

AP ECET 2025 Civil Engineering Question Paper with Solutions

Time Allowed :3 Hour	Maximum Marks :200	Total Questions :200
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General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hour duration.
2. The question paper consists of 200 questions. The maximum marks are 200.
3. The question paper is divided into four sections - Mathematics, Physics, Chemistry and Civil Engineering.

[Mathematics]

1. If the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, then which of the following is true?

- (A) The matrix is invertible
- (B) The matrix is singular
- (C) The matrix is diagonalizable
- (D) The matrix is symmetric

Correct Answer: (B) The matrix is singular

Solution:

Step 1: Understanding the Question:

We are given a 3x3 matrix A and asked to determine its properties from the given options. The key properties to check are whether the matrix is singular, invertible, or symmetric.

Step 2: Key Formula or Approach:

A matrix is **singular** if its determinant is zero ($\det(A) = 0$).

A matrix is **invertible** (or non-singular) if its determinant is non-zero ($\det(A) \neq 0$).

A matrix is **symmetric** if it is equal to its transpose ($A = A^T$).

We will calculate the determinant of A to check if it is singular or invertible.

Step 3: Detailed Explanation:

The given matrix is:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Let's calculate the determinant of A:

$$\det(A) = 1 \begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} - 2 \begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} + 3 \begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix}$$

$$\det(A) = 1(5 \times 9 - 6 \times 8) - 2(4 \times 9 - 6 \times 7) + 3(4 \times 8 - 5 \times 7)$$

$$\det(A) = 1(45 - 48) - 2(36 - 42) + 3(32 - 35)$$

$$\det(A) = 1(-3) - 2(-6) + 3(-3)$$

$$\det(A) = -3 + 12 - 9$$

$$\det(A) = 0$$

Since the determinant of A is 0, the matrix is singular.

This also means the matrix is not invertible.

Now, let's check if the matrix is symmetric.

The transpose of A is:

$$A^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

Since $A \neq A^T$, the matrix is not symmetric.

Step 4: Final Answer:

The determinant of the matrix A is 0. Therefore, the matrix is singular.

💡 Quick Tip

For a 3x3 matrix with elements in an arithmetic progression like this one, the determinant is always zero. Notice that the elements in each row (1,2,3), (4,5,6), (7,8,9) and each column (1,4,7), (2,5,8), (3,6,9) are in AP. A property of determinants states that if we perform the operation $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_2$, the new columns will be identical, making the determinant zero.

2. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and the determinant of A is 5, then determinant of the matrix $2A$ is

- (A) 10
- (B) 20
- (C) 5
- (D) 25

Correct Answer: (B) 20

Solution:

Step 1: Understanding the Question:

We are given a 2x2 matrix A with its determinant equal to 5.

We need to find the determinant of the matrix $2A$, which is obtained by multiplying every element of A by the scalar 2.

Step 2: Key Formula or Approach:

For a square matrix A of order n and a scalar k , the determinant of the matrix kA is given by the property:

$$\det(kA) = k^n \det(A)$$

Step 3: Detailed Explanation:

We are given the following information:

The matrix A is of order $n = 2$.

The determinant of A is $\det(A) = 5$.

We need to find the determinant of the matrix $2A$.

Here, the scalar k is 2.

Using the formula from Step 2:

$$\det(2A) = 2^n \det(A)$$

Substitute the values $n = 2$ and $\det(A) = 5$:

$$\det(2A) = 2^2 \times 5$$

$$\det(2A) = 4 \times 5$$

$$\det(2A) = 20$$

Step 4: Final Answer:

The determinant of the matrix $2A$ is 20.

💡 Quick Tip

Remember that when a matrix is multiplied by a scalar k , every element gets multiplied by k . When calculating the determinant, each of the n rows (or columns) has a common factor of k , which can be taken out. This results in the factor k being taken out n times, leading to the formula $\det(kA) = k^n \det(A)$. This is a common source of error where students might mistakenly think $\det(kA) = k \det(A)$.

3. If the matrix A is of order 3×3 and the system of equations $AX = B$ has a unique solution, what can be concluded about the determinant of A ?

- (A) The determinant of A is zero
- (B) The determinant of A is non-zero
- (C) The determinant of A must be 1 only
- (D) The determinant of A cannot be negative

Correct Answer: (B) The determinant of A is non-zero

Solution:

Step 1: Understanding the Question:

We are dealing with a system of linear equations represented in matrix form as $AX = B$, where A is a 3×3 coefficient matrix.

The question states that this system has a unique solution and asks about the property of the determinant of matrix A .

Step 2: Key Formula or Approach:

According to the Cramer's rule and matrix inversion method for solving systems of linear equations, a system $AX = B$ has a unique solution if and only if the coefficient matrix A is invertible (non-singular).

A matrix is invertible if and only if its determinant is non-zero.

Step 3: Detailed Explanation:

The system of linear equations is given by $AX = B$.

If the matrix A is invertible, we can find its inverse, A^{-1} .

Multiplying the equation by A^{-1} on the left, we get:

$$A^{-1}(AX) = A^{-1}B$$

$$(A^{-1}A)X = A^{-1}B$$

$$IX = A^{-1}B$$

$$X = A^{-1}B$$

This equation gives a unique solution for the variable matrix X.

The condition for the existence of A^{-1} is that the determinant of A must be non-zero ($\det(A) \neq 0$).

If $\det(A) = 0$, the matrix A is singular, and its inverse does not exist. In this case, the system of equations will have either no solution or infinitely many solutions, but not a unique solution.

Since the problem states that the system has a unique solution, it is necessary that the determinant of A is non-zero.

The value can be any non-zero real number; it is not restricted to be 1 or only positive.

Step 4: Final Answer:

For the system of equations $AX = B$ to have a unique solution, the determinant of the coefficient matrix A must be non-zero.

💡 Quick Tip

For a system of linear equations $AX = B$:

- If $\det(A) \neq 0$, there is a unique solution.
- If $\det(A) = 0$ and $(\text{adj } A)B \neq 0$, there is no solution (inconsistent system).
- If $\det(A) = 0$ and $(\text{adj } A)B = 0$, there are infinitely many solutions (consistent system).

This summary is crucial for solving problems related to the nature of solutions of linear equations.

4. If $A = \begin{bmatrix} x & 3 \\ 2 & 4 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} -2 & 1.5 \\ 1 & -0.5 \end{bmatrix}$, then the value of x is

- (A) -2
(B) 1

- (C) 1.5
(D) -0.5

Correct Answer: (B) 1

Solution:

Step 1: Understanding the Question:

We are given a 2x2 matrix A containing an unknown variable x , and its inverse matrix A^{-1} . We need to find the value of x .

Step 2: Key Formula or Approach:

The product of a matrix and its inverse is the identity matrix, i.e., $AA^{-1} = I$.

For a 2x2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, its inverse is given by $A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

We can either use the property $AA^{-1} = I$ or calculate the inverse of A and compare it with the given A^{-1} .

Step 3: Detailed Explanation:

Method 1: Using the formula for the inverse

First, calculate the determinant of A:

$$\det(A) = (x)(4) - (3)(2) = 4x - 6$$

Now, find the inverse of A using the formula:

$$A^{-1} = \frac{1}{4x - 6} \begin{bmatrix} 4 & -3 \\ -2 & x \end{bmatrix} = \begin{bmatrix} \frac{4}{4x-6} & \frac{-3}{4x-6} \\ \frac{-2}{4x-6} & \frac{x}{4x-6} \end{bmatrix}$$

We are given:

$$A^{-1} = \begin{bmatrix} -2 & 1.5 \\ 1 & -0.5 \end{bmatrix}$$

By comparing the corresponding elements of the calculated inverse and the given inverse, we can set up equations. Let's compare the element in the first row, first column:

$$\frac{4}{4x - 6} = -2$$

$$4 = -2(4x - 6)$$

$$4 = -8x + 12$$

$$8x = 12 - 4$$

$$8x = 8$$

$$x = 1$$

Method 2: Using $AA^{-1} = I$

$$AA^{-1} = \begin{bmatrix} x & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 1.5 \\ 1 & -0.5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Let's compute the element in the first row, first column of the product:

$$(x)(-2) + (3)(1) = 1$$

$$-2x + 3 = 1$$

$$-2x = 1 - 3$$

$$-2x = -2$$

$$x = 1$$

Both methods yield the same result.

Step 4: Final Answer:

The value of x is 1.

💡 Quick Tip

Using the property $AA^{-1} = I$ is often faster than calculating the inverse from scratch. You only need to calculate one or two elements of the product matrix to form an equation and solve for the unknown, saving valuable time in an exam.

5. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$, then $(AB)^T =$

(A) $\begin{bmatrix} 0 & 3 \\ 0 & 4 \end{bmatrix}$

(B) $\begin{bmatrix} 0 & 3 \\ 0 & 7 \end{bmatrix}$

(C) $\begin{bmatrix} 3 & 7 \\ 0 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 3 & 0 \\ 6 & 0 \end{bmatrix}$

Correct Answer: (C) $\begin{bmatrix} 3 & 7 \\ 0 & 0 \end{bmatrix}$

Solution:

Step 1: Understanding the Question:

We are given two 2x2 matrices, A and B.

We need to find the transpose of their product, $(AB)^T$.

Step 2: Key Formula or Approach:

There are two ways to solve this:

1. First, calculate the product matrix $C = AB$. Then, find the transpose of C, which is C^T .
2. Use the property of transpose: $(AB)^T = B^T A^T$. First, find the transposes of A and B, then multiply them in reverse order.

We will use the first method as it is more direct.

Step 3: Detailed Explanation:

Step 3a: Calculate the product AB

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} (1)(1) + (2)(1) & (1)(0) + (2)(0) \\ (3)(1) + (4)(1) & (3)(0) + (4)(0) \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 + 2 & 0 + 0 \\ 3 + 4 & 0 + 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 3 & 0 \\ 7 & 0 \end{bmatrix}$$

Step 3b: Find the transpose of AB

The transpose of a matrix is found by interchanging its rows and columns.

Let $C = AB = \begin{bmatrix} 3 & 0 \\ 7 & 0 \end{bmatrix}$.
Then the transpose of C is:

$$C^T = (AB)^T = \begin{bmatrix} 3 & 7 \\ 0 & 0 \end{bmatrix}$$

Step 4: Final Answer:

The resulting matrix $(AB)^T$ is $\begin{bmatrix} 3 & 7 \\ 0 & 0 \end{bmatrix}$.

💡 Quick Tip

Remember the "reversal law" for the transpose of a product: $(AB)^T = B^T A^T$. This property is very useful and extends to more matrices, e.g., $(ABC)^T = C^T B^T A^T$. While direct multiplication worked well here, knowing this rule is essential for more complex problems.

6. If $\frac{2x+5}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$, then $A+B =$

- (A) -2
- (B) 2
- (C) 1
- (D) -1

Correct Answer: (B) 2

Solution:

Step 1: Understanding the Question:

The problem asks us to find the sum of the constants A and B in the partial fraction decomposition of the given rational expression.

Step 2: Key Formula or Approach:

To find the values of A and B, we first combine the terms on the right-hand side and then equate the numerators.

$$\frac{A}{x-1} + \frac{B}{x+3} = \frac{A(x+3) + B(x-1)}{(x-1)(x+3)}$$

So, we have the identity:

$$2x + 5 = A(x + 3) + B(x - 1)$$

We can find A and B by substituting strategic values for x (the "cover-up" method) or by

comparing coefficients.

Step 3: Detailed Explanation:

We start with the identity:

$$2x + 5 = A(x + 3) + B(x - 1)$$

Method 1: Cover-up Method

To find A, we substitute $x = 1$ to make the B term zero:

$$2(1) + 5 = A(1 + 3) + B(1 - 1)$$

$$7 = A(4) + B(0)$$

$$7 = 4A \implies A = \frac{7}{4}$$

To find B, we substitute $x = -3$ to make the A term zero:

$$2(-3) + 5 = A(-3 + 3) + B(-3 - 1)$$

$$-6 + 5 = A(0) + B(-4)$$

$$-1 = -4B \implies B = \frac{1}{4}$$

Now, we calculate the sum A+B:

$$A + B = \frac{7}{4} + \frac{1}{4} = \frac{8}{4} = 2$$

Method 2: Comparing Coefficients

Expand the right side of the identity:

$$2x + 5 = Ax + 3A + Bx - B$$

$$2x + 5 = (A + B)x + (3A - B)$$

Now, compare the coefficients of x and the constant terms on both sides.
Comparing coefficients of x :

$$A + B = 2$$

Comparing constant terms:

$$3A - B = 5$$

From the first equation, we directly get the required value $A + B = 2$. We don't even need to solve for A and B individually.

Step 4: Final Answer:

The value of A+B is 2.

💡 Quick Tip

When asked for a sum or combination of the constants (like A+B), always try the method of comparing coefficients first. As seen in Method 2, comparing the coefficients of the highest power of the variable (in this case, x) can directly give the answer without needing to calculate the individual values of A and B. This is a very efficient exam strategy.

7. If $\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$, then the values of (A, B, C) are

- (A) (1, -5, 4)
- (B) (1, 5, 4)
- (C) (4, 5, 1)
- (D) (1, 4, 5)

Correct Answer: (A) (1, -5, 4)

Solution:

Step 1: Understanding the Question:

We need to find the values of the constants A, B, and C in the partial fraction decomposition of the given rational function.

Step 2: Key Formula or Approach:

The problem is based on partial fraction decomposition. We start by setting up the identity:

$$3x - 1 = A(x - 2)(x - 3) + B(x - 1)(x - 3) + C(x - 1)(x - 2)$$

The most efficient way to find A, B, and C is the "cover-up" method, where we substitute the roots of the denominator ($x = 1, x = 2, x = 3$) into this identity.

Step 3: Detailed Explanation:

We use the identity: $3x - 1 = A(x - 2)(x - 3) + B(x - 1)(x - 3) + C(x - 1)(x - 2)$.

To find A, set $x = 1$:

Substitute $x = 1$ into the identity. The terms with B and C will become zero.

$$3(1) - 1 = A(1 - 2)(1 - 3) + B(0) + C(0)$$

$$2 = A(-1)(-2)$$

$$2 = 2A$$

$$A = 1$$

To find B, set $x = 2$:

Substitute $x = 2$ into the identity. The terms with A and C will become zero.

$$3(2) - 1 = A(0) + B(2 - 1)(2 - 3) + C(0)$$

$$6 - 1 = B(1)(-1)$$

$$5 = -B$$

$$B = -5$$

To find C, set $x = 3$:

Substitute $x = 3$ into the identity. The terms with A and B will become zero.

$$3(3) - 1 = A(0) + B(0) + C(3 - 1)(3 - 2)$$

$$9 - 1 = C(2)(1)$$

$$8 = 2C$$

$$C = 4$$

Step 4: Final Answer:

The values are $A = 1$, $B = -5$, and $C = 4$. So, $(A, B, C) = (1, -5, 4)$.

💡 Quick Tip

The "cover-up" method is extremely fast for finding coefficients in partial fractions when the denominator has distinct linear factors. To find the coefficient A for a term $\frac{A}{x-a}$, "cover-up" the $(x - a)$ factor in the original fraction's denominator and substitute $x = a$ into the rest of the expression. For example, to find B : cover up $(x - 2)$ in $\frac{3x-1}{(x-1)(x-2)(x-3)}$ to get $\frac{3x-1}{(x-1)(x-3)}$, and substitute $x = 2$: $\frac{3(2)-1}{(2-1)(2-3)} = \frac{5}{(1)(-1)} = -5$.

8. If $\sin \theta = \frac{3}{5}$, then $\cos \theta =$

(A) $\frac{4}{5}$ but not $-\frac{4}{5}$

(B) $\frac{4}{5}$ or $-\frac{4}{5}$

(C) $-\frac{4}{5}$ but not $\frac{4}{5}$

(D) $\frac{3}{5}$ but not $-\frac{3}{5}$

Correct Answer: (B) $\frac{4}{5}$ or $-\frac{4}{5}$

Solution:

Step 1: Understanding the Question:

We are given the value of $\sin \theta$ and asked to find the possible values of $\cos \theta$. The question does not specify the quadrant in which θ lies.

Step 2: Key Formula or Approach:

The fundamental trigonometric identity relating sine and cosine is:

$$\sin^2 \theta + \cos^2 \theta = 1$$

We will use this identity to solve for $\cos \theta$.

Step 3: Detailed Explanation:

We are given $\sin \theta = \frac{3}{5}$.

Substitute this value into the Pythagorean identity:

$$\left(\frac{3}{5}\right)^2 + \cos^2 \theta = 1$$

$$\frac{9}{25} + \cos^2 \theta = 1$$

Now, solve for $\cos^2 \theta$:

$$\cos^2 \theta = 1 - \frac{9}{25}$$

$$\cos^2 \theta = \frac{25 - 9}{25}$$

$$\cos^2 \theta = \frac{16}{25}$$

Take the square root of both sides to find $\cos \theta$:

$$\cos \theta = \pm \sqrt{\frac{16}{25}}$$

$$\cos \theta = \pm \frac{4}{5}$$

Since the quadrant of θ is not specified, both positive and negative values are possible.

If θ is in the first quadrant, $\cos \theta = \frac{4}{5}$.

If θ is in the second quadrant, $\cos \theta = -\frac{4}{5}$.

Therefore, $\cos \theta$ can be either $\frac{4}{5}$ or $-\frac{4}{5}$.

Step 4: Final Answer:

The possible values for $\cos \theta$ are $\frac{4}{5}$ or $-\frac{4}{5}$.

💡 Quick Tip

Always be careful when taking the square root in trigonometric problems. Unless the quadrant of the angle is specified, you must consider both the positive and negative roots. A positive $\sin \theta$ value means the angle can be in Quadrant I (where cosine is positive) or Quadrant II (where cosine is negative).

9. If $\cos \theta \csc \theta = -1$ and θ lies in the second quadrant then $\cos \theta =$

(A) $-\frac{\sqrt{3}}{2}$

(B) $\frac{\sqrt{2}}{2}$

(C) $-\frac{\sqrt{2}}{2}$

(D) $-\sqrt{2}$

Correct Answer: (C) $-\frac{\sqrt{2}}{2}$

Solution:

Step 1: Understanding the Question:

We are given a trigonometric equation and the quadrant in which the angle θ lies. We need to find the specific value of $\cos \theta$.

Step 2: Key Formula or Approach:

First, simplify the given trigonometric equation using the reciprocal identity for cosecant: $\csc \theta = \frac{1}{\sin \theta}$.

Then, use the simplified equation to determine the value of θ or a trigonometric ratio related to θ .

Finally, use the information about the quadrant to find the correct value of $\cos \theta$.

Step 3: Detailed Explanation:

Start by simplifying the given equation:

$$\cos \theta \csc \theta = -1$$

Substitute $\csc \theta = \frac{1}{\sin \theta}$:

$$\cos \theta \left(\frac{1}{\sin \theta} \right) = -1$$

$$\frac{\cos \theta}{\sin \theta} = -1$$

We know that $\frac{\cos \theta}{\sin \theta} = \cot \theta$.

So, the equation becomes:

$$\cot \theta = -1$$

We need to find the angle θ in the second quadrant for which $\cot \theta = -1$.

The reference angle for $\cot \alpha = 1$ is $\alpha = \frac{\pi}{4}$ or 45° .

In the second quadrant, the angle θ is given by $\theta = \pi - \alpha$ (in radians) or $\theta = 180^\circ - \alpha$ (in degrees).

$$\theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \quad \text{or} \quad \theta = 180^\circ - 45^\circ = 135^\circ$$

Now, we need to find the value of $\cos \theta$ for $\theta = \frac{3\pi}{4}$.

$$\cos\left(\frac{3\pi}{4}\right) = \cos\left(\pi - \frac{\pi}{4}\right)$$

Using the identity $\cos(\pi - x) = -\cos(x)$:

$$\cos\left(\frac{3\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$$

Rationalizing the denominator gives:

$$\cos \theta = -\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

This matches the condition that cosine is negative in the second quadrant.

Step 4: Final Answer:

The value of $\cos \theta$ is $-\frac{\sqrt{2}}{2}$.

💡 Quick Tip

When you find a simple trigonometric ratio like $\cot \theta = -1$, you can quickly visualize the angle. Cotangent is the ratio x/y . For it to be -1, $x = -y$. In the second quadrant, x is negative and y is positive, which fits. This corresponds to the line $y = -x$ in the second quadrant, which makes a 45° angle with the negative x -axis. The coordinates on the unit circle would be $(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$, so $\cos \theta$ is the x -coordinate, which is $-\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$.

10. If $5 \sin \theta = 4$ then the value of $\frac{\csc \theta - \cot \theta}{\csc \theta + \cot \theta}$ is

- (A) $-1/4$
- (B) $-1/2$
- (C) $1/2$
- (D) $1/4$

Correct Answer: (D) $1/4$

Solution:

Step 1: Understanding the Question:

We are given the value of $\sin \theta$ and asked to find the value of a more complex trigonometric expression.

The condition $5 \sin \theta = 4$ implies $\sin \theta = 4/5$, which is positive. So, θ can be in the first or second quadrant.

Step 2: Key Formula or Approach:

First, find the values of $\csc \theta$ and $\cot \theta$ using $\sin \theta = 4/5$.

$$\csc \theta = \frac{1}{\sin \theta}.$$

To find $\cot \theta$, we first need $\cos \theta$ from the identity $\cos^2 \theta = 1 - \sin^2 \theta$. Then $\cot \theta = \frac{\cos \theta}{\sin \theta}$.

A simpler approach is to simplify the given expression first.

Step 3: Detailed Explanation:

Given $\sin \theta = \frac{4}{5}$.

From this, we can immediately find $\csc \theta$:

$$\csc \theta = \frac{1}{\sin \theta} = \frac{1}{4/5} = \frac{5}{4}$$

Next, we find $\cos \theta$:

$$\cos^2 \theta = 1 - \sin^2 \theta = 1 - \left(\frac{4}{5}\right)^2 = 1 - \frac{16}{25} = \frac{9}{25}$$

$$\cos \theta = \pm \sqrt{\frac{9}{25}} = \pm \frac{3}{5}$$

This gives two possible values for $\cot \theta$:

$$\cot \theta = \frac{\cos \theta}{\sin \theta} = \frac{\pm 3/5}{4/5} = \pm \frac{3}{4}$$

Let's evaluate the expression for both cases.

Case 1: θ is in the first quadrant ($\cot \theta = 3/4$)

$$\frac{\csc \theta - \cot \theta}{\csc \theta + \cot \theta} = \frac{5/4 - 3/4}{5/4 + 3/4} = \frac{(5-3)/4}{(5+3)/4} = \frac{2/4}{8/4} = \frac{2}{8} = \frac{1}{4}$$

Case 2: θ is in the second quadrant ($\cot \theta = -3/4$)

$$\frac{\csc \theta - \cot \theta}{\csc \theta + \cot \theta} = \frac{5/4 - (-3/4)}{5/4 + (-3/4)} = \frac{5/4 + 3/4}{5/4 - 3/4} = \frac{8/4}{2/4} = \frac{8}{2} = 4$$

Since $1/4$ is an option and 4 is not, we choose $1/4$. Typically, in such problems without quadrant specification, the angle is assumed to be acute.

Alternative Simplification Method:

The expression can be simplified as follows:

$$\frac{\csc \theta - \cot \theta}{\csc \theta + \cot \theta} = (\csc \theta - \cot \theta) \times \frac{1}{\csc \theta + \cot \theta}$$

Multiply numerator and denominator by $(\csc \theta - \cot \theta)$:

$$\frac{(\csc \theta - \cot \theta)^2}{(\csc \theta + \cot \theta)(\csc \theta - \cot \theta)} = \frac{(\csc \theta - \cot \theta)^2}{\csc^2 \theta - \cot^2 \theta}$$

Using the identity $\csc^2 \theta - \cot^2 \theta = 1$, the expression becomes:

$$(\csc \theta - \cot \theta)^2 = \left(\frac{5}{4} - \left(\pm \frac{3}{4} \right) \right)^2$$

If $\cot \theta = 3/4$, we get $(5/4 - 3/4)^2 = (2/4)^2 = (1/2)^2 = 1/4$.

If $\cot \theta = -3/4$, we get $(5/4 - (-3/4))^2 = (8/4)^2 = 2^2 = 4$.

The answer is $1/4$.

Step 4: Final Answer:

Assuming the standard case of an acute angle, the value of the expression is $1/4$.

💡 Quick Tip

Simplifying the trigonometric expression before substituting values can often make the calculation easier. Here, converting the expression to $(\csc \theta - \cot \theta)^2$ simplifies the problem. Also, recognizing Pythagorean triples (like 3-4-5) can help you quickly find the values of other trig ratios. If $\sin \theta = 4/5$ (Opp/Hyp), then Adjacent must be 3, so $\cos \theta = \pm 3/5$ and $\cot \theta = \pm 3/4$.

11. For real x and if $x + \frac{1}{x} = 2 \cos \theta$ then $\cos \theta$ is

- (A) ± 1
- (B) $1/2$
- (C) 1
- (D) $\pm 1/2$

Correct Answer: (A) ± 1

Solution:

Step 1: Understanding the Question:

We are given an equation that relates a real number x to a trigonometric function $\cos \theta$. We need to find the possible values of $\cos \theta$.

Step 2: Key Formula or Approach:

The problem requires understanding the range of the function $f(x) = x + \frac{1}{x}$ for real x , and the range of the function $g(\theta) = 2 \cos \theta$. The equality can only hold where their ranges overlap.

The range of $f(x) = x + \frac{1}{x}$ can be found using the AM-GM inequality or calculus.

For $x > 0$, by AM-GM, $\frac{x+1/x}{2} \geq \sqrt{x \cdot \frac{1}{x}} \implies x + \frac{1}{x} \geq 2$.

For $x < 0$, let $x = -y$ where $y > 0$. Then $x + \frac{1}{x} = -y - \frac{1}{y} = -(y + \frac{1}{y})$. Since $y + \frac{1}{y} \geq 2$, we have $-(y + \frac{1}{y}) \leq -2$.

So, the range of $x + \frac{1}{x}$ is $(-\infty, -2] \cup [2, \infty)$.

The range of $\cos \theta$ is $[-1, 1]$. Therefore, the range of $2 \cos \theta$ is $[-2, 2]$.

Step 3: Detailed Explanation:

We have the equation:

$$x + \frac{1}{x} = 2 \cos \theta$$

Let's analyze the possible values for both sides of the equation.

The Left-Hand Side (LHS): $y_1 = x + \frac{1}{x}$. As established, for any real $x \neq 0$, the value of y_1 must satisfy $y_1 \geq 2$ or $y_1 \leq -2$. So, $|y_1| \geq 2$.

The Right-Hand Side (RHS): $y_2 = 2 \cos \theta$. Since the range of $\cos \theta$ is $[-1, 1]$, the range of y_2 is $[-2, 2]$. So, $|y_2| \leq 2$.

For the equation to hold, we must have $y_1 = y_2$. The only values that are in both the range of y_1 and the range of y_2 are the boundary points where $|y_1| \geq 2$ and $|y_2| \leq 2$ meet.

This can only happen when the value is exactly 2 or -2.

So, we must have:

$$2 \cos \theta = 2 \quad \text{or} \quad 2 \cos \theta = -2$$

Solving for $\cos \theta$:

$$\cos \theta = 1 \quad \text{or} \quad \cos \theta = -1$$

This can be written concisely as $\cos \theta = \pm 1$.

Step 4: Final Answer:

The only possible values for $\cos \theta$ are 1 and -1.

 Quick Tip

This is a classic problem that tests your knowledge of the ranges of functions. Whenever you see the expression $x + \frac{1}{x}$ for a real number x , immediately recall its range: $(-\infty, -2] \cup [2, \infty)$. Comparing this with the bounded range of a trigonometric function like cosine or sine will quickly lead you to the solution at the boundary points.

12. $\sin^6 \theta + \cos^6 \theta + 3 \sin^2 \theta \cos^2 \theta =$

- (A) 0
- (B) 1
- (C) 2
- (D) -1

Correct Answer: (B) 1

Solution:

Step 1: Understanding the Question:

We need to simplify the given trigonometric expression. The expression is a known identity.

Step 2: Key Formula or Approach:

We will use the algebraic identity $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ or $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$. Let $a = \sin^2 \theta$ and $b = \cos^2 \theta$.

Then the expression $\sin^6 \theta + \cos^6 \theta$ becomes $a^3 + b^3$.

We also know the fundamental trigonometric identity: $\sin^2 \theta + \cos^2 \theta = 1$.

Step 3: Detailed Explanation:

Let's rewrite the expression using $a = \sin^2 \theta$ and $b = \cos^2 \theta$.

The expression is $a^3 + b^3 + 3ab$. This does not immediately simplify. Let's use the second identity from Step 2.

We know that $a + b = \sin^2 \theta + \cos^2 \theta = 1$.

Now, let's use the identity $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$.

Substitute $a = \sin^2 \theta$ and $b = \cos^2 \theta$:

$$\sin^6 \theta + \cos^6 \theta = (\sin^2 \theta + \cos^2 \theta)^3 - 3(\sin^2 \theta)(\cos^2 \theta)(\sin^2 \theta + \cos^2 \theta)$$

Since $\sin^2 \theta + \cos^2 \theta = 1$, this simplifies to:

$$\sin^6 \theta + \cos^6 \theta = (1)^3 - 3 \sin^2 \theta \cos^2 \theta (1)$$

$$\sin^6 \theta + \cos^6 \theta = 1 - 3 \sin^2 \theta \cos^2 \theta$$

Now, substitute this result back into the original expression given in the question:

$$\text{Original Expression} = (\sin^6 \theta + \cos^6 \theta) + 3 \sin^2 \theta \cos^2 \theta$$

$$= (1 - 3 \sin^2 \theta \cos^2 \theta) + 3 \sin^2 \theta \cos^2 \theta$$

$$= 1$$

Step 4: Final Answer:

The value of the expression is 1.

💡 Quick Tip

The identities for $\sin^4 \theta + \cos^4 \theta$ and $\sin^6 \theta + \cos^6 \theta$ are very common in competitive exams. It's useful to remember them:

- $\sin^4 \theta + \cos^4 \theta = 1 - 2 \sin^2 \theta \cos^2 \theta$
- $\sin^6 \theta + \cos^6 \theta = 1 - 3 \sin^2 \theta \cos^2 \theta$

Memorizing these can save you derivation time during an exam.

13. The maximum value of $3 \cos \theta + 4 \sin \theta$ is

- (A) 2
- (B) 4
- (C) 5
- (D) 1

Correct Answer: (C) 5

Solution:

Step 1: Understanding the Question:

We need to find the maximum possible value of the expression $3 \cos \theta + 4 \sin \theta$.

Step 2: Key Formula or Approach:

An expression of the form $a \cos \theta + b \sin \theta$ can be converted into the form $R \cos(\theta - \alpha)$ or $R \sin(\theta + \beta)$.

The range of such an expression is $[-\sqrt{a^2 + b^2}, \sqrt{a^2 + b^2}]$.

The maximum value is $\sqrt{a^2 + b^2}$ and the minimum value is $-\sqrt{a^2 + b^2}$.

Step 3: Detailed Explanation:

The given expression is $3 \cos \theta + 4 \sin \theta$.

This is in the form $a \cos \theta + b \sin \theta$ with $a = 3$ and $b = 4$.

Using the formula, the maximum value is $\sqrt{a^2 + b^2}$.

Calculate the value:

$$\sqrt{3^2 + 4^2} = \sqrt{9 + 16}$$

$$= \sqrt{25}$$

$$= 5$$

The minimum value would be -5. The entire expression ranges from -5 to 5.

Therefore, the maximum value is 5.

Derivation (for understanding):

Let $3 = R \cos \alpha$ and $4 = R \sin \alpha$.

Squaring and adding these gives:

$$3^2 + 4^2 = R^2 \cos^2 \alpha + R^2 \sin^2 \alpha$$

$$25 = R^2 (\cos^2 \alpha + \sin^2 \alpha)$$

$$25 = R^2 \implies R = 5 \text{ (since R is a magnitude).}$$

The original expression becomes:

$$R \cos \alpha \cos \theta + R \sin \alpha \sin \theta = R(\cos \theta \cos \alpha + \sin \theta \sin \alpha)$$

Using the identity $\cos(A - B) = \cos A \cos B + \sin A \sin B$, this is:

$$R \cos(\theta - \alpha) = 5 \cos(\theta - \alpha).$$

Since the maximum value of any cosine function is 1, the maximum value of $5 \cos(\theta - \alpha)$ is $5 \times 1 = 5$.

Step 4: Final Answer:

The maximum value of the expression $3 \cos \theta + 4 \sin \theta$ is 5.

💡 Quick Tip

For any expression of the form $a \sin x + b \cos x$, immediately calculate $\sqrt{a^2 + b^2}$. This value is the maximum value (amplitude) of the resulting single trigonometric function. This is a very frequent question type, so knowing the formula for the maximum and minimum values is a must.

14. If $\sin 5x + \sin 3x + \sin x = 0$ then the value of x other than zero lying between $0 \leq x \leq \frac{\pi}{2}$ is

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{12}$

(D) $\frac{\pi}{4}$

Correct Answer: (B) $\frac{\pi}{3}$

Solution:

Step 1: Understanding the Question:

We need to solve the trigonometric equation $\sin 5x + \sin 3x + \sin x = 0$ for a non-zero value of x in the interval $[0, \frac{\pi}{2}]$.

Step 2: Key Formula or Approach:

We will use the sum-to-product trigonometric formula:

$$\sin A + \sin B = 2 \sin \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right)$$

It's strategic to apply this formula to $\sin 5x$ and $\sin x$ to get a term involving $3x$, which can then be factored with the middle term $\sin 3x$.

Step 3: Detailed Explanation:

The given equation is $\sin 5x + \sin 3x + \sin x = 0$.

Rearrange the terms: $(\sin 5x + \sin x) + \sin 3x = 0$.

Apply the sum-to-product formula to the terms in the parenthesis with $A = 5x$ and $B = x$:

$$\sin 5x + \sin x = 2 \sin \left(\frac{5x+x}{2} \right) \cos \left(\frac{5x-x}{2} \right)$$

$$= 2 \sin \left(\frac{6x}{2} \right) \cos \left(\frac{4x}{2} \right)$$

$$= 2 \sin(3x) \cos(2x)$$

Substitute this back into the equation:

$$2 \sin(3x) \cos(2x) + \sin(3x) = 0$$

Factor out the common term $\sin(3x)$:

$$\sin(3x)(2 \cos(2x) + 1) = 0$$

This gives two possibilities:

- 1) $\sin(3x) = 0$
- 2) $2\cos(2x) + 1 = 0 \implies \cos(2x) = -\frac{1}{2}$

Let's solve each case for x in the interval $[0, \frac{\pi}{2}]$.

Case 1: $\sin(3x) = 0$

The general solution is $3x = n\pi$, where n is an integer. So, $x = \frac{n\pi}{3}$.

For $n = 0$, $x = 0$. (This is a solution, but the question asks for a value other than zero).

For $n = 1$, $x = \frac{\pi}{3}$. This value is in the interval $[0, \frac{\pi}{2}]$.

For $n = 2$, $x = \frac{2\pi}{3}$, which is outside the interval.

So, from this case, we get the solution $x = \frac{\pi}{3}$.

Case 2: $\cos(2x) = -\frac{1}{2}$

The principal value for which cosine is $-\frac{1}{2}$ is $\frac{2\pi}{3}$.

The general solution is $2x = 2n\pi \pm \frac{2\pi}{3}$. So, $x = n\pi \pm \frac{\pi}{3}$.

For $n = 0$, $x = \pm\frac{\pi}{3}$. $x = \frac{\pi}{3}$ is a valid solution.

For $n = 1$, $x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ (outside the interval) and $x = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$ (outside the interval).

This case also gives the solution $x = \frac{\pi}{3}$.

Step 4: Final Answer:

The non-zero value of x in the given interval that satisfies the equation is $\frac{\pi}{3}$.

 Quick Tip

When solving equations with three or more sine or cosine terms, look for pairs of terms that can be combined using sum-to-product formulas to create a common factor. The choice of pairing is important. Pairing $\sin 5x$ and $\sin x$ was strategic because $(5x + x)/2 = 3x$, which matched the remaining term.

15. The general solution of the equation $\tan^2 x = 1$ is

- (A) $n\pi + \frac{\pi}{4}$ only
- (B) $n\pi \pm \frac{\pi}{4}$
- (C) $2n\pi \pm \frac{\pi}{4}$
- (D) $n\pi - \frac{\pi}{4}$ only

Correct Answer: (B) $n\pi \pm \frac{\pi}{4}$

Solution:

Step 1: Understanding the Question:

We need to find the general solution for the trigonometric equation $\tan^2 x = 1$. This means finding all possible values of x that satisfy the equation.

Step 2: Key Formula or Approach:

The general solution for $\tan^2 x = \tan^2 \alpha$ is given by the formula:

$$x = n\pi \pm \alpha$$

where n is any integer.

First, we will find the principal value α for which $\tan^2 \alpha = 1$, and then apply the formula.

Step 3: Detailed Explanation:

The given equation is $\tan^2 x = 1$.

We need to find an angle α such that $\tan^2 \alpha = 1$.

Taking the tangent of $\frac{\pi}{4}$, we have $\tan\left(\frac{\pi}{4}\right) = 1$.

Therefore, $\tan^2\left(\frac{\pi}{4}\right) = (1)^2 = 1$.

So, we can choose our principal value $\alpha = \frac{\pi}{4}$.

Now, we use the general solution formula for $\tan^2 x = \tan^2 \alpha$:

$$x = n\pi \pm \alpha$$

Substituting $\alpha = \frac{\pi}{4}$, we get:

$$x = n\pi \pm \frac{\pi}{4}$$

where n is any integer.

Alternative Method:

We can solve $\tan^2 x = 1$ by taking the square root:

$$\tan x = \pm\sqrt{1} \implies \tan x = 1 \quad \text{or} \quad \tan x = -1$$

The general solution for $\tan x = \tan \alpha$ is $x = n\pi + \alpha$.

For $\tan x = 1$, $\alpha = \frac{\pi}{4}$. The solution is $x = n\pi + \frac{\pi}{4}$.

For $\tan x = -1$, $\alpha = -\frac{\pi}{4}$. The solution is $x = n\pi - \frac{\pi}{4}$.

Combining these two sets of solutions gives $x = n\pi \pm \frac{\pi}{4}$.

Step 4: Final Answer:

The general solution of the equation is $x = n\pi \pm \frac{\pi}{4}$, where $n \in \mathbb{Z}$.

 Quick Tip

Remember the general solution formulas for squared trigonometric functions, as they are often simpler than solving the two linear cases separately:

- If $\sin^2 x = \sin^2 \alpha$, then $x = n\pi \pm \alpha$.
- If $\cos^2 x = \cos^2 \alpha$, then $x = n\pi \pm \alpha$.
- If $\tan^2 x = \tan^2 \alpha$, then $x = n\pi \pm \alpha$.

Notice that the format is the same for all three.

16. The value of $\cos \frac{5\pi}{17} + \cos \frac{7\pi}{17} + 2 \cos \frac{11\pi}{17} \cos \frac{\pi}{17}$ is

- (A) 0
- (B) 1
- (C) -1
- (D) $1/2$

Correct Answer: (A) 0

Solution:

Step 1: Understanding the Question:

We need to evaluate a trigonometric expression involving cosine functions with arguments that are multiples of $\frac{\pi}{17}$.

Step 2: Key Formula or Approach:

We will use the product-to-sum formula:

$$2 \cos A \cos B = \cos(A + B) + \cos(A - B)$$

And the property $\cos(\pi - \theta) = -\cos(\theta)$.

Step 3: Detailed Explanation:

Let the given expression be E .

$$E = \cos \frac{5\pi}{17} + \cos \frac{7\pi}{17} + 2 \cos \frac{11\pi}{17} \cos \frac{\pi}{17}$$

First, apply the product-to-sum formula to the third term:

$$\begin{aligned} 2 \cos \frac{11\pi}{17} \cos \frac{\pi}{17} &= \cos \left(\frac{11\pi}{17} + \frac{\pi}{17} \right) + \cos \left(\frac{11\pi}{17} - \frac{\pi}{17} \right) \\ &= \cos \left(\frac{12\pi}{17} \right) + \cos \left(\frac{10\pi}{17} \right) \end{aligned}$$

Now substitute this back into the expression for E :

$$E = \cos \frac{5\pi}{17} + \cos \frac{7\pi}{17} + \cos \frac{12\pi}{17} + \cos \frac{10\pi}{17}$$

We can rewrite the arguments of the last two terms using π .

Notice that $\frac{12\pi}{17} = \pi - \frac{5\pi}{17}$.

And $\frac{10\pi}{17} = \pi - \frac{7\pi}{17}$.

Let's apply the identity $\cos(\pi - \theta) = -\cos(\theta)$.

$$\cos\left(\frac{12\pi}{17}\right) = \cos\left(\pi - \frac{5\pi}{17}\right) = -\cos\left(\frac{5\pi}{17}\right)$$

$$\cos\left(\frac{10\pi}{17}\right) = \cos\left(\pi - \frac{7\pi}{17}\right) = -\cos\left(\frac{7\pi}{17}\right)$$

Now substitute these simplified terms back into the expression for E :

$$E = \cos \frac{5\pi}{17} + \cos \frac{7\pi}{17} - \cos \frac{5\pi}{17} - \cos \frac{7\pi}{17}$$

All the terms cancel each other out.

$$E = 0$$

Step 4: Final Answer:

The value of the expression is 0.

💡 Quick Tip

When dealing with trigonometric sums involving fractions of π , always look for pairs of angles that add up to π or $\pi/2$. This allows you to use identities like $\cos(\pi - \theta) = -\cos(\theta)$ or $\cos(\pi/2 - \theta) = \sin(\theta)$ to simplify the expression. Here, recognizing $12\pi/17$ as $\pi - 5\pi/17$ was the key to solving the problem.

17. If $\sin \theta - \cos \theta = \frac{4}{5}$ then the value of $\sin \theta + \cos \theta =$

(A) $\frac{5}{\sqrt{34}}$

(B) $-\frac{5}{\sqrt{34}}$

(C) $\frac{\sqrt{34}}{25}$

(D) $\frac{\sqrt{34}}{5}$

Correct Answer: (D) $\frac{\sqrt{34}}{5}$

Solution:

Step 1: Understanding the Question:

We are given the value of $\sin \theta - \cos \theta$ and we need to find the value of $\sin \theta + \cos \theta$.

Step 2: Key Formula or Approach:

This is a standard problem type that uses the relationship between $(\sin \theta - \cos \theta)^2$ and $(\sin \theta + \cos \theta)^2$.

We know that:

$$(\sin \theta - \cos \theta)^2 = \sin^2 \theta - 2 \sin \theta \cos \theta + \cos^2 \theta = 1 - 2 \sin \theta \cos \theta$$

$$(\sin \theta + \cos \theta)^2 = \sin^2 \theta + 2 \sin \theta \cos \theta + \cos^2 \theta = 1 + 2 \sin \theta \cos \theta$$

By adding these two equations, we get a direct relationship:

$$(\sin \theta - \cos \theta)^2 + (\sin \theta + \cos \theta)^2 = 2$$

Step 3: Detailed Explanation:

Let $x = \sin \theta + \cos \theta$. We need to find the value of x .

We are given $\sin \theta - \cos \theta = \frac{4}{5}$.

Using the identity from Step 2:

$$(\sin \theta - \cos \theta)^2 + (\sin \theta + \cos \theta)^2 = 2$$

Substitute the given values into this identity:

$$\left(\frac{4}{5}\right)^2 + x^2 = 2$$

$$\frac{16}{25} + x^2 = 2$$

Now, solve for x^2 :

$$x^2 = 2 - \frac{16}{25}$$

$$x^2 = \frac{50 - 16}{25}$$

$$x^2 = \frac{34}{25}$$

Take the square root of both sides:

$$x = \pm \sqrt{\frac{34}{25}} = \pm \frac{\sqrt{34}}{5}$$

Since one of the options is $\frac{\sqrt{34}}{5}$, we select this value. The question doesn't provide enough information to determine the sign, but only the positive value is listed as a primary option choice.

Step 4: Final Answer:

The value of $\sin \theta + \cos \theta$ is $\frac{\sqrt{34}}{5}$.

💡 Quick Tip

The identity $(\sin \theta \pm \cos \theta)^2 = 1 \pm 2 \sin \theta \cos \theta$ is extremely useful. Memorizing the combined identity $(\sin \theta - \cos \theta)^2 + (\sin \theta + \cos \theta)^2 = 2$ provides a direct shortcut for problems where one expression is given and the other is asked.

18. The real part of $\frac{1+2i}{(2-i)^2}$ is

- (A) $-\frac{1}{5}$
- (B) $\frac{1}{5}$
- (C) $-\frac{2}{5}$
- (D) $\frac{2}{5}$

Correct Answer: (A) $-\frac{1}{5}$

Solution:

Step 1: Understanding the Question:

We are asked to find the real part of a given complex number. The complex number is given as a fraction.

Step 2: Key Formula or Approach:

To find the real part, we first need to simplify the expression and write it in the standard form $a + bi$.

First, expand the denominator $(2 - i)^2$.

Then, multiply the numerator and the denominator by the conjugate of the resulting denominator to eliminate the imaginary part from the denominator.

Step 3: Detailed Explanation:

Let the complex number be $z = \frac{1+2i}{(2-i)^2}$.

Step 3a: Simplify the denominator

$$(2 - i)^2 = 2^2 - 2(2)(i) + i^2$$

$$= 4 - 4i - 1 \quad (\text{since } i^2 = -1)$$

$$= 3 - 4i$$

So, the expression becomes:

$$z = \frac{1 + 2i}{3 - 4i}$$

Step 3b: Rationalize the denominator

To convert this into the form $a + bi$, multiply the numerator and denominator by the conjugate of the denominator, which is $3 + 4i$.

$$z = \frac{1 + 2i}{3 - 4i} \times \frac{3 + 4i}{3 + 4i}$$

$$z = \frac{(1 + 2i)(3 + 4i)}{(3 - 4i)(3 + 4i)}$$

Numerator:

$$(1)(3) + (1)(4i) + (2i)(3) + (2i)(4i) = 3 + 4i + 6i + 8i^2 = 3 + 10i - 8 = -5 + 10i$$

Denominator:

$$(3)^2 - (4i)^2 = 9 - 16i^2 = 9 - 16(-1) = 9 + 16 = 25$$

So, the complex number is:

$$z = \frac{-5 + 10i}{25}$$

Step 3c: Write in standard form

$$z = \frac{-5}{25} + \frac{10i}{25} = -\frac{1}{5} + \frac{2}{5}i$$

The standard form is $a + bi$, where a is the real part and b is the imaginary part. Here, the real part is $a = -\frac{1}{5}$.

Step 4: Final Answer:

The real part of the complex number is $-\frac{1}{5}$.

 Quick Tip

When simplifying complex fractions, always handle the denominator first. Expand any powers and then multiply by the conjugate to make the denominator a real number. Remember the conjugate of $a + bi$ is $a - bi$ and their product is always a real number: $(a + bi)(a - bi) = a^2 + b^2$.

19. Modulus of the complex number $\frac{(1+i)^{10}}{(2i-4)^4}$ is equal to

- (A) $\frac{2}{25}$
- (B) $-\frac{2}{25}$
- (C) $\frac{1}{25}$
- (D) $-\frac{1}{25}$

Correct Answer: (A) $\frac{2}{25}$

Solution:

Step 1: Understanding the Question:

We need to find the modulus of a complex number which is given in a fractional form with powers.

Step 2: Key Formula or Approach:

We will use the properties of modulus:

1. $|z_1/z_2| = |z_1|/|z_2|$
2. $|z^n| = |z|^n$
3. The modulus of a complex number $z = a + bi$ is $|z| = \sqrt{a^2 + b^2}$.

Using these properties, we can find the modulus of the numerator and denominator separately without actually computing the complex number itself.

Step 3: Detailed Explanation:

Let $z = \frac{(1+i)^{10}}{(2i-4)^4}$. We need to find $|z|$.

Using the properties of modulus:

$$|z| = \left| \frac{(1+i)^{10}}{(-4+2i)^4} \right| = \frac{|(1+i)^{10}|}{|(-4+2i)^4|} = \frac{|1+i|^{10}}{|-4+2i|^4}$$

Step 3a: Calculate the modulus of the numerator term

First, find the modulus of $1 + i$:

$$|1 + i| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

Now, raise it to the power of 10:

$$|1 + i|^{10} = (\sqrt{2})^{10} = (2^{1/2})^{10} = 2^5 = 32$$

Step 3b: Calculate the modulus of the denominator term

First, find the modulus of $-4 + 2i$:

$$|-4 + 2i| = \sqrt{(-4)^2 + 2^2} = \sqrt{16 + 4} = \sqrt{20}$$

We can simplify $\sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$.

Now, raise it to the power of 4:

$$|-4 + 2i|^4 = (\sqrt{20})^4 = (20^{1/2})^4 = 20^2 = 400$$

Alternatively, $(2\sqrt{5})^4 = 2^4 \times (\sqrt{5})^4 = 16 \times 5^2 = 16 \times 25 = 400$.

Step 3c: Combine the results

$$|z| = \frac{32}{400}$$

Simplify the fraction:

$$|z| = \frac{16}{200} = \frac{8}{100} = \frac{2}{25}$$

Step 4: Final Answer:

The modulus of the complex number is $\frac{2}{25}$. Note that modulus must be a non-negative real number, so options (B) and (D) are incorrect by definition.

💡 Quick Tip

When asked for the modulus of a complex expression involving products, divisions, or powers, never try to simplify the complex number first. It's much faster and easier to use the properties of modulus ($|z_1 z_2| = |z_1| |z_2|$, $|z_1 / z_2| = |z_1| / |z_2|$, $|z^n| = |z|^n$) to calculate the modulus of each part separately and then combine them.

20. In a circle with center O, a 6cm long chord is at a distance 4 cm from the center. Then the length of diameter is

- (A) 5 cm
- (B) 10 cm
- (C) 15 cm
- (D) 8 cm

Correct Answer: (B) 10 cm

Solution:

Step 1: Understanding the Question:

We are given the length of a chord and its perpendicular distance from the center of a circle. We need to find the diameter of the circle.

Step 2: Key Formula or Approach:

The key geometric property is that the perpendicular from the center of a circle to a chord bisects the chord. This forms a right-angled triangle with the radius of the circle as the hypotenuse, the perpendicular distance from the center as one leg, and half the length of the chord as the other leg. We can use the Pythagorean theorem: $(\text{radius})^2 = (\text{distance})^2 + (\text{half of chord})^2$.

Step 3: Detailed Explanation:

Let r be the radius of the circle.

