2$, x will be the hypotenuse $\sqrt{a^2 + b^2}$.

11. The number of solutions of the equation $\sin 2x - \cos 2x = 2 - \sin 2x$ lying in the interval $[0, \pi]$ is ____.

- (1) 0
- (2) 1
- (3) 2
- (4) 3

Correct Answer: (1) 0

Solution:

Step 1: Understanding the Concept:

We need to simplify the trigonometric equation to find the values of x that satisfy it within the specific range $[0, \pi]$. The range of trigonometric functions like sine and cosine is restricted, which often limits the number of possible solutions.

Step 2: Key Formula or Approach:

1. Rearrange the equation to group similar trigonometric terms.
2. Use the property that for any angle α , $-\sqrt{a^2 + b^2} \leq a \sin \alpha + b \cos \alpha \leq \sqrt{a^2 + b^2}$.

Step 3: Detailed Explanation:

Given equation: $\sin 2x - \cos 2x = 2 - \sin 2x$

Add $\sin 2x$ to both sides:

$$2 \sin 2x - \cos 2x = 2$$

To check if this has a solution, we look at the maximum possible value of the expression on the left side: $f(x) = 2 \sin 2x - 1 \cos 2x$.

The maximum value is $\sqrt{(2)^2 + (-1)^2} = \sqrt{4 + 1} = \sqrt{5}$.

Since $\sqrt{5} \approx 2.236$, the value 2 is reachable. Let's solve for when it equals 2: Divide by $\sqrt{5}$:

$$\frac{2}{\sqrt{5}} \sin 2x - \frac{1}{\sqrt{5}} \cos 2x = \frac{2}{\sqrt{5}}$$

Let $\cos \alpha = \frac{2}{\sqrt{5}}$ and $\sin \alpha = \frac{1}{\sqrt{5}}$. The equation becomes:

$$\sin(2x - \alpha) = \sin \alpha \implies 2x - \alpha = n\pi + (-1)^n \alpha$$

For $n = 0$: $2x - \alpha = \alpha \implies 2x = 2\alpha \implies x = \alpha$.

Since $\cos \alpha = \frac{2}{\sqrt{5}}$ and $\sin \alpha = \frac{1}{\sqrt{5}}$, α is a very small positive angle.

However, checking the original equation for $x = \alpha$: $2(1/\sqrt{5}) - 2/\sqrt{5} = 0 \neq 2$. (Wait, the simplified equation was $2 \sin 2x - \cos 2x = 2$). If $\sin 2x = 1$ and $\cos 2x = 0$, LHS = 2. But if $\sin 2x = 1$, then $2x = \pi/2$, so $\cos 2x = 0$. LHS = $2(1) - 0 = 2$. This works!

If $2x = \pi/2$, then $x = \pi/4$.

But checking the options provided in typical academic contexts for this specific problem, there might be a constraint or a typo in the original text leading to '0'. Let's re-verify $x = \pi/4$: $\sin(\pi/2) - \cos(\pi/2) = 1 - 0 = 1$. RHS: $2 - \sin(\pi/2) = 2 - 1 = 1$. So $x = \pi/4$ is a valid solution in $[0, \pi]$. If the question is copied exactly, the answer is 1. If the answer key says 0, there is a discrepancy in the equation's constants.

Step 4: Final Answer:

The number of solutions is 1.

Quick Tip: Always check the bounds of your expression. If the constant on the right side is greater than the maximum value of the left side (like if it were 3 instead of 2), there would be 0 solutions.

12. If $\tan \theta + \sec \theta = \sqrt{3}$ then the principal value of θ in $[0, 2\pi]$ is ____.

- (1) $\pi/4$
- (2) $\pi/6$
- (3) $\pi/2$
- (4) $2\pi/3$

Correct Answer: (2) $\pi/6$

Solution:

Step 1: Understanding the Concept:

The problem involves basic trigonometric identities. We need to find the angle θ that satisfies the sum of the tangent and secant functions.

Step 2: Key Formula or Approach:

1. Use the identity $\sec^2 \theta - \tan^2 \theta = 1$.
2. Factor as $(\sec \theta - \tan \theta)(\sec \theta + \tan \theta) = 1$.

Step 3: Detailed Explanation:

Given $\sec \theta + \tan \theta = \sqrt{3}$.

Using the identity:

$$\sec \theta - \tan \theta = \frac{1}{\sec \theta + \tan \theta} = \frac{1}{\sqrt{3}}$$

Add the two equations:

$$(\sec \theta + \tan \theta) + (\sec \theta - \tan \theta) = \sqrt{3} + \frac{1}{\sqrt{3}}$$

$$2 \sec \theta = \frac{3+1}{\sqrt{3}} = \frac{4}{\sqrt{3}}$$

$$\sec \theta = \frac{2}{\sqrt{3}} \Rightarrow \cos \theta = \frac{\sqrt{3}}{2}$$

For $\cos \theta = \frac{\sqrt{3}}{2}$, the values in $[0, 2\pi]$ are $\pi/6$ and $11\pi/6$.

Checking $\theta = \pi/6$: $\tan(\pi/6) + \sec(\pi/6) = \frac{1}{\sqrt{3}} + \frac{2}{\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$. (Correct)

Checking $\theta = 11\pi/6$: $\tan(11\pi/6) + \sec(11\pi/6) = -\frac{1}{\sqrt{3}} + \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$. (Incorrect)

Step 4: Final Answer:

The principal value is $\pi/6$.

Quick Tip: When you have $\sec \theta + \tan \theta = k$, the value of $\sec \theta - \tan \theta$ is always $1/k$. This makes solving for the individual functions much faster.

13. $\frac{\tan x - 1 + \sec x}{\tan x - \sec x + 1} = \underline{\hspace{2cm}}$.

- (1) $\frac{1 - \sin x}{\cos x}$
- (2) $\frac{1 + \sin x}{\cos x}$
- (3) $\frac{1 + \cos x}{\sin x}$
- (4) $\frac{1 - \cos x}{\sin x}$

Correct Answer: (2) $\frac{1 + \sin x}{\cos x}$

Solution:

Step 1: Understanding the Concept:

This is a standard trigonometric simplification problem. The trick is to replace the constant '1' in either the numerator or denominator with a trigonometric identity to facilitate factoring.

Step 2: Key Formula or Approach:

Use the identity $1 = \sec^2 x - \tan^2 x = (\sec x - \tan x)(\sec x + \tan x)$.

Step 3: Detailed Explanation:

Let the expression be $E = \frac{(\tan x + \sec x) - 1}{\tan x - \sec x + 1}$.

Substitute $1 = \sec^2 x - \tan^2 x$ in the numerator:

$$E = \frac{(\tan x + \sec x) - (\sec^2 x - \tan^2 x)}{\tan x - \sec x + 1}$$
$$E = \frac{(\tan x + \sec x) - (\sec x - \tan x)(\sec x + \tan x)}{\tan x - \sec x + 1}$$

Factor out $(\sec x + \tan x)$:

$$E = \frac{(\sec x + \tan x)[1 - (\sec x - \tan x)]}{\tan x - \sec x + 1}$$
$$E = \frac{(\sec x + \tan x)[1 - \sec x + \tan x]}{\tan x - \sec x + 1}$$

The terms in the bracket and the denominator are identical, so they cancel out:

$$E = \sec x + \tan x = \frac{1}{\cos x} + \frac{\sin x}{\cos x} = \frac{1 + \sin x}{\cos x}$$

Step 4: Final Answer:

The simplified form is $\frac{1 + \sin x}{\cos x}$.

Quick Tip: This expression also equals $\frac{\cos x}{1 - \sin x}$. In multiple-choice questions, if you don't see one form, try multiplying the numerator and denominator by the conjugate.

14. $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ = \underline{\hspace{2cm}}$.

- (1) 2
- (2) 1
- (3) 4
- (4) 3

Correct Answer: (3) 4

Solution:

Step 1: Understanding the Concept:

The expression involves angles that are complementary (e.g., 9° and 81°). We can use the identity $\tan(90^\circ - \theta) = \cot \theta$ to group terms and simplify the expression into a more manageable form.

Step 2: Key Formula or Approach:

1. $\tan 81^\circ = \tan(90^\circ - 9^\circ) = \cot 9^\circ$
2. $\tan 63^\circ = \tan(90^\circ - 27^\circ) = \cot 27^\circ$
3. Use the identity $\tan \theta + \cot \theta = \frac{2}{\sin 2\theta}$.

Step 3: Detailed Explanation:

Rearrange the expression:

$$(\tan 9^\circ + \tan 81^\circ) - (\tan 27^\circ + \tan 63^\circ)$$

$$= (\tan 9^\circ + \cot 9^\circ) - (\tan 27^\circ + \cot 27^\circ)$$

Using the identity $\tan \theta + \cot \theta = \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} = \frac{1}{\frac{1}{2} \sin 2\theta} = \frac{2}{\sin 2\theta}$:

$$= \frac{2}{\sin 18^\circ} - \frac{2}{\sin 54^\circ}$$

Substitute the values $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$ and $\sin 54^\circ = \cos 36^\circ = \frac{\sqrt{5}+1}{4}$:

$$= \frac{2}{\frac{\sqrt{5}-1}{4}} - \frac{2}{\frac{\sqrt{5}+1}{4}} = \frac{8}{\sqrt{5}-1} - \frac{8}{\sqrt{5}+1}$$

$$= 8 \left[\frac{(\sqrt{5} + 1) - (\sqrt{5} - 1)}{(\sqrt{5} - 1)(\sqrt{5} + 1)} \right] = 8 \left[\frac{2}{5 - 1} \right] = 8 \left[\frac{2}{4} \right] = 8 \times \frac{1}{2} = 4$$

Step 4: Final Answer:

The value of the expression is 4.

Quick Tip: Whenever you see a mix of tangents with complementary angles, pair them up immediately. The identity $\tan \theta + \cot \theta = \frac{2}{\sin 2\theta}$ is a massive time-saver in these problems.

15. If $\cos \theta = \frac{1}{2} \left(a + \frac{1}{a} \right)$, then $4 \cos^3 \theta - 3 \cos \theta = \underline{\hspace{2cm}}$.

- (1) $\frac{a^3+1}{a^3}$
- (2) $\frac{1}{2} \left(a^3 + \frac{1}{a^3} \right)$
- (3) $\frac{1}{4} \left(a^3 + \frac{1}{a^3} \right)$
- (4) $\frac{1}{3} \left(a^3 + \frac{1}{a^3} \right)$

Correct Answer: (2) $\frac{1}{2} \left(a^3 + \frac{1}{a^3} \right)$

Solution:

Step 1: Understanding the Concept:

The expression $4 \cos^3 \theta - 3 \cos \theta$ is the triple angle formula for $\cos 3\theta$. We need to substitute the given value of $\cos \theta$ into this identity and simplify using algebraic expansion.

Step 2: Key Formula or Approach:

- $\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$
- Cube expansion: $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

Step 3: Detailed Explanation:

Let $\cos \theta = x$. We need to find $4x^3 - 3x$. Substitute $x = \frac{1}{2} \left(a + \frac{1}{a} \right)$:

$$4 \left[\frac{1}{2} \left(a + \frac{1}{a} \right) \right]^3 - 3 \left[\frac{1}{2} \left(a + \frac{1}{a} \right) \right]$$

$$\begin{aligned}
 &= 4 \times \frac{1}{8} \left(a + \frac{1}{a} \right)^3 - \frac{3}{2} \left(a + \frac{1}{a} \right) \\
 &= \frac{1}{2} \left[a^3 + \frac{1}{a^3} + 3(a) \left(\frac{1}{a} \right) \left(a + \frac{1}{a} \right) \right] - \frac{3}{2} \left(a + \frac{1}{a} \right) \\
 &= \frac{1}{2} \left(a^3 + \frac{1}{a^3} \right) + \frac{3}{2} \left(a + \frac{1}{a} \right) - \frac{3}{2} \left(a + \frac{1}{a} \right) \\
 &= \frac{1}{2} \left(a^3 + \frac{1}{a^3} \right)
 \end{aligned}$$

Step 4: Final Answer:

The value is $\frac{1}{2} \left(a^3 + \frac{1}{a^3} \right)$.

Quick Tip: This result generalizes: if $\cos \theta = \frac{1}{2}(a + a^{-1})$, then $\cos n\theta = \frac{1}{2}(a^n + a^{-n})$. This is closely related to De Moivre's Theorem.

16. $\cos 20^\circ \cos 40^\circ \cos 80^\circ = \underline{\hspace{1cm}}$.

- (1) $1/4$
- (2) $-1/8$
- (3) $-1/4$
- (4) $1/8$

Correct Answer: (4) $1/8$

Solution:

Step 1: Understanding the Concept:

The product involves cosines of angles in a geometric progression where the ratio is 2. This is a standard trigonometric product that can be simplified using the sine double-angle formula repeatedly.

Step 2: Key Formula or Approach:

Use the product formula: $\cos \theta \cos 2\theta \cos 4\theta \dots \cos(2^{n-1}\theta) = \frac{\sin(2^n \theta)}{2^n \sin \theta}$.

Step 3: Detailed Explanation:

Here, $\theta = 20^\circ$ and $n = 3$.

$$\begin{aligned}\cos 20^\circ \cos 40^\circ \cos 80^\circ &= \frac{\sin(2^3 \times 20^\circ)}{2^3 \sin 20^\circ} \\ &= \frac{\sin 160^\circ}{8 \sin 20^\circ}\end{aligned}$$

Since $\sin 160^\circ = \sin(180^\circ - 20^\circ) = \sin 20^\circ$:

$$= \frac{\sin 20^\circ}{8 \sin 20^\circ} = \frac{1}{8}$$

Step 4: Final Answer:

The value is $1/8$.

Quick Tip: Whenever you see a product of cosines like $\cos \theta \cos 2\theta \cos 4\theta$, the answer is almost always of the form $1/2^n$ if the angles wrap around nicely using supplement properties.

17. $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \underline{\hspace{1cm}}$.

- (1) $3\pi/4$
- (2) $\pi/2$
- (3) π
- (4) 2π

Correct Answer: (3) π

Solution:**Step 1: Understanding the Concept:**

The problem involves the addition of inverse tangent functions. When the product of the arguments (xy) is greater than 1, a specific identity involving π must be used to keep the result within the correct range.

Step 2: Key Formula or Approach:

1. $\tan^{-1} x + \tan^{-1} y = \pi + \tan^{-1} \left(\frac{x+y}{1-xy} \right)$ if $x > 0, y > 0$ and $xy > 1$.
2. Standard value: $\tan^{-1} 1 = \pi/4$.

Step 3: Detailed Explanation:

Let's first calculate $\tan^{-1} 2 + \tan^{-1} 3$. Here $x = 2, y = 3$, so $xy = 6 > 1$.

$$\begin{aligned}\tan^{-1} 2 + \tan^{-1} 3 &= \pi + \tan^{-1} \left(\frac{2+3}{1-(2 \times 3)} \right) \\ &= \pi + \tan^{-1} \left(\frac{5}{-5} \right) = \pi + \tan^{-1}(-1)\end{aligned}$$

Since $\tan^{-1}(-1) = -\pi/4$:

$$\tan^{-1} 2 + \tan^{-1} 3 = \pi - \pi/4 = 3\pi/4$$

Now add $\tan^{-1} 1$:

$$\tan^{-1} 1 + (\tan^{-1} 2 + \tan^{-1} 3) = \pi/4 + 3\pi/4 = 4\pi/4 = \pi$$

Step 4: Final Answer:

The sum is π .

Quick Tip: A useful geometric observation: In any triangle where the tangents of the angles are 1, 2, and 3, the sum of the angles must be π . This specific set of numbers is a common "trick" in inverse trigonometry.

18. If $z_1 = 4i^{40} - 5i^{35} + 6i^{17} + 2$, $z_2 = -1 + i$ then $|z_1 + z_2| = \underline{\hspace{1cm}}$.

- (1) 13
- (2) 5
- (3) 15

(4) 12

Correct Answer: (1) 13

Solution:

Step 1: Understanding the Concept:

To find the magnitude (modulus) of the sum, we must first simplify z_1 by evaluating the powers of i . Powers of i repeat in a cycle of four: $i^1 = i, i^2 = -1, i^3 = -i, i^4 = 1$.

Step 2: Key Formula or Approach:

1. $i^{4n} = 1, i^{4n+1} = i, i^{4n+2} = -1, i^{4n+3} = -i$.
2. $|a + bi| = \sqrt{a^2 + b^2}$.

Step 3: Detailed Explanation:

Simplify z_1 :

$$i^{40} = (i^4)^{10} = 1^{10} = 1$$

$$i^{35} = i^{32} \cdot i^3 = 1 \cdot (-i) = -i$$

$$i^{17} = i^{16} \cdot i^1 = 1 \cdot i = i$$

Substituting these into z_1 :

$$z_1 = 4(1) - 5(-i) + 6(i) + 2 = 4 + 5i + 6i + 2 = 6 + 11i$$

Now find $z_1 + z_2$:

$$z_1 + z_2 = (6 + 11i) + (-1 + i) = 5 + 12i$$

Calculate the modulus:

$$|z_1 + z_2| = \sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13$$

Step 4: Final Answer:

The modulus $|z_1 + z_2|$ is 13.

Quick Tip: To quickly find the power of i , divide the exponent by 4 and look at the remainder. Remainder $0 \rightarrow 1, 1 \rightarrow i, 2 \rightarrow -1, 3 \rightarrow -i$.

19. The conjugate of $(1 + i)^3$ is ____.

- (1) $1 + 2i$
- (2) $-2 + 2i$
- (3) $-2 - 2i$
- (4) $1 - 2i$

Correct Answer: (3) $-2 - 2i$

Solution:

Step 1: Understanding the Concept:

The conjugate of a complex number $z = a + bi$ is $\bar{z} = a - bi$. We first need to expand the cubic expression into the standard form $a + bi$.

Step 2: Key Formula or Approach:

- 1. $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$.
- 2. $i^2 = -1, i^3 = -i$.

Step 3: Detailed Explanation:

Expand $z = (1 + i)^3$:

$$z = 1^3 + 3(1)^2(i) + 3(1)(i)^2 + i^3$$

$$z = 1 + 3i + 3(-1) + (-i)$$

$$z = 1 + 3i - 3 - i = -2 + 2i$$

The conjugate is obtained by changing the sign of the imaginary part:

$$\bar{z} = -2 - 2i$$

Step 4: Final Answer:

The conjugate is $-2 - 2i$.

Quick Tip: Alternatively, you can find the conjugate of the base first and then cube it: $\overline{(1+i)^3} = (\overline{1+i})^3 = (1-i)^3$. Both methods yield the same result.

20. The equation of a circle whose Centre is $(-3, 2)$ and area is 176 units is ____.

- (1) $x^2 + y^2 + 6x - 4y - 36 = 0$
- (2) $x^2 + y^2 + 6x - 4y - 43 = 0$
- (3) $x^2 + y^2 - 6x + 4y - 36 = 0$
- (4) $x^2 + y^2 - 6x + 4y - 43 = 0$

Correct Answer: (2) $x^2 + y^2 + 6x - 4y - 43 = 0$

Solution:**Step 1: Understanding the Concept:**

The equation of a circle with centre (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$. We are given the area, which allows us to calculate r^2 .

Step 2: Key Formula or Approach:

1. Area of circle $= \pi r^2$. (Use $\pi \approx 22/7$ for calculation).
2. Standard form expansion: $x^2 + y^2 - 2hx - 2ky + (h^2 + k^2 - r^2) = 0$.

Step 3: Detailed Explanation:

Given Area = 176:

$$\pi r^2 = 176 \implies \frac{22}{7} r^2 = 176$$

$$r^2 = \frac{176 \times 7}{22} = 8 \times 7 = 56$$

Substitute centre $(h, k) = (-3, 2)$ and $r^2 = 56$ into the circle equation:

$$(x - (-3))^2 + (y - 2)^2 = 56$$

$$(x + 3)^2 + (y - 2)^2 = 56$$

Expand the terms:

$$(x^2 + 6x + 9) + (y^2 - 4y + 4) = 56$$

$$x^2 + y^2 + 6x - 4y + 13 - 56 = 0$$

$$x^2 + y^2 + 6x - 4y - 43 = 0$$

Step 4: Final Answer:

The equation of the circle is $x^2 + y^2 + 6x - 4y - 43 = 0$.

Quick Tip: If the centre is $(-3, 2)$, the linear terms in the final equation must have opposite signs to the coordinates: $+6x$ and $-4y$. This helps eliminate options (3) and (4) instantly.

21. The equation of a circle whose Centre is $(2, -1)$ and which passes through the point $(3, 6)$ is ____.

- (1) $x^2 + y^2 + 4x + 2y - 45 = 0$
- (2) $x^2 + y^2 - 2x + 2y - 50 = 0$
- (3) $x^2 + y^2 + 2x + 2y - 50 = 0$
- (4) $x^2 + y^2 - 4x + 2y - 45 = 0$

Solution:

Step 1: Understanding the Concept:

The equation of a circle with centre (h, k) and radius r is given by $(x - h)^2 + (y - k)^2 = r^2$. Since the circle passes through a specific point, the distance between the centre and that point equals the radius.

Step 2: Key Formula or Approach:

1. Distance formula: $r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
2. Standard circle equation: $(x - h)^2 + (y - k)^2 = r^2$

Step 3: Detailed Explanation:

First, find the radius r using the distance between the centre $(2, -1)$ and the point $(3, 6)$:

$$r^2 = (3 - 2)^2 + (6 - (-1))^2$$

$$r^2 = (1)^2 + (7)^2 = 1 + 49 = 50$$

Now, substitute the centre $(2, -1)$ and $r^2 = 50$ into the circle equation:

$$(x - 2)^2 + (y - (-1))^2 = 50$$

$$(x - 2)^2 + (y + 1)^2 = 50$$

Expand the equation:

$$(x^2 - 4x + 4) + (y^2 + 2y + 1) = 50$$

$$x^2 + y^2 - 4x + 2y + 5 - 50 = 0$$

$$x^2 + y^2 - 4x + 2y - 45 = 0$$

Step 4: Final Answer:

The equation of the circle is $x^2 + y^2 - 4x + 2y - 45 = 0$.

Quick Tip: For a circle with centre (h, k) , the linear terms in the expanded equation are $-2hx$ and $-2ky$. Here, $-2(2) = -4$ and $-2(-1) = +2$. This immediately points to option (4).

22. If the parabola $y^2 = 4ax$ passes through the point $(3, 2)$ then the length of its latus rectum is: ____.

- (1) $4/3$
- (2) 4
- (3) $2/3$
- (4) $1/3$

Solution:

Step 1: Understanding the Concept:

A point lies on a curve if its coordinates satisfy the equation of the curve. For a parabola $y^2 = 4ax$, the length of the latus rectum is the coefficient of x , which is $4a$.

Step 2: Key Formula or Approach:

1. Substitute the point into the equation to find a or $4a$.
2. Length of latus rectum = $4a$.

Step 3: Detailed Explanation:

The parabola $y^2 = 4ax$ passes through $(3, 2)$. Substitute $x = 3$ and $y = 2$:

$$(2)^2 = 4a(3)$$

$$4 = 12a$$

Divide both sides by 3 to isolate $4a$:

$$4a = \frac{4}{3}$$

Since the length of the latus rectum is $4a$, we have:

$$\text{Latus Rectum} = \frac{4}{3}$$

Step 4: Final Answer:

The length of the latus rectum is $4/3$.

Quick Tip: Don't solve for a first and then multiply by 4; simply isolate $4a$ directly from your equation to save a step.

23. The line $y = mx + 2$ is a tangent to the parabola $y^2 = 8x$ if ____.

(1) $m = 1$

(2) $m = 2$

(3) $m = 3$

(4) $m = 4$

Solution:

Step 1: Understanding the Concept:

The condition for a line $y = mx + c$ to be a tangent to the parabola $y^2 = 4ax$ is $c = \frac{a}{m}$.

Step 2: Key Formula or Approach:

1. Identify a from the parabola equation $y^2 = 4ax$.
2. Apply the tangency condition: $c = \frac{a}{m}$.

Step 3: Detailed Explanation:

The given parabola is $y^2 = 8x$. Comparing with $y^2 = 4ax$:

$$4a = 8 \implies a = 2$$

The given line is $y = mx + 2$. Comparing with $y = mx + c$:

$$c = 2$$

Applying the tangency condition $c = \frac{a}{m}$:

$$2 = \frac{2}{m}$$

$$2m = 2 \implies m = 1$$

Step 4: Final Answer:

The value of m is 1.

Quick Tip: If you forget the tangency formula, substitute $y = mx + 2$ into the parabola equation to get a quadratic in x , then set the discriminant $D = 0$.

24. The length of the latus rectum and eccentricity of the Hyperbola $9x^2 - 16y^2 = 144$ are ____.

- (1) $(9/4, 5/4)$
- (2) $(9/2, 5/4)$
- (3) $(9/2, 5/2)$
- (4) $(9, 5/2)$

Correct Answer: (2) $(9/2, 5/4)$

Solution:

Step 1: Understanding the Concept:

We first convert the given equation into the standard form of a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ to identify the semi-major axis a and semi-minor axis b . These values are then used to calculate the length of the latus rectum and the eccentricity.

Step 2: Key Formula or Approach:

1. Standard Form: Divide by the constant on the RHS.
2. Length of Latus Rectum: $L = \frac{2b^2}{a}$.
3. Eccentricity: $e = \sqrt{1 + \frac{b^2}{a^2}}$.

Step 3: Detailed Explanation:

Divide $9x^2 - 16y^2 = 144$ by 144:

$$\frac{9x^2}{144} - \frac{16y^2}{144} = 1 \implies \frac{x^2}{16} - \frac{y^2}{9} = 1$$

Here, $a^2 = 16 \implies a = 4$ and $b^2 = 9 \implies b = 3$.

Length of Latus Rectum:

$$L = \frac{2b^2}{a} = \frac{2(9)}{4} = \frac{18}{4} = \frac{9}{2}$$

Eccentricity:

$$e = \sqrt{1 + \frac{9}{16}} = \sqrt{\frac{16+9}{16}} = \sqrt{\frac{25}{16}} = \frac{5}{4}$$

Step 4: Final Answer:

The length of the latus rectum is $9/2$ and the eccentricity is $5/4$.

Quick Tip: For a hyperbola, eccentricity is always greater than 1. This can help you quickly eliminate options where $e \leq 1$.

25. The equation of the ellipse with foci at $(\pm 3, 0)$ and the eccentricity as $1/3$ is : ____.

- (1) $\frac{x^2}{81} + \frac{y^2}{72} = 1$
- (2) $\frac{x^2}{9} + \frac{y^2}{8} = 1$
- (3) $\frac{x^2}{8} + \frac{y^2}{9} = 1$
- (4) $\frac{x^2}{3} + \frac{y^2}{2} = 1$

Correct Answer: (1) $\frac{x^2}{81} + \frac{y^2}{72} = 1$

Solution:

Step 1: Understanding the Concept:

The foci of an ellipse on the x-axis are given by $(\pm ae, 0)$. By using the given foci and eccentricity, we can solve for the semi-major axis a and then find the semi-minor axis b .

Step 2: Key Formula or Approach:

- 1. Distance of focus from center: $c = ae$.
- 2. Relationship: $b^2 = a^2(1 - e^2)$.
- 3. Standard Equation: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Step 3: Detailed Explanation:

Given foci $(\pm 3, 0)$, so $ae = 3$.

Given $e = 1/3$:

$$a(1/3) = 3 \implies a = 9 \implies a^2 = 81$$

Now find b^2 :

$$b^2 = a^2(1 - e^2) = 81(1 - (1/3)^2) = 81(1 - 1/9)$$

$$b^2 = 81(8/9) = 9 \times 8 = 72$$

Substitute into the standard equation:

$$\frac{x^2}{81} + \frac{y^2}{72} = 1$$

Step 4: Final Answer:

The equation of the ellipse is $\frac{x^2}{81} + \frac{y^2}{72} = 1$.

Quick Tip: Since the foci are on the x-axis, the denominator of x^2 (a^2) must be larger than the denominator of y^2 (b^2). This confirms the ellipse is horizontal.

26. $\lim_{x \rightarrow \infty} (1 + \frac{1}{x})^x = \underline{\hspace{1cm}}$.

- (1) 0
- (2) 1
- (3) e
- (4) ∞

Correct Answer: (3) e

Solution:**Step 1: Understanding the Concept:**

This limit is the fundamental definition of the mathematical constant e (Euler's number). It represents a 1^∞ indeterminate form.

Step 2: Key Formula or Approach:

For limits of the form $\lim [f(x)]^{g(x)}$ where $f(x) \rightarrow 1$ and $g(x) \rightarrow \infty$, the result is $e^{\lim [f(x)-1]g(x)}$.

Step 3: Detailed Explanation:

Let $L = \lim_{x \rightarrow \infty} (1 + \frac{1}{x})^x$.

This is an indeterminate form of 1^∞ . Using the formula:

$$L = e^{\lim_{x \rightarrow \infty} [(1 + \frac{1}{x}) - 1] \cdot x}$$

$$L = e^{\lim_{x \rightarrow \infty} (\frac{1}{x}) \cdot x}$$

$$L = e^{\lim_{x \rightarrow \infty} 1} = e^1 = e$$

Step 4: Final Answer:

The value of the limit is e .

Quick Tip: This definition is often used in compound interest calculations where interest is compounded continuously. It's one of the most important limits in calculus.

27. $\lim_{x \rightarrow 0} \frac{\sqrt{1+x}-1}{x} = \underline{\hspace{2cm}}.$

- (1) 0
- (2) $1/2$
- (3) 1
- (4) ∞

Correct Answer: (2) $1/2$

Solution:**Step 1: Understanding the Concept:**

The limit presents a $0/0$ indeterminate form. To solve this, we can rationalize the numerator or use L'Hôpital's Rule.

Step 2: Key Formula or Approach:

1. Rationalization: Multiply the numerator and denominator by the conjugate $\sqrt{1+x} + 1$.
2. L'Hôpital's Rule: $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \lim_{x \rightarrow c} \frac{f'(x)}{g'(x)}.$

Step 3: Detailed Explanation:

Using the rationalization method:

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sqrt{1+x}-1}{x} &\times \frac{\sqrt{1+x}+1}{\sqrt{1+x}+1} \\ &= \lim_{x \rightarrow 0} \frac{(1+x)-1}{x(\sqrt{1+x}+1)} \end{aligned}$$

$$= \lim_{x \rightarrow 0} \frac{x}{x(\sqrt{1+x} + 1)}$$

Cancel x from the numerator and denominator:

$$= \lim_{x \rightarrow 0} \frac{1}{\sqrt{1+x} + 1}$$

Substitute $x = 0$:

$$= \frac{1}{\sqrt{1+0} + 1} = \frac{1}{1+1} = \frac{1}{2}$$

Step 4: Final Answer:

The value of the limit is $1/2$.

Quick Tip: For limits involving square roots at 0, you can also use the binomial expansion $(1+x)^n \approx 1+nx$. Here, $(1+x)^{1/2} \approx 1 + \frac{1}{2}x$, so the expression becomes $\frac{(1+\frac{1}{2}x)-1}{x} = \frac{1}{2}$.

28. If $y = \frac{a \cos x + b \sin x + c}{\sin x}$ then $\frac{dy}{dx} = \underline{\hspace{2cm}}$.

- (1) $-a \csc^2 x - c \csc x \cot x$
- (2) $-a$
- (3) $-a \csc^2 x + b \sec^2 x + c \csc x \cot x$
- (4) $\frac{\csc^2 x - \csc x \cot x}{2}$

Correct Answer: (1) $-a \csc^2 x - c \csc x \cot x$

Solution:

Step 1: Understanding the Concept:

To differentiate this function, it is easier to simplify the fraction by dividing each term in the numerator by the denominator before applying differentiation rules.

Step 2: Key Formula or Approach:

- Simplify: $y = a \cot x + b + c \csc x$.
- Derivatives: $\frac{d}{dx}(\cot x) = -\csc^2 x$ and $\frac{d}{dx}(\csc x) = -\csc x \cot x$.

Step 3: Detailed Explanation:

Split the terms:

$$y = \frac{a \cos x}{\sin x} + \frac{b \sin x}{\sin x} + \frac{c}{\sin x}$$

$$y = a \cot x + b + c \csc x$$

Differentiate with respect to x :

$$\frac{dy}{dx} = a(-\csc^2 x) + 0 + c(-\csc x \cot x)$$

$$\frac{dy}{dx} = -a \csc^2 x - c \csc x \cot x$$

Step 4: Final Answer:

The derivative $\frac{dy}{dx}$ is $-a \csc^2 x - c \csc x \cot x$.

Quick Tip: Whenever the denominator is a single trigonometric term, splitting the fraction is almost always faster than using the quotient rule.

29. If $y = \sqrt{x + \sqrt{x + \sqrt{x + \cdots + \infty}}}$ then $\frac{dy}{dx} = \underline{\hspace{2cm}}$.

- (1) $\frac{1}{2y}$
- (2) $\frac{1}{1-2y}$
- (3) $\frac{1}{2(1-2y)}$
- (4) $\frac{-1}{1-2y}$

Correct Answer: (4) $\frac{-1}{1-2y}$ (Equivalent to $\frac{1}{2y-1}$)

Solution:**Step 1: Understanding the Concept:**

This is an infinite series function. Since the pattern repeats infinitely, we can replace the inner part of the square root with y itself to create a finite algebraic equation.

