AP EAPCET Engineering 2026 May 12 S1

Question Paper with Solutions

Conducted by JNTUH

General Instructions

(i) **Duration:** The total duration of the examination is 3 hours (180 minutes).

(ii) **Total Questions:** This paper consists of 160 questions.

(iii) **Maximum Marks:** The paper carries a maximum of 160 marks.

(iv) **Subject-wise Distribution:**

- **Physics:** 40 Questions
- **Chemistry:** 40 Questions
- **Mathematics:** 80 Questions

(v) **Question Type:** Each question has four options, out of which only one is correct.

(vi) **Correct Answer:** Each correct answer carries +1 mark.

(vii) **Negative Marking:** There is no negative marking for incorrect answers.

(viii) **Unattempted Questions:** No marks will be awarded or deducted for unattempted questions.

1. If $f:[1,\infty)\to[1,\infty)$ is defined by $f(x)=2^{x(x-1)}$, then $f^{-1}(x)=$

(A) $\frac{1}{2}[1+\sqrt{1+4\log_2 x}]$

(B) $\frac{1}{2}[1-\sqrt{1+4\log_2 x}]$

(C) $\frac{1}{2}[1+\sqrt{1-4\log_2 x}]$

(D) $\frac{1}{2}[1-\sqrt{1-4\log_2 x}]$

Correct Answer: (A) $\frac{1}{2}[1+\sqrt{1+4\log_2 x}]$

Solution:

Step 1: Understanding the Concept:

To find the inverse of a function $f(x)$, we follow a standard procedure: replace $f(x)$ with y, solve the resulting equation for x in terms of y, and finally swap x and y.

Given that the domain of the function is $[1, \infty)$, we must ensure that the inverse function returns values only within this specific range.

Key Formula or Approach:

The given function is $y = 2^{x(x-1)}$.

Since the function involves an exponent with base 2, we take the logarithm to the base 2 on both sides to isolate the power.

Step 2: Detailed Explanation:

Taking $\log_2$ on both sides of $y = 2^{x^2-x}$:

$$\log_2 y = x(x-1)$$

Expanding the right side gives:

$$x^2 - x - \log_2 y = 0$$

This is a quadratic equation in terms of x of the form $ax^2 + bx + c = 0$, where $a = 1$, $b = -1$, and $c = -\log_2 y$.

Applying the quadratic formula $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$:

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-\log_2 y)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{1+4\log_2 y}}{2}$$

We are given that the domain of f is $x \geq 1$.

If we choose the negative sign, $x = \frac{1-\sqrt{1+4\log_2 y}}{2}$, then since $y \geq 1 \implies \log_2 y \geq 0$, the numerator $1-\sqrt{1+\ldots}$ would be ≤ 0, meaning $x \leq 0$.

This contradicts our domain $x \geq 1$.

Therefore, we must choose the positive square root branch:

$$x = \frac{1+\sqrt{1+4\log_2 y}}{2}$$

Step 3: Final Answer:

Replacing y with x to get the final expression for the inverse function:

$$f^{-1}(x) = \frac{1}{2}[1+\sqrt{1+4\log_2 x}]$$

This matches Option (A).

Quick Tip

Quick Tip: In inverse function problems involving quadratics, always check the domain constraints to decide between the $+$ and $-$ sign of the square root. For $x \geq 1$, the expression must result in a value ≥ 1.

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2. The domain of the function $f(x) = \sqrt{\log_{10}\left(\frac{5x-x^2}{4}\right)}$ is:

(A) $[1,4]$

(B) $(1,4)$

(C) $[0,5]$

(D) $(0,5)$

Correct Answer: (A) $[1,4]$

Solution:

Step 1: Understanding the Concept:

The domain of a function is the set of all real values of x for which the function is defined.

For the square root $\sqrt{g(x)}$, we require $g(x) \geq 0$.

For the logarithm $\log_a(h(x))$, we require $h(x) > 0$.

Key Formula or Approach:

We must satisfy two conditions simultaneously:

1. The argument of the log must be positive: $\frac{5x-x^2}{4} > 0$.
2. The term inside the square root must be non-negative: $\log_{10}\left(\frac{5x-x^2}{4}\right) \geq 0$.

Step 2: Detailed Explanation:

Condition 1:

$$\frac{5x-x^2}{4} > 0 \implies x(5-x) > 0$$

This inequality holds when x lies between the roots 0 and 5.

Interval: $x \in (0, 5)$

Condition 2:

$$\log_{10}\left(\frac{5x-x^2}{4}\right) \geq 0$$

Using the definition of logarithms $\log_a b \geq c \implies b \geq a^c$ (since base $10 > 1$):

$$\frac{5x-x^2}{4} \geq 10^0 \implies \frac{5x-x^2}{4} \geq 1$$

$$5x-x^2 \geq 4 \implies x^2-5x+4 \leq 0$$

Factoring the quadratic:

$$(x-1)(x-4) \leq 0$$

The product is non-positive between the roots 1 and 4.

Interval: $x \in [1,4]$

Intersection:

The final domain is the intersection of $(0,5)$ and $[1,4]$.

Since $[1,4]$ is entirely contained within $(0,5)$, the intersection is $[1,4]$.

Step 3: Final Answer:

The domain of the function is the closed interval $[1,4]$.

Thus, the correct option is (A).

Quick Tip

Quick Tip: Always test boundary values. At $x=1$, the expression becomes $\sqrt{\log(1)} = \sqrt{0} = 0$, which is defined. Hence, 1 and 4 must be included in the interval (square brackets). This immediately rules out Option (B).

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3. If $a_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$ and $b_n = \sum_{r=0}^{n} \frac{r}{{}^nC_r}$, then $\frac{b_n}{a_n} =$

(A) n

(B) $\frac{n}{2}$

(C) $\frac{n}{3}$

(D) $n-1$

Correct Answer: (B) $\frac{n}{2}$

Solution:

Step 1: Understanding the Concept:

This problem involves the properties of binomial coefficients, specifically the symmetry

property ${}^nC_r = {}^nC_{n-r}$.

Summations involving binomial coefficients can often be solved by reversing the order of the terms and adding them to the original sum.

Key Formula or Approach:

We use the substitution $r = n - r$ inside the summation for b_n.

Step 2: Detailed Explanation:

Given $b_n = \sum_{r=0}^{n} \frac{r}{{}^nC_r}$.

Using the property ${}^nC_r = {}^nC_{n-r}$, we can rewrite the sum by replacing r with $(n-r)$:

$$b_n = \sum_{r=0}^{n} \frac{n-r}{{}^nC_{n-r}} = \sum_{r=0}^{n} \frac{n-r}{{}^nC_r}$$

Now, let's add the original expression for b_n and this new expression:

$$2b_n = \sum_{r=0}^{n} \frac{r}{{}^nC_r} + \sum_{r=0}^{n} \frac{n-r}{{}^nC_r}$$

Combine the summations over the common denominator:

$$2b_n = \sum_{r=0}^{n} \frac{r+n-r}{{}^nC_r} = \sum_{r=0}^{n} \frac{n}{{}^nC_r}$$

Since n is independent of the summation variable r, it can be taken outside:

$$2b_n = n \sum_{r=0}^{n} \frac{1}{{}^nC_r}$$

We recognize that $\sum_{r=0}^{n} \frac{1}{{}^nC_r}$ is exactly a_n.

$$2b_n = n \cdot a_n$$

Rearranging for the ratio:

$$\frac{b_n}{a_n} = \frac{n}{2}$$

Step 3: Final Answer:

The ratio $\frac{b_n}{a_n}$ evaluates to $\frac{n}{2}$.

The correct option is (B).

Quick Tip

Quick Tip: For questions involving properties of n, you can test small values.

Let $n = 1$:

$a_1 = \frac{1}{{}^1C_0} + \frac{1}{{}^1C_1} = 1 + 1 = 2$.

$b_1 = \frac{0}{{}^1C_0} + \frac{1}{{}^1C_1} = 0 + 1 = 1$.

Ratio $\frac{b_1}{a_1} = \frac{1}{2}$, which matches $\frac{n}{2}$.

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4. If $[x]$ denotes the greatest integer less than or equal to x, then the value of $\sum_{r=1}^{100} \left[\frac{r}{5}\right]$ is:

(A) 980

(B) 950

(C) 960

(D) 970

Correct Answer: (D) 970

Solution:

Step 1: Understanding the Concept:

The Greatest Integer Function $[x]$ remains constant for all x within a unit interval. For an expression $\left[\frac{r}{k}\right]$, the value will stay the same for k consecutive integers of r.

Key Formula or Approach:

We group the terms from $r = 1$ to 100 into blocks of 5.

Step 2: Detailed Explanation:

Let's analyze the values of $\left[\frac{r}{5}\right]$:

- For $r = 1, 2, 3, 4$, the value is 0 (4 terms).
- For $r = 5, 6, 7, 8, 9$, the value is 1 (5 terms).
- For $r = 10, 11, 12, 13, 14$, the value is 2 (5 terms).
- ...
- For $r = 95, 96, 97, 98, 99$, the value is 19 (5 terms).
- For $r = 100$, the value is $[100/5] = 20$ (1 term).

The sum S can be written as:

$$S = (0 \times 4) + (1 \times 5) + (2 \times 5) + \cdots + (19 \times 5) + 20$$

Factor out 5 from the middle terms:

$$S = 5(1 + 2 + 3 + \cdots + 19) + 20$$

Using the sum of first n natural numbers formula $\frac{n(n+1)}{2}$ for $n = 19$:

$$\sum_{i=1}^{19} i = \frac{19 \times 20}{2} = 190$$

Now substitute back into the sum:

$$S = 5(190) + 20$$

$$S = 950 + 20 = 970$$

Step 3: Final Answer:

The value of the summation is 970.

The correct option is (D).

Quick Tip

Quick Tip: General formula for $\sum_{r=1}^{nk}\left[\frac{r}{k}\right]$:
The sum is $k \cdot \frac{(n-1)n}{2} + n$.
Here $k = 5$ and $nk = 100 \implies n = 20$.
Sum $= 5 \cdot \frac{19\times 20}{2} + 20 = 950 + 20 = 970$.

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5. If the system of linear equations $x + y + z = 1$, $x + 2y + 4z = \eta$, $x + 4y + 10z = \eta^2$ has a solution, then the value of η is:

(A) 1 or 2
(B) 1 or -2
(C) 2 or -2
(D) 1 or 3

Correct Answer: (A) 1 or 2

Solution:

Step 1: Understanding the Concept:
A system of linear equations is consistent (has a solution) if and only if the rank of the coefficient matrix A is equal to the rank of the augmented matrix $[A|B]$.

Key Formula or Approach:
We use elementary row operations to reduce the augmented matrix to row-echelon form and check the condition for consistency.

Step 2: Detailed Explanation:
The augmented matrix is:

$$[A|B] = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & \eta \\ 1 & 4 & 10 & \eta^2 \end{array}\right)$$

Performing $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$:

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \eta - 1 \\ 0 & 3 & 9 & \eta^2 - 1 \end{array}\right)$$

Now performing $R_3 \to R_3 - 3R_2$:

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \eta - 1 \\ 0 & 0 & 0 & (\eta^2 - 1) - 3(\eta - 1) \end{array}\right)$$

For the system to have a solution, the rank of the augmented matrix must not increase. This requires the last element of the third row to be zero:

$$(\eta^2 - 1) - 3(\eta - 1) = 0$$

Factor out $(\eta - 1)$:

$$(\eta - 1)[(\eta + 1) - 3] = 0$$

$$(\eta - 1)(\eta - 2) = 0$$

Solving for η:

$$\eta = 1 \text{ or } \eta = 2$$

Step 3: Final Answer:

The system is consistent when η is 1 or 2.

The correct option is (A).

Quick Tip

Quick Tip: Check for linear dependence between rows of the coefficient matrix.

Notice that $3 \times (\text{Row } 2) - 2 \times (\text{Row } 1) = \text{Row } 3$.

For the system to be consistent, the same must hold for the constants:

$3(\eta) - 2(1) = \eta^2 \implies \eta^2 - 3\eta + 2 = 0$.

Solving gives $\eta = 1, 2$.

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6. If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, **then** $A^2 - 5A - 2I =$

(A) O

(B) I

(C) A

(D) $2A$

Correct Answer: (A) O

Solution:

Step 1: Understanding the Concept:

According to the Cayley-Hamilton Theorem, every square matrix satisfies its own characteristic equation. For a matrix A, the characteristic equation is given by $\det(A - \lambda I) = 0$.

Key Formula or Approach:

For a 2×2 matrix A, the characteristic equation is:

$$\lambda^2 - \text{Tr}(A)\lambda + \det(A) = 0$$

Substituting matrix A in place of λ gives:

$$A^2 - \text{Tr}(A)A + \det(A)I = O$$

Step 2: Detailed Explanation:

Given $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$.

1. Calculate the Trace of A (sum of diagonal elements):

$$\text{Tr}(A) = 1 + 4 = 5$$

2. Calculate the Determinant of A:

$$\det(A) = (1)(4) - (2)(3) = 4 - 6 = -2$$

3. Write the characteristic equation based on the theorem:

$$\lambda^2 - 5\lambda + (-2) = 0 \implies \lambda^2 - 5\lambda - 2 = 0$$

According to Cayley-Hamilton theorem, the matrix A satisfies this:

$$A^2 - 5A - 2I = O$$

Where O is the null matrix.

Step 3: Final Answer:

The expression $A^2 - 5A - 2I$ results in the zero matrix O.

The correct option is (A).

Quick Tip

Quick Tip: Do not waste time computing A^2 manually. For any 2×2 matrix A, the identity $A^2 - (\text{Tr}\, A)A + (\det A)I = O$ always holds true. Identify trace and determinant to solve instantly.

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7. If the conjugate of a complex number z is $\frac{1}{z-i}$, then z can be:

(A) $i\left(\frac{1+\sqrt{5}}{2}\right)$
(B) $i\left(\frac{1-\sqrt{5}}{2}\right)$
(C) $\frac{1+i\sqrt{5}}{2}$
(D) $\frac{1-i\sqrt{5}}{2}$

Correct Answer: (A) $i\left(\frac{1+\sqrt{5}}{2}\right)$

Solution:

Step 1: Understanding the Concept:

A fundamental property of complex conjugates is that $z\bar{z} = |z|^2$, which is always a purely real number. We use this to simplify the given relation $\bar{z} = \frac{1}{z-i}$.