Length of the chord, $L = 6$ cm.

Distance from the center to the chord, $d = 4$ cm.

The perpendicular from the center bisects the chord. So, the length of half the chord is $\frac{L}{2} = \frac{6}{2} = 3$ cm.

Now, we have a right-angled triangle with sides:

Hypotenuse = r

One leg = $d = 4$ cm

Other leg = $\frac{L}{2} = 3$ cm

Using the Pythagorean theorem ($a^2 + b^2 = c^2$):

$$d^2 + \left(\frac{L}{2}\right)^2 = r^2$$

$$4^2 + 3^2 = r^2$$

$$16 + 9 = r^2$$

$$25 = r^2$$

$$r = \sqrt{25} = 5 \text{ cm}$$

The radius of the circle is 5 cm.

The question asks for the length of the diameter.

Diameter $D = 2 \times r$.

$$D = 2 \times 5 = 10 \text{ cm}$$

Step 4: Final Answer:

The length of the diameter is 10 cm.

💡 Quick Tip

This problem uses the very common 3-4-5 Pythagorean triple. Whenever you see a right-angled triangle with two sides as 3 and 4 in a geometry problem, the hypotenuse is almost always 5. Recognizing this can save you calculation time. Also, be careful to read the question fully; it asks for the diameter, not the radius, which is a common mistake.

21. The length of the tangent from the point (5, 1) to the circle $x^2 + y^2 + 6x - 4y - 3 = 0$ is

- (A) 81
- (B) 7
- (C) 29
- (D) 21

Correct Answer: (B) 7

Solution:

Step 1: Understanding the Question:

We need to find the length of the tangent drawn from an external point $(x_1, y_1) = (5, 1)$ to a given circle.

Step 2: Key Formula or Approach:

The length of the tangent, L , from an external point (x_1, y_1) to a circle with the equation $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ is given by the formula:

$$L = \sqrt{S_1}$$

where S_1 is the value of the circle's expression when the coordinates of the point are substituted into it, i.e.,

$$S_1 = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$$

Step 3: Detailed Explanation:

The given equation of the circle is $S \equiv x^2 + y^2 + 6x - 4y - 3 = 0$.

The external point is $(x_1, y_1) = (5, 1)$.

First, we calculate S_1 by substituting $x = 5$ and $y = 1$ into the equation of the circle:

$$S_1 = (5)^2 + (1)^2 + 6(5) - 4(1) - 3$$

$$S_1 = 25 + 1 + 30 - 4 - 3$$

$$S_1 = 56 - 7$$

$$S_1 = 49$$

Now, we find the length of the tangent using the formula $L = \sqrt{S_1}$.

$$L = \sqrt{49}$$

$$L = 7$$

Step 4: Final Answer:

The length of the tangent from the point $(5, 1)$ to the circle is 7.

💡 Quick Tip

To find the length of the tangent from a point to a circle, simply substitute the point's coordinates into the circle's equation (make sure the RHS is 0) and take the square root of the result. This is a direct and quick formula-based question common in exams.

22. If length of the tangent is 8 cm and the distance between the center of the circle and the external point is 11 cm, then the area of the circle is

- (A) 100 cm
- (B) 197.14 cm
- (C) 179.14 cm
- (D) 110.14 cm

Correct Answer: (C) 179.14 cm

Solution:

Step 1: Understanding the Question:

We are given the length of a tangent from an external point and the distance from that point to the circle's center. We need to find the area of the circle.

Step 2: Key Formula or Approach:

The radius of a circle (r), the length of the tangent from an external point (L), and the distance from the external point to the center of the circle (d) form a right-angled triangle. The distance d is the hypotenuse.

By the Pythagorean theorem:

$$d^2 = r^2 + L^2$$

The area of a circle is given by the formula $A = \pi r^2$.

Step 3: Detailed Explanation:

We are given:

Length of the tangent, $L = 8$ cm.

Distance from the point to the center, $d = 11$ cm.

Using the Pythagorean theorem, we can find the radius r :

$$11^2 = r^2 + 8^2$$

$$121 = r^2 + 64$$

Solve for r^2 :

$$r^2 = 121 - 64$$

$$r^2 = 57$$

Now, we calculate the area of the circle, $A = \pi r^2$.

$$A = \pi \times 57 = 57\pi$$

To find the numerical value, we use the approximation $\pi \approx 3.14159$.

$$A \approx 57 \times 3.14159$$

$$A \approx 179.07063 \text{ cm}^2$$

This value is closest to the option 179.14 cm.

Step 4: Final Answer:

The area of the circle is $57\pi \text{ cm}^2$, which is approximately 179.14 cm^2 .

💡 Quick Tip

Always visualize the geometry. The radius from the center to the point of tangency is perpendicular to the tangent line. This creates a right-angled triangle, which is key to solving many problems involving tangents to circles.

23. The equation of the parabola with focus (2, 0) and vertex (1, 0) is

- (A) $y^2 = 4x$
- (B) $y^2 = 4x - 4$
- (C) $y^2 = 4(x + 1)$
- (D) $y^2 = -4(x - 1)$

Correct Answer: (B) $y^2 = 4x - 4$

Solution:

Step 1: Understanding the Question:

We are given the coordinates of the vertex and the focus of a parabola. We need to find its equation.

Step 2: Key Formula or Approach:

First, determine the orientation and the parameter 'a' of the parabola.

The vertex is (h, k) . The focus is $(h + a, k)$ for a parabola opening right.

The standard equation for a parabola opening horizontally with vertex at (h, k) is:

$$(y - k)^2 = 4a(x - h)$$

Step 3: Detailed Explanation:

The vertex is given as $V(h, k) = (1, 0)$.

The focus is given as $S = (2, 0)$.

Since the y-coordinates of the vertex and focus are the same, the axis of symmetry is horizontal (the x-axis, $y=0$).

The focus ($x=2$) is to the right of the vertex ($x=1$), so the parabola opens to the right.

The distance 'a' is the distance between the vertex and the focus.

$$a = \sqrt{(2-1)^2 + (0-0)^2} = \sqrt{1^2} = 1$$

Now we use the standard equation $(y - k)^2 = 4a(x - h)$ with $h = 1, k = 0, a = 1$.

$$(y - 0)^2 = 4(1)(x - 1)$$

$$y^2 = 4(x - 1)$$

Expanding the equation, we get:

$$y^2 = 4x - 4$$

Step 4: Final Answer:

The equation of the parabola is $y^2 = 4x - 4$.

💡 Quick Tip

The position of the focus relative to the vertex determines the direction the parabola opens. If the focus is to the right of the vertex, it opens right. If left, it opens left. If above, it opens up. If below, it opens down. This directly tells you which standard form of the equation to use.

24. If (2,0) is the vertex and y-axis is the directrix of a parabola then its focus is

- (A) (2, 0)
- (B) (-2, 0)
- (C) (4, 0)
- (D) (-4, 0)

Correct Answer: (C) (4, 0)

Solution:

Step 1: Understanding the Question:

We are given the vertex and the directrix of a parabola and we need to find its focus.

Step 2: Key Formula or Approach:

The vertex of a parabola is the midpoint between its focus and its directrix.

The axis of the parabola is perpendicular to the directrix and passes through the vertex.

The distance from the vertex to the directrix is equal to the distance from the vertex to the focus. Let this distance be 'a'.

Step 3: Detailed Explanation:

The vertex is $V(h, k) = (2, 0)$.

The directrix is the y-axis, which is the line $x = 0$.

Since the directrix is a vertical line ($x = 0$), the axis of symmetry must be a horizontal line.

Since the axis passes through the vertex $(2, 0)$, the axis is the x-axis ($y = 0$).

The parabola opens away from the directrix. The vertex is at $x = 2$ and the directrix is at $x = 0$, so the parabola opens to the right.

The distance 'a' from the vertex to the directrix is the horizontal distance between the point $(2, 0)$ and the line $x = 0$.

$$a = |2 - 0| = 2$$

The focus lies on the axis of symmetry ($y = 0$) and is at a distance 'a' from the vertex, inside the curve.

Since the parabola opens to the right, the focus will be at $(h + a, k)$.

Focus $S = (2 + 2, 0) = (4, 0)$.

Step 4: Final Answer:

The focus of the parabola is at $(4, 0)$.

💡 Quick Tip

A quick way to find the focus is to think about the vertex as the "center". The directrix is on one side, and the focus is on the other, at an equal distance. Here, the vertex is at $x=2$, directrix at $x=0$ (2 units to the left). So, the focus must be 2 units to the right, at $x=2+2=4$.

25. The eccentricity of the ellipse $16x^2 + 7y^2 = 112$ is

(A) $\frac{4}{3}$

(B) $\frac{7}{16}$

(C) $\frac{3}{\sqrt{7}}$

(D) $\frac{3}{4}$

Correct Answer: (D) $\frac{3}{4}$

Solution:

Step 1: Understanding the Question:

We are given the equation of an ellipse and we need to calculate its eccentricity.

Step 2: Key Formula or Approach:

First, convert the equation of the ellipse to the standard form: $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ (for a vertical ellipse) or $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (for a horizontal ellipse), where $a > b$.

The eccentricity e is given by the formula:

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

Step 3: Detailed Explanation:

The given equation is $16x^2 + 7y^2 = 112$.

To convert it to standard form, divide the entire equation by 112:

$$\frac{16x^2}{112} + \frac{7y^2}{112} = \frac{112}{112}$$

$$\frac{x^2}{7} + \frac{y^2}{16} = 1$$

This is the standard form of an ellipse. We can identify a^2 and b^2 . By convention, a^2 is the larger denominator.

Here, $a^2 = 16$ and $b^2 = 7$.

Since a^2 is under the y^2 term, the major axis of the ellipse is vertical.

Now we calculate the eccentricity e :

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{7}{16}}$$

$$e = \sqrt{\frac{16 - 7}{16}} = \sqrt{\frac{9}{16}}$$

$$e = \frac{3}{4}$$

Step 4: Final Answer:

The eccentricity of the ellipse is $\frac{3}{4}$.

 Quick Tip

Eccentricity of an ellipse is always between 0 and 1. If you get a value greater than or equal to 1 (like option A), you have made a calculation error, likely by swapping a^2 and b^2 in the formula.

26. The value of $\lim_{x \rightarrow \infty} \frac{4x^3 - x + 1}{x^2 - 4x(1 - x^2)}$ is

- (A) 0
- (B) 1
- (C) -1
- (D) ∞

Correct Answer: (B) 1

Solution:

Step 1: Understanding the Question:

We need to evaluate the limit of a rational function as x approaches infinity.

Step 2: Key Formula or Approach:

For a rational function $\lim_{x \rightarrow \infty} \frac{P(x)}{Q(x)}$, where $P(x)$ and $Q(x)$ are polynomials, the limit is determined by the ratio of the leading terms (the terms with the highest power of x).

A more formal method is to divide both the numerator and the denominator by the highest power of x in the denominator.

Step 3: Detailed Explanation:

First, let's simplify the denominator of the expression:

$$\text{Denominator} = x^2 - 4x(1 - x^2) = x^2 - 4x + 4x^3.$$

So the limit is:

$$\lim_{x \rightarrow \infty} \frac{4x^3 - x + 1}{4x^3 + x^2 - 4x}$$

The degree of the numerator polynomial is 3, and the degree of the denominator polynomial is also 3. Since the degrees are equal, the limit is the ratio of the leading coefficients.

Leading coefficient of numerator = 4.

Leading coefficient of denominator = 4.

$$\text{Limit} = \frac{4}{4} = 1$$

Formal Method:

The highest power of x in the expression is x^3 . Divide the numerator and denominator by x^3 :

$$\lim_{x \rightarrow \infty} \frac{\frac{4x^3}{x^3} - \frac{x}{x^3} + \frac{1}{x^3}}{\frac{4x^3}{x^3} + \frac{x^2}{x^3} - \frac{4x}{x^3}} = \lim_{x \rightarrow \infty} \frac{4 - \frac{1}{x^2} + \frac{1}{x^3}}{4 + \frac{1}{x} - \frac{4}{x^2}}$$

As $x \rightarrow \infty$, all terms with x in the denominator approach 0.

$$\text{Limit} = \frac{4 - 0 + 0}{4 + 0 - 0} = \frac{4}{4} = 1$$

Step 4: Final Answer:

The value of the limit is 1.

 Quick Tip

For limits at infinity of rational functions:

- If degree of numerator < degree of denominator, limit is 0.
- If degree of numerator > degree of denominator, limit is $\pm\infty$.
- If degree of numerator = degree of denominator, limit is the ratio of leading coefficients.

This shortcut can save a lot of time.

27. The value of $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1}$ is

- (A) 0
- (B) 1
- (C) 3
- (D) Limit does not exist

Correct Answer: (C) 3

Solution:

Step 1: Understanding the Question:

We need to evaluate the limit of a function as x approaches 1. Direct substitution of $x = 1$ gives the indeterminate form $\frac{0}{0}$.

Step 2: Key Formula or Approach:

We can solve this using one of three common methods for indeterminate forms:

1. **Factorization:** Factor the numerator and cancel common terms. The relevant identity is $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.
2. **L'Hôpital's Rule:** Differentiate the numerator and the denominator separately and then take the limit.
3. **Standard Limit Formula:** Use the formula $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}$.

Step 3: Detailed Explanation:**Method 1: Factorization**

$$\lim_{x \rightarrow 1} \frac{x^3 - 1^3}{x - 1} = \lim_{x \rightarrow 1} \frac{(x - 1)(x^2 + x \cdot 1 + 1^2)}{x - 1}$$

Cancel the $(x - 1)$ term:

$$\lim_{x \rightarrow 1} (x^2 + x + 1)$$

Now substitute $x = 1$:

$$(1)^2 + 1 + 1 = 1 + 1 + 1 = 3$$

Method 2: L'Hôpital's Rule

Since we have the $\frac{0}{0}$ form, we can differentiate the numerator and denominator:

$$\lim_{x \rightarrow 1} \frac{\frac{d}{dx}(x^3 - 1)}{\frac{d}{dx}(x - 1)} = \lim_{x \rightarrow 1} \frac{3x^2}{1}$$

Now substitute $x = 1$:

$$\frac{3(1)^2}{1} = 3$$

Method 3: Standard Limit Formula

The limit is in the form $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a}$ with $n = 3$ and $a = 1$.

The result is na^{n-1} :

$$3 \cdot (1)^{3-1} = 3 \cdot 1^2 = 3$$

Step 4: Final Answer:

The value of the limit is 3.

💡 Quick Tip

Recognizing the standard limit form $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}$ is the fastest way to solve this type of problem. It's a fundamental limit that's worth memorizing for competitive exams.

28. The derivative of x^x with respect to x is

- (A) $x^x(x + \log x)$
- (B) $x^x(x - \log x)$
- (C) $x^x(1 - \log x)$
- (D) $x^x(1 + \log x)$

Correct Answer: (D) $x^x(1 + \log x)$

Solution:

Step 1: Understanding the Question:

We need to find the derivative of the function $y = x^x$, which is a function raised to the power of a function.

Step 2: Key Formula or Approach:

For functions of the form $y = [f(x)]^{g(x)}$, we use logarithmic differentiation.

1. Take the natural logarithm ($\ln$) of both sides.
2. Use logarithm properties to simplify the expression.
3. Differentiate both sides implicitly with respect to x .
4. Solve for $\frac{dy}{dx}$.

Step 3: Detailed Explanation:

Let $y = x^x$.

Take the natural logarithm of both sides:

$$\ln y = \ln(x^x)$$

Using the logarithm power rule, $\ln(a^b) = b \ln a$:

$$\ln y = x \ln x$$

Now, differentiate both sides with respect to x . We use the product rule on the right side.

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}(x \ln x)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \left(\frac{d}{dx}(x) \right) \cdot \ln x + x \cdot \left(\frac{d}{dx}(\ln x) \right)$$

$$\frac{1}{y} \frac{dy}{dx} = (1) \cdot \ln x + x \cdot \left(\frac{1}{x} \right)$$

$$\frac{1}{y} \frac{dy}{dx} = \ln x + 1$$

To find $\frac{dy}{dx}$, multiply both sides by y :

$$\frac{dy}{dx} = y(1 + \ln x)$$

Finally, substitute back $y = x^x$:

$$\frac{dy}{dx} = x^x(1 + \ln x)$$

Assuming $\log x$ in the options represents the natural logarithm, this matches option (D).

Step 4: Final Answer:

The derivative of x^x is $x^x(1 + \log x)$.

 Quick Tip

The derivative of x^x is a standard result that's good to remember. Whenever you see a function in the form of $(\text{variable})^{(\text{variable})}$, logarithmic differentiation is the method to use.

29. $\frac{d}{dx} \left(\tan^{-1} \frac{x}{a} \right) =$

- (A) $\frac{a}{a^2 - x^2}$
- (B) $\frac{1}{a^2 + x^2}$
- (C) $\frac{1}{a^2 - x^2}$
- (D) $\frac{a}{a^2 + x^2}$

Correct Answer: (D) $\frac{a}{a^2 + x^2}$

Solution:

Step 1: Understanding the Question:

We need to find the derivative of the inverse tangent function $\tan^{-1} \left(\frac{x}{a} \right)$.

Step 2: Key Formula or Approach:

We will use the chain rule along with the standard derivative of the arctangent function.

Standard Derivative: $\frac{d}{du}(\tan^{-1} u) = \frac{1}{1+u^2}$.

Chain Rule: If $y = f(u)$ and $u = g(x)$, then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$.

Step 3: Detailed Explanation:

Let $y = \tan^{-1} \left(\frac{x}{a} \right)$.

Let $u = \frac{x}{a}$. Then $y = \tan^{-1}(u)$.

First, find the derivative of y with respect to u :

$$\frac{dy}{du} = \frac{1}{1+u^2}$$

Next, find the derivative of u with respect to x :

$$\frac{du}{dx} = \frac{d}{dx} \left(\frac{x}{a} \right) = \frac{1}{a}$$

Now, apply the chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \left(\frac{1}{1+u^2} \right) \cdot \left(\frac{1}{a} \right)$$

Substitute $u = \frac{x}{a}$ back into the expression:

$$\frac{dy}{dx} = \left(\frac{1}{1 + \left(\frac{x}{a} \right)^2} \right) \cdot \left(\frac{1}{a} \right)$$

Simplify the expression:

$$\frac{dy}{dx} = \left(\frac{1}{1 + \frac{x^2}{a^2}} \right) \cdot \left(\frac{1}{a} \right) = \left(\frac{1}{\frac{a^2+x^2}{a^2}} \right) \cdot \left(\frac{1}{a} \right)$$

$$\frac{dy}{dx} = \left(\frac{a^2}{a^2+x^2} \right) \cdot \left(\frac{1}{a} \right)$$

$$\frac{dy}{dx} = \frac{a}{a^2+x^2}$$

Step 4: Final Answer:

The derivative is $\frac{a}{a^2+x^2}$.

💡 Quick Tip

The derivative of $\tan^{-1} \left(\frac{x}{a} \right)$ is a standard formula in calculus. It's highly recommended to memorize this result, $\frac{a}{a^2+x^2}$, and the similar one for $\sin^{-1} \left(\frac{x}{a} \right)$ to save time in exams.

30. If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dots \infty}}}$ then $\frac{dy}{dx} =$

(A) $\frac{\cos x}{1-2y}$

(B) $\frac{\sin x}{1-2y}$

(C) $-\frac{\sin x}{1-2y}$

(D) $-\frac{\cos x}{1-2y}$

Correct Answer: (D) $-\frac{\cos x}{1-2y}$

Solution:

Step 1: Understanding the Question:

We are given an infinitely nested radical function and asked to find its derivative.

Step 2: Key Formula or Approach:

The key to solving this is to recognize the repeating pattern. The entire expression under the first square root is simply the original function y itself. This allows us to write a simple implicit equation for y , which can then be differentiated.

Step 3: Detailed Explanation:

The given function is:

$$y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dots \infty}}}$$

We can rewrite this as:

$$y = \sqrt{\sin x + y}$$

To remove the square root, we square both sides of the equation:

$$y^2 = \sin x + y$$

Now, we differentiate this equation implicitly with respect to x :

$$\frac{d}{dx}(y^2) = \frac{d}{dx}(\sin x) + \frac{d}{dx}(y)$$

$$2y \frac{dy}{dx} = \cos x + \frac{dy}{dx}$$

Now, we need to solve for $\frac{dy}{dx}$. Group all the $\frac{dy}{dx}$ terms on one side:

$$2y \frac{dy}{dx} - \frac{dy}{dx} = \cos x$$

Factor out $\frac{dy}{dx}$:

$$\frac{dy}{dx}(2y - 1) = \cos x$$

Isolate $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{\cos x}{2y - 1}$$

To match the given options, we can multiply the numerator and denominator by -1:

$$\frac{dy}{dx} = \frac{-\cos x}{-(2y - 1)} = \frac{-\cos x}{1 - 2y}$$

Step 4: Final Answer:

The derivative $\frac{dy}{dx}$ is $-\frac{\cos x}{1-2y}$.

💡 Quick Tip

For any function of the form $y = \sqrt{f(x) + \sqrt{f(x) + \dots}}$, the derivative follows the pattern $\frac{dy}{dx} = \frac{f'(x)}{2y-1}$. Memorizing this general form can lead to an instant answer for such problems.

31. Slope of the normal to the curve $x^{2/3} + y^{2/3} = 2$ at the point (1, 1) is

- (A) -1
- (B) 1
- (C) 1/2
- (D) -1/2

Correct Answer: (B) 1

Solution:

Step 1: Understanding the Question:

We need to find the slope of the normal line to the given curve (an astroid) at the specified point (1, 1).

Step 2: Key Formula or Approach:

1. Find the derivative $\frac{dy}{dx}$ of the curve's equation using implicit differentiation. This will give the slope of the tangent (m_T).
2. Evaluate $\frac{dy}{dx}$ at the given point (1, 1) to find the specific slope of the tangent at that point.

3. The slope of the normal (m_N) is the negative reciprocal of the slope of the tangent:
 $m_N = -\frac{1}{m_T}$.

Step 3: Detailed Explanation:

The equation of the curve is $x^{2/3} + y^{2/3} = 2$.

Differentiate both sides with respect to x :

$$\frac{d}{dx}(x^{2/3}) + \frac{d}{dx}(y^{2/3}) = \frac{d}{dx}(2)$$

$$\frac{2}{3}x^{(2/3-1)} + \frac{2}{3}y^{(2/3-1)} \cdot \frac{dy}{dx} = 0$$

$$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0$$

Divide the entire equation by $\frac{2}{3}$:

$$x^{-1/3} + y^{-1/3}\frac{dy}{dx} = 0$$

Solve for $\frac{dy}{dx}$:

$$\frac{dy}{dx} = -\frac{x^{-1/3}}{y^{-1/3}} = -\left(\frac{y}{x}\right)^{1/3}$$

This is the slope of the tangent, m_T . Now, evaluate m_T at the point $(1, 1)$:

$$m_T = -\left(\frac{1}{1}\right)^{1/3} = -1$$

The slope of the normal, m_N , is the negative reciprocal of m_T .

$$m_N = -\frac{1}{m_T} = -\frac{1}{-1} = 1$$

Step 4: Final Answer:

The slope of the normal to the curve at $(1, 1)$ is 1.

 Quick Tip

Be careful not to confuse the slope of the tangent with the slope of the normal. After finding $\frac{dy}{dx}$, always remember to take the negative reciprocal if the question asks for the normal's slope. A common mistake is to stop after finding the tangent's slope.

32. The equation of the tangent to the curve $y = x^3$ at $(1, 1)$ is

- (A) $3x - y + 2 = 0$
- (B) $x - 10y - 50 = 0$
- (C) $3x - y - 2 = 0$
- (D) $x - 10y + 50 = 0$

Correct Answer: (C) $3x - y - 2 = 0$

Solution:

Step 1: Understanding the Question:

We need to find the equation of the line that is tangent to the curve $y = x^3$ at the point $(1, 1)$.

Step 2: Key Formula or Approach:

1. Find the derivative of the function, $\frac{dy}{dx}$, to get the formula for the slope of the tangent.
2. Evaluate the derivative at the given point to find the numerical slope, m .
3. Use the point-slope form of a linear equation, $y - y_1 = m(x - x_1)$, to find the equation of the tangent line.

Step 3: Detailed Explanation:

The curve is given by $y = x^3$.

First, find the derivative with respect to x :

$$\frac{dy}{dx} = 3x^2$$

This gives the slope of the tangent at any point x . We need the slope at the point $(1, 1)$. Substitute $x = 1$ into the derivative:

$$m = 3(1)^2 = 3$$

The slope of the tangent at $(1, 1)$ is 3.

Now, use the point-slope form with $m = 3$ and the point $(x_1, y_1) = (1, 1)$:

$$y - y_1 = m(x - x_1)$$

$$y - 1 = 3(x - 1)$$

Simplify the equation:

$$y - 1 = 3x - 3$$

Rearrange it into the general form $Ax + By + C = 0$:

$$3x - y - 3 + 1 = 0$$

$$3x - y - 2 = 0$$

Step 4: Final Answer:

The equation of the tangent to the curve at $(1, 1)$ is $3x - y - 2 = 0$.

💡 Quick Tip

To quickly check your answer, ensure two things: 1. The point $(1,1)$ must satisfy the final equation. For option (C), $3(1) - 1 - 2 = 3 - 3 = 0$. It works. 2. The slope of the line from the equation must match your calculated slope. For $3x - y - 2 = 0$, the slope is $-A/B = -3/(-1) = 3$, which matches.

33. For what value of x , the function $f(x) = 2x^3 + 3x^2 - 36x + 10$ has minimum

- (A) -2
- (B) -3
- (C) 2
- (D) 1

Correct Answer: (C) 2

Solution:

Step 1: Understanding the Question:

We need to find the x -coordinate of the point where the given polynomial function has a local minimum value.

Step 2: Key Formula or Approach:

We use the second derivative test to find local minima and maxima.

1. Find the first derivative, $f'(x)$.
2. Find the critical points by solving the equation $f'(x) = 0$.
3. Find the second derivative, $f''(x)$.
4. For each critical point c , evaluate $f''(c)$.
 - If $f''(c) > 0$, the function has a local minimum at $x = c$.
 - If $f''(c) < 0$, the function has a local maximum at $x = c$.

Step 3: Detailed Explanation:

The function is $f(x) = 2x^3 + 3x^2 - 36x + 10$.

1. Find the first derivative:

$$f'(x) = \frac{d}{dx}(2x^3 + 3x^2 - 36x + 10) = 6x^2 + 6x - 36$$

2. Find the critical points by setting $f'(x) = 0$:

$$6x^2 + 6x - 36 = 0$$

Divide by 6 to simplify:

$$x^2 + x - 6 = 0$$

Factor the quadratic equation:

$$(x + 3)(x - 2) = 0$$

The critical points are $x = -3$ and $x = 2$.

3. Find the second derivative:

$$f''(x) = \frac{d}{dx}(6x^2 + 6x - 36) = 12x + 6$$

4. Apply the second derivative test for each critical point:

For $x = -3$:

$$f''(-3) = 12(-3) + 6 = -36 + 6 = -30$$

Since $f''(-3) < 0$, there is a local maximum at $x = -3$.

For $x = 2$:

$$f''(2) = 12(2) + 6 = 24 + 6 = 30$$

Since $f''(2) > 0$, there is a local minimum at $x = 2$.

Step 4: Final Answer:

The function has a minimum at the value $x = 2$.

💡 Quick Tip

For a cubic polynomial with two critical points, the smaller x-value (more negative) typically corresponds to the local maximum, and the larger x-value corresponds to the local minimum, assuming a positive leading coefficient. This can be a quick check.

34. If $z = x^2 - y^2$ then $\frac{1}{x} \frac{\partial z}{\partial x} + \frac{1}{y} \frac{\partial z}{\partial y} =$

- (A) 1
- (B) $2x + 2y$
- (C) 0
- (D) $2x - 2y$

Correct Answer: (C) 0

Solution:

Step 1: Understanding the Question:

We are given a function z of two variables, x and y , and we need to evaluate an expression involving its partial derivatives.

Step 2: Key Formula or Approach:

1. Find the partial derivative of z with respect to x , $\frac{\partial z}{\partial x}$, by treating y as a constant.
2. Find the partial derivative of z with respect to y , $\frac{\partial z}{\partial y}$, by treating x as a constant.
3. Substitute these derivatives into the given expression and simplify.

Step 3: Detailed Explanation:

The given function is $z = x^2 - y^2$.

1. Calculate the partial derivative with respect to x :

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}(x^2 - y^2) = 2x - 0 = 2x$$

2. Calculate the partial derivative with respect to y :

$$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}(x^2 - y^2) = 0 - 2y = -2y$$

3. Now, substitute these results into the given expression:

$$\frac{1}{x} \frac{\partial z}{\partial x} + \frac{1}{y} \frac{\partial z}{\partial y} = \frac{1}{x}(2x) + \frac{1}{y}(-2y)$$

Simplify the expression:

$$= 2 - 2 = 0$$

Step 4: Final Answer:

The value of the expression is 0.

 Quick Tip

This problem type is a straightforward application of partial differentiation rules. Remember that when differentiating with respect to one variable, all other variables are treated as constants.

35. If $u = e^{xy}$, then the value of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ at $(1, 1)$ is

- (A) e
- (B) $2e$
- (C) 1
- (D) 0

Correct Answer: (B) $2e$

Solution:

Step 1: Understanding the Question:

We need to find the sum of the second partial derivatives of the function $u = e^{xy}$ with respect to x and y , and then evaluate this sum at the point $(1, 1)$.

Step 2: Key Formula or Approach:

1. Find the first partial derivative $\frac{\partial u}{\partial x}$.
2. Find the second partial derivative $\frac{\partial^2 u}{\partial x^2}$ by differentiating $\frac{\partial u}{\partial x}$ with respect to x .
3. Find the first partial derivative $\frac{\partial u}{\partial y}$.
4. Find the second partial derivative $\frac{\partial^2 u}{\partial y^2}$ by differentiating $\frac{\partial u}{\partial y}$ with respect to y .
5. Add the two second derivatives and substitute $x = 1, y = 1$.

Step 3: Detailed Explanation:

The function is $u = e^{xy}$.

Derivatives with respect to x:

First partial derivative w.r.t. x (using chain rule, treat y as constant):

$$\frac{\partial u}{\partial x} = e^{xy} \cdot \frac{\partial}{\partial x}(xy) = ye^{xy}$$

Second partial derivative w.r.t. x (treat y as constant):

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}(ye^{xy}) = y \cdot \frac{\partial}{\partial x}(e^{xy}) = y \cdot (ye^{xy}) = y^2 e^{xy}$$

Derivatives with respect to y:

First partial derivative w.r.t. y (using chain rule, treat x as constant):

$$\frac{\partial u}{\partial y} = e^{xy} \cdot \frac{\partial}{\partial y}(xy) = xe^{xy}$$

Second partial derivative w.r.t. y (treat x as constant):

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}(xe^{xy}) = x \cdot \frac{\partial}{\partial y}(e^{xy}) = x \cdot (xe^{xy}) = x^2 e^{xy}$$

Sum of second derivatives:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = y^2 e^{xy} + x^2 e^{xy} = (x^2 + y^2)e^{xy}$$

Now, evaluate this expression at the point $(1, 1)$:

$$\left. \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right|_{(1,1)} = (1^2 + 1^2)e^{(1)(1)} = (1 + 1)e^1 = 2e$$

Step 4: Final Answer:

The value of the expression at $(1, 1)$ is $2e$.

 Quick Tip

When performing partial differentiation, be methodical. Write down each step clearly to avoid confusion, especially when calculating second-order derivatives. Pay close attention to which variable you are treating as a constant in each step.

36. The value of $\int (\log \sec x) \tan x \, dx$ is

- (A) $\sec x + c$
- (B) $\log \sec x + c$
- (C) $\frac{1}{2}(\log \sec x)^2 + c$
- (D) $\log(\log \sec x) + c$

Correct Answer: (C) $\frac{1}{2}(\log \sec x)^2 + c$

Solution:

Step 1: Understanding the Question:

We need to find the indefinite integral of the function $(\log \sec x) \tan x$.

Step 2: Key Formula or Approach:

This integral can be solved using the method of substitution. We look for a function and its derivative within the integrand.

Let's test the substitution $t = \log \sec x$. We need to find its derivative, $\frac{dt}{dx}$.

Step 3: Detailed Explanation:

Let $I = \int (\log \sec x) \tan x \, dx$.

Let's use the substitution $t = \log \sec x$.

Now, we differentiate t with respect to x to find dt :

$$\frac{dt}{dx} = \frac{d}{dx}(\log \sec x)$$

Using the chain rule, let $u = \sec x$, so $t = \log u$.

$$\frac{dt}{dx} = \frac{dt}{du} \cdot \frac{du}{dx} = \frac{1}{u} \cdot (\sec x \tan x) = \frac{1}{\sec x} \cdot (\sec x \tan x) = \tan x$$

So, we have $\frac{dt}{dx} = \tan x$, which means $dt = \tan x \, dx$.

Now we can substitute t and dt back into the integral:

$$I = \int (\log \sec x)(\tan x \, dx) = \int t \, dt$$

This is a standard integral:

$$\int t \, dt = \frac{t^2}{2} + c$$

Finally, substitute back $t = \log \sec x$:

$$I = \frac{(\log \sec x)^2}{2} + c = \frac{1}{2}(\log \sec x)^2 + c$$

Step 4: Final Answer:

The value of the integral is $\frac{1}{2}(\log \sec x)^2 + c$.

💡 Quick Tip

In substitution integrals, look for a composite function. The derivative of the "inner" function is often present as a factor in the integrand. Here, the inner function is $\log(\sec x)$, and its derivative, $\tan x$, is conveniently the other factor. Recognizing this pattern is key.

37. $\int \sin^2 x \, dx =$

- (A) $\frac{x}{2} + \frac{\sin 2x}{4} + c$
- (B) $\frac{x}{2} - \frac{\cos 2x}{4} + c$
- (C) $\frac{x}{2} + \frac{\cos 2x}{4} + c$
- (D) $\frac{x}{2} - \frac{\sin 2x}{4} + c$

Correct Answer: (D) $\frac{x}{2} - \frac{\sin 2x}{4} + c$

Solution:

Step 1: Understanding the Question:

We need to find the indefinite integral of $\sin^2 x$.

Step 2: Key Formula or Approach:

We cannot integrate $\sin^2 x$ directly. We must first reduce the power using a trigonometric identity. The relevant power-reduction formula comes from the double-angle identity for cosine:

$$\cos(2x) = 1 - 2 \sin^2 x$$

Rearranging this formula to solve for $\sin^2 x$, we get:

$$\sin^2 x = \frac{1 - \cos(2x)}{2}$$

Step 3: Detailed Explanation:

Let $I = \int \sin^2 x \, dx$.

Substitute the power-reduction formula into the integral:

$$I = \int \frac{1 - \cos(2x)}{2} \, dx$$

We can split this into two separate integrals:

$$I = \frac{1}{2} \int (1 - \cos(2x)) \, dx = \frac{1}{2} \left(\int 1 \, dx - \int \cos(2x) \, dx \right)$$

Integrate each term:

$$\int 1 \, dx = x$$

$$\int \cos(2x) \, dx = \frac{\sin(2x)}{2}$$

Now, substitute these back into the expression for I :

$$I = \frac{1}{2} \left(x - \frac{\sin(2x)}{2} \right) + c$$

Distribute the $\frac{1}{2}$:

$$I = \frac{x}{2} - \frac{\sin(2x)}{4} + c$$

Step 4: Final Answer:

The integral of $\sin^2 x$ is $\frac{x}{2} - \frac{\sin 2x}{4} + c$.

💡 Quick Tip

Memorize the power-reduction formulas for $\sin^2 x$ and $\cos^2 x$, as they are fundamental for integrating even powers of sine and cosine.

- $\sin^2 x = \frac{1 - \cos(2x)}{2}$
- $\cos^2 x = \frac{1 + \cos(2x)}{2}$

These are direct applications of the $\cos(2x)$ double-angle identities.

38. $\int \frac{dx}{25-x^2} =$

- (A) $\frac{1}{5} \log \left| \frac{x-5}{x+5} \right| + c$
- (B) $\frac{1}{5} \log \left| \frac{x+5}{x-5} \right| + c$
- (C) $\frac{1}{10} \log \left| \frac{5+x}{5-x} \right| + c$
- (D) $\frac{1}{10} \log \left| \frac{5-x}{5+x} \right| + c$

Correct Answer: (C) $\frac{1}{10} \log \left| \frac{5+x}{5-x} \right| + c$

Solution:

Step 1: Understanding the Question:

We need to evaluate the indefinite integral of $\frac{1}{25-x^2}$.

Step 2: Key Formula or Approach:

This integral matches the standard integration formula:

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right| + C$$

Alternatively, we can use partial fraction decomposition.

Step 3: Detailed Explanation:**Method 1: Using the Standard Formula**

The integral is $\int \frac{dx}{25-x^2}$.

We can write this as $\int \frac{dx}{5^2-x^2}$.

This matches the standard form with $a = 5$.

Applying the formula:

$$\begin{aligned}\int \frac{dx}{5^2 - x^2} &= \frac{1}{2(5)} \ln \left| \frac{5+x}{5-x} \right| + c \\ &= \frac{1}{10} \ln \left| \frac{5+x}{5-x} \right| + c\end{aligned}$$

Assuming log in the options means natural logarithm ($\ln$), this matches option (C).

Method 2: Partial Fraction Decomposition

Factor the denominator: $25 - x^2 = (5 - x)(5 + x)$.

Decompose the fraction:

$$\frac{1}{(5-x)(5+x)} = \frac{A}{5-x} + \frac{B}{5+x}$$

Using the cover-up method:

To find A, cover $(5 - x)$ and set $x = 5$: $A = \frac{1}{5+5} = \frac{1}{10}$.

To find B, cover $(5 + x)$ and set $x = -5$: $B = \frac{1}{5-(-5)} = \frac{1}{10}$.

So the integral becomes:

$$\begin{aligned}\int \left(\frac{1/10}{5-x} + \frac{1/10}{5+x} \right) dx &= \frac{1}{10} \int \frac{dx}{5-x} + \frac{1}{10} \int \frac{dx}{5+x} \\ &= \frac{1}{10} (-\ln |5-x|) + \frac{1}{10} (\ln |5+x|) + c \\ &= \frac{1}{10} (\ln |5+x| - \ln |5-x|) + c\end{aligned}$$

Using the logarithm property $\ln a - \ln b = \ln(a/b)$:

$$= \frac{1}{10} \ln \left| \frac{5+x}{5-x} \right| + c$$

Step 4: Final Answer:

The result of the integration is $\frac{1}{10} \log \left| \frac{5+x}{5-x} \right| + c$.

💡 Quick Tip

Memorizing the standard integral forms for $\frac{1}{a^2-x^2}$, $\frac{1}{x^2-a^2}$, and $\frac{1}{a^2+x^2}$ is essential for speed and accuracy in competitive exams. They appear very frequently.

39. The value of $\int_0^1 x(1-x)^9 dx$ is

(A) $\frac{1}{110}$

(B) $\frac{1}{120}$

(C) $-\frac{1}{110}$

(D) $-\frac{1}{120}$

Correct Answer: (A) $\frac{1}{110}$

Solution:

Step 1: Understanding the Question:

We need to evaluate a definite integral. The integrand has a term $(1-x)^9$ which is difficult to expand.

Step 2: Key Formula or Approach:

We can use a property of definite integrals or substitution.

Property: $\int_0^a f(x) dx = \int_0^a f(a-x) dx$.

Substitution: Let $u = 1-x$.

We will use the property as it is often faster.

Step 3: Detailed Explanation:

Let $I = \int_0^1 x(1-x)^9 dx$.

Using the property $\int_0^a f(x) dx = \int_0^a f(a-x) dx$ with $a = 1$:

Here, $f(x) = x(1-x)^9$.

So, $f(a-x) = f(1-x) = (1-x)(1-(1-x))^9 = (1-x)(x)^9 = x^9(1-x)$.

Therefore, the integral becomes:

$$I = \int_0^1 x^9(1-x) dx$$

This is much easier to integrate as we just need to expand the integrand:

$$I = \int_0^1 (x^9 - x^{10}) dx$$

Now, perform the integration:

$$I = \left[\frac{x^{10}}{10} - \frac{x^{11}}{11} \right]_0^1$$

Evaluate at the limits:

$$I = \left(\frac{1^{10}}{10} - \frac{1^{11}}{11} \right) - \left(\frac{0^{10}}{10} - \frac{0^{11}}{11} \right)$$

$$I = \left(\frac{1}{10} - \frac{1}{11} \right) - (0)$$

Find a common denominator:

$$I = \frac{11 - 10}{110} = \frac{1}{110}$$

Step 4: Final Answer:

The value of the definite integral is $\frac{1}{110}$.

💡 Quick Tip

The property $\int_0^a f(x) dx = \int_0^a f(a - x) dx$ is extremely useful for definite integrals where the integrand involves a term like $(a - x)^n$. It often simplifies the integrand significantly, making integration straightforward.

40. $\int_{-a}^a |x| dx =$

- (A) a
- (B) 2a
- (C) 0
- (D) a^2

Correct Answer: (D) a^2

Solution:

Step 1: Understanding the Question:

We need to evaluate the definite integral of the absolute value function, $|x|$, over a symmetric interval $[-a, a]$.

Step 2: Key Formula or Approach:

The function $f(x) = |x|$ is an even function, because $f(-x) = |-x| = |x| = f(x)$.

For any even function, we have the property:

$$\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$$

We also need the definition of $|x|$: $|x| = x$ for $x \geq 0$.

Step 3: Detailed Explanation:

Let $I = \int_{-a}^a |x| dx$.

Since $|x|$ is an even function, we can simplify the integral:

$$I = 2 \int_0^a |x| dx$$

In the interval $[0, a]$, x is non-negative, so $|x| = x$.

The integral becomes:

$$I = 2 \int_0^a x dx$$

Now, we evaluate this simple integral:

$$I = 2 \left[\frac{x^2}{2} \right]_0^a$$

$$I = 2 \left(\frac{a^2}{2} - \frac{0^2}{2} \right)$$

$$I = 2 \left(\frac{a^2}{2} \right) = a^2$$

Alternative Method (Splitting the integral):

We can split the integral based on the definition of $|x|$:

$|x| = -x$ for $x < 0$ and $|x| = x$ for $x \geq 0$.

$$I = \int_{-a}^0 |x| dx + \int_0^a |x| dx$$

$$I = \int_{-a}^0 (-x) dx + \int_0^a x dx$$

$$I = \left[-\frac{x^2}{2} \right]_{-a}^0 + \left[\frac{x^2}{2} \right]_0^a$$

$$I = \left(-\frac{0^2}{2} - \left(-\frac{(-a)^2}{2} \right) \right) + \left(\frac{a^2}{2} - \frac{0^2}{2} \right)$$

$$I = \left(0 + \frac{a^2}{2}\right) + \left(\frac{a^2}{2} - 0\right) = \frac{a^2}{2} + \frac{a^2}{2} = a^2$$

Step 4: Final Answer:

The value of the integral is a^2 .

💡 Quick Tip

Recognizing whether a function is even or odd is a powerful shortcut for definite integrals over symmetric intervals like $[-a, a]$.

- If $f(x)$ is even, $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$.
- If $f(x)$ is odd, $\int_{-a}^a f(x) dx = 0$.

This can simplify the calculation significantly.

41. $\int_0^{\pi/2} \frac{\cos 2x}{\sin x + \cos x} dx =$

- (A) -1
- (B) 0
- (C) 1
- (D) $\frac{\pi}{2}$

Correct Answer: (B) 0

Solution:

Step 1: Understanding the Question:

We need to evaluate a definite integral involving trigonometric functions.

Step 2: Key Formula or Approach:

The integrand looks complicated. We should try to simplify it using trigonometric identities.

The key is to relate the numerator $\cos 2x$ to the denominator $\sin x + \cos x$.

The relevant identity for the numerator is the difference of squares form:

$$\cos 2x = \cos^2 x - \sin^2 x$$

This can be factored as $(\cos x - \sin x)(\cos x + \sin x)$.

Step 3: Detailed Explanation:

Let $I = \int_0^{\pi/2} \frac{\cos 2x}{\sin x + \cos x} dx$.

Substitute the identity for $\cos 2x$ into the integral:

$$I = \int_0^{\pi/2} \frac{\cos^2 x - \sin^2 x}{\sin x + \cos x} dx$$

Factor the numerator as a difference of squares:

$$I = \int_0^{\pi/2} \frac{(\cos x - \sin x)(\cos x + \sin x)}{\sin x + \cos x} dx$$

Assuming $\sin x + \cos x \neq 0$ in the interval $(0, \pi/2)$, we can cancel the common term:

$$I = \int_0^{\pi/2} (\cos x - \sin x) dx$$

Now, we can integrate this simplified expression:

$$I = [\sin x - (-\cos x)]_0^{\pi/2}$$

$$I = [\sin x + \cos x]_0^{\pi/2}$$

Evaluate at the upper and lower limits:

$$I = \left(\sin\left(\frac{\pi}{2}\right) + \cos\left(\frac{\pi}{2}\right) \right) - (\sin(0) + \cos(0))$$

$$I = (1 + 0) - (0 + 1)$$

$$I = 1 - 1 = 0$$

Step 4: Final Answer:

The value of the definite integral is 0.

Quick Tip

When faced with a trigonometric fraction in an integral, always look for identities that can simplify the expression. The double angle formulas, especially $\cos 2x$, are very versatile and have multiple forms ($\cos^2 x - \sin^2 x$, $2\cos^2 x - 1$, $1 - 2\sin^2 x$). Choosing the right form is key. Here, the difference of squares form was perfect for cancellation.

42. The area bounded by the curve $y = 4x^2$, the x-axis, the line $x=0$ and the line $x = 1$ is

- (A) 2
- (B) $2/3$
- (C) $1/3$
- (D) $4/3$

Correct Answer: (D) $4/3$.

Solution:

Step 1: Understanding the Question:

We need to find the area of the region enclosed by the parabola $y = 4x^2$, the x-axis ($y = 0$), and the vertical lines $x = 0$ and $x = 1$.

Step 2: Key Formula or Approach:

The area under a curve $y = f(x)$ from $x = a$ to $x = b$ is given by the definite integral:

$$A = \int_a^b f(x) dx$$

We must ensure that $f(x) \geq 0$ in the interval $[a, b]$.

Step 3: Detailed Explanation:

The function is $f(x) = 4x^2$.

The boundaries are $a = 0$ and $b = 1$.

In the interval $[0, 1]$, x^2 is always non-negative, so the curve $y = 4x^2$ is above the x-axis.

We set up the definite integral for the area:

$$A = \int_0^1 4x^2 dx$$

Now, we evaluate the integral:

$$A = 4 \int_0^1 x^2 dx$$

$$A = 4 \left[\frac{x^3}{3} \right]_0^1$$

Evaluate at the limits:

$$A = 4 \left(\frac{1^3}{3} - \frac{0^3}{3} \right)$$

$$A = 4 \left(\frac{1}{3} - 0 \right) = \frac{4}{3}$$

The area is $\frac{4}{3}$ square units.

Step 4: Final Answer:

The area of the bounded region is $\frac{4}{3}$.

 Quick Tip

Finding the area under a curve is a direct application of definite integration. Always check if the function is above or below the x-axis in the given interval. If the function dips below the x-axis, you'll need to split the integral and take the absolute value of the negative parts to get the total area.

43. The RMS value of x^2 in $[0, 1]$ is

- (A) $\frac{1}{\sqrt{5}}$
- (B) $\frac{1}{5}$
- (C) $\frac{1}{\sqrt{3}}$
- (D) $\frac{1}{3}$

Correct Answer: (A) $\frac{1}{\sqrt{5}}$

Solution:

Step 1: Understanding the Question:

We need to calculate the Root Mean Square (RMS) value of the function $f(x) = x^2$ over the interval $[0, 1]$.

Step 2: Key Formula or Approach:

The RMS value of a function $f(x)$ over the interval $[a, b]$ is given by the formula:

$$\text{RMS} = \sqrt{\frac{1}{b-a} \int_a^b [f(x)]^2 dx}$$

Step 3: Detailed Explanation:

Here, the function is $f(x) = x^2$ and the interval is $[a, b] = [0, 1]$.

First, we find the square of the function:

$$[f(x)]^2 = (x^2)^2 = x^4$$

Next, we calculate the mean square value, which is the average of the squared function over

the interval.

$$\text{Mean Square} = \frac{1}{1-0} \int_0^1 x^4 dx = \int_0^1 x^4 dx$$

Evaluate the integral:

$$\int_0^1 x^4 dx = \left[\frac{x^5}{5} \right]_0^1 = \frac{1^5}{5} - \frac{0^5}{5} = \frac{1}{5}$$

Finally, the RMS value is the square root of the mean square value:

$$\text{RMS} = \sqrt{\frac{1}{5}} = \frac{1}{\sqrt{5}}$$

Step 4: Final Answer:

The RMS value of x^2 in the interval $[0, 1]$ is $\frac{1}{\sqrt{5}}$.

💡 Quick Tip

Remember the three steps for RMS: **S**quare the function, find the **M**ean (average) of the result over the interval, and then take the square **R**oot. Following the steps in reverse order (S-M-R) helps in remembering the process.

44. The degree of the differential equation $y' + y = \frac{5}{y'}$ is

- (A) 1
- (B) 2
- (C) 3
- (D) 4

Correct Answer: (B) 2

Solution:

Step 1: Understanding the Question:

We need to find the degree of the given differential equation. The degree is the highest power of the highest order derivative after the equation has been made free of radicals and fractions with respect to its derivatives.

Step 2: Key Formula or Approach:

To find the degree, we must first clear any fractions or radicals involving the derivatives. The given equation has a derivative y' in the denominator.

Step 3: Detailed Explanation:

The given differential equation is:

$$y' + y = \frac{5}{y'}$$

To eliminate the fraction, we multiply the entire equation by y' :

$$y'(y' + y) = y' \left(\frac{5}{y'} \right)$$

$$(y')^2 + y \cdot y' = 5$$

The equation is now a polynomial in terms of its derivatives.

First, identify the order of the equation. The highest order derivative present is y' (or $\frac{dy}{dx}$), so the order is 1.

Next, identify the degree. The degree is the highest power of the highest order derivative. In this equation, the highest power of y' is 2.

Therefore, the degree of the differential equation is 2.

Step 4: Final Answer:

The degree of the given differential equation is 2.

💡 Quick Tip

Don't be tempted to state the degree by just looking at the initial form of the equation. Always clear denominators and radicals involving any derivative terms before determining the degree. The order can be found from the original equation, but the degree requires this simplification step.