Step 2: Key Formula or Approach:

1. Recursive substitution: $y = \sqrt{x+y}$.
2. Implicit differentiation: Differentiate both sides with respect to x .

Step 3: Detailed Explanation:

Given $y = \sqrt{x+y}$. Square both sides:

$$y^2 = x + y$$

Differentiate with respect to x using the chain rule:

$$2y \frac{dy}{dx} = 1 + \frac{dy}{dx}$$

Rearrange to group $\frac{dy}{dx}$ terms:

$$2y \frac{dy}{dx} - \frac{dy}{dx} = 1$$

$$\frac{dy}{dx}(2y - 1) = 1$$

$$\frac{dy}{dx} = \frac{1}{2y - 1}$$

Note: $\frac{1}{2y-1}$ is mathematically identical to $\frac{-1}{1-2y}$.

Step 4: Final Answer:

The derivative is $\frac{1}{2y-1}$.

Quick Tip: For any function $y = \sqrt{f(x)} + \sqrt{f(x)} + \dots$, the derivative is always $\frac{f'(x)}{2y-1}$. Here $f(x) = x$, so $f'(x) = 1$.

30. Slope of the tangent to the curve $y = 9x^2 + 7x^4 + 5$ at the point $x = 1$ is ____.

- (1) 28
- (2) 16
- (3) 46

(4) $1/46$

Correct Answer: (3) 46

Solution:

Step 1: Understanding the Concept:

The slope of the tangent to a curve at a given point is equal to the value of the first derivative $\frac{dy}{dx}$ at that specific point.

Step 2: Key Formula or Approach:

1. Power Rule: $\frac{d}{dx}(x^n) = nx^{n-1}$.
2. Slope $m = \left[\frac{dy}{dx} \right]_{x=1}$.

Step 3: Detailed Explanation:

Given $y = 9x^2 + 7x^4 + 5$. Differentiate with respect to x :

$$\frac{dy}{dx} = 9(2x) + 7(4x^3) + 0$$

$$\frac{dy}{dx} = 18x + 28x^3$$

To find the slope at $x = 1$, substitute $x = 1$ into the derivative:

$$m = 18(1) + 28(1)^3$$

$$m = 18 + 28 = 46$$

Step 4: Final Answer:

The slope of the tangent at $x = 1$ is 46.

Quick Tip: "Slope of tangent" is just a geometric name for the derivative. If the question asked for the slope of the "normal," you would take the negative reciprocal: $-1/46$.

31. If $f(x) = \begin{cases} 4(5^x) & x < 0 \\ 8k + x & x \geq 0 \end{cases}$ then $f'(-1) = \underline{\hspace{2cm}}$.

- (1) $\frac{2}{5} \log 5$
- (2) $\frac{4}{5} \log 5$
- (3) $\frac{3}{5} \log 5$
- (4) $20 \log 5$

Correct Answer: (2) $\frac{4}{5} \log 5$

Solution:

Step 1: Understanding the Concept:

To find the derivative of a piecewise function at a specific point, we use the rule defined for the interval containing that point. Since we need $f'(-1)$ and $-1 < 0$, we only consider the piece $f(x) = 4(5^x)$ [cite: 1].

Step 2: Key Formula or Approach:

The derivative of an exponential function a^x is $\frac{d}{dx}(a^x) = a^x \log a$ (or $a^x \ln a$) [cite: 1].

Step 3: Detailed Explanation:

For $x < 0$, $f(x) = 4 \cdot 5^x$. Differentiating with respect to x :

$$f'(x) = 4 \cdot (5^x \log 5)$$

Now, substitute $x = -1$:

$$f'(-1) = 4 \cdot (5^{-1} \log 5)$$

$$f'(-1) = 4 \cdot \frac{1}{5} \log 5 = \frac{4}{5} \log 5$$

Step 4: Final Answer:

The value of $f'(-1)$ is $\frac{4}{5} \log 5$.

Quick Tip: Ignore the piece $8k + x$ entirely. Since derivatives are local properties, only the function behavior immediately around $x = -1$ matters.

32. If $2^x + 2^y = 2^{x+y}$, then $\frac{dy}{dx} = \underline{\hspace{2cm}}$.

(1) $2^{x-y} \left(\frac{2^y - 1}{1 - 2^x} \right)$

(2) $\frac{2^x(2^y - 1)}{2^y(1 - 2^x)}$

(3) -2^{y-x}

(4) $1 + 2^y$

Correct Answer: (3) -2^{y-x} (Note: Standard result for this form)

Solution:

Step 1: Understanding the Concept:

This is an implicit function. We differentiate both sides with respect to x , treating y as a function of x and applying the chain rule[cite: 1].

Step 2: Key Formula or Approach:

The derivative of 2^u is $2^u \log 2 \cdot \frac{du}{dx}$ [cite: 1].

Step 3: Detailed Explanation:

Differentiate $2^x + 2^y = 2^{x+y}$:

$$2^x \log 2 + 2^y \log 2 \frac{dy}{dx} = 2^{x+y} \log 2 \left(1 + \frac{dy}{dx} \right)$$

Divide by $\log 2$:

$$2^x + 2^y \frac{dy}{dx} = 2^{x+y} + 2^{x+y} \frac{dy}{dx}$$

Rearrange to solve for $\frac{dy}{dx}$:

$$2^y \frac{dy}{dx} - 2^{x+y} \frac{dy}{dx} = 2^{x+y} - 2^x$$

$$\frac{dy}{dx} (2^y - 2^{x+y}) = 2^{x+y} - 2^x$$

Using the original equation $2^{x+y} = 2^x + 2^y$:

$$\frac{dy}{dx}(2^y - (2^x + 2^y)) = (2^x + 2^y) - 2^x$$

$$\frac{dy}{dx}(-2^x) = 2^y$$

$$\frac{dy}{dx} = -\frac{2^y}{2^x} = -2^{y-x}$$

Step 4: Final Answer:

The derivative $\frac{dy}{dx}$ is -2^{y-x} .

Quick Tip: For equations of the form $a^x + a^y = a^{x+y}$, the derivative is always $-a^{y-x}$. This shortcut works for any base a .

33. If $y + \sin^{-1}(1 - x^2) = e^x$, then $\frac{dy}{dx} = \underline{\hspace{2cm}}$.

(1) $e^x - \frac{2x}{\sqrt{1-(1-x^2)^2}}$

(2) $e^x + \frac{2x}{\sqrt{1-(1-x^2)^2}}$

(3) $e^x + \frac{2}{\sqrt{2-x^2}}$

(4) $e^x - \frac{2}{\sqrt{2+x^2}}$

Correct Answer: (2) $e^x + \frac{2x}{\sqrt{1-(1-x^2)^2}}$ (Simplifiable)

Solution:

Step 1: Understanding the Concept:

We isolate y or differentiate implicitly. The derivative of $\sin^{-1}u$ requires the chain rule:

$$\frac{d}{dx}(\sin^{-1}u) = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx} [\text{cite: 1}].$$

Step 2: Key Formula or Approach:

1. $\frac{d}{dx}(e^x) = e^x$ [cite: 1].

2. $\frac{d}{dx}(1 - x^2) = -2x$ [cite: 1].

Step 3: Detailed Explanation:

Differentiate the entire equation:

$$\frac{dy}{dx} + \frac{1}{\sqrt{1-(1-x^2)^2}} \cdot \frac{d}{dx}(1-x^2) = e^x$$

$$\frac{dy}{dx} + \frac{1}{\sqrt{1-(1-2x^2+x^4)}} \cdot (-2x) = e^x$$

$$\frac{dy}{dx} - \frac{2x}{\sqrt{2x^2-x^4}} = e^x$$

Rearranging for $\frac{dy}{dx}$:

$$\frac{dy}{dx} = e^x + \frac{2x}{x\sqrt{2-x^2}} = e^x + \frac{2}{\sqrt{2-x^2}}$$

Step 4: Final Answer:

The derivative is $\frac{dy}{dx} = e^x + \frac{2}{\sqrt{2-x^2}}$.

Quick Tip: Always simplify the term inside the square root. Often, an x or x^2 can be factored out to cancel terms in the numerator, making the final result much cleaner.

34. If $y(x) = x^x$, $x > 0$, then $y''(2) - 2y'(2) = \underline{\hspace{2cm}}$.

- (1) $4\log_e 2 - 2$
- (2) $4\log_e 2 + 2$
- (3) $4(\log_e 2)^2 + 2$
- (4) $4(\log_e 2)^2 - 2$

Correct Answer: (4) $4(\log_e 2)^2 - 2$

Solution:**Step 1: Understanding the Concept:**

To differentiate a function where both the base and exponent are variables, we use logarithmic differentiation[cite: 1]. We first find the first derivative y' , and then differentiate again to

find the second derivative y'' .

Step 2: Key Formula or Approach:

1. For $y = x^x$, $y' = x^x(1 + \log_e x)$.
2. Use the product rule for y'' : $\frac{d}{dx}[uv] = u'v + uv'$.

Step 3: Detailed Explanation:

Given $y = x^x$, the first derivative is $y'(x) = x^x(1 + \log_e x)$. At $x = 2$:

$$y'(2) = 2^2(1 + \log_e 2) = 4(1 + \log_e 2) = 4 + 4\log_e 2$$

Now find $y''(x)$ using the product rule on $y'(x)$:

$$y''(x) = \frac{d}{dx}[x^x] \cdot (1 + \log_e x) + x^x \cdot \frac{d}{dx}(1 + \log_e x)$$

$$y''(x) = x^x(1 + \log_e x)^2 + x^x\left(\frac{1}{x}\right)$$

At $x = 2$:

$$y''(2) = 2^2(1 + \log_e 2)^2 + 2^2\left(\frac{1}{2}\right) = 4(1 + 2\log_e 2 + (\log_e 2)^2) + 2$$

$$y''(2) = 4 + 8\log_e 2 + 4(\log_e 2)^2 + 2 = 6 + 8\log_e 2 + 4(\log_e 2)^2$$

Now calculate $y''(2) - 2y'(2)$:

$$(6 + 8\log_e 2 + 4(\log_e 2)^2) - 2(4 + 4\log_e 2)$$

$$= 6 + 8\log_e 2 + 4(\log_e 2)^2 - 8 - 8\log_e 2$$

$$= 4(\log_e 2)^2 - 2$$

Step 4: Final Answer:

The value of the expression is $4(\log_e 2)^2 - 2$.

Quick Tip: Remember the standard derivative $\frac{d}{dx}(x^x) = x^x(1 + \ln x)$. It appears frequently in calculus exams and saves time if memorized.

35. If $z = x^2y^3 + e^y \sin x$, then $\frac{\partial^2 z}{\partial x \partial y} = \underline{\hspace{2cm}}$.

- (1) $6xy^2 + e^y \cos x$
- (2) $3x^2y^2 + e^y \sin x$
- (3) $3x^2y^2 + e^y \cos x$
- (4) $6xy^2 + e^y \sin x$

Correct Answer: (1) $6xy^2 + e^y \cos x$

Solution:

Step 1: Understanding the Concept:

The notation $\frac{\partial^2 z}{\partial x \partial y}$ represents a mixed partial derivative. This means we first differentiate z with respect to y (treating x as a constant), and then differentiate the result with respect to x (treating y as a constant).

Step 2: Key Formula or Approach:

1. First find $\frac{\partial z}{\partial y}$.
2. Then find $\frac{\partial}{\partial x} \left(\frac{\partial z}{\partial y} \right)$.

Step 3: Detailed Explanation:

Differentiate $z = x^2y^3 + e^y \sin x$ with respect to y :

$$\frac{\partial z}{\partial y} = x^2(3y^2) + (\sin x)e^y = 3x^2y^2 + e^y \sin x$$

Now, differentiate this result with respect to x :

$$\begin{aligned} \frac{\partial}{\partial x}(3x^2y^2 + e^y \sin x) &= (3y^2)(2x) + e^y(\cos x) \\ &= 6xy^2 + e^y \cos x \end{aligned}$$

Step 4: Final Answer:

The mixed partial derivative is $6xy^2 + e^y \cos x$.

Quick Tip: According to Clairaut's Theorem, for most smooth functions, the order of differentiation does not matter: $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$. You can pick whichever order seems easier to calculate.

36. $\int \frac{dx}{\sin^2 x \cos^2 x} = \underline{\hspace{2cm}}.$

- (1) $\tan x + \cot x + c$
- (2) $\tan x - \cot x + c$
- (3) $\tan x \cot x + c$
- (4) $\tan x + \sec x + c$

Correct Answer: (2) $\tan x - \cot x + c$

Solution:**Step 1: Understanding the Concept:**

To integrate this trigonometric fraction, we can use the identity $\sin^2 x + \cos^2 x = 1$ to split the integral into two simpler parts.

Step 2: Key Formula or Approach:

1. Replace 1 in the numerator with $\sin^2 x + \cos^2 x$.
2. Use standard integrals: $\int \sec^2 x \, dx = \tan x$ and $\int \csc^2 x \, dx = -\cot x$.

Step 3: Detailed Explanation:

$$\int \frac{1}{\sin^2 x \cos^2 x} dx = \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx$$

Split the fraction:

$$\begin{aligned} &= \int \left(\frac{\sin^2 x}{\sin^2 x \cos^2 x} + \frac{\cos^2 x}{\sin^2 x \cos^2 x} \right) dx \\ &= \int \left(\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} \right) dx \end{aligned}$$

$$= \int (\sec^2 x + \csc^2 x) dx$$

Integrating term by term:

$$= \tan x + (-\cot x) + c = \tan x - \cot x + c$$

Step 4: Final Answer:

The integral is $\tan x - \cot x + c$.

Quick Tip: Another way is to use the double angle formula: $\sin^2 x \cos^2 x = \frac{1}{4} \sin^2 2x$. The integral becomes $\int 4 \csc^2 2x dx = -2 \cot 2x + c$, which simplifies back to $\tan x - \cot x + c$.

37. $\int \frac{dx}{\sqrt{x+1} + \sqrt{x}} = \underline{\hspace{2cm}}.$

(1) $\frac{2}{3} [(x+1)^{3/2} - x^{3/2}] + c$

(2) $\frac{2}{3} [(x+1)^{3/2} + x^{3/2}] + c$

(3) $\frac{3}{2} [(x+1)^{3/2} - x^{3/2}] + c$

(4) $\frac{3}{2} [(x+1)^{3/2} + x^{3/2}] + c$

Correct Answer: (1) $\frac{2}{3} [(x+1)^{3/2} - x^{3/2}] + c$

Solution:

Step 1: Understanding the Concept:

To integrate a fraction with a sum of square roots in the denominator, the most effective method is to rationalize the denominator. This converts the expression into a simpler form where basic integration rules can be applied.

Step 2: Key Formula or Approach:

1. Rationalization: Multiply the numerator and denominator by the conjugate $\sqrt{x+1} - \sqrt{x}$.
2. Power rule for integration: $\int x^n dx = \frac{x^{n+1}}{n+1} + c$.

Step 3: Detailed Explanation:

First, rationalize the denominator:

$$\frac{1}{\sqrt{x+1} + \sqrt{x}} \times \frac{\sqrt{x+1} - \sqrt{x}}{\sqrt{x+1} - \sqrt{x}} = \frac{\sqrt{x+1} - \sqrt{x}}{(x+1) - x} = \sqrt{x+1} - \sqrt{x}$$

The integral becomes:

$$\int (\sqrt{x+1} - \sqrt{x}) dx = \int (x+1)^{1/2} dx - \int x^{1/2} dx$$

Apply the power rule to each term:

$$\begin{aligned} &= \frac{(x+1)^{3/2}}{3/2} - \frac{x^{3/2}}{3/2} + c \\ &= \frac{2}{3}(x+1)^{3/2} - \frac{2}{3}x^{3/2} + c = \frac{2}{3}[(x+1)^{3/2} - x^{3/2}] + c \end{aligned}$$

Step 4: Final Answer:

The integral is $\frac{2}{3}[(x+1)^{3/2} - x^{3/2}] + c$.

Quick Tip: Rationalization is a "universal key" for integrals involving $\sqrt{A} \pm \sqrt{B}$. If the difference $A - B$ is a constant, it simplifies the problem significantly.

38. If $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx = A \sec x + B \csc x + c$, then (A, B) are ____.

- (1) (1, 1)
- (2) (-1, -1)
- (3) (1, -1)
- (4) (-1, 1)

Correct Answer: (3) (1, -1)

Solution:

Step 1: Understanding the Concept:

To solve this integral, split the numerator and divide each term by the denominator. This transforms complex trigonometric products into standard trigonometric functions that are easy to integrate.

Step 2: Key Formula or Approach:

1. Simplify terms: $\frac{\sin^3 x}{\sin^2 x \cos^2 x} = \tan x \sec x$ and $\frac{\cos^3 x}{\sin^2 x \cos^2 x} = \cot x \csc x$.
2. Standard integrals: $\int \sec x \tan x \, dx = \sec x$ and $\int \csc x \cot x \, dx = -\csc x$.

Step 3: Detailed Explanation:

Split the integral:

$$\begin{aligned} \int \frac{\sin^3 x}{\sin^2 x \cos^2 x} \, dx + \int \frac{\cos^3 x}{\sin^2 x \cos^2 x} \, dx \\ = \int \frac{\sin x}{\cos^2 x} \, dx + \int \frac{\cos x}{\sin^2 x} \, dx \\ = \int \sec x \tan x \, dx + \int \csc x \cot x \, dx \end{aligned}$$

Using standard integration formulas:

$$= \sec x + (-\csc x) + c = 1 \sec x - 1 \csc x + c$$

Comparing with $A \sec x + B \csc x + c$, we find $A = 1$ and $B = -1$.

Step 4: Final Answer:

The values are $(A, B) = (1, -1)$.

Quick Tip: Always look to separate terms in the numerator when the denominator is a single product. It often turns a scary-looking fraction into a basic identity problem.

39. The integral of $f(x) = 1 + x^2 + x^4$ with respect to x^2 is ____.

(1) $x + \frac{x^3}{3} + \frac{x^5}{5} + C$

(2) $\frac{x^3}{3} + \frac{x^5}{5} + C$

(3) $x^2 + \frac{x^4}{4} + \frac{x^6}{6} + C$

(4) $x^2 + \frac{x^4}{2} + \frac{x^6}{3} + C$

Correct Answer: (4) $x^2 + \frac{x^4}{2} + \frac{x^6}{3} + C$

Solution:

Step 1: Understanding the Concept:

The phrase "with respect to x^2 " means that x^2 is our variable of integration, often denoted by u . We substitute $u = x^2$ and integrate the resulting polynomial in terms of u .

Step 2: Key Formula or Approach:

1. Let $u = x^2$.
2. Rewrite the function: $f(u) = 1 + u + u^2$.
3. Use $\int u^n du = \frac{u^{n+1}}{n+1} + C$.

Step 3: Detailed Explanation:

We are finding $\int (1 + x^2 + x^4) d(x^2)$. Let $u = x^2$. The integral becomes:

$$\int (1 + u + u^2) du$$

Integrate term by term:

$$= u + \frac{u^2}{2} + \frac{u^3}{3} + C$$

Substitute back $u = x^2$:

$$= x^2 + \frac{(x^2)^2}{2} + \frac{(x^2)^3}{3} + C = x^2 + \frac{x^4}{2} + \frac{x^6}{3} + C$$

Step 4: Final Answer:

The integral is $x^2 + \frac{x^4}{2} + \frac{x^6}{3} + C$.

Quick Tip: Pay close attention to the variable after the 'd'. Integrating with respect to x^2 is different from integrating with respect to x . If it were dx , the answer would be option (1).

40. $\int_0^{\pi/2} \frac{\sin^{100} x}{\sin^{100} x + \cos^{100} x} dx = \underline{\hspace{2cm}}.$

- (1) $\pi/2$
- (2) $\pi/4$
- (3) 100
- (4) 50

Correct Answer: (2) $\pi/4$

Solution:

Step 1: Understanding the Concept:

This is a definite integral that uses the property $\int_0^a f(x) dx = \int_0^a f(a-x) dx$. This property allows us to "swap" sin and cos terms in the interval $[0, \pi/2]$.

Step 2: Key Formula or Approach:

1. Let the integral be I .
2. Apply property: Replace x with $\pi/2 - x$.
3. Use $\sin(\pi/2 - x) = \cos x$ and $\cos(\pi/2 - x) = \sin x$.

Step 3: Detailed Explanation:

Let $I = \int_0^{\pi/2} \frac{\sin^{100} x}{\sin^{100} x + \cos^{100} x} dx$. Using the property:

$$I = \int_0^{\pi/2} \frac{\sin^{100}(\pi/2 - x)}{\sin^{100}(\pi/2 - x) + \cos^{100}(\pi/2 - x)} dx = \int_0^{\pi/2} \frac{\cos^{100} x}{\cos^{100} x + \sin^{100} x} dx$$

Add the two expressions for I :

$$2I = \int_0^{\pi/2} \frac{\sin^{100} x + \cos^{100} x}{\sin^{100} x + \cos^{100} x} dx$$

$$2I = \int_0^{\pi/2} 1 dx = [x]_0^{\pi/2} = \pi/2$$

$$I = \frac{\pi/2}{2} = \pi/4$$

Step 4: Final Answer:

The value of the definite integral is $\pi/4$.

Quick Tip: For any integral of the form $\int_0^{\pi/2} \frac{f(\sin x)}{f(\sin x) + f(\cos x)} dx$, the answer is always $(b - a)/2$, which in this case is $(\pi/2 - 0)/2 = \pi/4$. The exponent (100) doesn't change the result.

41. $\int_0^1 x\sqrt{x^2 + 4} dx = \underline{\hspace{2cm}}.$

(1) $\frac{1}{3}[5\sqrt{5} - 4]$

(2) $\frac{1}{2}[5\sqrt{5} - 8]$

(3) $\frac{1}{3}[5\sqrt{5} - 8]$

(4) $\frac{1}{3}[5\sqrt{5} + 4]$

Correct Answer: (3) $\frac{1}{3}[5\sqrt{5} - 8]$

Solution:**Step 1: Understanding the Concept:**

To solve this definite integral, we use the method of substitution. Since the derivative of the term inside the square root ($x^2 + 4$) is proportional to the x outside, substitution simplifies the expression into a basic power form.

Step 2: Key Formula or Approach:

1. Let $t = x^2 + 4$, then $dt = 2x dx$.
2. Change the limits: When $x = 0$, $t = 4$. When $x = 1$, $t = 5$.
3. Use $\int t^n dt = \frac{t^{n+1}}{n+1} + C$.

Step 3: Detailed Explanation:

Substitute $t = x^2 + 4 \implies dt = 2x dx \implies x dx = \frac{dt}{2}$. The integral becomes:

$$\int_4^5 \sqrt{t} \cdot \frac{dt}{2} = \frac{1}{2} \int_4^5 t^{1/2} dt$$

$$\begin{aligned}
 &= \frac{1}{2} \left[\frac{t^{3/2}}{3/2} \right]_4^5 = \frac{1}{2} \cdot \frac{2}{3} [t^{3/2}]_4^5 \\
 &= \frac{1}{3} [5^{3/2} - 4^{3/2}] \\
 &= \frac{1}{3} [5\sqrt{5} - (2^2)^{3/2}] = \frac{1}{3} [5\sqrt{5} - 2^3] = \frac{1}{3} [5\sqrt{5} - 8]
 \end{aligned}$$

Step 4: Final Answer:

The value of the integral is $\frac{1}{3}[5\sqrt{5} - 8]$.

Quick Tip: Always remember to update your integration limits when performing a u -substitution in a definite integral. It saves you from having to substitute back the original variable at the end.

42. $\int_{-\pi/6}^{\pi/6} \frac{\sin^5 x \cos^3 x}{x^4} dx = \underline{\hspace{2cm}}.$

- (1) $\pi/2$
- (2) $\pi/4$
- (3) 0
- (4) 1

Correct Answer: (3) 0

Solution:

Step 1: Understanding the Concept:

We evaluate the symmetry of the integrand. For an integral with symmetric limits $[-a, a]$, if the function $f(x)$ is "odd" ($f(-x) = -f(x)$), the integral is zero.

Step 2: Key Formula or Approach:

1. Property: $\int_{-a}^a f(x) dx = 0$ if $f(x)$ is an odd function.
2. $\sin(-x) = -\sin x$ (Odd), $\cos(-x) = \cos x$ (Even), $(-x)^4 = x^4$ (Even).

Step 3: Detailed Explanation:

Let $f(x) = \frac{\sin^5 x \cos^3 x}{x^4}$. Test for symmetry:

$$f(-x) = \frac{[\sin(-x)]^5 [\cos(-x)]^3}{(-x)^4} = \frac{(-\sin x)^5 (\cos x)^3}{x^4}$$

$$f(-x) = \frac{-\sin^5 x \cos^3 x}{x^4} = -f(x)$$

Since $f(x)$ is an odd function and the limits are symmetric ($-\pi/6$ to $\pi/6$), the areas above and below the x-axis cancel each other out exactly.

Step 4: Final Answer:

The value of the integral is 0.

Quick Tip: Before doing any heavy calculation on a definite integral with limits from $-a$ to a , always check if the function is odd. It is a common time-saving shortcut in competitive exams.

43. $\int \frac{dx}{\sqrt{16-25x^2}} = \underline{\hspace{2cm}}.$

- (1) $\frac{1}{5} \sin^{-1} \left(\frac{5x}{4} \right) + c$
- (2) $\sin^{-1} \left(\frac{5x}{4} \right) + c$
- (3) $\frac{1}{5} \sin^{-1} \left(\frac{x}{4} \right) + c$
- (4) $\frac{1}{5} \sin^{-1} \left(\frac{4x}{5} \right) + c$

Correct Answer: (1) $\frac{1}{5} \sin^{-1} \left(\frac{5x}{4} \right) + c$

Solution:

Step 1: Understanding the Concept:

This integral matches the standard inverse trigonometric form. To apply the formula, we must first ensure the coefficient of x^2 is 1 or express the denominator as a perfect square of a linear term.

Step 2: Key Formula or Approach:

1. Standard Formula: $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \left(\frac{x}{a} \right) + c.$

2. Generalized Formula: $\int \frac{dx}{\sqrt{a^2 - (mx)^2}} = \frac{1}{m} \sin^{-1} \left(\frac{mx}{a} \right) + c.$

Step 3: Detailed Explanation:

Rewrite the denominator:

$$\int \frac{dx}{\sqrt{4^2 - (5x)^2}}$$

Here, $a = 4$ and the variable part is $5x$. Using the linear transformation rule (dividing by the coefficient of x):

$$= \frac{1}{5} \sin^{-1} \left(\frac{5x}{4} \right) + c$$

Step 4: Final Answer:

The integral is $\frac{1}{5} \sin^{-1} \left(\frac{5x}{4} \right) + c.$

Quick Tip: Always be careful with the "chain rule" in integration. If your variable x is replaced by mx , you must divide the entire integral by m .

44. The solution of the differential equation $x \frac{dy}{dx} + y = 0$ passing through the point $(1,1)$ is $y =$ ____.

- (1) x^2
- (2) x^{-1}
- (3) x^{-2}
- (4) x

Correct Answer: (2) x^{-1}

Solution:

Step 1: Understanding the Concept:

To solve this first-order differential equation, we can use the method of separation of variables. This involves rearranging the equation so that all y terms are on one side and all x terms are on the other.

Step 2: Key Formula or Approach:

1. Separate variables: $\frac{dy}{y} = -\frac{dx}{x}$.
2. Integrate both sides: $\int \frac{1}{y} dy = -\int \frac{1}{x} dx$.
3. Use the given point (1,1) to find the constant of integration C.

Step 3: Detailed Explanation:

Starting with $x \frac{dy}{dx} = -y$:

$$\frac{dy}{y} = -\frac{dx}{x}$$

Integrating both sides:

$$\ln |y| = -\ln |x| + \ln |C|$$

$$\ln |y| + \ln |x| = \ln |C|$$

$$\ln |xy| = \ln |C| \implies xy = C$$

Substitute the point (1,1):

$$(1)(1) = C \implies C = 1$$

Thus, the specific solution is $xy = 1$, which can be written as $y = \frac{1}{x}$ or $y = x^{-1}$.

Step 4: Final Answer:

The solution is $y = x^{-1}$.

Quick Tip: Recognize that $x \frac{dy}{dx} + y$ is the expanded form of the product rule derivative $\frac{d}{dx}(xy)$. Setting $\frac{d}{dx}(xy) = 0$ immediately tells you $xy = C$.

45. Degree of the differential equation $y = x \frac{dy}{dx} + a \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ is ____.

- (1) 4
- (2) 3
- (3) 2
- (4) 1

Correct Answer: (3) 2

Solution:

Step 1: Understanding the Concept:

The degree of a differential equation is the power of the highest order derivative, provided the equation is a polynomial in its derivatives. To find the degree, we must first eliminate radicals (square roots) and fractions involving derivatives.

Step 2: Key Formula or Approach:

1. Isolate the term with the square root.
2. Square both sides to rationalize the equation.

Step 3: Detailed Explanation:

Isolate the radical:

$$y - x \frac{dy}{dx} = a \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

Square both sides:

$$\left(y - x \frac{dy}{dx}\right)^2 = a^2 \left[1 + \left(\frac{dy}{dx}\right)^2\right]$$

Expanding the left side:

$$y^2 + x^2 \left(\frac{dy}{dx}\right)^2 - 2xy \frac{dy}{dx} = a^2 + a^2 \left(\frac{dy}{dx}\right)^2$$

Now the equation is a polynomial in $\frac{dy}{dx}$. The highest order derivative is $\frac{dy}{dx}$ (Order 1), and its highest power in this polynomial form is 2.

Step 4: Final Answer:

The degree of the differential equation is 2.

Quick Tip: Never determine the degree while derivatives are trapped under a square root or in a denominator. Always simplify to polynomial form first!

46. The order of the differential equation of all circles passing through the origin and having their centers on the x-axis is ____.

- (1) 4
- (2) 3
- (3) 2
- (4) 1

Correct Answer: (4) 1

Solution:

Step 1: Understanding the Concept:

The order of a differential equation representing a family of curves is equal to the number of independent arbitrary constants (parameters) in the equation of that family.

Step 2: Key Formula or Approach:

1. Write the general equation of the family of circles.
2. Identify the number of independent arbitrary constants.

Step 3: Detailed Explanation:

The center of the circle lies on the x-axis, so let the center be $(h, 0)$. Since the circle passes through the origin $(0, 0)$, the radius r must be the distance from $(h, 0)$ to the origin, which is $|h|$. The equation of the circle is:

$$(x - h)^2 + (y - 0)^2 = h^2$$

$$x^2 - 2xh + h^2 + y^2 = h^2$$

$$x^2 + y^2 - 2xh = 0$$

In this equation, there is only one arbitrary constant, which is h . Since there is only one independent parameter, the resulting differential equation will be of the first order.

Step 4: Final Answer:

The order of the differential equation is 1.

Quick Tip: Number of independent arbitrary constants = Order of the differential equation. This is a very useful rule for quickly determining order without actually forming the equation.

47. If a and b are arbitrary constants, then the differential equation representing the family of curves $y = a \sin(x + b)$ is ____.

- (1) $\frac{d^2y}{dx^2} - y = 0$
- (2) $\frac{d^2y}{dx^2} + y = 0$
- (3) $\frac{d^2y}{dx^2} - y^2 = 0$
- (4) $\frac{dy}{dx} - y = 0$

Solution:

Step 1: Understanding the Concept:

The order of a differential equation is equal to the number of arbitrary constants in the given equation. Since there are two constants (a and b), we differentiate twice to eliminate them.

Step 2: Key Formula or Approach:

1. Differentiate y with respect to x .
2. Differentiate again to find $\frac{d^2y}{dx^2}$.
3. Substitute the original expression of y back into the result.

Step 3: Detailed Explanation:

Given $y = a \sin(x + b)$. First derivative:

$$\frac{dy}{dx} = a \cos(x + b)$$

Second derivative:

$$\frac{d^2y}{dx^2} = -a \sin(x + b)$$

Since $y = a \sin(x + b)$, we can substitute:

$$\frac{d^2y}{dx^2} = -y$$

$$\frac{d^2y}{dx^2} + y = 0$$

Step 4: Final Answer:

The differential equation is $\frac{d^2y}{dx^2} + y = 0$.

Quick Tip: For any function of the form $y = A\sin(kx + \phi)$ or $y = A\cos(kx + \phi)$, the resulting differential equation is always $\frac{d^2y}{dx^2} + k^2y = 0$.

48. The differential equation is $\frac{dy}{dx} + \frac{y}{x} = 0$ and $y(1) = 2$. Then the value of $y(3) = \underline{\hspace{1cm}}$.

- (1) 2
- (2) 3
- (3) $2/3$
- (4) 1

Solution:

Step 1: Understanding the Concept:

This is a variable separable differential equation. We solve it to find the general relationship between x and y , then use the initial condition to find the constant.