Detailed Explanation:

Given $\bar{z} = \frac{1}{z-i}$.

Multiplying both sides by $(z-i)$:

$$\bar{z}(z-i) = 1 \implies z\bar{z} - i\bar{z} = 1$$

Since $z\bar{z} = |z|^2$:

$$|z|^2 - i\bar{z} = 1 \implies i\bar{z} = |z|^2 - 1$$

The term $|z|^2 - 1$ is purely real. Thus, $i\bar{z}$ must be purely real.

Let $z = x + iy$, then $\bar{z} = x - iy$.

$$i(x - iy) = ix - i^2 y = y + ix$$

For $y + ix$ to be purely real, the imaginary part must be zero:

$$x = 0$$

This means z is a purely imaginary number, $z = iy$.
Substitute $z = iy$ back into the original equation:

$$-iy = \frac{1}{iy - i} \implies -iy = \frac{1}{i(y-1)}$$

Multiply both sides by $i(y-1)$:

$$-i^2 y(y-1) = 1 \implies y(y-1) = 1$$

$$y^2 - y - 1 = 0$$

Using the quadratic formula for y:

$$y = \frac{1 \pm \sqrt{1 - 4(1)(-1)}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

Thus, $z = iy = i\left(\frac{1 \pm \sqrt{5}}{2}\right)$.

Step 3: Final Answer:
Looking at the options, $i\left(\frac{1+\sqrt{5}}{2}\right)$ is a valid solution.
The correct option is (A).

Quick Tip

Quick Tip: Since $i\bar{z}$ = Real Number, $\bar{z}$ must be of the form $-i \times$ Real Number so that $i \times (-i) = 1$. This immediately tells you that z is purely imaginary. Options (C) and (D) can be ruled out at a glance.

8. If $z = \frac{\sqrt{3}+i}{2}$, then $z^{101} + z^{103} =$

(A) $-\sqrt{3}$

(B) $\sqrt{3}$

(C) $-i\sqrt{3}$

(D) $i\sqrt{3}$

Correct Answer: (A) $-\sqrt{3}$

Solution:

Step 1: Understanding the Concept:

To handle large powers of complex numbers, it is best to convert the number into polar (Euler) form: $z = re^{i\theta}$.

Key Formula or Approach:

Identify r and θ. Then use $e^{i\theta} = \cos\theta + i\sin\theta$.

Step 2: Detailed Explanation:

Given $z = \frac{\sqrt{3}}{2} + \frac{1}{2}i$.

Here, $r = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = 1$.

And $\tan\theta = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}} \implies \theta = \frac{\pi}{6}$.

So, $z = e^{i\pi/6}$.

The expression required is $z^{101} + z^{103}$.

Factor out z^{102}:

$$z^{101} + z^{103} = z^{102}(z^{-1} + z)$$

Calculation of z^{102}:

$$z^{102} = (e^{i\pi/6})^{102} = e^{i(102\pi/6)} = e^{i17\pi}$$

Since 17π is an odd multiple of π, $e^{i17\pi} = \cos(17\pi) + i\sin(17\pi) = -1$.

Calculation of $z + z^{-1}$:

$$z + z^{-1} = z + \bar{z} \text{ (since } |z| = 1)$$

$$z + \bar{z} = 2\text{Re}(z) = 2\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$$

Finally:

$$z^{101} + z^{103} = (-1)(\sqrt{3}) = -\sqrt{3}$$

Step 3: Final Answer:

The value of the expression is $-\sqrt{3}$.

The correct option is (A).

Quick Tip

Quick Tip: Notice that $z^6 = (e^{i\pi/6})^6 = e^{i\pi} = -1$.

Then $z^{102} = (z^6)^{17} = (-1)^{17} = -1$.

This simplification makes the power calculation much faster.

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9. If the roots of the quadratic equation $x^2 - 2px + q^2 = 0$ are real and distinct, then:

(A) $|p| > |q|$

(B) $|p| < |q|$

(C) $p^2 \geq q^2$

(D) $p^2 \leq q^2$

Correct Answer: (A) $|p| > |q|$

Solution:

Step 1: Understanding the Concept:

For a quadratic equation $ax^2 + bx + c = 0$, the nature of the roots is determined by the discriminant $D = b^2 - 4ac$.

- Roots are real and distinct if $D > 0$.
- Roots are real and equal if $D = 0$.
- Roots are non-real (complex) if $D < 0$.

Key Formula or Approach:

For the equation $x^2 - 2px + q^2 = 0$, we have $a = 1, b = -2p, c = q^2$.

Step 2: Detailed Explanation:

The condition for real and distinct roots is:

$$D > 0$$

$$(-2p)^2 - 4(1)(q^2) > 0$$

$$4p^2 - 4q^2 > 0$$

Dividing by 4:

$$p^2 - q^2 > 0 \implies p^2 > q^2$$

Taking the square root on both sides:

$$\sqrt{p^2} > \sqrt{q^2} \implies |p| > |q|$$

Step 3: Final Answer:

The condition for the roots to be real and distinct is $|p| > |q|$.

The correct option is (A).

Quick Tip

Quick Tip: Remember the phrase "distinct" implies a strict inequality ($>$). If the question said "real roots", the answer would have been $p^2 \geq q^2$. Always read the wording carefully to distinguish between $>$ and $\geq$.

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10. If α, β are the roots of the quadratic equation $x^2 - 2x + 4 = 0$, then the value of $\alpha^n + \beta^n$ is:

(A) $2^{n+1} \cos \frac{n\pi}{3}$

(B) $2^{n+1} \sin \frac{n\pi}{3}$

(C) $2^n \cos \frac{n\pi}{3}$

(D) $2^n \sin \frac{n\pi}{3}$

Correct Answer: (A) $2^{n+1} \cos \frac{n\pi}{3}$

Solution:

Step 1: Understanding the Concept:

To find the powers of the roots of a quadratic equation, it is efficient to solve the equation, convert roots to polar form, and apply De Moivre's Theorem $(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$.

Step 2: Detailed Explanation:

Given equation: $x^2 - 2x + 4 = 0$.

Using quadratic formula $x = \frac{2 \pm \sqrt{4-16}}{2} = \frac{2 \pm \sqrt{-12}}{2} = \frac{2 \pm 2i\sqrt{3}}{2} = 1 \pm i\sqrt{3}$.

So, $\alpha = 1 + i\sqrt{3}$ and $\beta = 1 - i\sqrt{3}$.

Convert α to polar form:

$r = \sqrt{1^2 + (\sqrt{3})^2} = 2$.

$\theta = \tan^{-1}(\sqrt{3}/1) = \frac{\pi}{3}$.

$$\alpha = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) \text{ and } \beta = 2\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)$$

Now, calculate $\alpha^n + \beta^n$:

$$\alpha^n = 2^n\left(\cos\frac{n\pi}{3} + i\sin\frac{n\pi}{3}\right)$$

$$\beta^n = 2^n\left(\cos\frac{n\pi}{3} - i\sin\frac{n\pi}{3}\right)$$

Adding them:

$$\alpha^n + \beta^n = 2^n\left(\cos\frac{n\pi}{3} + i\sin\frac{n\pi}{3} + \cos\frac{n\pi}{3} - i\sin\frac{n\pi}{3}\right)$$

$$\alpha^n + \beta^n = 2^n\left(2\cos\frac{n\pi}{3}\right) = 2^{n+1}\cos\frac{n\pi}{3}$$

Step 3: Final Answer:

The expression evaluates to $2^{n+1}\cos\frac{n\pi}{3}$.

The correct option is (A).

Quick Tip

Quick Tip: Test with $n = 1$:

$\alpha^1 + \beta^1 =$ sum of roots $= -b/a = 2$.

Check Option (A) with $n = 1$: $2^{1+1}\cos(\pi/3) = 4 \times 0.5 = 2$.

This confirms the answer without needing full derivation.

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11. If the roots of the equation $x^3 - 7x^2 + 14x - 8 = 0$ are in geometric progression, then the

common ratio can be:

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (B) 2

Solution:

Step 1: Understanding the Concept:

For a cubic equation $ax^3 + bx^2 + cx + d = 0$, if the roots are in Geometric Progression (GP), they can be represented as $\frac{a}{r}, a,$ and ar, where a is the middle term and r is the common ratio. We use Vieta's formulas relating the coefficients of the polynomial to the product and sum of its roots.

Key Formula or Approach:

The product of the roots is given by $-\frac{d}{a}$.

The sum of the roots is given by $-\frac{b}{a}$.

Step 2: Detailed Explanation:

The given equation is $x^3 - 7x^2 + 14x - 8 = 0$.

Let the roots be $\frac{a}{r}, a, ar$.

Using the product of roots:

$$\left(\frac{a}{r}\right) \cdot a \cdot (ar) = -(-8)$$

$$a^3 = 8 \implies a = 2$$

Now, using the sum of the roots:

$$\frac{a}{r} + a + ar = -(-7)$$

$$\frac{2}{r} + 2 + 2r = 7$$

$$\frac{2}{r} + 2r = 5$$

Multiplying by r to form a quadratic equation:

$$2 + 2r^2 = 5r \implies 2r^2 - 5r + 2 = 0$$

Factoring the quadratic:

$$2r^2 - 4r - r + 2 = 0$$

$$2r(r-2) - 1(r-2) = 0 \implies (2r-1)(r-2) = 0$$

This gives $r = 2$ or $r = \frac{1}{2}$.

Step 3: Final Answer:

The possible common ratio for the geometric progression of the roots is 2.

Quick Tip: If the roots of a cubic equation are in GP, the middle term a is always the cube root of the constant term (with sign changed if the leading coefficient is 1). Here, $\sqrt[3]{8} = 2$.

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12. If the number of permutations of n different things taken all at a time is 5040, then $n =$

(A) 5

(B) 6

(C) 7

(D) 8

Correct Answer: (C) 7

Solution:

Step 1: Understanding the Concept:

A permutation is an arrangement of objects in a specific order.

The number of ways to arrange n distinct objects taking all of them at a time is represented by $n!$ (n-factorial).

Step 2: Detailed Explanation:

We are given that the number of permutations is 5040.

According to the formula for permutations of n objects:

$$n! = 5040$$

To find n, we evaluate factorials of consecutive integers:

$$1! = 1$$

$$2! = 2$$

$$3! = 6$$

$$4! = 24$$

$$5! = 120$$

$$6! = 720$$

$$7! = 720 \times 7 = 5040$$

Since 7! equals 5040, the value of n must be 7.

Step 3: Final Answer:

The value of n is 7.

Quick Tip: Memorize the first few factorials: $5! = 120$, $6! = 720$, $7! = 5040$. This saves calculation time during competitive exams.

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13. If ${}^nC_{12} = {}^nC_8$, then ${}^nC_{17} =$

(A) 1140

(B) 1150

(C) 1160

(D) 1170

Correct Answer: (A) 1140

Solution:

Step 1: Understanding the Concept:

This problem uses the complement property of combinations: ${}^nC_x = {}^nC_y \implies x = y$ or $x + y = n$.

Key Formula or Approach:

First, determine n from the given equality ${}^nC_{12} = {}^nC_8$.

Then, calculate ${}^nC_{17}$ using the formula ${}^nC_r = \frac{n!}{r!(n-r)!}$.

Step 2: Detailed Explanation:

Given ${}^nC_{12} = {}^nC_8$.

Since $12 \neq 8$, we must have:

$$n = 12 + 8 = 20$$

Now, we need to find ${}^{20}C_{17}$.

Using the property ${}^nC_r = {}^nC_{n-r}$:

$${}^{20}C_{17} = {}^{20}C_{20-17} = {}^{20}C_3$$

Calculating the value:

$${}^{20}C_3 = \frac{20 \times 19 \times 18}{3 \times 2 \times 1}$$

$${}^{20}C_3 = 20 \times 19 \times 3 = 1140$$

Step 3: Final Answer:

The value of ${}^nC_{17}$ is 1140.

Quick Tip: Always simplify combinations using ${}^nC_r = {}^nC_{n-r}$ before computing to minimize arithmetic operations (${}^{20}C_{17} \to {}^{20}C_3$).

---

14. The number of terms in the expansion of $(x+y+z)^{10}$ is:

(A) 55

(B) 66

(C) 45

(D) 78

Correct Answer: (B) 66

Solution:

Step 1: Understanding the Concept:

The number of terms in a multinomial expansion follows a specific combinatorics formula. For an expression with r variables raised to the power n, the number of distinct terms in the expansion is given by the formula for combinations with repetition.

Key Formula or Approach:

The formula for the number of terms in the expansion of $(x_1 + x_2 + \cdots + x_r)^n$ is ${}^{n+r-1}C_{r-1}$.

Step 2: Detailed Explanation:

The given expression is $(x+y+z)^{10}$.

Here, the number of variables is $r = 3$ (which are $x, y,$ and z).

The power is $n = 10$.

Substitute these values into the formula:

$$\text{Number of terms} = {}^{10+3-1}C_{3-1} = {}^{12}C_2$$

Calculating the combination:

$$^{12}C_2 = \frac{12 \times 11}{2 \times 1}$$

$$^{12}C_2 = 6 \times 11 = 66$$

Step 3: Final Answer:

The number of terms in the expansion is 66.

Quick Tip: For a trinomial $(x + y + z)^n$, the number of terms simplifies to $\frac{(n+1)(n+2)}{2}$. For $n = 10$: $\frac{11 \times 12}{2} = 66$.

---

15. If $\frac{3x+4}{(x-1)(x-2)^2} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$, then $A + B + C =$

(A) 10

(B) 20

(C) 17

(D) 14

Correct Answer: (A) 10

Solution:

Step 1: Understanding the Concept:

Partial fraction decomposition allows us to break down a complex rational function into simpler parts.

By multiplying by the common denominator, we create an identity that holds true for all values of x, allowing us to solve for unknown coefficients.

Step 2: Detailed Explanation:

Given the identity:

$$\frac{3x+4}{(x-1)(x-2)^2} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

Multiply both sides by $(x-1)(x-2)^2$:

$$3x+4 = A(x-2)^2 + B(x-1)(x-2) + C(x-1)$$

To find A, substitute $x=1$:

$$3(1)+4 = A(1-2)^2 \implies 7 = A(1) \implies A = 7$$

To find C, substitute $x=2$:

$$3(2)+4 = C(2-1) \implies 10 = C(1) \implies C = 10$$

To find B, compare the coefficients of x^2 on both sides.

The coefficient of x^2 on the left is 0.