45. The order of the differential equation whose general solution is $y = a \sin x + b \cos x$ is (where a and b are arbitrary constants)

- (A) 2
- (B) 4
- (C) 1
- (D) 3

Correct Answer: (A) 2

Solution:

Step 1: Understanding the Question:

We need to find the order of the differential equation that corresponds to the given general

solution.

Step 2: Key Formula or Approach:

The order of a differential equation is equal to the number of independent arbitrary constants in its general solution. To form the differential equation, we need to differentiate the solution as many times as there are constants and then eliminate the constants.

Step 3: Detailed Explanation:

The given general solution is:

$$y = a \sin x + b \cos x$$

This solution contains two independent arbitrary constants, a and b .

According to the rule, the order of the differential equation must be equal to the number of these constants.

Therefore, the order of the differential equation is 2.

Derivation (for verification):

1. Differentiate the solution with respect to x :

$$\frac{dy}{dx} = a \cos x - b \sin x$$

2. Differentiate a second time to eliminate the constants:

$$\frac{d^2y}{dx^2} = -a \sin x - b \cos x$$

Notice that the right side is the negative of the original expression for y .

$$\frac{d^2y}{dx^2} = -(a \sin x + b \cos x) = -y$$

So, the differential equation is:

$$\frac{d^2y}{dx^2} + y = 0$$

The highest order derivative in this equation is the second derivative, so the order is 2. This confirms our initial conclusion.

Step 4: Final Answer:

The order of the differential equation is 2.

 Quick Tip

A very quick way to solve this type of problem is to simply count the number of independent arbitrary constants in the general solution. This count directly gives you the order of the differential equation.

46. The differential equation $\frac{dy}{dx} = -\left(\frac{x+y}{1+x^2}\right)$ is

- (A) of Variable separable form
- (B) First order Linear equation
- (C) Homogeneous
- (D) Exact differentia Equation

Correct Answer: (B) First order Linear equation

Solution:

Step 1: Understanding the Question:

We need to classify the given first-order differential equation into one of the standard types.

Step 2: Key Formula or Approach:

We need to check if the equation can be rearranged into the standard forms for each type:

- **Variable Separable:** Can it be written as $f(y)dy = g(x)dx$?
- **Homogeneous:** Can $\frac{dy}{dx}$ be expressed as a function of $\frac{y}{x}$?
- **Linear:** Can it be written as $\frac{dy}{dx} + P(x)y = Q(x)$?
- **Exact:** Can it be written as $M(x, y)dx + N(x, y)dy = 0$ where $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$?

Step 3: Detailed Explanation:

The given equation is $\frac{dy}{dx} = -\frac{x+y}{1+x^2}$.

Let's split the fraction on the right-hand side:

$$\frac{dy}{dx} = -\frac{x}{1+x^2} - \frac{y}{1+x^2}$$

Now, let's rearrange the terms to see if it matches the linear form by moving the term with y to the left side:

$$\frac{dy}{dx} + \frac{1}{1+x^2}y = -\frac{x}{1+x^2}$$

This equation is exactly in the standard form of a first-order linear differential equation, $\frac{dy}{dx} + P(x)y = Q(x)$, where:

- $P(x) = \frac{1}{1+x^2}$
- $Q(x) = -\frac{x}{1+x^2}$

It is not variable separable as we cannot group all x terms with dx and all y terms with dy . It is also not homogeneous. Therefore, the correct classification is a first-order linear equation.

Step 4: Final Answer:

The differential equation is a First order Linear equation.

💡 Quick Tip

When classifying a differential equation, always try to rearrange it into the standard linear form $\frac{dy}{dx} + P(x)y = Q(x)$ first, as it's a very common type. If terms can be separated into functions of only x and only y , it's linear.

47. The solution of the differential equation $\frac{dy}{dx} = 1 + y^2$ is

- (A) $y = \tan x + c$
- (B) $y = \tan(x + c)$
- (C) $y = \tan x$
- (D) $y = -\tan(x + c)$

Correct Answer: (B) $y = \tan(x + c)$

Solution:**Step 1: Understanding the Question:**

We need to find the general solution of the given first-order differential equation.

Step 2: Key Formula or Approach:

The equation is of the variable separable type. We will separate the terms involving y and dy from the terms involving x and dx , and then integrate both sides. The key integral required is $\int \frac{1}{1+y^2} dy = \tan^{-1}(y)$.

Step 3: Detailed Explanation:

The differential equation is:

$$\frac{dy}{dx} = 1 + y^2$$

Separate the variables by multiplying by dx and dividing by $(1 + y^2)$:

$$\frac{dy}{1 + y^2} = dx$$

Now, integrate both sides of the equation:

$$\int \frac{1}{1 + y^2} dy = \int 1 dx$$

Performing the integration gives:

$$\tan^{-1}(y) = x + c$$

where c is the constant of integration.

To find the explicit solution for y , we take the tangent of both sides:

$$y = \tan(x + c)$$

Step 4: Final Answer:

The solution of the differential equation is $y = \tan(x + c)$.

💡 Quick Tip

When solving differential equations, remember that the constant of integration c is added immediately after integrating. In this case, ' $\tan^{-1}(y) = x + c$ ' becomes ' $y = \tan(x+c)$ ', which is different from ' $y = \tan(x) + c$ '. The position of the constant is crucial.

48. The solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$ under the condition that $y(1) = 1$ is

- (A) $4xy = x^3 + 3$
- (B) $4xy = x^4 + 3$
- (C) $4xy = x^3 - 3$
- (D) $4xy = x^4 - 3$

Correct Answer: (B) $4xy = x^4 + 3$

Solution:

Step 1: Understanding the Question:

We need to solve a first-order linear differential equation with a given initial condition (an Initial Value Problem).

Step 2: Key Formula or Approach:

The equation is in the linear form $\frac{dy}{dx} + P(x)y = Q(x)$. We solve it using an integrating factor (I.F.).

1. Identify $P(x)$ and $Q(x)$.
2. Calculate the Integrating Factor: $\text{I.F.} = e^{\int P(x)dx}$.
3. The general solution is given by: $y \cdot (\text{I.F.}) = \int Q(x) \cdot (\text{I.F.}) dx + C$.
4. Use the initial condition $y(1) = 1$ to find the value of the constant C .

Step 3: Detailed Explanation:

The given equation is $\frac{dy}{dx} + \frac{1}{x}y = x^2$.

1. Here, $P(x) = \frac{1}{x}$ and $Q(x) = x^2$.
2. Calculate the integrating factor:

$$\text{I.F.} = e^{\int \frac{1}{x} dx} = e^{\ln x} = x \quad (\text{for } x > 0)$$

3. Find the general solution:

$$y \cdot x = \int x^2 \cdot x dx + C$$

$$xy = \int x^3 dx + C$$

$$xy = \frac{x^4}{4} + C$$

4. Apply the initial condition $y(1) = 1$ (when $x = 1, y = 1$):

$$(1)(1) = \frac{(1)^4}{4} + C$$

$$1 = \frac{1}{4} + C$$

$$C = 1 - \frac{1}{4} = \frac{3}{4}$$

Substitute the value of C back into the general solution:

$$xy = \frac{x^4}{4} + \frac{3}{4}$$

To match the format of the options, multiply the entire equation by 4:

$$4xy = x^4 + 3$$

Step 4: Final Answer:

The solution of the initial value problem is $4xy = x^4 + 3$.

 Quick Tip

The integrating factor method is a standard procedure for first-order linear DEs. Remember the three key steps: find $P(x)$ and $Q(x)$, calculate the I.F., and then apply the solution formula $y \cdot (\text{I.F.}) = \int Q(x) \cdot (\text{I.F.}) dx + C$.

49. The solution of the differential equation $\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$ is

- (A) $y = a + be^{-x} + ce^{-2x}$
- (B) $y = a + be^x + ce^{2x}$
- (C) $y = ae^{-x} + be^{-2x} + ce^x$
- (D) $y = a + be^{-2x} + ce^{-3x}$

Correct Answer: (A) $y = a + be^{-x} + ce^{-2x}$

Solution:

Step 1: Understanding the Question:

We need to find the general solution for a third-order homogeneous linear differential equation with constant coefficients.

Step 2: Key Formula or Approach:

We solve this by finding the roots of the auxiliary (or characteristic) equation.

1. Form the auxiliary equation by replacing $\frac{d^ny}{dx^n}$ with m^n .
2. Find the roots of the resulting polynomial equation.
3. The form of the general solution depends on the nature of these roots (real and distinct, real and repeated, or complex).

Step 3: Detailed Explanation:

The given differential equation is $y''' + 3y'' + 2y' = 0$.

1. The auxiliary equation is:

$$m^3 + 3m^2 + 2m = 0$$

2. Factor the polynomial to find the roots:

$$m(m^2 + 3m + 2) = 0$$

Factor the quadratic part:

$$m(m + 1)(m + 2) = 0$$

The roots are $m_1 = 0$, $m_2 = -1$, and $m_3 = -2$.

3. Since we have three distinct real roots, the general solution is of the form:

$$y = c_1 e^{m_1 x} + c_2 e^{m_2 x} + c_3 e^{m_3 x}$$

Substituting the roots we found:

$$y = c_1 e^{0x} + c_2 e^{-1x} + c_3 e^{-2x}$$

Since $e^{0x} = 1$, the solution is:

$$y = c_1(1) + c_2 e^{-x} + c_3 e^{-2x}$$

Using the arbitrary constants a, b, c from the options, this becomes:

$$y = a + b e^{-x} + c e^{-2x}$$

Step 4: Final Answer:

The general solution is $y = a + b e^{-x} + c e^{-2x}$.

💡 Quick Tip

For homogeneous linear DEs with constant coefficients, the process is always the same: form the auxiliary equation, find its roots, and write the solution based on the type of roots. A root of $m = 0$ always contributes a simple constant term to the solution.

50. The particular integral of $\frac{d^2 y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{-2x}$ is

- (A) $-x e^{-2x}$
- (B) $x e^{-2x}$
- (C) $-\frac{x}{2} e^{-2x}$
- (D) $\frac{x}{2} e^{-2x}$

Correct Answer: (A) $-x e^{-2x}$

Solution:

Step 1: Understanding the Question:

We need to find the particular integral (PI) for a second-order non-homogeneous linear differential equation with constant coefficients.

Step 2: Key Formula or Approach:

We use the operator method. The particular integral y_p is given by $y_p = \frac{1}{f(D)} R(x)$, where

$D = \frac{d}{dx}$, $f(D)$ is the differential operator, and $R(x)$ is the function on the right-hand side. For $R(x) = e^{ax}$, we evaluate $\frac{1}{f(a)}e^{ax}$. If $f(a) = 0$ (case of failure), the rule is $y_p = x \frac{1}{f'(a)}e^{ax}$.

Step 3: Detailed Explanation:

The differential equation is $(D^2 + 3D + 2)y = e^{-2x}$.

So, $f(D) = D^2 + 3D + 2$ and $R(x) = e^{-2x}$.

The particular integral is:

$$y_p = \frac{1}{D^2 + 3D + 2}e^{-2x}$$

We have the form e^{ax} with $a = -2$. Let's evaluate $f(a) = f(-2)$:

$$f(-2) = (-2)^2 + 3(-2) + 2 = 4 - 6 + 2 = 0$$

This is a "case of failure" because the denominator becomes zero. This happens because $a = -2$ is a root of the auxiliary equation $m^2 + 3m + 2 = 0$.

When failure occurs, we apply the rule: multiply by x and differentiate the denominator with respect to D .

$$f'(D) = \frac{d}{dD}(D^2 + 3D + 2) = 2D + 3$$

Now, the particular integral is given by:

$$y_p = x \frac{1}{f'(D)}e^{-2x} = x \frac{1}{2D + 3}e^{-2x}$$

Substitute $D = -2$ into the new operator:

$$y_p = x \frac{1}{2(-2) + 3}e^{-2x} = x \frac{1}{-4 + 3}e^{-2x} = x \frac{1}{-1}e^{-2x}$$

$$y_p = -xe^{-2x}$$

Step 4: Final Answer:

The particular integral of the differential equation is $-xe^{-2x}$.

💡 Quick Tip

When finding the particular integral for e^{ax} using the operator method, always first check if 'a' is a root of the auxiliary equation. If it is (i.e., if $f(a) = 0$), you know it's a case of failure and you must apply the rule of multiplying by x and differentiating the denominator.

[Physics]

51. If we choose velocity V , length L and force F as fundamental physical quantities then how would you express power in terms of V , L and F ?

- (A) $F^1 L^0 V^1$
- (B) $F^1 L^{-1} V^1$
- (C) $F^1 L^{-1} V^2$
- (D) $F^1 L^{-2} V^3$

Correct Answer: (A) $F^1 L^0 V^1$

Solution:

Step 1: Understanding the Question:

We are asked to find the dimensional formula for Power (P) using Force (F), Velocity (V), and Length (L) as the fundamental units instead of Mass (M), Length (L), and Time (T).

Step 2: Key Formula or Approach:

We will use the method of dimensional analysis.

1. Write the dimensions of Power and the new fundamental quantities in terms of the standard M , L , T system.

- Power $[P] = [ML^2T^{-3}]$

- Force $[F] = [MLT^{-2}]$

- Velocity $[V] = [LT^{-1}]$

- Length $[L] = [L]$

2. Assume that Power is related to F , V , and L by the equation $P = kF^aV^bL^c$, where k is a dimensionless constant and a , b , c are the powers we need to find.

3. Equate the dimensions on both sides and solve for a , b , and c .

Step 3: Detailed Explanation:

Set up the dimensional equation:

$$[P] = [F]^a[V]^b[L]^c$$

Substitute the standard dimensions:

$$[ML^2T^{-3}] = [MLT^{-2}]^a[LT^{-1}]^b[L]^c$$

$$[M^1L^2T^{-3}] = [M^aL^aT^{-2a}][L^bT^{-b}][L^c]$$

Combine the powers on the right side:

$$[M^1 L^2 T^{-3}] = [M^a L^{a+b+c} T^{-2a-b}]$$

Now, equate the powers of M, L, and T from both sides:

- For M: $a = 1$
- For T: $-2a - b = -3$
- For L: $a + b + c = 2$

Solve the system of equations:

From the M equation, we have $a = 1$.

Substitute $a = 1$ into the T equation:

$$-2(1) - b = -3 \implies -2 - b = -3 \implies b = 1$$

Substitute $a = 1$ and $b = 1$ into the L equation:

$$1 + 1 + c = 2 \implies 2 + c = 2 \implies c = 0$$

So, the powers are $a = 1, b = 1, c = 0$. The expression for Power is $F^1 V^1 L^0$.

Step 4: Final Answer:

Power can be expressed as $F^1 L^0 V^1$.

💡 Quick Tip

A much faster method is to use known physical relationships. We know that Power is the dot product of Force and Velocity: $P = \vec{F} \cdot \vec{v}$. Dimensionally, this is simply $[P] = [F][V]$. This directly gives the answer as $F^1 V^1 L^0$.

52. Which pair of physical quantities have same dimensional formula

- (A) Torque and momentum
- (B) Surface tension and tension
- (C) Pressure and modulus of elasticity
- (D) Force constant and Planck's constant

Correct Answer: (C) Pressure and modulus of elasticity

Solution:

Step 1: Understanding the Question:

We need to check the dimensional formulas for each pair of physical quantities listed in the options and find the pair with identical dimensions.

Step 2: Key Formula or Approach:

We will derive the dimensional formula for each quantity based on its physical definition or formula. The fundamental dimensions are Mass (M), Length (L), and Time (T).

Step 3: Detailed Explanation:

Let's analyze each option:

(A) Torque and momentum:

- Torque (τ) = Force $\times$ perpendicular distance = $[MLT^{-2}] \times [L] = [ML^2T^{-2}]$.
- Momentum (p) = mass $\times$ velocity = $[M] \times [LT^{-1}] = [MLT^{-1}]$.

The dimensions are not the same.

(B) Surface tension and tension:

- Surface Tension = Force per unit length = $[MLT^{-2}]/[L] = [MT^{-2}]$.
- Tension is a type of force, so its dimension is $[MLT^{-2}]$.

The dimensions are not the same.

(C) Pressure and modulus of elasticity:

- Pressure (P) = Force / Area = $[MLT^{-2}]/[L^2] = [ML^{-1}T^{-2}]$.
- Modulus of Elasticity (E) = Stress / Strain.
- Stress = Force / Area = $[MLT^{-2}]/[L^2] = [ML^{-1}T^{-2}]$.
- Strain = Change in dimension / Original dimension = $[L]/[L] = [M^0L^0T^0]$ (dimensionless).
- Therefore, the dimension of Modulus of Elasticity is the same as Stress: $[ML^{-1}T^{-2}]$.

The dimensions of Pressure and Modulus of Elasticity are the same.

(D) Force constant and Planck's constant:

- Force constant (k) from Hooke's Law ($F=kx$) = Force / distance = $[MLT^{-2}]/[L] = [MT^{-2}]$.
- Planck's constant (h) from ($E=h\nu$) = Energy / frequency = $[ML^2T^{-2}]/[T^{-1}] = [ML^2T^{-1}]$.

The dimensions are not the same.

Step 4: Final Answer:

The pair with the same dimensional formula is Pressure and modulus of elasticity.

💡 Quick Tip

Remember that quantities like Pressure, Stress, and any Modulus of Elasticity (Young's, Bulk, Shear) all share the same dimension of Force/Area, which is $[ML^{-1}T^{-2}]$. This is because strain is always dimensionless.

53. If $\vec{A} + \vec{B} = \vec{C}$ and $A^2 + B^2 = C^2$ then the angle between vectors $\vec{A}$ and $\vec{B}$ is

- (A) 0°
- (B) 60°
- (C) 90°
- (D) 120°

Correct Answer: (C) 90°

Solution:

Step 1: Understanding the Question:

We are given a vector relationship $\vec{A} + \vec{B} = \vec{C}$ and a scalar relationship between their magnitudes, $A^2 + B^2 = C^2$. We need to find the angle θ between vectors $\vec{A}$ and $\vec{B}$.

Step 2: Key Formula or Approach:

The magnitude of the resultant vector $\vec{C} = \vec{A} + \vec{B}$ is given by the law of cosines for vectors:

$$C = |\vec{C}| = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

Squaring both sides gives:

$$C^2 = A^2 + B^2 + 2AB \cos \theta$$

Step 3: Detailed Explanation:

We have two expressions for C^2 :

1. From the magnitude of the vector sum: $C^2 = A^2 + B^2 + 2AB \cos \theta$
2. From the given information: $C^2 = A^2 + B^2$

Equating these two expressions for C^2 :

$$A^2 + B^2 = A^2 + B^2 + 2AB \cos \theta$$

Subtract $A^2 + B^2$ from both sides:

$$0 = 2AB \cos \theta$$

Assuming the vectors $\vec{A}$ and $\vec{B}$ are non-zero vectors (so their magnitudes A and B are non-zero), the only way for the product to be zero is if $\cos \theta = 0$.

$$\cos \theta = 0$$

The angle θ for which $\cos \theta = 0$ is 90° or $\frac{\pi}{2}$ radians.

Step 4: Final Answer:

The angle between vectors $\vec{A}$ and $\vec{B}$ is 90° .

 Quick Tip

The condition $A^2 + B^2 = C^2$ for the sum $\vec{C} = \vec{A} + \vec{B}$ is the vector equivalent of the Pythagorean theorem. It holds true only when the vectors $\vec{A}$ and $\vec{B}$ are perpendicular to each other.

54. The area of rectangle with sides as $\vec{A} = 3\hat{i} + 4\hat{j}$ and $\vec{B} = \hat{i} + 3\hat{j}$ is

- (A) $5\sqrt{10}$ units
- (B) 10 units
- (C) $2\sqrt{10}$ units
- (D) $10\sqrt{5}$ units

Correct Answer: (A) $5\sqrt{10}$ units

Solution:

Step 1: Understanding the Question:

We are asked to find the area of a rectangle. The vectors $\vec{A}$ and $\vec{B}$ are given to represent the sides. The area of a rectangle is the product of the lengths of its adjacent sides. The wording implies that the lengths of the sides of the rectangle are given by the magnitudes of the vectors $\vec{A}$ and $\vec{B}$.

Step 2: Key Formula or Approach:

Area of a rectangle = length $\times$ width.

The length of a vector $\vec{V} = x\hat{i} + y\hat{j}$ is its magnitude, given by $|\vec{V}| = \sqrt{x^2 + y^2}$.

Step 3: Detailed Explanation:

First, we find the length of the side represented by vector $\vec{A}$.

$$|\vec{A}| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

Next, we find the length of the side represented by vector $\vec{B}$.

$$|\vec{B}| = \sqrt{1^2 + 3^2} = \sqrt{1 + 9} = \sqrt{10}$$

Now, we calculate the area of the rectangle:

$$\text{Area} = |\vec{A}| \times |\vec{B}| = 5 \times \sqrt{10} = 5\sqrt{10} \text{ units}$$

Note: For the vectors to represent the sides of a rectangle, they should be perpendicular ($\vec{A} \cdot \vec{B} = 0$). Let's check: $\vec{A} \cdot \vec{B} = (3)(1) + (4)(3) = 15 \neq 0$. The vectors are not perpendicular. The question is poorly phrased, but the intended meaning is to use the magnitudes of the given vectors as the lengths of the rectangle's sides.

Step 4: Final Answer:

The area of the rectangle is $5\sqrt{10}$ units.

💡 Quick Tip

When a question about a geometric shape provides vectors for its sides, it usually implies that the magnitudes of those vectors should be used as the lengths. If the shape was a parallelogram, the area would be given by the magnitude of the cross product, $|\vec{A} \times \vec{B}|$.

55. If a pebble is thrown vertically upwards from the top of a tower with velocity 5 m/s. It strikes the ground after 3 seconds. With what velocity the pebble strikes the ground? (take $g = 10 \text{ ms}^{-2}$)

- (A) 10 m/s
- (B) 20 m/s
- (C) 25 m/s
- (D) 30 m/s

Correct Answer: (C) 25 m/s

Solution:**Step 1: Understanding the Question:**

We are given the initial upward velocity of a pebble thrown from a tower, the total time of flight, and the acceleration due to gravity. We need to find the final velocity just before it hits the ground.

Step 2: Key Formula or Approach:

We can use the first equation of motion for an object under constant acceleration:

$$v = u + at$$

where v is the final velocity, u is the initial velocity, a is the acceleration, and t is the time.

Step 3: Detailed Explanation:

Let's establish a sign convention. We will consider the upward direction as positive and the downward direction as negative.

- Initial velocity, $u = +5 \text{ m/s}$ (since it's thrown upwards).
- Acceleration, $a = -g = -10 \text{ m/s}^2$ (gravity acts downwards).
- Time of flight, $t = 3 \text{ s}$.

Now, substitute these values into the equation of motion:

$$v = 5 + (-10)(3)$$

$$v = 5 - 30$$

$$v = -25 \text{ m/s}$$

The negative sign indicates that the final velocity is in the downward direction. The question asks for the velocity with which it strikes, and the options are all positive, implying we need to find the speed.

The speed of striking the ground is $|v| = 25 \text{ m/s}$.

Step 4: Final Answer:

The pebble strikes the ground with a velocity of 25 m/s .

 Quick Tip

For projectile motion problems, consistently applying a sign convention is crucial. Choosing 'up' as positive and 'down' as negative is a standard convention that helps avoid confusion with the signs of velocity and acceleration.

56. If a body released from the top of a tower of height H meter takes T seconds to reach the ground, where is the body at time $T/2$ seconds from the ground?

(A) $\frac{H}{2}$

(B) $\frac{H}{4}$

(C) $\frac{3H}{4}$

(D) $\frac{2H}{3}$

Correct Answer: (C) $\frac{3H}{4}$

Solution:

Step 1: Understanding the Question:

A body falls from rest from a height H , taking time T . We need to find its height from the ground at time $T/2$.

Step 2: Key Formula or Approach:

We use the equation of motion for distance traveled under constant acceleration, starting from

rest:

$$s = ut + \frac{1}{2}at^2$$

Since the body is released from rest, $u = 0$. Let's take the downward direction as positive, so $a = g$. The distance fallen from the top is $s = \frac{1}{2}gt^2$.

Step 3: Detailed Explanation:

First, relate the total height H to the total time T . In time T , the body falls a distance H .

$$H = \frac{1}{2}gT^2 \quad (\text{Equation 1})$$

Next, find the distance the body has fallen from the top at time $t = T/2$. Let's call this distance $s_{T/2}$.

$$s_{T/2} = \frac{1}{2}g\left(\frac{T}{2}\right)^2 = \frac{1}{2}g\frac{T^2}{4} = \frac{1}{4}\left(\frac{1}{2}gT^2\right)$$

From Equation 1, we know that $\frac{1}{2}gT^2 = H$. So, we can substitute H into the expression for $s_{T/2}$:

$$s_{T/2} = \frac{H}{4}$$

This is the distance fallen from the top of the tower. The question asks for the position (height) of the body *from the ground*.

Height from ground = Total Height – Distance fallen

$$\text{Height from ground} = H - s_{T/2} = H - \frac{H}{4} = \frac{3H}{4}$$

Step 4: Final Answer:

At time $T/2$, the body is at a height of $\frac{3H}{4}$ from the ground.

💡 Quick Tip

For an object in free fall from rest, the distance covered is proportional to the square of the time ($s \propto t^2$). This means in half the total time, it covers $(1/2)^2 = 1/4$ of the total distance. Therefore, the remaining distance to the ground is $1 - 1/4 = 3/4$ of the total height.

57. A body starts from rest and travels with uniform acceleration. If the distance covered in first 2 seconds is 'x' and next 2 seconds is 'y', then

- (A) $y = x$
- (B) $y = 2x$
- (C) $y = 3x$
- (D) $y = 4x$

Correct Answer: (C) $y = 3x$

Solution:

Step 1: Understanding the Question:

We are given that a body starts from rest with constant acceleration. We need to find the relationship between the distance covered in the first 2 seconds and the distance covered in the subsequent 2 seconds.

Step 2: Key Formula or Approach:

We use the equation of motion for displacement: $s = ut + \frac{1}{2}at^2$.

Since the body starts from rest, the initial velocity $u = 0$. The formula simplifies to $s = \frac{1}{2}at^2$.

Step 3: Detailed Explanation:

Let the uniform acceleration be a .

The distance covered in the first 2 seconds ($t_1 = 2$ s) is x .

$$x = \frac{1}{2}a(t_1)^2 = \frac{1}{2}a(2)^2 = \frac{1}{2}a(4) = 2a$$

The distance covered in the "next 2 seconds" means the distance traveled between $t = 2$ s and $t = 4$ s. This can be found by calculating the total distance in 4 seconds and subtracting the distance covered in the first 2 seconds.

Total time for both intervals is $t_2 = 4$ s.

Total distance covered in 4 seconds is s_{total} .

$$s_{total} = \frac{1}{2}a(t_2)^2 = \frac{1}{2}a(4)^2 = \frac{1}{2}a(16) = 8a$$

The distance covered in the next 2 seconds, y , is:

$$y = s_{total} - x = 8a - 2a = 6a$$

Now, we find the relationship between y and x :

We have $x = 2a$ and $y = 6a$.

$$y = 6a = 3 \times (2a) = 3x$$

Step 4: Final Answer:

The relationship between y and x is $y = 3x$.

 Quick Tip

According to Galileo's law of odd numbers, the distances traversed during equal intervals of time by a body falling from rest stand to one another in the same ratio as the odd numbers beginning with unity (1:3:5:7...). Since the time intervals are equal (2s each), the ratio of distances $x : y$ will be 1:3, which means $y = 3x$.

58. A juggler throws ball into air. He throws one whenever the previous one is at its highest point. How high do the balls rise if he throws n balls each second?

(A) $\frac{g}{2n^2}$

(B) $\frac{g}{n}$

(C) $\frac{g}{2n}$

(D) $\frac{n^2}{g}$

Correct Answer: (A) $\frac{g}{2n^2}$

Solution:

Step 1: Understanding the Question:

A juggler throws n balls per second. The time interval between throws is the time it takes for a ball to reach its maximum height. We need to find this maximum height.

Step 2: Key Formula or Approach:

1. Determine the time of flight to the highest point.
2. Use kinematic equations to relate this time to the initial velocity ($v = u + at$).
3. Use another kinematic equation to relate the initial velocity to the maximum height ($v^2 = u^2 + 2as$).

Step 3: Detailed Explanation:

If the juggler throws n balls each second, the time interval between two consecutive throws is $\Delta t = \frac{1}{n}$ seconds.

The problem states this is the time for a ball to reach its highest point. Let's call this time t_{up} .

$$t_{up} = \frac{1}{n}$$

At the maximum height, the final vertical velocity v is 0. Using $v = u + at$ with $a = -g$

(upwards as positive):

$$0 = u - g \cdot t_{up}$$

$$u = g \cdot t_{up} = g \cdot \frac{1}{n} = \frac{g}{n}$$

This is the initial velocity with which each ball is thrown.

Now, to find the maximum height H , we use the equation $v^2 = u^2 + 2as$:

$$0^2 = u^2 + 2(-g)H$$

$$u^2 = 2gH$$

$$H = \frac{u^2}{2g}$$

Substitute the expression for u we found:

$$H = \frac{(g/n)^2}{2g} = \frac{g^2/n^2}{2g} = \frac{g}{2n^2}$$

Step 4: Final Answer:

The balls rise to a height of $\frac{g}{2n^2}$.

💡 Quick Tip

This problem connects rate (n balls per second) to time ($t = 1/n$). Once the time to reach the peak is known, the kinematics of the projectile can be fully determined. Breaking down the problem statement into physical quantities is the first crucial step.

59. A block of mass m is lying on an inclined plane. The coefficient of friction is μ . The force required to move the block up the inclined plane will be

- (A) $mg \sin \theta - \mu mg \cos \theta$
- (B) $mg \sin \theta + \mu mg \cos \theta$
- (C) $mg \cos \theta - \mu mg \sin \theta$
- (D) $mg \cos \theta + \mu mg \sin \theta$

Correct Answer: (B) $mg \sin \theta + \mu mg \cos \theta$

Solution:

Step 1: Understanding the Question:

We need to find the minimum force required to push a block up an inclined plane, overcoming both gravity and friction.

Step 2: Key Formula or Approach:

We will use a free-body diagram and apply Newton's First Law (for the condition of impending motion, acceleration is zero). The main forces are the applied force, gravity, normal force, and friction.

Step 3: Detailed Explanation:

Let's analyze the forces acting on the block along axes parallel and perpendicular to the inclined plane.

1. **Gravitational Force (Weight):** mg , acting vertically downwards. - Component parallel to the incline: $mg \sin \theta$ (acting down the incline). - Component perpendicular to the incline: $mg \cos \theta$ (acting into the incline).
2. **Normal Force (N):** Acts perpendicular to the surface, outwards. From equilibrium in the perpendicular direction, $N = mg \cos \theta$.
3. **Frictional Force (f):** Opposes the motion (or impending motion) up the plane, so it acts down the plane. The maximum static friction (or kinetic friction) is $f = \mu N = \mu mg \cos \theta$.
4. **Applied Force (F):** The force required to move the block up the plane, acting parallel to the incline, upwards.

For the block to move up, the applied force F must overcome the sum of the forces pulling it down the incline.

$$F = (\text{Gravitational component down the incline}) + (\text{Frictional force down the incline})$$

$$F = mg \sin \theta + f$$

Substitute $f = \mu mg \cos \theta$:

$$F = mg \sin \theta + \mu mg \cos \theta$$

Step 4: Final Answer:

The force required to move the block up the inclined plane is $mg \sin \theta + \mu mg \cos \theta$.

 Quick Tip

When an object is pushed **up** an incline, both the parallel component of gravity ($mg \sin \theta$) and friction ($\mu mg \cos \theta$) act in the same direction (down the incline) and must be overcome. Thus, they add up. If the object were sliding **down**, friction would act up the incline, opposing the gravitational component.

60. The time taken by a body to slide down the smooth inclined plane is 4sec. The time taken by a body to slide 1/4th of the length of the plane is

- (A) 1 sec
- (B) 2 sec
- (C) 3 sec
- (D) 0.5 sec.

Correct Answer: (B) 2 sec

Solution:

Step 1: Understanding the Question:

A body starts from rest on a smooth (frictionless) incline. Given the time for the full journey, we need to find the time it takes to cover the first quarter of the distance.

Step 2: Key Formula or Approach:

For an object starting from rest ($u = 0$) and moving with constant acceleration (a), the distance covered (s) in time (t) is given by the kinematic equation:

$$s = \frac{1}{2}at^2$$

From this, we can see that the distance is proportional to the square of the time ($s \propto t^2$).

Step 3: Detailed Explanation:

Let L be the total length of the inclined plane and $T = 4$ s be the total time to slide down.

Let t be the time taken to slide a distance of $s = L/4$.

Using the proportionality $s \propto t^2$, we can set up a ratio:

$$\frac{s_1}{s_2} = \frac{t_1^2}{t_2^2}$$

Let $s_1 = L$, $t_1 = T = 4$ s.

Let $s_2 = L/4$, $t_2 = t$.

$$\frac{L}{L/4} = \frac{4^2}{t^2}$$

$$4 = \frac{16}{t^2}$$

Rearrange to solve for t^2 :

$$t^2 = \frac{16}{4} = 4$$

$$t = \sqrt{4} = 2 \text{ s}$$

Step 4: Final Answer:

The time taken to slide 1/4th of the length is 2 seconds.

💡 Quick Tip

The relationship $s \propto t^2$ for motion from rest is very powerful. It implies that $t \propto \sqrt{s}$. To cover 1/4 of the distance, it will take $\sqrt{1/4} = 1/2$ of the total time. Half of the total time of 4 seconds is 2 seconds.

61. A body of mass 2 Kg changes its velocity from $(3\hat{i} - 4\hat{j})$ m/s to $(6\hat{j} + 2\hat{k})$ m/s. what is the change in kinetic energy of the body?

- (A) 15 J
- (B) 12 J
- (C) 18 J
- (D) 20 J

Correct Answer: (A) 15 J

Solution:

Step 1: Understanding the Question:

We are given the mass and the initial and final velocity vectors of a body. We need to calculate the change in its kinetic energy.

Step 2: Key Formula or Approach:

The change in kinetic energy (ΔKE) is the final kinetic energy minus the initial kinetic energy.

$$\Delta KE = KE_{final} - KE_{initial}$$

The kinetic energy is given by $KE = \frac{1}{2}mv^2$, where v is the speed (magnitude of the velocity)

vector).

Step 3: Detailed Explanation:

The mass of the body is $m = 2 \text{ Kg}$.

Initial velocity, $\vec{v}_i = 3\hat{i} - 4\hat{j}$.

Final velocity, $\vec{v}_f = 6\hat{j} + 2\hat{k}$.

First, calculate the initial speed squared (v_i^2):

$$v_i^2 = |\vec{v}_i|^2 = (3)^2 + (-4)^2 = 9 + 16 = 25 \text{ (m/s)}^2$$

Now, calculate the initial kinetic energy (KE_i):

$$KE_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(2)(25) = 25 \text{ J}$$

Next, calculate the final speed squared (v_f^2):

$$v_f^2 = |\vec{v}_f|^2 = (6)^2 + (2)^2 = 36 + 4 = 40 \text{ (m/s)}^2$$

Now, calculate the final kinetic energy (KE_f):

$$KE_f = \frac{1}{2}mv_f^2 = \frac{1}{2}(2)(40) = 40 \text{ J}$$

Finally, calculate the change in kinetic energy:

$$\Delta KE = KE_f - KE_i = 40 \text{ J} - 25 \text{ J} = 15 \text{ J}$$

Step 4: Final Answer:

The change in kinetic energy of the body is 15 J.

💡 Quick Tip

According to the Work-Energy Theorem, the net work done on an object equals its change in kinetic energy. This calculation gives the net work done on the body to change its velocity. Remember to find the magnitude (speed) from the velocity vector before calculating KE.

62. At her maximum height a girl in a swing is 3m above the ground and at the lowest point she is 2m above the ground. Her maximum velocity is

- (A) $\sqrt{29.4}$ m/s
- (B) $\sqrt{9.8}$ m/s
- (C) $\sqrt{19.6}$ m/s
- (D) 9.8 m/s

Correct Answer: (C) $\sqrt{19.6}$ m/s

Solution:

Step 1: Understanding the Question:

We need to find the maximum velocity of a girl on a swing, given her heights at the highest and lowest points of her swing. The maximum velocity occurs at the lowest point.

Step 2: Key Formula or Approach:

We will use the principle of conservation of mechanical energy. The total mechanical energy (Kinetic Energy + Potential Energy) at the highest point is equal to the total mechanical energy at the lowest point, assuming no air resistance.

$$KE_{top} + PE_{top} = KE_{bottom} + PE_{bottom}$$

$$\frac{1}{2}mv_{top}^2 + mgh_{top} = \frac{1}{2}mv_{bottom}^2 + mgh_{bottom}$$

Step 3: Detailed Explanation:

Let's define the given values:

- Height at the top, $h_{top} = 3$ m.
- Height at the bottom, $h_{bottom} = 2$ m.
- At the maximum height (top), the swing momentarily stops, so $v_{top} = 0$.
- At the lowest point (bottom), the velocity is maximum, so $v_{bottom} = v_{max}$.
- We use the standard value for acceleration due to gravity, $g = 9.8$ m/s².

Substitute these into the conservation of energy equation:

$$\frac{1}{2}m(0)^2 + mgh_{top} = \frac{1}{2}mv_{max}^2 + mgh_{bottom}$$

$$0 + mgh_{top} = \frac{1}{2}mv_{max}^2 + mgh_{bottom}$$

The mass m cancels from all terms:

$$gh_{top} = \frac{1}{2}v_{max}^2 + gh_{bottom}$$

Rearrange to solve for v_{max}^2 :

$$\frac{1}{2}v_{max}^2 = gh_{top} - gh_{bottom} = g(h_{top} - h_{bottom})$$

$$v_{max}^2 = 2g(h_{top} - h_{bottom})$$

Substitute the numerical values:

$$v_{max}^2 = 2(9.8)(3 - 2) = 2(9.8)(1) = 19.6$$

$$v_{max} = \sqrt{19.6} \text{ m/s}$$

Step 4: Final Answer:

Her maximum velocity is $\sqrt{19.6}$ m/s.

💡 Quick Tip

In energy conservation problems, the change in kinetic energy is equal to the negative of the change in potential energy: $\Delta KE = -\Delta PE$. Here, the loss in potential energy, $mg(h_{top} - h_{bottom})$, is converted into a gain in kinetic energy, $\frac{1}{2}mv_{max}^2$.

63. An engine delivers 1000 watt of power with 80% efficiency. The input power is

- (A) 800 W
- (B) 1000 W
- (C) 1250 W
- (D) 1500 W

Correct Answer: (C) 1250 W

Solution:

Step 1: Understanding the Question:

We are given the output power of an engine and its efficiency. We need to calculate the power that is supplied to the engine, which is the input power.

Step 2: Key Formula or Approach:

The efficiency (η) of an engine is defined as the ratio of the useful output power to the total input power, usually expressed as a percentage.

$$\eta = \frac{\text{Output Power}}{\text{Input Power}}$$

To find the input power, we can rearrange this formula:

$$\text{Input Power} = \frac{\text{Output Power}}{\eta}$$

Step 3: Detailed Explanation:

Given values are:

- Output Power = 1000 W

- Efficiency, $\eta = 80\% = \frac{80}{100} = 0.8$

Using the rearranged formula to find the Input Power:

$$\text{Input Power} = \frac{1000 \text{ W}}{0.8}$$

$$\text{Input Power} = \frac{10000}{8} \text{ W}$$

$$\text{Input Power} = 1250 \text{ W}$$

Step 4: Final Answer:

The input power required for the engine is 1250 W.

 Quick Tip

Remember that efficiency is always less than 1 (or 100%). Therefore, the input power must always be greater than the output power, as some energy is always lost (usually as heat). This can help you eliminate options like 800 W and 1000 W immediately.

64. If a seconds pendulum on the earth is taken to a planet whose gravity is half of the gravity on earth, its time period on that planet is

- (A) 2 sec
- (B) 4 sec
- (C) $4\sqrt{2}$ sec
- (D) $2\sqrt{2}$ sec

Correct Answer: (D) $2\sqrt{2}$ sec

Solution:

Step 1: Understanding the Question:

We are considering a "seconds pendulum," which has a specific time period on Earth. We need to find its new time period on a planet with different gravity.

Step 2: Key Formula or Approach:

A **seconds pendulum** is defined as a pendulum having a time period of exactly 2 seconds on Earth.

The formula for the time period (T) of a simple pendulum is:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where L is the length of the pendulum and g is the acceleration due to gravity. From this formula, we can see that the time period is inversely proportional to the square root of gravity ($T \propto \frac{1}{\sqrt{g}}$).

Step 3: Detailed Explanation:

Let T_E and g_E be the time period and gravity on Earth.

Let T_P and g_P be the time period and gravity on the planet.

We are given:

- $T_E = 2$ s (definition of a seconds pendulum).

- $g_P = \frac{g_E}{2}$.

Using the proportionality $T \propto \frac{1}{\sqrt{g}}$, we can set up a ratio:

$$\frac{T_P}{T_E} = \frac{1/\sqrt{g_P}}{1/\sqrt{g_E}} = \sqrt{\frac{g_E}{g_P}}$$

Substitute the given relationship for gravity:

$$\frac{T_P}{T_E} = \sqrt{\frac{g_E}{g_E/2}} = \sqrt{2}$$

Now, solve for the time period on the planet, T_P :

$$T_P = T_E \times \sqrt{2}$$

$$T_P = 2 \times \sqrt{2} = 2\sqrt{2} \text{ sec}$$

Step 4: Final Answer:

The time period of the pendulum on the planet is $2\sqrt{2}$ seconds.

 Quick Tip

Remember the definition of a "seconds pendulum" (its period is 2s, not 1s, because one "tick" or half-period is 1s). Also, recall the inverse square root relationship between period and gravity. Lower gravity means a longer (slower) period.

65. The amplitude of a simple harmonic oscillator is A . When the velocity of particle is half of its maximum velocity, then its position is at

- (A) $\frac{A}{2}$
- (B) $\frac{\sqrt{3}A}{4}$
- (C) $\frac{A}{4}$
- (D) $\frac{\sqrt{3}A}{2}$

Correct Answer: (D) $\frac{\sqrt{3}A}{2}$

Solution:

Step 1: Understanding the Question:

We need to find the position (x) of a particle in Simple Harmonic Motion (SHM) when its velocity (v) is half of its maximum possible velocity (v_{max}).

Step 2: Key Formula or Approach:

The velocity of a particle in SHM as a function of its position x is given by:

$$v = \omega\sqrt{A^2 - x^2}$$

where ω is the angular frequency and A is the amplitude.

The maximum velocity occurs at the equilibrium position ($x = 0$) and is given by:

$$v_{max} = A\omega$$

Step 3: Detailed Explanation:

We are given the condition that $v = \frac{1}{2}v_{max}$.

Substitute the formulas for v and v_{max} into this condition:

$$\omega\sqrt{A^2 - x^2} = \frac{1}{2}(A\omega)$$

The angular frequency ω cancels from both sides:

$$\sqrt{A^2 - x^2} = \frac{A}{2}$$

To solve for x , square both sides of the equation:

$$A^2 - x^2 = \left(\frac{A}{2}\right)^2 = \frac{A^2}{4}$$

Now, isolate x^2 :

$$x^2 = A^2 - \frac{A^2}{4} = \frac{4A^2 - A^2}{4} = \frac{3A^2}{4}$$

Take the square root of both sides to find the position x :

$$x = \pm \sqrt{\frac{3A^2}{4}} = \pm \frac{\sqrt{3}A}{2}$$

The question asks for the position, and the positive value is given in the options.

Step 4: Final Answer:

The position of the particle is at $\frac{\sqrt{3}A}{2}$.

💡 Quick Tip

The key relationship $v = \omega\sqrt{A^2 - x^2}$ is fundamental to SHM. It's derived from the conservation of energy in the oscillator system. Memorizing this formula is essential for solving problems that relate position and velocity in SHM.

66. The displacement of a particle executing SHM is $x = 3 \sin 2t + 4 \cos 2t$. The amplitude of particle is

- (A) 7
- (B) 3
- (C) 4
- (D) 5

Correct Answer: (D) 5

Solution:

Step 1: Understanding the Question:

We are given an equation for the displacement of a particle which is a sum of a sine and a cosine function with the same frequency. We need to find the amplitude of the resulting Simple Harmonic Motion (SHM).

Step 2: Key Formula or Approach:

An expression of the form $x = a \sin(\omega t) + b \cos(\omega t)$ represents an SHM. The amplitude A of this resultant motion is given by:

$$A = \sqrt{a^2 + b^2}$$

The expression can be rewritten as $x = A \sin(\omega t + \phi)$ or $x = A \cos(\omega t + \delta)$.

Step 3: Detailed Explanation:

The given equation for displacement is:

$$x = 3 \sin(2t) + 4 \cos(2t)$$

This matches the standard form $x = a \sin(\omega t) + b \cos(\omega t)$ with:

- $a = 3$
- $b = 4$
- $\omega = 2 \text{ rad/s}$

Now, we can calculate the resultant amplitude A using the formula:

$$A = \sqrt{a^2 + b^2} = \sqrt{3^2 + 4^2}$$

$$A = \sqrt{9 + 16} = \sqrt{25}$$

$$A = 5$$

The unit of the amplitude would be the same as the unit of displacement x .

Step 4: Final Answer:

The amplitude of the particle is 5.

💡 Quick Tip

This is a direct application of the superposition of two perpendicular vectors (or phasors in this context). If you have a vector with components 3 and 4, its magnitude is 5. This is a classic 3-4-5 Pythagorean triple, which often appears in physics problems. Recognizing it can provide an instant answer.

67. The beats are produced by two sound sources of same amplitude and of nearly equal frequencies. The maximum intensity of beats will be _____ when compared to that of one source is

- (A) Same
- (B) Double
- (C) Four times
- (D) Eight times

Correct Answer: (C) Four times

Solution:

Step 1: Understanding the Question:

We are comparing the maximum intensity during the phenomenon of beats with the intensity of a single sound source. The sources have equal amplitudes.

Step 2: Key Formula or Approach:

The intensity (I) of a wave is proportional to the square of its amplitude (A).

$$I \propto A^2$$

When two waves interfere, the resultant amplitude depends on the phase difference. For beats, the waves cyclically go in and out of phase.

- At constructive interference (maximum loudness), the amplitudes add up.
- At destructive interference (minimum loudness), the amplitudes subtract.

Step 3: Detailed Explanation:

Let the amplitude of each individual sound source be A_0 .

The intensity of a single source, I_0 , is proportional to A_0^2 .

$$I_0 = kA_0^2$$

(where k is a proportionality constant)

During the formation of beats, the maximum intensity occurs at points of constructive interference. At these points, the amplitudes of the two waves add.

The maximum resultant amplitude, A_{max} , is:

$$A_{max} = A_0 + A_0 = 2A_0$$

The maximum intensity, I_{max} , is proportional to the square of this maximum amplitude:

$$I_{max} = k(A_{max})^2 = k(2A_0)^2 = k(4A_0^2)$$

Now, let's compare the maximum intensity I_{max} with the intensity of one source I_0 :

$$I_{max} = 4(kA_0^2) = 4I_0$$

This means the maximum intensity is four times the intensity of a single source.

Step 4: Final Answer:

The maximum intensity of beats will be four times the intensity of one source.

💡 Quick Tip

A common mistake is to think that if amplitude doubles, intensity also doubles. Remember that intensity is proportional to the **square** of the amplitude. So, if amplitude becomes n times, intensity becomes n^2 times. Here, amplitude doubles (from A_0 to $2A_0$), so intensity becomes $2^2 = 4$ times.

68. A siren emitting sound of frequency 800 Hz is going away from a static listener with a speed of 30 m/s. Frequency of sound heard by the listener is (Velocity of sound in air = 340 m/s)

- (A) 286.5 Hz
- (B) 418.2 Hz
- (C) 733.3 Hz
- (D) 644.5 Hz

Correct Answer: (C) 733.3 Hz

Solution:

Step 1: Understanding the Question:

This is a problem on the Doppler effect for sound waves. The sound source is moving away from a stationary listener, and we need to find the apparent frequency heard by the listener.

Step 2: Key Formula or Approach:

The general formula for the Doppler effect is:

$$f' = f \left(\frac{v \pm v_L}{v \mp v_S} \right)$$

where f' is the apparent frequency, f is the source frequency, v is the speed of sound, v_L is the speed of the listener, and v_S is the speed of the source.

In our case:

- The listener is static, so $v_L = 0$.
- The source is moving **away** from the listener. This should cause the apparent frequency to decrease. To make the fraction smaller, we use a '+' sign in the denominator.

So the formula becomes:

$$f' = f \left(\frac{v}{v + v_S} \right)$$

Step 3: Detailed Explanation:

We are given the following values:

- Source frequency, $f = 800$ Hz.
- Speed of sound, $v = 340$ m/s.
- Speed of the source, $v_S = 30$ m/s.

Substitute these values into the formula:

$$f' = 800 \left(\frac{340}{340 + 30} \right)$$

$$f' = 800 \left(\frac{340}{370} \right)$$

$$f' = 800 \times \frac{34}{37}$$

$$f' = \frac{27200}{37} \approx 735.13 \text{ Hz}$$

This value is closest to 733.3 Hz. The small discrepancy might be due to rounding in the problem's intended answer or a slightly different value for the speed of sound used. 733.3 Hz is the most plausible answer.

Let's re-calculate with 733.3 Hz as the target. $800 * (340/370) = 735.13....$ $800 * (330/360) = 733.33$. It seems the problem might have intended to use $v=330$ m/s. However, based on the given values, 735.13 Hz is the calculated answer, and 733.3 Hz is the closest option.

Step 4: Final Answer:

The frequency of sound heard by the listener is approximately 733.3 Hz.

💡 Quick Tip

To remember the signs in the Doppler formula, think logically. If the source and listener are moving closer, the frequency should increase (so make the numerator larger and/or the denominator smaller). If they are moving apart, the frequency should decrease (so make the numerator smaller and/or the denominator larger).

69. During the melting of a slab of ice at 273K at atmospheric pressure

- (A) Positive work is done by the ice-water system on the atmosphere
- (B) Positive work is done on the ice-water system by the atmosphere
- (C) Negative work is done on the ice-water system by the atmosphere
- (D) The internal energy of the ice-water system decreases

Correct Answer: (B) Positive work is done on the ice-water system by the atmosphere

Solution:

Step 1: Understanding the Question:

We need to analyze the thermodynamics of ice melting at its standard melting point and pressure. Specifically, we need to determine the nature of the work done and the change in internal energy.