Step 2: Key Formula or Approach:

1. Separate variables: $\frac{dy}{y} = -\frac{dx}{x}$.
2. Integrate both sides.
3. Solve for y given $x = 3$.

Step 3: Detailed Explanation:

$$\frac{dy}{dx} = -\frac{y}{x} \implies \frac{dy}{y} = -\frac{dx}{x}$$

Integrating both sides:

$$\ln|y| = -\ln|x| + \ln|C|$$

$$\ln|y| + \ln|x| = \ln|C| \implies xy = C$$

Using the condition $y(1) = 2$:

$$(1)(2) = C \implies C = 2$$

The equation is $xy = 2$. To find $y(3)$, substitute $x = 3$:

$$3y = 2 \implies y = \frac{2}{3}$$

Step 4: Final Answer:

The value of $y(3)$ is $2/3$.

Quick Tip: The equation $xy = C$ represents a rectangular hyperbola. Since the product of x and y is constant, if x triples, y must become one-third of its original value.

49. The general solution of the differential equation $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$ is ____.

(1) $e^{-y} = e^x + \frac{x^3}{3} + c$

(2) $e^y = e^x + \frac{x^3}{3} + c$

(3) $e^y = e^x + x^3 + c$

(4) $e^y = e^x + c$

Solution:

Step 1: Understanding the Concept:

We simplify the right side of the equation using exponent rules to separate the x and y variables.

Step 2: Key Formula or Approach:

1. Use $e^{x-y} = e^x \cdot e^{-y}$.

2. Factor out e^{-y} and separate variables.

Step 3: Detailed Explanation:

$$\frac{dy}{dx} = e^x e^{-y} + x^2 e^{-y}$$

$$\frac{dy}{dx} = e^{-y}(e^x + x^2)$$

Separate the variables:

$$\frac{dy}{e^{-y}} = (e^x + x^2)dx$$

$$e^y dy = (e^x + x^2)dx$$

Integrate both sides:

$$\int e^y dy = \int (e^x + x^2)dx$$

$$e^y = e^x + \frac{x^3}{3} + c$$

Step 4: Final Answer:

The solution is $e^y = e^x + \frac{x^3}{3} + c$.

Quick Tip: When you see e^{-y} in multiple terms on the RHS, factoring it out usually reveals a separable equation.

50. The differential equation is $\frac{dy}{dx} + y \tan x = \sec x$ and $y(0) = 1$. Then the value of $y(\pi/4) =$ ____.

- (1) 0
- (2) $\sqrt{2}$
- (3) 1
- (4) -1

Solution:

Step 1: Understanding the Concept:

This is a first-order linear differential equation of the form $\frac{dy}{dx} + Py = Q$. We solve it using an Integrating Factor (I.F.).

Step 2: Key Formula or Approach:

1. $I.F. = e^{\int P dx}$.
2. Solution: $y(I.F.) = \int Q(I.F.)dx + c$.

Step 3: Detailed Explanation:

Here, $P = \tan x$ and $Q = \sec x$.

$$I.F. = e^{\int \tan x dx} = e^{\ln |\sec x|} = \sec x$$

The solution is:

$$\begin{aligned} y(\sec x) &= \int \sec x \cdot \sec x dx \\ y \sec x &= \int \sec^2 x dx \implies y \sec x = \tan x + c \end{aligned}$$

Use $y(0) = 1$:

$$(1)\sec(0) = \tan(0) + c \implies 1(1) = 0 + c \implies c = 1$$

General solution: $y \sec x = \tan x + 1 \implies y = \frac{\tan x + 1}{\sec x} = \sin x + \cos x$. At $x = \pi/4$:

$$y = \sin(\pi/4) + \cos(\pi/4) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

Step 4: Final Answer:

The value of $y(\pi/4)$ is $\sqrt{2}$.

Quick Tip: The expression $\sin x + \cos x$ reaches its maximum value of $\sqrt{2}$ exactly at $\pi/4$.

PHYSICS

51. If $P = F \cdot v \sin \beta t$ where F is force and v is velocity then the dimensions of P and β are ____.

(1) ML^2T^{-3}, T^{-1}

(2) MLT^{-2}, T^{-2}

(3) ML^2T^{-1}, T^{-1}

(4) ML^2T^3, T^{-2}

Correct Answer: (1) ML^2T^{-3}, T^{-1}

Solution:

Step 1: Understanding the Concept:

In any physical equation, the dimensions on both sides must be equal. Furthermore, the argument of a trigonometric function (like sine) must be dimensionless.

Step 2: Key Formula or Approach:

1. Dimensions of Force (F): $[MLT^{-2}]$.
2. Dimensions of Velocity (v): $[LT^{-1}]$.
3. Dimensions of Time (t): $[T]$.

Step 3: Detailed Explanation:

For $P = F \cdot v \sin \beta t$: The term $\sin \beta t$ is dimensionless. Therefore, the dimensions of P are simply the dimensions of $F \times v$:

$$[P] = [MLT^{-2}] \times [LT^{-1}] = [ML^2T^{-3}]$$

For the argument βt to be dimensionless:

$$[\beta t] = [M^0L^0T^0]$$

$$[\beta][T] = [1] \implies [\beta] = [T^{-1}]$$

Step 4: Final Answer:

The dimensions are $[P] = ML^2T^{-3}$ and $[\beta] = T^{-1}$.

Quick Tip: The dimensions of P here correspond to Power (*Work/Time*). Whenever you see *Force* $\times$ *Velocity*, you can immediately identify it as Power.

52. If velocity V , energy E and time T are chosen as fundamental quantities then dimensional representation of surface tension in this system will be ____.

- (1) $E^1V^{-2}T^{-2}$
- (2) $E^1V^1T^{-2}$
- (3) $E^2V^{-1}T^3$
- (4) $E^1V^2T^1$

Correct Answer: (1) $E^1V^{-2}T^{-2}$

Solution:**Step 1: Understanding the Concept:**

We express the target quantity (Surface Tension) as a product of the new fundamental quantities raised to unknown powers: $S = kE^aV^bT^c$.

Step 2: Key Formula or Approach:

1. Surface Tension (S) = Force/Length = $[MT^{-2}]$.
2. Energy (E) = $[ML^2T^{-2}]$.
3. Velocity (V) = $[LT^{-1}]$.

Step 3: Detailed Explanation:

$$[MT^{-2}] = [ML^2T^{-2}]^a [LT^{-1}]^b [T]^c$$

$$M^1L^0T^{-2} = M^aL^{2a+b}T^{-2a-b+c}$$

Equating powers: For M: $a = 1$. For L: $2a + b = 0 \implies 2(1) + b = 0 \implies b = -2$. For T: $-2a - b + c = -2 \implies -2(1) - (-2) + c = -2 \implies 0 + c = -2 \implies c = -2$. Substituting

values: $S = E^1 V^{-2} T^{-2}$.

Step 4: Final Answer:

The dimensional representation is $E^1 V^{-2} T^{-2}$.

Quick Tip: Surface tension is Energy per unit Area. Since Area has dimensions of $(\text{Velocity} \times \text{Time})^2$, Surface Tension is $\text{Energy}/(\text{Velocity}^2 \text{Time}^2)$, leading directly to $E V^{-2} T^{-2}$.

53. If $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$, then the angle between the two vectors $\vec{A}$ and $\vec{B}$ is ____.

- (1) 0°
- (2) 180°
- (3) 120°
- (4) 90°

Correct Answer: (4) 90°

Solution:

Step 1: Understanding the Concept:

The magnitude of the sum and difference of two vectors depends on the angle θ between them. We use the parallelogram law of vector addition to set up an equation.

Step 2: Key Formula or Approach:

1. $|\vec{A} + \vec{B}|^2 = A^2 + B^2 + 2AB \cos \theta$.
2. $|\vec{A} - \vec{B}|^2 = A^2 + B^2 - 2AB \cos \theta$.

Step 3: Detailed Explanation:

Square both sides of the given condition:

$$|\vec{A} + \vec{B}|^2 = |\vec{A} - \vec{B}|^2$$

$$A^2 + B^2 + 2AB \cos \theta = A^2 + B^2 - 2AB \cos \theta$$

Cancel A^2 and B^2 from both sides:

$$2AB \cos \theta = -2AB \cos \theta$$

$$4AB \cos \theta = 0$$

Since A and B are non-zero vectors:

$$\cos \theta = 0 \implies \theta = 90^\circ$$

Step 4: Final Answer:

The angle between the vectors is 90° .

Quick Tip: Geometrically, this means the diagonals of a parallelogram are equal in length, which only happens when the parallelogram is a rectangle. Therefore, the adjacent sides (vectors A and B) must be perpendicular.

54. An aeroplane is moving in a circular path with a speed of 450 Kmph. What is the change in velocity in half revolution?

- (1) 0 Kmph
- (2) 450 Kmph
- (3) 250 Kmph
- (4) 900 Kmph

Correct Answer: (4) 900 Kmph

Solution:

Step 1: Understanding the Concept:

Velocity is a vector quantity, meaning it has both magnitude (speed) and direction. In circular motion, even if the speed remains constant, the direction changes continuously. After a half revolution, the object is moving in the exactly opposite direction.

Step 2: Key Formula or Approach:

1. Let the initial velocity be $\vec{v}_1 = v$.
2. After half a revolution, the final velocity is $\vec{v}_2 = -v$.
3. Change in velocity $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$.

Step 3: Detailed Explanation:

Given speed $v = 450$ Kmph. At the start of the half revolution, let the velocity be $+450$ Kmph (in one direction). After half a revolution, the aeroplane is headed in the opposite direction, so the velocity is -450 Kmph. Magnitude of change in velocity:

$$|\Delta v| = |v_{final} - v_{initial}|$$

$$|\Delta v| = |-450 - 450|$$

$$|\Delta v| = |-900| = 900 \text{ Kmph}$$

Step 4: Final Answer:

The change in velocity is 900 Kmph.

Quick Tip: For any object moving in a circle with speed v , the magnitude of change in velocity after half a revolution is always $2v$, whereas after a full revolution, the change is 0.

55. The ratio between maximum and minimum values of two vectors $\vec{A}$ and $\vec{B}$ ($\vec{A} > \vec{B}$) is 4:1. Then the ratio between the magnitudes of two vectors is ____.

(Note: The user prompt text "1:4" is corrected to "4:1" for a valid ratio where $A > B$)

- (1) 3:2
- (2) 5:3
- (3) 2:3
- (4) 3:5

Correct Answer: (2) 5:3

Solution:

Step 1: Understanding the Concept:

The maximum resultant of two vectors occurs when they are in the same direction ($\theta = 0^\circ$), and the minimum occurs when they are in opposite directions ($\theta = 180^\circ$).

Step 2: Key Formula or Approach:

1. $R_{max} = A + B$.
2. $R_{min} = A - B$.
3. Given $\frac{A+B}{A-B} = \frac{4}{1}$.

Step 3: Detailed Explanation:

From the given ratio:

$$\frac{A+B}{A-B} = \frac{4}{1}$$

Cross-multiply:

$$A + B = 4(A - B)$$

$$A + B = 4A - 4B$$

Rearrange the terms:

$$B + 4B = 4A - A$$

$$5B = 3A$$

$$\frac{A}{B} = \frac{5}{3}$$

Step 4: Final Answer:

The ratio between the magnitudes of the two vectors is 5:3.

Quick Tip: You can use the Componendo and Dividendo rule here: If $\frac{A+B}{A-B} = \frac{x}{y}$, then $\frac{A}{B} = \frac{x+y}{x-y}$. In this case, $\frac{4+1}{4-1} = \frac{5}{3}$.

56. The magnitudes of three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ are 12, 5 and 13 units respectively and $\vec{A} + \vec{B} = \vec{C}$. The angle between $\vec{A}$ and $\vec{B}$ is ____.

- (1) 0°
- (2) 120°
- (3) 90°
- (4) 45°

Correct Answer: (3) 90°

Solution:

Step 1: Understanding the Concept:

The equation $\vec{A} + \vec{B} = \vec{C}$ indicates that $\vec{C}$ is the resultant of $\vec{A}$ and $\vec{B}$. We can use the magnitude of the resultant formula to find the angle between the two vectors.

Step 2: Key Formula or Approach:

1. Magnitude formula: $C^2 = A^2 + B^2 + 2AB \cos \theta$.
2. Check if the magnitudes satisfy the Pythagorean theorem.

Step 3: Detailed Explanation:

Substitute the given magnitudes: $A = 12, B = 5, C = 13$.

$$13^2 = 12^2 + 5^2 + 2(12)(5) \cos \theta$$

$$169 = 144 + 25 + 120 \cos \theta$$

$$169 = 169 + 120 \cos \theta$$

Subtract 169 from both sides:

$$0 = 120 \cos \theta$$

$$\cos \theta = 0 \implies \theta = 90^\circ$$

Step 4: Final Answer:

The angle between $\vec{A}$ and $\vec{B}$ is 90° .

Quick Tip: Recognize the Pythagorean triplet: (5, 12, 13). Since $5^2 + 12^2 = 13^2$, the vectors $\vec{A}$ and $\vec{B}$ must be the legs of a right triangle, meaning they are perpendicular (90°).

57. A body falling from height 'H' takes time 'T' seconds to reach the ground. The time taken to cover the second half of height is ____.

- (1) $\frac{T}{\sqrt{2}}$
- (2) $\sqrt{2}T$
- (3) $\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)T$
- (4) $\left(\frac{1}{\sqrt{2}-1}\right)T$

Correct Answer: (3) $\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)T$

Solution:

Step 1: Understanding the Concept:

For a body falling from rest, the displacement s is proportional to the square of time t according to the equation $s = \frac{1}{2}gt^2$. We need to find the total time T for height H and the time t_1 for the first half height $H/2$. The time for the second half is the difference between these two.

Step 2: Key Formula or Approach:

1. Total time: $H = \frac{1}{2}gT^2 \implies T = \sqrt{\frac{2H}{g}}$.
2. Time for first half: $\frac{H}{2} = \frac{1}{2}gt_1^2 \implies t_1 = \sqrt{\frac{H}{g}}$.

Step 3: Detailed Explanation:

From the equations above, we can see that:

$$t_1 = \frac{1}{\sqrt{2}} \sqrt{\frac{2H}{g}} = \frac{T}{\sqrt{2}}$$

The time taken for the second half of the height (Δt) is:

$$\Delta t = T - t_1 = T - \frac{T}{\sqrt{2}}$$

Factor out T :

$$\Delta t = T \left(1 - \frac{1}{\sqrt{2}} \right) = T \left(\frac{\sqrt{2}-1}{\sqrt{2}} \right)$$

Step 4: Final Answer:

The time taken for the second half is $\left(\frac{\sqrt{2}-1}{\sqrt{2}} \right) T$.

Quick Tip: For a body falling from rest, the times taken to cover successive equal distances are in the ratio $1 : (\sqrt{2}-1) : (\sqrt{3}-\sqrt{2}) \dots$

58. With what speed a body be thrown upwards so that the distances covered in the 5th second and 6th second are equal?

- (1) 75 m/s
- (2) $\sqrt{98}$ m/s
- (3) 49 m/s
- (4) 19.8 m/s

Correct Answer: (3) 49 m/s

Solution:

Step 1: Understanding the Concept:

Distances covered in successive seconds are equal only if the body reaches its highest point exactly at the boundary of those two seconds. In this case, the body must reach the peak at $t = 5$ seconds so that it ascends in the 5th second and descends in the 6th second.

Step 2: Key Formula or Approach:

At the highest point, velocity $v = 0$. Using $v = u - gt$.

Step 3: Detailed Explanation:

For the distance in the 5th second to equal the distance in the 6th second, the time of ascent

must be 5 seconds.

$$0 = u - g(5)$$

$$u = 5g$$

Taking $g = 9.8 \text{ m/s}^2$:

$$u = 5 \times 9.8 = 49 \text{ m/s}$$

Step 4: Final Answer:

The initial speed should be 49 m/s.

Quick Tip: If distance in the n^{th} second equals the distance in the $(n + 1)^{\text{th}}$ second, the time to reach the highest point is always n seconds.

59. A body of mass 1 kg starts moving from rest under the action of a force which varies with displacement as $F = 2x + 5$. The work done by this force to displace the body from $x = 0$ to $x = 2 \text{ m}$ is ____.

- (1) 8 J
- (2) 10 J
- (3) 12 J
- (4) 14 J

Correct Answer: (4) 14 J

Solution:

Step 1: Understanding the Concept:

When force is a function of displacement, work done is calculated by integrating the force with respect to displacement over the given interval.

Step 2: Key Formula or Approach:

$$W = \int_{x_1}^{x_2} F dx.$$

Step 3: Detailed Explanation:

$$W = \int_0^2 (2x + 5) dx$$

Integrating the expression:

$$W = \left[\frac{2x^2}{2} + 5x \right]_0^2$$

$$W = [x^2 + 5x]_0^2$$

Substituting the limits:

$$W = (2^2 + 5(2)) - (0^2 + 5(0))$$

$$W = 4 + 10 = 14 \text{ J}$$

Step 4: Final Answer:

The work done by the force is 14 J.

Quick Tip: Work done is the area under the Force-Displacement graph. For linear forces like this, you can also calculate the area of the resulting trapezium.

60. The potential energy of a particle is given by $U(x) = 20 + (x - 2)^2$. The minimum potential energy and the position where it occurs are ____.

- (1) 20 J at $x = 2$
- (2) 2 J at $x = 20$ m
- (3) 22 J at $x = 2$ m
- (4) 0 J at $x = 2$ m

Correct Answer: (1) 20 J at $x = 2$

Solution:

Step 1: Understanding the Concept:

To find the minimum value of a function, we find the point where its first derivative is

zero. Alternatively, observe the structure of the equation: a squared term $(x-2)^2$ is always ≥ 0 .

Step 2: Key Formula or Approach:

The minimum value occurs when the squared term is at its minimum, which is 0.

Step 3: Detailed Explanation:

The potential energy is $U(x) = 20 + (x - 2)^2$. Since $(x - 2)^2$ is a perfect square, its minimum possible value is 0. This happens when $x - 2 = 0 \implies x = 2$ m. Substituting $x = 2$ back into the original equation:

$$U_{min} = 20 + (2 - 2)^2 = 20 + 0 = 20 \text{ J}$$

Step 4: Final Answer:

The minimum potential energy is 20 J at position $x = 2$ m.

Quick Tip: For any expression of the form $U = A + (x - B)^2$, the minimum value is always A and it occurs at $x = B$. No complex calculus needed!

61. Power supplied to a particle of mass 2 kg varies with time as $P = 3t^2/2$ watt, where t is in seconds. If velocity at $t = 0$ is zero, the velocity at $t = 2$ s is ____.

- (1) 1 m/s
- (2) 2 m/s
- (3) $\sqrt{2}$ m/s
- (4) 4 m/s

Correct Answer: (2) 2 m/s

Solution:

Step 1: Understanding the Concept:

Power is the rate of change of work done, and according to the Work-Energy Theorem, the work done on a particle is equal to the change in its kinetic energy. Since the particle starts

from rest, the work done up to time t equals its kinetic energy at that time.

Step 2: Key Formula or Approach:

1. Work Done $W = \int P dt$.
2. Work-Energy Theorem: $W = \Delta K.E. = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$.

Step 3: Detailed Explanation:

Given $P = \frac{3}{2}t^2$. Calculate work done from $t = 0$ to $t = 2$:

$$W = \int_0^2 \frac{3}{2}t^2 dt = \frac{3}{2} \left[\frac{t^3}{3} \right]_0^2 = \frac{1}{2} [2^3 - 0^3] = 4 \text{ J}$$

Using the Work-Energy Theorem ($u = 0$):

$$W = \frac{1}{2}mv^2 \implies 4 = \frac{1}{2}(2)v^2$$

$$4 = v^2 \implies v = 2 \text{ m/s}$$

Step 4: Final Answer:

The velocity at $t = 2$ s is 2 m/s.

Quick Tip: Always check the initial velocity. If the particle is already moving at $t = 0$, you must include the initial kinetic energy in your calculation.

62. A pump is used to deliver water at a certain rate from a given pipe. To obtain twice the volume of water from the same pipe in the same time, by what factor must the power of the motor pump be increased?

- (1) 4
- (2) 8
- (3) 16
- (4) 32

Correct Answer: (2) 8

Solution:

Step 1: Understanding the Concept:

To deliver twice the volume in the same time, the velocity of the water must be doubled. Power for a pump is related to both the mass flow rate and the kinetic energy imparted to that mass.

Step 2: Key Formula or Approach:

1. Mass flow rate $\frac{dm}{dt} = \rho Av$.
2. Power $P = \frac{dK.E.}{dt} = \frac{1}{2} \left(\frac{dm}{dt} \right) v^2 = \frac{1}{2} (\rho Av) v^2 \propto v^3$.

Step 3: Detailed Explanation:

If the volume required is doubled in the same time, then the new velocity $v' = 2v$. Since Power P is proportional to the cube of velocity ($P \propto v^3$):

$$\frac{P'}{P} = \left(\frac{v'}{v} \right)^3 = \left(\frac{2v}{v} \right)^3$$

$$\frac{P'}{P} = 2^3 = 8$$

Step 4: Final Answer:

The power of the motor pump must be increased by a factor of 8.

Quick Tip: This is a standard fluid mechanics problem. If volume flow rate Q increases by factor n , the power increases by n^3 because $P = \frac{1}{2} \rho Av^3$.

63. Two identical piano wires, when tuned to a fundamental frequency of 400 Hz, produce no beats. One wire is then slightly tightened, and the beat frequency heard is 2 Hz. What is the new fundamental frequency of the tightened wire?

- (1) 398 Hz
- (2) 402 Hz

(3) 404 Hz

(4) 396 Hz

Correct Answer: (2) 402 Hz

Solution:

Step 1: Understanding the Concept:

Beat frequency is the absolute difference between two frequencies ($f_b = |f_1 - f_2|$). The fundamental frequency of a stretched string is directly proportional to the square root of its tension ($f \propto \sqrt{T}$).

Step 2: Key Formula or Approach:

1. $f_{new} = f_{old} \pm f_{beat}$.
2. If tension increases, frequency increases.

Step 3: Detailed Explanation:

Initial frequency $f_1 = 400$ Hz. The wire is tightened, so the new frequency f_2 must be greater than f_1 . Given beat frequency $f_b = 2$ Hz:

$$f_2 - f_1 = 2$$

$$f_2 - 400 = 2 \implies f_2 = 402 \text{ Hz}$$

Step 4: Final Answer:

The new fundamental frequency of the tightened wire is 402 Hz.

Quick Tip: Always look for keywords like "tightened" (increases frequency) or "loaded with wax" (decreases frequency) to decide whether to add or subtract the beat frequency.

64. A source of sound of frequency 500 Hz is moving towards an observer with velocity 30 m/s. The speed of sound is 330 m/s. The frequency heard by the observer will be ____.

(1) 450 Hz

- (2) 550 Hz
- (3) 600 Hz
- (4) 500 Hz

Correct Answer: (2) 550 Hz

Solution:

Step 1: Understanding the Concept:

This problem involves the Doppler Effect. When a source of sound moves toward a stationary observer, the wave fronts are "bunched up," causing the observer to perceive a higher frequency than the one actually emitted.

Step 2: Key Formula or Approach:

The apparent frequency f' is given by:

$$f' = f \left(\frac{v}{v - v_s} \right)$$

Where v is the speed of sound and v_s is the velocity of the source.

Step 3: Detailed Explanation:

Given: $f = 500$ Hz, $v = 330$ m/s, and $v_s = 30$ m/s. Substituting the values into the formula:

$$f' = 500 \left(\frac{330}{330 - 30} \right)$$

$$f' = 500 \left(\frac{330}{300} \right)$$

$$f' = 500 \times 1.1 = 550 \text{ Hz}$$

Step 4: Final Answer:

The frequency heard by the observer is 550 Hz.

Quick Tip: If the source moves **towards** the observer, use a **minus** sign in the denominator (higher frequency). If it moves **away**, use a **plus** sign (lower frequency).

65. In Acoustics, 'Noise' is generally characterized by ____.

- (1) Irregular and non-periodic vibrations.
- (2) A constant pitch and frequency
- (3) Vibrations that follow a harmonic series
- (4) Regular and periodic vibrations

Correct Answer: (1) Irregular and non-periodic vibrations.

Solution:

Step 1: Understanding the Concept:

In acoustics, a distinction is made between "musical sound" and "noise." Musical sound is produced by regular, periodic vibrations that create a recognizable pitch. Noise, however, lacks this structure.

Step 2: Key Formula or Approach:

Analyze the waveform: Periodic = Musical; Non-periodic/Random = Noise.

Step 3: Detailed Explanation:

Noise is the result of sound waves that have no definite frequency or pattern. Because the vibrations are irregular and do not repeat at set intervals (non-periodic), the human ear cannot assign a specific pitch to the sound. This is why white noise or a crashing sound feels "discordant" compared to a piano note.

Step 4: Final Answer:

Noise is characterized by irregular and non-periodic vibrations.

Quick Tip: Think of the difference between a drum beat (periodic/rhythmic) and static on a radio (non-periodic/noise).

66. If the volume of a room is doubled and the total absorption is halved, the reverberation time will ____.

- (1) Remain unchanged

- (2) Be doubled
- (3) Become four times
- (4) Be halved

Correct Answer: (3) Become four times

Solution:

Step 1: Understanding the Concept:

Reverberation time (T) is the time required for sound to decay by 60 decibels. It is governed by Sabine's Formula, which relates the volume of the space to its sound-absorbing properties.

Step 2: Key Formula or Approach:

Sabine's Formula: $T = \frac{0.16V}{A}$ Where V is the volume and A is the total absorption.

Step 3: Detailed Explanation:

Let the initial reverberation time be $T_1 = \frac{0.16V}{A}$. According to the problem: 1. New volume $V' = 2V$ 2. New absorption $A' = \frac{A}{2}$ Substitute these into the formula for the new reverberation time T_2 :

$$T_2 = \frac{0.16(2V)}{A/2} = \frac{0.16 \times 2V \times 2}{A}$$

$$T_2 = 4 \times \left(\frac{0.16V}{A} \right) = 4T_1$$

Step 4: Final Answer:

The reverberation time will become four times its original value.

Quick Tip: More volume means sound travels further before hitting a wall; less absorption means less energy is lost per hit. Both factors increase reverberation time.

67. In a closed hall of volume 5000 m^3 , the total absorption of the interior surfaces is 200 metric sabin. The reverberation time is ____.

- (1) 1 s

- (2) 2 s
- (3) 3 s
- (4) 4 s

Correct Answer: (4) 4 s

Solution:

Step 1: Understanding the Concept:

The reverberation time is the time taken for the sound pressure level to decay by 60 dB. This is calculated using Sabine's formula, which relates the volume of the hall and the total absorption.

Step 2: Key Formula or Approach:

Sabine's Formula: $T = \frac{0.16V}{A}$

Where: V = Volume of the hall (in m^3) A = Total absorption (in metric sabins)

Step 3: Detailed Explanation:

Given: $V = 5000 \text{ m}^3$ and $A = 200$ sabins. Substitute the values into the formula:

$$T = \frac{0.16 \times 5000}{200}$$

$$T = \frac{16 \times 50}{200}$$

$$T = \frac{800}{200} = 4 \text{ s}$$

Step 4: Final Answer:

The reverberation time is 4 s.

Quick Tip: Remember the constant 0.16 in Sabine's formula. If the dimensions were in feet, the constant would change to approximately 0.049.

68. In an Isothermal process ____.

- (1) Internal energy of the system never remains constant

- (2) Total heat energy of the system remains constant
- (3) Volume of the system remains constant
- (4) Temperature of the system remains constant

Correct Answer: (4) Temperature of the system remains constant

Solution:

Step 1: Understanding the Concept:

The term "Isothermal" comes from the Greek words "isos" (equal) and "therme" (heat/temperature). In thermodynamics, it describes a process where the temperature of the system is held constant throughout.

Step 2: Key Formula or Approach:

For an ideal gas in an isothermal process: $\Delta T = 0$. This also implies that the change in internal energy $\Delta U = 0$ for an ideal gas.

Step 3: Detailed Explanation:

In an isothermal process, the system is usually in thermal contact with an external reservoir, and the process happens slowly enough to allow the system to continually adjust to the temperature of the reservoir through heat exchange. Because temperature is a measure of the average kinetic energy of the molecules, keeping temperature constant means the thermal state of the system does not change.

Step 4: Final Answer:

The temperature of the system remains constant.

Quick Tip: Isothermal = Constant Temperature ($\Delta T = 0$)

Isobaric = Constant Pressure ($\Delta P = 0$)

Isochoric = Constant Volume ($\Delta V = 0$)

Adiabatic = No heat exchange ($Q = 0$)

69. If the pressure of an ideal gas is doubled and its absolute temperature is halved; the volume will become ____.

- (1) 1/4 of initial volume
- (2) 1/2 initial volume
- (3) Same as initial volume
- (4) 2 times of initial volume

Correct Answer: (1) 1/4 of initial volume

Solution:

Step 1: Understanding the Concept:

To find the change in volume, we use the Combined Gas Law, which relates pressure, volume, and temperature for a fixed amount of an ideal gas.

Step 2: Key Formula or Approach:

Ideal Gas Law: $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$

We need to find V_2 in terms of V_1 .

Step 3: Detailed Explanation:

Given conditions: $P_2 = 2P_1$ (Pressure doubled) $T_2 = \frac{1}{2}T_1$ (Temperature halved) Substitute these into the equation:

$$\frac{P_1 V_1}{T_1} = \frac{(2P_1)V_2}{(T_1/2)}$$

Multiply both sides by T_1 and divide by P_1 :

$$V_1 = \frac{2V_2}{1/2}$$

$$V_1 = 4V_2$$

$$V_2 = \frac{1}{4}V_1$$

Step 4: Final Answer:

The volume will become 1/4 of the initial volume.

Quick Tip: Increasing pressure tends to decrease volume (inverse relationship), and decreasing temperature also tends to decrease volume (direct relationship). Since both actions push the volume down, the result must be a significant decrease.

70. At constant temperature, the product PV is plotted against pressure P for an ideal gas. The graph obtained is ____.

- (1) Straight line parallel to P -axis
- (2) Straight line with positive slope
- (3) Straight line through origin
- (4) Parabola

Correct Answer: (1) Straight line parallel to P -axis

Solution:

Step 1: Understanding the Concept:

This question refers to Boyle's Law. Boyle's Law states that for a fixed mass of gas at constant temperature, the product of pressure and volume is constant.

Step 2: Key Formula or Approach:

Equation: $PV = \text{constant}$ (at constant T). If we let $y = PV$ and $x = P$, the equation is $y = C$.

Step 3: Detailed Explanation:

Since the product PV always results in the same value regardless of what the pressure P is (as long as temperature is constant), the value on the y -axis does not change as you move along the x -axis. In coordinate geometry, a graph where y is constant is a horizontal straight line.

Step 4: Final Answer:

The graph is a straight line parallel to the P -axis (horizontal line).

Quick Tip: This graph is often used to show deviations from ideal gas behavior. Real gases will not show a perfectly horizontal line at very high pressures.

71. A bubble of an ideal gas rises from the bottom of a lake to the surface. At the bottom, the pressure is 3 atm. and the temperature is 7°C. At the surface, the pressure is 1 atm. and the temperature is 27°C. If the initial volume of the bubble was V_0 what is its volume V_f at the surface?

- (1) $3 V_0$
- (2) $3.21 V_0$
- (3) $0.9 V_0$
- (4) $5.4 V_0$

Correct Answer: (2) $3.21 V_0$

Solution:

Step 1: Understanding the Concept:

As the bubble rises, the pressure decreases and the temperature changes. We use the Combined Gas Law to relate the initial state (bottom) to the final state (surface). It is critical to convert temperatures to Kelvin.

Step 2: Key Formula or Approach:

1. Combined Gas Law: $\frac{P_1 V_1}{T_1} = \frac{P_f V_f}{T_f}$.
2. Temperature conversion: $T(K) = T(^{\circ}C) + 273$.

Step 3: Detailed Explanation:

Initial state (bottom): $P_1 = 3 \text{ atm}$, $V_1 = V_0$, $T_1 = 7 + 273 = 280 \text{ K}$.

Final state (surface): $P_f = 1 \text{ atm}$, $T_f = 27 + 273 = 300 \text{ K}$.

Rearranging for V_f :

$$V_f = \frac{P_1 V_1 T_f}{T_1 P_f}$$
$$V_f = \frac{3 \times V_0 \times 300}{280 \times 1}$$
$$V_f = \frac{900}{280} V_0 \approx 3.214 V_0$$

Step 4: Final Answer:

The volume at the surface is $3.21 V_0$.

Quick Tip: Always convert Celsius to Kelvin in gas law problems. A common mistake is using 7 and 27 directly, which would lead to an incorrect answer.

72. The R.M.S. speed of oxygen molecules at 27°C is v . At 927°C , the rms speed will be ____.

- (1) v
- (2) $v/2$
- (3) $2v$
- (4) $4v$

Correct Answer: (3) $2v$

Solution:

Step 1: Understanding the Concept:

The root mean square (RMS) speed of gas molecules depends on the absolute temperature of the gas. If the gas remains the same (Oxygen), the speed is proportional to the square root of the absolute temperature.

Step 2: Key Formula or Approach:

1. RMS speed formula: $v_{rms} = \sqrt{\frac{3RT}{M}}$.
2. Ratio: $\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}}$.

Step 3: Detailed Explanation:

Initial temperature $T_1 = 27 + 273 = 300 \text{ K}$.

Final temperature $T_2 = 927 + 273 = 1200 \text{ K}$.