On the right, x^2 comes from $A(x-2)^2$ and $B(x-1)(x-2)$:

$$0 = A + B$$

$$B = -A = -7$$

Now, calculate the required sum $A+B+C$:

$$A+B+C = 7+(-7)+10 = 10$$

Step 3: Final Answer:

The sum $A+B+C$ is equal to 10.

Quick Tip: To find the sum of coefficients in a partial fraction identity of this type, you can sometimes substitute strategic values, but here, the direct determination of A, B, C is extremely fast.

---

16. If $\sin\theta+\cos\theta=\sqrt{2}\cos\theta$**, then** $\cos\theta-\sin\theta=$

(A) $\sqrt{2}\sin\theta$

(B) $-\sqrt{2}\sin\theta$

(C) $\sqrt{2}\cos\theta$

(D) $-\sqrt{2}\cos\theta$

Correct Answer: (A) $\sqrt{2}\sin\theta$

Solution:

Step 1: Understanding the Concept:

Trigonometric equations can be manipulated algebraically to find relationships between different combinations of sine and cosine functions.

Step 2: Detailed Explanation:

The given equation is $\sin\theta+\cos\theta=\sqrt{2}\cos\theta$.

Isolate $\sin\theta$:

$$\sin\theta=\sqrt{2}\cos\theta-\cos\theta$$

$$\sin\theta=(\sqrt{2}-1)\cos\theta$$

Multiply both sides of the equation by $(\sqrt{2}+1)$:

$$(\sqrt{2}+1)\sin\theta = (\sqrt{2}+1)(\sqrt{2}-1)\cos\theta$$

$$\sqrt{2}\sin\theta + \sin\theta = ((\sqrt{2})^2 - 1^2)\cos\theta$$

$$\sqrt{2}\sin\theta + \sin\theta = (2-1)\cos\theta$$

$$\sqrt{2}\sin\theta + \sin\theta = \cos\theta$$

Rearranging to find $\cos\theta - \sin\theta$:

$$\cos\theta - \sin\theta = \sqrt{2}\sin\theta$$

Step 3: Final Answer:

The expression $\cos\theta - \sin\theta$ is equal to $\sqrt{2}\sin\theta$.

Quick Tip: There is a famous identity: if $a\cos\theta + b\sin\theta = c$, then $b\cos\theta - a\sin\theta = \pm\sqrt{a^2+b^2-c^2}$. Here, $1^2 + 1^2 - (\sqrt{2}\cos\theta)^2 = 2 - 2\cos^2\theta = 2\sin^2\theta$, giving $\sqrt{2}\sin\theta$.

---

17. The maximum value of $3\sin x + 4\cos x + 5$ is:

(A) 5

(B) 10

(C) 12

(D) 15

Correct Answer: (B) 10

Solution:

Step 1: Understanding the Concept:

The expression of the form $a\sin x + b\cos x$ varies between a specific range.

The range of $f(x) = a\sin x + b\cos x$ is $[-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}]$.

Step 2: Detailed Explanation:

The given expression is $3\sin x + 4\cos x + 5$.

First, we find the maximum value of the part $3\sin x + 4\cos x$.

Here, $a = 3$ and $b = 4$.

$$\text{Maximum value of } (3\sin x + 4\cos x) = \sqrt{3^2+4^2}$$

$$= \sqrt{9+16} = \sqrt{25} = 5$$

Now, to find the maximum value of the entire expression, we add the constant term 5:

$$\text{Total Maximum Value} = 5 + 5 = 10$$

Step 3: Final Answer:

The maximum value of the function is 10.

Quick Tip: $(3, 4, 5)$ is a standard Pythagorean triple, so $\sqrt{3^2+4^2}$ is instantly 5. Add 5 to get 10.

---

18. If $\tan^{-1}(x) + \tan^{-1}(y) + \tan^{-1}(z) = \frac{\pi}{2}$**, then** $xy + yz + zx =$

(A) 0

(B) 1

(C) -1

(D) xyz

Correct Answer: (B) 1

Solution:

Step 1: Understanding the Concept:

We use the addition formula for three inverse tangent functions.

The sum of $\tan^{-1} x, \tan^{-1} y,$ and $\tan^{-1} z$ results in a new inverse tangent of a rational function.

Key Formula or Approach:

The identity is:

$$\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \tan^{-1}\left(\frac{x+y+z-xyz}{1-(xy+yz+zx)}\right)$$

Step 2: Detailed Explanation:

We are given:

$$\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}$$

Substituting the identity:

$$\tan^{-1}\left(\frac{x+y+z-xyz}{1-(xy+yz+zx)}\right) = \frac{\pi}{2}$$

For the tangent of an angle to approach $\frac{\pi}{2}$, the argument of the inverse tangent function must approach infinity.

A fraction approaches infinity when its denominator is zero.

Therefore:

$$1-(xy+yz+zx) = 0$$

$$xy + yz + zx = 1$$

Step 3: Final Answer:

The value of $xy + yz + zx$ is 1.

Quick Tip: If $\sum \tan^{-1} x = \pi/2$, then $\sum xy = 1$. If $\sum \tan^{-1} x = \pi$, then $\sum x = xyz$.

---

19. If $\sinh x = \frac{3}{4}$, then $\cosh 2x =$

(A) $\frac{17}{8}$

(B) $\frac{15}{8}$

(C) $\frac{9}{8}$

(D) $\frac{25}{8}$

Correct Answer: (A) $\frac{17}{8}$

Solution:

Step 1: Understanding the Concept:

Hyperbolic functions have double-angle identities similar to trigonometric functions.

To find $\cosh 2x$ from $\sinh x$, we use the specific identity relating these two functions.

Key Formula or Approach:

The double-angle formula for hyperbolic cosine is $\cosh 2x = 1 + 2\sinh^2 x$.

Step 2: Detailed Explanation:

Given $\sinh x = \frac{3}{4}$.

Substitute this value into the identity:

$$\cosh 2x = 1 + 2\left(\frac{3}{4}\right)^2$$

$$\cosh 2x = 1 + 2\left(\frac{9}{16}\right)$$

Simplify the expression:

$$\cosh 2x = 1 + \frac{9}{8}$$

$$\cosh 2x = \frac{8+9}{8} = \frac{17}{8}$$

Step 3: Final Answer:

The value of $\cosh 2x$ is $\frac{17}{8}$.

Quick Tip: Hyperbolic trigonometric identities are very similar to standard ones but watch out for sign differences ($\cosh^2 x - \sinh^2 x = 1$ and $\cosh 2x = 1 + 2\sinh^2 x$).

---

20. In a triangle ABC, if $a = 13$, $b = 14$, $c = 15$, then the area of the triangle is:

(A) 84

(B) 48

(C) 36

(D) 96

Correct Answer: (A) 84

Solution:

Step 1: Understanding the Concept:

The area of a triangle with three known side lengths can be calculated using Heron's Formula. This method involves calculating the semi-perimeter of the triangle first.

Key Formula or Approach:

Area $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{a+b+c}{2}$.

Step 2: Detailed Explanation:

Given side lengths: $a = 13, b = 14, c = 15$.

1. Calculate the semi-perimeter s:

$$s = \frac{13+14+15}{2} = \frac{42}{2} = 21$$

2. Substitute the values into Heron's Formula:

$$\Delta = \sqrt{21(21-13)(21-14)(21-15)}$$

$$\Delta = \sqrt{21 \times 8 \times 7 \times 6}$$

3. Simplify the product inside the square root by prime factorization:

$$\Delta = \sqrt{(3 \times 7) \times (2^3) \times 7 \times (2 \times 3)}$$

$$\Delta = \sqrt{3^2 \times 7^2 \times 2^4}$$

$$\Delta = 3 \times 7 \times 2^2$$

$$\Delta = 21 \times 4 = 84$$

Step 3: Final Answer:

The area of the triangle is 84 square units.

Quick Tip: A triangle with sides $13, 14, 15$ is a standard classic triangle in geometry problems; its area is always 84, and its semi-perimeter is 21. Remembering this saves valuable time.

---

21. If $\vec{a} = 2\vec{i} + 3\vec{j} - \vec{k}$, $\vec{b} = -\vec{i} + 2\vec{j} - 4\vec{k}$ and $\vec{c} = \vec{i} + \vec{j} + \vec{k}$, then $(\vec{a} \times \vec{b}) \cdot (\vec{a} \times \vec{c}) =$

(A) -74

(B) 74

(C) -42

(D) 42

Correct Answer: (A) -74

Solution:

Step 1: Understanding the Concept:

The dot product of two cross products can be computed efficiently using a vector identity rather than calculating the cross products individually. This identity relates the dot products of the involved vectors.

Key Formula or Approach:

The vector identity used is:

$$(\vec{a} \times \vec{b}) \cdot (\vec{u} \times \vec{v}) = (\vec{a} \cdot \vec{u})(\vec{b} \cdot \vec{v}) - (\vec{a} \cdot \vec{v})(\vec{b} \cdot \vec{u})$$

For this problem, we set $\vec{u} = \vec{a}$ and $\vec{v} = \vec{c}$.

Step 2: Detailed Explanation:

Given $\vec{a} = 2\vec{i} + 3\vec{j} - \vec{k}$, $\vec{b} = -\vec{i} + 2\vec{j} - 4\vec{k}$, and $\vec{c} = \vec{i} + \vec{j} + \vec{k}$.

The expression is $(\vec{a} \times \vec{b}) \cdot (\vec{a} \times \vec{c}) = (\vec{a} \cdot \vec{a})(\vec{b} \cdot \vec{c}) - (\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{a})$.

1. Calculate the dot products:

- $\vec{a} \cdot \vec{a} = 2^2 + 3^2 + (-1)^2 = 4 + 9 + 1 = 14$.
- $\vec{b} \cdot \vec{c} = (-1)(1) + (2)(1) + (-4)(1) = -1 + 2 - 4 = -3$.
- $\vec{a} \cdot \vec{c} = (2)(1) + (3)(1) + (-1)(1) = 2 + 3 - 1 = 4$.
- $\vec{b} \cdot \vec{a} = (-1)(2) + (2)(3) + (-4)(-1) = -2 + 6 + 4 = 8$.

2. Substitute into the identity:

$$\text{Value} = (14)(-3) - (4)(8)$$

$$\text{Value} = -42 - 32 = -74$$

Step 3: Final Answer:

The value of $(\vec{a} \times \vec{b}) \cdot (\vec{a} \times \vec{c})$ is -74.

Quick Tip: Using Lagrange's identity for cross products saves significant time compared to computing the cross products $\vec{a} \times \vec{b}$ and $\vec{a} \times \vec{c}$ individually.

---

22. Let $\vec{a}$ and $\vec{b}$ be two unit vectors. If the angle between them is θ, then $\cos(\theta/2) =$

(A) $\frac{1}{2}|\vec{a} + \vec{b}|$

(B) $\frac{1}{2}|\vec{a} - \vec{b}|$

(C) $|\vec{a} + \vec{b}|$

(D) $|\vec{a} - \vec{b}|$

Correct Answer: (A) $\frac{1}{2}|\vec{a}+\vec{b}|$

Solution:

Step 1: Understanding the Concept:

For unit vectors $\vec{a}$ and $\vec{b}$, the magnitude of their sum is directly related to the angle θ between them through the law of cosines and trigonometric half-angle identities.

Key Formula or Approach:

Magnitude squared: $|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b}$.

Half-angle identity: $1+\cos\theta = 2\cos^2(\theta/2)$.

Step 2: Detailed Explanation:

Since $\vec{a}$ and $\vec{b}$ are unit vectors, $|\vec{a}| = 1$ and $|\vec{b}| = 1$.

The dot product is $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta = \cos\theta$.

Substitute these into the magnitude formula:

$$|\vec{a}+\vec{b}|^2 = 1^2 + 1^2 + 2\cos\theta$$

$$|\vec{a}+\vec{b}|^2 = 2 + 2\cos\theta = 2(1+\cos\theta)$$

Using the identity $1+\cos\theta = 2\cos^2(\theta/2)$:

$$|\vec{a}+\vec{b}|^2 = 2\cdot 2\cos^2(\theta/2) = 4\cos^2(\theta/2)$$

Taking the square root:

$$|\vec{a}+\vec{b}| = 2\cos(\theta/2)$$

Rearrange to solve for $\cos(\theta/2)$:

$$\cos(\theta/2) = \frac{1}{2}|\vec{a} + \vec{b}|$$

Step 3: Final Answer:

The expression for $\cos(\theta/2)$ is $\frac{1}{2}|\vec{a} + \vec{b}|$.

Quick Tip: For unit vectors, remember the standard forms: $|\vec{a}+\vec{b}| = 2\cos(\theta/2)$ and $|\vec{a}-\vec{b}| = 2\sin(\theta/2)$.

---

23. If $\vec{a} = \vec{i} + \vec{j} + \vec{k}$, $\vec{b} = 4\vec{i} + 3\vec{j} + 4\vec{k}$ and $\vec{c} = \vec{i} + \alpha\vec{j} + \beta\vec{k}$ are linearly dependent vectors and $|\vec{c}| = \sqrt{3}$, then:

(A) $\alpha = 1, \beta = -1$

(B) $\alpha = 1, \beta = 1$

(C) $\alpha = -1, \beta = -1$

(D) $\alpha = 2, \beta = 1$

Correct Answer: (B) $\alpha = 1, \beta = 1$

Solution:

Step 1: Understanding the Concept:

Three vectors are linearly dependent in 3D space if and only if they are coplanar. This occurs when their scalar triple product, represented by the determinant of their components, is zero.

Step 2: Detailed Explanation:

Given $\vec{a} = \vec{i} + \vec{j} + \vec{k}$, $\vec{b} = 4\vec{i} + 3\vec{j} + 4\vec{k}$, and $\vec{c} = \vec{i} + \alpha\vec{j} + \beta\vec{k}$.

1. Apply linear dependency condition:

$$\begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 4 \\ 1 & \alpha & \beta \end{vmatrix} = 0$$

Expanding along the first row:

$$1(3\beta - 4\alpha) - 1(4\beta - 4) + 1(4\alpha - 3) = 0$$

$$3\beta - 4\alpha - 4\beta + 4 + 4\alpha - 3 = 0$$

$$-\beta + 1 = 0 \implies \beta = 1$$

2. Use the magnitude condition $|\vec{c}| = \sqrt{3}$:

$$\sqrt{1^2 + \alpha^2 + \beta^2} = \sqrt{3}$$

Squaring both sides:

$$1 + \alpha^2 + \beta^2 = 3$$

Substitute $\beta = 1$:

$$1 + \alpha^2 + 1^2 = 3 \implies \alpha^2 = 1 \implies \alpha = \pm 1$$

From the options, the set $\alpha = 1, \beta = 1$ is present.