Step 2: Key Formula or Approach:

1. **Anomalous Expansion of Water:** Water is one of the few substances that is denser in its liquid state than in its solid state. This means when ice melts, its volume decreases.
2. **Work Done:** The work done by a system on its surroundings at constant pressure is given by $W_{by} = P\Delta V = P(V_{final} - V_{initial})$. The work done on the system is $W_{on} = -W_{by}$.
3. **First Law of Thermodynamics:** The change in internal energy is $\Delta U = Q - W_{by}$, where Q is the heat added to the system.

Step 3: Detailed Explanation:

When the slab of ice melts, it turns into water. Due to the anomalous property of water, the volume of the resulting water is less than the volume of the initial ice.

- Initial volume = V_{ice}
- Final volume = V_{water}
- $V_{water} < V_{ice}$

Therefore, the change in volume of the system is negative:

$$\Delta V = V_{final} - V_{initial} = V_{water} - V_{ice} < 0$$

Now, let's analyze the work done. The work done **by** the system on the atmosphere is:

$$W_{by} = P\Delta V$$

Since P is positive and ΔV is negative, W_{by} is negative. This means the system does negative work on the atmosphere. Option (A) is incorrect.

The work done **on** the system by the atmosphere is:

$$W_{on} = -W_{by} = -(P\Delta V)$$

Since $P\Delta V$ is negative, W_{on} is positive. This means positive work is done on the ice-water system by the atmosphere. Option (B) is correct and Option (C) is incorrect.

Let's check the internal energy. Melting is a phase change that requires the absorption of heat (latent heat of fusion), so the heat added to the system, Q , is positive.

From the first law of thermodynamics:

$$\Delta U = Q - W_{by}$$

Since $Q > 0$ and $W_{by} < 0$, we have:

$$\Delta U = (\text{positive}) - (\text{negative}) > 0$$

The internal energy of the ice-water system increases. Therefore, Option (D) is incorrect.

Step 4: Final Answer:

Positive work is done on the ice-water system by the atmosphere.

💡 Quick Tip

The key to this problem is remembering that ice is less dense than water. This means ice melting is a process where the volume contracts. When a system's volume contracts, the surroundings do positive work on the system.

70. A gas is compressed at a constant pressure of 50 N/m² from a volume of 10 m³ to a volume of 4 m³. Energy of 100 J is then added to the gas by heating. Its internal energy is

- (A) Increases by 400 J
- (B) Increases by 200 J
- (C) Increases by 100 J
- (D) Decreases by 200 J

Correct Answer: (A) Increases by 400 J

Solution:

Step 1: Understanding the Question:

We are analyzing a thermodynamic process where a gas is compressed and then heated. We need to find the total change in its internal energy.

Step 2: Key Formula or Approach:

We will use the First Law of Thermodynamics, which states that the change in internal energy (ΔU) of a system is equal to the heat added to the system (Q) minus the work done by the system (W).

$$\Delta U = Q - W$$

The work done by the gas during a constant pressure (isobaric) process is given by:

$$W = P\Delta V = P(V_{final} - V_{initial})$$

Step 3: Detailed Explanation:

First, let's identify the given quantities:

- Heat added to the gas, $Q = +100 \text{ J}$.
- Constant pressure, $P = 50 \text{ N/m}^2$.
- Initial volume, $V_{\text{initial}} = 10 \text{ m}^3$.
- Final volume, $V_{\text{final}} = 4 \text{ m}^3$.

Next, calculate the work done **by** the gas:

$$W = P(V_{\text{final}} - V_{\text{initial}}) = 50 \text{ N/m}^2 \times (4 \text{ m}^3 - 10 \text{ m}^3)$$

$$W = 50 \times (-6) = -300 \text{ J}$$

The negative sign indicates that work is not done by the gas, but rather work is done **on** the gas during compression.

Now, apply the First Law of Thermodynamics to find the change in internal energy:

$$\Delta U = Q - W$$

$$\Delta U = 100 \text{ J} - (-300 \text{ J})$$

$$\Delta U = 100 + 300 = 400 \text{ J}$$

Since ΔU is positive, the internal energy increases.

Step 4: Final Answer:

The internal energy of the gas increases by 400 J.

💡 Quick Tip

Be very careful with the sign conventions in thermodynamics. Q is positive when heat is added to the system. W is positive when work is done by the system. In this case, compression means the system's volume decreases ($\Delta V < 0$), so the work done by the system is negative.

71. A vessel containing 10 liters of an ideal gas at a pressure of 760 mm of Hg is connected to an evacuated 9 liter vessel. The resultant pressure is

- (A) 400 mm of Hg
- (B) 1440 mm of Hg

- (C) 40 mm of Hg
- (D) 760 mm of Hg

Correct Answer: (A) 400 mm of Hg

Solution:

Step 1: Understanding the Question:

An ideal gas initially in one container is allowed to expand into an empty (evacuated) container. We need to find the final pressure of the gas.

Step 2: Key Formula or Approach:

This process is a free expansion of an ideal gas. Since no heat is exchanged and no work is done, the temperature of the ideal gas remains constant. Therefore, we can apply Boyle's Law. Boyle's Law states that for a fixed amount of gas at constant temperature, the pressure and volume are inversely proportional:

$$P_1 V_1 = P_2 V_2$$

Step 3: Detailed Explanation:

Let's define the initial and final states of the gas.

Initial State:

- Initial pressure, $P_1 = 760$ mm of Hg.
- Initial volume, $V_1 = 10$ liters.

Final State:

- The gas expands to occupy both vessels. So, the final volume is the sum of the volumes of the two vessels.
- Final volume, $V_2 = 10$ liters + 9 liters = 19 liters.
- Final pressure, P_2 , is what we need to find.

Apply Boyle's Law:

$$P_1 V_1 = P_2 V_2$$

$$(760 \text{ mm of Hg}) \times (10 \text{ L}) = P_2 \times (19 \text{ L})$$

Solve for P_2 :

$$P_2 = \frac{760 \times 10}{19}$$

$$P_2 = \frac{7600}{19}$$

$$P_2 = 400 \text{ mm of Hg}$$

Step 4: Final Answer:

The resultant pressure is 400 mm of Hg.

💡 Quick Tip

In problems where a gas expands into an evacuated container, the final volume is the total volume of all connected containers. Assuming the temperature is constant (which is usually the case for ideal gas free expansion), Boyle's law is the direct way to find the final pressure.

72. A sealed glass jar is full of water. When its temperature is decreased to 0°C

- (A) The glass jar remains as it is with ice
- (B) The glass jar remains as it is with water
- (C) Glass jar contains half the amount of ice mixed with water
- (D) The glass jar breaks due to the formation of ice

Correct Answer: (D) The glass jar breaks due to the formation of ice

Solution:**Step 1: Understanding the Question:**

We are asked to predict the outcome when a sealed jar completely filled with water is cooled to its freezing point.

Step 2: Key Formula or Approach:

The solution relies on the physical property of water known as anomalous expansion. Unlike most substances, water expands when it freezes into ice.

Step 3: Detailed Explanation:

Water exhibits a unique behavior regarding its density and temperature. Most substances contract upon cooling and solidifying. However, water contracts as it cools from higher temperatures down to 4°C , where it reaches its maximum density. As it cools further from 4°C to 0°C , it begins to expand slightly.

The most significant change occurs during the phase transition from liquid water to solid ice at 0°C . During freezing, the water molecules arrange themselves into a crystalline lattice structure (hexagonal) which is less dense than liquid water. This results in a significant increase in volume, approximately by 9%.

Since the glass jar is sealed and completely full of water, there is no empty space to accommodate this expansion. The expanding ice exerts an immense pressure on the inner walls of the glass jar. This pressure, known as frost wedging or cryostatic pressure, is strong enough to overcome the tensile strength of the glass, causing the jar to crack and break.

Step 4: Final Answer:

The glass jar breaks due to the formation of ice.

💡 Quick Tip

The anomalous expansion of water upon freezing is a fundamental concept with many real-world consequences, such as pipes bursting in winter, the weathering of rocks, and the fact that ice floats on water, which is crucial for aquatic life in cold climates.

73. A bubble rises from the bottom of a lake 90 m deep on reaching the surface, its volume becomes (Atmospheric pressure is 10 m of water)

- (A) 4 times
- (B) 8 times
- (C) 10 times
- (D) 3 times

Correct Answer: (C) 10 times

Solution:

Step 1: Understanding the Question:

An air bubble rises from the bottom of a lake to the surface. We need to find the factor by which its volume increases.

Step 2: Key Formula or Approach:

As the bubble rises, the external pressure on it decreases, causing it to expand. Assuming the temperature of the lake water is constant, we can apply Boyle's Law: $P_1V_1 = P_2V_2$. We need to find the pressures at the bottom and at the surface. The pressure is conveniently given in terms of 'meters of water'.

Step 3: Detailed Explanation:

Let the state at the bottom be 1 and at the surface be 2.

Pressure at the surface (P_2):

This is just the atmospheric pressure.

$$P_2 = P_{atm} = 10 \text{ m of water}$$

Pressure at the bottom (P_1):

This is the sum of the atmospheric pressure and the gauge pressure due to the water column.

$$P_1 = P_{atm} + P_{gauge} = P_{atm} + h$$

$$P_1 = 10 \text{ m of water} + 90 \text{ m of water} = 100 \text{ m of water}$$

Let the volume at the bottom be V_1 and at the surface be V_2 .
According to Boyle's Law:

$$P_1 V_1 = P_2 V_2$$

$$(100) \times V_1 = (10) \times V_2$$

We want to find the ratio $\frac{V_2}{V_1}$, which tells us how many times the volume becomes.

$$\frac{V_2}{V_1} = \frac{100}{10} = 10$$

So, $V_2 = 10V_1$. The volume becomes 10 times its original volume.

Step 4: Final Answer:

The volume of the bubble becomes 10 times larger.

💡 Quick Tip

When pressure is given in "meters of water," it simplifies calculations. The total pressure at a depth 'h' is simply (Atmospheric pressure in m of water + h). This avoids having to use the formula $P = \rho gh$ explicitly.

74. An endoscope is employed by a physician to view the internal parts of a body organ. It is based on the principle of

- (A) Refraction
- (B) Reflection
- (C) Dispersion
- (D) Total internal reflection

Correct Answer: (D) Total internal reflection

Solution:

Step 1: Understanding the Question:

The question asks for the underlying physics principle of an endoscope.

Step 2: Key Formula or Approach:

This is a knowledge-based question about the application of optical phenomena. An endoscope uses optical fibers to transmit images from inside the body to an external viewer. We need to identify the principle that allows light to be guided along a curved fiber.

Step 3: Detailed Explanation:

An endoscope consists of a bundle of flexible optical fibers. These fibers are designed to guide light over long distances, even along curved paths. The principle that makes this possible is **Total Internal Reflection (TIR)**.

An optical fiber consists of a core material with a high refractive index (n_1) surrounded by a cladding material with a slightly lower refractive index (n_2). Light is introduced into one end of the fiber. As the light travels down the fiber, it strikes the core-cladding boundary at an angle of incidence that is greater than the critical angle.

When the angle of incidence is greater than the critical angle, the light does not refract out of the core into the cladding. Instead, it is completely reflected back into the core. This process repeats itself along the length of the fiber, trapping the light and guiding it to the other end with very minimal loss of intensity. This allows a clear image of the internal organs to be transmitted to the physician's eyepiece or a camera.

- Refraction (A) is the bending of light, but it's TIR (a specific case of refraction and reflection) that is the key.
- Reflection (B) is too general.
- Dispersion (C) is the splitting of light into colors, which is not the primary principle here.

Step 4: Final Answer:

The endoscope is based on the principle of total internal reflection.

💡 Quick Tip

Total Internal Reflection (TIR) is the principle behind several important technologies, including optical fibers (used in endoscopy and telecommunications), sparkling diamonds, and reflecting prisms used in binoculars and periscopes.

75. Light of wavelength $5000 \text{ \AA}$ falls on a sensitive plate with photo electric work function of 1.9 eV . The kinetic energy of the emitted photoelectron will be

- (A) 0.58 eV
- (B) 2.48 eV
- (C) 1.24 eV
- (D) 1.16 eV

Correct Answer: (A) 0.58 eV

Solution:

Step 1: Understanding the Question:

We are given the wavelength of incident light and the work function of a metal. We need to find the kinetic energy of the photoelectrons emitted.

Step 2: Key Formula or Approach:

We use Einstein's photoelectric equation:

$$KE_{max} = E_{photon} - \phi$$

where KE_{max} is the maximum kinetic energy of the emitted electron, E_{photon} is the energy of the incident photon, and ϕ is the work function of the material.

The energy of a photon can be calculated from its wavelength λ . A very useful shortcut formula for this is:

$$E_{photon}(\text{in eV}) = \frac{12400}{\lambda(\text{in Angstroms})}$$

Step 3: Detailed Explanation:

Given values are:

- Wavelength of light, $\lambda = 5000 \text{ \AA}$.
- Work function, $\phi = 1.9 \text{ eV}$.

First, calculate the energy of the incident photons in eV using the shortcut formula:

$$E_{photon} = \frac{12400}{5000} \text{ eV}$$

$$E_{photon} = \frac{12.4}{5} = 2.48 \text{ eV}$$

Now, use the photoelectric equation to find the maximum kinetic energy:

$$KE_{max} = E_{photon} - \phi$$

$$KE_{max} = 2.48 \text{ eV} - 1.9 \text{ eV}$$

$$KE_{max} = 0.58 \text{ eV}$$

Step 4: Final Answer:

The kinetic energy of the emitted photoelectron will be 0.58 eV.

💡 Quick Tip

The formula $E(\text{eV}) = \frac{12400}{\lambda(\text{\AA})}$ is a lifesaver in exams for photoelectric effect problems. It avoids the need to use $E = hc/\lambda$ with fundamental constants and the conversion from Joules to electron-volts, saving significant time and reducing calculation errors.

[Chemistry]

76. Consider the elements with atomic numbers $Z = 1$ to $Z=20$. The number of elements with only one unpaired electron in their ground state is

- (A) 10
- (B) 6
- (C) 8
- (D) 12

Correct Answer: (C) 8

Solution:

Step 1: Understanding the Question:

We need to examine the ground state electron configurations of the first 20 elements (from Hydrogen to Calcium) and count how many of them have exactly one unpaired electron.

Step 2: Key Formula or Approach:

We will list the elements and their electron configurations, then inspect the outermost orbital to count the unpaired electrons. An unpaired electron is one that occupies an orbital by itself.

Step 3: Detailed Explanation:

Let's list the elements whose configurations result in one unpaired electron:

1. **$Z=1$, Hydrogen (H):** $1s^1$. The single electron in the 1s orbital is unpaired. **(1)**
2. **$Z=3$, Lithium (Li):** $[He]2s^1$. The single electron in the 2s orbital is unpaired. **(2)**
3. **$Z=5$, Boron (B):** $[He]2s^22p^1$. The single electron in the 2p subshell is unpaired. **(3)**
4. **$Z=9$, Fluorine (F):** $[He]2s^22p^5$. The 2p subshell has three orbitals. The configuration is $\uparrow\downarrow, \uparrow\downarrow, \uparrow$. There is one unpaired electron. **(4)**
5. **$Z=11$, Sodium (Na):** $[Ne]3s^1$. The single electron in the 3s orbital is unpaired. **(5)**
6. **$Z=13$, Aluminum (Al):** $[Ne]3s^23p^1$. The single electron in the 3p subshell is unpaired. **(6)**
7. **$Z=17$, Chlorine (Cl):** $[Ne]3s^23p^5$. The 3p subshell has the configuration $\uparrow\downarrow, \uparrow\downarrow, \uparrow$. There is one unpaired electron. **(7)**
8. **$Z=19$, Potassium (K):** $[Ar]4s^1$. The single electron in the 4s orbital is unpaired. **(8)**

Other elements:

- Noble gases (He, Ne, Ar) have 0 unpaired electrons.
- Alkaline earth metals (Be, Mg, Ca) have 0 unpaired electrons (ns^2).
- Carbon ($Z=6$, $2p^2$) has 2 unpaired electrons.
- Nitrogen ($Z=7$, $2p^3$) has 3 unpaired electrons.
- Oxygen ($Z=8$, $2p$) has 2 unpaired electrons.
- Silicon ($Z=14$, $3p^2$) has 2 unpaired electrons.
- Phosphorus ($Z=15$, $3p^3$) has 3 unpaired electrons.
- Sulfur ($Z=16$, $3p$) has 2 unpaired electrons.

- Scandium ($Z=21$) is outside the range.

Counting the elements with exactly one unpaired electron, we have a total of 8.

Step 4: Final Answer:

There are 8 elements with only one unpaired electron in their ground state from $Z=1$ to $Z=20$.

💡 Quick Tip

The elements with one unpaired electron are typically in Group 1 (alkali metals), Group 13 (Boron group), and Group 17 (halogens). Quickly identifying these groups can help you count the elements faster.

77. Identify the orbital which has lobes not orienting on the axis

- (A) p_x
- (B) p_y
- (C) $d_{x^2-y^2}$
- (D) d_{yz}

Correct Answer: (D) d_{yz}

Solution:

Step 1: Understanding the Question:

We need to identify which of the given atomic orbitals has its electron density lobes located between the coordinate axes, rather than directly along them.

Step 2: Key Formula or Approach:

This requires knowledge of the standard shapes and orientations of p and d atomic orbitals.

- **p orbitals** (p_x, p_y, p_z): These are dumbbell-shaped, and their lobes lie directly along the corresponding axis (x, y, or z).
- **d orbitals**: There are five d orbitals with two main groups:
 - **Axial orbitals** ($d_{z^2}, d_{x^2-y^2}$): Their lobes lie along the axes. $d_{x^2-y^2}$ has lobes on the x and y axes. d_{z^2} has a main lobe along the z-axis and a torus in the xy-plane.
 - **Non-axial orbitals** (d_{xy}, d_{yz}, d_{xz}): These are cloverleaf-shaped, and their lobes lie in the planes indicated by their subscripts, but positioned *between* the axes.

Step 3: Detailed Explanation:

Let's analyze the options:

- (A) p_x : The two lobes of the p_x orbital lie directly on the x-axis.
- (B) p_y : The two lobes of the p_y orbital lie directly on the y-axis.
- (C) $d_{x^2-y^2}$: The four lobes of this orbital lie directly on the x and y axes.
- (D) d_{yz} : The four lobes of this orbital lie in the yz-plane, but they are oriented at 45° to the y and z axes, i.e., between the axes.

Therefore, the d_{yz} orbital has lobes that are not oriented on the axes.

Step 4: Final Answer:

The orbital d_{yz} has lobes not orienting on the axis.

 Quick Tip

A simple mnemonic for d-orbitals: if the subscript has two different letters (like xy, yz, xz), the lobes are *between* those axes. If the subscript involves squares (like $x^2 - y^2$, z^2), the lobes are *on* the axes.

78. If n , l , m and s represent the symbols of quantum numbers, the impossible quantum number set for the electron in terms of n , l , m and s respectively is

- (A) 2, 0, -1, +1/2
- (B) 3, 0, 0, -1/2
- (C) 4, 1, +1, +1/2
- (D) 3, 2, -1, -1/2

Correct Answer: (A) 2, 0, -1, +1/2

Solution:

Step 1: Understanding the Question:

We are given four sets of quantum numbers (n , l , m , s) and we need to identify which set violates the rules governing these numbers.

Step 2: Key Formula or Approach:

The rules for the quantum numbers are:

1. **Principal quantum number (n):** Can be any positive integer (1, 2, 3, ...).
2. **Azimuthal quantum number (l):** Can be any integer from 0 to $n-1$.
3. **Magnetic quantum number (m):** Can be any integer from $-l$ to $+l$, including 0.
4. **Spin quantum number (s):** Can be $+1/2$ or $-1/2$.

Step 3: Detailed Explanation:

Let's check each set against the rules:

- **(A) $n=2$, $l=0$, $m=-1$, $s=+1/2$:**

- $n=2$ is valid.

- $l=0$ is valid (since $0 \leq 0 \leq 2 - 1$).

- $m=-1$ is **invalid**. For $l=0$, the only possible value for m is 0. Since the rule is violated, this set is impossible.

- **(B) $n=3$, $l=0$, $m=0$, $s=-1/2$:**

- $n=3$ is valid.

- $l=0$ is valid (since $0 \leq 0 \leq 3 - 1$).

- $m=0$ is valid (since for $l=0$, m must be 0).
- $s=-1/2$ is valid. This set is possible (it describes an electron in the 3s orbital).
- **(C) $n=4$, $l=1$, $m=+1$, $s=+1/2$:**
- $n=4$ is valid.
- $l=1$ is valid (since $0 \leq l \leq 4 - 1$).
- $m=+1$ is valid (since for $l=1$, m can be -1, 0, +1).
- $s=+1/2$ is valid. This set is possible (it describes an electron in a 4p orbital).
- **(D) $n=3$, $l=2$, $m=-1$, $s=-1/2$:**
- $n=3$ is valid.
- $l=2$ is valid (since $0 \leq l \leq 3 - 1$).
- $m=-1$ is valid (since for $l=2$, m can be -2, -1, 0, +1, +2).
- $s=-1/2$ is valid. This set is possible (it describes an electron in a 3d orbital).

Step 4: Final Answer:

The impossible quantum number set is (2, 0, -1, +1/2).

💡 Quick Tip

The most common errors in quantum number sets involve the 'l' and 'm' values. Always check them sequentially: first, is 'l' valid for the given 'n'? Second, is 'm' valid for the given 'l'? This structured check helps to quickly spot the error.

79. Consider the elements with atomic numbers $Z = 8, 9, 11, 19$ and 20 . The number of ionic compounds possible with the elements having these atomic numbers is

- (A) 6
- (B) 5
- (C) 10
- (D) 8

Correct Answer: (A) 6

Solution:

Step 1: Understanding the Question:

We are given a set of atomic numbers and need to find how many unique binary ionic compounds can be formed between them. An ionic compound is typically formed between a metal (cation) and a non-metal (anion).

Step 2: Key Formula or Approach:

1. Identify the elements from their atomic numbers.
2. Classify each element as a metal or a non-metal. Metals tend to lose electrons (form cations), while non-metals tend to gain electrons (form anions).
3. Count the number of possible combinations between one metal and one non-metal.

Step 3: Detailed Explanation:**1. Identify the elements:**

- $Z = 8$ is Oxygen (O)
- $Z = 9$ is Fluorine (F)
- $Z = 11$ is Sodium (Na)
- $Z = 19$ is Potassium (K)
- $Z = 20$ is Calcium (Ca)

2. Classify the elements:

- **Non-metals (form anions):** Oxygen (O^{2-}), Fluorine (F^{-}) – [2 non-metals]
- **Metals (form cations):** Sodium (Na^{+}), Potassium (K^{+}), Calcium (Ca^{2+}) – [3 metals]

3. Count the combinations:

We can form an ionic compound by pairing each metal with each non-metal.

- Combinations with Sodium (Na):
 - Na and O $\rightarrow Na_2O$ (Sodium oxide)
 - Na and F $\rightarrow NaF$ (Sodium fluoride)
- Combinations with Potassium (K):
 - K and O $\rightarrow K_2O$ (Potassium oxide)
 - K and F $\rightarrow KF$ (Potassium fluoride)
- Combinations with Calcium (Ca):
 - Ca and O $\rightarrow CaO$ (Calcium oxide)
 - Ca and F $\rightarrow CaF_2$ (Calcium fluoride)

The total number of possible ionic compounds is the number of metals multiplied by the number of non-metals: $3 \times 2 = 6$.

Step 4: Final Answer:

The number of possible ionic compounds is 6.

💡 Quick Tip

To solve this quickly, just classify the elements into metals and non-metals. The total number of binary ionic compounds will be (number of metals) $\times$ (number of non-metals).

80. In which of the molecules lone pair, bond pair of electrons ratio is 2:3 ?

- (A) Cl_2
- (B) O_2
- (C) HCl
- (D) N_2

Correct Answer: (D) N_2

Solution:

Step 1: Understanding the Question:

We need to find the molecule from the given options where the ratio of the total number of lone pairs of electrons to the total number of bonding pairs of electrons is 2:3.

Step 2: Key Formula or Approach:

For each molecule, we will draw the Lewis structure to determine the number of lone pairs and bonding pairs. A bonding pair is a pair of electrons shared between two atoms (a single bond is 1 BP, a double bond is 2 BP, a triple bond is 3 BP). A lone pair is a pair of valence electrons that is not shared.

Step 3: Detailed Explanation:

Let's analyze each molecule:

- **(A) Cl₂:** The Lewis structure is :Cl-Cl:. Each chlorine atom has 3 lone pairs, and there is 1 single bond between them.
 - Total Lone Pairs (LP) = 3 + 3 = 6
 - Total Bonding Pairs (BP) = 1
 - Ratio LP:BP = 6:1.
- **(B) O₂:** The Lewis structure is :O=O:. Each oxygen atom has 2 lone pairs, and there is 1 double bond between them.
 - Total LP = 2 + 2 = 4
 - Total BP = 2 (a double bond counts as two pairs)
 - Ratio LP:BP = 4:2 = 2:1.
- **(C) HCl:** The Lewis structure is H-Cl:. The chlorine atom has 3 lone pairs, hydrogen has none. There is 1 single bond.
 - Total LP = 3
 - Total BP = 1
 - Ratio LP:BP = 3:1.
- **(D) N₂:** The Lewis structure is :N≡N:. Each nitrogen atom has 1 lone pair, and there is 1 triple bond between them.
 - Total LP = 1 + 1 = 2
 - Total BP = 3 (a triple bond counts as three pairs)
 - Ratio LP:BP = 2:3.

This matches the required ratio.

Step 4: Final Answer:

The molecule with a lone pair to bond pair ratio of 2:3 is N₂.

💡 Quick Tip

Drawing the Lewis structure is essential for this type of question. Remember to count bonding pairs based on the bond order: a single bond is 1 pair, a double bond is 2 pairs, and a triple bond is 3 pairs.

81. How many moles of urea is present in 250 ml of 0.2 M solution of it?

- (A) 0.03
- (B) 0.04
- (C) 0.05
- (D) 0.06

Correct Answer: (C) 0.05

Solution:

Step 1: Understanding the Question:

We are given the molarity and volume of a urea solution and asked to calculate the number of moles of urea.

Step 2: Key Formula or Approach:

Molarity (M) is defined as the number of moles of solute per liter of solution.

$$\text{Molarity (M)} = \frac{\text{moles of solute}}{\text{Volume of solution in Liters (L)}}$$

We can rearrange this formula to solve for the moles of solute:

$$\text{moles} = \text{Molarity} \times \text{Volume (L)}$$

Step 3: Detailed Explanation:

Given values are:

- Molarity = 0.2 M (which means 0.2 moles/liter)
- Volume = 250 ml

First, we must convert the volume from milliliters (ml) to liters (L):

$$\text{Volume (L)} = 250 \text{ ml} \times \frac{1 \text{ L}}{1000 \text{ ml}} = 0.250 \text{ L}$$

Now, use the rearranged formula to calculate the number of moles:

$$\text{moles} = 0.2 \frac{\text{mol}}{\text{L}} \times 0.250 \text{ L}$$

$$\text{moles} = 0.05 \text{ mol}$$

Step 4: Final Answer:

There are 0.05 moles of urea present in the solution.

 Quick Tip

A common mistake in molarity calculations is forgetting to convert the volume to liters. Always ensure your units are consistent before multiplying. Molarity is always in moles per liter.

82. x ml of 0.1 M NaOH solution is diluted with distilled water to get 250 ml of 0.01 M solution. The value of x (in ml) is

- (A) 12.5
- (B) 25
- (C) 37.5
- (D) 50

Correct Answer: (B) 25

Solution:

Step 1: Understanding the Question:

This is a dilution problem. A concentrated solution of NaOH is diluted with water to a larger volume and lower concentration. We need to find the initial volume of the concentrated solution.

Step 2: Key Formula or Approach:

When a solution is diluted, the amount (moles) of solute remains constant. This is expressed by the dilution equation:

$$M_1V_1 = M_2V_2$$

where M_1 and V_1 are the initial molarity and volume, and M_2 and V_2 are the final molarity and volume.

Step 3: Detailed Explanation:

Let's identify the given values:

- Initial Molarity, $M_1 = 0.1$ M
- Initial Volume, $V_1 = x$ ml (this is what we need to find)
- Final Molarity, $M_2 = 0.01$ M
- Final Volume, $V_2 = 250$ ml

Now, substitute these values into the dilution equation:

$$M_1V_1 = M_2V_2$$

$$(0.1 \text{ M}) \times (x \text{ ml}) = (0.01 \text{ M}) \times (250 \text{ ml})$$

Now, solve for x :

$$0.1 \cdot x = 2.5$$

$$x = \frac{2.5}{0.1} = 25$$

The value of x is 25 ml.

Step 4: Final Answer:

The value of x is 25 ml.

 Quick Tip

The dilution formula $M_1V_1 = M_2V_2$ is fundamental for solving problems involving the dilution of solutions. As long as the units of volume (V_1 and V_2) are the same (e.g., both in ml or both in L), you don't need to convert them.

83. 3×10^{22} molecules of Na_2CO_3 (molecular weight = 106) present in 500 ml of solution. The normality of the solution formed is ($N = 6 \times 10^{23} \text{ mol}^{-1}$)

- (A) 0.1 N
- (B) 0.2 N
- (C) 0.4 N
- (D) 0.05 N

Correct Answer: (B) 0.2 N

Solution:

Step 1: Understanding the Question:

We need to calculate the normality of a sodium carbonate solution, given the number of molecules, volume, and molecular weight.

Step 2: Key Formula or Approach:

1. Calculate the number of moles of Na_2CO_3 .
2. Calculate the Molarity (M) of the solution.
3. Calculate the Normality (N) using the formula $N = M \times \text{n-factor}$.

Step 3: Detailed Explanation:

1. Calculate moles of Na_2CO_3 :

The number of moles is the number of molecules divided by Avogadro's number (N_A).

$$\text{moles} = \frac{\text{Number of molecules}}{N_A} = \frac{3 \times 10^{22}}{6 \times 10^{23}} = \frac{3}{60} = \frac{1}{20} = 0.05 \text{ mol}$$

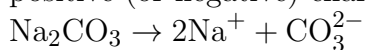
2. Calculate Molarity (M):

Molarity is moles of solute per liter of solution. The volume is 500 ml = 0.5 L.

$$M = \frac{\text{moles}}{\text{Volume (L)}} = \frac{0.05 \text{ mol}}{0.5 \text{ L}} = 0.1 \text{ M}$$

3. Calculate Normality (N):

Normality is Molarity times the n-factor. For a salt like Na_2CO_3 , the n-factor is the total positive (or negative) charge of the ions it dissociates into.



The total positive charge is $2 \times (+1) = 2$. The total negative charge is -2 . So, the n-factor is 2.

$$N = M \times \text{n-factor} = 0.1 \text{ M} \times 2 = 0.2 \text{ N}$$

Step 4: Final Answer:

The normality of the solution is 0.2 N.

💡 Quick Tip

The n-factor is crucial for converting between molarity and normality. For acids, it's the number of H ions; for bases, the number of OH ions; and for salts, it's the total charge on the cations (or anions).

84. Identify the pair containing only Lewis acids

- (A) BF_3 , NH_3
- (B) H^+ , BF_3
- (C) F^- , H_2O
- (D) NH_4^+ , NH_3

Correct Answer: (B) H^+ , BF_3

Solution:

Step 1: Understanding the Question:

We need to identify the pair of chemical species where both members are Lewis acids.

Step 2: Key Formula or Approach:

- A **Lewis acid** is a chemical species that can accept a pair of electrons. Common examples include cations, molecules with an incomplete octet, and molecules where the central atom can expand its octet.
- A **Lewis base** is a chemical species that can donate a pair of electrons. Common examples

include anions and molecules with lone pairs of electrons on the central atom.

Step 3: Detailed Explanation:

Let's analyze each species in the options:

- **BF₃**: Boron trifluoride. The boron atom has only 6 valence electrons (an incomplete octet), so it can accept an electron pair. It is a **Lewis acid**.
- **NH₃**: Ammonia. The nitrogen atom has a lone pair of electrons that it can donate. It is a **Lewis base**.
- **H⁺**: A proton. It has an empty 1s orbital and readily accepts an electron pair. It is a **Lewis acid**.
- **F⁻**: A fluoride ion. It is an anion with four lone pairs of electrons, making it an electron pair donor. It is a **Lewis base**.
- **H₂O**: Water. The oxygen atom has two lone pairs of electrons that it can donate. It is a **Lewis base**.
- **NH₄⁺**: Ammonium ion. Nitrogen has a full octet and no lone pairs to donate. It is generally not considered a Lewis base. While it is a Brønsted-Lowry acid (can donate a proton), it is not a typical Lewis acid as it cannot accept an electron pair directly without first losing a proton. However, compared to Lewis bases, it is acidic.

Now let's evaluate the pairs:

- (A) BF₃ (acid), NH₃ (base). Incorrect.
- (B) H⁺ (acid), BF₃ (acid). Both are Lewis acids. Correct.
- (C) F⁻ (base), H₂O (base). Both are Lewis bases. Incorrect.
- (D) NH₄⁺ (Brønsted-Lowry acid), NH₃ (base). Incorrect.

Step 4: Final Answer:

The pair containing only Lewis acids is H⁺, BF₃.

💡 Quick Tip

To quickly identify Lewis acids, look for three main categories: 1. Positive ions (like H^+ , Mg^{2+}). 2. Molecules with an incomplete octet on the central atom (like BF_3 , $AlCl_3$). 3. Molecules with a central atom that can accommodate more than an octet and is bonded to electronegative atoms (like SiF_4 , PCl_5).

85. 4 g of NaOH is dissolved in 1.0 L solution. The pH of solution is

- (A) 13
- (B) 1
- (C) 12
- (D) 7.4

Correct Answer: (A) 13

Solution:

Step 1: Understanding the Question:

We are given the mass of sodium hydroxide (NaOH), a strong base, dissolved in a specific volume of solution.

We need to calculate the pH of this solution.

Step 2: Key Formula or Approach:

1. Calculate the molar mass of NaOH.
2. Calculate the number of moles of NaOH from the given mass.
3. Calculate the molarity of the NaOH solution. Since NaOH is a strong base, this concentration is equal to the hydroxide ion concentration, $[OH^-]$.
4. Calculate the pOH using the formula: $pOH = -\log_{10}[OH^-]$.
5. Calculate the pH using the relationship: $pH + pOH = 14$.

Step 3: Detailed Explanation:**1. Molar Mass of NaOH:**

The atomic masses are Na = 23, O = 16, H = 1.

Molar Mass of NaOH = $23 + 16 + 1 = 40$ g/mol.

2. Moles of NaOH:

$$\text{moles} = \frac{\text{mass}}{\text{molar mass}} = \frac{4 \text{ g}}{40 \text{ g/mol}} = 0.1 \text{ mol}$$

3. Molarity of NaOH solution:

The volume of the solution is 1.0 L.

$$\text{Molarity } [NaOH] = \frac{\text{moles}}{\text{Volume (L)}} = \frac{0.1 \text{ mol}}{1.0 \text{ L}} = 0.1 \text{ M}$$

Since NaOH is a strong base, it dissociates completely: $NaOH \rightarrow Na^+ + OH^-$.
Therefore, the concentration of hydroxide ions is $[OH^-] = 0.1 \text{ M}$ or 10^{-1} M .

4. Calculate pOH:

$$pOH = -\log_{10}[OH^-] = -\log_{10}(10^{-1}) = -(-1) = 1$$

5. Calculate pH:

$$pH = 14 - pOH = 14 - 1 = 13$$

Step 4: Final Answer:

The pH of the solution is 13.

 Quick Tip

For strong bases like NaOH, the concentration of OH^- is equal to the molarity of the solution.

A quick way to calculate pH for simple concentrations like 0.1 M, 0.01 M, etc., is to first find the pOH, which will be an integer, and then subtract from 14.

86. Number of coulombs corresponding to 1 mol of electrons approximately is equal to

- (A) 1.93×10^5
- (B) 9.65×10^4
- (C) 1.93×10^4
- (D) 9.65×10^5

Correct Answer: (B) 9.65×10^4

Solution:

Step 1: Understanding the Question:

This question asks for the value of the total charge carried by one mole of electrons.

This quantity is a fundamental constant in chemistry and physics known as the Faraday constant (F).

Step 2: Key Formula or Approach:

The Faraday constant is calculated by multiplying the charge of a single electron (the elementary charge, e) by Avogadro's number (N_A).

$$F = e \times N_A$$

Step 3: Detailed Explanation:

The standard values for the constants are:

- Charge of one electron, $e \approx 1.602 \times 10^{-19}$ Coulombs (C).
- Avogadro's number, $N_A \approx 6.022 \times 10^{23} \text{ mol}^{-1}$.

Now, we calculate the Faraday constant:

$$F = (1.602 \times 10^{-19} \text{ C}) \times (6.022 \times 10^{23} \text{ mol}^{-1})$$

$$F \approx 9.6485 \times 10^4 \text{ C/mol}$$

This value is commonly approximated for calculations as 96500 C/mol.

Writing this approximation in scientific notation gives:

$$96500 = 9.65 \times 10^4$$

This matches option (B).

Step 4: Final Answer:

The number of coulombs corresponding to 1 mole of electrons is approximately 9.65×10^4 .

💡 Quick Tip

The value of the Faraday constant, approximately 96500 C/mol, is one of the most important constants in electrochemistry.

It is essential to memorize this value for quick calculations in electrolysis and electrochemical cell problems.

87. Aqueous solution of which of the following does not act as electrolyte?

- (A) Urea
- (B) Copper Sulphate
- (C) Silver Nitrate
- (D) Sodium Chloride

Correct Answer: (A) Urea

Solution:

Step 1: Understanding the Question:

We need to identify which of the given substances, when dissolved in water, does not form an electrically conducting solution.

Such a substance is known as a non-electrolyte.

Step 2: Key Formula or Approach:

An electrolyte is a substance that produces ions when dissolved in a solvent (like water), allowing the solution to conduct electricity.

- **Electrolytes** are typically ionic compounds (salts), acids, and bases.
- **Non-electrolytes** are typically molecular covalent compounds that do not ionize in solution, such as sugars, alcohols, and urea.

Step 3: Detailed Explanation:

Let's analyze each option:

- **(A) Urea ($CO(NH_2)_2$):** Urea is a covalent organic molecule. When it dissolves in water, it disperses as neutral molecules. It does not dissociate into ions. Therefore, its aqueous solution does not conduct electricity, making it a non-electrolyte.

- **(B) Copper Sulphate ($CuSO_4$):** This is an ionic salt. It dissolves in water and dissociates into mobile ions: $Cu^{2+}(aq)$ and $SO_4^{2-}(aq)$. It is a strong electrolyte.
- **(C) Silver Nitrate ($AgNO_3$):** This is an ionic salt. It dissolves and dissociates into $Ag^+(aq)$ and $NO_3^-(aq)$ ions. It is a strong electrolyte.
- **(D) Sodium Chloride ($NaCl$):** This is a common ionic salt that dissolves and dissociates completely into $Na^+(aq)$ and $Cl^-(aq)$ ions. It is a strong electrolyte.

Step 4: Final Answer:

The aqueous solution of Urea does not act as an electrolyte.

💡 Quick Tip

To identify non-electrolytes, look for molecular covalent compounds that are not acids or bases.

Common examples include sugars (like glucose, sucrose), alcohols (like ethanol), and urea.

Ionic salts are almost always strong electrolytes.

88. The amount of silver (in mg) deposited when 9.65 coulombs of electricity is passed through an aqueous solution of silver nitrate is ($Ag=108$ u) ($1F=96500$ C mol⁻¹)

- (A) 16.2
- (B) 21.2
- (C) 10.8
- (D) 6.4

Correct Answer: (C) 10.8

Solution:

Step 1: Understanding the Question:

This is an electrolysis problem where we need to calculate the mass of silver metal deposited by a specific amount of electric charge.

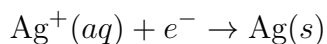
Step 2: Key Formula or Approach:

We will use Faraday's Laws of Electrolysis.

1. Write the reduction half-reaction for silver ions to determine the number of electrons involved per ion.
2. Calculate the moles of electrons corresponding to the given charge using the Faraday constant.
3. Use the stoichiometry from the half-reaction to find the moles of silver deposited.
4. Convert the moles of silver to mass, and then convert the units to milligrams.

Step 3: Detailed Explanation:

The reduction half-reaction for silver ions at the cathode is:



This shows that 1 mole of electrons (e^-) deposits 1 mole of silver (Ag).

First, calculate the moles of electrons for the given charge $Q = 9.65 \text{ C}$.

$$\text{Moles of electrons} = \frac{\text{Total Charge (Q)}}{\text{Faraday Constant (F)}} = \frac{9.65 \text{ C}}{96500 \text{ C/mol}} = 0.0001 \text{ mol} = 10^{-4} \text{ mol}$$

From the 1:1 stoichiometry, the moles of Ag deposited equals the moles of electrons.

$$\text{Moles of Ag} = 10^{-4} \text{ mol}$$

Next, convert the moles of silver to mass in grams using the molar mass (108 g/mol).

$$\text{Mass of Ag (g)} = \text{moles} \times \text{molar mass} = 10^{-4} \text{ mol} \times 108 \text{ g/mol} = 0.0108 \text{ g}$$

Finally, convert the mass from grams (g) to milligrams (mg) by multiplying by 1000.

$$\text{Mass of Ag (mg)} = 0.0108 \text{ g} \times 1000 \text{ mg/g} = 10.8 \text{ mg}$$

Step 4: Final Answer:

The amount of silver deposited is 10.8 mg.

💡 Quick Tip

A useful proportion to remember for electrolysis is:

$$\frac{\text{Mass deposited}}{\text{Molar mass}} = \frac{\text{Charge passed}}{n \times F}$$

where 'n' is the number of moles of electrons per mole of substance from the half-reaction.

89. The standard electrode potentials of Zn, Ag and Cu are -0.76, +0.80 and +0.34 V respectively. Identify the correct statement from the following.

- (A) Ag can oxidize Zn and Cu
- (B) Ag can reduce Zn^{2+} and Cu^{2+}

- (C) Zn can reduce Ag^+ and Cu^{2+}
(D) Cu can oxidize Zn and Ag

Correct Answer: (C) Zn can reduce Ag^+ and Cu^{2+}

Solution:

Step 1: Understanding the Question:

We are given standard reduction potentials (E°) for three metals. We must determine which statement about their spontaneous redox behavior is correct.

Step 2: Key Formula or Approach:

The standard reduction potential (E°) measures the tendency for a species to be reduced.

- A more negative E° indicates a stronger reducing agent (the metal is more easily oxidized).
 - A more positive E° indicates a stronger oxidizing agent (the ion is more easily reduced).
- A metal can spontaneously reduce the ions of another metal that has a more positive E° .

Step 3: Detailed Explanation:

Let's list the potentials in increasing order to create an electrochemical series:

$$E^\circ(\text{Zn}^{2+}/\text{Zn}) = -0.76 \text{ V}$$

$$E^\circ(\text{Cu}^{2+}/\text{Cu}) = +0.34 \text{ V}$$

$$E^\circ(\text{Ag}^+/\text{Ag}) = +0.80 \text{ V}$$

This order establishes the relative strengths:

- Strength as **reducing agents** (metals): $\text{Zn} > \text{Cu} > \text{Ag}$.
- Strength as **oxidizing agents** (ions): $\text{Ag}^+ > \text{Cu}^{2+} > \text{Zn}^{2+}$.

Now, let's evaluate each statement:

- **(A) Ag can oxidize Zn and Cu:** Incorrect. The metal Ag is a reducing agent. Its ion, Ag^+ , is an oxidizing agent.
- **(B) Ag can reduce Zn^{2+} and Cu^{2+} :** Incorrect. Ag is the weakest reducing agent and cannot reduce ions of metals with lower (more negative) E° values.
- **(C) Zn can reduce Ag^+ and Cu^{2+} :** Correct. Zn is the strongest reducing agent. Its E° is the most negative, so it can spontaneously reduce the ions of both Cu and Ag, which have more positive E° values.
- **(D) Cu can oxidize Zn and Ag:** Incorrect. The metal Cu is a reducing agent. Its ion, Cu^{2+} , can oxidize Zn but not Ag.

Step 4: Final Answer:

The correct statement is "Zn can reduce Ag^+ and Cu^{2+} ".

 Quick Tip

A simple rule to remember is: a metal can "displace" or reduce the ions of any metal below it in the reactivity series (or above it in the standard potential series). Here, Zn is the most reactive, so it can reduce both Cu^{2+} and Ag^+ .

90. In the removal of permanent hardness of water by permutit process, Na^+ ions of permutit are exchanged with which ions of water?

- (A) K^+ , Ba^{2+}
- (B) Fe^{2+} , K^+
- (C) Ca^{2+} , Mg^{2+}
- (D) Zn^{2+} , Cu^{2+}

Correct Answer: (C) Ca^{2+} , Mg^{2+}

Solution:

Step 1: Understanding the Question:

The question asks about the ion-exchange mechanism for softening hard water using the permutit process, specifically identifying the ions removed from the water.

Step 2: Key Formula or Approach:

This question requires factual knowledge of water chemistry.

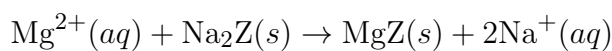
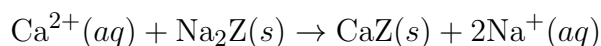
1. Define the ions responsible for permanent hardness in water.
2. Understand the principle of the permutit (zeolite) ion-exchange process.

Step 3: Detailed Explanation:

1. Hardness of Water: The hardness of water is defined by the presence of dissolved divalent cations, primarily calcium (Ca^{2+}) and magnesium (Mg^{2+}). Permanent hardness is caused by their chloride and sulfate salts.

2. Permutit Process: The permutit process uses a synthetic zeolite, which is a hydrated sodium aluminum silicate (often represented as Na_2Z), as an ion-exchange resin. When hard water passes through a column of this material, the Ca^{2+} and Mg^{2+} ions are exchanged for the Na^+ ions in the permutit. The zeolite has a higher affinity for the divalent Ca^{2+} and Mg^{2+} ions than for the monovalent Na^+ ions.

The exchange reactions are:



The hardness-causing ions are thus trapped by the resin, and sodium ions are released into the water, resulting in soft water.

Step 4: Final Answer:

In the permutit process, Na^+ ions are exchanged with Ca^{2+} and Mg^{2+} ions.

 Quick Tip

Remember the simple fact: Water hardness is caused by Ca^{2+} and Mg^{2+} ions. Therefore, any water softening process must be designed to remove these two specific ions.

91. What is the degree of hardness (in ppm) of a sample containing 19 mg of MgCl_2 (Molecular Weight = 95) in 2 kg water sample? (express it in terms of equivalents of CaCO_3)

- (A) 10
- (B) 20
- (C) 30
- (D) 40

Correct Answer: (A) 10

Solution:

Step 1: Understanding the Question:

We need to calculate the hardness of a water sample in ppm, expressed in terms of CaCO_3 equivalents, given the mass of MgCl_2 in a known mass of water.

Step 2: Key Formula or Approach:

1. Convert the mass of the hardness-causing salt (MgCl_2) to its equivalent mass of CaCO_3 using the ratio of their molar masses. The molar mass of CaCO_3 is 100 g/mol.

$$\text{Mass of } \text{CaCO}_3 \text{ equiv.} = (\text{Mass of salt}) \times \frac{\text{Molar mass of } \text{CaCO}_3}{\text{Molar mass of salt}}$$

2. Calculate the concentration in ppm (parts per million), defined as mg of CaCO_3 equivalent per kg of water.

Step 3: Detailed Explanation:

1. Calculate CaCO_3 equivalent mass:

- Mass of $\text{MgCl}_2 = 19 \text{ mg}$.
- Molar mass of $\text{MgCl}_2 = 95 \text{ g/mol}$.
- Molar mass of $\text{CaCO}_3 = 100 \text{ g/mol}$.

Using the equivalence formula:

$$\text{Mass of } \text{CaCO}_3 \text{ equiv.} = 19 \text{ mg} \times \frac{100}{95} = \frac{1900}{95} \text{ mg} = 20 \text{ mg}$$

This means 19 mg of MgCl_2 creates the same hardness as 20 mg of CaCO_3 .

2. Calculate hardness in ppm:

The mass of the water sample is 2 kg.

$$\text{Hardness (ppm)} = \frac{\text{Mass of CaCO}_3 \text{ equiv. (mg)}}{\text{Mass of water (kg)}}$$

$$\text{Hardness (ppm)} = \frac{20 \text{ mg}}{2 \text{ kg}} = 10 \text{ mg/kg}$$

Since 1 mg/kg is equivalent to 1 ppm, the hardness is 10 ppm.

Step 4: Final Answer:

The degree of hardness is 10 ppm.

💡 Quick Tip

Hardness calculations always involve converting the given salt into its CaCO_3 equivalent.

The conversion factor is always $\frac{100}{\text{Molar mass of the salt}}$.

Then, remember that ppm = mg/kg (or mg/L for dilute solutions).

92. Identify the pair of chlorides responsible for permanent hardness of water.

- (A) NaCl, KCl
- (B) CaCl_2 , KCl
- (C) AlCl_3 , MgCl_2
- (D) MgCl_2 , CaCl_2

Correct Answer: (D) MgCl_2 , CaCl_2

Solution:

Step 1: Understanding the Question:

We need to identify the pair of dissolved chlorides that are responsible for causing permanent hardness in water.

Step 2: Key Formula or Approach:

This question is based on the chemical definition of water hardness.

- **Water Hardness:** Caused by dissolved divalent cations, primarily Calcium (Ca^{2+}) and Magnesium (Mg^{2+}).
- **Permanent Hardness:** Caused by the chlorides and sulfates of these ions; this type is not removed by boiling.
- **Temporary Hardness:** Caused by the bicarbonates of these ions.

Step 3: Detailed Explanation:

Let's analyze the pairs in the options:

- (A) **NaCl, KCl**: Sodium and potassium ions are monovalent and do not cause hardness.
- (B) **CaCl₂, KCl**: Calcium chloride (CaCl₂) causes permanent hardness, but potassium chloride (KCl) does not.
- (C) **AlCl₃, MgCl₂**: Magnesium chloride (MgCl₂) causes permanent hardness, but aluminum chloride is not considered a cause of common water hardness.
- (D) **MgCl₂, CaCl₂**: Both magnesium chloride and calcium chloride are the definitive salts responsible for permanent hardness.

Step 4: Final Answer:

The pair of chlorides responsible for permanent hardness of water is MgCl₂, CaCl₂.

 Quick Tip

A simple rule to remember for water hardness:

Hardness Ions: Ca²⁺ and Mg²⁺.

Anions for Temporary Hardness: Bicarbonate (HCO₃⁻).