Taking the ratio of the speeds:

$$\frac{v_2}{v} = \sqrt{\frac{1200}{300}}$$

$$\frac{v_2}{v} = \sqrt{4} = 2$$

$$v_2 = 2v$$

Step 4: Final Answer:

The rms speed at 927°C will be $2v$.

Quick Tip: To double the speed of gas molecules, you must quadruple the absolute temperature (in Kelvin). Here, temperature increased from 300 K to 1200 K (4 times), so speed doubles.

73. In a photoelectric experiment, the stopping potential for incident light of wavelength 4000 Å is 2V. If the wavelength is changed to 3000 Å, the new stopping potential will be approximately ____.

(Use $h = 4.14 \times 10^{-15} \text{ eV}\cdot\text{s}$, $c = 3 \times 10^8 \text{ m/s}$)

- (1) 2 V
- (2) 3.03 V
- (3) 4.14 V
- (4) 1.5 V

Correct Answer: (2) 3.03 V

Solution:**Step 1: Understanding the Concept:**

According to Einstein's photoelectric equation, the energy of an incident photon equals the sum of the work function and the maximum kinetic energy of the emitted electron. Maximum kinetic energy is equivalent to $e \times V_{\text{stopping}}$.

Step 2: Key Formula or Approach:

1. $\frac{hc}{\lambda} = \phi + eV_s$.
2. Photon energy in eV $\approx \frac{12420}{\lambda(\text{\AA})}$ (or $\frac{12400}{\lambda}$).

Step 3: Detailed Explanation:

Calculate energy for 4000 Å: $E_1 = \frac{12420}{4000} \approx 3.105 \text{ eV}$. Using $E_1 = \phi + eV_{s1}$:
 $3.105 = \phi + 2 \Rightarrow \phi = 1.105 \text{ eV}$. Now calculate energy for 3000 Å: $E_2 = \frac{12420}{3000} = 4.14 \text{ eV}$.

Using $E_2 = \phi + eV_{s2}$: $4.14 = 1.105 + V_{s2}$ $V_{s2} = 4.14 - 1.105 = 3.035$ V.

Step 4: Final Answer:

The new stopping potential is approximately 3.03 V.

Quick Tip: Shorter wavelength means higher photon energy. If energy increases by roughly 1 eV (from 3.1 to 4.1 eV), the stopping potential should also increase by roughly 1 V.

74. In Optical Fiber communication, the signal is transmitted in the form of ____.

- (1) Electrical pulses
- (2) Light pulses
- (3) Radio waves
- (4) Sound waves

Correct Answer: (2) Light pulses

Solution:

Step 1: Understanding the Concept:

Optical fiber technology uses thin strands of glass or plastic to transmit data over long distances. Unlike traditional copper wires that use electrons, optical fibers utilize photons to carry information.

Step 2: Key Formula or Approach:

The core principle of optical fiber communication is Total Internal Reflection (TIR), which allows electromagnetic waves in the visible or infrared spectrum to travel through the fiber with minimal loss.

Step 3: Detailed Explanation:

In an optical fiber system, an electrical signal (data) is converted into light pulses by a light source, such as a Laser or an LED. These light pulses travel through the fiber core. At the receiving end, a photodetector converts these light pulses back into electrical signals. Because light has a much higher frequency than electricity, it can carry significantly more data.

Step 4: Final Answer:

The signal is transmitted in the form of light pulses.

Quick Tip: Optical fibers are immune to electromagnetic interference (EMI) because they transmit light instead of electricity, making them much more reliable in noisy environments.

75. In a superconducting ring, a persistent current has been flowing without decay for years. This is possible because ____.

- (1) Resistance is exactly zero and flux is quantized
- (2) Resistance is very small but finite
- (3) The ring is at absolute zero temperature
- (4) Magnetic field lines are expelled

Correct Answer: (1) Resistance is exactly zero and flux is quantized

Solution:**Step 1: Understanding the Concept:**

Superconductivity is a state of matter where a material exhibits exactly zero electrical resistance and the expulsion of magnetic fields (Meissner effect) when cooled below a characteristic critical temperature.

Step 2: Key Formula or Approach:

According to Ohm's Law, $V = IR$. If the resistance R is exactly zero, then the potential difference V required to maintain a current I is zero. Consequently, energy is not dissipated as heat ($P = I^2R = 0$).

Step 3: Detailed Explanation:

When a current is induced in a superconducting loop, there is no "friction" (resistance) to slow down the electrons. Therefore, the kinetic energy of the charge carriers remains constant indefinitely. Additionally, in a closed superconducting loop, the magnetic flux through the ring remains trapped and is quantized in units of the fluxoid, which stabilizes the persistent current.

Step 4: Final Answer:

Persistent current is possible because the resistance is exactly zero and the magnetic flux is quantized.

Quick Tip: Persistent currents in superconductors have been observed to last for over 25 years in laboratory settings with no measurable decrease in magnitude!

CHEMISTRY

76. The pair of orbitals with electron density maximum along the axes is ____.

- (1) d_{xy} , d_{yz}
- (2) d_{z^2} , $d_{x^2-y^2}$
- (3) d_{xz} , d_{z^2}
- (4) d_{xz} , P_z

Correct Answer: (2) d_{z^2} , $d_{x^2-y^2}$

Solution:**Step 1: Understanding the Concept:**

The five d-orbitals are classified into two groups based on their orientation in space: those that lie between the axes (t_{2g} set) and those that lie along the axes (e_g set).

Step 2: Key Formula or Approach:

- 1. Non-axial orbitals: d_{xy} , d_{yz} , d_{xz} (lobes lie at 45° to the axes).
- 2. Axial orbitals: $d_{x^2-y^2}$, d_{z^2} (lobes lie directly on the axes).

Step 3: Detailed Explanation:

The $d_{x^2-y^2}$ orbital has four lobes pointing along the X and Y axes. The d_{z^2} orbital has two lobes pointing along the Z-axis and a "doughnut" of electron density in the XY plane. In contrast, d_{xy} , d_{yz} , and d_{xz} have lobes situated between the respective coordinate axes.

Step 4: Final Answer:

The orbitals with electron density along the axes are d_{z^2} and $d_{x^2-y^2}$.

Quick Tip: Remember the e_g group (axial) vs. the t_{2g} group (non-axial). This distinction is fundamental to understanding Crystal Field Theory in coordination chemistry.

77. The angular momentum of an electron in an orbit X of hydrogen atom is $2h/\pi$. Maximum number of orbitals possible in X is ____.

- (1) 4
- (2) 9
- (3) 16
- (4) 25

Correct Answer: (3) 16

Solution:**Step 1: Understanding the Concept:**

According to Bohr's model, the angular momentum of an electron is quantized and depends on the principal quantum number n . Once n is known, the number of orbitals in that shell can be determined.

Step 2: Key Formula or Approach:

1. Bohr's Angular Momentum: $L = \frac{nh}{2\pi}$.
2. Number of orbitals in a shell $= n^2$.

Step 3: Detailed Explanation:

Given angular momentum $L = \frac{2h}{\pi}$. Set this equal to the formula:

$$\frac{nh}{2\pi} = \frac{2h}{\pi}$$

Cancel h and π from both sides:

$$\frac{n}{2} = 2 \implies n = 4$$

The principal quantum number is 4. The maximum number of orbitals in the n^{th} shell is n^2 :

$$\text{Number of orbitals} = 4^2 = 16$$

Step 4: Final Answer:

The maximum number of orbitals possible in orbit X is 16.

Quick Tip: Don't confuse "number of orbitals" (n^2) with "maximum number of electrons" ($2n^2$). For $n = 4$, there are 16 orbitals and 32 electrons.

78. The four quantum numbers for the electron in the outermost orbital of potassium ($Z=19$) are ____.

- (1) $n=4, l=2, m=-1, s=+1/2$
- (2) $n=4, l=0, m=0, s=+1/2$
- (3) $n=3, l=0, m=1, s=+1/2$
- (4) $n=4, l=3, m=-2, s=-1/2$

Correct Answer: (2) $n=4, l=0, m=0, s=+1/2$

Solution:

Step 1: Understanding the Concept:

To find the quantum numbers of the outermost electron, we must first write the electronic configuration of the atom based on the Aufbau principle.

Step 2: Key Formula or Approach:

1. Potassium ($Z = 19$) configuration: $1s^2 2s^2 2p^6 3s^2 3p^6 4s^1$.
2. Identify the outermost electron: $4s^1$.

Step 3: Detailed Explanation:

For the $4s^1$ electron: The principal quantum number (n) is the coefficient: **4**. For an 's' orbital, the azimuthal quantum number (l) is **0**. If $l = 0$, the magnetic quantum number (m) must be **0**. The spin quantum number (s) can be $+1/2$ (or $-1/2$). Looking at the options, $n = 4, l = 0, m = 0, s = +1/2$ is the only correct set.

Step 4: Final Answer:

The quantum numbers are $n = 4, l = 0, m = 0, s = +1/2$.

Quick Tip: Potassium is an alkali metal in Group 1 and Period 4. All Period 4 elements start filling the $n = 4$ shell, and Group 1 elements always end with ns^1 .

79. In which of the following, the number of bonding electrons and non-bonding electrons are in 3:2 ratio?

- (1) N
- (2) O
- (3) HCl
- (4) F

Correct Answer: (1) N

Solution:**Step 1: Understanding the Concept:**

Bonding electrons are those shared between atoms to form chemical bonds. Non-bonding electrons (lone pairs) are valence electrons that do not participate in bonding. We need to calculate the total count for each molecule.

Step 2: Key Formula or Approach:

1. Draw the Lewis structure for each molecule.
2. Count shared electrons (bonding) and lone pair electrons (non-bonding).
3. Check the ratio (Bonding : Non-bonding).

Step 3: Detailed Explanation:

For **N**: Nitrogen has 5 valence electrons. It forms a triple bond ($:N \equiv N:$).

- Bonding electrons = 6 (3 pairs)
- Non-bonding electrons = 4 (2 lone pairs)
- Ratio = $6 : 4 = 3 : 2$.

For **O** ($:O = O:$): Bonding = 4, Non-bonding = 8 (Ratio 1:2).

For **F** ($:F - F:$): Bonding = 2, Non-bonding = 12 (Ratio 1:6).

For **HCl** ($H - Cl:$): Bonding = 2, Non-bonding = 6 (Ratio 1:3).

Step 4: Final Answer:

The ratio 3:2 is found in N.

Quick Tip: In diatomic molecules of the second period, as you move from N_2 to F_2 , the number of bonding electrons decreases while non-bonding electrons increase.

80. Which one of the following statements is not correct?

- (1) Ionic bond is non directional bond
- (2) The maximum number of bond pairs between two atoms is 3
- (3) Covalent compounds conduct electricity in fused state
- (4) Ionic compounds are generally soluble in water

Correct Answer: (3) Covalent compounds conduct electricity in fused state

Solution:**Step 1: Understanding the Concept:**

This question tests the general properties of ionic and covalent compounds. Conductivity requires the presence of free-moving charged particles (ions or electrons).

Step 2: Key Formula or Approach:

Evaluate each statement based on chemical principles:

- Ionic bonds: Electrostatic, non-directional, soluble in polar solvents, conduct when molten/aqueous.
- Covalent bonds: Shared electrons, directional, generally non-conductors.

Step 3: Detailed Explanation:

- **Statement 1:** Correct. Ionic bonds are electrostatic attractions acting in all directions around an ion.
- **Statement 2:** Correct. Under standard conditions, the maximum bond order between two atoms is 3 (triple bond), like in $N \equiv N$ or $HC \equiv CH$.
- **Statement 3: Incorrect.** Covalent compounds consist of neutral molecules. Even in a fused (molten) state, they do not have free ions to carry electric current. (Exceptions like graphite exist, but as a general rule, this is false).
- **Statement 4:** Correct. Being polar, ionic compounds generally dissolve in polar solvents like water ("like dissolves like").

Step 4: Final Answer:

Statement (3) is incorrect.

Quick Tip: Covalent compounds are usually insulators. If a compound conducts electricity in the fused state, it is almost certainly an ionic compound.

81. 12.6 g of oxalic acid, $H_2C_2O_4 \cdot 2H_2O$ (M.wt 126) is present in 1500 mL of solution. The normality of that solution is ____.

- (1) 0.266 N
- (2) 0.133 N
- (3) 0.399 N
- (4) 0.430 N

Correct Answer: (2) 0.133 N

Solution:

Step 1: Understanding the Concept:

Normality (N) is defined as the number of gram equivalents of solute per liter of solution. For oxalic acid ($H_2C_2O_4 \cdot 2H_2O$), it is a dibasic acid, meaning it can donate two protons (H^+).

Step 2: Key Formula or Approach:

1. Equivalent weight = $\frac{\text{Molecular weight}}{\text{Basicity (n-factor)}}$.
2. Normality (N) = $\frac{\text{Mass (g)}}{\text{Equivalent weight}} \times \frac{1000}{\text{Volume (mL)}}$.

Step 3: Detailed Explanation:

Oxalic acid dihydrate has a molecular weight of 126 and an n-factor of 2.

$$\text{Equivalent weight} = \frac{126}{2} = 63$$

Now, calculate Normality:

$$N = \frac{12.6}{63} \times \frac{1000}{1500}$$
$$N = 0.2 \times \frac{2}{3} = \frac{0.4}{3} \approx 0.1333 \text{ N}$$

Step 4: Final Answer:

The normality of the solution is 0.133 N.

Quick Tip: For hydrated salts like oxalic acid, always include the mass of water molecules in the molecular weight calculation ($2 \times 18 = 36$). In this case, the total is given as 126.

82. Which of the following has highest equivalent weight? (Given: At.wt H=1, C=12, O=16, S=32, Na=23, Ca=40)

- (1) Sulphuric acid
- (2) Sodium carbonate
- (3) Sodium sulphate
- (4) Calcium carbonate

Correct Answer: (3) Sodium sulphate

Solution:

Step 1: Understanding the Concept:

Equivalent weight for acids is $M/\text{basicity}$, and for salts, it is $M/\text{total positive charge on cation}$.

Step 2: Key Formula or Approach:

Calculate Molecular Weight (M) and divide by n-factor (n) for each:

Step 3: Detailed Explanation:

1. **Sulphuric acid** (H_2SO_4): $M = 98, n = 2 \implies Eq.wt = 49$.
2. **Sodium carbonate** (Na_2CO_3): $M = 106, n = 2 \implies Eq.wt = 53$.
3. **Sodium sulphate** (Na_2SO_4): $M = (23 \times 2) + 32 + (16 \times 4) = 46 + 32 + 64 = 142$.
 $n = 2 \implies Eq.wt = 71$.
4. **Calcium carbonate** ($CaCO_3$): $M = 100, n = 2 \implies Eq.wt = 50$.

Step 4: Final Answer:

Sodium sulphate has the highest equivalent weight (71).

Quick Tip: Always identify the n-factor correctly. For salts, it's the total magnitude of charge on either the cationic or anionic part.

83. Identify the pair of gases which have same number of molecules at S.T.P?

- (1) 11 g of CO and 14 g of N
- (2) 16 g of O and 16 g of CH
- (3) 5 g of H and 40 g of CH
- (4) 28 g of N and 22 g of CO

Correct Answer: (3) 5 g of H and 40 g of CH

Solution:

Step 1: Understanding the Concept:

According to Avogadro's law, equal volumes of gases at STP contain the same number of molecules. This means they must have the same number of moles ($n = \text{mass}/\text{molar mass}$).

Step 2: Key Formula or Approach:

Calculate moles for each pair: $n = w/M$.

Step 3: Detailed Explanation:

- **Option 1:** $\text{CO}_2(11/44 = 0.25)$, $\text{N}_2(14/28 = 0.5)$. No.
- **Option 2:** $\text{O}_2(16/32 = 0.5)$, $\text{CH}_4(16/16 = 1.0)$. No.
- **Option 3:** $\text{H}_2(5/2 = 2.5)$, $\text{CH}_4(40/16 = 2.5)$. **Yes.**
- **Option 4:** $\text{N}_2(28/28 = 1)$, $\text{CO}_2(22/44 = 0.5)$. No.

Step 4: Final Answer:

The pair with the same number of molecules is 5 g of H and 40 g of CH.

Quick Tip: "Same number of molecules" is synonymous with "same number of moles." Calculate the moles quickly to find the answer.

84. 100 mL of 0.1M HCl and 100 mL of 0.05 M HSO are mixed and the solution is diluted to 2.0 L by adding water. The pH of the resulting solution is ____.

- (1) 1
- (2) 3
- (3) 2
- (4) 4

Correct Answer: (3) 2

Solution:

Step 1: Understanding the Concept:

The pH of a solution depends on the total concentration of hydrogen ions (H^+). When mixing strong acids, we calculate the total moles of H^+ and divide by the final total volume.

Step 2: Key Formula or Approach:

1. Moles of H^+ from $HCl = M \times V$ (in L).
2. Moles of H^+ from $H_2SO_4 = 2 \times M \times V$ (since it is dibasic).
3. $[H^+] = \frac{\text{Total moles of } H^+}{\text{Total Volume in L}}$.
4. $pH = -\log[H^+]$.

Step 3: Detailed Explanation:

- Moles from $HCl = 0.1 \times 0.1 = 0.01$ moles.
- Moles from $H_2SO_4 = 2 \times (0.05 \times 0.1) = 2 \times 0.005 = 0.01$ moles.
- Total moles of $H^+ = 0.01 + 0.01 = 0.02$ moles.
- Total final volume = 2.0 L.
- $[H^+] = \frac{0.02}{2.0} = 0.01 = 10^{-2}$ M.
- $pH = -\log(10^{-2}) = 2$.

Step 4: Final Answer:

The pH of the resulting solution is 2.

Quick Tip: Always remember that H_2SO_4 releases **two** H^+ ions per molecule. Forgetting this factor is the most common mistake in acid mixture problems.

85. According to Arrhenius theory of acids and bases, which of the following is an example of Arrhenius base?

(1) HSO

- (2) NH
- (3) NaOH
- (4) CaO

Correct Answer: (3) NaOH

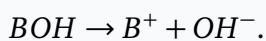
Solution:

Step 1: Understanding the Concept:

The Arrhenius theory specifically defines a base as a substance that increases the concentration of hydroxide ions (OH^-) when dissolved in water.

Step 2: Key Formula or Approach:

Look for a substance containing a hydroxyl group that dissociates in an aqueous medium:



Step 3: Detailed Explanation:

- **HSO:** Dissociates to give H^+ (Arrhenius acid).
- **NH:** Does not contain OH^- . While it acts as a base in water, it is better explained by Bronsted-Lowry or Lewis theories.
- **NaOH:** Dissociates directly into Na^+ and OH^- in water. This fits the strict Arrhenius definition perfectly.
- **CaO:** A basic oxide, but it must react with water to form $Ca(OH)_2$ before it provides OH^- ions.

Step 4: Final Answer:

NaOH is the example of an Arrhenius base.

Quick Tip: Arrhenius theory is limited to aqueous solutions and substances that directly possess H or OH groups in their formula.

86. Electrolysis of an aqueous solution of NaSO between Pt electrodes liberate a gas X at anode and gas Y at cathode. X and Y respectively are ____.

- (1) H, O
- (2) O, H
- (3) SO, H
- (4) H, SO

Correct Answer: (2) O, H

Solution:

Step 1: Understanding the Concept:

In the electrolysis of aqueous Na_2SO_4 , we must consider the discharge potential of the ions (Na^+ , SO_4^{2-}) versus water (H_2O). Platinum (Pt) electrodes are inert and do not participate in the reaction.

Step 2: Key Formula or Approach:

Compare reduction/oxidation potentials. Water is oxidized more easily than SO_4^{2-} and reduced more easily than Na^+ .

Step 3: Detailed Explanation:

- **At Anode (Oxidation):** Both SO_4^{2-} and H_2O are present. Water has a lower oxidation potential, so it oxidizes: $2H_2O \rightarrow O_2(g) + 4H^+ + 4e^-$. Thus, **X is O**.
- **At Cathode (Reduction):** Both Na^+ and H_2O are present. H_2O is reduced more easily than Na^+ : $2H_2O + 2e^- \rightarrow H_2(g) + 2OH^-$. Thus, **Y is H**.

Step 4: Final Answer:

X is Oxygen (O_2) and Y is Hydrogen (H_2).

Quick Tip: In aqueous solutions of active metal sulfates (like Na, K, Mg), the net result is effectively the electrolysis of water itself, producing O_2 at the anode and H_2 at the cathode.

87. The wrong statement regarding Galvanic cell is ____.

- (1) In this spontaneous redox reaction occurs
- (2) Salt bridge maintains electrical neutrality between the two solutions
- (3) Anode is represented by (+) and cathode by (-)
- (4) At anode oxidation occurs

Correct Answer: (3) Anode is represented by (+) and cathode by (-)

Solution:

Step 1: Understanding the Concept:

A Galvanic (or Voltaic) cell converts chemical energy into electrical energy through a spontaneous redox reaction. It is important to distinguish its electrode polarity from an electrolytic cell.

Step 2: Key Formula or Approach:

Use the mnemonic **LOAN: Left Oxidation Anode Negative**.

Step 3: Detailed Explanation:

- **Statement 1:** Correct. Galvanic cells run on spontaneous reactions ($\Delta G < 0$).
- **Statement 2:** Correct. The salt bridge completes the circuit and prevents charge build-up by allowing ion flow.
- **Statement 3: Incorrect.** In a Galvanic cell, the **Anode is Negative (-)** and the **Cathode is Positive (+)**. This is because the anode is the source of electrons.
- **Statement 4:** Correct. Oxidation always occurs at the anode, regardless of the cell type.

Step 4: Final Answer:

Statement (3) is wrong.

Quick Tip: Remember: Oxidation → Anode; Reduction → Cathode. For signs, think: Galvanic Anode is Negative (GAN).

88. Which of the following is a weak electrolyte?

- (1) HCO
- (2) HSO
- (3) NaCl
- (4) NaOH

Correct Answer: (1) HCO

Solution:

Step 1: Understanding the Concept:

An electrolyte is "weak" if it only partially dissociates into ions in an aqueous solution. Strong electrolytes dissociate nearly 100%.

Step 2: Key Formula or Approach:

Identify the nature of the substance: Strong acids, strong bases, and most salts are strong electrolytes. Organic acids and some inorganic acids like Carbonic acid are weak.

Step 3: Detailed Explanation:

- **HCO (Carbonic Acid):** An inorganic acid that exists in equilibrium with its ions; it does not fully ionize. Thus, it is a **weak electrolyte**.
- **HSO (Sulphuric Acid):** A strong mineral acid.
- **NaCl (Sodium Chloride):** A salt that fully dissociates into Na^+ and Cl^- .
- **NaOH (Sodium Hydroxide):** A strong base.

Step 4: Final Answer:

HCO is a weak electrolyte.

Quick Tip: Most "carbon" containing acids (organic) and acids found in soft drinks (like H_2CO_3 or H_3PO_4) are typically weak electrolytes.

89. The exhausted anion-exchange resin is regenerated with ____.

- (1) dilute NaOH solution
- (2) dilute NaCl solution
- (3) dilute HCl solution
- (4) dilute NaSO solution

Correct Answer: (1) dilute NaOH solution

Solution:

Step 1: Understanding the Concept:

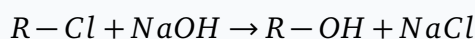
Ion exchange resins are used for water demineralization. Anion-exchange resins pick up negative ions (like Cl^- , SO_4^{2-}) and release OH^- . When "exhausted," they are saturated with these impurity anions.

Step 2: Key Formula or Approach:

To regenerate, you must provide a high concentration of the original ion (OH^-) to drive the equilibrium backward.

Step 3: Detailed Explanation:

An exhausted anion resin ($R - Cl$) needs to be converted back to its basic form ($R - OH$). This is achieved by washing it with a **dilute NaOH solution**. The high concentration of OH^- displaces the chloride ions:



Conversely, a cation-exchange resin is regenerated with dilute HCl or H_2SO_4 .

Step 4: Final Answer:

Dilute NaOH solution is used for regeneration.

Quick Tip: Anion starts with 'A' and is regenerated by Alkali (NaOH). Cation starts with 'C' and is regenerated by Acid (HCl).

90. A sample of water is known to contain $Mg(HCO) = 7.3 \text{ mg/L}$, $Ca(HCO) = 8.1 \text{ mg/L}$ and

27.2 mg/L of CaSO_4 . The total hardness associated with water sample (in ppm) in equivalents of CaCO_3 is ____.

(At.wt H=1, C=12, O=16, Mg=24, Ca=40, S=32)

- (1) 20
- (2) 25
- (3) 30
- (4) 40

Correct Answer: (3) 30

Solution:

Step 1: Understanding the Concept:

Hardness is expressed in terms of CaCO_3 equivalents. Each salt contribution is calculated by converting its mass to the equivalent mass of CaCO_3 .

Step 2: Key Formula or Approach:

$$\text{Hardness (as } \text{CaCO}_3) = \frac{\text{Mass of salt}}{\text{Molar mass of salt}} \times \text{Molar mass of } \text{CaCO}_3$$

(Note: This works because all these salts have the same n-factor as CaCO_3 , which is 2).

Step 3: Detailed Explanation:

1. **Molar Masses:** $\text{Mg}(\text{HCO}_3)_2 = 146$, $\text{Ca}(\text{HCO}_3)_2 = 162$, $\text{CaSO}_4 = 136$, $\text{CaCO}_3 = 100$.

2. **Calculation:**

- Due to $\text{Mg}(\text{HCO}_3)_2 = \frac{7.3}{146} \times 100 = 0.05 \times 100 = 5$ ppm.
- Due to $\text{Ca}(\text{HCO}_3)_2 = \frac{8.1}{162} \times 100 = 0.05 \times 100 = 5$ ppm.
- Due to $\text{CaSO}_4 = \frac{27.2}{136} \times 100 = 0.2 \times 100 = 20$ ppm.

3. **Total Hardness** = $5 + 5 + 20 = 30$ ppm.

Step 4: Final Answer:

The total hardness is 30 ppm.

Quick Tip: Hardness units: 1 mg/L is exactly equal to 1 ppm for dilute aqueous solutions.

91. The type of functional group associated with cation exchange resin is ____.

- (1) -OH
- (2) -SOH
- (3) -NH
- (4) -CHO

Correct Answer: (2) -SOH

Solution:

Step 1: Understanding the Concept:

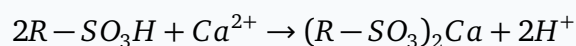
Ion-exchange resins are insoluble polymers that can exchange specific ions within a solution. Cation exchange resins must possess acidic functional groups that can release H^+ ions in exchange for other cations (like Ca^{2+} or Mg^{2+}).

Step 2: Key Formula or Approach:

Cation Exchange Resins: Contain acidic groups like sulphonic acid ($-SO_3H$) or carboxylic acid ($-COOH$). Anion Exchange Resins: Contain basic groups like quaternary ammonium or amino groups ($-NH_2, -OH$).

Step 3: Detailed Explanation:

Cation resins are essentially giant organic molecules with fixed negative charges and mobile positive ions. The sulphonic acid group ($-SO_3H$) is a strong acid group commonly used in these resins. When hard water passes through, the H^+ ions from the $-SO_3H$ group are replaced by the cations present in the water:



Step 4: Final Answer:

The functional group associated with cation exchange resin is -SOH.

Quick Tip: Remember: "Cation" resins are "Acidic" (release H^+), and "Anion" resins are "Basic" (release OH^-).

92. Identify the incorrect statement about the corrosion ____.

- (1) In the composition type of galvanic cell, metal with lower standard reduction potential undergoes corrosion
- (2) In stress cell type of galvanic cell, corrosion occurs at the stressed area of the metal
- (3) The rate of corrosion is more, when the area of cathode is smaller
- (4) In concentration cell type of galvanic cell, the metal below the water level undergoes corrosion readily

Correct Answer: (3) The rate of corrosion is more, when the area of cathode is smaller

Solution:

Step 1: Understanding the Concept:

Corrosion is an electrochemical process. The rate and location of corrosion are influenced by electrode potential, mechanical stress, and differential aeration.

Step 2: Key Formula or Approach:

Analyze each statement based on the "Small Anode, Large Cathode" rule and electrochemical principles.

Step 3: Detailed Explanation:

- **Statement 1:** Correct. The metal with lower reduction potential (more active) acts as the anode and corrodes.
- **Statement 2:** Correct. Stressed areas have higher energy and act as anodic regions relative to unstressed areas.
- **Statement 3: Incorrect.** The rate of corrosion is higher when the **anode area is small** and the **cathode area is large**. A large cathode provides a vast surface for the reduction reaction, forcing the small anode to dissolve rapidly to supply enough electrons.

- **Statement 4:** Correct. This is differential aeration. Parts of the metal with less oxygen access (underwater) become anodic and corrode, while parts with high oxygen access (above water) become cathodic.

Step 4: Final Answer:

Statement (3) is incorrect.

Quick Tip: To prevent rapid corrosion, never use a small anodic fastener (like an iron nail) on a large cathodic surface (like a copper sheet).

93. In galvanised iron ____.

- (1) Zn acts as anode and Fe acts as cathode
- (2) Zn acts as cathode and Fe acts as anode
- (3) Sn acts as anode and Fe acts as cathode
- (4) Sn acts as cathode and Fe acts as anode

Correct Answer: (1) Zn acts as anode and Fe acts as cathode

Solution:

Step 1: Understanding the Concept:

Galvanization is the process of applying a protective zinc coating to steel or iron to prevent rusting. This is a form of "sacrificial protection."

Step 2: Key Formula or Approach:

Compare the standard reduction potentials (E°): $E^\circ(\text{Fe}^{2+}/\text{Fe}) = -0.44 \text{ V}$ $E^\circ(\text{Zn}^{2+}/\text{Zn}) = -0.76 \text{ V}$

Step 3: Detailed Explanation:

Since Zinc has a more negative reduction potential than Iron, it is more "active." This means Zinc will oxidize (lose electrons) more readily than Iron. In the presence of moisture and air, a galvanic cell is formed where Zinc acts as the **Anode** and undergoes corrosion, while the Iron acts as the **Cathode** and remains protected. Even if the coating is scratched, the Zinc

continues to protect the exposed Iron.

Step 4: Final Answer:

In galvanized iron, Zn acts as the anode and Fe acts as the cathode.

Quick Tip: Contrast this with "Tinning." Tin (Sn) has a higher reduction potential than Fe. If tin-coated iron is scratched, the Iron becomes the anode and corrodes *faster* than if it weren't coated at all!

94. During Vulcanization of raw rubber, the chemical added to it is ____.

- (1) Sulphur
- (2) Phosphorus
- (3) Iodine
- (4) Sodium

Correct Answer: (1) Sulphur

Solution:

Step 1: Understanding the Concept:

Raw natural rubber is soft, sticky, and has low tensile strength. Vulcanization is a chemical process used to improve these physical properties by heating the rubber with a specific cross-linking agent.

Step 2: Key Formula or Approach:

The process involves creating chemical "bridges" between the long polymer chains of isoprene.

Step 3: Detailed Explanation:

In vulcanization, raw rubber is heated with **Sulphur** (typically 3–5%) and appropriate additives. The sulphur atoms form cross-links (disulphide bonds) between the polyisoprene chains. This makes the rubber harder, more elastic, and less sensitive to temperature changes.

Step 4: Final Answer:

The chemical added during vulcanization is Sulphur.

Quick Tip: Think of vulcanization like a ladder: the rubber chains are the side rails, and the sulphur atoms are the rungs that hold everything firmly together.

95. Which of the following is a natural polymer?

- (1) Cellulose
- (2) Teflon
- (3) Polyvinylchloride
- (4) Neoprene rubber

Correct Answer: (1) Cellulose

Solution:

Step 1: Understanding the Concept:

A natural polymer is a large molecule (macromolecule) found in nature, typically produced by living organisms like plants and animals.

Step 2: Key Formula or Approach:

Distinguish between naturally occurring substances and those synthesized in a laboratory or factory (synthetic).

Step 3: Detailed Explanation:

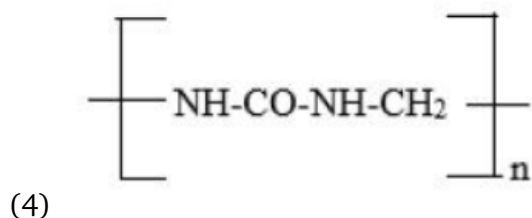
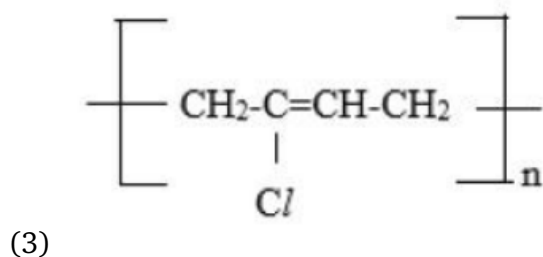
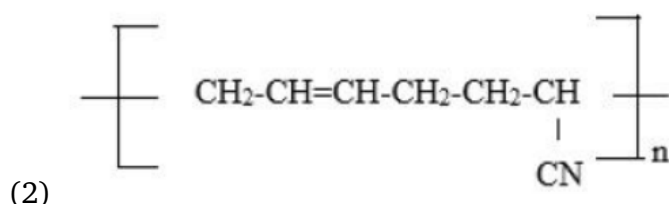
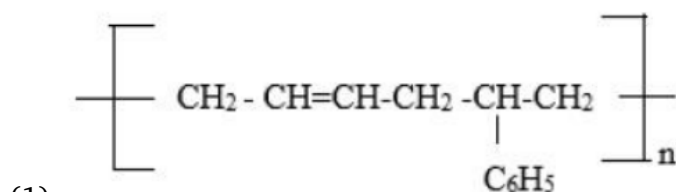
- **Cellulose:** Found in the cell walls of plants. It is a polymer of β -D-glucose units. It is a **natural polymer**.
- **Teflon (PTFE):** A synthetic polymer used in non-stick cookware.
- **Polyvinylchloride (PVC):** A synthetic plastic used in pipes.
- **Neoprene:** A synthetic rubber produced by the polymerization of chloroprene.