Step 3: Final Answer:

The values are $\alpha = 1$ and $\beta = 1$.

Quick Tip: Linear dependency of three vectors in $\mathbb{R}^3$ is equivalent to coplanarity. Always set the determinant to 0.

24. The variance of the first n even natural numbers is:

(A) $\frac{n^2-1}{12}$

(B) $\frac{n^2-1}{3}$

(C) $\frac{n^2+1}{3}$

(D) $\frac{n^2-1}{4}$

Correct Answer: (B) $\frac{n^2-1}{3}$

Solution:

Step 1: Understanding the Concept:

The variance σ^2 of a set of data is the average of the squared deviations from the mean. It can be calculated using the formula $\frac{\sum x_i^2}{n} - (\bar{x})^2$.

Step 2: Detailed Explanation:

The first n even natural numbers are $2, 4, 6, \ldots, 2n$.

1. Find the Mean $(\bar{x})$:

$$\bar{x} = \frac{2+4+6+\cdots+2n}{n} = \frac{2(1+2+3+\cdots+n)}{n}$$

$$\bar{x} = \frac{2 \cdot \frac{n(n+1)}{2}}{n} = n+1$$

2. Find the sum of squares of the terms:

$$\sum x_i^2 = 2^2 + 4^2 + 6^2 + \cdots + (2n)^2 = 2^2(1^2 + 2^2 + \cdots + n^2)$$

$$\sum x_i^2 = 4 \cdot \frac{n(n+1)(2n+1)}{6} = \frac{2n(n+1)(2n+1)}{3}$$

3. Calculate Variance:

$$\sigma^2 = \frac{1}{n}\left[\frac{2n(n+1)(2n+1)}{3}\right] - (n+1)^2$$

$$\sigma^2 = \frac{2(n+1)(2n+1)}{3} - (n+1)^2$$

Factor out $(n+1)$:

$$\sigma^2 = (n+1)\left[\frac{4n+2-3(n+1)}{3}\right] = (n+1)\left[\frac{n-1}{3}\right] = \frac{n^2-1}{3}$$

Step 3: Final Answer:

The variance of the first n even natural numbers is $\frac{n^2-1}{3}$.

Quick Tip: Since the first n natural numbers have a variance of $\frac{n^2-1}{12}$, scaling them by 2 (even numbers) scales the variance by $2^2 = 4$. Thus, $4 \times \frac{n^2-1}{12} = \frac{n^2-1}{3}$.

---

25. If the mean deviation of the numbers $1, 1+d, 1+2d, \ldots, 1+100d$ from their mean is 255, then $d =$

(A) 10.1

(B) 10

(C) 5.05

(D) 5.1

Correct Answer: (A) 10.1

Solution:

Step 1: Understanding the Concept:

Mean deviation is the average of the absolute differences between each data point and the

mean. For an arithmetic progression (AP), the mean is the middle term if the number of terms is odd.

Step 2: Detailed Explanation:

The sequence is $1, 1+d, 1+2d, \ldots, 1+100d$.

This sequence has $N = 101$ terms.

1. The mean $\bar{x}$ is the 51^{st} term:

$$\bar{x} = a + 50d = 1 + 50d$$

2. Calculate deviations $|x_i - \bar{x}|$:

The deviations are $|50d|, |49d|, \ldots, |d|, 0, |d|, \ldots, |50d|$.

3. Sum of deviations:

$$\sum |x_i - \bar{x}| = 2 \cdot |d| \sum_{k=1}^{50} k = 2|d| \cdot \frac{50 \times 51}{2} = 2550|d|$$

4. Mean Deviation (MD):

$$MD = \frac{2550|d|}{101} = 255$$

Solving for $|d|$:

$$\frac{10|d|}{101} = 1 \implies |d| = \frac{101}{10} = 10.1$$

Step 3: Final Answer:

The value of the common difference d is 10.1.

Quick Tip: The mean deviation of an arithmetic progression of $(2n+1)$ terms with common difference d is $\frac{n(n+1)}{2n+1}|d|$. Here, $n = 50$, so $\frac{50 \times 51}{101}|d| = 255 \implies |d| = 10.1$.

26. A bag contains 5 red and 4 black balls. Three balls are drawn at random from the bag. The probability that two of them are red and one is black is:

(A) $\frac{5}{21}$

(B) $\frac{10}{21}$

(C) $\frac{5}{14}$

(D) $\frac{25}{84}$

Correct Answer: (B) $\frac{10}{21}$

Solution:

Step 1: Understanding the Concept:

The probability is calculated as the ratio of favorable outcomes to the total number of possible outcomes (classical definition).

Step 2: Detailed Explanation:

The bag contains 5 red and 4 black balls. Total balls $= 9$.

We draw 3 balls at random.

1. Total number of ways to choose 3 balls out of 9:

$$n(S) = {}^9C_3 = \frac{9 \times 8 \times 7}{3 \times 2 \times 1} = 84$$

2. Number of ways to choose exactly 2 red and 1 black ball:

- Choose 2 red balls from 5: ${}^5C_2 = 10$.

- Choose 1 black ball from 4: ${}^4C_1 = 4$.

- Total favorable outcomes $n(E) = 10 \times 4 = 40$.

3. Calculate Probability:

$$P(E) = \frac{n(E)}{n(S)} = \frac{40}{84}$$

Divide both by 4:

$$P(E) = \frac{10}{21}$$

Step 3: Final Answer:

The probability is $\frac{10}{21}$.

Quick Tip: Always simplify combinations first: ${}^5C_2 = 10$ and ${}^9C_3 = 84$. This makes dividing $\frac{40}{84}$ to get $\frac{10}{21}$ straightforward.

---

27. If A and B are two events such that $P(A) = 0.4$, $P(B) = 0.8$ and $P(B|A) = 0.6$, then $P(\bar{A} \cap B) =$

(A) 0.56

(B) 0.24

(C) 0.16

(D) 0.32

Correct Answer: (A) 0.56

Solution:

Step 1: Understanding the Concept:

This problem involves conditional probability and set theory identities. We need to find the probability of event B occurring without event A.

Key Formula or Approach:

Identity: $P(\bar{A} \cap B) = P(B) - P(A \cap B)$.

Conditional Prob: $P(A \cap B) = P(B|A) \cdot P(A)$.

Step 2: Detailed Explanation:

Given $P(A) = 0.4$, $P(B) = 0.8$, and $P(B|A) = 0.6$.

1. Calculate the intersection:

$$P(A \cap B) = 0.6 \times 0.4 = 0.24$$

2. Calculate the required probability:

$$P(\bar{A} \cap B) = P(B) - P(A \cap B)$$

$$P(\bar{A} \cap B) = 0.8 - 0.24 = 0.56$$

Step 3: Final Answer:

The value of $P(\bar{A} \cap B)$ is 0.56.

Quick Tip: $P(\bar{A} \cap B)$ represents the probability of "only B". Visualizing this on a Venn diagram makes the relation $P(B) - P(A \cap B)$ immediately clear.

---

28. Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others. The probability that all three apply for the same house is:

(A) $\frac{2}{9}$

(B) $\frac{1}{9}$

(C) $\frac{4}{9}$

(D) $\frac{1}{27}$

Correct Answer: (B) $\frac{1}{9}$

Solution:

Step 1: Understanding the Concept:

When people apply for houses independently, each person's choice is a separate event. The probability is the ratio of the number of favorable house-assignment combinations to the total possible combinations.

Step 2: Detailed Explanation:

There are 3 houses and 3 persons.

1. Total possible ways (each of 3 people can pick any of 3 houses):

$$n(S) = 3 \times 3 \times 3 = 3^3 = 27$$

2. Favorable ways (all three people choose the same house):

- They all pick House 1, OR they all pick House 2, OR they all pick House 3.

- $n(E) = 3$.

3. Calculate Probability:

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{27} = \frac{1}{9}$$

Step 3: Final Answer:

The probability that all three apply for the same house is $\frac{1}{9}$.

Quick Tip: Alternatively: the first person can choose any house (prob = 1). The second and third persons must choose that same house, each with probability 1/3. Total probability $= 1 \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$.

---

29. A random variable X has the following probability distribution:

$X = x$: 0, 1, 2, 3, 4, 5

$P(X = x)$: k, $3k$, $5k$, $7k$, $9k$, $11k$

Then the value of k is:

(A) $\frac{1}{36}$

(B) $\frac{1}{18}$

(C) $\frac{1}{12}$

(D) $\frac{1}{6}$

Correct Answer: (A) $\frac{1}{36}$

Solution:

Step 1: Understanding the Concept:

For any valid probability distribution, the sum of all individual probabilities across the entire sample space must equal 1.

Step 2: Detailed Explanation:

The random variable X has values $0, 1, 2, 3, 4, 5$.

The corresponding probabilities are $k, 3k, 5k, 7k, 9k, 11k$.

1. Sum the probabilities:

$$\sum P(X = x) = k + 3k + 5k + 7k + 9k + 11k$$

$$\sum P(X = x) = 36k$$

2. Set the sum to 1:

$$36k = 1$$

$$k = \frac{1}{36}$$

Step 3: Final Answer:

The value of k is $\frac{1}{36}$.

Quick Tip: The coefficients $1, 3, 5, 7, 9, 11$ are the first 6 odd natural numbers. The sum of the first n odd natural numbers is always n^2. Here, $6^2 = 36$, so $36k = 1 \implies k = 1/36$.

---

30. In a binomial distribution, the mean is 4 and the variance is 3. Then the number of trials n is:

(A) 8

(B) 12

(C) 16

(D) 20

Correct Answer: (C) 16

Solution:

Step 1: Understanding the Concept:

In a binomial distribution $B(n, p)$, the parameters are n (number of trials) and p (probability of success). The mean and variance are defined in terms of these parameters.

Key Formula or Approach:

Mean $= np$.

Variance $= npq$, where $q = 1 - p$.

Step 2: Detailed Explanation:

Given Mean $= 4$ and Variance $= 3$.

1. Find q:

$$\frac{\text{Variance}}{\text{Mean}} = \frac{npq}{np} = q$$

$$q = \frac{3}{4}$$

2. Find p:

$$p = 1 - q = 1 - \frac{3}{4} = \frac{1}{4}$$

3. Find n:

Substitute $p = \frac{1}{4}$ into the mean equation $np = 4$:

$$n \cdot \frac{1}{4} = 4$$

$$n = 16$$

Step 3: Final Answer:

The number of trials n is 16.

Quick Tip: In a binomial distribution, the variance is always less than the mean. The ratio Variance/Mean gives q directly, so $p = 1 - q$, and then $n = \text{Mean}/p$.

---

31. When the origin is shifted to (2,3) by translation of axes, the coordinates of a point P become (1,2). The original coordinates of P are:

(A) (3, 1)

(B) (1, 5)

(C) (3,5)

(D) (1, 5)

Correct Answer: (A) (3, 1)

Solution:

Step 1: Understanding the Concept:

Translation of axes involves shifting the origin to a new point without changing the direction

of the coordinate axes.

The relationship between original coordinates and translated (new) coordinates is linear.

Key Formula or Approach:

If the origin is shifted to (h, k), and the new coordinates are (x', y'), the original coordinates (x, y) are given by:

$$x = x' + h$$

$$y = y' + k$$

Step 2: Detailed Explanation:

In this problem, the new origin is $(h, k) = (2, 3)$.

The translated coordinates of point P are $(x', y') = (1, -2)$.

Substituting these values into the translation formulas:

For the x-coordinate:

$$x = 1 + 2 = 3$$

For the y-coordinate:

$$y = -2 + 3 = 1$$

The original coordinates are calculated by simply adding the shift to the new coordinates.

Step 3: Final Answer:

The original coordinates of the point P are $(3, 1)$.

This matches Option (A).

Quick Tip

Quick Tip: To find original coordinates, remember the intuitive rule: Original = New + Shift.
To find new coordinates, use: New = Original − Shift.

---

32. If the straight lines $x+2y-9=0$, $3x+5y-5=0$ and $ax+by-1=0$ are concurrent, then the straight line $22x-35y=1$ passes through the point:

(A) (a, b)

(B) (b, a)

(C) (a,b)

(D) (b,a)

Correct Answer: (B) (b, a)

Solution:

Step 1: Understanding the Concept:

Three lines are concurrent if they all pass through the same point.

We find the intersection of the first two lines and substitute it into the third line to find a relationship between a and b.

Key Formula or Approach:

1. Solve the system of equations for the first two lines.
2. Substitute the intersection point (x, y) into $ax+by-1=0$.
3. Compare the resulting equation with the line $22x-35y=1$.

Step 2: Detailed Explanation:

Consider the lines:

(1) $x+2y=9$

(2) $3x+5y=5$

From (1), $x=9-2y$. Substitute this into (2):

$$3(9-2y)+5y=5$$

$$27 - 6y + 5y = 5 \implies -y = -22 \implies y = 22$$

Substituting $y = 22$ back into (1):

$$x = 9 - 2(22) = 9 - 44 = -35$$

The point of intersection is $(-35, 22)$.

Since the third line $ax + by - 1 = 0$ is concurrent, it must pass through $(-35, 22)$:

$$a(-35) + b(22) - 1 = 0 \implies 22b - 35a = 1$$

Now, we look at the line $22x - 35y = 1$.

If we substitute $x = b$ and $y = a$, we get:

$$22(b) - 35(a) = 1$$

This is exactly the relationship we derived from the concurrency condition.

Step 3: Final Answer:

The line $22x - 35y = 1$ passes through the point (b, a).

This matches Option (B).

Quick Tip

Quick Tip: Once you get the relation $22b - 35a = 1$, look at the equation of the line $22x - 35y = 1$. By direct comparison, x corresponds to b and y corresponds to a.

---

33. The distance between the parallel lines $5x + 12y - 3 = 0$ and $5x + 12y + 10 = 0$ is:

(A) 1

(B) 2

(C) $\frac{13}{17}$

(D) $\frac{7}{13}$

Correct Answer: (A) 1

Solution:

Step 1: Understanding the Concept:

The distance between two parallel lines is the constant perpendicular distance between them.

Parallel lines have the same coefficients for x and y.