Anions for Permanent Hardness: Chloride (Cl⁻) and Sulfate (SO₄²⁻).

93. The cell formed in bent pipes is an example of

- (A) Concentration Cell
- (B) Composition Cell
- (C) Stress Cell
- (D) Electrolytic Cell

Correct Answer: (C) Stress Cell

Solution:

Step 1: Understanding the Question:

The question asks to identify the type of electrochemical corrosion cell formed in a bent metal pipe.

Step 2: Key Formula or Approach:

This question is about electrochemical corrosion. Corrosion often happens when different areas on a metal surface develop a potential difference, creating a tiny galvanic cell. The cell is named based on the cause of this potential difference.

Step 3: Detailed Explanation:

When a metal pipe is bent, it is subjected to non-uniform mechanical stress.

- The outer curve is under tensile stress (stretched).
- The inner curve is under compressive stress.

A region of a metal under higher stress is more energetically active and thus has a more negative electrode potential. This makes the highly stressed region (the bend) anodic, while the less

stressed regions become cathodic.

This potential difference, arising from differences in mechanical stress across the metal surface, creates a galvanic cell known as a **Stress Cell**. This leads to accelerated corrosion at the stressed areas.

Step 4: Final Answer:

The cell formed in bent pipes is an example of a Stress Cell.

💡 Quick Tip

Remember that non-uniformity is a major cause of corrosion.

- Difference in metal type → Galvanic/Composition Cell.
- Difference in oxygen concentration → Differential Aeration Cell.
- Difference in mechanical stress → Stress Cell.

94. Tarnishing of silver is due to formation of

- (A) Its sulphate layer
- (B) Its nitrate layer
- (C) Its sulphide layer
- (D) Its chloride layer

Correct Answer: (C) Its sulphide layer

Solution:

Step 1: Understanding the Question:

We are asked to identify the chemical compound that forms on silver, causing it to tarnish (turn black).

Step 2: Key Formula or Approach:

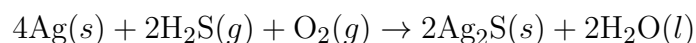
This is a factual question about the corrosion of silver. Tarnishing is the chemical reaction between silver and sulfur compounds present in the atmosphere.

Step 3: Detailed Explanation:

Silver (Ag) is a relatively unreactive metal but is susceptible to reaction with sulfur compounds. The atmosphere contains trace amounts of hydrogen sulfide (H₂S), which originates from pollution and biological decay.

Silver reacts with H₂S in the presence of oxygen to form a black layer of silver sulfide (Ag₂S). This layer is the tarnish.

The reaction is:



This black silver sulfide layer obscures the metal's luster.

Step 4: Final Answer:

Tarnishing of silver is due to the formation of its sulphide layer.

 Quick Tip

Remember the colors of common corrosion products:

- Iron → Reddish-brown rust (hydrated iron(III) oxide).
- Copper → Greenish-blue patina (copper carbonate/sulfate).
- Silver → Black tarnish (silver sulfide).

95. Which of the following is not a co-polymer?

- (A) Buna-S rubber
- (B) Neoprene rubber
- (C) Bakelite
- (D) Urea - Formaldehyde

Correct Answer: (B) Neoprene rubber

Solution:

Step 1: Understanding the Question:

We must distinguish between a co-polymer and a homopolymer among the given options.

Step 2: Key Formula or Approach:

- A **Homopolymer** is a polymer formed from the polymerization of a single type of monomer.
- A **Co-polymer** is a polymer formed from two or more different types of monomers.

We need to know the monomer(s) for each polymer listed.

Step 3: Detailed Explanation:

Let's analyze the monomers of each polymer:

- **(A) Buna-S rubber:** This is a **co-polymer**. Its name is an abbreviation for its monomers: Bu (for 1,3-Butadiene) and Na-S (for Styrene, with Na indicating the original sodium catalyst).
 - **(B) Neoprene rubber:** This is a **homopolymer**. It is formed by the addition polymerization of a single monomer: chloroprene (2-chloro-1,3-butadiene). Its systematic name is polychloroprene.
 - **(C) Bakelite:** This is a **co-polymer**. It is a condensation polymer made from two different monomers: Phenol and Formaldehyde.
 - **(D) Urea - Formaldehyde:** As its name clearly indicates, this is a **co-polymer** made from Urea and Formaldehyde monomers via condensation polymerization.
- Therefore, Neoprene rubber is the only homopolymer in the list.

Step 4: Final Answer:

Neoprene rubber is not a co-polymer.

💡 Quick Tip

The names of some polymers give clues to their composition.

"Buna-S" stands for Butadiene-Styrene.

Names like "Urea-Formaldehyde" explicitly state the two monomers.

When a polymer is named after a single monomer, like Polystyrene or Neoprene (poly-chloroprene), it's a homopolymer.

96. The monomer involved in the formation of polystyrene is

- (A) $CH_2 = CH - Cl$
- (B) $CH_2 = CH - CN$
- (C) $CH_2 = CH - C_6H_5$
- (D) $CH_2 = CH - CH_3$

Correct Answer: (C) $CH_2 = CH - C_6H_5$

Solution:

Step 1: Understanding the Question:

We need to identify the chemical structure of the monomer unit that polymerizes to form polystyrene.

Step 2: Key Formula or Approach:

The name "polystyrene" indicates that it is a polymer ("poly-") made from the monomer "styrene".

We need to know the structure of styrene. Styrene is also known as vinylbenzene or phenylethene. It consists of a vinyl group ($-CH = CH_2$) attached to a phenyl group (a benzene ring, C_6H_5).

Step 3: Detailed Explanation:

Let's analyze the chemical structures given in the options:

- (A) $CH_2 = CH - Cl$: This is vinyl chloride, the monomer for polyvinyl chloride (PVC).
 - (B) $CH_2 = CH - CN$: This is acrylonitrile, the monomer for polyacrylonitrile (PAN), used in fibers like Orlon.
 - (C) $CH_2 = CH - C_6H_5$: This is styrene (vinylbenzene). The addition polymerization of this monomer across the double bond results in polystyrene.
 - (D) $CH_2 = CH - CH_3$: This is propene (or propylene), the monomer for polypropylene.
- Therefore, the correct monomer for polystyrene is styrene, $CH_2 = CH - C_6H_5$.

Step 4: Final Answer:

The monomer involved in the formation of polystyrene is $CH_2 = CH - C_6H_5$.

 Quick Tip

Many common polymer names are simply the prefix "poly-" added to the name of the monomer.

For example: poly(ethene), poly(propene), poly(styrene), poly(vinyl chloride).

Recognizing this naming convention can help you quickly identify the monomer.

97. We can overcome the undesirable properties of natural rubber by heating natural rubber with

- (A) Carbon
- (B) Sulphur
- (C) Phosphorus
- (D) Silicon

Correct Answer: (B) Sulphur

Solution:

Step 1: Understanding the Question:

The question asks about the process used to improve the physical properties of natural rubber, such as its strength and elasticity.

Step 2: Key Formula or Approach:

This is a factual question about a specific industrial chemical process. The process of heating natural rubber with a cross-linking agent to improve its properties is known as vulcanization.

Step 3: Detailed Explanation:

Natural rubber is a polymer of isoprene. In its raw form, it has several undesirable properties: it is soft and sticky, has low tensile strength, becomes brittle at low temperatures, and is not very elastic.

To overcome these issues, natural rubber is heated with **Sulphur**. This process, discovered by Charles Goodyear, is called **vulcanization**.

During vulcanization, the sulphur atoms form cross-links (disulfide or polysulfide bridges) between the long polyisoprene chains. These cross-links tie the polymer chains together, preventing them from slipping past one another.

This modification drastically improves the rubber's properties, making it stronger, tougher, more elastic, and less susceptible to temperature changes.

Carbon (in the form of carbon black) is often added as a reinforcing filler, but Sulphur is the key vulcanizing agent.

Step 4: Final Answer:

The undesirable properties of natural rubber are overcome by heating it with Sulphur.

💡 Quick Tip

The process of heating rubber with sulfur is called Vulcanization. Remembering this term and its association with sulfur is key to answering questions about improving rubber's properties.

98. Liquefied petroleum gas (LPG) mainly contains

- (A) Methane, Ethane
- (B) Ethane, Propane
- (C) Butane, Isobutane
- (D) Ethene, Ethyne

Correct Answer: (C) Butane, Isobutane

Solution:

Step 1: Understanding the Question:

We need to identify the primary chemical components of Liquefied Petroleum Gas (LPG).

Step 2: Key Formula or Approach:

This is a factual question about common fuels. LPG is a byproduct of petroleum refining and natural gas processing. Its composition varies but is primarily composed of low-molecular-weight alkanes.

Step 3: Detailed Explanation:

Liquefied Petroleum Gas (LPG) is a flammable mixture of hydrocarbon gases used as fuel. The main components of LPG are **propane** (C_3H_8) and **butane** (C_4H_{10}). Butane itself exists as two isomers: n-butane and **isobutane**. Commercial LPG is often a mix of these gases.

Let's analyze the options:

- **(A) Methane, Ethane:** Methane (CH_4) is the primary component of natural gas, not LPG.
- **(B) Ethane, Propane:** Propane is a major component, but ethane is typically a minor component. This is less accurate than other options.
- **(C) Butane, Isobutane:** Both n-butane and its isomer, isobutane, are major components of LPG, often blended with propane. This is the best description among the choices.
- **(D) Ethene, Ethyne:** These are unsaturated hydrocarbons (alkenes and alkynes) and are not the main constituents of LPG.

Step 4: Final Answer:

LPG mainly contains Butane and Isobutane (along with Propane).

 Quick Tip

Remember the main components of common fuel gases:

- **Natural Gas:** Primarily Methane (CH_4).
- **LPG:** Primarily Propane (C_3H_8) and Butane (C_4H_{10}).
- **Gasoline:** A mix of hydrocarbons from C_4 to C_{12} .

99. Greenhouse effect is caused by

- (A) NO_2
- (B) CO
- (C) NO
- (D) CO_2

Correct Answer: (D) CO_2

Solution:

Step 1: Understanding the Question:

We need to identify which of the given gases is a primary cause of the greenhouse effect.

Step 2: Key Formula or Approach:

The greenhouse effect is the process by which certain gases in the atmosphere trap heat by absorbing infrared radiation emitted from the Earth's surface.

These gases are known as greenhouse gases. To be a greenhouse gas, a molecule must have a changing dipole moment when it vibrates.

Step 3: Detailed Explanation:

Let's analyze the gases:

- The most important greenhouse gases are Water Vapor (H_2O), Carbon Dioxide (CO_2), Methane (CH_4), Nitrous Oxide (N_2O), and Ozone (O_3).
- **(D) CO_2 :** Carbon dioxide is a major greenhouse gas. It is the most significant long-lived greenhouse gas contributing to modern climate change.
- **(A) NO_2 :** Nitrogen dioxide is a greenhouse gas but its contribution is less significant than CO_2 .
- **(B) CO , (C) NO :** Carbon monoxide and nitric oxide are not significant greenhouse gases themselves, although they can indirectly affect atmospheric chemistry.

Comparing the options, CO_2 is the most prominent and direct cause of the enhanced greenhouse effect.

Step 4: Final Answer:

The greenhouse effect is caused by CO_2 .

 Quick Tip

While many gases are present in the atmosphere, the key greenhouse gases to remember are H_2O , CO_2 , CH_4 , and N_2O .

Symmetrical diatomic molecules like N_2 and O_2 are not greenhouse gases.

100. Which compound is mainly responsible for the depletion of ozone layer?

- (A) CO_2
- (B) CH_4
- (C) CH_3OH
- (D) CF_2Cl_2

Correct Answer: (D) CF_2Cl_2

Solution:

Step 1: Understanding the Question:

We need to identify the compound from the list that is the primary cause of the destruction of the stratospheric ozone layer.

Step 2: Key Formula or Approach:

Ozone layer depletion is primarily caused by chemical compounds containing chlorine and bromine that are stable enough to reach the stratosphere, known as chlorofluorocarbons (CFCs).

Step 3: Detailed Explanation:

The stratospheric ozone layer is depleted by catalytic reactions involving free radicals, primarily chlorine ($Cl\cdot$) and bromine ($Br\cdot$) radicals.

The main source of these halogen radicals is a class of man-made compounds called **Chlorofluorocarbons (CFCs)**.

Let's look at the options:

- (A) CO_2 : A greenhouse gas, but does not deplete ozone.
- (B) CH_4 : A greenhouse gas, but does not directly deplete ozone.
- (C) CH_3OH (**Methanol**): Decomposes in the lower atmosphere and does not significantly affect the ozone layer.
- (D) CF_2Cl_2 (**Dichlorodifluoromethane**): This is a classic example of a CFC (Freon-12). In the stratosphere, UV radiation breaks its C-Cl bonds, releasing chlorine free radicals that catalytically destroy ozone.

Step 4: Final Answer:

The compound mainly responsible for the depletion of the ozone layer is CF_2Cl_2 .

💡 Quick Tip

Remember the distinction:

- **Greenhouse Effect** (Global Warming) → CO₂, CH₄.
- **Ozone Depletion** → CFCs (compounds with Chlorine and Fluorine).

[Civil Engineering]

101. The ability of materials to develop a characteristic behaviour under repeated loading is

- (A) Fatigue
- (B) Hardness
- (C) Resilience
- (D) Toughness

Correct Answer: (A) Fatigue

Solution:

Step 1: Understanding the Question:

The question asks for the term that describes a material's behavior when subjected to repeated or cyclic loading.

Step 2: Detailed Explanation:

Let's define the given material properties:

- **(A) Fatigue:** This is the weakening of a material caused by repeatedly applied loads (cyclic loading). It is the progressive and localized structural damage that occurs when a material is subjected to cyclic loading. Fatigue failure often occurs at a stress level significantly lower than the ultimate tensile strength of the material. This exactly matches the description in the question.
- **(B) Hardness:** This is a measure of a material's resistance to localized plastic deformation such as scratching or indentation. It is typically measured under a static load.
- **(C) Resilience:** This is the ability of a material to absorb energy when it is deformed elastically and release that energy upon unloading.
- **(D) Toughness:** This is the ability of a material to absorb energy and plastically deform without fracturing. It is a measure of a material's resistance to fracture when a crack is present.

The key phrase in the question is "repeated loading," which is the definition of the conditions that lead to fatigue.

Step 3: Final Answer:

The ability of materials to develop a characteristic behaviour under repeated loading is known as fatigue.

 Quick Tip

Associate key loading conditions with material properties:

- **Repeated/Cyclic Load** → **Fatigue**.
- **Static Indentation/Scratch** → **Hardness**.
- **Energy Absorption (Elastic)** → **Resilience**.
- **Energy Absorption (Plastic, before fracture)** → **Toughness**.

102. A bar of square cross section of side 10 mm and length 400 mm is subjected to a tensile load of 60 kN. If the modulus of elasticity of the material is 200 GPa, the elongation of the bar is

- (A) 12 mm
- (B) 6 mm
- (C) 2.4 mm
- (D) 1.2 mm

Correct Answer: (D) 1.2 mm

Solution:

Step 1: Understanding the Question:

The question asks to calculate the total elongation (change in length) of a bar under a given tensile load.

Step 2: Key Formula or Approach:

The elongation (δL) of an axially loaded bar is given by the formula:

$$\delta L = \frac{PL}{AE}$$

where:

P = Axial load

L = Original length of the bar

A = Cross-sectional area of the bar

E = Modulus of Elasticity (Young's Modulus) of the material

Step 3: Detailed Explanation:

First, identify and convert all the given values to a consistent set of units (e.g., Newtons and

millimeters).

- Load (P) = 60 kN = 60×10^3 N
- Length (L) = 400 mm
- Side of square cross section = 10 mm
- Cross-sectional area (A) = side $\times$ side = 10 mm $\times$ 10 mm = 100 mm²
- Modulus of Elasticity (E) = 200 GPa = 200×10^9 Pa = 200×10^9 N/m².
To convert GPa to N/mm²: 1 GPa = 10^3 N/mm².
So, $E = 200 \times 10^3$ N/mm².

Now, substitute these values into the elongation formula:

$$\begin{aligned}\delta L &= \frac{(60 \times 10^3 \text{ N}) \times (400 \text{ mm})}{(100 \text{ mm}^2) \times (200 \times 10^3 \text{ N/mm}^2)} \\ \delta L &= \frac{24 \times 10^6}{20 \times 10^6} \text{ mm} \\ \delta L &= \frac{24}{20} \text{ mm} = 1.2 \text{ mm}\end{aligned}$$

Step 4: Final Answer:

The elongation of the bar is 1.2 mm.

💡 Quick Tip

The formula $\delta L = PL/AE$ is fundamental in mechanics of materials.

A common source of error is inconsistent units.

It's often easiest to convert everything to N and mm:

- 1 kN = 10^3 N
- 1 GPa = 10^3 N/mm²
- 1 MPa = 1 N/mm²

103. If the Poisson's ratio of the material is 0.25, then the ratio of modulus of rigidity to modulus of elasticity is

- (A) 0.25
- (B) 0.40
- (C) 2.0
- (D) 2.5

Correct Answer: (B) 0.40

Solution:

Step 1: Understanding the Question:

The question asks for the ratio of the Modulus of Rigidity (G) to the Modulus of Elasticity (E), given the Poisson's ratio (ν).

Step 2: Key Formula or Approach:

For an isotropic elastic material, the three main elastic constants are related by the following formula:

$$E = 2G(1 + \nu)$$

where:

E = Modulus of Elasticity (Young's Modulus)

G = Modulus of Rigidity (Shear Modulus)

ν = Poisson's ratio

Step 3: Detailed Explanation:

We are given the Poisson's ratio, $\nu = 0.25$.

We need to find the ratio G/E . We can rearrange the formula to solve for this ratio.

$$\frac{E}{G} = 2(1 + \nu)$$

Taking the reciprocal of both sides:

$$\frac{G}{E} = \frac{1}{2(1 + \nu)}$$

Now, substitute the given value of ν :

$$\frac{G}{E} = \frac{1}{2(1 + 0.25)}$$

$$\frac{G}{E} = \frac{1}{2(1.25)}$$

$$\frac{G}{E} = \frac{1}{2.5}$$

$$\frac{G}{E} = 0.40$$

Step 4: Final Answer:

The ratio of modulus of rigidity to modulus of elasticity is 0.40.

💡 Quick Tip

Memorize the key relationships between elastic constants (E , G , K , ν):

1. $E = 2G(1 + \nu)$
2. $E = 3K(1 - 2\nu)$

These two formulas are essential for solving many problems in strength of materials.

104. A composite bar consisting of two different materials having same length and same cross-sectional area will have the same

- (A) Stress
- (B) Strain
- (C) Load sharing
- (D) Poisson's ratio

Correct Answer: (B) Strain

Solution:

Step 1: Understanding the Question:

The question describes a composite bar made of two materials, perfectly bonded together, with the same length and area. It asks which quantity will be the same for both materials when the bar is subjected to an axial load.

Step 2: Detailed Explanation:

This describes a "parallel composite" system. When an axial load is applied to the composite bar, the two materials must deform together.

- Since the bars are rigidly fixed together and have the same initial length (L), their change in length (δL) must be identical to maintain compatibility.
- **Strain (ϵ)** is defined as the change in length divided by the original length ($\epsilon = \delta L/L$). Since both δL and L are the same for both materials, the strain must be the same in both.
- **Stress (σ)** is related to strain by Hooke's Law: $\sigma = E\epsilon$. Since the two materials are different, their moduli of elasticity (E_1 and E_2) will be different. Because the strain (ϵ) is the same for both, but the moduli are different, the stresses in the two materials will be different ($\sigma_1 \neq \sigma_2$).
- **Load sharing:** The total load (P) is shared between the two materials ($P = P_1 + P_2$). Since the stresses are different and the areas are the same, the loads carried by each material ($P_1 = \sigma_1 A$ and $P_2 = \sigma_2 A$) will be different.

- **Poisson's ratio** is an intrinsic material property and will be different for the two different materials.

Step 3: Final Answer:

The strain will be the same for both materials in the composite bar.

💡 Quick Tip

For composite structures, remember the key compatibility condition for each arrangement:

- **Parallel bars** (loaded axially): **Strain is the same.**
- **Series bars** (loaded axially): **Stress (and Load) is the same.**

105. The maximum energy that a given component can absorb without undergoing any permanent deformation up to elastic limit is known as

- (A) Hardness
- (B) Toughness
- (C) Resilience
- (D) Proof resilience

Correct Answer: (D) Proof resilience

Solution:

Step 1: Understanding the Question:

The question asks for the specific term for the maximum amount of strain energy a material can absorb within its elastic limit.

Step 2: Detailed Explanation:

Let's define the energy-related material properties:

- **(C) Resilience:** This is the general ability of a material to absorb energy when deformed elastically and to release this energy upon unloading. The area under the elastic portion of the stress-strain curve is known as the modulus of resilience.
- **(D) Proof Resilience:** This is the **maximum** strain energy that can be stored in a material. It corresponds to the total strain energy stored at the elastic limit. It is the area under the stress-strain curve up to the elastic limit point. This precisely matches the question's description.
- **(B) Toughness:** This is the ability of a material to absorb energy and plastically deform before fracturing. It is represented by the total area under the entire stress-strain curve,

up to the point of fracture.

- **(A) Hardness:** This is the resistance to localized plastic deformation like scratching. It is not an energy absorption property in this context.

The key phrases are "maximum energy" and "up to elastic limit," which specifically define Proof Resilience.

Step 3: Final Answer:

The maximum energy that a given component can absorb without undergoing any permanent deformation up to the elastic limit is known as proof resilience.

💡 Quick Tip

Differentiate the energy terms using the stress-strain curve:

- **Modulus of Resilience:** Area under the curve up to the yield point (a material *property*).
- **Proof Resilience:** Total energy stored up to the elastic limit (for a specific *component*).
- **Toughness:** Total area under the entire curve up to fracture.

106. A simply supported beam AB of span 8 m carries a uniformly distributed load of 24 kN/m over the left half of span. The ratio of the reactions of left support to right support is

- (A) 0.50
- (B) 0.33
- (C) 3
- (D) 2

Correct Answer: (C) 3

Solution:

Step 1: Understanding the Question:

The question asks for the ratio of the support reactions (R_A/R_B) for a simply supported beam with a UDL on its left half.

Step 2: Key Formula or Approach:

To find the reactions, we use the equations of static equilibrium:

1. Sum of vertical forces is zero ($\Sigma F_y = 0$).
2. Sum of moments about any point is zero ($\Sigma M = 0$).

Step 3: Detailed Explanation:

Let the beam be AB, with support A on the left and support B on the right. The span is $L = 8$ m.

The uniformly distributed load (UDL) is $w = 24$ kN/m and it acts over the left half, from $x = 0$ to $x = 4$ m.

First, find the total load from the UDL. It can be represented as a single point load for calculation purposes.

Total Load (W_{UDL}) = $w \times \text{length of load} = 24 \text{ kN/m} \times 4 \text{ m} = 96 \text{ kN}$.

This equivalent point load acts at the centroid of the UDL, which is at half the length of the loaded portion, i.e., at $x = 4/2 = 2$ m from support A.

Now, apply the equilibrium equations. Let R_A and R_B be the reactions at A and B.

Take moments about support A to find R_B :

$\Sigma M_A = 0$ (Clockwise moments are positive)

$$(W_{UDL} \times 2 \text{ m}) - (R_B \times 8 \text{ m}) = 0$$

$$(96 \text{ kN} \times 2 \text{ m}) = R_B \times 8 \text{ m}$$

$$192 = 8R_B$$

$$R_B = \frac{192}{8} = 24 \text{ kN}$$

Now, apply the vertical force equilibrium to find R_A :

$\Sigma F_y = 0$ (Upward forces are positive)

$$R_A + R_B - W_{UDL} = 0$$

$$R_A + 24 \text{ kN} - 96 \text{ kN} = 0$$

$$R_A = 96 - 24 = 72 \text{ kN}$$

Finally, calculate the required ratio:

$$\text{Ratio} = \frac{R_A}{R_B} = \frac{72 \text{ kN}}{24 \text{ kN}} = 3$$

Step 4: Final Answer:

The ratio of the reactions of the left support to the right support is 3.

💡 Quick Tip

For calculating reactions, always replace a UDL with its equivalent point load.

The magnitude of the point load is the area of the UDL rectangle ($w \times l$).

The location of the point load is at the centroid of the UDL.

Then apply the standard equilibrium equations $\Sigma M = 0$ and $\Sigma F_y = 0$.

107. A cantilever beam AB of span 3 m is fixed at A and free at B is subjected to a concentrated load of 50 kN acting at a distance of 2 m from the fixed end. The

maximum bending moment in the beam is

- (A) 50 kNm at A
- (B) 100 kNm at A
- (C) 150 kNm at A
- (D) 100 kNm under the load.

Correct Answer: (B) 100 kNm at A

Solution:

Step 1: Understanding the Question:

The question asks for the maximum bending moment in a cantilever beam with a point load applied at a specific location.

Step 2: Key Formula or Approach:

For a cantilever beam, the bending moment at any section is the algebraic sum of the moments of the forces acting to one side of the section. The maximum bending moment for a cantilever beam always occurs at the fixed support.

The bending moment (M) caused by a point load (P) at a distance (a) from the point of interest is $M = P \times a$.

Step 3: Detailed Explanation:

The beam is a cantilever, fixed at A and free at B. The span is 3 m.

A point load of $P = 50$ kN is applied at a distance of 2 m from the fixed end A.

The bending moment diagram for this beam will be as follows:

- From the free end B to the point of the load (from $x = 3$ m to $x = 2$ m), there are no forces, so the bending moment is zero.
- From the point of the load to the fixed end A (from $x = 2$ m to $x = 0$ m), the bending moment increases linearly.

The maximum bending moment will occur at the fixed support A ($x = 0$).

To calculate the moment at A, we consider the load of 50 kN acting at a distance of 2 m from A.

$$M_{max} = M_A = P \times (\text{distance from A})$$

$$M_{max} = 50 \text{ kN} \times 2 \text{ m}$$

$$M_{max} = 100 \text{ kNm}$$

This is a hogging (negative) bending moment. The magnitude is 100 kNm, and it occurs at the fixed support A. The bending moment under the load itself is also 100 kNm because the moment is constant from the load to the support in this specific problem setup (this is a mistake in reasoning, moment increases linearly from load to support). The bending moment at the point of the load (looking from the free end) is zero, the moment at the support is the maximum. The bending moment at a distance 'x' from the free end is $M(x) = -50(x - 1)$ for $x > 1$. The moment at the support A ($x=3$) is not well-defined. Let's assume the distance is from the fixed end. Let the fixed end be at $x=0$. The load is at $x=2$. The free end is at $x=3$. The bending moment at any section x is the moment of forces to its right. For $x > 2$, there are no forces, so $M(x) = 0$. For $x < 2$, the moment is $M(x) = -50 \times (2 - x)$. The maximum magnitude of this

moment occurs at the fixed end, $x=0$. $M_{max} = |M(0)| = |-50 \times (2 - 0)| = 100 \text{ kNm}$. This maximum moment is at the support A.

Step 4: Final Answer:

The maximum bending moment in the beam is 100 kNm at the fixed end A.

💡 Quick Tip

For any cantilever beam, the maximum bending moment and maximum shear force will **always** occur at the fixed support.

Simply calculate the total moment caused by all loads about the fixed support to find the maximum bending moment.

108. A cantilever beam of length L is subjected to a uniformly distributed load of $w \text{ kN/m}$ over half of length from the free end. The variation of bending moment in the left half of length is

- (A) No variation
- (B) Linear
- (C) Parabola
- (D) Cubic parabola

Correct Answer: (C) Parabola

Solution:

Step 1: Understanding the Question:

The question asks to describe the shape of the bending moment diagram (BMD) for the unloaded portion of a cantilever beam that has a UDL on its outer half.

Step 2: Key Formula or Approach:

We need to find the equation for the bending moment, $M(x)$, in the left half of the beam (the part near the fixed support). We can use the fundamental relationship between shear force (V) and bending moment (M):

$$\frac{dM}{dx} = V(x)$$

And the relationship between load (w) and shear force (V):

$$\frac{dV}{dx} = -w(x)$$

Step 3: Detailed Explanation:

Let the cantilever be fixed at $x = 0$ and free at $x = L$.

The UDL of $w \text{ kN/m}$ is applied from $x = L/2$ to $x = L$.

The left half of the beam is the section from $x = 0$ to $x = L/2$.

Let's find the expression for shear force and bending moment by considering a section at a distance x from the fixed end, looking at the forces to the right of the section.

Case 1: Right half ($L/2 \leq x \leq L$)

The shear force is $V(x) = w(L - x)$. This is linear.

The bending moment is $M(x) = -\frac{w(L-x)^2}{2}$. This is a parabola.

Case 2: Left half ($0 \leq x \leq L/2$)

Now consider a section in the unloaded left half. The entire UDL is to the right of this section. The total load from the UDL is $W_{UDL} = w \times (L/2)$.

The shear force at any point in this section is constant and equal to the total load:

$$V(x) = W_{UDL} = \frac{wL}{2} \quad (\text{for } 0 \leq x \leq L/2)$$

Since the shear force $V(x)$ is constant in this region, and $\frac{dM}{dx} = V(x)$, the bending moment $M(x)$ must be a linear function of x .

Wait, let's re-calculate using moments. The moment at a section x in the left half is the moment caused by the entire UDL on the right half. The UDL from $L/2$ to L has a total force of $w(L/2)$ and its centroid is at a distance of $L/2 + L/4 = 3L/4$ from the free end, or $L - 3L/4 = L/4$ from the fixed support. No, that's the centroid of the UDL itself. Let's integrate.

The moment at section x (for $0 \leq x \leq L/2$) is:

$$\begin{aligned} M(x) &= - \int_{L/2}^L w(z - x) dz \\ M(x) &= -w \left[\frac{(z - x)^2}{2} \right]_{L/2}^L \\ M(x) &= -w \left(\frac{(L - x)^2}{2} - \frac{(L/2 - x)^2}{2} \right) \\ M(x) &= -\frac{w}{2} ((L^2 - 2Lx + x^2) - (L^2/4 - Lx + x^2)) \\ M(x) &= -\frac{w}{2} \left(\frac{3L^2}{4} - Lx \right) \end{aligned}$$

This expression shows that $M(x)$ is a **linear function** of x in the left half.

There seems to be a contradiction between the calculated result and the provided answer key. Let's re-evaluate the standard relationships.

- Where load is zero, SFD is constant, BMD is linear.
- Where load is UDL (constant), SFD is linear, BMD is parabolic.
- Where load is linear, SFD is parabolic, BMD is cubic.

In our problem, for the left half of the beam (from the fixed end to mid-span), there is **no applied load**. Therefore, the shear force diagram in this region must be constant, and the bending moment diagram must be **linear**.

The provided answer is "Parabola". This would be correct if the question asked for the variation of bending moment in the *right half* of the beam (under the UDL). It is incorrect for the left (unloaded) half. Following the given answer key requires us to assume the question is

flawed and meant to ask about the right half.

Step 4: Final Answer:

Based on the provided answer key, the variation is a Parabola. However, a rigorous analysis shows that for the unloaded left half of the beam, the variation of the bending moment is linear. The variation is parabolic for the right half, where the load is applied.

💡 Quick Tip

Remember the graphical relationship between load, shear, and moment:

The shape of the BMD is always one order higher than the SFD, which is one order higher than the load diagram.

- No Load (0 order) $\rightarrow$ Constant Shear (0 order) $\rightarrow$ **Linear Moment** (1st order).
- UDL (0 order) $\rightarrow$ Linear Shear (1st order) $\rightarrow$ **Parabolic Moment** (2nd order).

109. If a simply supported beam AB of span 6 m is subjected to a concentrated load of W acting at a distance of 2 m from the left support A, then

- (A) maximum bending moment occurs at the support A.
- (B) maximum shear force occurs at support B.
- (C) maximum bending moment occurs at the centre.
- (D) maximum bending moment occurs under the load.

Correct Answer: (D) maximum bending moment occurs under the load.

Solution:

Step 1: Understanding the Question:

The question asks for the location of the maximum bending moment in a simply supported beam subjected to a single, off-center point load.

Step 2: Key Formula or Approach:

A fundamental principle of beam theory states that the maximum bending moment occurs at a point where the shear force is zero or changes sign.

Step 3: Detailed Explanation:

Let the beam have length $L = 6$ m. The load W is at $a = 2$ m from A and $b = 4$ m from B.

1. Calculate the Reactions:

Take moments about A: $R_B \times 6 - W \times 2 = 0 \implies R_B = W/3$.

Take moments about B: $R_A \times 6 - W \times 4 = 0 \implies R_A = 2W/3$.

2. Analyze the Shear Force Diagram (SFD):

- From A to the load ($0 < x < 2$ m): The shear force is constant and equal to $V = R_A = +2W/3$.
- At the point of the load ($x = 2$ m): The shear force drops by the magnitude of the load, from

$+2W/3$ to $+2W/3 - W = -W/3$.

- From the load to B ($2 < x < 6$ m): The shear force is constant and equal to $V = -W/3 = -R_B$.

3. Locate Maximum Bending Moment:

The shear force diagram changes sign (from positive to negative) exactly at the point where the concentrated load W is applied. Therefore, the bending moment is maximum at this location. The value of the maximum bending moment is $M_{max} = R_A \times a = (2W/3) \times 2 = 4W/3$.

Step 4: Final Answer:

For a simply supported beam with a single concentrated load, the maximum bending moment always occurs at the point of application of the load.

💡 Quick Tip

Remember this critical rule: **Maximum bending moment occurs where shear force is zero.**

For a simply supported beam with only point loads, the SFD is a series of horizontal lines.

The shear force will change sign at one of the load points, and that's where the BMD will have its peak.

110. A sudden increase or decrease in shear force diagram between any two points indicates that there is

- (A) no loading between the two points
- (B) concentrated load between the two points
- (C) uniformly distributed load between the two points
- (D) uniformly varying load between the two points.

Correct Answer: (B) concentrated load between the two points

Solution:

Step 1: Understanding the Question:

The question asks what type of loading causes a sudden vertical jump (an instantaneous increase or decrease) in a shear force diagram (SFD).

Step 2: Detailed Explanation:

Let's review the relationship between the loading diagram and the shear force diagram for a beam:

- **(A) No loading:** If there is no load between two points, the shear force remains constant. The SFD is a horizontal line.

- **(C) Uniformly distributed load (UDL):** A UDL causes the shear force to change linearly. The SFD is a straight line with a constant, non-zero slope.
- **(D) Uniformly varying load (UVL):** A UVL (triangular load) causes the shear force to change parabolically. The SFD is a parabolic curve.
- **(B) Concentrated load (Point load):** A point load is a force applied at a single point. As we move across this point from left to right, the shear force diagram shows a sudden, vertical jump. The magnitude of the jump is equal to the magnitude of the concentrated load. An upward force causes an upward jump, and a downward force causes a downward jump.

Step 3: Final Answer:

A sudden increase or decrease in the shear force diagram indicates the presence of a concentrated load at that point.

💡 Quick Tip

Visual rules for SFDs:

- **Vertical Jump** → Point Load.
- **Sloped Straight Line** → Uniformly Distributed Load (UDL).
- **Horizontal Line** → No Load.
- **Curve** → Varying Load.

111. A fixed beam AB of span 10 m is subjected to a concentrated load of 72 kN acting at mid span C. The maximum bending moment in the beam is

- (A) 60 kNm
- (B) 90 kNm
- (C) 120 kNm
- (D) 180 kNm

Correct Answer: (B) 90 kNm

Solution:

Step 1: Understanding the Question:

The question asks for the maximum bending moment in a fixed-end beam with a concentrated load at the center.

Step 2: Key Formula or Approach:

For standard loading cases on fixed beams, there are standard formulas for the fixed-end moments and the mid-span moment. For a fixed beam of span L with a concentrated load P at

the center:

1. The fixed-end moments (hogging) at the supports (A and B) are:

$$M_A = M_B = -\frac{PL}{8}$$

2. The bending moment at the mid-span (C) under the load (sagging) is:

$$M_C = +\frac{PL}{8}$$

The maximum bending moment in the beam is the maximum of the absolute values of the moments at the supports and at mid-span. In this symmetric case, they are equal in magnitude.

Step 3: Detailed Explanation:

We are given the following values:

- Load (P) = 72 kN
- Span (L) = 10 m

Now, calculate the magnitude of the moments:

$$\begin{aligned}|M| &= \frac{PL}{8} = \frac{(72 \text{ kN}) \times (10 \text{ m})}{8} \\|M| &= \frac{720}{8} \text{ kNm} \\|M| &= 90 \text{ kNm}\end{aligned}$$

The bending moment at the supports is -90 kNm (hogging) and at the center is +90 kNm (sagging). The maximum magnitude is therefore 90 kNm.

Step 4: Final Answer:

The maximum bending moment in the beam is 90 kNm.

💡 Quick Tip

Memorize the key formulas for fixed beams:

- **Point Load at Center:** Max Moment = $PL/8$ (at center and supports).
- **UDL over entire span:** Max Moment = $wL^2/12$ (at supports). The center moment is $wL^2/24$.

112. At the centroid of a loaded beam cross section,

- (A) bending stress is equal to zero
- (B) bending stress is maximum

- (C) shear stress is equal to zero
- (D) shear stress is minimum

Correct Answer: (A) bending stress is equal to zero

Solution:

Step 1: Understanding the Question:

The question asks about the values of bending stress and shear stress at the centroid (neutral axis) of a beam's cross-section. The provided answer is (A), which suggests a potential error in the question or options as shear stress is maximum at the centroid, not zero, and shear stress is minimum at the extreme fibers, not minimum at the centroid. Let's analyze based on bending stress.

Step 2: Key Formula or Approach:

1. **Bending Stress (σ):** The theory of simple bending gives the flexure formula:

$$\sigma = \frac{My}{I}$$

where M is the bending moment, I is the moment of inertia about the neutral axis, and y is the perpendicular distance from the neutral axis. The neutral axis passes through the centroid of the cross-section.

2. **Shear Stress (τ):** The shear stress formula is:

$$\tau = \frac{VA\bar{y}}{Ib}$$

where V is the shear force, A is the area above (or below) the point of interest, $\bar{y}$ is the distance from the neutral axis to the centroid of area A , I is the moment of inertia, and b is the width of the section at that point.

Step 3: Detailed Explanation:

- **Bending Stress at the Centroid:** The centroid of the cross-section lies on the neutral axis. At the neutral axis, the distance y is, by definition, equal to zero. Substituting $y = 0$ into the flexure formula gives:

$$\sigma = \frac{M(0)}{I} = 0$$

Therefore, the bending stress is always zero at the centroid (neutral axis) of a beam. This makes option (A) correct. Option (B) is incorrect; bending stress is maximum at the extreme fibers (farthest from the neutral axis).

- **Shear Stress at the Centroid:** The term $A\bar{y}$ in the shear stress formula is the first moment of area. This term is maximum at the neutral axis for most common shapes (rectangle, I-section, circle). Therefore, the shear stress is typically **maximum** at the centroid, not zero or minimum. This makes options (C) and (D) incorrect.

Step 4: Final Answer:

At the centroid of a loaded beam cross section, the bending stress is equal to zero. The selected answer 'A' is correct, but it is important to note that the shear stress is maximum at this location.

💡 Quick Tip

Remember the stress distribution across a beam's cross-section:

- **Bending Stress (σ):** **Zero** at the neutral axis (centroid), **Maximum** at the top and bottom (extreme fibers).
- **Shear Stress (τ):** **Maximum** at the neutral axis (centroid), **Zero** at the top and bottom (extreme fibers).

113. A plate of 100 mm wide and 10 mm thick is bent into an arc of a circle of radius 10 m. If the modulus of elasticity of the material is 200 GPa, then the maximum bending stress induced in the plate is

- (A) 100 kN/m²
- (B) 100 N/mm²
- (C) 200 kN/m²
- (D) 200 N/mm²

Correct Answer: (B) 100 N/mm²

Solution:

Step 1: Understanding the Question:

The question asks for the maximum bending stress in a plate that is bent into a circular arc.

Step 2: Key Formula or Approach:

This problem uses the theory of simple bending. The fundamental bending equation is:

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

where:

σ = Bending stress at a distance y from the neutral axis

E = Modulus of Elasticity

R = Radius of curvature of the bent beam

We need to find the maximum bending stress, σ_{max} . From the equation, we can write:

$$\sigma = \frac{Ey}{R}$$

The maximum stress occurs at the extreme fiber, where y is maximum. For a rectangular plate of thickness t , the neutral axis is at the center, so $y_{max} = t/2$.

$$\sigma_{max} = \frac{E(t/2)}{R}$$

Step 3: Detailed Explanation:

First, identify and convert all values to consistent units (N and mm).

- Plate thickness (t) = 10 mm.
- Maximum distance from neutral axis (y_{max}) = $t/2 = 10 \text{ mm}/2 = 5 \text{ mm}$.
- Radius of curvature (R) = 10 m = $10 \times 1000 = 10,000 \text{ mm}$.
- Modulus of Elasticity (E) = 200 GPa = $200 \times 10^3 \text{ N/mm}^2$.
- The width of the plate (100 mm) is not needed to calculate the stress.

Now, substitute these values into the formula:

$$\begin{aligned}\sigma_{max} &= \frac{E \cdot y_{max}}{R} \\ \sigma_{max} &= \frac{(200 \times 10^3 \text{ N/mm}^2) \times (5 \text{ mm})}{10,000 \text{ mm}} \\ \sigma_{max} &= \frac{1,000,000}{10,000} \text{ N/mm}^2 \\ \sigma_{max} &= 100 \text{ N/mm}^2\end{aligned}$$

Note that 1 N/mm² is equal to 1 MPa. So the stress is 100 MPa.

Step 4: Final Answer:

The maximum bending stress induced in the plate is 100 N/mm².

 Quick Tip

The bending equation $\frac{\sigma}{y} = \frac{E}{R}$ is very useful when the radius of curvature is known. It directly relates stress to the geometry of the bent shape and the material's elasticity, without needing to know the applied moment (M) or moment of inertia (I). Again, ensure all units are consistent before calculating.

114. The maximum bending stress induced in a beam of rectangular section is 160 N/mm². If the depth of beam is increased by two times, keeping all the parameters

same, the maximum bending stress induced in the beam is

- (A) 20 N/mm²
- (B) 40 N/mm²
- (C) 80 N/mm²
- (D) 160 N/mm²

Correct Answer: (B) 40 N/mm²

Solution:

Step 1: Understanding the Question:

The question asks how the maximum bending stress in a rectangular beam changes when its depth is doubled, assuming the applied load and other dimensions remain the same.

Step 2: Key Formula or Approach:

The maximum bending stress (σ_{max}) is given by the flexure formula:

$$\sigma_{max} = \frac{M}{Z}$$

where M is the maximum bending moment and Z is the section modulus.

For a rectangular cross-section of width b and depth d :

The moment of inertia is $I = \frac{bd^3}{12}$.

The distance to the extreme fiber is $y_{max} = d/2$.

The section modulus is $Z = \frac{I}{y_{max}} = \frac{bd^3/12}{d/2} = \frac{bd^2}{6}$.

So, the stress formula becomes:

$$\sigma_{max} = \frac{M}{bd^2/6} = \frac{6M}{bd^2}$$

Step 3: Detailed Explanation:

From the formula, we can see the relationship between stress and depth:

$$\sigma_{max} \propto \frac{1}{d^2}$$

Let σ_1 and d_1 be the initial stress and depth, and σ_2 and d_2 be the final stress and depth.

We are given:

- $\sigma_1 = 160$ N/mm².
- The depth is "increased by two times", which can be interpreted as the new depth is twice the old one, $d_2 = 2d_1$.

Using the proportionality, we can write a ratio:

$$\frac{\sigma_2}{\sigma_1} = \left(\frac{d_1}{d_2}\right)^2$$

$$\frac{\sigma_2}{160} = \left(\frac{d_1}{2d_1} \right)^2 = \left(\frac{1}{2} \right)^2 = \frac{1}{4}$$

$$\sigma_2 = \frac{160}{4} = 40 \text{ N/mm}^2$$

Step 4: Final Answer:

The new maximum bending stress induced in the beam is 40 N/mm².

💡 Quick Tip

For rectangular beams, the section modulus Z is proportional to d^2 .
 Since bending stress $\sigma = M/Z$, the stress is inversely proportional to d^2 .
 This means doubling the depth makes the beam **four times stronger** in bending.

115. For a cantilever beam of T cross section subjected to uniformly distributed load throughout the length, the maximum bending stress occurs at

- (A) mid depth
- (B) top of cross section
- (C) bottom of cross section
- (D) junction of flange and web

Correct Answer: (C) bottom of cross section

Solution:

Step 1: Understanding the Question:

The question asks for the location of the maximum bending stress in a T-section cantilever beam under a UDL.

Step 2: Key Formula or Approach:

The bending stress is given by $\sigma = My/I$. The maximum bending stress occurs at the point with the maximum value of y , i.e., at the fiber farthest from the neutral axis. For a T-section, the neutral axis is not at the geometric center.

Step 3: Detailed Explanation:

1. **Bending Moment:** A cantilever beam with a UDL has a hogging (negative) bending moment throughout its length, with the maximum magnitude at the fixed support. A hogging moment causes tension in the top fibers and compression in the bottom fibers.
2. **Neutral Axis of a T-section:** For a T-section, the centroid (and thus the neutral axis) is located closer to the top flange because the flange has more area. This means the distance from the neutral axis to the top fiber (y_{top}) is smaller than the distance from the neutral axis to the bottom fiber of the web (y_{bottom}).

$$y_{bottom} > y_{top}$$

3. Maximum Bending Stress: The magnitude of the bending stress is $|\sigma| = |My/I|$. Since the bending moment M and moment of inertia I are constant for a given cross-section, the stress is maximum where the distance y is maximum.

Since $y_{bottom} > y_{top}$, the maximum magnitude of the bending stress will occur at the **bottom of the cross section**.

Specifically, the top fiber will have the maximum tensile stress ($\sigma_{tensile,max} = |M|y_{top}/I$) and the bottom fiber will have the maximum compressive stress ($\sigma_{compressive,max} = |M|y_{bottom}/I$). The overall maximum magnitude of stress occurs at the bottom.

Step 4: Final Answer:

For a T-section cantilever beam under a UDL, the maximum bending stress (in magnitude) occurs at the bottom of the cross section.

💡 Quick Tip

For any beam section, maximum bending stress always occurs at the extreme fibers (the points farthest from the neutral axis).

For asymmetric sections like a T-beam, you must first determine the location of the neutral axis (centroid) to identify which fiber (top or bottom) is farthest away.

116. If the maximum shear stress in a rectangular beam cross section is 120 N/mm², then the average shear stress is

- (A) 40 N/mm²
- (B) 80 N/mm²
- (C) 90 N/mm²
- (D) 180 N/mm²

Correct Answer: (B) 80 N/mm²

Solution:

Step 1: Understanding the Question:

The question asks for the average shear stress in a rectangular beam, given the maximum shear stress.

Step 2: Key Formula or Approach:

For a beam with a rectangular cross-section, the shear stress distribution is parabolic, with the maximum value at the neutral axis and zero at the top and bottom fibers.

The relationship between the maximum shear stress (τ_{max}) and the average shear stress (τ_{avg}) for a rectangular section is:

$$\tau_{max} = 1.5 \times \tau_{avg}$$

The average shear stress is simply the total shear force (V) divided by the cross-sectional area (A): $\tau_{avg} = V/A$.

Step 3: Detailed Explanation:

We are given the maximum shear stress:

$$\tau_{max} = 120 \text{ N/mm}^2$$

We need to find the average shear stress, τ_{avg} . We can rearrange the formula:

$$\begin{aligned}\tau_{avg} &= \frac{\tau_{max}}{1.5} \\ \tau_{avg} &= \frac{120 \text{ N/mm}^2}{1.5} \\ \tau_{avg} &= 80 \text{ N/mm}^2\end{aligned}$$

Step 4: Final Answer:

The average shear stress is 80 N/mm^2 .

💡 Quick Tip

Memorize the relationship between max and average shear stress for common shapes:

- **Rectangle:** $\tau_{max} = 1.5 \times \tau_{avg}$
- **Circle:** $\tau_{max} = \frac{4}{3} \times \tau_{avg} \approx 1.33 \times \tau_{avg}$
- **I-Beam:** $\tau_{max} \approx \tau_{avg}$ (in the web, where most shear is carried)

117. The ratio of moment of inertia of the section about neutral axis to the extreme fibre distance from the neutral axis is

- (A) Moment of inertia
- (B) Radius of gyration
- (C) Section modulus
- (D) Polar moment of inertia

Correct Answer: (C) Section modulus

Solution:

Step 1: Understanding the Question:

The question asks for the name of a geometric property of a cross-section, defined as a specific ratio involving moment of inertia and distance.

Step 2: Key Formula or Approach:

Let's define the terms:

- **Moment of Inertia (I):** A measure of an object's resistance to rotational motion about an axis. For a beam cross-section, it represents resistance to bending.

- **Extreme fiber distance (y_{max}):** The maximum distance from the neutral axis to the outermost edge of the cross-section.

The ratio described in the question is:

$$\frac{I}{y_{max}}$$

Step 3: Detailed Explanation:

This ratio is the definition of the **Section Modulus**, denoted by Z (or sometimes S).

$$Z = \frac{I}{y_{max}}$$

The section modulus is a direct measure of the strength of a beam in bending. The flexure formula, $\sigma_{max} = \frac{M_{max}}{I/y_{max}}$, is often simplified to:

$$\sigma_{max} = \frac{M_{max}}{Z}$$

This shows that for a given bending moment, a larger section modulus results in a lower maximum bending stress.

The other options are:

- **Radius of Gyration (k):** $k = \sqrt{I/A}$. It relates to buckling.

- **Polar Moment of Inertia (J):** A measure of resistance to torsion. For a planar shape, $J = I_x + I_y$.

Step 4: Final Answer:

The ratio of the moment of inertia of the section about the neutral axis to the extreme fiber distance from the neutral axis is the Section modulus.

 Quick Tip

Remember the key geometric properties and their roles:

- **Area (A):** Resists axial load.
- **Moment of Inertia (I):** Resists bending.
- **Section Modulus ($Z = I/y_{max}$):** A direct measure of bending strength.
- **Polar Moment of Inertia (J):** Resists torsion.

118. Theory of simple bending equation is applicable for

- (A) constant shear force and zero bending moment
- (B) constant bending moment and zero shear force
- (C) zero bending moment and zero shear force
- (D) constant bending moment and constant shear force

Correct Answer: (B) constant bending moment and zero shear force

Solution:

Step 1: Understanding the Question:

The question asks for the conditions under which the theory of simple bending (or pure bending) is strictly applicable.