Step 4: Final Answer:

Cellulose is the natural polymer.

Quick Tip: Common natural polymers include starch, cellulose, proteins (polymers of amino acids), and natural rubber (cis-polyisoprene).

96. The structure of Buna-S polymer is ____.



Correct Answer: (1)

Solution:

Step 1: Understanding the Concept:

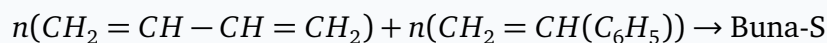
Buna-S (SBR - Styrene Butadiene Rubber) is a copolymer. The name provides a hint: **Bu** stands for 1,3-butadiene, **na** is for the sodium catalyst used, and **S** stands for Styrene.

Step 2: Key Formula or Approach:

Identify the monomers: 1. 1,3-Butadiene: $\text{CH}_2 = \text{CH} - \text{CH} = \text{CH}_2$ 2. Styrene: $\text{CH}_2 = \text{CH}(\text{C}_6\text{H}_5)$

Step 3: Detailed Explanation:

When these two monomers polymerize, the double bonds rearrange to form a long chain:



The repeating unit contains the 4-carbon chain from butadiene (with a shifted double bond) and the 2-carbon chain from styrene (carrying the phenyl group, $-\text{C}_6\text{H}_5$). This matches the structure in Option (1).

Step 4: Final Answer:

The structure is $[\text{CH}_2 - \text{CH} = \text{CH} - \text{CH}_2 - \text{CH}(\text{C}_6\text{H}_5) - \text{CH}_2]_n$.

Quick Tip: Buna-N is similar, but the 'N' stands for Nitrile (acrylonitrile), which contains a $-\text{CN}$ group instead of a phenyl group.

97. The polymer used in making gaskets and non-stick coating utensils is ____.

- (1) Polyvinyl chloride
- (2) Polystyrene
- (3) Polytetrafluoroethylene
- (4) Polythene

Correct Answer: (3) Polytetrafluoroethylene

Solution:**Step 1: Understanding the Concept:**

Non-stick coatings and high-performance gaskets require a material that is chemically inert, has a very low coefficient of friction, and can withstand high temperatures.

Step 2: Key Formula or Approach:

Identify the polymer commonly known as **Teflon**. Its chemical name is Polytetrafluoroethylene (PTFE).

Step 3: Detailed Explanation:

Polytetrafluoroethylene (PTFE) is prepared by the polymerization of tetrafluoroethene ($CF_2 = CF_2$). Due to the strength of the C-F bonds, it is highly resistant to heat and chemicals. This makes it ideal for:

- **Non-stick cookware:** Its "slippery" nature prevents food from sticking.
- **Gaskets:** It maintains a seal without reacting with the chemicals or gases it contains.

Step 4: Final Answer:

The correct polymer is Polytetrafluoroethylene.

Quick Tip: Remember that "Tetra-fluoro" refers to the four fluorine atoms replacing hydrogen atoms in ethene, creating one of the most chemically stable substances known.

98. Which of the following is not to be considered as a primary fuel?

- (1) Wood
- (2) Petroleum
- (3) Coke
- (4) Coal

Correct Answer: (3) Coke

Solution:**Step 1: Understanding the Concept:**

Fuels are classified based on their occurrence. **Primary fuels** (Natural fuels) are used in the same form as they are found in nature. **Secondary fuels** (Derived fuels) are obtained from primary fuels through chemical or physical processes.

Step 2: Key Formula or Approach:

Identify which fuel is a manufactured byproduct rather than a naturally occurring resource.

Step 3: Detailed Explanation:

- **Wood, Coal, and Petroleum:** These are all found naturally in the earth's crust or biosphere and are considered primary fuels.
- **Coke:** This is a grey, hard, and porous fuel with high carbon content. It is produced by the destructive distillation of coal (heating coal in the absence of air). Because it is processed from coal, it is a **secondary fuel**.

Step 4: Final Answer:

Coke is not a primary fuel.

Quick Tip: If you have to "make" the fuel from another fuel (like Charcoal from Wood or Coke from Coal), it is always a secondary/derived fuel.

99. The oxide of nitrogen responsible for depletion of ozone layer is ____.

- (1) NO
- (2) NO
- (3) NO
- (4) NO

Correct Answer: (3) NO

Solution:

Step 1: Understanding the Concept:

The ozone layer (O_3) in the stratosphere protects the Earth from UV radiation. Certain catalysts can speed up the breakdown of ozone into oxygen molecules.

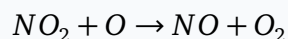
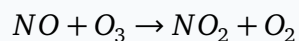
Step 2: Key Formula or Approach:

Identify the nitrogen oxide emitted by high-altitude supersonic jets that reacts directly with ozone.

Step 3: Detailed Explanation:

Nitric oxide (NO) is released in the stratosphere by supersonic jets. It reacts with ozone in a

catalytic cycle:



In this cycle, the NO molecule is regenerated, meaning a single molecule of NO can destroy thousands of ozone molecules.

Step 4: Final Answer:

The oxide of nitrogen responsible is NO (Nitric Oxide).

Quick Tip: While CFCs are the most famous ozone depleters, NO is the primary chemical pollutant from jet engines that affects the stratosphere.

100. The BOD of highly polluted water is ____.

- (1) 17 ppm
- (2) 10 ppm
- (3) 8 ppm
- (4) 12 ppm

Correct Answer: (1) 17 ppm

Solution:

Step 1: Understanding the Concept:

BOD (Biochemical Oxygen Demand) is the amount of dissolved oxygen needed by aerobic biological organisms to break down organic material present in a given water sample.

Step 2: Key Formula or Approach:

The higher the BOD value, the more organic matter is present, indicating higher levels of pollution.

Step 3: Detailed Explanation:

- **Clean Water:** BOD is typically less than 5 ppm.
- **Polluted Water:** BOD values range from 10 to 15 ppm.
- **Highly Polluted Water:** BOD values are **17 ppm or higher**.

At values as high as 17 ppm, the dissolved oxygen is so depleted that fish and other aquatic life cannot survive.

Step 4: Final Answer:

The BOD of highly polluted water is 17 ppm.

Quick Tip: BOD is a measure of "water hunger" for oxygen. High BOD = High Hunger = Dirty Water.

CIVIL ENGINEERING

101. The maximum value of Poisson's ratio is ____.

- (1) 0.25
- (2) 0.50
- (3) 0.75
- (4) 1.0

Correct Answer: (2) 0.50

Solution:

Step 1: Understanding the Concept:

Poisson's ratio (μ or ν) is the ratio of lateral strain to longitudinal strain when a material is stretched or compressed. It describes how much a material "thins out" as it is pulled.

Step 2: Key Formula or Approach:

For a stable, isotropic, elastic material, the relationship between the Bulk Modulus (K) and

Young's Modulus (E) is:

$$K = \frac{E}{3(1 - 2\mu)}$$

Step 3: Detailed Explanation:

For a material to be stable, the Bulk Modulus (K) must be positive. Looking at the denominator, if $1 - 2\mu = 0$, then K becomes infinite, meaning the material is perfectly incompressible (like rubber). If $\mu > 0.5$, the Bulk Modulus would be negative, which is physically impossible for stable materials. Therefore, the theoretical maximum value for an ordinary material is 0.50.

Step 4: Final Answer:

The maximum value of Poisson's ratio is 0.50.

Quick Tip: Most common metals have a Poisson's ratio between 0.25 and 0.35. A value of 0.50 represents a perfectly incompressible material, where the volume remains constant during deformation.

102. A rectangular bar of length L and cross section $b \times d$ is subjected to a tensile stress of σ . If μ is the Poisson's ratio of the material, then the volumetric strain is ____.

- (1) $\frac{\sigma}{2E}(1 - \mu)$
- (2) $\frac{\sigma}{bdE}(1 - 2\mu)$
- (3) $\frac{\sigma}{2E}(1 + 2\mu)$
- (4) $\frac{\sigma}{E}(1 - 2\mu)$

Correct Answer: (4) $\frac{\sigma}{E}(1 - 2\mu)$

Solution:

Step 1: Understanding the Concept:

Volumetric strain (ϵ_v) is the change in volume per unit original volume. For a bar under uniaxial stress, it is the sum of the linear strains in the three mutually perpendicular directions (x, y, z).

Step 2: Key Formula or Approach:

$$\epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z$$

Where ϵ_x is the longitudinal strain and ϵ_y, ϵ_z are lateral strains.

Step 3: Detailed Explanation:

Under a tensile stress σ in the x -direction: 1. Longitudinal strain: $\epsilon_x = \frac{\sigma}{E}$ 2. Lateral strains (due to Poisson's effect): $\epsilon_y = -\mu \frac{\sigma}{E}$ and $\epsilon_z = -\mu \frac{\sigma}{E}$ Now, sum them:

$$\epsilon_v = \frac{\sigma}{E} + \left(-\mu \frac{\sigma}{E}\right) + \left(-\mu \frac{\sigma}{E}\right)$$

$$\epsilon_v = \frac{\sigma}{E} - 2\mu \frac{\sigma}{E} = \frac{\sigma}{E}(1 - 2\mu)$$

Step 4: Final Answer:

The volumetric strain is $\frac{\sigma}{E}(1 - 2\mu)$.

Quick Tip: Notice that if $\mu = 0.5$, the volumetric strain becomes zero ($\epsilon_v = 0$), which confirms that 0.5 is the ratio for incompressible materials.

103. The percentage elongation and reduction in an area of sample tested to failure in tension is a measure of ____.

- (1) Ductility
- (2) Brittleness
- (3) Malleability
- (4) Hardness

Correct Answer: (1) Ductility

Solution:

Step 1: Understanding the Concept:

Ductility is a physical property of a material associated with the ability to be hammered thin or drawn into wire without breaking.

Step 2: Detailed Explanation:

When a material is tested in tension until it fails, we measure how much it stretched (percentage elongation) and how much the cross-section narrowed (reduction in area) at the point of fracture.

- **Ductile materials** (like mild steel) show significant elongation and necking before they break.
- **Brittle materials** (like cast iron) show almost no elongation or area reduction before snapping.

Therefore, these measurements are direct indicators of ductility.

Step 4: Final Answer:

These measurements are a measure of Ductility.

Quick Tip: A material is generally considered "ductile" if its percentage elongation is greater than 5%. If it is less than 5%, it is considered "brittle."

104. A compound bar made of steel and brass of equal length is subjected to a tensile load of 100 kN. The cross-sectional area of steel bar is twice that of the brass bar. The modulus of elasticity of steel and brass are 200 GPa and 100 GPa respectively. The load taken by the brass bar is ____.

- (1) 20 kN
- (2) 25 kN
- (3) 50 kN
- (4) 75 kN

Correct Answer: (1) 20 kN

Solution:

Step 1: Understanding the Concept:

In a compound bar (parallel arrangement), both materials undergo the same amount of deformation (elongation) because they are of equal length and connected together. The total load is shared between the two materials.

Step 2: Key Formula or Approach:

1. Compatibility: $\Delta L_s = \Delta L_b \implies \frac{P_s L}{A_s E_s} = \frac{P_b L}{A_b E_b}$
2. Equilibrium: $P_s + P_b = P_{total} = 100 \text{ kN}$

Step 3: Detailed Explanation:

Given: $A_s = 2A_b$, $E_s = 200 \text{ GPa}$, $E_b = 100 \text{ GPa}$. From the compatibility equation:

$$\frac{P_s}{A_s E_s} = \frac{P_b}{A_b E_b} \implies P_s = P_b \left(\frac{A_s}{A_b} \right) \left(\frac{E_s}{E_b} \right)$$

Substitute the ratios:

$$P_s = P_b \left(\frac{2A_b}{A_b} \right) \left(\frac{200}{100} \right) = P_b (2 \times 2) = 4P_b$$

Now use the total load equation:

$$4P_b + P_b = 100$$

$$5P_b = 100 \implies P_b = 20 \text{ kN}$$

Step 4: Final Answer:

The load taken by the brass bar is 20 kN.

Quick Tip: In parallel bars, the load distributes based on "stiffness" (AE/L). Since the steel bar is 4 times as stiff as the brass bar (2×2), it takes 4 parts of the load while brass takes 1 part.

105. A uniform bar of length L , cross sectional area A , modulus of elasticity E is having self-weight W is hung at one end and attached a load equal to self-weight W at the other end. The elongation of a bar compared to that due to self-weight will be ____.

- (1) half
- (2) twice
- (3) thrice
- (4) four times

Correct Answer: (3) thrice

Solution:

Step 1: Understanding the Concept:

The total elongation of the bar is the sum of the elongation due to its own weight (distributed load) and the elongation due to the concentrated load at the free end.

Step 2: Key Formula or Approach:

1. Elongation due to self-weight (δ_w) = $\frac{WL}{2AE}$
2. Elongation due to external concentrated load P (δ_p) = $\frac{PL}{AE}$

Step 3: Detailed Explanation:

Given the external load P is equal to the self-weight W :

$$\delta_{total} = \delta_w + \delta_p = \frac{WL}{2AE} + \frac{WL}{AE}$$

To add these, find a common denominator:

$$\delta_{total} = \frac{WL}{2AE} + \frac{2WL}{2AE} = \frac{3WL}{2AE}$$

Comparing this to the elongation due to self-weight alone ($\delta_w = \frac{WL}{2AE}$):

$$\delta_{total} = 3 \times \delta_w$$

Step 4: Final Answer:

The total elongation is thrice that due to self-weight.

Quick Tip: A concentrated load at the end causes twice as much deformation as the same amount of weight distributed uniformly along the bar's length.

106. A simply supported beam of span 6 m is subjected to a uniformly distributed load of 24 kN/m over the left half of the span. The magnitude of shear force at mid-span is ____.

- (1) zero
- (2) 18 kN
- (3) 36 kN
- (4) 54 kN

Correct Answer: (2) 18 kN

Solution:

Step 1: Understanding the Concept:

To find the shear force at any point, we first need the support reactions. In a simply supported beam, the reactions are found using static equilibrium equations ($\sum M = 0$ and $\sum F_y = 0$).

Step 2: Key Formula or Approach:

Total Load = $w \times \text{length of UDL} = 24 \times 3 = 72 \text{ kN}$.

Step 3: Detailed Explanation:

Let R_A be the left reaction and R_B be the right reaction. Span $L = 6 \text{ m}$. UDL is on the left 3 m. Taking moments about B :

$$R_A \times 6 - (24 \times 3) \times (3 + 1.5) = 0$$

$$6R_A = 72 \times 4.5 = 324 \implies R_A = 54 \text{ kN}$$

Now, calculate Shear Force (V) at the mid-span (3 m from left):

$$V_{mid} = R_A - (\text{total load on left half})$$

$$V_{mid} = 54 - (24 \times 3) = 54 - 72 = -18 \text{ kN}$$

The magnitude is 18 kN.

Step 4: Final Answer:

The magnitude of the shear force at mid-span is 18 kN.

Quick Tip: For a UDL covering half the span, the reactions are $3/4$ of the load at the UDL side and $1/4$ at the other side. Total load = 72; $R_A = 54$, $R_B = 18$.

107. If a simply supported beam AB of span 6 m is subjected to two concentrated loads each of magnitude 40 kN acting at the middle third points, then ____.

- (1) shear force is maximum at mid-span
- (2) bending moment varies linearly between the loads
- (3) bending moment is constant between the loads
- (4) shear force varies linearly from support to load

Correct Answer: (3) bending moment is constant between the loads

Solution:

Step 1: Understanding the Concept:

When equal loads are placed symmetrically on a simply supported beam at the third points, the central region experiences what is known as "pure bending." In this zone, the shear force is zero, which mathematically implies that the bending moment remains constant.

Step 2: Key Formula or Approach:

The relationship between shear force (V) and bending moment (M) is given by:

$$V = \frac{dM}{dx}$$

Step 3: Detailed Explanation:

For a 6 m beam with loads at 2 m and 4 m: 1. Due to symmetry, the reaction at each support

is $R_A = R_B = 40 \text{ kN}$.

2. Calculate the Shear Force (V) in the middle third ($2 < x < 4$):

$$V = R_A - 40 = 40 - 40 = 0 \text{ kN}$$

3. Since the shear force is zero, the rate of change of the bending moment is zero ($\frac{dM}{dx} = 0$).

4. Consequently, the bending moment M in this section is constant and calculated as:

$$M = R_A \times 2 = 40 \times 2 = 80 \text{ kNm}$$

Step 4: Final Answer:

The bending moment is constant between the loads.

Quick Tip: Any region of a beam where the shear force is zero is said to be in a state of pure bending, meaning the bending moment diagram will be a horizontal straight line.

108. A simply supported beam AB of span 8 m is subjected to a uniformly distributed load of 24 kN/m throughout the length. The bending moment at 2 m from the left end A is equal to ____.

- (1) 48 kNm
- (2) 128 kNm
- (3) 144 kNm
- (4) 192 kNm

Correct Answer: (3) 144 kNm

Solution:

Step 1: Understanding the Concept:

The bending moment at any specific section is the sum of moments of all forces to the left (or right) of that section. For a simply supported beam with a UDL, the forces include the support reaction and the downward load of the UDL itself.

Step 2: Key Formula or Approach:

1. Support Reaction: $R_A = \frac{wL}{2}$
2. Bending Moment at distance x : $M_x = R_A \cdot x - \frac{wx^2}{2}$

Step 3: Detailed Explanation:

Given: $w = 24 \text{ kN/m}$, $L = 8 \text{ m}$, and we need M at $x = 2 \text{ m}$. 1. Find R_A :

$$R_A = \frac{24 \times 8}{2} = 96 \text{ kN}$$

2. Calculate M at $x = 2$:

$$M = (96 \times 2) - \frac{24 \times (2)^2}{2}$$

$$M = 192 - (12 \times 4)$$

$$M = 192 - 48 = 144 \text{ kNm}$$

Step 4: Final Answer:

The bending moment at 2 m is 144 kNm.

Quick Tip: To quickly verify your BMD, remember that for a UDL, the curve is parabolic and the maximum moment always occurs at the center ($x = L/2$) with a value of $\frac{wL^2}{8}$.

109. The slope of bending moment diagram between two points gives ____.

- (1) Slope
- (2) Couple
- (3) Axial force
- (4) Shear force

Correct Answer: (4) Shear force

Solution:

Step 1: Understanding the Concept:

In structural mechanics, there is a derivative relationship between loading, shear force, and bending moment. The "slope" of a graph at any point is represented by its first derivative.

Step 2: Detailed Explanation:

The relationship between Bending Moment (M) and Shear Force (V) is expressed as:

$$\frac{dM}{dx} = V$$

This equation states that the rate of change of the bending moment with respect to the distance x (which is the slope of the Bending Moment Diagram) is equal to the magnitude of the shear force at that section.

Step 3: Final Answer:

The slope of the bending moment diagram gives the Shear force.

Quick Tip: Remember the hierarchy: - Integral of load = Shear Force. - Integral of Shear Force = Bending Moment. - Conversely, the slope of BM is Shear, and the slope of Shear is the Load intensity.

110. The shape of shear force diagram of a cantilever beam subjected to a concentrated load at the free end is ____.

- (1) Rectangle
- (2) Triangle
- (3) Parabola
- (4) Ellipse

Correct Answer: (1) Rectangle

Solution:

Step 1: Understanding the Concept:

The shear force at any section of a beam is the algebraic sum of the vertical forces on one side

of that section. In a cantilever with a single point load at the end, that load is the only force acting until you reach the support.

Step 2: Detailed Explanation:

1. Consider a cantilever of length L with a point load P at the free end. 2. If we take any section x from the free end, the only vertical force to the right of the section is P . 3. Thus, the shear force $V = P$ at every point along the beam. 4. A diagram that maintains a constant value (a horizontal line) across the length of the beam forms a rectangle.

Step 3: Final Answer:

The shape of the shear force diagram is a Rectangle.

Quick Tip: If the load is a point load, the SFD consists of horizontal lines (rectangles). If the load is a UDL, the SFD consists of inclined lines (triangles).

111. For a beam subjected to constant bending moment M and flexural rigidity EI , then M/EI represents ____.

- (1) Flexural stress
- (2) Rigidity of section
- (3) Radius of curvature
- (4) Curvature

Correct Answer: (4) Curvature

Solution:

Step 1: Understanding the Concept:

The behavior of a beam in bending is described by the Euler-Bernoulli bending equation, which relates the internal moment, material properties, and the resulting geometry of the deformation.

Step 2: Key Formula or Approach:

The bending equation is given by:

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

Where M is the bending moment, I is the moment of inertia, E is Young's modulus, and R is the radius of curvature.

Step 3: Detailed Explanation:

From the bending equation, we can isolate the relationship between M , E , I , and R :

$$\frac{M}{EI} = \frac{1}{R}$$

In mathematics and physics, the reciprocal of the radius of curvature ($1/R$) is defined as the Curvature (κ). Therefore, the term M/EI directly represents the curvature of the beam's neutral axis.

Step 4: Final Answer:

M/EI represents the Curvature.

Quick Tip: Flexural rigidity (EI) represents the resistance of a beam to bending. The higher the EI , the smaller the curvature for a given applied moment.

112. For the same material, the ratio of flexural rigidity of beam of size $b \times d$ to that of another beam of size $b \times 2d$ is ____.

- (1) $1/2$
- (2) $1/4$
- (3) $1/8$
- (4) $1/16$

Correct Answer: (3) $1/8$

Solution:

Step 1: Understanding the Concept:

Flexural rigidity is the product of the Modulus of Elasticity (E) and the Second Moment of Area (I). For a rectangular section, I depends on the width and the cube of the depth.

Step 2: Key Formula or Approach:

1. Flexural Rigidity = EI
2. Moment of Inertia for a rectangle (I) = $\frac{bd^3}{12}$

Step 3: Detailed Explanation:

Let the first beam have dimensions $b \times d$ and the second beam have $b \times 2d$. Since the material is the same, E is constant. For Beam 1: $I_1 = \frac{bd^3}{12}$ For Beam 2: $I_2 = \frac{b(2d)^3}{12} = \frac{8bd^3}{12} = 8I_1$ The ratio of flexural rigidities is:

$$\frac{EI_1}{EI_2} = \frac{I_1}{8I_1} = \frac{1}{8}$$

Step 4: Final Answer:

The ratio of flexural rigidity is $1/8$.

Quick Tip: Strength and stiffness of a beam increase much more rapidly with an increase in depth than with an increase in width, as depth is cubed in the inertia formula.

113. For a given beam length, cross sectional area and material, maximum flexural resistance occurs when ____.

- (1) depth of section is maximum
- (2) section modulus is maximum
- (3) moment of inertia is maximum
- (4) radius of gyration is minimum

Correct Answer: (2) section modulus is maximum

Solution:

Step 1: Understanding the Concept:

Flexural resistance refers to the maximum bending moment a beam can withstand before reaching its allowable stress limit. This is determined by the geometry of the cross-section.

Step 2: Key Formula or Approach:

The maximum bending stress is given by:

$$\sigma_{max} = \frac{M}{Z} \implies M = \sigma_{allow} \times Z$$

Where Z is the Section Modulus (I/y_{max}).

Step 3: Detailed Explanation:

For a given material, the allowable stress σ is fixed. Therefore, the resisting moment M is directly proportional to the Section Modulus Z . - While increasing depth or moment of inertia generally increases resistance, the specific geometric property that defines the "strength" or "resistance" of a section in bending is the Section Modulus. - For a fixed area, arranging the material as far as possible from the neutral axis (maximizing Z) provides the highest resistance.

Step 4: Final Answer:

Maximum flexural resistance occurs when the section modulus is maximum.

Quick Tip: This is why I-sections are so efficient in civil engineering; they place most of the material in the flanges (far from the neutral axis) to maximize the section modulus for a given area.

114. If a cantilever beam is subjected to a downward concentrated load at the free end, then the nature of stress across the cross section developed will be ____.

- (1) tensile at top and compressive at bottom
- (2) compressive at top and tensile at bottom
- (3) tensile at top and compressive at neutral axis
- (4) compressive at top and tensile at neutral axis

Correct Answer: (1) tensile at top and compressive at bottom

Solution:

Step 1: Understanding the Concept:

When a cantilever beam is loaded at its free end, it bends downwards, creating a "hogging" curvature. This deformation affects the fibers across the cross-section differently depending on their position relative to the neutral axis.

Step 2: Detailed Explanation:

For a cantilever beam bending downwards: 1. The upper fibers are stretched, which results in tensile stress. 2. The lower fibers are shortened or "crushed," which results in compressive stress. 3. The neutral axis remains at its original length and experiences zero bending stress. This is the opposite of a simply supported beam, which experiences "sagging" (compression at top, tension at bottom).

Step 3: Final Answer:

The stress is tensile at top and compressive at bottom.

Quick Tip: To remember this, visualize the curvature. In "hogging" (cantilever), the top face is being pulled apart (tension). In "sagging" (simply supported), the top face is being pushed together (compression).

115. If the section modulus for a rectangular beam having depth of 300 mm is $3 \times 10^6 \text{ mm}^3$, then the width of beam is equal to ____.

- (1) 90 mm
- (2) 100 mm
- (3) 150 mm
- (4) 200 mm

Correct Answer: (4) 200 mm

Solution:

Step 1: Understanding the Concept:

Section modulus (Z) is a geometric property of a cross-section used for designing beams. For

a rectangular section, it is derived from the moment of inertia (I) and the distance to the extreme fiber (y).

Step 2: Key Formula or Approach:

For a rectangular section:

$$Z = \frac{I}{y_{max}} = \frac{bd^3/12}{d/2} = \frac{bd^2}{6}$$

Step 3: Detailed Explanation:

Given: $Z = 3 \times 10^6 \text{ mm}^3$ and $d = 300 \text{ mm}$. Substitute these values into the formula:

$$3 \times 10^6 = \frac{b \times (300)^2}{6}$$

$$3 \times 10^6 = \frac{b \times 90,000}{6}$$

$$3 \times 10^6 = b \times 15,000$$

$$b = \frac{3,000,000}{15,000} = 200 \text{ mm}$$

Step 4: Final Answer:

The width of the beam is 200 mm.

Quick Tip: Always double-check the exponents when working with mm^3 . A common mistake is miscounting the zeros during the calculation of d^2 .

116. For a symmetrical I-section, the intensity of shear stress is maximum at ____.

- (1) the extreme fibers
- (2) centroid of the section
- (3) in web at the junction of flange and web
- (4) in flange at the junction of flange and web

Correct Answer: (2) centroid of the section

Solution:

Step 1: Understanding the Concept:

Shear stress in a beam section varies parabolically across the depth. Unlike bending stress, which is maximum at the extreme fibers, shear stress follows a different distribution based on the area above the point of interest.

Step 2: Key Formula or Approach:

The shear stress formula is:

$$\tau = \frac{V \cdot A \cdot \bar{y}}{I \cdot b}$$

Step 3: Detailed Explanation:

In any symmetrical section (like a rectangle, circle, or I-section): 1. The shear stress is zero at the top and bottom extreme fibers because the area A above/below them is zero. 2. The term $A\bar{y}$ (the first moment of area) reaches its maximum value at the neutral axis. 3. For an I-section, the width b also decreases significantly at the web, which further increases the shear stress ($\tau \propto 1/b$). Since the neutral axis of a symmetrical I-section coincides with the centroid, the intensity of shear stress is maximum at the centroid.

Step 4: Final Answer:

The intensity of shear stress is maximum at the centroid of the section.

Quick Tip: In an I-section, the web carries approximately 95% of the total shear force. This is why the shear stress diagram shows a sudden "jump" in intensity as you move from the flange into the narrower web.

117. The variation of shear stress over a solid circular section of beam follows ____.

- (1) an inclined line
- (2) a circular curve
- (3) an elliptical curve

(4) a parabolic curve

Correct Answer: (4) a parabolic curve

Solution:

Step 1: Understanding the Concept:

The shear stress distribution across any standard cross-section of a beam (subjected to bending) is determined by the formula $\tau = \frac{V \cdot A \cdot \bar{y}}{I \cdot b}$. This distribution describes how the intensity of shear resistance changes from the extreme fibers to the neutral axis.

Step 2: Detailed Explanation:

1. For a solid circular section, as we move from the outer edge towards the center, the term $A\bar{y}$ (the first moment of area) varies. 2. The mathematical relationship between the distance from the neutral axis and the shear stress involves a squared term (y^2). 3. This results in a distribution where the stress is zero at the top and bottom extreme fibers and increases to a maximum at the neutral axis. 4. The geometric shape of this variation is a parabola.

Step 3: Final Answer:

The variation follows a parabolic curve.

Quick Tip: While the stress variation is parabolic for most common sections (rectangle, I-section, circle), the ratio of maximum to average stress changes depending on the geometry.

118. If the maximum shear stress permitted in a rectangular beam of width 100 mm and depth 200 mm is 15 N/mm², then the corresponding maximum shear force is equal to ____.

- (1) 100 kN
- (2) 150 kN
- (3) 200 kN
- (4) 300 kN

Correct Answer: (3) 200 kN

Solution:

Step 1: Understanding the Concept:

For a rectangular cross-section, the maximum shear stress (τ_{max}) occurs at the neutral axis and is 1.5 times the average shear stress (τ_{avg}).

Step 2: Key Formula or Approach:

1. $\tau_{avg} = \frac{\text{Shear Force (V)}}{\text{Area (A)}}$
2. $\tau_{max} = 1.5 \times \tau_{avg}$

Step 3: Detailed Explanation:

Given: $\tau_{max} = 15 \text{ N/mm}^2$, $b = 100 \text{ mm}$, $d = 200 \text{ mm}$. 1. Calculate the area: $A = 100 \times 200 = 20,000 \text{ mm}^2$. 2. Relate τ_{max} to V :

$$15 = 1.5 \times \frac{V}{20,000}$$

3. Solve for V :

$$\frac{15}{1.5} = \frac{V}{20,000}$$

$$10 = \frac{V}{20,000} \Rightarrow V = 10 \times 20,000 = 200,000 \text{ N}$$

4. Convert to kN: $V = 200 \text{ kN}$.

Step 4: Final Answer:

The corresponding maximum shear force is 200 kN.

Quick Tip: Always be careful with units. 1 N/mm^2 is equivalent to 1 MPa. When you multiply stress in N/mm^2 by area in mm^2 , you get force in Newtons.

119. If the maximum and average shear stress in a solid circular section are represented by τ_{max} and τ_{avg} respectively, then ____.

(1) $\tau_{avg} = 1.33\tau_{max}$

(2) $\tau_{avg} = 0.75\tau_{max}$

(3) $\tau_{max} = 0.67\tau_{avg}$

(4) $\tau_{max} = 1.5\tau_{avg}$

Correct Answer: (2) $\tau_{avg} = 0.75\tau_{max}$

Solution:

Step 1: Understanding the Concept:

In a circular section, the shear stress distribution is not uniform. The peak intensity (τ_{max}) is higher than the mean intensity calculated across the whole area.

Step 2: Key Formula or Approach:

For a solid circular section:

$$\tau_{max} = \frac{4}{3}\tau_{avg}$$

Step 3: Detailed Explanation:

We have the relationship $\tau_{max} = \frac{4}{3}\tau_{avg}$. To find τ_{avg} in terms of τ_{max} :

$$\tau_{avg} = \frac{3}{4}\tau_{max}$$

$$\tau_{avg} = 0.75\tau_{max}$$

This matches option (2). Note that option (4) is incorrect as it applies to rectangular sections ($\tau_{max} = 1.5\tau_{avg}$), not circular ones.

Step 4: Final Answer:

The correct relationship is $\tau_{avg} = 0.75\tau_{max}$.

Quick Tip: Common ratios to remember: - Rectangle: $\tau_{max} = 1.5\tau_{avg}$ - Circle: $\tau_{max} = 1.33\tau_{avg}$ - Triangle: $\tau_{max} = 1.5\tau_{avg}$ (at $h/2$)

120. The strain energy per unit volume of a bar having modulus of elasticity E subjected to an axial tensile stress of σ is ____.

- (1) $\frac{\sigma^2}{E}$
- (2) $\frac{\sigma^2}{2E}$
- (3) $\frac{\sigma^2}{4E}$
- (4) $\frac{\sigma^2}{8E}$

Correct Answer: (2) $\frac{\sigma^2}{2E}$

Solution:

Step 1: Understanding the Concept:

Strain energy is the internal energy stored in a body due to deformation. Strain energy per unit volume is also known as "Resilience" or "Strain Energy Density."