Key Formula or Approach:

The distance d between parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is:

$$d = \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$

Step 2: Detailed Explanation:

The given equations are $5x + 12y - 3 = 0$ and $5x + 12y + 10 = 0$.

Here, $a = 5$, $b = 12$, $c_1 = -3$, and $c_2 = 10$.

Substitute these values into the formula:

$$d = \frac{|-3 - 10|}{\sqrt{5^2 + 12^2}}$$

$$d = \frac{|-13|}{\sqrt{25 + 144}}$$

$$d = \frac{13}{\sqrt{169}}$$

$$d = \frac{13}{13} = 1$$

Step 3: Final Answer:

The distance between the parallel lines is 1 unit.

This matches Option (A).

Quick Tip

Quick Tip: Identify the Pythagorean triplet $(5, 12, 13)$.

This means $\sqrt{5^2 + 12^2}$ is instantly 13.

The difference in constants is $10 - (-3) = 13$.

Distance $= 13/13 = 1$.

---

34. If the angle between the pair of lines $x^2 - 2cxy - 7y^2 = 0$ is $\frac{\pi}{3}$, then the value of c^2 is:

(A) 20

(B) 10

(C) 5

(D) 15

Correct Answer: (A) 20

Solution:

Step 1: Understanding the Concept:

A second-degree homogeneous equation $ax^2 + 2hxy + by^2 = 0$ represents a pair of lines passing through the origin.

The angle θ between these lines depends on the coefficients $a, b,$ and h.

Key Formula or Approach:

The angle θ is given by:

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{|a + b|}$$

Step 2: Detailed Explanation:

Given the equation $x^2 - 2cxy - 7y^2 = 0$.

Comparing with $ax^2 + 2hxy + by^2 = 0$:

$a = 1$, $b = -7$, and $2h = -2c \implies h = -c$.

The angle $\theta = \frac{\pi}{3} = 60°$.

Substitute these into the formula:

$$\tan(60°) = \frac{2\sqrt{(-c)^2 - (1)(-7)}}{|1 + (-7)|}$$

$$\sqrt{3} = \frac{2\sqrt{c^2 + 7}}{|-6|}$$

$$\sqrt{3} = \frac{2\sqrt{c^2 + 7}}{6} = \frac{\sqrt{c^2 + 7}}{3}$$

Cross-multiply and square both sides:

$$3\sqrt{3} = \sqrt{c^2 + 7}$$

$$(3\sqrt{3})^2 = c^2 + 7$$

$$27 = c^2 + 7$$

$$c^2 = 20$$

Step 3: Final Answer:

The value of c^2 is 20.

This matches Option (A).

Quick Tip

Quick Tip: Always simplify the denominator $|a+b|$ before squaring to keep calculations clean.

Squaring at the last step prevents unnecessary large numbers.

---

35. If the lines joining the origin to the points of intersection of the line $y = mx + 1$ and the circle $x^2 + y^2 = 1$ are perpendicular to each other, then the value of m^2 is:

(A) 1

(B) 2

(C) 0

(D) 3

Correct Answer: (A) 1

Solution:

Step 1: Understanding the Concept:

To find the equation of lines joining the origin to intersection points of a curve and a line, we use the process of homogenization.

We make the curve's equation a homogeneous second-degree equation using the line's equation.

Key Formula or Approach:

1. Rewrite the line as $y - mx = 1$.
2. Homogenize the circle equation $x^2 + y^2 = 1$ using this relationship.
3. Use the condition for perpendicular lines: Coefficient of x^2 + Coefficient of $y^2 = 0$.

Step 2: Detailed Explanation:

The given circle is $x^2 + y^2 = 1$.

The line is $y = mx + 1 \implies y - mx = 1$.

Homogenizing the circle equation:

$$x^2 + y^2 = 1 \cdot (1)^2$$

Substitute $1 = y - mx$:

$$x^2 + y^2 = (y - mx)^2$$

$$x^2 + y^2 = y^2 + m^2x^2 - 2mxy$$

Rearrange to set equal to zero:

$$x^2 - m^2x^2 + 2mxy = 0$$

$$x^2(1 - m^2) + 2mxy + 0y^2 = 0$$

For the pair of lines to be perpendicular:

$$\text{Coefficient of } x^2 + \text{Coefficient of } y^2 = 0$$

$$(1 - m^2) + 0 = 0$$

$$m^2 = 1$$

Step 3: Final Answer:

The value of m^2 is 1.

This matches Option (A).

Quick Tip

Quick Tip: In homogenization problems, if the circle is $x^2 + y^2 = a^2$ and the line is $lx + my = n$, the perpendicularity condition often leads to a simple relationship between coefficients.

Always verify if the constant term in the line is 1 to simplify homogenization.

---

36. If the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ passes through the origin, has radius 3, and its center lies on the line $x + y = 4$, then $g + f =$

(A) 4

(B) 4

(C) 2

(D) 2

Correct Answer: (A) 4

Solution:

Step 1: Understanding the Concept:

The general equation of a circle is $x^2 + y^2 + 2gx + 2fy + c = 0$.

The coordinates of its center are $(-g, -f)$.

Key Formula or Approach:

The center $(-g, -f)$ must satisfy the equation of the line it lies on.

Step 2: Detailed Explanation:

The problem states that the center of the circle lies on the line $x + y = 4$.

Substitute $x = -g$ and $y = -f$ into the line equation:

$$(-g) + (-f) = 4$$

Factor out the negative sign:

$$-(g+f)=4$$

Multiply by -1 on both sides:

$$g+f=-4$$

Note: While the radius and passing through the origin determine the values of $g, f,$ and c, the value of $g+f$ is directly determined by the center's location on the line.

Step 3: Final Answer:

The value of $g+f$ is -4.

This matches Option (A).

Quick Tip

Quick Tip: Don't get distracted by extra data! The question only asks for $g+f$, which is immediately found from the line equation containing the center. Using the radius or origin conditions is unnecessary for this specific query.

---

37. If the circle $x^2+y^2-4x-6y+\lambda=0$ touches the x-axis, then the value of λ is:

(A) 4

(B) 9

(C) 13

(D) 16

Correct Answer: (A) 4

Solution:

Step 1: Understanding the Concept:

When a circle touches the x-axis, the length of its intercept on the x-axis is zero.

This implies a specific relationship between the constants in the circle's general equation.

Key Formula or Approach:

For a circle $x^2 + y^2 + 2gx + 2fy + c = 0$, the condition to touch the x-axis is:

$$g^2 = c$$

Step 2: Detailed Explanation:

Given equation: $x^2 + y^2 - 4x - 6y + \lambda = 0$.

Comparing with the general equation:

$2g = -4 \implies g = -2$.

$2f = -6 \implies f = -3$.

$c = \lambda$.

Using the condition $g^2 = c$:

$$(-2)^2 = \lambda$$

$$4 = \lambda$$

$$\lambda = 4$$

Step 3: Final Answer:

The value of λ is 4.

This matches Option (A).

Quick Tip

Quick Tip: Remember the conditions:

Touching x-axis: $g^2 = c$.

Touching y-axis: $f^2 = c$.

Touching both axes: $g^2 = f^2 = c$.

---

38. The equation of the common chord of the circles $x^2 + y^2 - 4x - 4y = 0$ and $x^2 + y^2 - 6x - 8y + 10 = 0$ is:

(A) $x + 2y - 5 = 0$

(B) $2x + y - 5 = 0$

(C) $x - 2y + 5 = 0$

(D) $2x - y + 5 = 0$

Correct Answer: (A) $x + 2y - 5 = 0$

Solution:

Step 1: Understanding the Concept:

The common chord of two intersecting circles is the straight line passing through their points of intersection.

Mathematically, it represents the locus where the powers of a point with respect to both circles are equal.

Key Formula or Approach:

The equation of the common chord is given by:

$$S_1 - S_2 = 0$$

where $S_1 = 0$ and $S_2 = 0$ are the equations of the circles with coefficients of x^2 and y^2 as unity.

Step 2: Detailed Explanation:

Let $S_1 = x^2 + y^2 - 4x - 4y = 0$.

Let $S_2 = x^2 + y^2 - 6x - 8y + 10 = 0$.

Subtract S_2 from S_1:

$$(x^2 + y^2 - 4x - 4y) - (x^2 + y^2 - 6x - 8y + 10) = 0$$

Cancel the quadratic terms:

$$-4x - 4y + 6x + 8y - 10 = 0$$

Group like terms:

$$(-4x + 6x) + (-4y + 8y) - 10 = 0$$

$$2x + 4y - 10 = 0$$

Divide the entire equation by 2:

$$x + 2y - 5 = 0$$

Step 3: Final Answer:

The equation of the common chord is $x + 2y - 5 = 0$.

This matches Option (A).

Quick Tip

Quick Tip: Always ensure the coefficients of x^2 and y^2 are exactly 1 in both equations before subtracting. If one has $2x^2$, divide that whole circle equation by 2 first.

39. If the circle $x^2+y^2+2x-2y+c=0$ cuts the circle $x^2+y^2-4x-6y+11=0$ orthogonally, then the value of c is:

(A) 9

(B) 9

(C) 13

(D) 13

Correct Answer: (A) 9

Solution:

Step 1: Understanding the Concept:

Two circles are said to be orthogonal if their tangents at the points of intersection are perpendicular.

This results in a specific algebraic condition involving their parameters $g, f,$ and c.

Key Formula or Approach:

The condition for two circles to cut each other orthogonally is:

$$2g_1g_2+2f_1f_2=c_1+c_2$$

Step 2: Detailed Explanation:

Circle 1: $x^2+y^2+2x-2y+c=0$.

Parameters: $g_1=1, f_1=-1, c_1=c$.

Circle 2: $x^2+y^2-4x-6y+11=0$.

Parameters: $g_2=-2, f_2=-3, c_2=11$.

Apply the orthogonality condition:

$$2(1)(-2)+2(-1)(-3)=c+11$$

$$-4+6=c+11$$

$$2 = c + 11$$

$$c = 2 - 11$$

$$c = -9$$

Step 3: Final Answer:

The value of c is -9.

This matches Option (A).

Quick Tip

Quick Tip: Be very careful with signs when identifying g and f.

Remember that $2gx$ and $2fy$ are used in the formula, so g is half the coefficient of x.

---

40. The equation of the parabola with focus at (3, 0) and directrix $x + 3 = 0$ is:

(A) $y^2 = 12x$

(B) $y^2 = -12x$

(C) $x^2 = 12y$

(D) $x^2 = -12y$

Correct Answer: (A) $y^2 = 12x$

Solution:

Step 1: Understanding the Concept:

A parabola is defined as the locus of points equidistant from a fixed point (focus) and a fixed

line (directrix).

The axis of the parabola is the line passing through the focus and perpendicular to the directrix.

Key Formula or Approach:

For a focus at $(a, 0)$ and directrix $x = -a$, the standard equation is:

$$y^2 = 4ax$$

Step 2: Detailed Explanation:

The given focus is $(3, 0)$, which lies on the x-axis.

The given directrix is $x + 3 = 0 \implies x = -3$.

Comparing with the standard forms:

$a = 3$.

Since the focus is on the positive x-axis and the directrix is $x = -a$, this is a rightward-opening parabola.

Substitute $a = 3$ into the standard equation:

$$y^2 = 4(3)x$$

$$y^2 = 12x$$

Verification: The vertex is the midpoint between the focus $(3, 0)$ and the point on the directrix $(-3, 0)$, which is $(0, 0)$. This confirms the standard form.

Step 3: Final Answer:

The equation of the parabola is $y^2 = 12x$.

This matches Option (A).

Quick Tip

Quick Tip: If the focus is $(a, 0)$ and the directrix is $x = -a$, the equation is always $y^2 = 4ax$.
If focus is $(0, a)$ and directrix $y = -a$, the equation is $x^2 = 4ay$.

---

41. If the distance between the foci of an ellipse is 6 and the distance between its directrices is 12, then the length of its latus rectum is:

(A) $3\sqrt{2}$
(B) $\sqrt{2}$
(C) $2\sqrt{2}$
(D) $4\sqrt{2}$

Correct Answer: (A) $3\sqrt{2}$

Solution:

Step 1: Understanding the Concept:

In a standard ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (where $a > b$), the distance between the foci is given by $2ae$ and the distance between the directrices is given by $\frac{2a}{e}$.
The length of the latus rectum is defined as $\frac{2b^2}{a}$.

Key Formula or Approach:

1. Use the given distances to find a and e.
2. Use the relation $b^2 = a^2(1 - e^2)$ to find b^2.
3. Calculate the latus rectum using $\frac{2b^2}{a}$.

Step 2: Detailed Explanation:

Given:
Distance between foci: $2ae = 6 \implies ae = 3$ (Eq. 1)
Distance between directrices: $\frac{2a}{e} = 12 \implies \frac{a}{e} = 6$ (Eq. 2)
Multiply Eq. 1 and Eq. 2:

$$(ae) \cdot \left(\frac{a}{e}\right) = 3 \times 6 \implies a^2 = 18 \implies a = \sqrt{18} = 3\sqrt{2}$$

Divide Eq. 1 by Eq. 2:

$$\frac{ae}{a/e} = \frac{3}{6} \implies e^2 = \frac{1}{2}$$

Now, find b^2 using the ellipse property:

$$b^2 = a^2(1-e^2) = 18\left(1-\frac{1}{2}\right) = 18\left(\frac{1}{2}\right) = 9$$

Length of latus rectum:

$$L.R. = \frac{2b^2}{a} = \frac{2(9)}{3\sqrt{2}} = \frac{18}{3\sqrt{2}} = \frac{6}{\sqrt{2}} = 3\sqrt{2}$$

Step 3: Final Answer:

The length of the latus rectum is $3\sqrt{2}$.

This matches Option (A).

Quick Tip

Quick Tip: The product of focal distance $(2ae)$ and directrix distance $(2a/e)$ is always $4a^2$.

Calculation: $6 \times 12 = 72 = 4a^2 \implies a^2 = 18$. This is a faster way to find a.

---

42. The line $y = mx + c$ touches the ellipse $9x^2 + 16y^2 = 144$ if the value of c^2 is:

(A) $16m^2 + 9$

(B) $9m^2 + 16$

(C) $16m^2 - 9$

(D) $9m^2 - 16$

Correct Answer: (A) $16m^2 + 9$

Solution:

Step 1: Understanding the Concept:

For a line $y = mx + c$ to be a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, it must satisfy the condition of tangency.