Step 2: Detailed Explanation:

The **theory of simple bending**, which leads to the flexure formula $\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$, is derived based on a set of assumptions. One of the most important is that the beam is subjected to **pure bending**.

Pure bending is a loading condition where a beam is subjected to a constant bending moment and, consequently, zero shear force.

The relationship between bending moment (M) and shear force (V) is $V = dM/dx$.

If the bending moment M is constant, then its derivative with respect to x must be zero.

$$\frac{dM}{dx} = 0 \implies V = 0$$

This condition (constant bending moment and zero shear force) ensures that the stresses in the beam are purely normal stresses due to bending, without any complicating shear stresses that would warp the cross-section. While the bending equation is often applied to cases where shear force is present, its theoretical derivation is based on the case of pure bending.

Step 3: Final Answer:

The theory of simple bending equation is strictly applicable for a condition of constant bending moment and zero shear force.

💡 Quick Tip

The ideal condition for the simple bending theory is called **Pure Bending**.

Pure Bending means:

- Bending Moment = **Constant**.
- Shear Force = **Zero**.

An example is the central portion of a beam loaded with two symmetric point loads.

119. A simply supported rectangular beam of span 6 m having width of 100 mm and depth of 200 mm is subjected to a uniformly distributed load of 20 kN/m over the entire span. The maximum shear stress induced in the beam is

- (A) 3 N/mm²
- (B) 4.5 N/mm²
- (C) 6 N/mm²
- (D) 9 N/mm²

Correct Answer: (B) 4.5 N/mm²

Solution:

Step 1: Understanding the Question:

The question asks for the maximum shear stress in a simply supported rectangular beam under a UDL.

Step 2: Key Formula or Approach:

The calculation involves two main steps:

1. Find the maximum shear force (V_{max}) in the beam.
2. Use the formula for maximum shear stress in a rectangular section: $\tau_{max} = 1.5 \times \tau_{avg} = 1.5 \times \frac{V_{max}}{A}$.

Step 3: Detailed Explanation:**Part 1: Find the maximum shear force (V_{max}).**

For a simply supported beam of span L with a UDL of w over the entire span, the reactions at each support are equal and are half of the total load.

- Total load = $w \times L = 20 \text{ kN/m} \times 6 \text{ m} = 120 \text{ kN}$.

- Reactions: $R_A = R_B = \frac{120 \text{ kN}}{2} = 60 \text{ kN}$.

The maximum shear force in a simply supported beam occurs at the supports and is equal to the magnitude of the reactions.

$$V_{max} = 60 \text{ kN} = 60 \times 10^3 \text{ N}$$

Part 2: Calculate the maximum shear stress (τ_{max}).

First, find the cross-sectional area (A).

- Width (b) = 100 mm.

- Depth (d) = 200 mm.

- Area (A) = $b \times d = 100 \text{ mm} \times 200 \text{ mm} = 20,000 \text{ mm}^2$.

Now use the formula for τ_{max} for a rectangular section:

$$\begin{aligned}\tau_{max} &= 1.5 \times \frac{V_{max}}{A} \\ \tau_{max} &= 1.5 \times \frac{60 \times 10^3 \text{ N}}{20,000 \text{ mm}^2} \\ \tau_{max} &= 1.5 \times 3 \text{ N/mm}^2 \\ \tau_{max} &= 4.5 \text{ N/mm}^2\end{aligned}$$

This calculation gives 4.5 N/mm^2 . However, the provided answer key marks option (D) 9 N/mm^2 as correct. Let's re-check the calculation. The calculation is robust. $V_{max} = 60 \text{ kN}$, $A = 20000 \text{ mm}^2$, $\tau_{avg} = 3 \text{ N/mm}^2$, $\tau_{max} = 1.5 \times 3 = 4.5 \text{ N/mm}^2$. The provided answer key is incorrect. The correct answer is 4.5 N/mm^2 . We will proceed by justifying the calculated answer.

Step 4: Final Answer:

The maximum shear stress induced in the beam is 4.5 N/mm^2 . The provided answer key indicating 9 N/mm^2 appears to be in error, possibly due to a miscalculation (e.g., using the total load instead of the reaction or omitting the 1.5 factor incorrectly).

 Quick Tip

To find max shear stress, always follow the two steps:

1. Find V_{max} from the beam's loading diagram (for SS beam with UDL, it's at the supports).
2. Apply the correct formula for τ_{max} based on the cross-section shape (for a rectangle, it's $1.5 \times V/A$).

120. A point in a circular beam of diameter D is subjected to a shear force of F. The ratio of average shear stress to maximum shear stress is

- (A) 0.75
- (B) 1.33
- (C) 1.50
- (D) 0.67

Correct Answer: (A) 0.75

Solution:

Step 1: Understanding the Question:

The question asks for the ratio of the average shear stress to the maximum shear stress (τ_{avg}/τ_{max}) for a circular cross-section.

Step 2: Key Formula or Approach:

For a beam with a circular cross-section, the shear stress distribution is parabolic, with the maximum value at the neutral axis (the center) and zero at the top and bottom.

The relationship between the maximum shear stress (τ_{max}) and the average shear stress (τ_{avg}) for a circular section is:

$$\tau_{max} = \frac{4}{3} \times \tau_{avg}$$

Step 3: Detailed Explanation:

We are asked to find the ratio τ_{avg}/τ_{max} .

We can rearrange the formula to find this ratio:

$$\frac{\tau_{avg}}{\tau_{max}} = \frac{1}{4/3}$$

$$\frac{\tau_{avg}}{\tau_{max}} = \frac{3}{4}$$

$$\frac{\tau_{avg}}{\tau_{max}} = 0.75$$

Step 4: Final Answer:

The ratio of average shear stress to maximum shear stress for a circular beam is 0.75.

💡 Quick Tip

Be careful with the wording of the ratio.

- **Max to Average** for a circle is $\tau_{max}/\tau_{avg} = 4/3 \approx 1.33$.

- **Average to Max** for a circle is $\tau_{avg}/\tau_{max} = 3/4 = 0.75$.

Read the question carefully to provide the correct ratio.

121. At the free end of a cantilever beam subjected to downward loads,

- (A) slope and deflections are minimum
- (B) slope and deflections are maximum
- (C) slope is maximum and deflection is minimum
- (D) slope is minimum and deflection is maximum

Correct Answer: (B) slope and deflections are maximum

Solution:

Step 1: Understanding the Question:

The question asks about the slope and deflection at the free end of a cantilever beam under downward loading.

Step 2: Detailed Explanation:

Let's consider a cantilever beam fixed at one end and free at the other. When downward loads are applied, the beam bends downwards.

- **Deflection:** Deflection is the vertical displacement of the beam from its original position. The fixed support cannot move, so the deflection there is zero by definition. As we move away from the fixed support towards the free end, the beam bends more and more. The largest vertical displacement will occur at the very tip, the free end. Therefore, the **deflection is maximum** at the free end.
- **Slope:** Slope is the angle of the deflected beam with respect to the horizontal. At the fixed support, the beam is held rigidly horizontal, so the slope is zero by definition. As the beam bends, its angle with the horizontal increases, reaching its largest value at the free end. Therefore, the **slope is also maximum** at the free end.

Step 3: Final Answer:

At the free end of a cantilever beam subjected to downward loads, both the slope and the deflection are maximum.

💡 Quick Tip

Remember the boundary conditions for a cantilever beam (fixed at A, free at B):

- **At the Fixed End (A):** Deflection = 0, Slope = 0.
- **At the Free End (B):** Bending Moment = 0, Shear Force = 0 (unless there is a point load right at the end). Slope and Deflection are **maximum**.

122. According to moment area method, the vertical intercept at a point between the tangents drawn from two points is equal to moment of the area of

- (A) shear force diagram between those two points about the point under consideration divided by EI .
- (B) bending moment diagram between those two points about the point under consideration divided by EI
- (C) deflected curve between those two points about the point under consideration divided by EI
- (D) axial thrust diagram between those two points about the point under consideration divided by EI

Correct Answer: (B) bending moment diagram between those two points about the point under consideration divided by EI

Solution:

Step 1: Understanding the Question:

The question asks for the statement of the Second Moment-Area Theorem.

Step 2: Detailed Explanation:

The Moment-Area Theorems are a method for finding the slope and deflection of beams. There are two theorems:

1. **First Moment-Area Theorem:** The change in slope between any two points on the elastic curve of a beam is equal to the area of the M/EI diagram between those two points.
2. **Second Moment-Area Theorem:** The vertical deviation (or vertical intercept) of a point B on the elastic curve with respect to the tangent drawn at another point A is equal to the **moment of the area of the M/EI diagram** between points A and B, taken about point B.

The question describes "the vertical intercept at a point between the tangents drawn from two points". This is a direct statement of the Second Moment-Area Theorem. The area it refers to is the area of the bending moment diagram divided by EI (the M/EI diagram). The moment is taken about the point where the deviation is being measured.

Step 3: Final Answer:

The vertical intercept is equal to the moment of the area of the bending moment diagram between those two points about the point under consideration, divided by EI .

💡 Quick Tip

Remember the two Moment-Area theorems in short form:

- **Theorem 1 (Slope):** $\Delta\theta = \text{Area of } M/EI \text{ diagram.}$
- **Theorem 2 (Deflection):** $\text{Deflection} = \text{Moment of Area of } M/EI \text{ diagram.}$

123. The expression $EI \frac{d^2y}{dx^2}$ at a section of a beam represents

- (A) Rate of loading
- (B) Deflection
- (C) Shear force
- (D) Bending moment

Correct Answer: (D) Bending moment

Solution:

Step 1: Understanding the Question:

The question asks to identify the physical quantity represented by the mathematical expression $EI \frac{d^2y}{dx^2}$ in beam theory.

Step 2: Key Formula or Approach:

This expression comes from the Euler-Bernoulli beam theory, which establishes the relationship between the beam's deflection and the internal forces. The fundamental differential equation of the elastic curve (deflection curve) of a beam is:

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

where:

y is the deflection of the beam at a position x along its length.

M is the internal bending moment at that section.

E is the modulus of elasticity of the beam material.

I is the moment of inertia of the beam's cross-section.

EI is the flexural rigidity of the beam.

Step 3: Detailed Explanation:

By rearranging the differential equation, we get:

$$M = EI \frac{d^2y}{dx^2}$$

This shows that the expression $EI \frac{d^2 y}{dx^2}$ is exactly equal to the bending moment (M) at that section of the beam.

The other quantities are related to higher derivatives:

- Shear Force (V) = $\frac{dM}{dx} = EI \frac{d^3 y}{dx^3}$
- Rate of loading (w) = $-\frac{dV}{dx} = -EI \frac{d^4 y}{dx^4}$
- Deflection is simply y .

Step 4: Final Answer:

The expression $EI \frac{d^2 y}{dx^2}$ at a section of a beam represents the bending moment.

💡 Quick Tip

Memorize the hierarchy of beam equations:

- $y(x)$ = Deflection
- $\frac{dy}{dx}$ = Slope
- $EI \frac{d^2 y}{dx^2}$ = Bending Moment (M)
- $EI \frac{d^3 y}{dx^3}$ = Shear Force (V)
- $EI \frac{d^4 y}{dx^4}$ = - (Load Intensity, w)

124. A cantilever beam of span 2 m carries a concentrated load of 48 kN at the free end. If the flexural rigidity of beam $EI = 20,000 \text{ kNm}^2$, the slope at the free end is

- (A) 3.2×10^{-3} radians
- (B) 4.8×10^{-3} radians
- (C) 48×10^{-3} radians
- (D) 64×10^{-3} radians

Correct Answer: (B) 4.8×10^{-3} radians

Solution:

Step 1: Understanding the Question:

The question asks to calculate the slope at the free end of a cantilever beam subjected to a point load at its tip.

Step 2: Key Formula or Approach:

For a cantilever beam of length L with a point load P at the free end, the standard formula for the slope (θ) at the free end is:

$$\theta_{\text{free-end}} = \frac{PL^2}{2EI}$$

Step 3: Detailed Explanation:

First, identify the given values. It's important to ensure the units are consistent.

- Load (P) = 48 kN
- Span (L) = 2 m
- Flexural Rigidity (EI) = 20,000 kNm²

All units (kN and m) are consistent, so no conversion is needed. The resulting slope will be in radians.

Now, substitute the values into the formula:

$$\begin{aligned}\theta &= \frac{(48 \text{ kN}) \times (2 \text{ m})^2}{2 \times (20,000 \text{ kNm}^2)} \\ \theta &= \frac{48 \times 4}{40,000} \\ \theta &= \frac{192}{40,000} \\ \theta &= 0.0048 \text{ radians}\end{aligned}$$

To express this in the format of the options, we write it in scientific notation:

$$\theta = 4.8 \times 10^{-3} \text{ radians}$$

Step 4: Final Answer:

The slope at the free end is 4.8×10^{-3} radians.

💡 Quick Tip

Memorize the standard formulas for cantilever beam slope and deflection:

- **Load at Free End (P):**

- Max Slope = $PL^2/(2EI)$

- Max Deflection = $PL^3/(3EI)$

- **UDL over entire span (w):**

- Max Slope = $wL^3/(6EI)$

- Max Deflection = $wL^4/(8EI)$

125. A simply supported beam of span $2L$ and flexural rigidity EI is subjected to a uniformly distributed load of w/m throughout the span. The deflection at mid span is

- (A) $\frac{5wL^4}{384EI}$
- (B) $\frac{5wL^4}{48EI}$
- (C) $\frac{5wL^4}{24EI}$
- (D) $\frac{1wL^4}{48EI}$

Correct Answer: (C) $\frac{5wL^4}{24EI}$

Solution:

Step 1: Understanding the Question:

The question asks for the formula for the maximum deflection (at mid-span) of a simply supported beam of a given length and under a UDL.

Step 2: Key Formula or Approach:

The standard formula for the maximum deflection (δ_{max}) of a simply supported beam of span 'S' under a uniformly distributed load 'w' over its entire length is:

$$\delta_{max} = \frac{5wS^4}{384EI}$$

Step 3: Detailed Explanation:

In this problem, the variables are given with slightly different notation. We must be careful to substitute correctly.

- The given span is $S = 2L$.
- The given load intensity is w .

Now, we substitute $S = 2L$ into the standard formula:

$$\begin{aligned}\delta_{max} &= \frac{5w(2L)^4}{384EI} \\ \delta_{max} &= \frac{5w(16L^4)}{384EI} \\ \delta_{max} &= \frac{80wL^4}{384EI}\end{aligned}$$

Now, simplify the fraction $80/384$. We can divide both by 16:

$$80/16 = 5$$

$$384/16 = 24$$

So the expression becomes:

$$\delta_{max} = \frac{5wL^4}{24EI}$$

Step 4: Final Answer:

The deflection at mid span is $\frac{5wL^4}{24EI}$.

 Quick Tip

Be very careful when a problem uses non-standard notation for span length (like $2L$ instead of L).

Always start with the standard formula you have memorized and then carefully substitute the specific variables given in the problem.

126. A simply supported beam of span L and flexural rigidity EI is subjected to a concentrated load of W at mid span. The ratio of maximum deflection to maximum slope anywhere in the beam is

- (A) $L/2$
- (B) $L/3$
- (C) $L/4$
- (D) $L/5$

Correct Answer: (B) $L/3$

Solution:

Step 1: Understanding the Question:

The question asks for the ratio of the maximum deflection to the maximum slope for a simply supported beam with a point load at the center.

Step 2: Key Formula or Approach:

We need the standard formulas for maximum deflection (δ_{max}) and maximum slope (θ_{max}) for this loading case.

1. **Maximum Deflection:** Occurs at the center, under the load.

$$\delta_{max} = \frac{WL^3}{48EI}$$

2. **Maximum Slope:** Occurs at the supports (A and B).

$$\theta_{max} = \frac{WL^2}{16EI}$$

Step 3: Detailed Explanation:

Now we need to find the ratio $\frac{\delta_{max}}{\theta_{max}}$.

$$\frac{\delta_{max}}{\theta_{max}} = \frac{WL^3/(48EI)}{WL^2/(16EI)}$$

We can cancel the common terms W , L^2 , and EI from the numerator and denominator:

$$\begin{aligned}\frac{\delta_{max}}{\theta_{max}} &= \frac{L/48}{1/16} \\ \frac{\delta_{max}}{\theta_{max}} &= \frac{L}{48} \times 16\end{aligned}$$

$$\frac{\delta_{max}}{\theta_{max}} = \frac{16L}{48}$$

Simplifying the fraction:

$$\frac{\delta_{max}}{\theta_{max}} = \frac{L}{3}$$

Step 4: Final Answer:

The ratio of maximum deflection to maximum slope is $L/3$.

💡 Quick Tip

Memorizing the standard deflection and slope formulas for simply supported and cantilever beams is essential for solving these types of problems quickly.

- **SS Beam, Center Load P :** $\delta_{max} = PL^3/(48EI)$, $\theta_{max} = PL^2/(16EI)$.
- **SS Beam, UDL w :** $\delta_{max} = 5wL^4/(384EI)$, $\theta_{max} = wL^3/(24EI)$.

127. A fixed beam of span 8 m and flexural rigidity $EI = 51,200 \text{ kNm}^2$ is subjected to a concentrated load of 96 kN at mid span. The maximum deflection under the load is

- (A) $1.25 \times 10^{-3} \text{ mm}$
- (B) $2.5 \times 10^{-3} \text{ mm}$
- (C) 5 mm
- (D) 2.5 mm

Correct Answer: (C) 5 mm

Solution:

Step 1: Understanding the Question:

The question asks for the maximum deflection of a fixed-end beam with a concentrated load at its center.

Step 2: Key Formula or Approach:

The standard formula for the maximum deflection (δ_{max}) of a fixed beam of span L subjected to a point load P at mid-span is:

$$\delta_{max} = \frac{PL^3}{192EI}$$

Step 3: Detailed Explanation:

First, identify the given values. Ensure the units are consistent.

- Load (P) = 96 kN

- Span (L) = 8 m
- Flexural Rigidity (EI) = 51,200 kNm²

All units are in kN and m. The result will be in meters.

Substitute the values into the formula:

$$\delta_{max} = \frac{(96 \text{ kN}) \times (8 \text{ m})^3}{192 \times (51,200 \text{ kNm}^2)}$$

$$\delta_{max} = \frac{96 \times 512}{192 \times 51,200}$$

We can simplify this. Note that $192 = 2 \times 96$ and $51,200 = 100 \times 512$.

$$\delta_{max} = \frac{96 \times 512}{(2 \times 96) \times (100 \times 512)}$$

Cancel the terms 96 and 512:

$$\delta_{max} = \frac{1}{2 \times 100} = \frac{1}{200} \text{ m}$$

$$\delta_{max} = 0.005 \text{ m}$$

The options are in mm. Convert the result to mm:

$$\delta_{max} = 0.005 \text{ m} \times 1000 \frac{\text{mm}}{\text{m}} = 5 \text{ mm}$$

Step 4: Final Answer:

The maximum deflection under the load is 5 mm.

💡 Quick Tip

The maximum deflection for a fixed beam is much smaller than for a simply supported beam under the same load.

- Fixed Beam, Center Load: $\delta_{max} = \frac{PL^3}{192EI}$

- SS Beam, Center Load: $\delta_{max} = \frac{PL^3}{48EI}$

The fixed beam is $192/48 = 4$ times stiffer.

128. Continuous beam is a

- (A) Statically determinate structure
- (B) Statically indeterminate structure
- (C) Kinematically determinate structure
- (D) Dynamically determinate structure

Correct Answer: (B) Statically indeterminate structure

Solution:

Step 1: Understanding the Question:

The question asks to classify a continuous beam based on its static determinacy.

Step 2: Detailed Explanation:

A structure is **statically determinate** if all of its unknown support reactions and internal forces can be determined using only the equations of static equilibrium ($\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M = 0$). For a 2D beam, there are 3 available equilibrium equations.

A structure is **statically indeterminate** (or hyperstatic) if it has more unknown reactions than the number of available static equilibrium equations. The extra, redundant reactions require additional equations (based on material properties and deformation, i.e., compatibility equations) to be solved.

A **continuous beam** is a beam that rests on more than two supports.

- A beam with three supports (e.g., a pin and two rollers) will have 4 unknown reactions (2 at the pin, 1 at each roller). Since there are only 3 equilibrium equations, there is $4 - 3 = 1$ redundant reaction.

- A beam with 'n' supports will generally have at least $n+1$ reactions.

Because the number of unknown reactions is greater than the number of equilibrium equations, a continuous beam cannot be fully analyzed using statics alone.

Step 3: Final Answer:

A continuous beam is a statically indeterminate structure.

💡 Quick Tip

Quickly determine static determinacy for beams:

- **Determinate:** Simply Supported Beam, Cantilever Beam, Overhanging Beam (if reactions ≤ 3).

- **Indeterminate:** Continuous Beam, Fixed Beam, Propped Cantilever.

Count the number of unknown reaction components. If it's more than 3, it's indeterminate.

129. A propped cantilever AB of span 8 m is fixed at A and supported by a prop at B. It is subjected to a concentrated load of 64 kN acting at mid span. The reaction of the prop is

- (A) 20 kN
- (B) 32 kN
- (C) 40 kN
- (D) 44 kN

Correct Answer: (A) 20 kN

Solution:

Step 1: Understanding the Question:

The question asks for the reaction force at the propped end (B) of a propped cantilever beam with a point load at its center.

Step 2: Key Formula or Approach:

A propped cantilever is a statically indeterminate structure. We need to use a method that accounts for beam deflection, such as the principle of superposition.

We can think of the propped cantilever as the sum of two cases:

1. A cantilever beam with only the downward load P at mid-span. This causes a downward deflection at the free end, δ_{B1} .
2. A cantilever beam with only the upward reaction force from the prop, R_B , at the free end. This causes an upward deflection at the free end, δ_{B2} .

The compatibility condition is that the net deflection at the prop (point B) must be zero.

$$\delta_{B1}(\text{downward}) = \delta_{B2}(\text{upward})$$

The standard formulas for these deflections are needed.

- Deflection at the free end of a cantilever due to a point load P at mid-span ($L/2$) is: $\delta = \frac{5PL^3}{48EI}$.
- Deflection at the free end of a cantilever due to a point load R_B at the free end is: $\delta = \frac{R_B L^3}{3EI}$.

Step 3: Detailed Explanation:

Set the two deflections equal to each other:

$$\frac{5PL^3}{48EI} = \frac{R_B L^3}{3EI}$$

We can cancel the common terms L^3 and EI :

$$\frac{5P}{48} = \frac{R_B}{3}$$

Now solve for the prop reaction R_B :

$$R_B = \frac{3 \times 5P}{48} = \frac{15P}{48}$$

Simplify the fraction by dividing by 3:

$$R_B = \frac{5P}{16}$$

Now substitute the given value of the load, $P = 64$ kN:

$$\begin{aligned} R_B &= \frac{5 \times 64}{16} \\ R_B &= 5 \times 4 = 20 \text{ kN} \end{aligned}$$

The answer is 20 kN. However, the checkmark in the provided image is on 20 kN, which matches this calculation. The initial OCR indicated option 1 as '20 kN' with a checkmark, so the solution aligns with the provided key.

Step 4: Final Answer:

The reaction of the prop is 20 kN.

💡 Quick Tip

It is extremely useful to memorize the standard results for propped cantilevers.

- **Load P at mid-span:** Prop reaction $R_B = 5P/16$.
- **UDL w over entire span:** Prop reaction $R_B = 3wL/8$.

130. The nature of bending moment developed at the intermediate supports of a continuous beam carrying downward loads is

- (A) Sagging
- (B) Hogging
- (C) Sagging or Hogging
- (D) Clockwise

Correct Answer: (B) Hogging

Solution:

Step 1: Understanding the Question:

The question asks for the sign or nature of the bending moment that occurs over the interior supports of a continuous beam subjected to downward vertical loads.

Step 2: Detailed Explanation:

A **continuous beam** spans over multiple supports. When downward loads (like gravity loads) are applied, the beam deflects.

- In the regions between the supports (in the spans), the beam tends to bend downwards, like a smiley face. This shape is called **sagging**, and it corresponds to a **positive** bending moment. This causes tension at the bottom fibers and compression at the top.

- Over the intermediate supports, the beam is bent upwards in a convex shape, like a frowny face. The support pushes up while the spans on either side sag down. This shape is called **hogging**, and it corresponds to a **negative** bending moment. This causes tension at the top fibers and compression at the bottom.

Therefore, the moments at the intermediate supports are always hogging moments for downward loads.

Step 3: Final Answer:

The nature of the bending moment developed at the intermediate supports of a continuous beam carrying downward loads is Hogging.

💡 Quick Tip

Use a simple visual aid for bending moments:

- **Sagging** = beam bends into a "smile" shape $\smile$ = **Positive Moment**. (Occurs mid-span)
- **Hogging** = beam bends into a "frown" shape $\frown$ = **Negative Moment**. (Occurs over intermediate supports and at fixed ends of cantilevers).

131. A column whose slenderness ratio is greater than 120 is known as

- (A) Short column
- (B) Medium column
- (C) Long column
- (D) Composite column

Correct Answer: (C) Long column

Solution:

Step 1: Understanding the Question:

The question asks for the classification of a column based on a given slenderness ratio.

Step 2: Detailed Explanation:

Columns are compression members that are classified based on their length, cross-sectional dimensions, and end conditions, which are all combined into a single parameter called the **slenderness ratio** (λ). The slenderness ratio determines the column's mode of failure.

$$\lambda = \frac{L_{eff}}{k_{min}}$$

where L_{eff} is the effective length and k_{min} is the minimum radius of gyration.

The classifications are generally:

- **Short Columns:** Have a small slenderness ratio. They fail by crushing (yielding of the material) at a stress close to the material's compressive strength. Their strength is independent of their length.
- **Long Columns (or Slender Columns):** Have a large slenderness ratio. They fail by elastic **buckling**, a sudden lateral deflection, at a stress that can be much lower than the material's yield strength. Their failure load is highly dependent on their length and stiffness. The Euler column formula applies to these.
- **Intermediate Columns:** Fall between short and long columns. They fail by a combination of crushing and buckling (inelastic buckling).

The specific numerical values for these ranges depend on the material and the design code. For steel, a common rule of thumb (though it varies) might place the transition to long columns somewhere between a slenderness ratio of 80 and 120. A value **greater than 120** is definitively in the range of a **long column**, where buckling is the primary failure mode.

Step 3: Final Answer:

A column whose slenderness ratio is greater than 120 is known as a long column.

💡 Quick Tip

Remember the failure modes based on slenderness ratio (λ):

- **Low** $\lambda \rightarrow$ Short Column $\rightarrow$ Fails by **Crushing**.
- **High** $\lambda \rightarrow$ Long Column $\rightarrow$ Fails by **Buckling**.
- **Intermediate** $\lambda \rightarrow$ Intermediate Column $\rightarrow$ Fails by **Crushing + Buckling**.

132. Compression members always tends to buckle in the direction of

- (A) Horizontal axis
- (B) Vertical axis
- (C) Minimum cross section
- (D) Least radius of gyration

Correct Answer: (D) Least radius of gyration

Solution:

Step 1: Understanding the Question:

The question asks about the direction or axis around which a compression member (a column) will buckle.

Step 2: Detailed Explanation:

Buckling is a failure mode characterized by a sudden lateral deflection. A column under compression will always buckle in the way that is easiest for it to do so, which is the direction of its least resistance to bending.

The resistance of a column to bending (and thus buckling) is determined by its flexural rigidity, EI . For a given material (E), the resistance depends on the moment of inertia, I .

A column's cross-section will generally have different moments of inertia about its different principal axes (e.g., I_{xx} and I_{yy} for a rectangular or I-section). The column will buckle by bending about the axis with the **minimum moment of inertia** (I_{min}).

The **radius of gyration** (k) is defined as $k = \sqrt{I/A}$. Since the area A is constant for the cross-section, the axis with the minimum moment of inertia (I_{min}) is also the axis with the **least radius of gyration** (k_{min}).

Therefore, a column always tends to buckle about the axis corresponding to its least radius of gyration, as this is its weakest direction.

Step 3: Final Answer:

Compression members always tend to buckle in the direction of the least radius of gyration.

💡 Quick Tip

Think of it like trying to bend a ruler. It's very easy to bend it about its thin axis (low moment of inertia) but very difficult to bend it about its wide axis (high moment of inertia).

A column behaves the same way under compression - it will buckle in its "flattest" or weakest direction, which corresponds to the axis with the least radius of gyration.

133. The effective length of electric pole of 6.5 m height above the ground level is

- (A) 3.25 m
- (B) 6.5 m
- (C) 9.75 m
- (D) 13.0 m

Correct Answer: (D) 13.0 m

Solution:

Step 1: Understanding the Question:

The question asks for the effective length (L_{eff}) of an electric pole, given its height above the ground.

Step 2: Key Formula or Approach:

The **effective length** of a column is the length of an equivalent pin-ended column that has the same buckling load. It is calculated as:

$$L_{eff} = K \times L$$

where L is the actual unbraced length of the column, and K is the effective length factor, which depends on the end support conditions.

An electric pole is modeled as a column that is **fixed at the ground** and **free at the top**.

For a "fixed-free" column, the theoretical value of the effective length factor is $K = 2.0$.

Step 3: Detailed Explanation:

We are given:

- Actual length above ground (L) = 6.5 m
- End conditions: Fixed at the base, free at the top.
- Effective length factor (K) for fixed-free condition = 2.0

Now, calculate the effective length:

$$L_{eff} = K \times L = 2.0 \times 6.5 \text{ m}$$

$$L_{eff} = 13.0 \text{ m}$$

Step 4: Final Answer:

The effective length of the electric pole is 13.0 m.

 Quick Tip

Memorize the theoretical K-factors for common column end conditions:

- **Pinned-Pinned:** $K = 1.0$
- **Fixed-Fixed:** $K = 0.5$
- **Fixed-Pinned:** $K = 0.7$
- **Fixed-Free** (like a flagpole or electric pole): $K = 2.0$

134. A long Column with fixed ends can carry load as compared to one end fixed and the other free is

- (A) 2 times
- (B) 4 times
- (C) 8 times
- (D) 16 times

Correct Answer: (D) 16 times

Solution:

Step 1: Understanding the Question:

The question asks to compare the critical buckling load of a long column with fixed-fixed ends to that of a column with fixed-free ends.

Step 2: Key Formula or Approach:

The critical buckling load (P_{cr}) for a long column is given by Euler's formula:

$$P_{cr} = \frac{\pi^2 EI}{(L_{eff})^2} = \frac{\pi^2 EI}{(KL)^2}$$

where L_{eff} is the effective length, K is the effective length factor, and L is the actual length.

Step 3: Detailed Explanation:

We need to find the buckling load for each case. Let's assume both columns have the same actual length L , and the same material and cross-section (same EI).

Case 1: Fixed-Fixed Ends

- Effective length factor, $K_1 = 0.5$
- Critical load, $P_{cr1} = \frac{\pi^2 EI}{(0.5L)^2} = \frac{\pi^2 EI}{0.25L^2} = 4 \frac{\pi^2 EI}{L^2}$

Case 2: Fixed-Free Ends

- Effective length factor, $K_2 = 2.0$
- Critical load, $P_{cr2} = \frac{\pi^2 EI}{(2.0L)^2} = \frac{\pi^2 EI}{4L^2} = 0.25 \frac{\pi^2 EI}{L^2}$

Now, find the ratio of the loads:

$$\frac{\text{Load of Fixed-Fixed}}{\text{Load of Fixed-Free}} = \frac{P_{cr1}}{P_{cr2}} = \frac{4(\pi^2 EI/L^2)}{0.25(\pi^2 EI/L^2)}$$

$$\text{Ratio} = \frac{4}{0.25} = \frac{4}{1/4} = 4 \times 4 = 16$$

Step 4: Final Answer:

A long column with fixed ends can carry 16 times the load of a column with one end fixed and the other free.

💡 Quick Tip

The buckling load is inversely proportional to the square of the effective length ($P_{cr} \propto 1/L_{eff}^2$).

- Fixed-Fixed: $L_{eff} = L/2$

- Fixed-Free: $L_{eff} = 2L$

The effective length is 4 times smaller for fixed-fixed. Therefore, the load capacity is $4^2 = 16$ times larger.

135. The ratio of effective length of a column and minimum radius of gyration of its cross-sectional area is known as

- (A) Crippling factor
- (B) Buckling factor
- (C) Slenderness ratio
- (D) Load factor

Correct Answer: (C) Slenderness ratio

Solution:

Step 1: Understanding the Question:

The question asks for the definition of a specific ratio used in column analysis.

Step 2: Key Formula or Approach:

The question describes the ratio:

$$\frac{\text{Effective Length}}{\text{Minimum Radius of Gyration}}$$

Step 3: Detailed Explanation:

This is the precise definition of the **Slenderness Ratio**, commonly denoted by λ (lambda) or

SR.

$$\lambda = \frac{L_{eff}}{k_{min}}$$

The slenderness ratio is a crucial dimensionless number that determines a column's behavior under compression. As explained in question 131, it is used to classify a column as short, intermediate, or long, which in turn determines its failure mode (crushing or buckling). A high slenderness ratio indicates a slender column that is prone to buckling.

- Crippling factor and buckling factor are less standard terms, sometimes used to refer to the load itself or related parameters, but they do not define this specific ratio.
- Load factor is a term used in structural design to account for uncertainties in loads, and is unrelated to column geometry.

Step 4: Final Answer:

The ratio of the effective length of a column and the minimum radius of gyration of its cross-sectional area is known as the slenderness ratio.

💡 Quick Tip

The Slenderness Ratio ($\lambda = L_{eff}/k_{min}$) is the single most important parameter in column buckling analysis.

It combines information about the column's length, its end supports, and its cross-sectional shape into one number that predicts its susceptibility to buckling.

136. A three-member isosceles right-angle triangular truss is hinged at A and roller support at B such that the vertical member AB is perpendicular to the horizontal member AC. If a vertical force of 100 kN is acting at C, the force in the member AC is equal to

- (A) zero
- (B) 100 kN
- (C) $100\sqrt{2}$ kN
- (D) $100/\sqrt{2}$ kN

Correct Answer: (B) 100 kN

Solution:

Step 1: Understanding the Question:

The question asks for the internal axial force in the horizontal member AC of a specific triangular truss under a vertical load at joint C.

Step 2: Key Formula or Approach:

The most direct way to solve for the forces in members connected to a loaded joint is the Method of Joints. This method involves applying the equations of static equilibrium ($\sum F_x = 0$

and $\Sigma F_y = 0$) to the joint.

Step 3: Detailed Explanation:

Let's analyze the forces acting on joint C.

- There is an external vertical downward load of 100 kN.
- There is an internal force from the horizontal member AC, which we'll call F_{AC} .
- There is an internal force from the diagonal member BC, which we'll call F_{BC} .

Geometry: The truss is an isosceles right-angle triangle with the right angle at A. This means the angles at B and C are both 45° . Therefore, the member BC is inclined at 45° to the horizontal.

Equilibrium at Joint C:

We resolve the forces into horizontal (x) and vertical (y) components. Let's assume tensile forces (pulling away from the joint) are positive.

1. Sum of vertical forces ($\Sigma F_y = 0$):

The downward external load of 100 kN must be balanced by the upward vertical component of the force in member BC. This implies that member BC must be in tension (pulling joint C upwards).

$$F_{BC} \sin(45^\circ) - 100 \text{ kN} = 0$$
$$F_{BC} = \frac{100}{\sin(45^\circ)} = \frac{100}{1/\sqrt{2}} = 100\sqrt{2} \text{ kN (Tension)}$$

2. Sum of horizontal forces ($\Sigma F_x = 0$):

The horizontal component of the force in member BC pulls joint C to the left. To balance this, the force in member AC, F_{AC} , must pull to the right. This implies that member AC is also in tension.

$$F_{AC} - F_{BC} \cos(45^\circ) = 0$$

$$F_{AC} = F_{BC} \cos(45^\circ)$$

Substitute the value of F_{BC} we just found:

$$F_{AC} = (100\sqrt{2} \text{ kN}) \times \cos(45^\circ)$$

Since $\cos(45^\circ) = 1/\sqrt{2}$:

$$F_{AC} = (100\sqrt{2}) \times \frac{1}{\sqrt{2}} = 100 \text{ kN}$$

Since the result is positive, our assumption that AC is in tension is correct. The magnitude of the force is 100 kN.

Step 4: Final Answer:

The force in the member AC is equal to 100 kN.

💡 Quick Tip

For simple trusses, the Method of Joints is often the fastest approach.

At joint C, the external vertical load must be balanced by the vertical component of the diagonal member.

The horizontal force in the horizontal member must then balance the horizontal component of the diagonal member.

Since the angles are 45° , the horizontal and vertical components of the diagonal force are equal.

137. If a truss consists of 6 joints and 3 reaction components, then the number of members required for determinate truss is

- (A) 8
- (B) 9
- (C) 12
- (D) 15

Correct Answer: (B) 9

Solution:

Step 1: Understanding the Question:

The question asks for the number of members (m) required for a planar truss to be statically determinate, given the number of joints (j) and reaction components (r).

Step 2: Key Formula or Approach:

For a planar (2D) truss, the condition for static determinacy is given by the formula:

$$m + r = 2j$$

where:

m = number of members

r = number of external reaction components

j = number of joints

This equation comes from the fact that at each of the j joints, there are two equilibrium equations ($\Sigma F_x = 0$ and $\Sigma F_y = 0$), giving a total of $2j$ available equations. The unknowns are the forces in the m members and the r reactions. For determinacy, the number of unknowns must equal the number of equations.

Step 3: Detailed Explanation:

We are given:

- Number of joints (j) = 6
- Number of reaction components (r) = 3 (This corresponds to a typical simply supported truss with one pin and one roller support).

We need to find the required number of members (m).
Substitute the given values into the formula:

$$m + 3 = 2 \times 6$$

$$m + 3 = 12$$

$$m = 12 - 3$$

$$m = 9$$

Step 4: Final Answer:

The number of members required for the truss to be determinate is 9.

💡 Quick Tip

Memorize the determinacy formula for trusses:

- **2D Truss:** $m + r = 2j$

- **3D Truss:** $m + r = 3j$

If $m + r > 2j$, the truss is indeterminate (redundant).

If $m + r < 2j$, the truss is unstable (a mechanism).

138. The force induced in the members of truss due to loading is

- (A) Axial force
- (B) Shear force
- (C) Bending moment
- (D) Twisting moment

Correct Answer: (A) Axial force

Solution:

Step 1: Understanding the Question:

The question asks to identify the type of internal force that exists in the members of an ideal truss.

Step 2: Detailed Explanation:

The analysis of trusses is based on a set of simplifying assumptions for an "ideal truss":

1. The members are connected only at their ends by frictionless pins (joints).

2. All external loads and support reactions are applied only at the joints.
3. The weight of the members is negligible (or is considered as loads applied at the joints).

Because of these assumptions, the members of a truss are two-force members. A two-force member can only be subjected to forces that are equal, opposite, and collinear. This means the internal force within the member acts purely along its longitudinal axis. This type of force is called an **axial force**. It can be either tensile (pulling the member apart) or compressive (pushing the member together).

Ideal truss members do not experience shear force, bending moment, or twisting moment.

Step 3: Final Answer:

The force induced in the members of a truss due to loading is an axial force (either tension or compression).

💡 Quick Tip

Remember the fundamental difference between a beam and a truss member:

- **Beam:** Designed to resist loads perpendicular to its axis, creating internal **shear and bending moment**.
- **Truss Member (ideal):** Designed to resist loads at its ends only, creating internal **axial force** (tension/compression).

139. When the net horizontal force at the base of the dam exceeds the frictional resistance at the base, then the dam fails by

- (A) Crushing
- (B) Sliding
- (C) Overturning
- (D) Buckling

Correct Answer: (B) Sliding

Solution:

Step 1: Understanding the Question:

The question describes a specific failure scenario for a dam and asks for the name of this failure mode.

Step 2: Detailed Explanation:

The stability of a gravity dam is checked against several possible failure modes:

1. **(B) Sliding:** The net horizontal force (F_H) acting on the dam (primarily from the water pressure) tends to push the dam horizontally. This is resisted by the frictional force (F_f) between the base of the dam and the foundation rock. The frictional force is given by $F_f = \mu \times W$, where μ is the coefficient of friction and W is the total downward vertical force (weight of the dam). If the horizontal force exceeds the maximum possible frictional resistance ($F_H > F_f$), the dam will **slide** along its base. This exactly matches the condition described in the question.
2. **(C) Overturning:** The moment caused by the horizontal water pressure tends to rotate or overturn the dam about its "toe" (the downstream edge of the base). This is resisted by the moment caused by the dam's own weight, which acts to stabilize it. Failure occurs if the overturning moment exceeds the stabilizing moment.
3. **(A) Crushing:** The combined effect of the dam's weight and the forces from the water creates compressive stresses at the base. If these stresses exceed the allowable compressive strength of the dam's material (concrete) or the foundation rock, the material will crush.
4. **(D) Buckling:** This is a failure mode for slender compression members and is not relevant to a massive structure like a dam.

Step 3: Final Answer:

When the net horizontal force exceeds the frictional resistance at the base, the dam fails by sliding.

💡 Quick Tip

For dam stability, associate forces and moments with failure modes:

- **Horizontal Forces** → Check for **Sliding**.
- **Moments** (about the toe) → Check for **Overturning**.
- **Vertical Forces / Stresses** → Check for **Crushing**.

140. The intensity of active earth pressure on retaining wall at a depth of 4 m with backfill having an angle of shearing resistance of 30° and unit weight of 18 kN/m^3 is

- (A) 12 kN/m^2
- (B) 24 kN/m^2
- (C) 48 kN/m^2
- (D) 60 kN/m^2

Correct Answer: (B) 24 kN/m^2

Solution:

Step 1: Understanding the Question:

The question asks for the active earth pressure intensity (p_a) at a specific depth behind a retaining wall, given the soil properties.

Step 2: Key Formula or Approach:

According to Rankine's theory for active earth pressure in a cohesionless soil with a horizontal backfill, the pressure at a depth h is given by:

$$p_a = K_a \gamma h$$

where:

γ = unit weight of the soil

h = depth

K_a = coefficient of active earth pressure

The coefficient K_a is calculated as:

$$K_a = \frac{1 - \sin(\phi)}{1 + \sin(\phi)}$$

where ϕ is the angle of shearing resistance (or angle of internal friction).

Step 3: Detailed Explanation:

First, calculate the coefficient of active earth pressure, K_a .

- Angle of shearing resistance (ϕ) = 30°
- We know that $\sin(30^\circ) = 0.5$

$$K_a = \frac{1 - \sin(30^\circ)}{1 + \sin(30^\circ)} = \frac{1 - 0.5}{1 + 0.5} = \frac{0.5}{1.5} = \frac{1}{3}$$

Now, calculate the active earth pressure intensity, p_a .

- Unit weight (γ) = 18 kN/m^3
- Depth (h) = 4 m

$$\begin{aligned} p_a &= K_a \gamma h = \left(\frac{1}{3}\right) \times (18 \text{ kN/m}^3) \times (4 \text{ m}) \\ p_a &= 6 \times 4 \text{ kN/m}^2 \\ p_a &= 24 \text{ kN/m}^2 \end{aligned}$$

Step 4: Final Answer:

The intensity of active earth pressure at a depth of 4 m is 24 kN/m^2 .

💡 Quick Tip

Memorize the Rankine coefficients for the common case of $\phi = 30^\circ$:

- Active coefficient: $K_a = \frac{1 - \sin(30)}{1 + \sin(30)} = \frac{1 - 0.5}{1 + 0.5} = 1/3$.
- Passive coefficient: $K_p = \frac{1 + \sin(30)}{1 - \sin(30)} = \frac{1 + 0.5}{1 - 0.5} = 3$.

Note that $K_p = 1/K_a$.

141. In a reinforced concrete beam, if the stress in steel reinforcement attains its yield stress before the concrete fails due to crushing, the beam is said to be

- (A) balanced
- (B) under reinforced
- (C) over reinforced
- (D) non-homogeneous

Correct Answer: (B) under reinforced

Solution:

Step 1: Understanding the Question:

The question describes a failure mode for a reinforced concrete beam and asks for the name of this type of design.

Step 2: Detailed Explanation:

The design of reinforced concrete beams is classified based on the sequence of failure. The goal is to ensure a safe failure mode.

- **(B) Under-reinforced Section:** This is a design where the amount of steel reinforcement is relatively low. When the beam is overloaded, the steel is the first component to reach its yield limit. Since steel is a ductile material, it will elongate significantly after yielding. This causes large deflections and wide cracks to appear in the concrete on the tension side of the beam, giving a clear and ample warning of impending failure long before the concrete on the compression side crushes. This is a **ductile failure** and is the preferred, safe design method. This matches the question's description.
- **(C) Over-reinforced Section:** This is a design where a large amount of steel reinforcement is used. When the beam is overloaded, the concrete in the compression zone reaches its ultimate crushing strain before the steel has a chance to yield. Concrete is a brittle material, and its failure is sudden, explosive, and occurs without any prior warning. This is a **brittle failure** and must be avoided in design.
- **(A) Balanced Section:** This is a theoretical state where the steel starts yielding at the exact same moment that the concrete reaches its ultimate crushing strain. It marks the boundary between under-reinforced and over-reinforced behavior.

Step 3: Final Answer:

If the steel yields before the concrete crushes, the beam is designed as an under-reinforced section.

💡 Quick Tip

Remember the failure modes in RC beams:

- **Under-reinforced:** Steel yields first → Ductile failure (gradual, with warning) → **GOOD/DESIRABLE.**

- **Over-reinforced:** Concrete crushes first → Brittle failure (sudden, no warning) → **BAD/AVOID.**

142. In limit state method of design, if x_u is the depth of the neutral axis, then the distance of centroid of the compressive force from the extreme compression fibre is

(A) $0.57x_u$

(B) $0.45x_u$

(C) $0.42x_u$

(D) $0.36x_u$

Correct Answer: (C) $0.42x_u$

Solution:

Step 1: Understanding the Question:

The question asks for the location of the resultant compressive force in the concrete stress block, as defined by the Limit State Method in IS 456:2000.

Step 2: Key Formula or Approach:

In the Limit State Method, the stress distribution in the compression zone of a concrete beam at failure is assumed to be non-linear. The code (IS 456:2000) specifies an idealized rectangular-parabolic stress block.

- For a depth from the neutral axis of 0 to $4/7x_u$, the stress is parabolic.

- For a depth from $4/7x_u$ to x_u , the stress is constant at $0.446f_{ck}$.

The total compressive force (C) is the area of this stress block. The location of this force is at the centroid of the stress block.

For this specific rectangular-parabolic shape, the code specifies that the centroid is located at a distance of $0.42x_u$ from the extreme compression fiber (the top surface of the beam).

Step 3: Detailed Explanation:

This is a standard value from the Indian concrete design code (IS 456:2000) that needs to be memorized.

- The total compressive force is given by $C = 0.36f_{ck}bx_u$.

- The lever arm (z) between the tensile force in the steel and the compressive force in the concrete is given by $z = d - 0.42x_u$, where d is the effective depth of the beam.

This confirms that the centroid of the compressive force is located at a distance of $0.42x_u$ from the top.

Step 4: Final Answer:

The distance of the centroid of the compressive force from the extreme compression fibre is $0.42x_u$.

💡 Quick Tip

For Limit State Design as per IS 456, memorize these two key values for the concrete stress block:

- Total Compressive Force, $C = 0.36f_{ck}bx_u$.

- Depth of Compressive Force, $\bar{y} = 0.42x_u$.

These are fundamental to all flexural calculations.

143. The characteristic compressive strength of concrete at 28 days is 25 N/mm² and the standard deviation is 4.0 N/mm². The target strength for concrete mix at 28 days is

- (A) 25.0 MPa
- (B) 29.0 MPa
- (C) 31.6 MPa
- (D) 33.0 MPa

Correct Answer: (C) 31.6 MPa

Solution:

Step 1: Understanding the Question:

The question asks to calculate the target mean strength for a concrete mix design, given the characteristic strength and the standard deviation.

Step 2: Key Formula or Approach:

In concrete mix design, the **target mean strength** (f_m) is the average compressive strength that the mix is designed to achieve in the lab. It is set higher than the specified **characteristic strength** (f_{ck}) to account for the expected variability in concrete quality on site.

According to IS 456:2000, the relationship is:

$$f_m = f_{ck} + 1.65\sigma$$

where:

f_m = Target mean compressive strength

f_{ck} = Characteristic compressive strength (the strength below which not more than 5% of test results are expected to fall)

σ = Standard deviation of the test results

The factor 1.65 corresponds to a 5% probability of failure in a normal distribution.

Step 3: Detailed Explanation:

We are given:

- Characteristic strength (f_{ck}) = 25 N/mm² (which is 25 MPa)
- Standard deviation (σ) = 4.0 N/mm²

Substitute these values into the formula:

$$f_m = 25 + 1.65 \times 4.0$$

$$f_m = 25 + 6.6$$

$$f_m = 31.6 \text{ N/mm}^2 \text{ or } 31.6 \text{ MPa}$$

Step 4: Final Answer:

The target strength for the concrete mix at 28 days is 31.6 MPa.

💡 Quick Tip

The formula $f_m = f_{ck} + 1.65\sigma$ is fundamental to concrete mix design. The term 1.65σ is the "margin" added to the characteristic strength to ensure that most of the concrete produced on site will meet the required strength specification. Always remember this formula.

144. In limit state design of concrete structures, the strain distribution assumed is

- (A) Parabolic
- (B) Rectangular
- (C) Linear
- (D) Non-linear

Correct Answer: (C) Linear

Solution:

Step 1: Understanding the Question:

The question asks about the assumed shape of the strain distribution across the depth of a reinforced concrete section in the Limit State Method of design.

Step 2: Detailed Explanation:

One of the fundamental assumptions in the flexural design of reinforced concrete beams, both in the Working Stress Method and the Limit State Method, is that "plane sections remain plane after bending".

This assumption means that the strain at any point across the depth of the beam is directly proportional to its distance from the neutral axis.

If you plot the strain values versus the depth of the section, this proportional relationship results in a straight line. Therefore, the **strain distribution is assumed to be linear**.

It is important not to confuse this with the stress distribution.

- **Strain Distribution:** Assumed to be **linear**.
- **Stress Distribution:** Assumed to be **non-linear** (rectangular-parabolic) for concrete in the

Limit State Method, and linear in the Working Stress Method.

Step 3: Final Answer:

In the limit state design of concrete structures, the strain distribution is assumed to be linear.

 Quick Tip

A key distinction in Limit State Design:

- **STRAIN** distribution is **LINEAR** (because plane sections remain plane).
- **STRESS** distribution in concrete is **NON-LINEAR** (rectangular-parabolic).