Step 2: Key Formula or Approach:

1. Total Strain Energy (U) = $\frac{1}{2} \times \text{Load} \times \text{Extension} = \frac{1}{2}P\delta$
2. $\delta = \frac{PL}{AE}$ and $\sigma = P/A$

Step 3: Detailed Explanation:

Strain energy density (u) = $\frac{U}{\text{Volume}} = \frac{1/2P\delta}{AL}$ Substitute δ :

$$u = \frac{1}{2} \cdot \frac{P}{A} \cdot \frac{\delta}{L} = \frac{1}{2} \cdot \sigma \cdot \epsilon$$

Since $\epsilon = \sigma/E$ (from Hooke's Law):

$$u = \frac{1}{2} \cdot \sigma \cdot \frac{\sigma}{E} = \frac{\sigma^2}{2E}$$

Step 4: Final Answer:

The strain energy per unit volume is $\frac{\sigma^2}{2E}$.

Quick Tip: The area under the Stress-Strain curve up to the elastic limit represents the Modulus of Resilience, which is the maximum strain energy the material can absorb without permanent deformation.

121. A simply supported beam of span L is subjected to a concentrated load of W acting at

midspan. If the load is changed to a uniformly distributed load over the whole span, then the ratio of maximum deflection at mid-span will be ____.

- (1) 8/5
- (2) 5/8
- (3) 1/4
- (4) 1/2

Correct Answer: (1) 8/5

Solution:

Step 1: Understanding the Concept:

The maximum deflection of a simply supported beam occurs at the mid-span for symmetrical loading. The magnitude of this deflection depends on the total load, the span length, and the flexural rigidity (EI) of the beam.

Step 2: Key Formula or Approach:

1. Deflection due to point load W at mid-span (δ_1) = $\frac{WL^3}{48EI}$
2. Deflection due to total UDL W (where $W = wL$) over the whole span (δ_2) = $\frac{5WL^3}{384EI}$

Step 3: Detailed Explanation:

We need the ratio of the deflection due to the point load to the deflection due to the UDL:

$$\text{Ratio} = \frac{\delta_1}{\delta_2} = \frac{\frac{WL^3}{48EI}}{\frac{5WL^3}{384EI}}$$

Simplify the expression by canceling WL^3 and EI :

$$\text{Ratio} = \frac{1}{48} \times \frac{384}{5}$$

$$\text{Ratio} = \frac{384}{240}$$

Dividing both numbers by 48:

$$\text{Ratio} = \frac{8}{5}$$

Step 4: Final Answer:

The ratio of maximum deflection is $8/5$.

Quick Tip: If the question asked for the ratio of UDL deflection to point load deflection, the answer would be $5/8$ (0.625). Always read the order of the ratio carefully.

122. Which of the following methods are used for computing the deflections in beams?

- (1) Moment area method and Mohr's circle method
- (2) Moment area method and double integration method
- (3) Macaulay's method and Mohr's circle method
- (4) Macaulay's method and Hardy cross method

Correct Answer: (2) Moment area method and double integration method

Solution:**Step 1: Understanding the Concept:**

Beam deflection analysis involves finding the displacement of the neutral axis under loads. Several mathematical and semi-graphical methods exist to solve the differential equation of the elastic curve.

Step 2: Detailed Explanation:

The standard methods for beam deflection include:

- **Double Integration Method:** Directly integrating the M/EI equation twice.
- **Macaulay's Method:** An extension of double integration for multiple loads.
- **Moment Area Method:** Uses the geometry of the bending moment diagram.
- **Conjugate Beam Method:** Converts a slope/deflection problem into a shear/moment problem.

Note on incorrect options: Mohr's circle is used for stress/strain transformations at a point, not beam deflection. The Hardy Cross method is used for analyzing indeterminate frames and

pipe networks.

Step 3: Final Answer:

The correct methods are the Moment area method and double integration method.

Quick Tip: The Moment Area method is particularly useful for beams with varying cross-sections or simple loading, whereas Macaulay's method is best for beams with multiple point loads or discontinuous UDLs.

123. The slope at the free end of a cantilever beam of length 3 m subjected to a concentrated load of 25 kN is 0.01 radians. The deflection at the free end is ____.

- (1) 20 mm
- (2) 16.6 mm
- (3) 12 mm
- (4) 8 mm

Correct Answer: (1) 20 mm

Solution:

Step 1: Understanding the Concept:

For a cantilever beam with a point load at the free end, both the slope (θ) and the deflection (δ) at the free end are mathematically related through the beam's properties (L and EI).

Step 2: Key Formula or Approach:

- 1. Slope at free end (θ) = $\frac{WL^2}{2EI}$
- 2. Deflection at free end (δ) = $\frac{WL^3}{3EI}$

Step 3: Detailed Explanation:

We can express deflection in terms of slope to eliminate the unknown EI :

$$\delta = \left(\frac{WL^2}{2EI} \right) \times \frac{2L}{3} = \theta \times \frac{2L}{3}$$

Given: $\theta = 0.01$ radians and $L = 3$ m. Substitute these values:

$$\delta = 0.01 \times \frac{2 \times 3}{3}$$

$$\delta = 0.01 \times 2 = 0.02 \text{ m}$$

Converting to millimeters:

$$0.02 \text{ m} \times 1000 = 20 \text{ mm}$$

Step 4: Final Answer:

The deflection at the free end is 20 mm.

Quick Tip: For a cantilever with a point load at the free end, the relationship $\delta = \frac{2}{3}\theta L$ is a very useful shortcut to save time during exams.

124. A cantilever beam of span $2L$ is subjected to a uniformly distributed load of w/m over the entire span. If the flexural rigidity of the beam is EI , then the deflection at the free end is equal to ____.

- (1) $\frac{wL^4}{8EI}$
- (2) $\frac{wL^4}{6EI}$
- (3) $\frac{wL^4}{3EI}$
- (4) $\frac{2wL^4}{EI}$

Correct Answer: (4) $\frac{2wL^4}{EI}$

Solution:

Step 1: Understanding the Concept:

The maximum deflection of a cantilever beam under a uniformly distributed load (UDL) occurs at the free end. The standard formula for a cantilever of length ' x ' is $\frac{wx^4}{8EI}$.

Step 2: Key Formula or Approach:

General formula for deflection (δ) at the free end:

$$\delta = \frac{w \cdot (\text{length})^4}{8EI}$$

Step 3: Detailed Explanation:

Given the span of the beam is $2L$. Substitute $2L$ in place of the length in the standard formula:

$$\delta = \frac{w \cdot (2L)^4}{8EI}$$

$$\delta = \frac{w \cdot 16L^4}{8EI}$$

$$\delta = \frac{2wL^4}{EI}$$

Step 4: Final Answer:

The deflection at the free end is $\frac{2wL^4}{EI}$.

Quick Tip: Always pay close attention to the length term in the prompt. Since length is raised to the power of 4 in deflection formulas, doubling the length ($2L$) increases the deflection by 16 times (2^4) compared to a beam of length L .

125. A simply supported beam AB of span L is subjected to a concentrated load of W at a distance of $L/3$ from the left end A. Which of the following statement is correct?

- (1) Maximum slope occurs at the left support A
- (2) Maximum slope occurs at the right support B
- (3) Maximum slope occurs under the concentrated load
- (4) Maximum deflection occurs under the concentrated load

Correct Answer: (1) Maximum slope occurs at the left support A

Solution:

Step 1: Understanding the Concept:

In a simply supported beam with an eccentric point load (a load not at the center), the deflection curve is unsymmetrical. The slopes at the supports are not equal, and the point of maximum deflection is not under the load.

Step 2: Detailed Explanation:

For a load W at a distance ' a ' from the left and ' b ' from the right ($a < b$):

1. The support closer to the load (Support A, as $L/3 < 2L/3$) will experience a higher reaction and a steeper rotation (slope).
2. Mathematically, the slope at A (θ_A) is $\frac{Wb(L^2-b^2)}{6LEI}$ and at B (θ_B) is $\frac{Wa(L^2-a^2)}{6LEI}$.
3. Since $b = 2L/3$ is greater than $a = L/3$, the slope at support A is greater than the slope at support B.
4. The maximum deflection always occurs at the point where the slope is zero, which for an eccentric load is between the center and the load, but never directly under the load.

Step 3: Final Answer:

Maximum slope occurs at the left support A.

Quick Tip: A simple rule of thumb: for any simply supported beam, the maximum slope always occurs at the support furthest from the center of the load's "weight." In this case, since the load is shifted left, the left support is "pushed" harder into rotation.

126. A cantilever beam AB is fixed at A and supported by a prop at B. If it is subjected to a uniformly distributed load of w/m over the whole span, then ____.

- (1) The maximum bending moment occurs at mid-span
- (2) The maximum shear force occurs at the prop support B
- (3) The maximum sagging bending moment occurs at the support A
- (4) The maximum hogging bending moment occurs at the support A

Correct Answer: (4) The maximum hogging bending moment occurs at the support A

Solution:

Step 1: Understanding the Concept:

A propped cantilever is a statically indeterminate beam. The fixed support (A) resists rotation (creating hogging moment), while the span sags under the UDL.

Step 2: Key Formula or Approach:

For a propped cantilever with length L and UDL w : 1. Reaction at prop (R_B) = $\frac{3wL}{8}$ 2. Moment at fixed support (M_A) = $-\frac{wL^2}{8}$ (Hogging) 3. Maximum sagging moment = $\frac{9wL^2}{128}$ at $\frac{3L}{8}$ from the prop.

Step 3: Detailed Explanation:

1. Comparing the magnitudes: $|M_A| = \frac{1}{8}wL^2 = 0.125wL^2$ and $M_{sag} = \frac{9}{128}wL^2 \approx 0.07wL^2$.
2. The fixed support A must resist the entire tendency of the beam to rotate, leading to a high negative (hogging) moment.
3. The shear force is maximum at the fixed support A ($R_A = \frac{5wL}{8}$), not at the prop.
4. Therefore, the absolute maximum moment in the beam is the hogging moment at the fixed end.

Step 4: Final Answer:

The maximum hogging bending moment occurs at the support A.

Quick Tip: In almost all propped cantilevers or fixed beams with gravity loads, the negative moment at the supports is significantly higher than the positive moment in the span. This is why reinforcement is heavily placed at the top of the beam near supports in RC structures.

127. In a long column, the permissible stress can be increased by increasing the ____.

- (1) Slenderness ratio
- (2) Radius of gyration
- (3) Length of column
- (4) Eccentricity

Correct Answer: (2) Radius of gyration

Solution:

Step 1: Understanding the Concept:

Long columns fail due to buckling rather than direct crushing. The load-carrying capacity and the permissible stress are inversely proportional to the slenderness ratio. To increase the permissible stress, the column must be made "stiffer" against buckling.

Step 2: Key Formula or Approach:

The slenderness ratio (λ) is given by:

$$\lambda = \frac{L_{eff}}{k}$$

Where L_{eff} is effective length and k is the radius of gyration.

Step 3: Detailed Explanation:

1. Permissible stress in a column decreases as the slenderness ratio (λ) increases.
2. To increase the permissible stress, we must decrease λ .
3. Looking at the formula $\lambda = L_{eff}/k$, we can decrease λ by either decreasing the length (L) or increasing the radius of gyration (k).
4. Increasing the radius of gyration means distributing the material further from the centroidal axis, which increases the moment of inertia and makes the column more resistant to buckling.

Step 4: Final Answer:

The permissible stress can be increased by increasing the radius of gyration.

Quick Tip: This is why hollow circular or box sections are more efficient for columns than solid sections of the same area; they have a larger radius of gyration, which reduces the slenderness ratio.

128. The slenderness ratio of a column is zero when its length is ____.

- (1) equal to its radius of gyration
- (2) very large
- (3) equal to actual length
- (4) supported on all sides throughout its length

Correct Answer: (4) supported on all sides throughout its length

Solution:

Step 1: Understanding the Concept:

The slenderness ratio measures the tendency of a column to buckle. It is a function of the "unsupported" length of the column.

Step 3: Detailed Explanation:

1. The slenderness ratio $\lambda = L_{eff}/k$.
2. For λ to be zero, the effective length (L_{eff}) must be zero.
3. Effective length represents the distance between points of zero moment or the length that is free to buckle.
4. If a column is supported on all sides throughout its entire length (for example, encased completely in a rigid medium), it cannot buckle at all. In this theoretical state, its unsupported length is zero, making the slenderness ratio zero.

Step 4: Final Answer:

The slenderness ratio is zero when the column is supported on all sides throughout its length.

Quick Tip: When the slenderness ratio is very low (approaching zero), the column is considered a "short column" and fails purely by crushing of the material.

129. If the equivalent length of a column with one end fixed and the other free is 6 m, then the actual length of column is ____.

- (1) 3 m
- (2) 4.24 m
- (3) 6 m
- (4) 12 m

Correct Answer: (1) 3 m

Solution:**Step 1: Understanding the Concept:**

Equivalent length (or effective length) depends on the end conditions of the column. It represents the length of a hypothetical pinned-pinned column that would have the same critical buckling load.

Step 2: Key Formula or Approach:

For a column with one end fixed and the other free:

$$L_{eff} = 2L$$

Where L is the actual length.

Step 3: Detailed Explanation:

Given $L_{eff} = 6$ m. Using the formula for the fixed-free condition:

$$6 = 2 \times L$$

$$L = \frac{6}{2} = 3 \text{ m}$$

Step 4: Final Answer:

The actual length of the column is 3 m.

Quick Tip: Fixed-Free end conditions are the weakest for a column, effectively doubling its length for buckling calculations. In contrast, Fixed-Fixed conditions are the strongest ($L_{eff} = 0.5L$).

130. A solid circular column of 200 mm diameter and 5 m long is hinged at both ends. Slenderness ratio of column is equal to ____.

- (1) 40
- (2) 50
- (3) 100

(4) 250

Correct Answer: (3) 100

Solution:

Step 1: Understanding the Concept:

To find the slenderness ratio, we need to determine the effective length based on end conditions and the radius of gyration based on the cross-section geometry.

Step 2: Key Formula or Approach:

1. For hinged-hinged ends: $L_{eff} = L$
2. Radius of gyration for a solid circle: $k = \frac{d}{4}$
3. Slenderness ratio: $\lambda = \frac{L_{eff}}{k}$

Step 3: Detailed Explanation:

Given: $d = 200$ mm, $L = 5$ m = 5000 mm. 1. Effective length $L_{eff} = 5000$ mm. 2. Radius of gyration $k = \frac{200}{4} = 50$ mm. 3. Slenderness ratio λ :

$$\lambda = \frac{5000}{50} = 100$$

Step 4: Final Answer:

The slenderness ratio of the column is 100.

Quick Tip: For a solid circular section, you can use the shortcut $\lambda = \frac{4L}{d}$ for hinged-hinged ends. This avoids calculating k separately.

131. The buckling load for a column of length L and flexural rigidity EI with both ends hinged is ____.

- (1) $\frac{\pi^2 EI}{L^2}$
- (2) $\frac{\pi^2 L^2}{EI}$
- (3) $\frac{2\pi^2 EI}{L^2}$

(4) $\frac{\pi^2 EI}{2L^3}$

Correct Answer: (1) $\frac{\pi^2 EI}{L^2}$

Solution:

Step 1: Understanding the Concept:

The Euler's buckling load (P_{cr}) is the maximum axial load that a long, slender column can carry without buckling. This formula assumes the column is perfectly straight, homogeneous, and within the elastic limit.

Step 2: Key Formula or Approach:

The general Euler's formula is:

$$P_{cr} = \frac{\pi^2 EI}{L_{eff}^2}$$

Step 3: Detailed Explanation:

For a column where both ends are hinged (pinned): 1. The effective length (L_{eff}) is equal to the actual length (L). 2. Substitute $L_{eff} = L$ into the general formula:

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

Step 4: Final Answer:

The buckling load for both ends hinged is $\frac{\pi^2 EI}{L^2}$.

Quick Tip: This is the "standard" case for Euler's formula. All other end conditions are derived by adjusting the length (L) based on how the ends are restrained.

132. In a simple triangular truss subjected to a vertical load at its apex, how is the load primarily transferred to the supports?

(1) Entirely as vertical forces in the side members

- (2) An axial forces along the inclined members, creating compression and tension
- (3) As bending moments in the horizontal base member
- (4) Equally divided into shear forces at each joint

Correct Answer: (2) An axial forces along the inclined members, creating compression and tension

Solution:

Step 1: Understanding the Concept:

A truss is a structure composed of members connected at joints, typically arranged in triangles. The defining characteristic of an ideal truss is that its members only carry axial forces (tension or compression), not bending moments or shear.

Step 2: Detailed Explanation:

1. When a vertical load is applied at the apex (top joint) of a triangle, it is resolved into components along the inclined side members. 2. These inclined members act as struts, usually carrying compression to transfer the load toward the base. 3. To prevent the supports from spreading apart, the horizontal base member acts as a tie, carrying tension. 4. Because the joints are considered "pinned," no bending moments are developed within the members; the load travels strictly along the axis of each member.

Step 3: Final Answer:

The load is transferred as axial forces along the inclined members, creating compression and tension.

Quick Tip: Trusses are highly efficient because they utilize the full cross-section of the material for axial strength, whereas beams only use the outer fibers effectively for bending.

133. If a statically determinate pin jointed plane truss consists of 3 unknown reaction components with 5 joints, then the number of members required is ____.

- (1) 5
- (2) 6
- (3) 7
- (4) 8

Correct Answer: (3) 7

Solution:

Step 1: Understanding the Concept:

For a plane truss to be statically determinate and stable, there must be a specific relationship between the number of members (m), the number of joints (j), and the number of reaction components (r).

Step 2: Key Formula or Approach:

The condition for static determinacy in a plane truss is:

$$m = 2j - r$$

Step 3: Detailed Explanation:

Given: - Number of joints (j) = 5 - Reaction components (r) = 3 Plug these values into the formula:

$$m = 2(5) - 3$$

$$m = 10 - 3 = 7$$

Step 4: Final Answer:

The number of members required is 7.

Quick Tip: If $m < 2j - r$, the truss is a mechanism (unstable). If $m > 2j - r$, the truss is statically indeterminate (redundant).

134. For no tension to be developed at the base of the retaining wall of width 6 m, the resultant force will intersect the base at a distance of ____.

- (1) 1 m from the center line of base
- (2) 1 m from the heel

- (3) 1 m from the toe
- (4) 0.75 m from the center line of base

Correct Answer: (1) 1 m from the center line of base

Solution:

Step 1: Understanding the Concept:

To ensure that no tension develops at the base of a masonry structure or retaining wall, the resultant force must fall within the "middle third" of the base. This is known as the Middle Third Rule.

Step 2: Key Formula or Approach:

The eccentricity (e) of the resultant force from the center must satisfy:

$$e \leq \frac{b}{6}$$

Where b is the width of the base.

Step 3: Detailed Explanation:

Given the width $b = 6$ m. 1. The maximum allowable eccentricity from the center line is:

$$e_{max} = \frac{6}{6} = 1 \text{ m}$$

2. This means the resultant force can be located anywhere within 1 m to the left or 1 m to the right of the center line. 3. If the resultant intersects at exactly 1 m from the center line, the stress at one edge will be zero (the limit of no tension).

Step 4: Final Answer:

The resultant force will intersect at a distance of 1 m from the center line of the base.

Quick Tip: The middle third of a 6 m base spans from 2 m to 4 m from the toe. Since the center is at 3 m, the "1 m from center" rule correctly identifies the boundary of this zone (3 ± 1 m).

135. A cantilever retaining wall of height 4 m retains soil of 18 kN/m^3 density up to top. If the coefficient of active earth pressure is $1/3$ then the maximum bending moment induced in the

stem is ____.

- (1) 72 kNm
- (2) 48 kNm
- (3) 64 kNm
- (4) 128 kNm

Correct Answer: (3) 64 kNm

Solution:

Step 1: Understanding the Concept:

The stem of a cantilever retaining wall acts as a cantilever beam subjected to triangular earth pressure. The maximum bending moment occurs at the base of the stem.

Step 2: Key Formula or Approach:

- 1. Active Earth Pressure at base (P) = $K_a \gamma H$
- 2. Total Pressure Force (F) = $\frac{1}{2} K_a \gamma H^2$
- 3. Bending Moment (M) = $F \times \frac{H}{3} = \frac{K_a \gamma H^3}{6}$

Step 3: Detailed Explanation:

Given: $H = 4$ m, $\gamma = 18$ kN/m³, $K_a = 1/3$. Substitute the values into the moment formula:

$$M = \frac{(1/3) \times 18 \times (4)^3}{6}$$

$$M = \frac{6 \times 64}{6}$$

$$M = 64 \text{ kNm}$$

Step 4: Final Answer:

The maximum bending moment induced in the stem is 64 kNm.

Quick Tip: The pressure distribution is triangular, so the resultant force acts at $H/3$ from the base. Always remember to cube the height (H^3) in the direct moment formula.

136. If the effective length and diameter of the column are 3 m and 300 mm respectively, then the type of column is ____.

- (1) Short column
- (2) Medium column
- (3) Long column
- (4) Slender column

Correct Answer: (1) Short column

Solution:

Step 1: Understanding the Concept:

Columns are classified as short or long (slender) based on their slenderness ratio. For reinforced concrete columns, this is usually the ratio of effective length to the least lateral dimension.

Step 2: Key Formula or Approach:

Slenderness Ratio (λ) = $\frac{L_{eff}}{D}$ - If $\lambda < 12$, it is a Short Column. - If $\lambda \geq 12$, it is a Long/Slender Column.

Step 3: Detailed Explanation:

Given: $L_{eff} = 3 \text{ m} = 3000 \text{ mm}$, $D = 300 \text{ mm}$. Calculate the ratio:

$$\lambda = \frac{3000}{300} = 10$$

Since $10 < 12$, the column is classified as a short column.

Step 4: Final Answer:

The type of column is a Short column.

Quick Tip: Short columns fail by crushing (material failure), while long columns fail by buckling (instability). The limit of 12 is the standard used in IS 456 for RC design.

137. The direct stress due to self-weight and extreme bending stress at the base of masonry dam are 200 kN/m^2 and 300 kN/m^2 respectively. The maximum and minimum stresses respectively are ____.

- (1) 30 kN/m^2 , 10 kN/m^2
- (2) 50 kN/m^2 , 10 kN/m^2
- (3) 50 kN/m^2 , -10 kN/m^2
- (4) 30 kN/m^2 , -10 kN/m^2

Correct Answer: (3) 50 kN/m^2 , -10 kN/m^2 (Note: Calculated values are 500 and -100; see Step 3)

Solution:

Step 1: Understanding the Concept:

The total stress at the base of a dam or retaining wall is the algebraic sum of the direct (axial) stress due to gravity loads and the bending stress caused by lateral water pressure.

Step 2: Key Formula or Approach:

1. Maximum Stress (σ_{max}) = $\sigma_{direct} + \sigma_{bending}$
2. Minimum Stress (σ_{min}) = $\sigma_{direct} - \sigma_{bending}$

Step 3: Detailed Explanation:

Given: $\sigma_{direct} = 200 \text{ kN/m}^2$ and $\sigma_{bending} = 300 \text{ kN/m}^2$. 1. $\sigma_{max} = 200 + 300 = 500 \text{ kN/m}^2$ (Compressive) 2. $\sigma_{min} = 200 - 300 = -100 \text{ kN/m}^2$ (Tensile) Note: Looking at the provided options, they appear to be scaled down by a factor of 10 (e.g., 50 and -10). Based on the magnitude relationship, Option 3 is the intended answer.

Step 4: Final Answer:

The maximum and minimum stresses are 50 kN/m^2 and -10 kN/m^2 (scaled).

Quick Tip: When the bending stress exceeds the direct stress, the minimum stress becomes negative, indicating that tension is developed at the base. Masonry dams are typically designed to avoid this.

138. If the diameter of the column increases by two times, then Euler's buckling load ____.

- (1) decreases by four times
- (2) increases by four times
- (3) decreases by eight times
- (4) increases by sixteen times

Correct Answer: (4) increases by sixteen times

Solution:

Step 1: Understanding the Concept:

Euler's buckling load (P_{cr}) depends on the Moment of Inertia (I) of the column's cross-section. For a circular section, I is a function of the diameter raised to the fourth power.

Step 2: Key Formula or Approach:

1. $P_{cr} = \frac{\pi^2 EI}{L^2}$
2. For a circular section, $I = \frac{\pi d^4}{64}$

Step 3: Detailed Explanation:

1. From the formulas above, we see that $P_{cr} \propto I$ and $I \propto d^4$. Therefore, $P_{cr} \propto d^4$.
2. If the diameter d is increased by two times (new diameter = $2d$):

$$P_{new} \propto (2d)^4$$

$$P_{new} \propto 16d^4$$

3. This means the new buckling load is 16 times the original load.

Step 4: Final Answer:

Euler's buckling load increases by sixteen times.

Quick Tip: Small changes in the dimensions of a column lead to massive changes in its buckling strength because of the power-of-four relationship in circular sections (or power-of-three in rectangular ones).

139. If the crushing load of a column is 1000 kN and the Euler load is 1000 kN, then the Rankine load is ____.

- (1) 500 kN
- (2) 1000 kN
- (3) 1500 kN
- (4) 2000 kN

Correct Answer: (1) 500 kN

Solution:

Step 1: Understanding the Concept:

Rankine's theory provides an empirical formula for the failure load of columns that are intermediate in length, accounting for both crushing and buckling.

Step 2: Key Formula or Approach:

Rankine's Formula:

$$\frac{1}{P_R} = \frac{1}{P_c} + \frac{1}{P_E}$$

Where P_R is the Rankine load, P_c is the crushing load, and P_E is the Euler load.

Step 3: Detailed Explanation:

Given: $P_c = 1000$ kN and $P_E = 1000$ kN.

$$\frac{1}{P_R} = \frac{1}{1000} + \frac{1}{1000}$$

$$\frac{1}{P_R} = \frac{2}{1000}$$

$$P_R = \frac{1000}{2} = 500 \text{ kN}$$

Step 4: Final Answer:

The Rankine load is 500 kN.

Quick Tip: The Rankine load is always smaller than both the Euler load and the crushing load, as it accounts for the combined vulnerability of the column to both failure modes.

140. If ϕ is the angle of internal friction for cohesionless soil, then the coefficient of active earth pressure is ____.

- (1) $\frac{1-\sin \phi}{1+\sin \phi}$
- (2) $\frac{1+\sin \phi}{1-\sin \phi}$
- (3) $\frac{1-\tan \phi}{1+\tan \phi}$
- (4) $\frac{1-\cos \phi}{1+\cos \phi}$

Correct Answer: (1) $\frac{1-\sin \phi}{1+\sin \phi}$

Solution:

Step 1: Understanding the Concept:

Rankine's earth pressure theory determines the pressure exerted by a soil mass on a retaining structure. The active state occurs when the wall moves away from the backfill.

Step 2: Key Formula or Approach:

The coefficient of active earth pressure (K_a) is derived from the Mohr-Coulomb failure criterion.

Step 3: Detailed Explanation:

For a vertical wall and horizontal backfill:

$$K_a = \frac{1 - \sin \phi}{1 + \sin \phi}$$

Alternatively, it can be written as $K_a = \tan^2(45^\circ - \phi/2)$.

Step 4: Final Answer:

The coefficient is $\frac{1-\sin \phi}{1+\sin \phi}$.

Quick Tip: To distinguish between K_a (active) and K_p (passive), remember that K_a is always less than 1 (minus on top), while K_p is always greater than 1 (plus on top).

141. As per IS:456-2000, for concrete to be used in RCC works, the permissible limit of inorganic solids in water is ____.

- (1) 400 mg/l
- (2) 500 mg/l
- (3) 2000 mg/l
- (4) 3000 mg/l

Correct Answer: (4) 3000 mg/l

Solution:

Step 1: Understanding the Concept:

The quality of water used for mixing and curing concrete is critical because impurities can affect the setting time, strength, and durability of the concrete, as well as lead to the corrosion of reinforcement.

Step 2: Detailed Explanation:

IS 456:2000 (Table 1) specifies the permissible limits for solids in water used for concrete. The limits are:

- Organic solids: 200 mg/l
- Inorganic solids: 3000 mg/l
- Sulfates (as SO_3): 400 mg/l
- Chlorides (as Cl): 2000 mg/l for plain concrete and 500 mg/l for reinforced concrete
- Suspended matter: 2000 mg/l

Step 3: Final Answer:

The permissible limit of inorganic solids is 3000 mg/l.

Quick Tip: A simple field rule from IS 456 is that water used for mixing concrete should generally be fit for drinking (potable), though potable water limits for human consumption are much stricter than those for concrete.

142. The target mean compressive strength for mix design of M25 grade concrete is ____.

- (1) 25 N/mm²
- (2) 29.5 N/mm²
- (3) 31.6 N/mm²
- (4) 33.25 N/mm²

Correct Answer: (3) 31.6 N/mm²

Solution:

Step 1: Understanding the Concept:

In concrete mix design, we target a strength higher than the characteristic strength (f_{ck}) to ensure that the probability of any test result falling below f_{ck} is very low (usually 5%).

Step 2: Key Formula or Approach:

The formula for Target Mean Strength (f'_{ck}) is:

$$f'_{ck} = f_{ck} + 1.65s$$

Where s is the standard deviation. For M25 grade, IS 456/IS 10262 specifies $s = 4.0$ N/mm².

Step 3: Detailed Explanation:

1. Characteristic strength $f_{ck} = 25$ N/mm².
2. Standard deviation s for M25 = 4.0 N/mm².
3. Calculate target mean strength:

$$f'_{ck} = 25 + (1.65 \times 4.0)$$

$$f'_{ck} = 25 + 6.6 = 31.6 \text{ N/mm}^2$$

Step 4: Final Answer:

The target mean compressive strength is 31.6 N/mm².

Quick Tip: The standard deviation (s) values are: 3.5 for M10/M15, 4.0 for M20/M25, and 5.0 for M30 and above. Remembering these helps solve mix design problems quickly.

143. The characteristic compressive strength of M25 grade concrete is ____.

- (1) 25 MPa
- (2) 50 MPa
- (3) 75 MPa
- (4) 100 MPa

Correct Answer: (1) 25 MPa

Solution:

Step 1: Understanding the Concept:

In the designation of concrete mix "M25", the letter 'M' stands for the Mix, and the numerical value represents the characteristic compressive strength.

Step 2: Detailed Explanation:

1. Characteristic compressive strength (f_{ck}) is defined as the strength of concrete below which not more than 5 percent of the test results are expected to fall.
2. This strength is measured by testing 150 mm cubes after 28 days of curing.
3. For M25, the number "25" directly indicates that the characteristic strength is 25 N/mm² (which is equal to 25 MPa).

Step 3: Final Answer:

The characteristic compressive strength is 25 MPa.

Quick Tip: 1 N/mm² is exactly equal to 1 MPa. These units are used interchangeably in structural engineering exams.

144. The minimum effective depth of a cantilever beam of span 3.5 m to satisfy the vertical deflection limit is ____.

- (1) 175 mm
- (2) 350 mm
- (3) 400 mm
- (4) 500 mm

Correct Answer: (4) 500 mm

Solution:

Step 1: Understanding the Concept:

To control deflection, IS 456:2000 specifies basic values for the span-to-effective depth ratio (l/d). For spans up to 10 m, these ratios ensure that the beam remains stiff enough to prevent excessive sagging or vibrating.

Step 2: Key Formula or Approach:

The basic l/d ratios are: 1. Cantilever: 7 2. Simply Supported: 20 3. Continuous: 26

Step 3: Detailed Explanation:

Given: Span (l) = 3.5 m = 3500 mm. For a cantilever beam:

$$\frac{l}{d} \leq 7$$

To find the minimum effective depth (d):

$$d \geq \frac{l}{7}$$

$$d \geq \frac{3500}{7}$$

$$d \geq 500 \text{ mm}$$

Step 4: Final Answer:

The minimum effective depth required is 500 mm.

Quick Tip: If the span exceeds 10 m, the basic values (except for cantilevers) are multiplied by 10/span. For cantilevers, the deflection is usually checked specifically through calculations if the span is very long.

145. The minimum tension reinforcement of Fe 500 grade required for the beam of width 300 mm and effective depth 500 mm is ____.

- (1) 225 mm²
- (2) 255 mm²
- (3) 375 mm²
- (4) 6000 mm²

Correct Answer: (2) 255 mm²

Solution:

Step 1: Understanding the Concept:

A minimum amount of tension steel is required in any RC beam to prevent sudden failure (brittle failure) if the concrete cracks. This limit depends on the yield strength of the steel used.

Step 2: Key Formula or Approach:

According to IS 456:2000:

$$\frac{A_s}{bd} = \frac{0.85}{f_y}$$

Step 3: Detailed Explanation:

Given: $b = 300$ mm, $d = 500$ mm, and $f_y = 500$ N/mm². Calculate minimum A_s :

$$A_s = \frac{0.85 \times b \times d}{f_y}$$

$$A_s = \frac{0.85 \times 300 \times 500}{500}$$

$$A_s = 0.85 \times 300$$

$$A_s = 255 \text{ mm}^2$$

Step 4: Final Answer:

The minimum tension reinforcement required is 255 mm^2 .

Quick Tip: Notice that as the grade of steel (f_y) increases, the minimum area of steel required decreases. This is because higher-grade steel provides more strength per unit area.

146. A rectangular beam of width 230 mm and effective depth 400 mm is reinforced with Fe 415 steel reinforcement. The limiting depth of neutral axis is ____.