Key Formula or Approach:

The condition of tangency for an ellipse is:

$$c^2 = a^2m^2 + b^2$$

Step 2: Detailed Explanation:

The given equation is $9x^2 + 16y^2 = 144$.

Divide the entire equation by 144 to convert it to standard form:

$$\frac{9x^2}{144} + \frac{16y^2}{144} = 1$$

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

Comparing with standard form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$:

$a^2 = 16$ and $b^2 = 9$.

Now, substitute these into the condition of tangency:

$$c^2 = (16)m^2 + 9$$

$$c^2 = 16m^2 + 9$$

Step 3: Final Answer:

The line touches the ellipse if $c^2 = 16m^2 + 9$.

This matches Option (A).

Quick Tip

Quick Tip: Always reduce the ellipse equation to standard form before identifying a^2 and b^2.

In $Ax^2 + By^2 = C$, $a^2 = C/A$ and $b^2 = C/B$.

---

43. If the eccentricity of a hyperbola is $\sqrt{3}$, then the eccentricity of its conjugate hyperbola is:

(A) $\sqrt{\frac{3}{2}}$

(B) $\sqrt{3}$

(C) $\frac{\sqrt{3}}{2}$

(D) $\sqrt{2}$

Correct Answer: (A) $\sqrt{\frac{3}{2}}$

Solution:

Step 1: Understanding the Concept:

A hyperbola and its conjugate hyperbola share a fixed relationship between their eccentricities.

Key Formula or Approach:

If e_1 and e_2 are the eccentricities of a hyperbola and its conjugate hyperbola respectively, then:

$$\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1$$

Step 2: Detailed Explanation:

Given eccentricity of hyperbola $e_1 = \sqrt{3}$.

Substitute e_1 into the formula:

$$\frac{1}{(\sqrt{3})^2} + \frac{1}{e_2^2} = 1$$

$$\frac{1}{3} + \frac{1}{e_2^2} = 1$$

Isolate the term with e_2:

$$\frac{1}{e_2^2} = 1 - \frac{1}{3} = \frac{2}{3}$$

Take the reciprocal of both sides:

$$e_2^2 = \frac{3}{2}$$

Take the square root:

$$e_2 = \sqrt{\frac{3}{2}}$$

Step 3: Final Answer:

The eccentricity of the conjugate hyperbola is $\sqrt{\frac{3}{2}}$.

This matches Option (A).

Quick Tip

Quick Tip: Memory aid: For any hyperbola, $e_2 = \frac{e_1}{\sqrt{e_1^2 - 1}}$.

Using $e_1 = \sqrt{3}$: $e_2 = \frac{\sqrt{3}}{\sqrt{3-1}} = \frac{\sqrt{3}}{\sqrt{2}} = \sqrt{\frac{3}{2}}$.

---

44. The ratio in which the xy-plane divides the line segment joining the points (2,4,5) and (3, 5,4) is:

(A) $5:4$ internally

(B) $5:4$ externally

(C) $4:5$ internally

(D) $4:5$ externally

Correct Answer: (A) $5:4$ internally

Solution:

Step 1: Understanding the Concept:

When a coordinate plane divides a line segment joining two points (x_1, y_1, z_1) and (x_2, y_2, z_2), the ratio is determined by the coordinates perpendicular to that plane.

For the xy-plane, the perpendicular coordinate is z.

Key Formula or Approach:

The xy-plane (equation $z = 0$) divides the segment in the ratio:

$$m : n = -z_1 : z_2$$

If the ratio is positive, the division is internal. If negative, it is external.

Step 2: Detailed Explanation:

The given points are $P(2, 4, 5)$ and $Q(3, 5, -4)$.

Here, $z_1 = 5$ and $z_2 = -4$.

Substitute into the ratio formula:

$$\text{Ratio} = -5 : (-4)$$

$$\text{Ratio} = 5 : 4$$

Since the resulting ratio $\frac{5}{4}$ is a positive value, the division must be internal.

Physically, this means the points are on opposite sides of the xy-plane (one has positive z, the

other has negative z).

Step 3: Final Answer:

The ratio is $5:4$ internally.

This matches Option (A).

Quick Tip

Quick Tip: Plane yz: Ratio $-x_1 : x_2$.

Plane zx: Ratio $-y_1 : y_2$.

Plane xy: Ratio $-z_1 : z_2$.

---

45. If a line makes angles 45° and 60° with the positive x and y axes respectively, then the acute angle it makes with the z-axis is:

(A) 60°

(B) 30°

(C) 45°

(D) 90°

Correct Answer: (A) 60°

Solution:

Step 1: Understanding the Concept:

The direction cosines l, m, n of a line are defined as the cosines of the angles α, β, γ that the line makes with the coordinate axes.

Key Formula or Approach:

The fundamental identity for direction cosines is:

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

Step 2: Detailed Explanation:

Given:

Angle with x-axis: $\alpha = 45^\circ$

Angle with y-axis: $\beta = 60^\circ$

Let the angle with z-axis be γ.

Using the identity:

$$\cos^2(45^\circ) + \cos^2(60^\circ) + \cos^2\gamma = 1$$

Substitute the values $\cos 45^\circ = \frac{1}{\sqrt{2}}$ and $\cos 60^\circ = \frac{1}{2}$:

$$\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + \cos^2\gamma = 1$$

$$\frac{1}{2} + \frac{1}{4} + \cos^2\gamma = 1$$

$$\frac{3}{4} + \cos^2\gamma = 1$$

$$\cos^2\gamma = 1 - \frac{3}{4} = \frac{1}{4}$$

Take the square root:

$$\cos\gamma = \pm\frac{1}{2}$$

Since we need the acute angle, we take the positive root:

$$\cos\gamma = \frac{1}{2} \implies \gamma = 60^\circ$$

Step 3: Final Answer:

The angle with the z-axis is 60°.

This matches Option (A).

Quick Tip

Quick Tip: Direction cosines satisfy $l^2 + m^2 + n^2 = 1$.

Common angles to remember:

$\cos^2 30° = 3/4$, $\cos^2 45° = 1/2$, $\cos^2 60° = 1/4$.

---

46. The acute angle between the planes $2x - y + z = 6$ and $x + y + 2z = 3$ is:

(A) $\frac{\pi}{3}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{6}$

(D) $\frac{\pi}{2}$

Correct Answer: (A) $\frac{\pi}{3}$

Solution:

Step 1: Understanding the Concept:

The angle between two planes is equal to the angle between their normal vectors.

The coefficients of $x, y,$ and z in the plane equation form the components of the normal vector.

Key Formula or Approach:

For planes $\vec{n_1} \cdot \vec{r} = d_1$ and $\vec{n_2} \cdot \vec{r} = d_2$, the angle θ is:

$$\cos\theta = \frac{|\vec{n_1} \cdot \vec{n_2}|}{|\vec{n_1}||\vec{n_2}|}$$

Step 2: Detailed Explanation:

Plane 1: $2x - y + z = 6 \implies \vec{n_1} = (2, -1, 1)$.

Plane 2: $x + y + 2z = 3 \implies \vec{n_2} = (1, 1, 2)$.

1. Calculate dot product:

$$\vec{n_1} \cdot \vec{n_2} = (2)(1) + (-1)(1) + (1)(2) = 2 - 1 + 2 = 3$$

2. Calculate magnitudes:

$$|\vec{n_1}| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{6}$$

$$|\vec{n_2}| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{6}$$

3. Compute $\cos\theta$:

$$\cos\theta = \frac{3}{\sqrt{6} \cdot \sqrt{6}} = \frac{3}{6} = \frac{1}{2}$$

For $\cos\theta = 1/2$, the acute angle θ is 60° or $\frac{\pi}{3}$.

Step 3: Final Answer:

The acute angle between the planes is $\frac{\pi}{3}$.

This matches Option (A).

Quick Tip

Quick Tip: Always take the absolute value of the dot product to ensure you find the "acute" angle. If the result is negative, it represents the obtuse angle between the planes.

---

47. The value of $\lim_{x\to\infty} \left(\frac{x+6}{x+1}\right)^{x+4}$ is:

(A) e^5

(B) e^6

(C) e

(D) e^4

Correct Answer: (A) e^5

Solution:

Step 1: Understanding the Concept:

This limit is of the indeterminate form 1^∞.

As $x \to \infty$, the base $\frac{x+6}{x+1} \to 1$ and the exponent $x+4 \to \infty$.

Key Formula or Approach:

For a limit $\lim[f(x)]^{g(x)}$ of the form 1^∞, the value is:

$$e^{\lim[f(x)-1]g(x)}$$

Step 2: Detailed Explanation:

Let $f(x) = \frac{x+6}{x+1}$ and $g(x) = x+4$.

1. Calculate $f(x)-1$:

$$f(x)-1 = \frac{x+6}{x+1} - 1 = \frac{x+6-(x+1)}{x+1} = \frac{5}{x+1}$$

2. Calculate the limit in the exponent:

$$L = \lim_{x\to\infty} \left(\frac{5}{x+1}\right)(x+4)$$

$$L = \lim_{x\to\infty} \frac{5x+20}{x+1}$$

Divide numerator and denominator by x:

$$L = \lim_{x\to\infty} \frac{5+20/x}{1+1/x} = \frac{5+0}{1+0} = 5$$

3. The final result is $e^L = e^5$.

Step 3: Final Answer:

The limit value is e^5.

This matches Option (A).

Quick Tip

Quick Tip: Shortcut: For $\lim_{x\to\infty}\left(\frac{x+a}{x+b}\right)^{x+c}$, the result is always e^{a-b}.
Here $a=6, b=1$, so $e^{6-1}=e^5$. This saves significant calculation time!

---

48. If $f(x)=\frac{k\cos x}{\pi-2x}$ for $x\neq\frac{\pi}{2}$ and $f\left(\frac{\pi}{2}\right)=3$ is continuous at $x=\frac{\pi}{2}$, then the value of k is:

(A) 6

(B) 3

(C) 2

(D) 1.5

Correct Answer: (A) 6

Solution:

Step 1: Understanding the Concept:

A function $f(x)$ is continuous at a point a if the limit as $x\to a$ exists and is equal to the functional value $f(a)$.

Here, we must have $\lim_{x\to\pi/2} f(x) = f(\pi/2) = 3$.

Key Formula or Approach:

Use L'Hôpital's Rule for the limit as it results in the indeterminate form $0/0$.

Step 2: Detailed Explanation:

Set up the limit condition:

$$\lim_{x\to\frac{\pi}{2}} \frac{k\cos x}{\pi-2x} = 3$$

Since $\cos(\pi/2) = 0$ and $\pi - 2(\pi/2) = 0$, we apply L'Hôpital's Rule (differentiate top and bottom):

$$\lim_{x\to\frac{\pi}{2}} \frac{\frac{d}{dx}(k\cos x)}{\frac{d}{dx}(\pi-2x)} = 3$$

$$\lim_{x\to\frac{\pi}{2}} \frac{-k\sin x}{-2} = 3$$

$$\frac{k\sin(\pi/2)}{2} = 3$$

Substitute $\sin(\pi/2) = 1$:

$$\frac{k(1)}{2} = 3 \implies k = 6$$

Step 3: Final Answer:

The value of k is 6.

This matches Option (A).

Quick Tip

Quick Tip: Alternatively, use substitution $x = \pi/2 - h$.

$\cos(\pi/2 - h) = \sin h$ and $\pi - 2(\pi/2 - h) = 2h$.

Limit becomes $\lim_{h\to 0} \frac{k\sin h}{2h} = \frac{k}{2}\cdot 1 = 3 \implies k = 6$.

49. If $y = \tan^{-1}\left(\frac{\sin x+\cos x}{\cos x-\sin x}\right)$, then $\frac{dy}{dx} =$

(A) 1

(B) -1

(C) 1/2

(D) 0

Correct Answer: (A) 1

Solution:

Step 1: Understanding the Concept:

Differentiating inverse trigonometric functions is much easier if the internal expression can be simplified using trigonometric identities first.

Key Formula or Approach:

Use the identity: $\tan(\frac{\pi}{4} + x) = \frac{1+\tan x}{1-\tan x} = \frac{\cos x+\sin x}{\cos x-\sin x}$.

Step 2: Detailed Explanation:

Given:

$$y = \tan^{-1}\left(\frac{\sin x + \cos x}{\cos x - \sin x}\right)$$

Divide numerator and denominator by $\cos x$:

$$y = \tan^{-1}\left(\frac{\frac{\sin x}{\cos x} + \frac{\cos x}{\cos x}}{\frac{\cos x}{\cos x} - \frac{\sin x}{\cos x}}\right)$$

$$y = \tan^{-1}\left(\frac{\tan x + 1}{1 - \tan x}\right)$$

Using the tangent addition formula:

$$y = \tan^{-1}\left[\tan\left(\frac{\pi}{4} + x\right)\right]$$

Simplifying the expression:

$$y = \frac{\pi}{4} + x$$

Now, differentiate with respect to x:

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\pi}{4}\right) + \frac{d}{dx}(x)$$

$$\frac{dy}{dx} = 0 + 1 = 1$$

Step 3: Final Answer:

The derivative $\frac{dy}{dx}$ is equal to 1.

This matches Option (A).

Quick Tip

Quick Tip: Always look to convert terms involving $\sin x \pm \cos x$ into a single tangent term. It usually results in a linear function of x, making differentiation trivial.

---

50. If $x = a\cos^3 t$ and $y = a\sin^3 t$, then the value of $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$ is:

(A) -1

(B) 1

(C) $-\sqrt{3}$

(D) $\frac{1}{\sqrt{3}}$

Correct Answer: (A) -1

Solution:

Step 1: Understanding the Concept:

For functions given in parametric form $x = f(t)$ and $y = g(t)$, the derivative $\frac{dy}{dx}$ is calculated as the ratio of their derivatives with respect to the parameter t.

Key Formula or Approach:

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

Step 2: Detailed Explanation:

Given parametric equations:

$x = a\cos^3 t$

$y = a\sin^3 t$

1. Differentiate x with respect to t:

$$\frac{dx}{dt} = a \cdot 3\cos^2 t \cdot \frac{d}{dt}(\cos t) = -3a\cos^2 t \sin t$$

2. Differentiate y with respect to t:

$$\frac{dy}{dt} = a \cdot 3\sin^2 t \cdot \frac{d}{dt}(\sin t) = 3a\sin^2 t \cos t$$

3. Calculate the ratio:

$$\frac{dy}{dx} = \frac{3a\sin^2 t \cos t}{-3a\cos^2 t \sin t}$$

$$\frac{dy}{dx} = -\frac{\sin t}{\cos t} = -\tan t$$

4. Evaluate at $t = \frac{\pi}{4}$:

$$\left(\frac{dy}{dx}\right)_{t=\pi/4} = -\tan\left(\frac{\pi}{4}\right) = -1$$

Step 3: Final Answer:

The value of the derivative at the given point is -1.