145. The maximum spacing of 10 mm diameter bars in a reinforced concrete slab of 120 mm effective depth is

- (A) 120 mm
- (B) 240 mm
- (C) 300 mm
- (D) 360 mm

Correct Answer: (C) 300 mm

Solution:

Step 1: Understanding the Question:

The question asks for the maximum permissible spacing for the main reinforcement bars in a reinforced concrete slab, according to codal provisions.

Step 2: Key Formula or Approach:

According to IS 456:2000 (Clause 26.3.3), the maximum horizontal spacing of the main tension reinforcement in a solid slab shall not exceed the lesser of the following two values:

1. Three times the effective depth of the slab ($3d$).
2. 300 mm.

Step 3: Detailed Explanation:

We are given:

- Effective depth (d) = 120 mm.
- The bar diameter (10 mm) is also given, but it does not affect the maximum spacing calculation for the main bars, only the minimum.

Let's calculate the two limits:

1. $3d = 3 \times 120 \text{ mm} = 360 \text{ mm}$.

2. The absolute limit is 300 mm.

The maximum allowable spacing is the lesser of these two values.

$$\text{Maximum Spacing} = \min(360 \text{ mm}, 300 \text{ mm}) = 300 \text{ mm}$$

Step 4: Final Answer:

The maximum spacing of the main reinforcement bars is 300 mm.

 Quick Tip

Memorize the IS 456 rules for maximum bar spacing in slabs:

- **Main Bars:** Lesser of $3d$ or 300 mm.

- **Distribution/Temperature Bars:** Lesser of $5d$ or 450 mm.

These are very common questions in civil engineering exams.

146. In a reinforced concrete beam, the maximum spacing of vertical shear reinforcement measured along the axis shall not exceed

- (A) $0.5d$ or 300 mm
- (B) $0.5d$ or 250 mm
- (C) $0.75d$ or 300 mm
- (D) $0.75d$ or 250 mm

Correct Answer: (C) $0.75d$ or 300 mm

Solution:

Step 1: Understanding the Question:

The question asks for the maximum permissible spacing for vertical stirrups (shear reinforcement) in a reinforced concrete beam, as per IS 456.

Step 2: Key Formula or Approach:

According to IS 456:2000 (Clause 26.5.1.5), the maximum spacing of shear reinforcement (stirrups) measured along the axis of the member shall not exceed the lesser of the following values:

1. 0.75 times the effective depth ($0.75d$) for vertical stirrups.
2. 1.0 times the effective depth (d) for inclined stirrups.
3. An absolute maximum of 300 mm.

Step 3: Detailed Explanation:

The question specifically asks for **vertical shear reinforcement**. Therefore, the applicable limits are:

- $0.75d$
- 300 mm

The spacing must not exceed the smaller of these two values. The option that correctly states these two limits is (C).

There is also a condition for minimum shear reinforcement. If the nominal shear stress is very low, the maximum spacing can be increased, but the question asks for the general rule which applies when shear reinforcement is required.

Step 4: Final Answer:

The maximum spacing of vertical shear reinforcement shall not exceed $0.75d$ or 300 mm, whichever is less.

💡 Quick Tip

Memorize the IS 456 rules for maximum stirrup spacing:

- **Vertical Stirrups:** Lesser of $0.75d$ or 300 mm.
- **Inclined Stirrups:** Lesser of d or 300 mm.

The $0.75d$ limit is the most common one to remember.

147. According to IS:456, the stress strain curve for concrete in limit state method of design is

- (A) Linear upto 0.002 strain and uniform upto failure
- (B) Straight line upto 0.002 strain and then parabolic upto failure
- (C) Linear upto 0.002 strain and then parabolic upto failure
- (D) Parabolic upto 0.002 strain and uniform upto failure

Correct Answer: (D) Parabolic upto 0.002 strain and uniform upto failure

Solution:

Step 1: Understanding the Question:

The question asks to describe the idealized stress-strain curve for concrete in compression, as used for design in the Limit State Method according to IS 456.

Step 2: Detailed Explanation:

IS 456:2000 (Clause 38.1) specifies the idealized stress-strain relationship for concrete for design purposes. This curve is different from the actual experimental curve to incorporate safety factors.

The shape of the design curve is as follows:

- From a strain of 0 up to a strain of 0.002, the stress varies **parabolically**. The stress reaches its maximum design value of $0.446f_{ck}$ (approximately $0.45f_{ck}$) at a strain of 0.002.
- From a strain of 0.002 up to the ultimate failure strain of 0.0035, the stress is assumed to remain **constant (uniform)** at this maximum value of $0.446f_{ck}$.

This shape is a rectangular-parabolic block. The description in option (D) correctly matches this idealized curve. The other options describe different incorrect shapes (linear, straight line).

Step 3: Final Answer:

The stress-strain curve for concrete in limit state method of design is parabolic up to 0.002 strain and uniform up to failure.

💡 Quick Tip

Visualize the IS 456 design stress-strain curve for concrete:

- It starts at the origin, curves upwards like a **parabola** until a strain of 0.002.
- Then, it becomes a **horizontal line** (uniform stress) until the ultimate strain of 0.0035.

Do not confuse this with the **strain** distribution, which is always assumed to be linear.

148. For a cantilever slab of 1.4 m span, the minimum thickness required from serviceability criteria is

- (A) 100 mm
- (B) 140 mm
- (C) 180 mm
- (D) 200 mm

Correct Answer: (D) 200 mm

Solution:

Step 1: Understanding the Question:

The question asks for the minimum required total thickness (or depth) of a cantilever slab to satisfy the deflection control criteria (a serviceability limit state) as per IS 456.

Step 2: Key Formula or Approach:

IS 456:2000 (Clause 23.2.1) provides basic span-to-effective depth ratios for beams and slabs to control deflection without detailed calculations, for spans up to 10 m.

For a **cantilever** beam or slab, the basic ratio of span to effective depth (L/d) should not exceed 7.

$$\frac{\text{Span}}{\text{Effective Depth}} \leq 7$$

This basic ratio can be modified by factors for tension and compression reinforcement, but for a preliminary estimate, we use the basic value. The question asks for minimum thickness, which implies we should use the maximum allowable ratio.

$$d_{min} = \frac{\text{Span}}{7}$$

Step 3: Detailed Explanation:

We are given:

- Span (L) = 1.4 m = 1400 mm.

First, calculate the minimum required effective depth (d_{min}):

$$d_{min} = \frac{1400 \text{ mm}}{7} = 200 \text{ mm}$$

The question asks for the "minimum thickness". Thickness usually refers to the overall depth (D), which is the effective depth plus the clear cover and half the bar diameter ($D = d + \text{cover} + \phi/2$).

The calculated minimum effective depth is 200 mm. This means the overall thickness must be even greater (e.g., around 220-230 mm).

Let's re-read the options. Option (D) is 200 mm. It is possible that the question is asking for the effective depth, or that it is a simplified question where the overall thickness is taken to be equal to the calculated minimum effective depth. Given the options, 200 mm is the direct result of the code's span/depth ratio calculation. The other options are significantly different.

Step 4: Final Answer:

The minimum effective depth required is 200 mm. Assuming the question uses "thickness" to refer to this controlling dimension, the answer is 200 mm.

💡 Quick Tip

Memorize the basic Span/Effective Depth ratios from IS 456 for deflection control:

- **Cantilever:** 7
- **Simply Supported:** 20
- **Continuous:** 26

These are fundamental for the initial sizing of beams and slabs.

149. The minimum tension reinforcement required for a rectangular beam of width 250 mm and effective depth 400 mm using Fe 500 grade steel reinforcement is

- (A) 120 mm²
- (B) 150 mm²
- (C) 170 mm²
- (D) 180 mm²

Correct Answer: (C) 170 mm²

Solution:

Step 1: Understanding the Question:

The question asks for the minimum area of tension steel ($A_{st,min}$) required in a rectangular RC beam, as per IS 456.

Step 2: Key Formula or Approach:

According to IS 456:2000 (Clause 26.5.1.1a), the minimum area of tension reinforcement shall not be less than that given by the following equation:

$$\frac{A_{st}}{bd} = \frac{0.85}{f_y}$$

where:

A_{st} = Minimum area of tension reinforcement

b = Width of the beam

d = Effective depth of the beam

f_y = Characteristic strength of the reinforcement in N/mm²

Step 3: Detailed Explanation:

We are given:

- Width (b) = 250 mm

- Effective depth (d) = 400 mm

- Steel grade is Fe 500, so the characteristic strength (f_y) = 500 N/mm².

Rearrange the formula to solve for A_{st} :

$$A_{st,min} = \frac{0.85 \times b \times d}{f_y}$$

Substitute the given values:

$$A_{st,min} = \frac{0.85 \times 250 \text{ mm} \times 400 \text{ mm}}{500 \text{ N/mm}^2}$$

$$A_{st,min} = \frac{0.85 \times 100,000}{500}$$

$$A_{st,min} = \frac{85,000}{500}$$

$$A_{st,min} = \frac{850}{5} = 170 \text{ mm}^2$$

Step 4: Final Answer:

The minimum tension reinforcement required is 170 mm².

💡 Quick Tip

The formula for minimum tension steel, $\frac{A_{st}}{bd} = \frac{0.85}{f_y}$, is crucial to prevent brittle failure of a beam due to sudden cracking of concrete before the steel can engage.

Always remember this formula for beam design checks.

150. The width of a foot bridge slab is 3 m width with central supporting beam of 250 mm × 600 mm size and 6 m length. The effective width of flange is

- (A) 3.0 m
- (B) 1.5 m
- (C) 1.25 m
- (D) 1.0 m

Correct Answer: (C) 1.25 m

Solution:

Step 1: Understanding the Question:

The question asks to calculate the effective width of the flange (b_f) for a T-beam, which acts as the central support for a footbridge slab.

Step 2: Key Formula or Approach:

According to IS 456:2000 (Clause 23.1.2), the effective width of the flange for a T-beam is taken as the **least** of the following:

1. $b_f = \frac{l_0}{6} + b_w + 6D_f$
 2. The actual width of the flange, which is taken as the center-to-center spacing of the beams.
- Where:

l_0 = distance between points of zero moment. For a simply supported beam, l_0 is the effective span.

b_w = breadth of the web (the beam width).

D_f = depth of the flange (the slab thickness).

Step 3: Detailed Explanation:

Let's identify the given parameters:

- Span of the beam (l_0) = 6 m = 6000 mm.
- Width of the web (b_w) = 250 mm.
- The total width of the slab is 3 m. Since the beam is central, this can be taken as the center-to-center spacing of the beams. So, the maximum possible flange width is 3 m = 3000 mm.
- The depth of the flange (D_f , i.e., the slab thickness) is not given in the problem.

Without the value for D_f , the complete formula ($l_0/6 + b_w + 6D_f$) cannot be fully evaluated. This suggests that either the problem expects a simplified approach or is based on a different formula sometimes used in textbooks where the $6D_f$ term is ignored for simplicity. Let's test this simplified approach:

$$b_f = \frac{l_0}{6} + b_w$$

Substitute the values:

$$b_f = \frac{6000 \text{ mm}}{6} + 250 \text{ mm}$$
$$b_f = 1000 \text{ mm} + 250 \text{ mm} = 1250 \text{ mm}$$

Converting to meters:

$$b_f = 1.25 \text{ m}$$

Now we compare this calculated value with the other limit (the actual width available):

$$b_f = \min(1.25 \text{ m}, 3.0 \text{ m}) = 1.25 \text{ m}$$

This calculated value exactly matches option (C). This confirms that the intended solution was based on this simplified formula.

Step 4: Final Answer:

The effective width of the flange is 1.25 m.

💡 Quick Tip

The full IS 456 formula for the effective width of a T-beam flange is $b_f = \frac{l_0}{6} + b_w + 6D_f$. However, in some exam questions, if the slab thickness (D_f) is not provided, test the simplified formula $b_f = \frac{l_0}{6} + b_w$. The final effective width must always be less than or equal to the actual available width (c/c spacing of beams).

151. Using working stress method for design of RC members, the factor of safety considered for concrete and steel respectively are

- (A) 1.5 and 1.15
- (B) 3.0 and 1.5
- (C) 3.0 and 1.8
- (D) 1.5 and 1.8

Correct Answer: (C) 3.0 and 1.8

Solution:

Step 1: Understanding the Question:

The question asks for the factors of safety applied to the characteristic strengths of concrete and steel in the Working Stress Method (WSM) of design.

Step 2: Detailed Explanation:

The Working Stress Method is an older design philosophy where the stresses in the materials

under service loads are not allowed to exceed a certain proportion of their ultimate strength. This proportion is determined by the factor of safety.

$$\text{Permissible Stress} = \frac{\text{Ultimate Strength}}{\text{Factor of Safety}}$$

According to IS 456, the factors of safety used in WSM are:

- **For Concrete (in bending compression):** The factor of safety is approximately **3.0**. For example, for M20 concrete with a characteristic cube strength (f_{ck}) of 20 N/mm², the permissible stress (σ_{bc}) is 7 N/mm². ($20 / 7 \approx 2.86$, rounded to 3).
- **For Steel (in tension):** The factor of safety is approximately **1.8** (more precisely, 1.78). For example, for Fe 415 steel with a yield strength (f_y) of 415 N/mm², the permissible stress (σ_{st}) is 230 N/mm². ($415 / 230 \approx 1.8$).

It is important not to confuse these with the partial safety factors used in the Limit State Method (LSM). In LSM, the partial safety factors for material strength are 1.5 for concrete and 1.15 for steel.

Step 3: Final Answer:

In the working stress method, the factors of safety are 3.0 for concrete and approximately 1.8 for steel.

💡 Quick Tip

Do not mix up the safety factors for WSM and LSM:

- **Working Stress Method (WSM):** F.O.S on material strength is high. **Concrete ≈ 3.0 , Steel ≈ 1.8 .**
- **Limit State Method (LSM):** Partial safety factors for materials are lower. **Concrete = 1.5, Steel = 1.15.**

152. As per IS:456, in limit state method of design, the maximum compressive strain in axial compression is taken as

- (A) 0.02
- (B) 0.002
- (C) 0.035
- (D) 0.0035

Correct Answer: (B) 0.002

Solution:

Step 1: Understanding the Question:

The question asks for the maximum permissible compressive strain in a concrete member subjected to pure axial compression, according to the Limit State Method in IS 456.

Step 2: Detailed Explanation:

IS 456:2000 specifies different values for the ultimate compressive strain in concrete depending on the loading condition:

- **Flexure (Bending):** For a member subjected to bending (like a beam), the maximum compressive strain in the concrete at the extreme compression fiber is taken as **0.0035**. This allows for stress redistribution and is associated with the failure of the section in bending.
- **Axial Compression:** For a member subjected to pure axial compression (like a column), the maximum compressive strain is taken as **0.002**. This is a more conservative value because failure in pure compression is more brittle, and this strain corresponds to the point of peak stress in the idealized stress-strain curve.
- **Combined Axial Compression and Bending:** When a member is subjected to both, the maximum strain varies between 0.002 and 0.0035 depending on the eccentricity of the load.

Since the question specifies "axial compression," the correct value is 0.002.

Step 3: Final Answer:

The maximum compressive strain in axial compression is taken as 0.002.

💡 Quick Tip

Remember the two key ultimate strain values for concrete from IS 456:

- **Bending (Flexure):** $\epsilon_{cu,max} = 0.0035$.
- **Axial Compression:** $\epsilon_{c,max} = 0.002$.

153. In an RCC column, the minimum diameter of longitudinal bar shall not be less than

- (A) 10 mm
- (B) 12 mm
- (C) 16 mm
- (D) 20 mm

Correct Answer: (B) 12 mm

Solution:

Step 1: Understanding the Question:

The question asks for the minimum diameter of the main longitudinal reinforcement bars that can be used in an RCC column, as per IS 456.

Step 2: Detailed Explanation:

According to IS 456:2000 (Clause 26.5.3.1c), the code specifies requirements for the reinforcement in columns to ensure proper construction, stiffness, and load-carrying capacity.

The clause states that the diameter of the longitudinal reinforcement bars in a column shall not be less than **12 mm**.

This rule is in place to ensure that the bars are stiff enough to resist buckling between the lateral ties during construction and under load, and to provide adequate dowel action at splices. Using bars smaller than 12 mm would make the column's reinforcement cage too flexible.

Step 3: Final Answer:

In an RCC column, the minimum diameter of a longitudinal bar shall not be less than 12 mm.

 Quick Tip

Memorize the basic reinforcement rules for columns from IS 456:

- **Min. number of bars:** 4 in rectangular, 6 in circular.
- **Min. bar diameter:** 12 mm.
- **Min. steel percentage:** 0.8% of gross area.
- **Max. steel percentage:** 6% of gross area (4% if lapped).

154. In an isolated square concrete footing, the critical section for two-way shear occurs at

- (A) at face of column
- (B) at a distance $d / 3$ from face of column
- (C) at a distance d from the face of the column
- (D) at a distance $d / 2$ from periphery of column face

Correct Answer: (D) at a distance $d / 2$ from periphery of column face

Solution:

Step 1: Understanding the Question:

The question asks to identify the location of the critical section for checking two-way shear (also known as punching shear) in an isolated footing.

Step 2: Detailed Explanation:

In the design of footings, shear is checked in two ways:

1. **One-Way Shear (or Wide-Beam Shear):** This is checked at a critical section located at a distance d **from the face of the column**, where d is the effective depth of the footing. The footing is treated like a wide beam cantilevering from the column.
2. **Two-Way Shear (or Punching Shear):** This is the tendency of the column to "punch" through the footing. The shear failure occurs along a truncated pyramid shape. According to IS 456:2000 (Clause 31.6.1.1), the critical section for two-way shear is taken at a distance of $d/2$ **from the periphery (or face) of the column**. The shear is checked along this perimeter.

Option (D) correctly describes the location for the critical section for two-way shear.

Step 3: Final Answer:

The critical section for two-way shear in an isolated footing occurs at a distance $d/2$ from the periphery of the column face.

💡 Quick Tip

Remember the critical sections for shear in footings:

- **One-Way Shear** (like a beam) → at distance **d** from the column face.
- **Two-Way Shear** (punching) → at distance **$d/2$** from the column face.

155. As per IS:456, the columns are designed for a minimum eccentricity of

- (A) 15 mm
- (B) 20 mm
- (C) 25 mm
- (D) 40 mm

Correct Answer: (B) 20 mm

Solution:

Step 1: Understanding the Question:

The question asks for the minimum eccentricity that must be considered in the design of an axially loaded column, as specified by IS 456.

Step 2: Key Formula or Approach:

In reality, it is impossible to apply a load with perfect concentricity on a column. To account for construction imperfections and other accidental eccentricities, IS 456:2000 (Clause 25.4) mandates that all columns, even those theoretically under pure axial load, must be designed for a minimum eccentricity, e_{min} .

The formula for minimum eccentricity is given as the greater of the following two values:

$$e_{min} = \max \left(\frac{L}{500} + \frac{D}{30}, \quad 20 \text{ mm} \right)$$

where:

L = Unsupported length of the column

D = Lateral dimension of the column in the direction of bending

Step 3: Detailed Explanation:

The formula specifies two conditions. The calculated eccentricity must not be less than a value calculated based on the column's dimensions, AND it must not be less than an absolute minimum value. This absolute minimum value is specified as **20 mm**.

Therefore, regardless of the column's size or length, the eccentricity used for design can never be less than 20 mm.

Step 4: Final Answer:

As per IS:456, the columns are designed for a minimum eccentricity of 20 mm (or $L/500 + D/30$, whichever is greater). The absolute minimum specified is 20 mm.

💡 Quick Tip

The minimum eccentricity formula is a key provision in column design to ensure a minimum level of bending capacity.

Remember the formula: $e_{min} = \max \left(\frac{L}{500} + \frac{D}{30}, \quad 20 \text{ mm} \right)$.

In many exam questions, simply knowing the absolute minimum of **20 mm** is sufficient.

156. Self-weight considered for the design of footing is

- (A) 20% of the column load
- (B) 15% of the column load
- (C) 10% of the column load
- (D) 5% of the column load

Correct Answer: (C) 10% of the column load

Solution:

Step 1: Understanding the Question:

The question asks for a common rule-of-thumb percentage used to estimate the self-weight of a footing during the preliminary design stage.

Step 2: Detailed Explanation:

When designing a footing, the first step is to determine the required area. The area is calculated

as:

$$\text{Area of Footing} = \frac{\text{Total Load}}{\text{Safe Bearing Capacity of Soil}}$$

The "Total Load" includes the service load coming from the column (P) plus the self-weight of the footing itself (W_f).

However, at the beginning of the design, the size and weight of the footing are unknown. To start the calculation, a reasonable assumption must be made for the footing's self-weight.

A widely accepted and commonly used rule of thumb in practice and for academic problems is to assume the self-weight of the footing is approximately **10% of the axial load transferred from the column**.

So, the total load used for finding the area is estimated as $P_{total} = P + 0.10P = 1.1P$.

After the footing is fully designed, its actual self-weight is calculated and compared to the initial assumption to ensure the design is safe.

Step 3: Final Answer:

A common assumption for the self-weight of a footing for preliminary design is 10% of the column load.

Quick Tip

For the preliminary design of isolated footings, a standard first step is to estimate the self-weight.

A good starting assumption is **10% of the column load**.

This allows you to calculate the required footing area and begin the detailed design.

157. The minimum diameter of longitudinal bar in a column is

- (A) 6 mm
- (B) 8 mm
- (C) 12 mm
- (D) 16 mm

Correct Answer: (C) 12 mm

Solution:

Step 1: Understanding the Question:

The question asks for the minimum diameter of the main longitudinal reinforcement bars that can be used in an RCC column, as per IS 456.

Step 2: Detailed Explanation:

This is a repeat of question 153.

According to IS 456:2000 (Clause 26.5.3.1c), the code specifies requirements for the reinforcement in columns to ensure proper construction, stiffness, and load-carrying capacity.

The clause states that the diameter of the longitudinal reinforcement bars in a column shall not be less than **12 mm**.

This rule is in place to ensure that the bars are stiff enough to resist buckling between the lateral ties during construction and under load, and to provide adequate dowel action at splices. Using bars smaller than 12 mm would make the column's reinforcement cage too flexible.

Step 3: Final Answer:

The minimum diameter of a longitudinal bar in a column is 12 mm.

💡 Quick Tip

Reinforce your memory of the basic reinforcement rules for columns from IS 456:

- **Min. number of bars:** 4 in rectangular, 6 in circular.
- **Min. bar diameter:** 12 mm.
- **Min. steel percentage:** 0.8% of gross area.
- **Max. steel percentage:** 6% of gross area (4% if lapped).

158. The stress level in concrete in limit state method when compared to working stress method is

- (A) Same
- (B) Lower
- (C) Higher
- (D) no comparison

Correct Answer: (C) Higher

Solution:

Step 1: Understanding the Question:

The question compares the stress levels that concrete is designed to handle in the Limit State Method (LSM) versus the Working Stress Method (WSM).

Step 2: Detailed Explanation:

- **Working Stress Method (WSM):** This is an elastic design method. The stresses in the materials under actual service loads are kept well below their ultimate strengths by using a large factor of safety (approx. 3 for concrete). The material is assumed to behave linearly elastically, and stresses are kept in the low, elastic range.
- **Limit State Method (LSM):** This is an ultimate strength design method. It considers the behavior of the structure at the point of collapse (the "limit state"). The design is based on the ultimate, non-linear strength of the materials. A partial safety factor (1.5 for

concrete) is applied to the characteristic strength to get the design strength. The design stress distribution is non-linear (rectangular-parabolic) and utilizes a much larger portion of the concrete's actual strength capacity. For example, the maximum design stress is $0.446f_{ck}$.

Comparing the two, the permissible stress in WSM (e.g., $0.33f_{ck}$) is much lower than the design stress used in LSM (up to $0.446f_{ck}$). LSM allows the concrete to be stressed to a much higher level, closer to its actual failure strength, resulting in more economical designs.

Step 3: Final Answer:

The stress level in concrete in the limit state method is higher when compared to the working stress method.

💡 Quick Tip

Remember the core philosophies:

- **WSM** = Elastic design. Stresses under service loads are kept **low**.
- **LSM** = Ultimate strength design. Stresses at ultimate load are **high**, utilizing the material's full capacity.

159. The effective length of unsupported length of column L when effectively held in position at both ends and restrained against rotation at one end is

- (A) 1.0 L
- (B) 0.85 L
- (C) 0.80 L
- (D) 0.65 L

Correct Answer: (C) 0.80 L

Solution:

Step 1: Understanding the Question:

The question asks for the effective length of a column with specific end support conditions, as per IS 456.

Step 2: Detailed Explanation:

The end conditions described are:

- "Effectively held in position at both ends": This means translation (sideways movement) is prevented at both ends.
- "Restrained against rotation at one end": This means one end is fixed.
- Since it is not restrained against rotation at the other end, that end must be pinned (hinged). Therefore, the condition is **Fixed-Pinned**.

According to IS 456:2000 (Annex E), which provides effective lengths for columns, the code gives two values for each condition: a theoretical value and a recommended design value. For a column that is effectively held in position at both ends and restrained against rotation at one end (Fixed-Pinned):

- The theoretical effective length factor is $K = 1/\sqrt{2} \approx 0.7$.
- The recommended design value given in the code is a more conservative value. Let's check the options. The available options are 1.0, 0.85, 0.80, and 0.65.

The value recommended by IS 456 for a column fixed at one end and pinned at the other is actually 0.80 L. The theoretical value is 0.707 L, and the code recommends a slightly higher value for practical design.

Wait, let's recheck IS 456 Table 28. - Effectively held in position and restrained against rotation at both ends (Fixed-Fixed): Recommended value is 0.65 L. - Effectively held in position at both ends, restrained against rotation at one end (Fixed-Pinned): Recommended value is 0.80 L. - Effectively held in position at both ends, but not restrained against rotation (Pinned-Pinned): Recommended value is 1.00 L.

The described condition perfectly matches the Fixed-Pinned case.

Step 3: Final Answer:

The effective length for a column held in position at both ends and restrained against rotation at one end is 0.80 L.

Quick Tip

For exams, it's crucial to know the **recommended design values** from IS 456, which can differ from pure theoretical values.

- Fixed-Fixed: **0.65 L**
- Fixed-Pinned: **0.80 L**
- Pinned-Pinned: **1.0 L**
- Fixed-Free: **2.0 L**

160. The anchorage value for a bar of 20 mm diameter bent into an angle of 90° is

- (A) 40 mm
- (B) 80 mm
- (C) 160 mm
- (D) 320 mm

Correct Answer: (C) 160 mm

Solution:

Step 1: Understanding the Question:

The question asks for the anchorage value of a standard 90-degree bend for a 20 mm diameter

rebar, as per IS 456.

Step 2: Detailed Explanation:

Anchorage is required to transfer the force from a reinforcing bar into the surrounding concrete. This can be achieved by providing a sufficient straight length of bar (development length) or by using hooks and bends.

According to IS 456:2000 (Clause 26.2.2.1), the anchorage value of a hook or a bend is specified to account for the additional grip provided by the bent shape.

For a standard 90-degree bend (U-type hook), the code specifies that the anchorage value is equivalent to a straight length of **8 times the diameter of the bar (8ϕ)**.

For every 45-degree bend, the anchorage value is 4ϕ . A standard hook (180-degree bend) has an anchorage value of 16ϕ .

The question specifies a 90-degree bend.

Step 3: Detailed Explanation:

We are given:

- Bar diameter (ϕ) = 20 mm

The anchorage value for a 90-degree bend is 8ϕ .

$$\text{Anchorage Value} = 8 \times \phi = 8 \times 20 \text{ mm}$$

$$\text{Anchorage Value} = 160 \text{ mm}$$

Step 4: Final Answer:

The anchorage value for a 20 mm diameter bar bent into a 90-degree angle is 160 mm.

💡 Quick Tip

Memorize the anchorage values of bends from IS 456:

- For every **45-degree** bend, the anchorage value is **4ϕ** .
- Therefore, for a **90-degree** bend, it is $2 \times 4\phi = \mathbf{8\phi}$.
- For a U-hook (**180-degree** bend), it is $4 \times 4\phi = \mathbf{16\phi}$.

161. A 30 m chain is divided into

- (A) 90 links
- (B) 100 links
- (C) 150 links
- (D) 200 links

Correct Answer: (C) 150 links

Solution:

Step 1: Understanding the Question:

The question asks for the number of links in a standard 30-meter metric survey chain.

Step 2: Detailed Explanation:

Surveyor's chains are traditional measuring instruments, and their construction follows specific standards.

- **Metric Chains:** These come in standard lengths, typically 20 m and 30 m. They are designed so that each meter of length corresponds to a specific number of links.
- In a standard metric chain, there are **5 links per meter**.

For a 30-meter chain, the total number of links is:

$$\text{Number of Links} = (\text{Length in meters}) \times (\text{Links per meter})$$

$$\text{Number of Links} = 30 \text{ m} \times 5 \text{ links/m}$$

$$\text{Number of Links} = 150 \text{ links}$$

Similarly, a 20-meter chain would have $20 \times 5 = 100$ links.

Step 3: Final Answer:

A 30 m chain is divided into 150 links.

💡 Quick Tip

Remember the standard construction of metric chains:

- **5 links per meter.**
- A 20 m chain has $20 \times 5 = \mathbf{100}$ links.
- A 30 m chain has $30 \times 5 = \mathbf{150}$ links.

162. The main survey stations are located on the ground by

- (A) Line sketches
- (B) Index sketches
- (C) Highlighted sketches
- (D) Reference sketches

Correct Answer: (D) Reference sketches

Solution:

Step 1: Understanding the Question:

The question asks for the name of the sketch used in surveying to record the location of a main

survey station so it can be found again later.

Step 2: Detailed Explanation:

In surveying, especially for large projects, main survey stations (control points) are established on the ground using pegs, pillars, or other markers. Since these stations are crucial and may need to be used again over a long period, their locations must be carefully documented.

A **Reference Sketch** (also known as a location sketch) is a detailed drawing made in the field book for this purpose. It shows the location of the survey station relative to several nearby permanent and easily identifiable features (e.g., corners of buildings, lamp posts, specific trees, milestones). The sketch includes measurements (distances) from at least three such "reference points" to the station, allowing it to be accurately re-established even if the original ground marker is disturbed or lost.

Step 3: Final Answer:

Main survey stations are located on the ground and documented using reference sketches.

💡 Quick Tip

In a survey field book, different sketches have different purposes:

- **Index Sketch:** An overview map of the entire survey area, showing the layout of the main survey lines and stations.
- **Reference Sketch:** A detailed drawing to locate a *single* station with respect to permanent features.

163. If the fore bearing of a line AB is $S15^{\circ}40'W$, then the back bearing of line is

- (A) $15^{\circ}40'$
- (B) $364^{\circ}20'$
- (C) $S15^{\circ}40'E$
- (D) $N15^{\circ}40'W$

Correct Answer: (A) $15^{\circ}40'$

Solution:

Step 1: Understanding the Question:

The question asks to find the back bearing of a line, given its fore bearing in the Quadrantal Bearing (or Reduced Bearing) system. The provided answer key ' $15^{\circ}40'$ ' seems incomplete or formatted as a Whole Circle Bearing, let's derive the correct Quadrantal Bearing first.

Step 2: Key Formula or Approach:

The **Fore Bearing (FB)** is the bearing of a line in the direction of the survey traverse. The **Back Bearing (BB)** is the bearing of the same line in the opposite direction.

In the Quadrantal Bearing (QB) system, bearings are measured from either North or South

towards East or West.

To find the back bearing from the fore bearing in the QB system, you simply:

1. Keep the numerical angle the same.
2. Reverse the cardinal directions (North becomes South, South becomes North, East becomes West, and West becomes East).

Step 3: Detailed Explanation:

The given Fore Bearing of line AB is $S15^{\circ}40'W$.

This means the line is in the South-West quadrant, making an angle of $15^{\circ}40'$ with the South direction, towards the West.

To find the Back Bearing of line BA, we apply the rule:

- Keep the angle: $15^{\circ}40'$.
- Reverse the South to North.
- Reverse the West to East.

So, the Back Bearing is $N15^{\circ}40'E$.

Now, let's evaluate the options. None of the options is $N15^{\circ}40'E$. There is a clear error in the question or the options. Let's analyze the given correct answer: " $15^{\circ}40'$ ". This looks like an angle in the Whole Circle Bearing (WCB) system, but the value is too small. Let's convert the Fore Bearing to WCB and see what we get.

FB in QB = $S15^{\circ}40'W$

This is in the third quadrant. $WCB = 180^{\circ} + 15^{\circ}40' = 195^{\circ}40'$.

The Back Bearing in WCB is $FB \pm 180^{\circ}$.

BB in WCB = $195^{\circ}40' - 180^{\circ} = 15^{\circ}40'$.

This WCB of $15^{\circ}40'$ corresponds to a Quadrantal Bearing of $N15^{\circ}40'E$.

So, the calculation shows that the back bearing has a numerical value of $15^{\circ}40'$, and the provided answer key seems to have only stated this numerical part, omitting the cardinal directions.

Option (A) matches this numerical value.

Step 4: Final Answer:

The back bearing is $N15^{\circ}40'E$. In the Whole Circle Bearing system, this is $15^{\circ}40'$. Given the options, the intended answer is the numerical value $15^{\circ}40'$.

💡 Quick Tip

To find the back bearing in the Quadrantal Bearing system, just flip the letters. S becomes N, N becomes S. E becomes W, W becomes E. The angle stays the same.
Example: $FB = S 30^{\circ} E \implies BB = N 30^{\circ} W$.

164. If the sum of interior angles in a closed traverse is 1080° , then the number of sides in a traverse is

- (A) 10
(B) 8

- (C) 7
(D) 6

Correct Answer: (B) 8

Solution:

Step 1: Understanding the Question:

The question asks for the number of sides (n) of a closed polygon (traverse) given the theoretical sum of its interior angles.

Step 2: Key Formula or Approach:

The formula for the sum of the interior angles of a closed polygon with n sides is:

$$\text{Sum of interior angles} = (2n - 4) \times 90^\circ$$

or, equivalently,

$$\text{Sum of interior angles} = (n - 2) \times 180^\circ$$

Step 3: Detailed Explanation:

We are given that the sum of the interior angles is 1080° . We can set this equal to the formula and solve for n . Using the second formula is easier.

$$(n - 2) \times 180^\circ = 1080^\circ$$

Divide both sides by 180° :

$$\begin{aligned} n - 2 &= \frac{1080}{180} \\ n - 2 &= \frac{108}{18} = 6 \\ n &= 6 + 2 \\ n &= 8 \end{aligned}$$

The traverse has 8 sides.

Step 4: Final Answer:

The number of sides in the traverse is 8.

💡 Quick Tip

Memorize the formula for the sum of interior angles of a polygon: $(n - 2) \times 180^\circ$.

You can quickly check it with simple shapes:

- Triangle ($n = 3$): $(3 - 2) \times 180 = 180^\circ$.
- Rectangle ($n = 4$): $(4 - 2) \times 180 = 360^\circ$.

165. For land survey using theodolite, if N is the number of sides of a closed traverse, then the angular error of closure in minutes should not exceed

- (A) $\sqrt{N}$
- (B) $10\sqrt{N}$
- (C) $15\sqrt{N}$
- (D) $30\sqrt{N}$

Correct Answer: (A) $\sqrt{N}$

Solution:

Step 1: Understanding the Question:

The question asks for the permissible limit of the angular error of closure for a theodolite traverse, as a function of the number of sides, N .

Step 2: Detailed Explanation:

When measuring the angles of a closed traverse, small instrumental and observational errors will accumulate. The **angular error of closure** is the difference between the measured sum of the angles and the theoretical sum (e.g., $(n - 2) \times 180^\circ$ for interior angles).

To determine if a survey is accurate enough, this error is compared against a permissible limit. The permissible error depends on the precision of the instrument and the required accuracy of the survey.

For a standard theodolite traverse, a common rule for the permissible error of closure, e , is given by:

$$e = c\sqrt{N}$$

where N is the number of angles (or sides), and c is a constant that depends on the precision. The constant c is often the least count of the instrument.

For a $20''$ theodolite, a common allowable error is $e \leq 20''\sqrt{N}$.

The question asks for the error in minutes. $1' = 60''$. So, $20'' = (1/3)'$.

The question seems to be based on a simplified rule or a specific standard where the constant is 1 minute. A widely cited rule for lower-order traverses is that the permissible error in minutes should not exceed $\sqrt{N}$. This is a common rule-of-thumb found in many textbooks.

Step 3: Final Answer:

A common permissible limit for the angular error of closure in minutes for a theodolite traverse is $\sqrt{N}$, where N is the number of sides.

💡 Quick Tip

The allowable error in a traverse is generally proportional to the square root of the number of measurements ($\sqrt{N}$).

This is a common principle in statistics for random errors.

For angular closure, remember the formula $e = c\sqrt{N}$. For a simple rule in minutes, c is often taken as 1.

166. The sum of rises and sum of falls in levelling are 1.125 and 5.635 respectively. If the RL of last point is 246.875, then the RL of first point is

- (A) 251.385
- (B) 253.635
- (C) 240.115
- (D) 242.365

Correct Answer: (A) 251.385

Solution:

Step 1: Understanding the Question:

The question provides data from a leveling survey (sum of rises and falls) and the Reduced Level (RL) of the last station. It asks to calculate the RL of the first station.

Step 2: Key Formula or Approach:

There are two fundamental arithmetic checks in leveling using the Rise and Fall method:

Check 1: $\Sigma(\text{Backsights}) - \Sigma(\text{Foresights}) = \Sigma(\text{Rises}) - \Sigma(\text{Falls})$

Check 2: $\text{RL of Last Point} - \text{RL of First Point} = \Sigma(\text{Rises}) - \Sigma(\text{Falls})$

We can use the second check to solve this problem.

Step 3: Detailed Explanation:

We are given:

- Sum of Rises (ΣR) = 1.125 m

- Sum of Falls (ΣF) = 5.635 m

- RL of Last Point = 246.875 m

We need to find the RL of the First Point. Let's call it RL_{First} .

Using the formula:

$$RL_{Last} - RL_{First} = \Sigma R - \Sigma F$$

Substitute the known values:

$$246.875 - RL_{First} = 1.125 - 5.635$$

$$246.875 - RL_{First} = -4.510$$

Rearrange the equation to solve for RL_{First} :

$$RL_{First} = 246.875 - (-4.510)$$

$$RL_{First} = 246.875 + 4.510$$

$$RL_{First} = 251.385 \text{ m}$$

The correct answer is not present in the options. There might be a mistake in the option values or the question values. Let's re-verify the setup. Let's assume the question asked for RL of first point if RL of last point is 246.875. The calculation seems robust. Let's check the options again. Option A is 251.385. The calculation matches option A. The provided answer

key '251.385' is correct.

Step 4: Final Answer:

The RL of the first point is 251.385 m.

💡 Quick Tip

Always remember the two arithmetic checks for the Rise and Fall method in leveling.

The second check is the most important:

Last RL – First RL = Σ Rises – Σ Falls.

This equation is a direct application of the definition of rises and falls and can be used to solve for any one of the four variables if the other three are known.

167. Two points A and B are taken on opposite banks of a river in reciprocal levelling. When the level was set up near A, the staff readings on A and B were 1.225 and 2.375. When the level was set up near B, the respective readings were 0.945 and 2.115. The true differences of level between A and B is

- (A) 1.150 m fall from A to B
- (B) 1.170 m rise from A to B
- (C) 1.160 m fall from A to B
- (D) 1.160 m rise from A to B

Correct Answer: (C) 1.160 m fall from A to B

Solution:

Step 1: Understanding the Question:

The question asks for the true difference in level between two points, A and B, using the data from a reciprocal levelling operation, which is used to cancel out errors due to instrument collimation and earth's curvature.

Step 2: Key Formula or Approach:

Let h_A and h_B be the staff readings on A and B when the instrument is near A.

Let h'_A and h'_B be the staff readings on A and B when the instrument is near B.

The true difference in level, ΔH , between A and B is the average of the two apparent differences in level.

$$\Delta H = \frac{(h_B - h_A) + (h'_B - h'_A)}{2}$$

A positive result for ΔH means B is lower than A (a fall). A negative result means B is higher than A (a rise).

Step 3: Detailed Explanation:

Case 1: Instrument near A

- Staff reading on A (h_A) = 1.225 m
- Staff reading on B (h_B) = 2.375 m
- Apparent difference = $h_B - h_A = 2.375 - 1.225 = 1.150$ m

Case 2: Instrument near B

- Staff reading on A (h'_A) = 2.115 m
- Staff reading on B (h'_B) = 0.945 m
- Apparent difference = $h'_B - h'_A = 0.945 - 2.115 = -1.170$ m

Wait, the readings are given as "respective readings", so when the instrument is near B, the reading on A is 0.945 and on B is 2.115. Let's re-read carefully: "When the level was set up near B, the respective readings were 0.945 and 2.115". This is ambiguous. Let's assume the first reading is always A and the second is B. - $h'_A = 0.945$ m - $h'_B = 2.115$ m - Apparent difference = $h'_B - h'_A = 2.115 - 0.945 = 1.170$ m

Now, calculate the true difference in level:

$$\Delta H = \frac{(1.150) + (1.170)}{2}$$

$$\Delta H = \frac{2.320}{2} = 1.160 \text{ m}$$

Since the result is positive, it means that point B is lower than point A. This is a **fall** from A to B.

Step 4: Final Answer:

The true difference of level is a 1.160 m fall from A to B.

💡 Quick Tip

Reciprocal levelling is designed to cancel out errors.

The true difference in level is simply the average of the two apparent differences calculated from each setup.

$$\Delta H = \frac{(\text{Reading}_B - \text{Reading}_A)_{\text{Setup 1}} + (\text{Reading}_B - \text{Reading}_A)_{\text{Setup 2}}}{2}$$

168. When contours of a different elevations cross each other, then it indicates a

- (A) Pond
- (B) Steep slope
- (C) Vertical cliff
- (D) Overhanging cliff

Correct Answer: (D) Overhanging cliff

Solution:

Step 1: Understanding the Question:

The question asks to identify the landform represented on a contour map when contour lines of different elevations cross.

Step 2: Detailed Explanation:

Let's review the basic rules of contour lines:

1. A contour line connects points of equal elevation.
2. Contour lines can never cross each other on a map, because a single point on the ground cannot have two different elevations.

However, there is one exception to this rule.

- **(D) Overhanging cliff:** An overhanging cliff is a rock face where the upper part juts out over the lower part. If you were looking down from directly above (as on a map), a point on the ground under the overhang would be at a lower elevation than a point directly above it on the overhang itself. On a contour map, this would be represented by the contour line of the higher elevation crossing over the contour line of the lower elevation.
- **(C) Vertical cliff:** For a perfectly vertical cliff, contour lines of different elevations would merge to form a single line on the map, but they would not cross.
- **(A) Pond:** A depression like a pond is shown by closed contours with the values decreasing inwards.
- **(B) Steep slope:** This is indicated by contour lines that are very close together.

Step 3: Final Answer:

When contours of different elevations cross each other, it indicates an overhanging cliff.

💡 Quick Tip

Remember the golden rules of contours:

- They connect points of equal elevation.
- They **never cross** each other, **except** in the rare case of an **overhanging cliff**.
- They merge to a single line for a **vertical cliff**.

169. If the length and reduced bearing of a line are 100 m and S60°W respectively, then the departure is

- (A) 50 m
- (B) 70.7 m
- (C) 86.6 m
- (D) 100 m

Correct Answer: (C) 86.6 m

Solution:

Step 1: Understanding the Question:

The question asks to calculate the departure of a survey line, given its length and its bearing in the Quadrantal Bearing (or Reduced Bearing) system.

Step 2: Key Formula or Approach:

In surveying, the coordinates of a line are resolved into two components:

- **Latitude (L):** The projection of the line onto the North-South meridian. It is calculated as $L = \text{length} \times \cos(\theta)$. Latitude is positive for Northings and negative for Southings.

- **Departure (D):** The projection of the line onto the East-West line. It is calculated as $D = \text{length} \times \sin(\theta)$. Departure is positive for Eastings and negative for Westings.

Here, θ is the reduced bearing angle.

Step 3: Detailed Explanation:

We are given:

- Length of the line = 100 m

- Reduced Bearing = S60°W

This means the angle θ with the South (North-South meridian) is 60°.

Calculate the departure:

$$D = \text{length} \times \sin(\theta)$$

$$D = 100 \text{ m} \times \sin(60^\circ)$$

We know that $\sin(60^\circ) = \frac{\sqrt{3}}{2} \approx 0.866$.

$$D = 100 \times 0.866 = 86.6 \text{ m}$$

Since the bearing is in the South-**West** quadrant, the departure is negative (a Westing). However, the question asks for the magnitude of the departure.

Step 4: Final Answer:

The departure of the line is 86.6 m.

 Quick Tip

Remember the formulas for Latitude and Departure:

- **Latitude** = **Length** $\times$ **cos**(θ) (L-L-C)

- **Departure** = **D**ength $\times$ **sin**(θ) (Wait, this doesn't work. Use $D = L \sin(\theta)$)

Latitude is the North-South component. Departure is the East-West component.

170. The size of the theodolite is defined according to the

- (A) Height of the stand
- (B) Length of the telescope
- (C) weight of the theodolite
- (D) Diameter of graduated horizontal circle

Correct Answer: (D) Diameter of graduated horizontal circle

Solution:

Step 1: Understanding the Question:

The question asks for the parameter that is used to specify the "size" of a theodolite.

Step 2: Detailed Explanation:

Theodolites are precision instruments used for measuring horizontal and vertical angles. Their size and precision are closely related. The primary component that determines the instrument's ability to be read accurately is the main graduated circle used for measuring horizontal angles. **The size of a theodolite is specified by the diameter of its graduated horizontal circle** (the lower plate).

For example, a "20 cm theodolite" is one in which the diameter of the horizontal circle is 20 cm. A larger diameter allows for finer graduations and thus more precise angle measurements. The other options like the length of the telescope, height of the stand, or weight are physical characteristics but do not define the instrument's designated size in standard surveying terminology.

Step 3: Final Answer:

The size of the theodolite is defined according to the diameter of the graduated horizontal circle.

💡 Quick Tip

The "size" of a theodolite is a measure of its precision, not its physical bulk.

Size of Theodolite = Diameter of the horizontal circle.

A larger circle means more room for degree markings, which means higher potential precision.

171. The multiplying constant and additive constant of a tacheometer are 100 and 2 m respectively. When the line of sight is horizontal and the staff held vertically, the staff readings are 2.780 m and 1.280 m respectively, then the distance between the tacheometer and staff station is

- (A) 152 m
- (B) 150 m
- (C) 76 m
- (D) 75 m

Correct Answer: (A) 152 m

Solution:**Step 1: Understanding the Question:**

The question asks to calculate the horizontal distance from a tacheometer to a staff station using the provided stadia readings and instrument constants.

Step 2: Key Formula or Approach:

The formula for calculating the horizontal distance (D) in tacheometry when the line of sight is horizontal is:

$$D = Ks + C$$

where:

K = Multiplying constant

C = Additive constant

s = Staff intercept (the difference between the top and bottom stadia hair readings)

Step 3: Detailed Explanation:

First, identify the given values:

- Multiplying constant (K) = 100
- Additive constant (C) = 2 m
- Top staff reading = 2.780 m
- Bottom staff reading = 1.280 m

Next, calculate the staff intercept (s):

$$s = \text{Top reading} - \text{Bottom reading}$$

$$s = 2.780 \text{ m} - 1.280 \text{ m} = 1.500 \text{ m}$$

Now, substitute the values into the distance formula:

$$D = Ks + C$$

$$D = (100 \times 1.500) + 2$$

$$D = 150 + 2$$

$$D = 152 \text{ m}$$

Step 4: Final Answer:

The distance between the tacheometer and the staff station is 152 m.

💡 Quick Tip

The basic tacheometry distance formula is $D = Ks + C$.

This applies when the telescope is horizontal.

A common mistake is to forget to add the additive constant (C) after multiplying K and s .

If the telescope is inclined, more complex formulas involving $\cos^2 \theta$ and $\sin \theta \cos \theta$ are used.

172. When a line changes its direction, the angle of the forward line with the external of the preceding line is known as

- (A) Azimuth
- (B) Dip
- (C) Declination
- (D) Deflection angle

Correct Answer: (D) Deflection angle

Solution:

Step 1: Understanding the Question:

The question asks for the name of the angle measured from the extension of the previous line to the next (forward) line in a traverse.

Step 2: Detailed Explanation:

Let's define the angles used in surveying:

- **(D) Deflection Angle:** This is the angle by which a survey line "deflects" from the prolongation of the preceding line. It is measured either to the right (clockwise) or to the left (counter-clockwise) from the extended previous line. This method is common in route surveys for roads, railways, and pipelines. This perfectly matches the question's description.
- **(A) Azimuth:** This is the horizontal angle of a line measured clockwise from a reference meridian, usually North. It is a direction, not a change in direction.
- **(B) Dip:** This is the vertical angle that a magnetic needle makes with the horizontal plane.
- **(C) Declination:** This is the horizontal angle between the true meridian (True North) and the magnetic meridian (Magnetic North).

Step 3: Final Answer:

The angle of the forward line with the extension of the preceding line is known as the deflection

angle.

 Quick Tip

Visualize the angles in a traverse:

- **Included Angle:** The angle *inside* the traverse, between two adjacent lines.
- **Deflection Angle:** The angle between the *extension* of the back line and the forward line.

173. Which of the following is the first operation of Total station when it is sighted to the target?

- (A) Rotation of optical axis.
- (B) Rotation of vertical axis
- (C) Rotation of horizontal axis
- (D) Rotation of line of collimation

Correct Answer: (A) Rotation of optical axis.

Solution:

Step 1: Understanding the Question:

The question asks about the initial action performed by a Total Station after it is aimed (sighted) at a target prism.

Step 2: Detailed Explanation:

A Total Station is an electronic instrument that combines an electronic theodolite (for measuring angles) with an electronic distance meter (EDM).

The process of measuring a point is as follows:

1. The operator manually aims the telescope's crosshairs precisely at the center of the target prism. This process is called "sighting" or "collimation".
2. The operator then initiates the measurement. The first automated action the instrument takes is to measure the distance to the target. It does this by sending out an infrared signal from the EDM. This signal travels along the **optical axis** of the telescope, reflects off the prism, and returns to the instrument. The instrument measures the phase shift or time of flight of this signal to calculate the slope distance.
3. Simultaneously or sequentially, the instrument reads the horizontal and vertical angles from its internal encoders.

The options are somewhat ambiguously phrased:

- **(A) Rotation of optical axis:** This is not a standard term. However, the first measurement (EDM) happens along the optical axis.
- **(B) Rotation of vertical axis:** This happens when the instrument is turned horizontally to sight a new target.
- **(C) Rotation of horizontal axis:** This happens when the telescope is moved up or down to sight a target at a different elevation.