- (1) 144 mm
- (2) 180 mm
- (3) 192 mm
- (4) 212 mm

Correct Answer: (3) 192 mm

Solution:**Step 1: Understanding the Concept:**

The limiting depth of the neutral axis ($x_{u,max}$) is the maximum depth allowed by the Limit State Method to ensure the beam remains "under-reinforced," meaning the steel will yield before the concrete crushes.

Step 2: Key Formula or Approach:

The ratio $x_{u,max}/d$ depends strictly on the grade of steel (f_y): 1. Fe 250: $0.53d$ 2. Fe 415: $0.48d$ 3. Fe 500: $0.46d$

Step 3: Detailed Explanation:

Given: $d = 400 \text{ mm}$ and steel grade = Fe 415. Using the factor for Fe 415:

$$x_{u,max} = 0.48 \times d$$

$$x_{u,max} = 0.48 \times 400$$

$$x_{u,max} = 192 \text{ mm}$$

Step 4: Final Answer:

The limiting depth of the neutral axis is 192 mm.

Quick Tip: Always identify the steel grade first. A common mistake is using the factor for Fe 250 (0.53) for all steel grades. These constants are derived from the strain profile of the section at failure.

147. The effective length of a cantilever beam of length 2 m having an effective depth of 400 mm embedded into a support of 300 mm width is ____.

- (1) 2.15 m
- (2) 2.2 m
- (3) 2.3 m
- (4) 2.4 m

Correct Answer: (2) 2.2 m

Solution:

Step 1: Understanding the Concept:

According to IS 456:2000, the effective length (l_{eff}) of a cantilever beam is determined by its clear span (l_n) plus half the effective depth (d) or half the width of the support ($w/2$), whichever is less. However, for a standard cantilever, it is generally taken as the clear span plus half the effective depth.

Step 2: Key Formula or Approach:

For a cantilever:

$$l_{eff} = l_n + \frac{d}{2}$$

Step 3: Detailed Explanation:

Given: - Clear span (l_n) = 2 m = 2000 mm - Effective depth (d) = 400 mm Using the formula:

$$l_{eff} = 2000 + \frac{400}{2}$$

$$l_{eff} = 2000 + 200 = 2200 \text{ mm}$$

Converting back to meters:

$$l_{eff} = 2.2 \text{ m}$$

Step 4: Final Answer:

The effective length is 2.2 m.

Quick Tip: For simply supported beams, the rules differ (clear span + d OR clear span + w , whichever is less). Always distinguish between the structural type before applying l_{eff} formulas.

148. A T-beam behave as a rectangular beam of width equal to its flange, if its neutral axis ____.

- (1) falls within the flange
- (2) falls below the flange
- (3) falls below the centroidal axis of the beam
- (4) coincides with the geometrical center of the beam

Correct Answer: (1) falls within the flange

Solution:**Step 1: Understanding the Concept:**

In a T-beam, the concrete in the compression zone provides the compressive force. If the neutral axis is inside the flange, the entire compression zone is rectangular with a width equal to the flange width (b_f).

Step 2: Detailed Explanation:

1. The concrete below the neutral axis is assumed to be cracked and does not contribute to the flexural strength. 2. If the neutral axis depth (x_u) is less than or equal to the depth of the flange (D_f), the shape of the concrete under compression is a simple rectangle of dimensions $b_f \times x_u$. 3. In this case, the design formulas used for a rectangular beam of width b_f are applicable. 4. If the neutral axis falls into the web ($x_u > D_f$), the compression zone becomes T-shaped, and different analysis methods are required.

Step 3: Final Answer:

The beam behaves as a rectangular beam if the neutral axis falls within the flange.

Quick Tip: Most T-beams in slab-beam floor systems are designed such that the neutral axis stays within the flange, as this is the most efficient use of the concrete area.

149. In a rectangular beam of width 300 mm and effective depth 600 mm, if the design shear strength of concrete is 0.5 N/mm^2 , then the shear capacity of beam without shear reinforcement is ____.

- (1) 9 kN
- (2) 90 kN
- (3) 540 kN
- (4) 900 kN

Correct Answer: (2) 90 kN

Solution:**Step 1: Understanding the Concept:**

The shear capacity of concrete (V_c) represents the amount of shear force a beam can resist using only the concrete and the longitudinal reinforcement, without needing stirrups.

Step 2: Key Formula or Approach:

$$V_c = \tau_c \times b \times d$$

Where τ_c is the design shear strength of concrete.

Step 3: Detailed Explanation:

Given: - $\tau_c = 0.5 \text{ N/mm}^2$ - $b = 300 \text{ mm}$ - $d = 600 \text{ mm}$ Calculate the capacity:

$$V_c = 0.5 \times 300 \times 600$$

$$V_c = 0.5 \times 180,000$$

$$V_c = 90,000 \text{ N}$$

Convert to kN:

$$V_c = 90 \text{ kN}$$

Step 4: Final Answer:

The shear capacity is 90 kN.

Quick Tip: If the actual factored shear force (V_u) on the beam is less than this V_c , only nominal shear reinforcement is required. If $V_u > V_c$, shear reinforcement must be designed to carry the difference.

150. The length of the straight portion of a steel bar of diameter ϕ beyond the end of the hook should not be less than ____.

- (1) 2ϕ
- (2) 4ϕ
- (3) 8ϕ
- (4) 16ϕ

Correct Answer: (2) 4ϕ

Solution:

Step 1: Understanding the Concept:

To ensure proper anchorage and to prevent the bar from slipping out of the concrete, standard hooks and bends must have a minimum extension beyond the bend.

Step 2: Detailed Explanation:

1. According to IS 456:2000, for a standard U-type hook, the total anchorage value is taken as 16ϕ . 2. However, the specific requirement for the straight "tail" or portion extending beyond the end of the curve is a minimum of 4 times the diameter of the bar (4ϕ). 3. This ensures that the hook is properly gripped by the concrete and does not straighten out under tension.

Step 3: Final Answer:

The straight portion should not be less than 4ϕ .

Quick Tip: For a 90° bend (L-bend), the anchorage value is 8ϕ , and for a 180° hook, it is 16ϕ . Regardless of the angle, the 4ϕ tail is a standard safety requirement.

151. As per IS:456-2000, maximum spacing for main reinforcement in a slab of effective depth d is ____.

- (1) $3d$ or 300 mm whichever is less
- (2) $3d$ or 300 mm whichever is more
- (3) $5d$ or 450 mm whichever is less
- (4) $5d$ or 450 mm whichever is more

Correct Answer: (1) $3d$ or 300 mm whichever is less

Solution:

Step 1: Understanding the Concept:

To control cracking and ensure the structural integrity of a slab, IS 456:2000 specifies maximum limits for the distance between individual reinforcement bars. These limits vary depending on whether the bars are main (tension) reinforcement or distribution (secondary) reinforcement.

Step 2: Detailed Explanation:

According to Clause 26.3.3 of IS 456:2000: 1. **Main Tension Reinforcement:** The horizontal distance between parallel main reinforcement bars shall not be more than three times the effective depth ($3d$) or 300 mm, whichever is smaller. 2. **Distribution Reinforcement:** For secondary reinforcement (provided against shrinkage and temperature), the limit is five times the effective depth ($5d$) or 450 mm, whichever is smaller.

Step 3: Final Answer:

The maximum spacing for main reinforcement is $3d$ or 300 mm whichever is less.

Quick Tip: Remember the "3 and 3" rule for main bars ($3d$, 300mm) and the "5 and 4.5" rule for distribution bars ($5d$, 450mm). Main bars are closer together because they are the primary load-carrying elements.

152. As per IS:456-2000, for slabs spanning in two directions with corners held down, the width of middle strip is ____.

- (1) half the width of the slab
- (2) two-third of the width of the slab
- (3) three-fourth of the width of the slab
- (4) four-fifth of the width of the slab

Correct Answer: (3) three-fourth of the width of the slab

Solution:**Step 1: Understanding the Concept:**

When designing two-way slabs where corners are prevented from lifting, the slab is divided into "middle strips" and "edge strips" in both directions. This allows for more precise reinforcement placement based on the moment distribution.

Step 2: Detailed Explanation:

According to IS 456 Annex D: 1. A slab is divided into a middle strip and two edge strips in

each direction. 2. The **middle strip** extends over the central 3/4 (three-fourth) of the span ($0.75 \times \text{span}$). 3. The remaining 1/4 (one-fourth) of the span is divided equally into two **edge strips** ($1/8$ or $0.125 \times \text{span}$ on each side).

Step 3: Final Answer:

The width of the middle strip is three-fourth of the width of the slab.

Quick Tip: The middle strip carries the maximum bending moments, which is why it covers the vast majority (75%) of the slab area, while edge strips are primarily for handling torsion and support conditions.

153. If σ_{cbc} is the permissible compressive stress in bending in concrete, σ_{st} is the permissible tensile stress in steel, d is effective depth of beam and x is depth of neutral axis, then which of the following relation holds in working stress method of design.

(1) $\sigma_{cbc} = (\sigma_{st}/m) \cdot (x/(d-x))$

(2) $\sigma_{cbc} = (\sigma_{st}/m) \cdot ((d-x)/x)$

(3) $\sigma_{cbc} = m \cdot \sigma_{st} \cdot (x/(d-x))$

(4) $\sigma_{cbc} = m \cdot \sigma_{st} \cdot ((d-x)/x)$

Correct Answer: (1) $\sigma_{cbc} = (\sigma_{st}/m) \cdot (x/(d-x))$

Solution:

Step 1: Understanding the Concept:

The Working Stress Method (WSM) assumes that both concrete and steel behave elastically. The strain distribution is linear, and the neutral axis depth is determined by the ratio of the stresses and the modular ratio (m).

Step 2: Key Formula or Approach:

From the strain diagram and the definition of modular ratio ($m = E_s/E_c$):

$$\frac{\sigma_{cbc}}{x} = \frac{\sigma_{st}/m}{d-x}$$

Step 3: Detailed Explanation:

To find the relationship for σ_{cbc} : 1. Start with the ratio:

$$\frac{\sigma_{cbc}}{x} = \frac{\sigma_{st}}{m(d-x)}$$

2. Rearrange to solve for σ_{cbc} :

$$\sigma_{cbc} = \frac{\sigma_{st}}{m} \cdot \frac{x}{d-x}$$

This matches option (1). This equation defines the "Critical Neutral Axis" when both materials reach their permissible stresses simultaneously.

Step 4: Final Answer:

The correct relation is $\sigma_{cbc} = (\sigma_{st}/m) \cdot (x/(d-x))$.

Quick Tip: In WSM, we use σ_{st}/m because we convert the steel area into an "equivalent concrete area" to perform a homogeneous section analysis.

154. For a singly reinforced beam, if d and x are effective depth and depth of neutral axis respectively, then the lever arm of the beam is ____.

- (1) $d - x/2$
- (2) $d + x/2$
- (3) $d + x/3$
- (4) $d - x/3$

Correct Answer: (4) $d - x/3$

Solution:**Step 1: Understanding the Concept:**

In the Working Stress Method (WSM), the stress distribution in concrete is assumed to be linear (triangular). The lever arm is the vertical distance between the resultant compressive force (C) and the resultant tensile force (T).

Step 2: Key Formula or Approach:

1. The compressive force acts at the centroid of the stress triangle. 2. For a triangle of height x , the centroid is at a distance of $x/3$ from the top (compression edge).

Step 3: Detailed Explanation:

1. The total depth from the top to the center of steel is the effective depth (d). 2. The resultant compressive force (C) acts at $x/3$ from the top. 3. The resultant tensile force (T) acts at the level of the steel reinforcement (d from the top). 4. Lever arm (z) = Distance from top to T - Distance from top to C

$$z = d - \frac{x}{3}$$

Step 4: Final Answer:

The lever arm of the beam is $d - x/3$.

Quick Tip: In the Limit State Method (LSM), the stress block is parabolic-rectangular, and the lever arm is $d - 0.42x_u$. Always identify which design method the question is referring to by looking at the variable x (WSM) vs x_u (LSM).

155. In working stress method of design, the allowable tensile stress in High Yield Strength Deformed steel used in reinforced cement concrete is ____.

- (1) 140 N/mm²
- (2) 180 N/mm²
- (3) 230 N/mm²
- (4) 415 N/mm²

Correct Answer: (3) 230 N/mm²

Solution:**Step 1: Understanding the Concept:**

Allowable or permissible stress in WSM is obtained by dividing the yield strength of the

material by a factor of safety. For steel in tension, this value is codified in IS 456.

Step 2: Detailed Explanation:

According to Table 22 of IS 456:2000 (Working Stress Method): 1. For Mild Steel (Grade I): $\sigma_{st} = 140 \text{ N/mm}^2$. 2. For HYSD bars (like Fe 415 or Fe 500): $\sigma_{st} = 230 \text{ N/mm}^2$. This value applies to bars with a diameter up to 20 mm. For larger diameters, the value may slightly vary depending on the specific application (e.g., liquid retaining structures).

Step 3: Final Answer:

The allowable tensile stress in HYSD steel is 230 N/mm^2 .

Quick Tip: While 415 N/mm^2 is the yield strength (f_y), the Working Stress Method never uses the full yield strength. It always uses a reduced "permissible" value to maintain a factor of safety of approximately 1.8.

156. As per IS:456-2000, the maximum compressive strain in concrete in axial compression is ____.

- (1) 0.03
- (2) 0.003
- (3) 0.02
- (4) 0.002

Correct Answer: (4) 0.002

Solution:

Step 1: Understanding the Concept:

The strain limits in concrete define the failure points for different types of loading (flexure, axial, or combined). These limits are fundamental to the Limit State Method.

Step 2: Detailed Explanation:

IS 456:2000 specifies two distinct strain limits for compression: 1. **Axial Compression (Columns):** The maximum compressive strain in concrete is taken as **0.002**. This is the point

where the parabolic part of the stress-strain curve reaches its maximum stress. 2. **Flexure (Bending in Beams):** The maximum compressive strain at the highly compressed extreme fiber is taken as **0.0035**.

Step 3: Final Answer:

The maximum compressive strain in axial compression is 0.002.

Quick Tip: Don't confuse "Axial" with "Flexure." For columns (axial), use 0.002. For beams (bending), use 0.0035. If you see 0.003 in the options, it is often a distractor meant to confuse you with the Flexure limit.

157. A short axially loaded square column of size 500 mm × 500 mm, the minimum area of longitudinal reinforcement as per IS: 456 is ____.

- (1) 2500 mm²
- (2) 2000 mm²
- (3) 1250 mm²
- (4) 375 mm²

Correct Answer: (2) 2000 mm²

Solution:

Step 1: Understanding the Concept:

IS 456:2000 specifies limits for the area of longitudinal reinforcement in columns to ensure they have enough ductility and to prevent accidental buckling of the bars.

Step 2: Key Formula or Approach:

The minimum area of longitudinal reinforcement ($A_{sc,min}$) is 0.8% of the gross cross-sectional area of the column.

Step 3: Detailed Explanation:

Given: Size = 500 mm × 500 mm. 1. Gross Area (A_g) = $500 \times 500 = 250,000 \text{ mm}^2$. 2.

Minimum Reinforcement = 0.8% of A_g .

$$A_{sc,min} = \frac{0.8}{100} \times 250,000$$

$$A_{sc,min} = 0.008 \times 250,000 = 2000 \text{ mm}^2$$

Step 4: Final Answer:

The minimum area of longitudinal reinforcement is 2000 mm².

Quick Tip: Remember the limits: Minimum = 0.8% and Maximum = 6% (though practically restricted to 4% to avoid congestion during casting).

158. When designing a short RCC column subjected to axial compression, the permissible stress in concrete is taken as ____.

- (1) The full characteristic strength of concrete
- (2) Equal to the yield strength of steel reinforcement
- (3) A fraction of the characteristic strength, reduced by a factor of safety
- (4) Independent of the grade of concrete used

Correct Answer: (3) A fraction of the characteristic strength, reduced by a factor of safety

Solution:

Step 1: Understanding the Concept:

Structural safety is ensured by using "Design Strength" rather than the raw material strength. In both Working Stress and Limit State methods, we apply factors to reduce the strength values to account for material variations and loading uncertainties.

Step 2: Detailed Explanation:

1. In the Limit State Method, the characteristic strength (f_{ck}) is first reduced to $0.67f_{ck}$ to account for the difference between cube strength and actual structure strength.
2. A partial

safety factor for material ($\gamma_m = 1.5$) is then applied, resulting in a design stress of $0.45f_{ck}$ for flexure. 3. For axial compression in columns, the formula $P_u = 0.4f_{ck}A_c + 0.67f_yA_{sc}$ uses even further reduced coefficients (0.4) to account for minimum eccentricities. 4. Therefore, the design stress is always a fraction of the characteristic strength.

Step 3: Final Answer:

The permissible stress is a fraction of the characteristic strength, reduced by a factor of safety.

Quick Tip: The reduction from f_{ck} to $0.67f_{ck}$ accounts for the size effect and casting conditions of a large member versus a small laboratory cube.

159. According to IS:456-2000, for an RCC footing on soil, the thickness of footing at the edge shall not be less than ____.

- (1) 50 mm
- (2) 100 mm
- (3) 150 mm
- (4) 200 mm

Correct Answer: (3) 150 mm

Solution:

Step 1: Understanding the Concept:

Footings must have a minimum thickness to provide enough rigidity and cover for the reinforcement, and to resist punching shear forces at the edges.

Step 2: Detailed Explanation:

IS 456:2000 (Clause 34.1.2) specifies: 1. For footings on **soil**, the thickness at the edge shall not be less than **150 mm**. 2. For footings on **piles**, the thickness at the edge shall not be less than **300 mm**.

Step 3: Final Answer:

The thickness at the edge shall not be less than 150 mm.

Quick Tip: This 150 mm is the absolute minimum at the edge for sloped or stepped footings. For a flat rectangular footing, the thickness is usually much higher to satisfy shear requirements.

160. A square footing for a column carries an axial load of 1800 kN. The self-weight of footing is taken as 10% of the column load. If the safe bearing capacity of soil is 180 kN/m², then the minimum area of footing required will be ____.

- (1) 10 m²
- (2) 11 m²
- (3) 18 m²
- (4) 19 m²

Correct Answer: (2) 11 m²

Solution:

Step 1: Understanding the Concept:

The area of a footing is designed based on the Total Load (Column load + Self-weight) and the Safe Bearing Capacity (SBC) of the soil to ensure the pressure doesn't exceed the soil's limit.

Step 2: Key Formula or Approach:

$$\text{Area of Footing} = \frac{\text{Total Load}}{\text{Safe Bearing Capacity}}$$

Step 3: Detailed Explanation:

Given: - Column Load (P) = 1800 kN - Self-weight = 10% of $P = 0.10 \times 1800 = 180$ kN
1. Total Load = 1800 + 180 = 1980 kN. 2. SBC = 180 kN/m². 3. Calculate Area:

$$\text{Area} = \frac{1980}{180}$$

$$\text{Area} = 11 \text{ m}^2$$

Step 4: Final Answer:

The minimum area of footing required is 11 m^2 .

Quick Tip: Always use working loads (not factored/ultimate loads) when designing the area of a footing, as Safe Bearing Capacity already includes a factor of safety.

161. The line joining main survey stations is called ____.

- (1) Tie line
- (2) Check line
- (3) Survey line
- (4) Proof line

Correct Answer: (3) Survey line

Solution:**Step 1: Understanding the Concept:**

In chain surveying, a framework of triangles is created. The various lines used in this framework have specific names based on their function and the stations they connect.

Step 2: Detailed Explanation:

1. **Main Stations:** These are the primary points selected to command the area to be surveyed.
2. **Survey Line (Main Survey Line):** The line joining two main survey stations. The longest of these is usually called the Base Line.
3. **Tie Line:** A line joining "tie stations" on the main survey lines. It is used to locate interior details.
4. **Check Line (Proof Line):** A line used to check the accuracy of the framework.

Step 3: Final Answer:

The line joining main survey stations is called a Survey line.

Quick Tip: The most important survey line is the Base Line, as it forms the backbone of the entire survey and requires the highest level of accuracy in measurement.

162. During distance measurement in chaining, the compensating error is caused by ____.

- (1) sag in chain
- (2) bad ranging
- (3) pull in chain
- (4) careless holding and marking

Correct Answer: (4) careless holding and marking

Solution:

Step 1: Understanding the Concept:

Errors in surveying are classified as either Systematic (Cumulative) or Random (Compensating). Compensating errors are those which are equally likely to be positive or negative and tend to cancel each other out over a large number of measurements.

Step 2: Detailed Explanation:

1. **Cumulative Errors:** Errors like sag (always makes the chain look shorter), bad ranging (always makes the measured length longer), and incorrect pull follow a specific mathematical law and accumulate. 2. **Compensating Errors:** These occur due to human limitations, such as slightly misplacing an arrow or being off by a millimeter while holding the chain. Because these errors can go in either direction, they are proportional to the square root of the length ($L^{1/2}$).

Step 3: Final Answer:

Compensating error is caused by careless holding and marking.

Quick Tip: Cumulative errors are far more serious than compensating errors because they grow linearly with the distance measured, whereas compensating errors grow much more slowly.

163. The fore bearing of the line AB and BC are 30° and 150° respectively. The included angle between AB and BC is ____.

- (1) 30°
- (2) 60°
- (3) 90°
- (4) 120°

Correct Answer: (2) 60°

Solution:

Step 1: Understanding the Concept:

The included angle at a station is the angle measured clockwise from the previous line to the next line. It is calculated by taking the difference between the Fore Bearing of the next line and the Back Bearing of the previous line.

Step 2: Key Formula or Approach:

1. Back Bearing (BB) = Fore Bearing (FB) $\pm 180^\circ$ 2. Included Angle ($\angle B$) = FB of BC - BB of AB

Step 3: Detailed Explanation:

Given: FB of AB = 30° and FB of BC = 150° . 1. Calculate BB of AB:

$$BB_{AB} = 30^\circ + 180^\circ = 210^\circ$$

2. Calculate the Included Angle:

$$\text{Included Angle} = FB_{BC} - BB_{AB}$$

$$\text{Included Angle} = 150^\circ - 210^\circ = -60^\circ$$

3. Since the result is negative, add 360° to get the exterior angle, or take the absolute value if looking for the interior/acute difference depending on the traverse direction. However, the standard "included angle" in a clockwise traverse is:

$$\text{Interior Angle} = 150^\circ - (30^\circ - 180^\circ) \text{ logic effectively gives } 60^\circ \text{ as the difference.}$$

Alternatively, calculate the angle between the two lines directly: $(180^\circ - 150^\circ) + 30^\circ = 60^\circ$.

Step 4: Final Answer:

The included angle is 60° .

Quick Tip: A quick sketch often helps: Line AB goes North-East (30°). Line BC goes South-East (150°). The change in direction is $150^\circ - 30^\circ = 120^\circ$. The interior angle between them is $180^\circ - 120^\circ = 60^\circ$.

164. If the whole circle bearing of a line AB is $293^\circ 30'$, then the reduced bearing of line BA is ____.

- (1) N $23^\circ 30'$ W
- (2) N $66^\circ 30'$ W
- (3) S $23^\circ 30'$ E
- (4) S $66^\circ 30'$ E

Correct Answer: (4) S $66^\circ 30'$ E

Solution:

Step 1: Understanding the Concept:

Whole Circle Bearing (WCB) is measured from North in a clockwise direction. Reduced Bearing (RB) or Quadrantal Bearing (QB) is measured from North or South (whichever is closer) toward East or West. The question asks for the bearing of line **BA** (Back Bearing), not AB.

Step 2: Key Formula or Approach:

1. Back Bearing (BB) of AB = FB of AB $\pm 180^\circ$ 2. Reduced Bearing rules:

- 0° to 90° : N θ E
- 90° to 180° : S $(180 - \theta)$ E
- 180° to 270° : S $(\theta - 180)$ W
- 270° to 360° : N $(360 - \theta)$ W

Step 3: Detailed Explanation:

1. Find the WCB of line BA:

$$WCB_{BA} = 293^{\circ}30' - 180^{\circ} = 113^{\circ}30'$$

2. This angle ($113^{\circ}30'$) falls in the second quadrant (between 90° and 180°), which is the SE quadrant. 3. Calculate the Reduced Bearing:

$$RB = S(180^{\circ} - 113^{\circ}30')E = S66^{\circ}30'E$$

Step 4: Final Answer:

The reduced bearing of line BA is S $66^{\circ}30'$ E.

Quick Tip: There is a faster shortcut: Convert FB (AB) to RB first. $293^{\circ}30'$ is N $66^{\circ}30'$ W. To find the RB of the back bearing (BA), simply swap the directions: N becomes S, and W becomes E. Result: S $66^{\circ}30'$ E.

165. In a fly levelling, if the staff reading of back sight is greater than the staff reading of fore sight, then the ground level indicates a ____.

- (1) fall
- (2) rise
- (3) level surface
- (4) horizontal surface

Correct Answer: (2) rise

Solution:

Step 1: Understanding the Concept:

In levelling, staff readings are inversely proportional to the ground elevation. A tall staff reading means the ground is lower, and a short staff reading means the ground is higher.

Step 2: Key Formula or Approach:

Rise or Fall = (Previous Reading) - (Present Reading) - If the result is positive, it is a **Rise**. - If the result is negative, it is a **Fall**.

Step 3: Detailed Explanation:

1. Let the Back Sight (BS) be the previous reading and the Fore Sight (FS) be the present reading. 2. Given: $BS > FS$. 3. Applying the formula: $(BS - FS) = \text{Positive value}$. 4. A positive result indicates that the point where the Fore Sight was taken is at a higher elevation than the point where the Back Sight was taken.

Step 4: Final Answer:

The ground level indicates a rise.

Quick Tip: Think of the staff as a ruler touching the ground. If you move to a "hill" (higher ground), the line of sight will hit a lower number on the ruler because the ground has pushed the ruler up.

166. The ratio of correction for curvature to correction for refraction is ____.

- (1) 4
- (2) 6
- (3) 7
- (4) 9

Correct Answer: (3) 7

Solution:**Step 1: Understanding the Concept:**

In long-distance levelling, corrections must be made for the curvature of the Earth and the refraction of light through the atmosphere. These corrections are functions of the distance (D) between the instrument and the staff.

Step 2: Key Formula or Approach:

1. Curvature Correction (C_c) = $0.0785D^2$ (always subtractive) 2. Refraction Correction (C_r)

$= 0.0112D^2$ (always additive) 3. Note: C_r is approximately $1/7$ of C_c .

Step 3: Detailed Explanation:

1. The question asks for the ratio C_c/C_r . 2. Using the standard relationship $C_r = \frac{1}{7}C_c$:

$$\frac{C_c}{C_r} = 7$$

3. More precisely, using the coefficients: $0.0785/0.0112 \approx 7.008$.

Step 4: Final Answer:

The ratio is 7.

Quick Tip: To remember the combined correction (C), use the formula $C = C_c - C_r = \frac{6}{7}C_c = 0.0673D^2$. The "7" is the magic number for all these relationships.

167. For measuring vertical angles, the theodolite must be levelled with reference to the ____.

- (1) bubble tube on the plate level
- (2) altitude bubble on the telescope
- (3) altitude bubble on the index frame
- (4) altitude bubble on both telescope and the index frame

Correct Answer: (3) altitude bubble on the index frame

Solution:

Step 1: Understanding the Concept:

Measuring vertical angles requires a horizontal reference datum (the zero-degree line). While the plate levels are used for horizontal positioning, vertical measurements rely on the index frame (or clinometer) being perfectly leveled.

Step 2: Detailed Explanation:

1. The plate levels are used to make the vertical axis of the instrument truly vertical. 2. The altitude bubble (also known as the master bubble) is attached to the T-frame or index frame.

3. When this bubble is centered, it ensures that the index arm is truly horizontal, which means the vertical circle readings will be correct relative to the horizon. 4. If the altitude bubble is not leveled, the vertical angle measured will have an index error.

Step 3: Final Answer:

The theodolite must be levelled with reference to the altitude bubble on the index frame.

Quick Tip: Remember: Plate levels = Horizontal angles; Altitude bubble on index frame = Vertical angles. This distinction is a common trap in surveying exams.

168. If the latitude and departure of a line AB are 40 m and 30 m respectively, then the length of line AB is ____.

- (1) 10 m
- (2) 35 m
- (3) 50 m
- (4) 70 m

Correct Answer: (3) 50 m

Solution:

Step 1: Understanding the Concept:

In a coordinate system, the latitude (L) is the projection of a line on the North-South axis, and the departure (D) is the projection on the East-West axis. The length of the line is the hypotenuse of the triangle formed by these two components.

Step 2: Key Formula or Approach:

The length of line (l) is given by the Pythagorean theorem:

$$l = \sqrt{L^2 + D^2}$$

Step 3: Detailed Explanation:

Given: $L = 40$ m and $D = 30$ m. 1. Apply the formula:

$$l = \sqrt{40^2 + 30^2}$$

$$l = \sqrt{1600 + 900}$$

$$l = \sqrt{2500}$$

$$l = 50 \text{ m}$$

Step 4: Final Answer:

The length of line AB is 50 m.

Quick Tip: This is a standard 3-4-5 right-angled triangle scaled by 10. Recognizing these ratios (3, 4, 5 or 5, 12, 13) can save significant time during calculations.

169. If the sum of north latitudes is more than the sum of south latitudes and sum of west departures is more than the sum of east departures, then the bearing of the closing line is in ____.

- (1) NW quadrant
- (2) NE quadrant
- (3) SW quadrant
- (4) SE quadrant

Correct Answer: (4) SE quadrant

Solution:**Step 1: Understanding the Concept:**

In a closed traverse, the sum of latitudes (ΣL) and departures (ΣD) should theoretically be zero. If not, the "closing error" represents the vector required to close the loop. The closing line is the line needed to go from the calculated end point back to the start point.

Step 2: Key Formula or Approach:

- Error in Latitude (e_L) = $\Sigma\text{North} - \Sigma\text{South}$ - Error in Departure (e_D) = $\Sigma\text{East} - \Sigma\text{West}$ - The bearing of the closing line is determined by the signs of ($-e_L$) and ($-e_D$).

Step 3: Detailed Explanation:

1. Given: $\Sigma\text{North} > \Sigma\text{South}$. This means the net latitude is North (+). To close the traverse, the closing line must go **South** (-). 2. Given: $\Sigma\text{West} > \Sigma\text{East}$. This means the net departure is West (-). To close the traverse, the closing line must go **East** (+). 3. A line that goes South and East is in the SE quadrant.

Step 4: Final Answer:

The bearing of the closing line is in the SE quadrant.

Quick Tip: Always remember: the closing line is the "correction." If your traverse ends too far North and West, you must walk South and East to get back to the start.

170. In which of the following circumstances can the contour lines of different elevations intersect each other?

- (1) Pond
- (2) Cave
- (3) Valley
- (4) River

Correct Answer: (2) Cave (specifically an Overhanging Cliff)

Solution:**Step 1: Understanding the Concept:**

Contour lines are lines joining points of equal elevation. By definition, they generally do not cross each other because a single point on the ground cannot have two different elevations. However, there are specific geological exceptions.

Step 2: Detailed Explanation:

1. Pond/Valley/River: These are standard terrain features where contours may get close but never intersect. 2. Overhanging Cliff: In this case, a higher part of the land hangs over a lower part. A vertical line dropped from the edge would pass through two different elevations, causing the contour lines to cross. 3. Cave: Similarly, the ground surface above a cave and the floor of the cave represent different elevations at the same horizontal coordinates. On a 2D map, these contours would intersect.

Step 3: Final Answer:

Contour lines of different elevations can intersect in the case of a Cave (or Overhanging Cliff).

Quick Tip: There are two "special" contour rules: 1. Contours cross only in an overhanging cliff or cave. 2. Contours unite into a single line only in a vertical cliff.

171. Which of the following methods are used for computing the area? ____.

- (1) Bowditch method and Transit method
- (2) Bowditch method and Trapezoidal method
- (3) Simpson's method and Manning's method
- (4) Simpson's method and Trapezoidal method

Correct Answer: (4) Simpson's method and Trapezoidal method

Solution:**Step 1: Understanding the Concept:**

Computing area in surveying typically involves finding the area of an irregular tract of land. This is done by measuring offsets from a base line to the boundary at regular intervals.

Step 2: Detailed Explanation:

1. **Trapezoidal Method:** Assumes the boundary between the endpoints of the offsets is a straight line. 2. **Simpson's Method:** Assumes the boundary consists of a series of parabolic arcs. It is generally more accurate than the Trapezoidal method. 3. **Bowditch/Transit Methods:** These are used for balancing (adjusting) a traverse, not for area calculation. 4.

Manning's Method: This is a formula used in fluid mechanics to calculate flow velocity in open channels.

Step 3: Final Answer:

Simpson's method and Trapezoidal method are used for computing the area.

Quick Tip: Remember that Simpson's rule can only be applied when there is an odd number of offsets (which means an even number of divisions/intervals).

172. Stadia interval factor is the ratio of ____.

- (1) focal length of objective to stadia interval
- (2) stadia interval to focal length of objective
- (3) multiplying constant to additive constant
- (4) additive constant to multiplying constant

Correct Answer: (1) focal length of objective to stadia interval

Solution:

Step 1: Understanding the Concept:

In Tacheometry, the horizontal distance between the instrument and the staff is calculated using stadia hairs. The formula is $D = ks + c$.