This matches Option (A).

Quick Tip

Quick Tip: This set of equations represents an "astroid". The derivative of an astroid is always $-\tan t$ (or $-\sqrt[3]{y/x}$). Knowing this standard form can save time in exams.

---

51. If $y = e^{a\sin^{-1}x}$, then $(1-x^2)y_2 - xy_1 =$

(A) a^2y

(B) $-a^2y$

(C) ay

(D) $-ay$

Correct Answer: (A) a^2y

Solution:

Step 1: Understanding the Concept:

This problem involves successive differentiation of a function to find a relationship between its first derivative (y_1), second derivative (y_2), and the function itself.

This is a standard procedure in differential equations derived from algebraic or transcendental functions.

Key Formula or Approach:

1. Find the first derivative y_1 using the chain rule.
2. Rearrange and square the terms to eliminate the square root and radical parts.

3. Differentiate again with respect to x to find y_2.

Step 2: Detailed Explanation:

Given: $y = e^{a\sin^{-1}x}$.

Differentiating with respect to x:

$$y_1 = \frac{dy}{dx} = e^{a\sin^{-1}x} \cdot \frac{d}{dx}(a\sin^{-1}x)$$

$$y_1 = e^{a\sin^{-1}x} \cdot \frac{a}{\sqrt{1-x^2}}$$

Substitute y back into the derivative:

$$y_1 = \frac{ay}{\sqrt{1-x^2}}$$

Cross-multiply and square both sides to simplify:

$$y_1\sqrt{1-x^2} = ay$$

$$y_1^2(1-x^2) = a^2y^2$$

Differentiating both sides with respect to x using the product rule and power rule:

$$\frac{d}{dx}[y_1^2(1-x^2)] = \frac{d}{dx}(a^2y^2)$$

$$y_1^2(-2x) + (1-x^2)(2y_1y_2) = a^2(2yy_1)$$

Divide the entire equation by $2y_1$ (assuming $y_1 \neq 0$):

$$-xy_1 + (1-x^2)y_2 = a^2y$$

$$(1-x^2)y_2 - xy_1 = a^2y$$

Step 3: Final Answer:

The value of the expression is a^2y.

This matches Option (A).

Quick Tip

Quick Tip: For functions of the form $y = e^{af(x)}$, always try to substitute y back into the derivative expression immediately. Squaring before the second differentiation is a powerful trick to eliminate square roots from the denominator.

---

52. The slope of the normal to the curve $y = 2x^2 + 3\sin x$ at $x = 0$ is:

(A) $-\frac{1}{3}$

(B) $\frac{1}{3}$

(C) -3

(D) 3

Correct Answer: (A) $-\frac{1}{3}$

Solution:

Step 1: Understanding the Concept:

The slope of the tangent to a curve at a point is given by the derivative $\frac{dy}{dx}$ evaluated at that point.

The normal is a line perpendicular to the tangent. Therefore, the product of their slopes is -1.

Key Formula or Approach:

1. Find $\frac{dy}{dx}$.
2. Evaluate $m_t = \left(\frac{dy}{dx}\right)_{x=x_0}$.
3. Calculate the slope of the normal $m_n = -\frac{1}{m_t}$.

Step 2: Detailed Explanation:

Given curve: $y = 2x^2 + 3\sin x$.

Differentiate with respect to x:

$$\frac{dy}{dx} = \frac{d}{dx}(2x^2) + \frac{d}{dx}(3\sin x)$$

$$\frac{dy}{dx} = 4x + 3\cos x$$

Now, find the slope of the tangent at $x = 0$:

$$m_t = 4(0) + 3\cos(0)$$

$$m_t = 0 + 3(1) = 3$$

The slope of the normal is the negative reciprocal of the slope of the tangent:

$$m_n = -\frac{1}{m_t} = -\frac{1}{3}$$

Step 3: Final Answer:

The slope of the normal to the curve at $x = 0$ is $-\frac{1}{3}$.

This matches Option (A).

Quick Tip

Quick Tip: Always double-check if the question asks for the slope of the "tangent" or the "normal". Many students find the derivative and select that value (3 in this case), forgetting to take the negative reciprocal for the normal.

---

53. A balloon, which always remains spherical, has a variable radius. The rate at which its volume is increasing with respect to its radius r when $r = 5$ cm is:

(A) 100π

(B) 50π

(C) 25π

(D) 10π

Correct Answer: (A) 100π

Solution:

Step 1: Understanding the Concept:

This problem asks for the "rate of change of volume with respect to radius". This is mathematically equivalent to finding the derivative of the volume function V with respect to the variable r, denoted as $\frac{dV}{dr}$.

Key Formula or Approach:

The volume of a sphere is given by:

$$V = \frac{4}{3}\pi r^3$$

Step 2: Detailed Explanation:

We need to find the rate of change of V with respect to r:

$$\frac{dV}{dr} = \frac{d}{dr}\left(\frac{4}{3}\pi r^3\right)$$

Since $\frac{4}{3}\pi$ is a constant, we differentiate r^3:

$$\frac{dV}{dr} = \frac{4}{3}\pi \cdot (3r^2)$$

$$\frac{dV}{dr} = 4\pi r^2$$

Note that the rate of change of volume with respect to radius is exactly the surface area of the sphere.

Now, evaluate this at $r = 5$ cm:

$$\left(\frac{dV}{dr}\right)_{r=5} = 4\pi(5)^2$$

$$\left(\frac{dV}{dr}\right)_{r=5} = 4\pi(25) = 100\pi$$

Step 3: Final Answer:

The rate at which the volume is increasing with respect to its radius is 100π.

This matches Option (A).

Quick Tip

Quick Tip: Remember the geometric relationship: the derivative of the volume of a sphere with respect to its radius is its surface area ($4\pi r^2$), and the derivative of the area of a circle with respect to its radius is its circumference ($2\pi r$).

---

54. The minimum value of the function $f(x) = x^2 + \frac{250}{x}$ for $x > 0$ is:

(A) 75

(B) 50

(C) 25

(D) 100

Correct Answer: (A) 75

Solution:

Step 1: Understanding the Concept:

To find the minimum value of a function, we identify its critical points by setting the first derivative to zero and then verify the nature of the point using the second derivative test.

Key Formula or Approach:

1. Find $f'(x)$ and solve $f'(x) = 0$.
2. Check $f''(x) > 0$ at the critical point for a minimum.
3. Substitute the critical value of x into $f(x)$ to find the minimum value.

Step 2: Detailed Explanation:

Given: $f(x) = x^2 + 250x^{-1}$.

Differentiate with respect to x:

$$f'(x) = 2x - 250x^{-2} = 2x - \frac{250}{x^2}$$

To find critical points, set $f'(x) = 0$:

$$2x - \frac{250}{x^2} = 0 \implies 2x = \frac{250}{x^2}$$

$$2x^3 = 250 \implies x^3 = 125 \implies x = 5$$

Verification using second derivative:

$$f''(x) = 2 - 250(-2)x^{-3} = 2 + \frac{500}{x^3}$$

At $x = 5$, $f''(5) = 2 + \frac{500}{125} = 2 + 4 = 6$.

Since $f''(5) > 0$, the function has a local minimum at $x = 5$.

Calculating the minimum value:

$$f(5) = (5)^2 + \frac{250}{5}$$

$$f(5) = 25 + 50 = 75$$

Step 3: Final Answer:

The minimum value of the function is 75.

This matches Option (A).

Quick Tip

Quick Tip: For $x > 0$, you can use the AM-GM Inequality.
Split $x^2 + \frac{250}{x}$ as $x^2 + \frac{125}{x} + \frac{125}{x}$.
AM $\geq$ GM: $\frac{x^2+\frac{125}{x}+\frac{125}{x}}{3} \geq \sqrt[3]{x^2 \cdot \frac{125}{x} \cdot \frac{125}{x}}$.
$\frac{f(x)}{3} \geq \sqrt[3]{125^2} = 25 \implies f(x) \geq 75$.

---

55. The integral $\int \frac{1}{\cos^2 x(1-\tan x)^2} dx =$

(A) $\frac{1}{1-\tan x} + C$
(B) $-\frac{1}{1-\tan x} + C$
(C) $\frac{1}{(1-\tan x)^2} + C$
(D) $-\frac{1}{(1-\tan x)^2} + C$

Correct Answer: (A) $\frac{1}{1-\tan x} + C$

Solution:

Step 1: Understanding the Concept:

The integral involves trigonometric functions. The presence of $\cos^2 x$ in the denominator

suggests that it can be rewritten as $\sec^2 x$, which is the derivative of $\tan x$. This indicates that integration by substitution is the best approach.

Key Formula or Approach:

Use the identity $\frac{1}{\cos^2 x} = \sec^2 x$.

Substitution: Let $u = 1 - \tan x$.

Step 2: Detailed Explanation:

Rewrite the integral:

$$I = \int \frac{\sec^2 x}{(1-\tan x)^2} dx$$

Let $u = 1 - \tan x$.

Differentiating both sides:

$$du = -\sec^2 x \, dx \implies \sec^2 x \, dx = -du$$

Substitute into the integral:

$$I = \int \frac{-du}{u^2} = -\int u^{-2} \, du$$

Using the power rule for integration $\int x^n dx = \frac{x^{n+1}}{n+1}$:

$$I = -\left(\frac{u^{-1}}{-1}\right) + C$$

$$I = \frac{1}{u} + C$$

Now, substitute the value of u back:

$$I = \frac{1}{1-\tan x} + C$$

Step 3: Final Answer:

The indefinite integral evaluates to $\frac{1}{1-\tan x} + C$.

This matches Option (A).

Quick Tip

Quick Tip: Always look for a "function-derivative" pair in the integrand. If you see $\tan x$ and $\sec^2 x$ (or $1/\cos^2 x$), it is almost always a substitution problem.

---

56. The integral $\int \frac{e^x(1+x)}{\cos^2(xe^x)}dx =$

(A) $\tan(xe^x) + C$

(B) $-\tan(xe^x) + C$

(C) $\cot(xe^x) + C$

(D) $-\cot(xe^x) + C$

Correct Answer: (A) $\tan(xe^x) + C$

Solution:

Step 1: Understanding the Concept:

This is an integration by substitution problem. The argument of the trigonometric function is a product xe^x. We check if its derivative is present elsewhere in the integrand.

Key Formula or Approach:

Use the product rule for differentiation: $\frac{d}{dx}(uv) = uv' + vu'$.

Substitution: Let $u = xe^x$.

Step 2: Detailed Explanation:

Let $u = xe^x$.

Differentiating both sides with respect to x:

$$\frac{du}{dx} = x \cdot e^x + e^x \cdot 1$$

$$du = e^x(x+1)\,dx$$

This matches the numerator of the original integral exactly.

Rewrite the integral with the substitution:

$$I = \int \frac{du}{\cos^2 u}$$

$$I = \int \sec^2 u\, du$$

The standard integral of $\sec^2 u$ is $\tan u$:

$$I = \tan u + C$$

Substitute $u = xe^x$ back into the expression:

$$I = \tan(xe^x) + C$$

Step 3: Final Answer:

The result of the integration is $\tan(xe^x) + C$.

This matches Option (A).

Quick Tip

Quick Tip: Recognizing the derivative of xe^x as $e^x(x+1)$ is a very common requirement in calculus exams. Memorizing this specific product-derivative pair saves time.

---

57. The value of the definite integral $\int_0^{\pi/2} \frac{\sin^{3/2} x}{\sin^{3/2} x+\cos^{3/2} x} dx$ is:

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{2}$

(C) π

(D) 0

Correct Answer: (A) $\frac{\pi}{4}$

Solution:

Step 1: Understanding the Concept:

For definite integrals with limits from 0 to a, the property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ (often called the King's property) is extremely useful for simplifying ratios of symmetric trigonometric functions.

Step 2: Detailed Explanation:

Let $I = \int_0^{\pi/2} \frac{\sin^{3/2} x}{\sin^{3/2} x+\cos^{3/2} x} dx$ – (1)

Using the property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$:

$$I = \int_0^{\pi/2} \frac{\sin^{3/2}(\frac{\pi}{2}-x)}{\sin^{3/2}(\frac{\pi}{2}-x)+\cos^{3/2}(\frac{\pi}{2}-x)} dx$$

Using trigonometric identities $\sin(\frac{\pi}{2}-x) = \cos x$ and $\cos(\frac{\pi}{2}-x) = \sin x$:

$$I = \int_0^{\pi/2} \frac{\cos^{3/2} x}{\cos^{3/2} x + \sin^{3/2} x} dx$$

– (2)

Add equation (1) and equation (2):

$$2I = \int_0^{\pi/2} \left[\frac{\sin^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x} + \frac{\cos^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x} \right] dx$$

$$2I = \int_0^{\pi/2} \frac{\sin^{3/2} x + \cos^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x} dx$$

$$2I = \int_0^{\pi/2} 1\, dx$$

$$2I = [x]_0^{\pi/2} = \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$

Step 3: Final Answer:

The value of the definite integral is $\frac{\pi}{4}$.

This matches Option (A).

Quick Tip

Quick Tip: For any integral of the form $\int_0^{\pi/2} \frac{f(\sin x)}{f(\sin x)+f(\cos x)} dx$, the answer is always half the upper limit, i.e., $\frac{\pi}{4}$. The power (3/2) does not affect the final result.

---

58. The value of the definite integral $\int_{-\pi/2}^{\pi/2} (x^3 + x\cos x + \tan^5 x + 1) dx$ is:

(A) π

(B) $\frac{\pi}{2}$

(C) 0

(D) 2π

Correct Answer: (A) π

Solution:

Step 1: Understanding the Concept:

When integrating over a symmetric interval $[-a, a]$, we can use the properties of even and odd functions:

- $\int_{-a}^{a} f(x)dx = 0$ if $f(x)$ is an odd function ($f(-x) = -f(x)$).
- $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$ if $f(x)$ is an even function ($f(-x) = f(x)$).