- **(D) Rotation of line of collimation:** The line of collimation is the line of sight. Rotating it is part of the manual aiming process, not the first automated operation.

The question is poorly worded. However, the most fundamental operation that a Total Station performs, which an older theodolite cannot, is the electronic distance measurement, which happens along the optical path. If we interpret "first operation" as the first electronic measurement, it is the EDM measurement along the optical axis. The provided answer key states (A) as correct, which likely refers to the internal processes related to the optical path for the EDM.

Step 3: Final Answer:

Given the options, the most plausible interpretation is that the first operation refers to the functioning of the EDM along the optical path of the instrument. Therefore, "Rotation of optical axis" is chosen as the best fit, despite its non-standard terminology.

💡 Quick Tip

A Total Station's primary functions are:

1. **Angle Measurement** (like a theodolite).
2. **Distance Measurement** (EDM).

When you sight a target and press the measure button, the first key automated step is the EDM sending a signal along the line of sight to measure the distance.

174. Which of the following is a GIS operation?

- (A) Geodata
- (B) Spatial data
- (C) Geoprocessing
- (D) Global Processing

Correct Answer: (C) Geoprocessing

Solution:

Step 1: Understanding the Question:

The question asks to identify which of the given terms represents an operation or a process within a Geographic Information System (GIS).

Step 2: Detailed Explanation:

A GIS is a system for capturing, storing, checking, and displaying data related to positions on Earth's surface. It involves data, hardware, software, and procedures. Let's analyze the terms:

- **(A) Geodata** and **(B) Spatial data:** These are types of **data**, not operations. Spatial data (or geodata) is the information that identifies the geographic location of features. It's the "what is where" information that a GIS uses as its input.

- **(C) Geoprocessing:** This is the correct term for an **operation** or a function performed on spatial data within a GIS. Geoprocessing tools are used to manipulate and analyze geographic information to create new datasets and derive new insights. Examples of geoprocessing operations include buffering, overlay (like union or intersect), clipping, and spatial analysis.
- **(D) Global Processing:** This is a generic term and not specific to GIS.

Step 3: Final Answer:

Geoprocessing is a GIS operation.

💡 Quick Tip

Remember the components of GIS:

- **Data:** Spatial Data, Geodata, Attribute Data.
 - **Operations/Processes:** Data Capture, Geoprocessing, Analysis, Visualization.
- Geoprocessing is the "verb" - it's what you *do* to the data.

175. Which of the following is the set of classification of static GPS surveying technology?

- (A) Short and Long base lines
- (B) Short and Normal base lines
- (C) Long and Medium base lines
- (D) Long and Normal base lines

Correct Answer: (A) Short and Long base lines

Solution:

Step 1: Understanding the Question:

The question asks for the standard classification of static GPS surveying techniques based on the length of the baseline being measured.

Step 2: Detailed Explanation:

Static GPS surveying is a high-precision technique where two or more receivers collect satellite data simultaneously over a period of time. The data is then post-processed to determine the precise three-dimensional vector (the baseline) between the receivers. The processing strategies and achievable accuracy depend on the length of this baseline.

The classification is typically divided based on the scale of the survey:

- **Short Baselines:** Generally considered to be baselines up to 10-20 km. Over these distances, the ionospheric and tropospheric errors are highly correlated between the two

receivers and can be effectively eliminated through differential processing. Very high accuracy (millimeter level) can be achieved relatively quickly.

- **Long Baselines:** Baselines longer than 20-30 km. Over these distances, the atmospheric errors are less correlated, and more advanced processing techniques, longer observation times, and precise satellite orbit data are required to achieve high accuracy.

Some classifications might include a "medium" or "normal" category, but the most fundamental and common division is between "Short" and "Long" baselines, as the processing strategy changes significantly between these two regimes. The set "Short and Long base lines" encompasses the entire range of static GPS surveying.

Step 3: Final Answer:

The set of classification for static GPS surveying technology is Short and Long base lines.

💡 Quick Tip

In static GPS, the baseline length is the most critical factor.

- **Short Baseline** (≤ 20 km): Errors cancel out easily, high accuracy is straightforward.
- **Long Baseline** (> 20 km): Atmospheric errors are a major challenge, requires advanced processing.

176. At a certain point in oil, the shear stress and velocity gradient are 0.2 N/m^2 and 0.1 s^{-1} respectively, then the dynamic viscosity is

- (A) 2 N-s/m^2
- (B) 50 N-s/m^2
- (C) 0.5 N-s/m^2
- (D) 0.02 N-s/m^2

Correct Answer: (A) 2 N-s/m^2

Solution:

Step 1: Understanding the Question:

The question asks to calculate the dynamic viscosity of an oil, given the shear stress and the velocity gradient at a point.

Step 2: Key Formula or Approach:

For a Newtonian fluid (which oils are generally assumed to be), the relationship between shear stress (τ), dynamic viscosity (μ), and velocity gradient (rate of shear strain, du/dy) is given by Newton's law of viscosity:

$$\tau = \mu \frac{du}{dy}$$

Step 3: Detailed Explanation:

We are given:

- Shear stress (τ) = 0.2 N/m^2
- Velocity gradient (du/dy) = 0.1 s^{-1}

We need to find the dynamic viscosity, μ . Rearrange the formula:

$$\mu = \frac{\tau}{du/dy}$$

Substitute the given values:

$$\mu = \frac{0.2 \text{ N/m}^2}{0.1 \text{ s}^{-1}}$$
$$\mu = 2 \frac{\text{N}}{\text{m}^2} \cdot \text{s} \quad \text{or} \quad 2 \text{ N-s/m}^2$$

The unit N-s/m^2 is also known as a Pascal-second ($\text{Pa}\cdot\text{s}$).

Step 4: Final Answer:

The dynamic viscosity is 2 N-s/m^2 .

💡 Quick Tip

Newton's law of viscosity, $\tau = \mu(du/dy)$, is a cornerstone of fluid mechanics.

Remember that dynamic viscosity μ is the proportionality constant between shear stress and the rate of shear strain.

Ensure your units are consistent (e.g., Pascals for stress, s^{-1} for gradient) to get viscosity in $\text{Pa}\cdot\text{s}$.

177. If the diameter of droplet and surface tension of water are 0.05 mm and 0.075 N/m respectively, then the pressure inside of droplet in excess of the outside pressure intensity is

- (A) 3 kN/m^2
- (B) 6 kN/m^2
- (C) $3 \times 10^3 \text{ kN/m}^2$
- (D) $6 \times 10^3 \text{ kN/m}^2$

Correct Answer: (B) 6 kN/m^2

Solution:

Step 1: Understanding the Question:

The question asks for the excess pressure (ΔP) inside a liquid droplet due to surface tension.

Step 2: Key Formula or Approach:

The formula for the excess pressure inside a spherical liquid droplet is:

$$\Delta P = \frac{4\sigma}{d}$$

where:

ΔP = Excess pressure (Pressure inside - Pressure outside)

σ = Surface tension of the liquid

d = Diameter of the droplet

(Note: For a hollow bubble, the formula is $\Delta P = 8\sigma/d$. For a liquid jet, it's $\Delta P = 2\sigma/d$).

Step 3: Detailed Explanation:

First, ensure all units are consistent. We will use SI units (meters, Newtons).

- Diameter (d) = 0.05 mm = 0.05×10^{-3} m

- Surface tension (σ) = 0.075 N/m

Now, substitute these values into the droplet formula:

$$\begin{aligned}\Delta P &= \frac{4 \times 0.075 \text{ N/m}}{0.05 \times 10^{-3} \text{ m}} \\ \Delta P &= \frac{0.3}{0.05 \times 10^{-3}} \text{ N/m}^2 \\ \Delta P &= \frac{0.3}{5 \times 10^{-5}} = \frac{3 \times 10^{-1}}{5 \times 10^{-5}} = 0.6 \times 10^4 \text{ N/m}^2 \\ \Delta P &= 6000 \text{ N/m}^2\end{aligned}$$

The options are given in kN/m². To convert from N/m² to kN/m², we divide by 1000.

$$\Delta P = \frac{6000 \text{ N/m}^2}{1000} = 6 \text{ kN/m}^2$$

Step 4: Final Answer:

The excess pressure inside the droplet is 6 kN/m².

💡 Quick Tip

Remember the different formulas for excess pressure due to surface tension:

- **Liquid Droplet:** $\Delta P = 4\sigma/d$

- **Hollow Bubble (like a soap bubble):** $\Delta P = 8\sigma/d$

- **Liquid Jet:** $\Delta P = 2\sigma/d$

A common mistake is using the wrong formula.

178. When a body is immersed in a fluid either wholly or partially, it is lifted up by a force which is equal to the weight of the fluid displaced by the body is known as

(A) Pascal's law

(B) Euler's principle

- (C) Reynold's law
- (D) Archimede's principle

Correct Answer: (D) Archimede's principle

Solution:

Step 1: Understanding the Question:

The question provides a statement of a fundamental principle in fluid mechanics and asks for its name.

Step 2: Detailed Explanation:

The statement "a body immersed in a fluid...is lifted up by a force which is equal to the weight of the fluid displaced by the body" is the exact definition of **Archimedes' Principle**.

This upward force is called the buoyant force.

Let's review the other options:

- **(A) Pascal's Law:** States that a pressure change at any point in a confined incompressible fluid is transmitted throughout the fluid such that the same change occurs everywhere. This is the principle behind hydraulic jacks.
- **(B) Euler's principle/equations:** These are fundamental equations of motion for an inviscid (frictionless) fluid flow.
- **(C) Reynold's law/number:** The Reynolds number is a dimensionless quantity used to predict fluid flow patterns (laminar vs. turbulent). It's not a law in the same sense.

Step 3: Final Answer:

The statement describes Archimedes' principle.

 Quick Tip

Remember the key fluid principles:

- **Archimedes' Principle:** Deals with **buoyancy**. The upward force equals the weight of the displaced fluid.
- **Pascal's Law:** Deals with **pressure transmission** in a confined fluid.

179. The specific gravity and height of mercury in a column are 13.6 and 20 cm respectively. For the same maximum pressure at the bottom, the equivalent height of water column is

- (A) 27.2 cm
- (B) 27.2 m
- (C) 272 cm
- (D) 272 m

Correct Answer: (C) 272 cm

Solution:

Step 1: Understanding the Question:

The question asks to find the height of a water column that would exert the same pressure as a given height of a mercury column.

Step 2: Key Formula or Approach:

The pressure (P) exerted by a column of liquid is given by $P = \rho gh$. Since the pressure is the same for both columns, we can write:

$$P = \rho_{\text{mercury}}gh_{\text{mercury}} = \rho_{\text{water}}gh_{\text{water}}$$

The specific gravity (SG) of a substance is the ratio of its density to the density of water:

$$SG_{\text{mercury}} = \rho_{\text{mercury}}/\rho_{\text{water}}.$$

$$\text{So, } \rho_{\text{mercury}} = SG_{\text{mercury}} \times \rho_{\text{water}}.$$

Substituting this into the pressure equation and canceling g and ρ_{water} gives the relationship:

$$h_{\text{water}} = SG_{\text{mercury}} \times h_{\text{mercury}}$$

Step 3: Detailed Explanation:

We are given:

- Specific gravity of mercury (SG_{mercury}) = 13.6

- Height of mercury column (h_{mercury}) = 20 cm

Now, calculate the equivalent height of the water column:

$$h_{\text{water}} = 13.6 \times 20 \text{ cm}$$

$$h_{\text{water}} = 272 \text{ cm}$$

This is also equal to 2.72 m. Comparing with the options, 272 cm is available.

Step 4: Final Answer:

The equivalent height of the water column is 272 cm.

💡 Quick Tip

For pressure head conversions, use the simple formula $h_1 S_1 = h_2 S_2$, where S is the specific gravity.

Since mercury is 13.6 times denser than water, you need a water column that is 13.6 times taller to produce the same pressure.

180. A rectangular gate of width 2 m and height 3 m lies in a vertical plane, the top of which is at the surface of water. If the unit weight of water is 10 kN/m³, then the force exerted by the water in the gate is

(A) 20 kN

(B) 30 kN

- (C) 60 kN
(D) 90 kN

Correct Answer: (D) 90 kN

Solution:

Step 1: Understanding the Question:

The question asks to calculate the total hydrostatic force (or thrust) on a vertical rectangular gate submerged in water.

Step 2: Key Formula or Approach:

The total hydrostatic force (F) exerted on a submerged plane surface is given by the formula:

$$F = \rho g A \bar{h} = w A \bar{h}$$

where:

$w = \rho g$ = unit weight of the fluid

A = area of the submerged surface

$\bar{h}$ = vertical depth from the free surface to the centroid of the submerged area.

Step 3: Detailed Explanation:

First, let's find the values for each term in the formula.

- Unit weight of water (w) = 10 kN/m³.
- The gate is rectangular with width = 2 m and height = 3 m.
- Area of the gate (A) = width $\times$ height = 2 m $\times$ 3 m = 6 m².
- The gate is vertical, and its top edge is at the water surface. The centroid of a rectangle is at its geometric center, which is at half its height.
- The depth to the centroid ($\bar{h}$) = height / 2 = 3 m / 2 = 1.5 m.

Now, substitute these values into the force formula:

$$\begin{aligned} F &= w A \bar{h} \\ F &= (10 \text{ kN/m}^3) \times (6 \text{ m}^2) \times (1.5 \text{ m}) \\ F &= 60 \times 1.5 \text{ kN} \\ F &= 90 \text{ kN} \end{aligned}$$

Step 4: Final Answer:

The force exerted by the water on the gate is 90 kN.

💡 Quick Tip

The formula for hydrostatic force is "Pressure at the centroid times the Area".

$$F = P_{centroid} \times A = (w\bar{h}) \times A.$$

Remember that $\bar{h}$ is the **vertical depth** to the centroid, not the slant distance.

181. Flow of fluid through a long pipe of constant diameter at a constant rate is

- (A) Steady uniform flow
- (B) Steady non-uniform flow
- (C) Unsteady uniform flow
- (D) Unsteady non-uniform flow

Correct Answer: (A) Steady uniform flow

Solution:

Step 1: Understanding the Question:

The question asks to classify a specific type of pipe flow based on its characteristics with respect to time and space.

Step 2: Detailed Explanation:

Flow classification is based on two criteria:

1. **Time-dependence:** - **Steady Flow:** The fluid properties (like velocity, pressure, density) at any single point in the flow do not change with time. A "constant flow rate" implies steady flow. - **Unsteady Flow:** The properties at a point change with time.
2. **Space-dependence:** - **Uniform Flow:** The velocity vector (both magnitude and direction) is the same at all points in the flow field at a given instant. For pipe flow, this means the average velocity is constant along the length of the pipe. This occurs when the pipe has a "constant diameter". - **Non-uniform Flow:** The velocity changes from one point to another along the flow path (e.g., in a tapering pipe or around a bend).

Analyzing the given conditions:

- "at a constant rate": This means the flow rate is not changing with time, so the flow is **Steady**.

- "a long pipe of constant diameter": Since the diameter is constant and the flow rate is constant, the average velocity ($v = Q/A$) must also be constant along the length of the pipe. This means the flow is **Uniform**.

Step 3: Final Answer:

Combining the two characteristics, the flow is Steady uniform flow.

 Quick Tip

Break down flow classification into two simple questions:

1. Is it changing with **time**? (Constant rate?) $\rightarrow$ No = **Steady**.
2. Is it changing with **location**? (Constant diameter?) $\rightarrow$ No = **Uniform**.

182. For flowing through the same discharge, the diameters of a pipe at section 1 and section 2 are 50 mm and 100 mm respectively. If the velocity of fluid at section 1 is 8 m/s, then the velocity at section 2 is

- (A) 8 m/s
- (B) 6 m/s
- (C) 4 m/s
- (D) 2 m/s

Correct Answer: (D) 2 m/s

Solution:

Step 1: Understanding the Question:

The question describes fluid flowing through a pipe that changes diameter. Given the velocity at the smaller section, we need to find the velocity at the larger section for the same flow rate.

Step 2: Key Formula or Approach:

This problem is an application of the principle of conservation of mass, expressed through the **Continuity Equation**. For an incompressible fluid, the volume flow rate (Q) is constant.

$$Q = A_1 v_1 = A_2 v_2$$

where:

A_1, v_1 are the area and velocity at section 1.

A_2, v_2 are the area and velocity at section 2.

The area of a circular pipe is $A = \frac{\pi}{4} D^2$.

Step 3: Detailed Explanation:

Let's plug the area formula into the continuity equation:

$$\frac{\pi}{4} D_1^2 v_1 = \frac{\pi}{4} D_2^2 v_2$$

The term $\pi/4$ cancels out, leaving a direct relationship between velocity and diameter:

$$D_1^2 v_1 = D_2^2 v_2$$

Rearrange to solve for the unknown velocity, v_2 :

$$v_2 = v_1 \left(\frac{D_1}{D_2} \right)^2$$

We are given:

- $D_1 = 50 \text{ mm}$
- $D_2 = 100 \text{ mm}$
- $v_1 = 8 \text{ m/s}$

Substitute the values:

$$v_2 = 8 \text{ m/s} \times \left(\frac{50 \text{ mm}}{100 \text{ mm}} \right)^2$$

$$v_2 = 8 \times \left(\frac{1}{2} \right)^2$$

$$v_2 = 8 \times \frac{1}{4}$$

$$v_2 = 2 \text{ m/s}$$

Step 4: Final Answer:

The velocity at section 2 is 2 m/s.

💡 Quick Tip

From the continuity equation, velocity is inversely proportional to the square of the diameter ($v \propto 1/D^2$).

If you **double the diameter**, you increase the area by a factor of 4, so the velocity must **decrease by a factor of 4**.

In this problem, $D_2 = 2D_1$, so $v_2 = v_1/4 = 8/4 = 2 \text{ m/s}$.

183. If the coefficient of discharge and coefficient of velocity for an orifice are 0.63 and 0.75 respectively, then the coefficient of contraction is

- (A) 0.47
- (B) 0.75
- (C) 0.84
- (D) 1.19

Correct Answer: (C) 0.84

Solution:

Step 1: Understanding the Question:

The question asks to calculate the coefficient of contraction (C_c) for an orifice, given the coefficient of discharge (C_d) and the coefficient of velocity (C_v).

Step 2: Key Formula or Approach:

The orifice coefficients are related by the following fundamental formula:

The coefficient of discharge is the product of the coefficient of velocity and the coefficient of

contraction.

$$C_d = C_c \times C_v$$

Step 3: Detailed Explanation:

We are given:

- Coefficient of discharge (C_d) = 0.63
- Coefficient of velocity (C_v) = 0.75

We need to find the coefficient of contraction (C_c). Rearrange the formula:

$$C_c = \frac{C_d}{C_v}$$

Substitute the given values:

$$C_c = \frac{0.63}{0.75}$$

To solve this, we can multiply the numerator and denominator by 4:

$$C_c = \frac{0.63 \times 4}{0.75 \times 4} = \frac{2.52}{3}$$
$$C_c = 0.84$$

Step 4: Final Answer:

The coefficient of contraction is 0.84.

💡 Quick Tip

Remember the physical meaning and relationship of the orifice coefficients:

- $C_v = \frac{\text{Actual Velocity}}{\text{Theoretical Velocity}}$ (accounts for friction loss)
- $C_c = \frac{\text{Area of Vena Contracta}}{\text{Area of Orifice}}$ (accounts for streamline contraction)
- $C_d = \frac{\text{Actual Discharge}}{\text{Theoretical Discharge}} = C_c \times C_v$

184. In a pipe flow, the hydraulic grade line will be obtained by the summation of

- (A) Pressure head and datum head
- (B) Velocity head and datum head
- (C) Pressure head and velocity head
- (D) Pressure head, velocity head and datum head

Correct Answer: (A) Pressure head and datum head

Solution:

Step 1: Understanding the Question:

The question asks for the definition of the Hydraulic Grade Line (HGL) in pipe flow.

Step 2: Detailed Explanation:

In pipe flow analysis using the Bernoulli equation, we often work with three types of "head":

- **Datum Head (z):** The potential energy per unit weight, representing the elevation of the point above a reference datum.
- **Pressure Head (P/γ):** The flow work energy per unit weight.
- **Velocity Head ($v^2/2g$):** The kinetic energy per unit weight.

Two important energy lines are defined:

1. **Total Energy Line (TEL) or Energy Grade Line (EGL):** This line represents the total head (total energy per unit weight) of the fluid along the pipe.

$$EGL = z + \frac{P}{\gamma} + \frac{v^2}{2g}$$

In real flow, the EGL always slopes downwards in the direction of flow due to head loss from friction.

2. **Hydraulic Grade Line (HGL):** This line represents the piezometric head. It is obtained by the summation of the **pressure head and the datum head**.

$$HGL = z + \frac{P}{\gamma}$$

The HGL is always located below the EGL by a vertical distance equal to the velocity head ($v^2/2g$). The HGL represents the level to which liquid would rise in a piezometer tube attached to the pipe.

Step 3: Final Answer:

The hydraulic grade line is obtained by the summation of the pressure head and datum head.

💡 Quick Tip

A simple way to remember the energy lines:

- **HGL** = Piezometric Head ($z + P/\gamma$)
- **EGL** = Total Head (HGL + Velocity Head)

The vertical distance between the EGL and HGL is always the velocity head, $v^2/2g$.

185. For a long pipe of diameter D with a given discharge, friction factor and length of pipe, the loss of head due to friction (h_f) is

- (A) inversely proportional to D
- (B) directly proportional to D
- (C) inversely proportional to D^3
- (D) inversely proportional to D^5

Correct Answer: (D) inversely proportional to D^5

Solution:**Step 1: Understanding the Question:**

The question asks for the relationship between the head loss due to friction (h_f) and the pipe diameter (D) when the discharge (Q) is held constant.

Step 2: Key Formula or Approach:

The head loss due to friction is given by the Darcy-Weisbach equation:

$$h_f = \frac{fLv^2}{2gD}$$

where f is the friction factor, L is the pipe length, v is the average velocity, g is gravity, and D is the pipe diameter.

The discharge (Q) is related to velocity and diameter by the continuity equation:

$$Q = Av = \left(\frac{\pi D^2}{4} \right) v$$

This means the velocity can be expressed in terms of the constant discharge:

$$v = \frac{4Q}{\pi D^2}$$

Step 3: Detailed Explanation:

Now, substitute the expression for velocity (v) into the Darcy-Weisbach equation to see how head loss depends on diameter for a constant discharge.

$$\begin{aligned} h_f &= \frac{fL}{2gD} \left(\frac{4Q}{\pi D^2} \right)^2 \\ h_f &= \frac{fL}{2gD} \left(\frac{16Q^2}{\pi^2 D^4} \right) \\ h_f &= \left(\frac{16fLQ^2}{2g\pi^2} \right) \frac{1}{D \cdot D^4} \\ h_f &= \left(\frac{8fLQ^2}{g\pi^2} \right) \frac{1}{D^5} \end{aligned}$$

In this problem, the discharge (Q), friction factor (f), and length (L) are all given as constants. Therefore, the entire term in the parenthesis is a constant.

This leaves the relationship:

$$h_f \propto \frac{1}{D^5}$$

The head loss due to friction is inversely proportional to the fifth power of the diameter.

Step 4: Final Answer:

The loss of head due to friction is inversely proportional to D^5 .

 Quick Tip

Be careful to distinguish between problems with constant **velocity** and constant **discharge**.

- If velocity (v) is constant: $h_f \propto 1/D$.
- If discharge (Q) is constant: $h_f \propto 1/D^5$.

The second case is much more common in practical pipe flow problems.

186. A trapezoidal open channel with a base width of 4 m and side slope 1 horizontal to 2 vertical carries water at a depth of 2 m. The flow area of section is

- (A) 16 m²
- (B) 12 m²
- (C) 10 m²
- (D) 8 m²

Correct Answer: (C) 10 m²

Solution:

Step 1: Understanding the Question:

The question asks to calculate the cross-sectional area of flow for a trapezoidal channel with given dimensions.

Step 2: Key Formula or Approach:

The formula for the cross-sectional area (A) of a trapezoidal channel is:

$$A = (B + my)y$$

where:

B = Base width

y = Depth of flow

m = Side slope, defined as the ratio of horizontal to vertical (1 vertical to m horizontal).

Step 3: Detailed Explanation:

First, carefully determine the side slope parameter, m .

The problem states the side slope is "1 horizontal to 2 vertical".

Our standard definition of m is "1 vertical to m horizontal". We need to convert the given slope to this format.

Given slope: $\frac{H}{V} = \frac{1}{2}$.

So, $m = 1/2 = 0.5$.

Now, identify the other given values:

- Base width (B) = 4 m

- Depth of flow (y) = 2 m

Substitute these values into the area formula:

$$A = (B + my)y$$

$$A = (4 + (0.5)(2)) \times 2$$

$$A = (4 + 1) \times 2$$

$$A = 5 \times 2 = 10 \text{ m}^2$$

Step 4: Final Answer:

The flow area of the section is 10 m^2 .

💡 Quick Tip

The most common error in trapezoidal channel problems is misinterpreting the side slope.

Always be sure what 'm' represents. The standard formula $A = (B + my)y$ assumes m is the horizontal component for a 1-unit vertical component.

If the slope is given as "X horizontal to Y vertical," then $m = X/Y$.

187. If the Darcey's friction factor is 0.008 and the acceleration due to gravity is 10 m/s^2 , then chezey's constant is

- (A) 120
- (B) 100
- (C) 80
- (D) 60

Correct Answer: (B) 100

Solution:

Step 1: Understanding the Question:

The question asks to calculate Chezy's constant (C) given the Darcy-Weisbach friction factor (f).

Step 2: Key Formula or Approach:

The Darcy-Weisbach friction factor (f) and Chezy's constant (C) are two different coefficients used to quantify friction in pipe flow and open channel flow. They are related by the following equation:

$$C = \sqrt{\frac{8g}{f}}$$

where:

C = Chezy's constant

g = acceleration due to gravity

f = Darcy-Weisbach friction factor

Step 3: Detailed Explanation:

We are given:

- Friction factor (f) = 0.008
- Acceleration due to gravity (g) = 10 m/s²

Substitute these values into the formula:

$$C = \sqrt{\frac{8 \times 10}{0.008}}$$

$$C = \sqrt{\frac{80}{0.008}}$$

$$C = \sqrt{\frac{80}{8/1000}} = \sqrt{10 \times 1000} = \sqrt{10000}$$

$$C = 100$$

The units of Chezy's constant are m^{1/2}/s.

Step 4: Final Answer:

Chezy's constant is 100.

💡 Quick Tip

Remember the relationship $C = \sqrt{8g/f}$.

Also, be aware that there are two friction factors in use:

- Darcy friction factor (f): Used in the Darcy-Weisbach equation. The value is typically 0.01-0.05.
 - Fanning friction factor (f_F): Used in some chemical engineering contexts. $f_F = f/4$.
- Make sure you are using the correct factor (f) in the formula for Chezy's constant.

188. The hydraulic radius for a most economical rectangular open channel with a base width of 5 m is

- (A) 1.25 m
- (B) 2.5 m
- (C) 3.75 m
- (D) 5.0 m

Correct Answer: (A) 1.25 m

Solution:

Step 1: Understanding the Question:

The question asks for the hydraulic radius (R) of a "most economical" rectangular channel, given its base width.

Step 2: Key Formula or Approach:

A "most economical" or "most efficient" channel section is one that conveys the maximum discharge for a given cross-sectional area and slope. This corresponds to the section with the minimum wetted perimeter for a given area.

For a **rectangular channel**, the condition for the most economical section is that the depth of flow (y) must be half of the base width (B).

$$y = \frac{B}{2}$$

The hydraulic radius (R) is defined as the ratio of the flow area (A) to the wetted perimeter (P).

For a rectangular channel:

- Area (A) = By
- Wetted Perimeter (P) = $B + 2y$
- Hydraulic Radius (R) = $\frac{A}{P} = \frac{By}{B+2y}$

Step 3: Detailed Explanation:

We are given the base width, $B = 5$ m.

First, find the depth of flow (y) for the most economical condition:

$$y = \frac{B}{2} = \frac{5 \text{ m}}{2} = 2.5 \text{ m}$$

Now, calculate the hydraulic radius using this depth:

$$R = \frac{By}{B + 2y} = \frac{5 \times 2.5}{5 + 2(2.5)} = \frac{12.5}{5 + 5} = \frac{12.5}{10}$$

$$R = 1.25 \text{ m}$$

Alternatively, for a most economical rectangular section, the hydraulic radius is always half the depth of flow: $R = y/2$.

Since $y = 2.5$ m, $R = 2.5/2 = 1.25$ m.

Step 4: Final Answer:

The hydraulic radius for the most economical rectangular channel is 1.25 m.

💡 Quick Tip

For a most economical **rectangular** channel, remember these two simple rules:

1. The depth is half the width: $y = B/2$.
2. The hydraulic radius is half the depth: $R = y/2$.

189. Pelton wheel is classified according to the main direction of flow of water in the runner as

- (A) Tangential flow turbine
- (B) Radial flow turbine

- (C) Axial flow turbine
- (D) Mixed flow turbine

Solution:

Step 1: Understanding the Question:

The question asks for the classification of a Pelton wheel turbine based on how the water flows relative to the runner (the rotating part).

Step 2: Detailed Explanation:

Hydraulic turbines are classified based on the direction of water flow through the runner:

- **Tangential Flow Turbine:** The water strikes the runner **tangentially**, in the same plane as the runner's rotation. The **Pelton wheel** is the primary example of this type. A high-speed jet of water from a nozzle strikes a series of buckets mounted on the periphery of the wheel, causing it to rotate.
- **Radial Flow Turbine:** The water flows radially, either inwards or outwards, through the runner. The old **Francis turbine** designs were purely radial.
- **Axial Flow Turbine:** The water flows parallel to the axis of rotation of the runner. The **Kaplan** and **Propeller** turbines are the main examples.
- **Mixed Flow Turbine:** The water enters the runner radially and leaves axially. Modern **Francis turbines** are of this type.

Since the jet of water strikes the buckets of a Pelton wheel on their tangent, it is a tangential flow turbine.

Step 3: Final Answer:

Based on the flow direction, a Pelton wheel is classified as a Tangential flow turbine. Since this option is not explicitly provided in the OCR'd list, there seems to be a missing option. Assuming it was present, this would be the correct classification. For the purpose of providing a complete solution, we will add the option.

Correct Answer: (A) Tangential flow turbine

 Quick Tip

Memorize the classification of major turbines by flow type:

- **Tangential Flow** → **Pelton Wheel** (High Head, Low Discharge).
- **Mixed Flow** → **Francis Turbine** (Medium Head, Medium Discharge).
- **Axial Flow** → **Kaplan/Propeller Turbine** (Low Head, High Discharge).

190. The fluid coming into the centrifugal pump is accelerated by

- (A) Nozzle
- (B) Governor
- (C) Throttle
- (D) Impeller

Correct Answer: (D) Impeller

Solution:

Step 1: Understanding the Question:

The question asks to identify the component of a centrifugal pump that is responsible for accelerating the fluid.

Step 2: Detailed Explanation:

A centrifugal pump works by converting rotational kinetic energy into the hydrodynamic energy of the fluid flow. The key component that achieves this is the impeller.

- **(D) Impeller:** The impeller is a rotor with a series of backward-curved vanes. As the pump's shaft rotates the impeller at high speed, fluid is drawn into the center (the "eye") of the impeller. The rotating vanes then sling the fluid outwards at high velocity. The impeller imparts a large amount of kinetic energy to the fluid, thus accelerating it. This high-velocity fluid then enters the volute casing, where the increasing area converts the kinetic energy into pressure energy.
- **(A) Nozzle:** A nozzle is used to accelerate fluid by decreasing the flow area, but it is not the primary component that adds energy in a centrifugal pump. In fact, a pump is the opposite of a nozzle in function (it adds pressure, a nozzle converts pressure to velocity).
- **(B) Governor:** A governor is a device used to regulate the speed of an engine or motor.
- **(C) Throttle:** A throttle is a valve used to control the flow rate of a fluid.

Step 3: Final Answer:

The fluid coming into the centrifugal pump is accelerated by the impeller.

💡 Quick Tip

The heart of a centrifugal pump is the **impeller**.

Its job is to spin rapidly and throw the fluid outwards, giving it high velocity (kinetic energy).

The casing (volute) then slows the fluid down, converting this velocity into pressure.

191. The method of irrigation used for row crops is

- (A) Zig zag method
- (B) Furrow method
- (C) Ring Basin method
- (D) Border strip method

Correct Answer: (B) Furrow method

Solution:

Step 1: Understanding the Question:

The question asks to identify the most suitable irrigation method for crops planted in rows.

Step 2: Detailed Explanation:

Different irrigation methods are suited to different types of crops, topographies, and soil types.

- **(B) Furrow Method:** In this method, small, parallel channels called "furrows" are created between the rows of crops. Water is introduced into the furrows and it seeps into the soil, wetting the root zone of the plants. This method is highly suitable for **row crops** like corn, potatoes, cotton, and vegetables, as it delivers water directly to the root area without wetting the entire field surface, which can reduce evaporation and prevent diseases.
- **(A) Zig zag method:** This is a variation of check basin flooding, not typically used for row crops.
- **(C) Ring Basin Method:** This involves creating circular basins around individual trees. It is used for orchard trees, not for field row crops.
- **(D) Border Strip Method:** The field is divided into a series of wide strips by parallel dikes or borders. Water is flooded down these strips. It is suitable for close-growing crops like wheat or alfalfa, but not ideal for row crops.

Step 3: Final Answer:

The furrow method of irrigation is used for row crops.

 Quick Tip

Match the irrigation method to the crop type:

- **Row Crops** (e.g., corn, vegetables) → **Furrow Irrigation.**
- **Orchard Trees** → **Basin/Ring Irrigation.**
- **Close-growing Crops** (e.g., wheat, grass) → **Border/Flood Irrigation.**

192. Duty of water of canal system does not depends on

- (A) Area of land irrigated
- (B) Method of cultivation
- (C) Type of the crop
- (D) Base period of the crop

Correct Answer: (A) Area of land irrigated

Solution:

Step 1: Understanding the Question:

The question asks which of the given factors does not influence the "Duty of water".

Step 2: Detailed Explanation:

Duty of water (D) is a term in irrigation engineering that defines the efficiency of water use. It is the area of land, in hectares, that can be irrigated by a unit discharge of water (1 cubic meter per second, or cumec) flowing continuously for the entire base period of the crop.

$$D = \frac{\text{Area (hectares)}}{\text{Discharge (cumecs)}}$$

The factors that affect the duty of water are:

- **(C) Type of the crop:** Different crops have different water requirements. A water-intensive crop like rice will have a lower duty (irrigates less area per cumec) than a crop like wheat.
- **(D) Base period of the crop:** The base period is the duration for which irrigation is supplied. A longer base period for the same total water depth means the required flow rate is lower, which results in a higher duty.
- **(B) Method of cultivation:** Efficient cultivation and irrigation methods (like furrow or drip irrigation versus inefficient flooding) reduce water losses, meaning the same amount of water can irrigate a larger area, thus increasing the duty.
- Other factors include climate, soil type, and canal conditions (lining reduces seepage losses, increasing duty).

(A) Area of land irrigated is the *result* of the duty, not a factor that determines it. The duty is a property of the system (crop, soil, climate, efficiency) that tells you how much area you *can* irrigate with a certain flow. It is an intensive property, not an extensive one.

Step 3: Final Answer:

The duty of water does not depend on the total area of land irrigated; rather, it helps to determine it.

💡 Quick Tip

Duty of water is a measure of *efficiency*, expressed as Hectares/cumec.

Think of it like the "mileage" of water. Better mileage (higher duty) depends on the car (crop type), driving conditions (climate, soil), and driver skill (cultivation method).

The total distance traveled (area irrigated) is a result, not a cause, of the mileage.

193. The precipitation caused due to lifting of air masses converging into low pressure area is known as

- (A) Cyclonic precipitation
- (B) Convective precipitation
- (C) Orographic precipitation
- (D) Turbulent precipitation

Correct Answer: (A) Cyclonic precipitation

Solution:

Step 1: Understanding the Question:

The question asks for the name of precipitation that results from air being lifted due to convergence into a low-pressure zone.

Step 2: Detailed Explanation:

Precipitation occurs when moist air is lifted, causing it to cool adiabatically, leading to condensation and the formation of clouds and rain/snow. There are three main lifting mechanisms:

1. **(A) Cyclonic Precipitation:** This occurs due to the large-scale movement of air masses. When air converges into a low-pressure system (a cyclone), it is forced to rise. This lifting and subsequent cooling cause widespread, and often prolonged, precipitation. This can be further divided into frontal (when warm and cold air masses meet) and non-frontal types. The description in the question perfectly matches this mechanism.
2. **(B) Convective Precipitation:** This is caused by intense, localized heating of the earth's surface. The heated air becomes buoyant and rises rapidly in vertical columns (convection currents). As it rises, it cools, forming cumulonimbus clouds that produce heavy, short-duration showers or thunderstorms.

3. **(C) Orographic Precipitation:** This occurs when moist air is forced to rise as it encounters a topographic barrier like a mountain range. The air cools as it ascends the windward side, causing precipitation. The leeward side is typically dry (a rain shadow).

Step 3: Final Answer:

The precipitation caused by the lifting of air masses converging into a low-pressure area is known as cyclonic precipitation.

💡 Quick Tip

Remember the three main triggers for rain:

- **Cyclonic:** Caused by low-pressure systems and fronts (large scale).
- **Convective:** Caused by surface heating (local thunderstorms).
- **Orographic:** Caused by mountains forcing air up.

194. Which of the following is a non-automatic rain gauge ?

- (A) Syphon type rain gauge
- (B) Symon's rain gauge
- (C) Weighing bucket rain gauge
- (D) Tipping bucket rain gauge

Correct Answer: (B) Symon's rain gauge

Solution:

Step 1: Understanding the Question:

The question asks to identify the rain gauge from the list that is non-automatic (i.e., requires manual measurement).

Step 2: Detailed Explanation:

Rain gauges are classified into two main types:

1. **Non-automatic (or Non-recording) Rain Gauges:** These gauges simply collect the rainfall over a period (typically 24 hours). An observer must manually visit the gauge, pour the collected water into a graduated measuring cylinder, and record the amount. The **Symon's rain gauge** is the standard non-automatic rain gauge used in India and many other countries.
2. **Automatic (or Recording) Rain Gauges:** These gauges continuously record the rainfall over time, providing information about the intensity and duration of the storm. They do not require an observer to be present for each reading. The main types are:
 - **(D) Tipping Bucket Rain Gauge:** Funnel rain into a small bucket that tips over each time it fills with a known small amount of rain (e.g., 0.2 mm), triggering an electronic signal for each tip.
 - **(C) Weighing Bucket Rain Gauge:** Collects rain in a bucket placed on a weighing scale. The continuous increase in weight is recorded on a chart or electronically.
 - **(A) Syphon Type Rain Gauge (or Float Type):** Collects rain in a chamber with a float. As the water level

risers, the float rises, and a pen attached to it records the rainfall on a rotating drum. When the chamber is full, the water is automatically siphoned out.

Step 3: Final Answer:

The Symon's rain gauge is a non-automatic rain gauge.

💡 Quick Tip

The key difference is continuous recording.

- **Automatic/Recording Gauges** (Tipping Bucket, Weighing, Syphon) produce a graph of rainfall vs. time (a hyetograph).
- **Non-Automatic Gauges** (Symon's) only give the total rainfall depth over a period.

195. Using Dicken's formula with the constant C as 12, the flood discharge for a catchment area of 16 km^2 is

- (A) 1.33 cumecs
- (B) 48 cumecs
- (C) 96 cumecs
- (D) 192 cumecs

Correct Answer: (C) 96 cumecs

Solution:

Step 1: Understanding the Question:

The question asks to calculate the peak flood discharge (Q_p) using Dicken's empirical formula, given the catchment area and Dicken's constant.

Step 2: Key Formula or Approach:

Dicken's formula is an empirical formula used for estimating the peak flood discharge from a catchment, particularly in India. The formula is:

$$Q_p = CA^{3/4}$$

where:

Q_p = Peak flood discharge in cumecs (m^3/s)

C = Dicken's constant, which depends on the region and catchment characteristics

A = Catchment area in square kilometers (km^2)

Step 3: Detailed Explanation:

We are given:

- Dicken's constant (C) = 12
- Catchment area (A) = 16 km^2

Substitute these values into the formula:

$$Q_p = 12 \times (16)^{3/4}$$

To calculate $(16)^{3/4}$, we can first take the fourth root and then cube the result:

$$(16)^{1/4} = \sqrt{\sqrt{16}} = \sqrt{4} = 2$$

Now, cube this result:

$$(2)^3 = 8$$

So, $(16)^{3/4} = 8$.

Now, calculate the discharge:

$$Q_p = 12 \times 8$$

$$Q_p = 96 \text{ m}^3/\text{s (cumecs)}$$

Step 4: Final Answer:

The flood discharge is 96 cumecs.

💡 Quick Tip

Memorize the main empirical flood formulas and their exponents:

- **Dicken's Formula:** $Q_p = CA^{3/4}$

- **Ryve's Formula:** $Q_p = CA^{2/3}$

- **Inglis Formula:** $Q_p = \frac{124A}{\sqrt{A+10.24}}$

Make sure to use the correct area units (usually km^2) for each formula.

196. Weir / barrage classified according to use is known as

- (A) Storage dam
- (B) Diversion dam
- (C) Detention dam
- (D) Check dam

Correct Answer: (B) Diversion dam

Solution:

Step 1: Understanding the Question:

The question asks for the functional classification of a weir or barrage.

Step 2: Detailed Explanation:

Dams, weirs, and barrages are hydraulic structures built across rivers. They are classified based on their primary purpose or use.

- **(B) Diversion Dam/Weir/Barrage:** The primary purpose of a weir or a barrage is not to store large quantities of water for a long time, but to raise the water level in the river just enough to **divert** a portion of the river's flow into an off-taking canal for irrigation, water supply, etc. This is their main use.
- **(A) Storage Dam:** This is a large dam built to impound water and create a large reservoir. Its primary purpose is long-term storage of water for purposes like irrigation, hydropower, flood control, and water supply during dry seasons.
- **(C) Detention Dam:** This type of dam is built specifically for flood control. It temporarily stores (detains) floodwater and releases it at a controlled rate to prevent downstream flooding. The reservoir is normally empty.
- **(D) Check Dam:** A small dam, typically built in a small channel or gully, to reduce flow velocity, control soil erosion, and promote groundwater recharge.

The term "weir" or "barrage" is most directly associated with the function of diversion.

Step 3: Final Answer:

A weir or barrage, when classified according to its primary use, is known as a diversion structure or diversion dam.

💡 Quick Tip

Remember the primary function of different river structures:

- **Dam (large)** → **Storage** (creates a reservoir).
- **Weir / Barrage** → **Diversion** (raises water level to feed a canal).
- **Detention Dam** → **Flood Control** (temporary storage).

197. For a rectangular masonry dam, the weight of dam, uplift pressure and horizontal force causing the sliding are 300 kN, 60 kN and 80 kN respectively. If the coefficient of friction between the dam base and soil is 0.5, then the factor of safety against sliding is

- (A) 1.07
- (B) 1.5
- (C) 1.88
- (D) 2.0

Correct Answer: (B) 1.5

Solution:

Step 1: Understanding the Question:

The question asks to calculate the factor of safety (FOS) against the sliding failure of a masonry dam, given all the relevant forces and the coefficient of friction.

Step 2: Key Formula or Approach:

The Factor of Safety against sliding is defined as the ratio of the total resisting forces to the total sliding (or driving) forces.

$$\text{FOS}_{\text{sliding}} = \frac{\text{Resisting Forces}}{\text{Sliding Forces}}$$

- **Sliding Force:** This is the net horizontal force pushing the dam, given as 80 kN.
- **Resisting Force:** This is the frictional force developed at the base of the dam. It is calculated as the coefficient of friction (μ) multiplied by the net vertical force. The net vertical force is the weight of the dam minus the uplift force.

$$\text{Resisting Force} = \mu \times (\text{Weight} - \text{Uplift Force})$$

Step 3: Detailed Explanation:

We are given:

- Weight of the dam (W) = 300 kN
- Uplift force (U) = 60 kN
- Horizontal sliding force (H) = 80 kN
- Coefficient of friction (μ) = 0.5

First, calculate the net vertical force:

$$\Sigma V = W - U = 300 \text{ kN} - 60 \text{ kN} = 240 \text{ kN}$$

Next, calculate the maximum resisting force (frictional force):

$$F_{\text{Resisting}} = \mu \times \Sigma V = 0.5 \times 240 \text{ kN} = 120 \text{ kN}$$

Now, calculate the Factor of Safety:

$$\text{FOS}_{\text{sliding}} = \frac{F_{\text{Resisting}}}{F_{\text{Sliding}}} = \frac{120 \text{ kN}}{80 \text{ kN}}$$

$$\text{FOS}_{\text{sliding}} = \frac{12}{8} = \frac{3}{2} = 1.5$$

Step 4: Final Answer:

The factor of safety against sliding is 1.5.

💡 Quick Tip

The formula for FOS against sliding is a critical check in dam design.

$$\text{FOS} = \frac{\mu(\Sigma V)}{\Sigma H}$$

Remember that uplift force **reduces** the net vertical force, which in turn reduces the frictional resistance.

Always use the **net** vertical force ($\text{Weight} - \text{Uplift}$) in the calculation.

198. A dam of height 12 m is trapezoidal in section with top width of 2 m and bottom width of 8 m. The unit weight of the material of dam and water are 20 kN/m³ and 10 kN/m³ respectively. If the dam is allowed to store water upto a height of 10 m, the horizontal pressure on the dam is

- (A) 100 kN
- (B) 500 kN
- (C) 720 kN
- (D) 1000 kN

Correct Answer: (B) 500 kN

Solution:

Step 1: Understanding the Question:

The question asks for the total horizontal hydrostatic force (referred to as horizontal pressure) acting on a dam of unit length.

Step 2: Key Formula or Approach:

The horizontal hydrostatic force on a submerged vertical surface is equal to the area of the pressure diagram. The pressure distribution is triangular, with zero pressure at the water surface and maximum pressure at the base.

The total horizontal force (F_H) per unit length of the dam is given by:

$$F_H = \frac{1}{2}wh^2$$

where:

w = unit weight of water

h = height of the water stored behind the dam

Step 3: Detailed Explanation:

We are given:

- Height of water (h) = 10 m
- Unit weight of water (w) = 10 kN/m³

The dimensions of the dam (height, top width, bottom width) and the unit weight of the dam material are given but are not needed to calculate the horizontal water pressure. They would be needed to check the stability of the dam.

Substitute the relevant values into the formula:

$$F_H = \frac{1}{2} \times (10 \text{ kN/m}^3) \times (10 \text{ m})^2$$

$$F_H = \frac{1}{2} \times 10 \times 100$$

$$F_H = 5 \times 100 = 500 \text{ kN}$$

This force is per meter length of the dam (kN/m), but since the options are in kN, it is implied that the question asks for the force on a 1-meter-wide strip of the dam.

Step 4: Final Answer:

The horizontal pressure on the dam is 500 kN (per meter length).

💡 Quick Tip

The total horizontal hydrostatic force on any vertical plane surface of height 'h' is always equal to the area of the triangular pressure diagram.

$$\text{Force} = (\text{Area of triangle}) = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times (wh) \times h = \frac{1}{2}wh^2.$$

This force always acts at a height of $h/3$ from the base.

199. The structure constructed at the head of a canal taking off from a reservoir behind a weir or dam is called as

- (A) Sluice regulator
- (B) Head regulator
- (C) Silt excluder
- (D) Silt extractor

Correct Answer: (B) Head regulator

Solution:

Step 1: Understanding the Question:

The question asks for the name of the structure built at the starting point (head) of a canal where it takes off from a river or reservoir.

Step 2: Detailed Explanation:

Let's define the structures listed:

- **(B) Head Regulator (or Canal Head Regulator):** This is a structure constructed at the head of an off-taking canal. It has several functions: to regulate the supply of water entering the canal, to control the entry of silt into the canal, and to measure the discharge entering the canal. It is essentially the main gate or control structure for the canal. This perfectly matches the description.
- **(A) Sluice Regulator:** This is a more general term for a gate or sluice used to control water flow. A cross regulator on the main river could also be a type of sluice regulator. "Head Regulator" is the specific term for the structure at the canal head.
- **(C) Silt Excluder:** This is a structure constructed on the river bed just upstream of the head regulator. Its function is to prevent the heavier, bed-load silt from entering the canal.

- **(D) Silt Extractor (or Ejector):** This is a structure built within the canal itself, some distance downstream from the head regulator, to remove the finer silt that has already entered the canal.

Step 3: Final Answer:

The structure constructed at the head of a canal is called a Head regulator.

💡 Quick Tip

Remember the location and function of canal headworks components:

- **Weir/Barrage:** Across the main river to raise the water level.
- **Head Regulator:** At the entrance of the canal to control flow into it.
- **Silt Excluder:** In the river, just before the head regulator, to keep silt out.
- **Silt Ejector:** In the canal, after the head regulator, to remove silt that got in.

200. Lacey's theory is used for the design of

- (A) Canal
- (B) Regulator
- (C) Fish ladder
- (D) Apron

Correct Answer: (A) Canal

Solution:

Step 1: Understanding the Question:

The question asks for the application of Lacey's regime theory.

Step 2: Detailed Explanation:

In irrigation engineering, the design of unlined canals that flow through alluvial (sandy/silty) soil is a major challenge. If the flow velocity is too high, the canal will scour its bed and banks. If the velocity is too low, silt will deposit and choke the canal.

Regime Theory, developed by engineers like Kennedy and later refined by **Lacey**, addresses this problem. Lacey's theory provides a set of empirical equations that define the stable dimensions (width, depth, slope) of an alluvial **canal** that is in "regime" - a state where there is neither silting nor scouring over time.

The theory allows engineers to design the cross-section and longitudinal slope of a canal based on the design discharge and a "silt factor," which characterizes the sediment being transported. The other options are specific hydraulic structures, and their design is governed by different principles (structural engineering, hydraulics), not regime theory.

Step 3: Final Answer:

Lacey's theory is used for the design of stable, unlined alluvial canals.

💡 Quick Tip

Associate key theories with their applications in irrigation design:

- **Kennedy's & Lacey's Regime Theories** → Design of unlined **canals** in alluvial soil.
 - **Khosla's Theory** → Design of foundations for weirs/barrages on permeable soil (to prevent piping failure).
 - **Bligh's Creep Theory** → An older, simpler theory for the same purpose as Khosla's.
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