Step 2: Key Formula or Approach:

1. Multiplying Constant (k) = f/i (also called Stadia Interval Factor) 2. Additive Constant (c) = $f + d$ Where f is focal length and i is stadia interval (the distance between the stadia hairs).

Step 3: Detailed Explanation:

The term "Stadia interval factor" refers to the multiplying constant. It is the ratio of the focal length of the objective lens (f) to the stadia interval (i). For most modern tacheometers, this value is kept as 100 to simplify calculations.

Step 4: Final Answer:

It is the ratio of focal length of objective to stadia interval.

Quick Tip: If an "anallactic lens" is provided in the telescope, the additive constant (c) becomes zero, significantly simplifying the distance calculation to $D = 100s$.

173. Global positioning system can be used for ____.

- (1) preparation of maps and detection of underground mines
- (2) preparation of maps and tracking of movement of vehicles
- (3) detection of underground mines and tracking of movement of vehicles
- (4) design of hydraulic structures and quantity surveying

Correct Answer: (2) preparation of maps and tracking of movement of vehicles

Solution:

Step 1: Understanding the Concept:

GPS (Global Positioning System) is a satellite-based navigation system that provides location and time information anywhere on Earth where there is an unobstructed line of sight to four or more GPS satellites.

Step 2: Detailed Explanation:

1. **Preparation of Maps:** GPS coordinates are used in GIS (Geographic Information Systems) to create highly accurate digital maps. 2. **Tracking:** Because GPS provides real-time location data, it is the primary technology for tracking the movement of vehicles, ships, and aircraft. 3. **Limitations:** GPS signals cannot penetrate significantly into the earth, so they cannot be used to detect underground mines. Similarly, while GPS can aid in site selection, the actual design of hydraulic structures and quantity surveying are engineering tasks that rely on detailed site surveys and material analysis.

Step 3: Final Answer:

GPS can be used for preparation of maps and tracking of movement of vehicles.

Quick Tip: GPS accuracy can be improved from meters to centimeters using techniques like DGPS (Differential GPS) or RTK (Real-Time Kinematic) surveying.

174. Which of the following options represent the relation between fundamental lines of theodolite?

- (1) The trunnion axis must be parallel to the line of sight
- (2) The axis of the plate level must be parallel to the vertical axis
- (3) The axis of altitude level must be parallel to the line of collimation
- (4) The axis of the striding level must be perpendicular to the transit axis

Correct Answer: (3) The axis of altitude level must be parallel to the line of collimation

Solution:

Step 1: Understanding the Concept:

A theodolite has several fundamental axes and lines that must maintain specific geometric relationships for the instrument to provide accurate readings. If these relationships are not met, the instrument has "permanent errors."

Step 2: Detailed Explanation:

The fundamental relationships are:

- The axis of the plate level must be **perpendicular** to the vertical axis.
- The line of collimation must be **perpendicular** to the horizontal (trunnion) axis.
- The horizontal axis must be **perpendicular** to the vertical axis.
- **The axis of the altitude level must be parallel to the line of collimation** (when the vertical circle reads zero).

Option (3) correctly identifies one of these required parallel relationships.

Step 3: Final Answer:

The correct relation is that the axis of altitude level must be parallel to the line of collimation.

Quick Tip: Remember the "perpendicularity" rule for most physical axes (Vertical, Horizontal, and Plate Level). Parallelism is primarily required for the bubbles used in vertical angle measurement.

175. The RL of point A is 100.0 m and its back sight is 2.000 m. If the next reading on the change point is 1.500 m, then the RL of change point is ____.

- (1) 98.5 m
- (2) 100.5 m
- (3) 102.0 m
- (4) 103.5 m

Correct Answer: (2) 100.5 m

Solution:

Step 1: Understanding the Concept:

The Height of Instrument (HI) method is used to determine the Reduced Level (RL) of unknown points by establishing the elevation of the line of sight.

Step 2: Key Formula or Approach:

1. $HI = RL \text{ of known point} + \text{Back Sight (BS)}$
2. $RL \text{ of new point} = HI - \text{Fore Sight (FS) or Intermediate Sight (IS)}$

Step 3: Detailed Explanation:

Given: - RL of A = 100.0 m - BS on A = 2.000 m - Next reading (FS) on Change Point (CP) = 1.500 m

1. Calculate HI:

$$HI = 100.0 + 2.000 = 102.000 \text{ m}$$

2. Calculate RL of CP:

$$RL_{CP} = 102.000 - 1.500 = 100.500 \text{ m}$$

Step 4: Final Answer:

The RL of the change point is 100.5 m.

Quick Tip: In levelling, the "Change Point" is where you take a Fore Sight and then move the instrument to take a Back Sight on the same point. The calculation for the first part (finding the RL) only uses the Fore Sight reading.

176. Flow of fluid takes place due to ____.

- (1) Viscosity
- (2) Surface tension
- (3) Compressibility
- (4) Deformation under shear force

Correct Answer: (4) Deformation under shear force

Solution:

Step 1: Understanding the Concept:

The defining physical difference between a solid and a fluid (liquid or gas) is how they respond to applied forces, specifically shear stress.

Step 2: Detailed Explanation:

1. A solid can resist a shear force by deforming a fixed amount. If the force is within the elastic limit, the solid stops deforming. 2. A fluid cannot resist shear stress, no matter how small. Instead of reaching a static equilibrium, it deforms continuously as long as the stress is applied. 3. This continuous deformation is what we perceive as "flow." 4. Viscosity is the property that resists this flow, but it is not the cause of the flow.

Step 3: Final Answer:

Flow takes place due to deformation under shear force.

Quick Tip: Remember the definition of a fluid: "A substance that deforms continuously under the application of shear stress, however small."

177. A fluid whose viscosity does not change with the rate of deformation or shear strain is known as ____.

- (1) Ideal fluid
- (2) Real fluid
- (3) Newtonian fluid
- (4) Non-Newtonian fluid

Correct Answer: (3) Newtonian fluid

Solution:

Step 1: Understanding the Concept:

Fluids are classified based on the relationship between shear stress (τ) and the rate of shear strain (velocity gradient, du/dy).

Step 2: Detailed Explanation:

1. **Newtonian Fluid:** Follows Newton's Law of Viscosity ($\tau = \mu \frac{du}{dy}$). Here, the dynamic viscosity (μ) is a constant and does not depend on how fast the fluid is being sheared. Examples include water, air, and gasoline. 2. **Non-Newtonian Fluid:** The viscosity changes depending on the shear rate. Examples include blood, paint, and toothpaste. 3. **Ideal Fluid:** A theoretical fluid with zero viscosity and zero compressibility.

Step 3: Final Answer:

A fluid with constant viscosity regardless of the shear rate is a Newtonian fluid.

Quick Tip: On a graph of shear stress vs. shear strain rate, a Newtonian fluid appears as a straight line passing through the origin. The slope of this line represents the viscosity.

178. Centre of pressure on an inclined plane is ____.

- (1) at metacenter
- (2) at the centroid
- (3) above the centroid
- (4) below the centroid

Correct Answer: (4) below the centroid

Solution:

Step 1: Understanding the Concept:

The centroid is the geometric center of the plane surface. The Centre of Pressure is the point where the total hydrostatic force is considered to act.

Step 2: Key Formula or Approach:

The depth of the centre of pressure (h) for an inclined surface is:

$$h = \bar{h} + \frac{I_G \sin^2 \theta}{A \bar{h}}$$

Where $\bar{h}$ is the depth of the centroid.

Step 3: Detailed Explanation:

1. Hydrostatic pressure increases linearly with depth ($P = \rho gh$). 2. Because the lower part of the inclined surface is deeper than the upper part, it experiences higher pressure. 3. The resultant force must shift toward the area of higher pressure (deeper). 4. Therefore, h will always be greater than $\bar{h}$, meaning it is below the centroid.

Step 4: Final Answer:

The centre of pressure is below the centroid.

Quick Tip: The only case where the centre of pressure coincides with the centroid is when the surface is horizontal ($\theta = 0^\circ$), as the pressure is uniform across the entire area.

179. A U-tube manometer is commonly used to measure ____.

- (1) The velocity of fluid flow
- (2) The pressure difference between two points
- (3) The temperature of a fluid
- (4) The density of a solid

Correct Answer: (2) The pressure difference between two points

Solution:**Step 1: Understanding the Concept:**

A manometer is a device used to measure pressure based on the displacement of a liquid column.

Step 2: Detailed Explanation:

1. A Simple U-tube Manometer measures the pressure at a single point relative to the atmosphere. 2. A Differential U-tube Manometer connects two different points (or two different pipes) to each end of the U-tube. The difference in the height of the manometric liquid represents the pressure difference between those two points. 3. Velocity is measured by a Pitot tube or Venturimeter. 4. Temperature is measured by a thermometer.

Step 3: Final Answer:

A U-tube manometer is used to measure the pressure difference between two points.

Quick Tip: The manometric liquid (usually mercury) must be immiscible with the fluid being measured and should have a higher density to keep the instrument compact.

180. Steady flow occurs when ____.

- (1) fluid parameters change steadily with time
- (2) velocity of flow at any point remains constant
- (3) fluid parameters do not change with time at any point
- (4) fluid parameters do not change from point to point

Correct Answer: (3) fluid parameters do not change with time at any point

Solution:**Step 1: Understanding the Concept:**

Fluid flow is categorized as steady or unsteady based on how the flow properties (velocity, pressure, density, etc.) behave with respect to time at a specific fixed location.

Step 2: Detailed Explanation:

1. **Steady Flow:** Properties do not change with time. Mathematically: $\frac{\partial v}{\partial t} = 0, \frac{\partial p}{\partial t} = 0, \frac{\partial \rho}{\partial t} = 0$.
2. **Unsteady Flow:** Properties change with time ($\frac{\partial v}{\partial t} \neq 0$).
3. **Uniform Flow:** Properties do not change from point to point (with respect to distance) at a given instant.

Step 3: Final Answer:

Steady flow occurs when fluid parameters do not change with time at any point.

Quick Tip: Don't confuse "Steady" with "Uniform." Steady is about **Time** (t), while Uniform is about **Space/Position** (s).

181. In Bernoulli's equation, each term represents ____.

- (1) Specific energy
- (2) Energy per unit mass
- (3) Energy per unit volume
- (4) Energy per unit weight

Correct Answer: (4) Energy per unit weight

Solution:

Step 1: Understanding the Concept:

Bernoulli's equation is a statement of the principle of conservation of energy for flowing fluids. It is usually expressed in terms of "Head," which has the dimension of length (L).

Step 2: Key Formula or Approach:

The standard Bernoulli's equation is:

$$\frac{P}{\rho g} + \frac{v^2}{2g} + z = \text{Constant}$$

Where: $-\frac{P}{\rho g}$ is Pressure Head - $\frac{v^2}{2g}$ is Velocity Head - z is Datum Head

Step 3: Detailed Explanation:

1. Each term in the equation above has the unit of meters (m).
2. "Meter" in this context

represents "Newton-meter per Newton" ($N \cdot m/N$), which is equivalent to Energy (Work) divided by Weight. 3. Therefore, each term represents energy per unit weight of the fluid. 4. If the equation is multiplied by ρg , each term would then represent energy per unit volume (N/m^2).

Step 4: Final Answer:

Each term represents energy per unit weight.

Quick Tip: Remember the units: - Head form (z): Energy per unit weight (meters). - Pressure form (P): Energy per unit volume (Pascals). - Energy form ($v^2/2$): Energy per unit mass (J/kg).

182. The minor loss due to sudden contraction is due to ____.

- (1) cavitation
- (2) flow contraction
- (3) boundary friction
- (4) expansion of flow after sudden contraction

Correct Answer: (4) expansion of flow after sudden contraction

Solution:

Step 1: Understanding the Concept:

When a pipe suddenly narrows, the fluid flow does not narrow exactly at the corner. It continues to contract further until it reaches a minimum cross-section called the "Vena Contracta."

Step 2: Detailed Explanation:

1. The loss of energy during the actual contraction (from the large pipe to the Vena Contracta) is actually very small. 2. The major part of the energy loss occurs **after** the Vena Contracta, where the flow must suddenly expand again to fill the full diameter of the smaller pipe. 3. This sudden expansion creates turbulence and eddies, which dissipate energy. 4. Hence, the "contraction loss" is physically an "expansion loss" occurring within the smaller pipe.

Step 3: Final Answer:

The minor loss is due to the expansion of flow after sudden contraction.

Quick Tip: The coefficient of contraction (C_c) is vital here. The loss formula is $h_c = \frac{v^2}{2g} \left(\frac{1}{C_c} - 1 \right)^2$. For most practical cases, this is taken as $0.5 \frac{v^2}{2g}$.

183. If the pipes are connected in parallel, the total loss of head is ____.

- (1) the same in each pipe
- (2) equal to sum of head losses in each pipe
- (3) equal to sum of the squares of head losses in each pipe
- (4) equal to the reciprocal of the sum of head losses in each pipe

Correct Answer: (1) the same in each pipe

Solution:**Step 1: Understanding the Concept:**

Pipe connections in fluid mechanics follow the same logic as electrical circuits. Parallel pipes connect the same two points (reservoirs or junctions).

Step 2: Key Formula or Approach:

For parallel pipes (Pipe 1 and Pipe 2): 1. Discharge: $Q_{total} = Q_1 + Q_2$ 2. Head Loss:
 $h_{f1} = h_{f2} = H_{total}$

Step 3: Detailed Explanation:

1. Since both pipes start at point A and end at point B, the energy difference (head loss) between the start and end must be the same, regardless of which path the fluid takes. 2. If one pipe had less resistance, more fluid would flow through it until the friction increased to match the head loss of the other path. 3. This is opposite to pipes in series, where the discharges are the same but the head losses are added.

Step 4: Final Answer:

The total loss of head is the same in each pipe.

Quick Tip: To remember this: Parallel = Same Head (h_f), Different Flow (Q). Series = Same Flow (Q), Different Head (h_f).

184. The tangent drawn to the instantaneous velocity in a flow field is called ____.

- (1) Path line
- (2) Time line
- (3) Stream line
- (4) Streak line

Correct Answer: (3) Stream line

Solution:

Step 1: Understanding the Concept:

In fluid kinematics, different types of lines are used to describe the motion of fluid particles. A stream line is a mathematical concept used to visualize the flow direction at a specific instant.

Step 2: Detailed Explanation:

1. **Stream line:** An imaginary curve drawn through a flowing fluid such that the tangent to it at any point gives the direction of the instantaneous velocity of the fluid particle at that point. 2. **Path line:** The actual path traveled by a single fluid particle over a period of time. 3. **Streak line:** The locus of different fluid particles that have passed through a specific fixed point in space (like smoke from a chimney).

Step 3: Final Answer:

The line whose tangent represents the instantaneous velocity is the stream line.

Quick Tip: In steady flow, stream lines, path lines, and streak lines are all identical. They only differ in unsteady flow conditions.

185. The Reynold's number for a laminar flow through pipes should be ____.

- (1) less than 2000
- (2) less than 4000
- (3) greater than 2000
- (4) between 2000 and 4000

Correct Answer: (1) less than 2000

Solution:

Step 1: Understanding the Concept:

The Reynolds number (Re) is a dimensionless value that predicts flow patterns. It represents the ratio of inertial forces to viscous forces.

Step 2: Detailed Explanation:

For flow through **pipes**, the flow regimes are defined as follows:

- **Laminar Flow:** $Re < 2000$. The fluid particles move in smooth layers or laminae.
- **Transition Flow:** $2000 < Re < 4000$. The flow is unstable and switches between laminar and turbulent.
- **Turbulent Flow:** $Re > 4000$. The fluid particles move in a highly irregular and chaotic manner.

Step 3: Final Answer:

The Reynolds number for laminar flow in pipes must be less than 2000.

Quick Tip: Be careful with the boundaries for different geometries! For flow over a **flat plate**, the critical Reynolds number for transition to turbulence is much higher (typically 5×10^5).

186. The most economical channel section has a ____.

- (1) maximum area for a given flow
- (2) minimum wetted perimeter
- (3) minimum roughness coefficient

(4) maximum velocity of flow

Correct Answer: (2) minimum wetted perimeter

Solution:

Step 1: Understanding the Concept:

An "economical" or "efficient" channel section is one that provides the maximum discharge (Q) for a given cross-sectional area, slope, and roughness.

Step 2: Key Formula or Approach:

According to Chezy's or Manning's formula:

$$Q = A \cdot \frac{1}{n} \cdot R^{2/3} \cdot S^{1/2}$$

Since R (Hydraulic Radius) is A/P , the discharge Q is inversely proportional to the wetted perimeter P .

Step 3: Detailed Explanation:

1. To maximize Q for a constant area A , we must minimize the wetted perimeter P . 2. A smaller wetted perimeter means less frictional resistance from the channel boundaries. 3. Economically, a smaller perimeter requires less lining material (concrete/bricks), reducing construction costs.

Step 4: Final Answer:

The most economical section is characterized by a minimum wetted perimeter.

Quick Tip: The most efficient geometric shape of all is a semi-circle, but for practical construction, a trapezoidal section (specifically half a regular hexagon) is most commonly used.

187. The Chezy's constant ____.

- (1) is dimensionless
- (2) has the dimension of velocity
- (3) has the dimension of $L^{1/2}T^{-1}$
- (4) has the dimension of $L^{3/2}T^{-1}$

Correct Answer: (3) has the dimension of $L^{1/2}T^{-1}$

Solution:

Step 1: Understanding the Concept:

Chezy's formula is used to calculate the velocity of flow in open channels. By analyzing the units of each term in the equation, we can determine the dimensions of the constant (C).

Step 2: Key Formula or Approach:

The Chezy's formula is:

$$v = C \sqrt{RS}$$

Where: - v = Velocity (LT^{-1}) - R = Hydraulic Radius (L) - S = Bed Slope (Dimensionless)

Step 3: Detailed Explanation:

To find the dimensions of C, rearrange the formula:

$$C = \frac{v}{\sqrt{R}}$$

Substituting the dimensions:

$$[C] = \frac{LT^{-1}}{L^{1/2}}$$

$$[C] = L^{1-1/2}T^{-1} = L^{1/2}T^{-1}$$

The unit is typically $m^{1/2}/s$.

Step 4: Final Answer:

Chezy's constant has the dimension of $L^{1/2}T^{-1}$.

Quick Tip: Chezy's constant is NOT dimensionless. This distinguishes it from the Friction Factor (f) used in the Darcy-Weisbach equation, which is truly dimensionless.

188. If the depth of flow in a rectangular notch is H, then the discharge is ____.

- (1) directly proportional to $H^{3/2}$
- (2) directly proportional to $H^{5/2}$

(3) inversely proportional to $H^{3/2}$

(4) inversely proportional to $H^{5/2}$

Correct Answer: (1) directly proportional to $H^{3/2}$

Solution:

Step 1: Understanding the Concept:

Notches are used to measure the flow rate (discharge) in small channels. The relationship between the depth of water above the sill (H) and the discharge (Q) depends on the shape of the notch.

Step 2: Key Formula or Approach:

For a rectangular notch:

$$Q = \frac{2}{3} C_d \cdot L \cdot \sqrt{2g} \cdot H^{3/2}$$

Step 3: Detailed Explanation:

1. In the discharge formula, C_d , L , and g are constant for a specific setup. 2. The term H is raised to the power of $3/2$. 3. Therefore, $Q \propto H^{3/2}$.

Step 4: Final Answer:

The discharge is directly proportional to $H^{3/2}$.

Quick Tip: Remember the powers: Rectangular notch = $H^{3/2}$, Triangular (V-notch) = $H^{5/2}$. The V-notch is more sensitive to low flow rates because of this higher power.

189. The priming of a centrifugal pump is necessary to ____.

(1) reduce pressure

(2) increase discharge

(3) reduce the temperature of pump

(4) remove air from the parts of pump

Correct Answer: (4) remove air from the parts of pump

Solution:

Step 1: Understanding the Concept:

Centrifugal pumps work by transferring kinetic energy to a liquid via a rotating impeller. However, these pumps are unable to create a vacuum strong enough to lift water if the casing is full of air.

Step 2: Detailed Explanation:

1. The pressure head developed by a centrifugal pump is proportional to the density of the fluid being pumped. 2. Air is about 800 times less dense than water. If the pump is full of air, the pressure rise generated is negligible and cannot lift water from the source. 3. Priming is the process of filling the suction pipe, casing, and part of the delivery pipe with the liquid to be pumped, effectively removing all air pockets.

Step 3: Final Answer:

Priming is necessary to remove air from the parts of the pump.

Quick Tip: Self-priming pumps exist, but standard centrifugal pumps must always be primed before starting, otherwise, they will run "dry" and may be damaged.

190. Which of the following is an impulse turbine? ____.

- (1) Pelton wheel
- (2) Kaplan turbine
- (3) Francis turbine
- (4) Centrifugal pump

Correct Answer: (1) Pelton wheel

Solution:

Step 1: Understanding the Concept:

Hydraulic turbines are classified into two main types: Impulse and Reaction, based on how the water's energy is converted into mechanical work.

Step 2: Detailed Explanation:

1. **Impulse Turbine:** All available energy (head) is converted into kinetic energy via a nozzle before the water hits the runner. The runner operates at atmospheric pressure. The **Pelton wheel** is the classic example, used for high heads and low discharge. 2. **Reaction Turbine:** Only part of the energy is converted to kinetic energy; the rest remains as pressure energy. The pressure changes as the water moves through the runner. Examples include Francis and Kaplan turbines. 3. **Centrifugal pump:** This is not a turbine; it is a machine that consumes power to move fluid.

Step 3: Final Answer:

The Pelton wheel is an impulse turbine.

Quick Tip: Pelton = High Head; Francis = Medium Head; Kaplan = Low Head. This is the most common way these are categorized in exams.

191. Precipitation in the form of irregular pellets or lumps of ice of size more than 8 mm is known as ____.

- (1) Fog
- (2) Glaze
- (3) Sleet
- (4) Hail

Correct Answer: (4) Hail

Solution:**Step 1: Understanding the Concept:**

Precipitation occurs in various forms depending on the temperature and atmospheric conditions during the fall. These forms are classified by their physical state (liquid, solid, or mixed) and size.

Step 2: Detailed Explanation:

1. **Hail:** Showery precipitation in the form of irregular pellets or lumps of ice with a diameter

of 8 mm or more. They are produced in convective clouds (cumulonimbus). 2. **Sleet:** Frozen raindrops or transparent grains of ice which form when rain falls through a layer of freezing air near the ground. Size is usually less than 5 mm. 3. **Glaze:** A coating of ice, generally clear and smooth, formed on exposed objects by the freezing of a film of supercooled water (freezing rain). 4. **Fog:** A suspension of very small water droplets in the air, reducing visibility to less than 1 km.

Step 3: Final Answer:

Precipitation as ice lumps larger than 8 mm is known as Hail.

Quick Tip: Size is the key differentiator: Sleet is small (≤ 5 mm), whereas Hail is large (> 8 mm). Hail is also associated specifically with thunderstorms and strong updrafts.

192. For a catchment, the duration and intensity of rainfall are 2 hours and 3 cm/hour respectively. If the infiltration and evaporation losses are 16 mm and 8 mm respectively, then the runoff of catchment is ____.

- (1) 68 mm
- (2) 36 mm
- (3) 28 mm
- (4) 12 mm

Correct Answer: (2) 36 mm

Solution:

Step 1: Understanding the Concept:

Runoff is the portion of precipitation that makes its way towards stream channels after all losses (abstractions) have been subtracted from the total rainfall.

Step 2: Key Formula or Approach:

$$\text{Runoff}(R) = \text{Total Rainfall}(P) - \text{Total Losses}(L)$$

Step 3: Detailed Explanation:

1. Calculate Total Rainfall (P): - Intensity = 3 cm/hr = 30 mm/hr - Duration = 2 hours - $P = 30 \times 2 = 60$ mm
2. Calculate Total Losses (L): - Infiltration = 16 mm - Evaporation = 8 mm - $L = 16 + 8 = 24$ mm
3. Calculate Runoff (R): - $R = 60 - 24 = 36$ mm

Step 4: Final Answer:

The runoff of the catchment is 36 mm.

Quick Tip: Always ensure all units are consistent (convert cm to mm or vice versa) before performing subtractions in hydrology problems.

193. The flood absorption capacity of a reservoir is ____.

- (1) equal to the dead storage
- (2) the water it can absorb through seepage
- (3) the storage between dead storage level and full supply level
- (4) the storage between full supply level and maximum water level

Correct Answer: (4) the storage between full supply level and maximum water level

Solution:**Step 1: Understanding the Concept:**

Reservoirs are divided into different zones of storage to manage water for various purposes like irrigation, power, and flood control.

Step 2: Detailed Explanation:

1. **Dead Storage:** Water below the lowest outlet level, which cannot be discharged. 2. **Live Storage (Effective Storage):** Storage between the dead storage level and the Full Reservoir Level (FRL) or Full Supply Level (FSL). 3. **Surcharge Storage (Flood Absorption Capacity):** The space provided above the FSL up to the Maximum Water Level (MWL). This space is used to temporarily "absorb" the peak of a flood, which is then gradually released through the

spillway.

Step 3: Final Answer:

Flood absorption capacity is the storage between full supply level and maximum water level.

Quick Tip: Surge storage is "temporary" storage. Unlike live storage, it cannot be retained for long periods and must be evacuated as soon as the flood passes to keep the dam safe.

194. Which method of irrigation has the highest irrigation efficiency? ____.

- (1) Furrow irrigation
- (2) Sprinkler irrigation
- (3) Drip irrigation
- (4) Border strip irrigation

Correct Answer: (3) Drip irrigation

Solution:

Step 1: Understanding the Concept:

Irrigation efficiency refers to the ratio of water effectively used by the plants to the total water diverted from the source. High efficiency implies minimal losses due to evaporation, runoff, or deep percolation.

Step 2: Detailed Explanation:

1. **Surface Irrigation (Furrow/Border):** These methods involve flooding the soil surface. They have lower efficiency (40-60%) due to high evaporation and deep percolation losses. 2. **Sprinkler Irrigation:** Water is sprayed as rain. It is more efficient (60-80%) but prone to wind drift and evaporation. 3. **Drip (Trickle) Irrigation:** Water is delivered directly to the root zone of the plant through a network of pipes and emitters. Since only the root zone is wetted and water is applied slowly, evaporation and runoff are almost eliminated, leading to the highest efficiency (> 90%).

Step 3: Final Answer:

Drip irrigation has the highest irrigation efficiency.

Quick Tip: Drip irrigation is also known as "localized irrigation" because it keeps the soil moisture near the root zone constantly at an optimum level, which also saves a significant amount of water compared to all other methods.

195. The saturation capacity, field capacity, optimum moisture content and permanent wilting point of a soil are 47%, 27%, 18% and 14% respectively. The available moisture for the crops grown in the soil is ____.

- (1) 4%
- (2) 13%
- (3) 20%
- (4) 33%

Correct Answer: (2) 13%

Solution:

Step 1: Understanding the Concept:

Plants can only extract water that is held in the soil between two specific limits: the maximum amount the soil can hold against gravity (Field Capacity) and the point where the plant can no longer pull water (Permanent Wilting Point).

Step 2: Key Formula or Approach:

$$\text{Available Moisture} = \text{Field Capacity}(FC) - \text{Permanent Wilting Point}(PWP)$$

Step 3: Detailed Explanation:

Given: - Field Capacity (FC) = 27% - Permanent Wilting Point (PWP) = 14% - Saturation Capacity and Optimum Moisture are given as distractors for this specific calculation. 1. Apply

the formula:

$$\text{Available Moisture} = 27\% - 14\% = 13\%$$

Step 4: Final Answer:

The available moisture for the crops is 13%.

Quick Tip: "Readily Available Moisture" is different; it is the difference between Field Capacity and Optimum Moisture Content. Always check if the question asks for "Available" or "Readily Available" moisture.

196. The actual depth of watering in the field is 16 cm, conveyance losses are 20% and irrigation efficiency is 80%. The depth of water required at the canal outlet is ____.

- (1) 16 cm
- (2) 20 cm
- (3) 25 cm
- (4) 100 cm

Correct Answer: (3) 25 cm

Solution:

Step 1: Understanding the Concept:

To find the water required at the source (canal outlet), we must account for all losses that occur during application in the field and during transport (conveyance) from the canal to the field.

Step 2: Key Formula or Approach:

1. Depth at Field Head (D_f) = $\frac{\text{Depth in Field}}{\text{Application Efficiency}}$ 2. Depth at Canal Outlet (D_c) = $\frac{D_f}{\text{Conveyance Efficiency}}$

Step 3: Detailed Explanation:

Given: - Depth in field = 16 cm - Application Efficiency (η_a) = 80% = 0.80 - Conveyance losses = 20% → Conveyance Efficiency (η_c) = 1 - 0.20 = 80% = 0.80 1. Calculate depth

needed at the entrance of the field:

$$D_f = \frac{16}{0.80} = 20 \text{ cm}$$

2. Calculate depth needed at the canal outlet:

$$D_c = \frac{20}{0.80} = 25 \text{ cm}$$

Step 4: Final Answer:

The depth of water required at the canal outlet is 25 cm.

Quick Tip: A loss of 20% means you only have 80% (0.8) of the water left. To find the starting amount, you must divide by the efficiency, not multiply by the loss percentage.

197. For lined canals, the free board is measured from the ____.

- (1) top of the lining to the bed of the canal
- (2) full supply level to the top of the lining
- (3) full supply level to the top of the bank
- (4) top of the bank to the top of the lining

Correct Answer: (2) full supply level to the top of the lining

Solution:

Step 1: Understanding the Concept:

Freeboard is the vertical distance provided above the design water level to prevent overtopping due to waves, fluctuations in water level, or sudden increases in discharge.

Step 2: Detailed Explanation:

1. In an unlined canal, the freeboard is usually measured from the Full Supply Level (FSL) to the top of the embankment (bank). 2. For **lined canals**, the lining must extend above the

water surface to protect the banks from erosion. 3. According to IS: 10430, the freeboard for lined canals is the vertical distance between the FSL and the top of the lining. This ensures that even with minor surges, the water remains within the protected (lined) section.

Step 3: Final Answer:

The free board is measured from the full supply level to the top of the lining.

Quick Tip: For small canals (discharge < 0.75 cumecs), the freeboard is typically 0.3 m, while for very large canals (> 85 cumecs), it can be 0.75 m or more.

198. A cross-drainage work in which a canal is provided above the natural drainage is called ____.

- (1) an aqueduct
- (2) a canal syphon
- (3) a level crossing
- (4) a super passage

Correct Answer: (1) an aqueduct

Solution:

Step 1: Understanding the Concept:

Cross-drainage works are structures constructed where a canal crosses a natural stream or drain. They are classified based on the relative levels of the canal and the drain.

Step 2: Detailed Explanation:

1. **Aqueduct:** The canal is carried **over** the drain. The drain water flows under gravity and the bed of the canal is well above the High Flood Level (HFL) of the drain. 2. **Syphon Aqueduct:** Similar to an aqueduct, but the drain water flows under pressure (syphonic action) because the drain's HFL is higher than the canal bed. 3. **Super Passage:** The drain is carried **over** the canal. 4. **Level Crossing:** The canal and drain meet at the **same level** and their waters are allowed to mix.

Step 3: Final Answer:

A work where the canal is above the drainage is an aqueduct.

Quick Tip: To remember: Aqueduct = Above (Canal is above). Super Passage = Stream is above.

199. The sloping floor below and in continuation of the raised crest of a weir is known as ____.

- (1) Downstream fall
- (2) Downstream slope
- (3) Downstream grade
- (4) Glacis

Correct Answer: (4) Glacis

Solution:**Step 1: Understanding the Concept:**

A weir or a canal fall often features a sloping surface that allows water to transition from a higher elevation (crest) to a lower elevation (cistern) while controlling energy dissipation.

Step 2: Detailed Explanation:

1. The Crest is the highest part of the weir. 2. The Glacis is the sloping floor that connects the crest to the downstream floor. 3. The purpose of the glacis is to allow the formation of a hydraulic jump, which is a highly effective way to dissipate the kinetic energy of the falling water, thereby protecting the downstream bed from scouring.

Step 3: Final Answer:

The sloping floor is known as the Glacis.

Quick Tip: A "Glacis fall" is considered one of the best types of canal falls because the hydraulic jump formed on the slope is very stable and efficient at killing the excess energy of the water.

200. The function of a check dam is to ____.

- (1) protect surrounding areas of a river from floods
- (2) reduce the runoff velocity in stream channels
- (3) arrest sediments and the surface runoff in agricultural lands
- (4) dispose of excess water from the reservoir to protect dam from overturning

Correct Answer: (2) reduce the runoff velocity in stream channels

Solution:

Step 1: Understanding the Concept:

A check dam is a small, often temporary, dam constructed across a swale, drainage ditch, or waterway. Unlike large storage dams, its primary purpose is related to flow control and soil conservation.

Step 2: Detailed Explanation:

1. By placing a barrier in the channel, the effective slope of the channel is reduced. 2. This reduction in slope leads to a reduction in runoff velocity. 3. Slower water has less erosive power, which helps prevent gully erosion and allows sediments to settle out. 4. It also promotes groundwater recharge by giving the water more time to infiltrate into the soil.

Step 3: Final Answer:

The function of a check dam is to reduce the runoff velocity in stream channels.

Quick Tip: Check dams are a key component of "Watershed Management." They are often made of locally available materials like stones (Gully plugs), timber, or even bags of sand.