Step 2: Detailed Explanation:

The integrand is $f(x) = x^3 + x\cos x + \tan^5 x + 1$.

Let's check the nature of each term:

1. $g_1(x) = x^3$: $g_1(-x) = (-x)^3 = -x^3 = -g_1(x)$. (Odd)
2. $g_2(x) = x\cos x$: $g_2(-x) = (-x)\cos(-x) = -x\cos x = -g_2(x)$. (Odd)
3. $g_3(x) = \tan^5 x$: $g_3(-x) = [\tan(-x)]^5 = (-\tan x)^5 = -\tan^5 x = -g_3(x)$. (Odd)
4. $g_4(x) = 1$: This is a constant term, which is an even function.

Integrating over $[-\pi/2, \pi/2]$:

$$I = \int_{-\pi/2}^{\pi/2} x^3 dx + \int_{-\pi/2}^{\pi/2} x\cos x dx + \int_{-\pi/2}^{\pi/2} \tan^5 x dx + \int_{-\pi/2}^{\pi/2} 1 dx$$

Since the first three integrals are of odd functions over a symmetric interval, they evaluate to 0:

$$I = 0 + 0 + 0 + \int_{-\pi/2}^{\pi/2} 1 dx$$

Evaluate the remaining constant integral:

$$I = [x]_{-\pi/2}^{\pi/2} = \frac{\pi}{2} - \left(-\frac{\pi}{2}\right) = \frac{\pi}{2} + \frac{\pi}{2} = \pi$$

Step 3: Final Answer:

The value of the definite integral is π.

This matches Option (A).

Quick Tip

Quick Tip: Recognize odd powers of x, $\sin x$, and $\tan x$ immediately as odd functions. In any sum being integrated over $[-a, a]$, these terms vanish completely, leaving only the even terms and constants to be calculated.

---

59. The area (in square units) of the region bounded by the parabola $y^2 = 4x$ and the line $y = 2x$ is:

(A) $\frac{1}{3}$
(B) $\frac{1}{6}$
(C) $\frac{2}{3}$
(D) $\frac{4}{3}$

Correct Answer: (A) $\frac{1}{3}$

Solution:

Step 1: Understanding the Concept:

The area between two curves is found by integrating the difference between the upper function and the lower function between their intersection points.

Key Formula or Approach:

Area $= \int_a^b |y_{upper} - y_{lower}| dx$.

First, find intersection points by solving the equations simultaneously.

Step 2: Detailed Explanation:

Equations: $y^2 = 4x$ and $y = 2x$.

Solving for x:

Substitute $y = 2x$ into the parabola equation:

$$(2x)^2 = 4x \implies 4x^2 = 4x$$

$$4x^2 - 4x = 0 \implies 4x(x-1) = 0$$

$$x = 0, x = 1$$

The intersection points are $(0,0)$ and $(1,2)$.

Between $x = 0$ and $x = 1$, for $x = 0.5$:

$y_{parabola} = \sqrt{4(0.5)} = \sqrt{2} \approx 1.41$

$y_{line} = 2(0.5) = 1$

Since $\sqrt{4x} > 2x$ in this interval, the parabola is the upper curve.

Area $A = \int_0^1 (\sqrt{4x} - 2x)dx$

$$A = \int_0^1 (2x^{1/2} - 2x)dx$$

Integrating:

$$A = \left[2 \cdot \frac{x^{3/2}}{3/2} - 2 \cdot \frac{x^2}{2}\right]_0^1$$

$$A = \left[\frac{4}{3}x^{3/2} - x^2\right]_0^1$$

$$A = \left(\frac{4}{3}(1) - (1)\right) - (0 - 0)$$

$$A = \frac{4}{3} - 1 = \frac{1}{3}$$

Step 3: Final Answer:

The area of the bounded region is $\frac{1}{3}$ square units.

This matches Option (A).

Quick Tip

Quick Tip: Shortcut: The area bounded by $y^2 = 4ax$ and $y = mx$ is given by $\frac{8a^2}{3m^3}$.
Here $4a = 4 \implies a = 1$ and $m = 2$.
Area $= \frac{8(1)^2}{3(2)^3} = \frac{8}{3 \cdot 8} = \frac{1}{3}$.

---

60. The order and degree of the differential equation $[1 + (\frac{dy}{dx})^2]^{3/2} = \frac{d^2y}{dx^2}$ are respectively:

(A) 2 and 2

(B) 2 and 3

(C) 1 and 2

(D) 2 and 1

Correct Answer: (A) 2 and 2

Solution:

Step 1: Understanding the Concept:

- **Order**: The order of a differential equation is the order of the highest derivative present in the equation.
- **Degree**: The degree is the power of the highest order derivative, provided the equation is in the form of a polynomial in derivatives (i.e., cleared of radicals and fractional powers).

Step 2: Detailed Explanation:

The given equation is:

$$[1 + (\frac{dy}{dx})^2]^{3/2} = \frac{d^2y}{dx^2}$$

This equation contains a fractional power 3/2. To define the degree, we must convert it into a polynomial form by squaring both sides:

$$\left([1+(\frac{dy}{dx})^2]^{3/2}\right)^2 = \left(\frac{d^2y}{dx^2}\right)^2$$

$$[1+(\frac{dy}{dx})^2]^3 = \left(\frac{d^2y}{dx^2}\right)^2$$

Now, let's analyze the derivatives:

1. The highest derivative present is $\frac{d^2y}{dx^2}$, which is of the second order. Thus, **Order = 2**.
2. The power of this highest derivative in the polynomial form is 2. Thus, **Degree = 2**.

Step 3: Final Answer:

The order and degree are 2 and 2 respectively.

This matches Option (A).

Quick Tip

Quick Tip: Always clear fractional indices first before determining the degree. The order is determined simply by looking at the equation, but the degree requires the equation to be free of radicals affecting the derivatives.

---

61. The general solution of the differential equation $\frac{dy}{dx} + y\cot x = 2\cos x$ is:

(A) $y\sin x = -\frac{1}{2}\cos 2x + C$

(B) $y\sin x = \frac{1}{2}\cos 2x + C$

(C) $y\sin x = -\cos 2x + C$

(D) $y\sin x = \sin 2x + C$

Correct Answer: (A) $y\sin x = -\frac{1}{2}\cos 2x + C$

Solution:

Step 1: Understanding the Concept:

This is a first-order linear differential equation of the standard form $\frac{dy}{dx} + P(x)y = Q(x)$.

The general solution for such equations is found by determining an Integrating Factor (I.F.) and applying it to both sides.

Key Formula or Approach:

1. Integrating Factor I.F. $= e^{\int P(x)dx}$.
2. The solution is $y \cdot (\text{I.F.}) = \int Q(x) \cdot (\text{I.F.})dx + C$.

Step 2: Detailed Explanation:

Given the equation: $\frac{dy}{dx} + (\cot x)y = 2\cos x$.

Here, $P(x) = \cot x$ and $Q(x) = 2\cos x$.

1. Find the Integrating Factor:

$$\text{I.F.} = e^{\int \cot x dx} = e^{\ln|\sin x|} = \sin x$$

2. Write the general solution:

$$y \cdot \sin x = \int (2\cos x)(\sin x)dx + C$$

3. Use the trigonometric identity $2\sin x \cos x = \sin 2x$:

$$y \sin x = \int \sin 2x dx + C$$

4. Integrate the right-hand side:

$$y \sin x = -\frac{\cos 2x}{2} + C$$

Step 3: Final Answer:

The general solution is $y \sin x = -\frac{1}{2}\cos 2x + C$.

This matches Option (A).

Quick Tip

Quick Tip: Standard integral $\int \cot x\,dx = \ln\sin x$ and $\int \tan x\,dx = \ln\sec x$.
Recognizing these identities instantly simplifies finding the integrating factor.

---

62. The integrating factor of the differential equation $(1+x^2)\frac{dy}{dx} + 2xy = \cos x$ is:

(A) e^{x^2}

(B) $1+x^2$

(C) $\ln(1+x^2)$

(D) $\frac{1}{1+x^2}$

Correct Answer: (B) $1+x^2$

Solution:

Step 1: Understanding the Concept:

To find the integrating factor, a linear differential equation must first be written in its standard form: $\frac{dy}{dx} + P(x)y = Q(x)$.

Key Formula or Approach:

Standardize the equation by dividing by the coefficient of $\frac{dy}{dx}$.
The Integrating Factor is I.F. $= e^{\int P(x)dx}$.

Step 2: Detailed Explanation:

The given equation is $(1+x^2)\frac{dy}{dx} + 2xy = \cos x$.
Divide the entire equation by $(1+x^2)$:

$$\frac{dy}{dx} + \left(\frac{2x}{1+x^2}\right)y = \frac{\cos x}{1+x^2}$$

Identifying $P(x)$:

$$P(x) = \frac{2x}{1+x^2}$$

Now, calculate the Integrating Factor:

$$\text{I.F.} = e^{\int \frac{2x}{1+x^2} dx}$$

To solve the integral $\int \frac{2x}{1+x^2} dx$, use substitution $u = 1 + x^2 \implies du = 2x dx$:

$$\int \frac{du}{u} = \ln u = \ln(1 + x^2)$$

Substitute this back into the power of e:

$$\text{I.F.} = e^{\ln(1+x^2)} = 1 + x^2$$

Step 3: Final Answer:

The integrating factor is $1 + x^2$.

This matches Option (B).

Quick Tip

Quick Tip: Shortcut: If the coefficient of y is the derivative of the coefficient of $\frac{dy}{dx}$, the integrating factor is simply the coefficient of $\frac{dy}{dx}$.

Here, $\frac{d}{dx}(1 + x^2) = 2x$, so I.F. $= 1 + x^2$.

---

63. The foot of the perpendicular from the point (1, 3) to the line $x + y - 2 = 0$ is:

(A) (0, 2)

(B) (1, 1)

(C) (2, 0)

(D) (1, 3)

Correct Answer: (A) (0, 2)

Solution:

Step 1: Understanding the Concept:

The foot of the perpendicular (h, k) from a point (x_1, y_1) to a line $ax + by + c = 0$ is the point on the line that is closest to (x_1, y_1).

Key Formula or Approach:

The coordinates (h, k) are given by:

$$\frac{h - x_1}{a} = \frac{k - y_1}{b} = -\frac{ax_1 + by_1 + c}{a^2 + b^2}$$

Step 2: Detailed Explanation:

Given point $(x_1, y_1) = (1, 3)$.

Line: $x + y - 2 = 0 \implies a = 1, b = 1, c = -2$.

1. Calculate the constant ratio:

$$\text{Ratio} = -\frac{1(1) + 1(3) - 2}{1^2 + 1^2} = -\frac{1 + 3 - 2}{2} = -\frac{2}{2} = -1$$

2. Solve for h:

$$\frac{h - 1}{1} = -1 \implies h - 1 = -1 \implies h = 0$$

3. Solve for k:

$$\frac{k - 3}{1} = -1 \implies k - 3 = -1 \implies k = 2$$

The point is $(0, 2)$.

Step 3: Final Answer:

The coordinates of the foot of the perpendicular are $(0, 2)$.

This matches Option (A).

Quick Tip

Quick Tip: Verification Trick: The foot of the perpendicular must lie on the given line.

Check Option (A): $0+2-2=0$. (Satisfied)

Check Option (B): $1+1-2=0$. (Satisfied)

Since multiple options satisfy the line, calculate the slope of line PQ. It must be $\frac{b}{a}$.

---

64. The length of the tangent from the point (3, 4) to the circle $x^2+y^2-2x-4y+1=0$ is:

(A) 2

(B) 4

(C) $\sqrt{2}$

(D) 3

Correct Answer: (A) 2

Solution:

Step 1: Understanding the Concept:

The length of a tangent from an external point to a circle is calculated by substituting the point's coordinates into the normalized circle equation and taking the square root.

Key Formula or Approach:

If the circle is $S \equiv x^2+y^2+2gx+2fy+c=0$, the length is $\sqrt{S_{11}}$.

$$\text{Length} = \sqrt{x_1^2+y_1^2+2gx_1+2fy_1+c}$$

Step 2: Detailed Explanation:

Given point: $(3,4)$.

Circle equation: $x^2+y^2-2x-4y+1=0$.

1. Substitute $x=3$ and $y=4$ into the equation:

$$S_{11} = (3)^2 + (4)^2 - 2(3) - 4(4) + 1$$

$$S_{11} = 9 + 16 - 6 - 16 + 1$$

$$S_{11} = 25 - 6 - 16 + 1$$

$$S_{11} = 19 - 16 + 1 = 4$$

2. Take the square root:

$$\text{Length} = \sqrt{4} = 2$$

Step 3: Final Answer:

The length of the tangent is 2 units.

This matches Option (A).

Quick Tip

Quick Tip: Always ensure the coefficients of x^2 and y^2 are 1 before using the $\sqrt{S_{11}}$ formula. If the equation was $2x^2 + 2y^2 \ldots$, divide the whole expression by 2 first.

---

65. The number of solutions of the trigonometric equation $\sin^2 x - \sin x - 2 = 0$ in the interval $[0, 2\pi]$ is:

(A) 1

(B) 2

(C) 3

(D) 0

Correct Answer: (A) 1

Solution:

Step 1: Understanding the Concept:

This is a quadratic equation in terms of $\sin x$. We treat $\sin x$ as a variable, find its possible values, and then determine how many angles x in the given range satisfy those values.

Step 2: Detailed Explanation:

Let $t = \sin x$.

The equation becomes: $t^2 - t - 2 = 0$.

1. Factorize the quadratic:

$$t^2 - 2t + t - 2 = 0$$

$$t(t-2) + 1(t-2) = 0$$

$$(t-2)(t+1) = 0$$

2. Possible values for t:

$$t = 2 \text{ or } t = -1$$

3. Analyze validity:

- Case 1: $\sin x = 2$. Since the range of the sine function is $[-1, 1]$, this value is impossible.
- Case 2: $\sin x = -1$. In the interval $[0, 2\pi]$, the sine function equals -1 only at $x = \frac{3\pi}{2}$.

4. Count the solutions:

There is exactly one value of x that satisfies the equation in the given range.

Step 3: Final Answer:

The number of solutions is 1.

This matches Option (A).

Quick Tip

Quick Tip: In trigonometric quadratic equations, always discard roots that fall outside the range $[-1, 1]$ for sine and cosine. They are "extraneous" roots.

---

66. If $\vec{a}, \vec{b},$ and $\vec{c}$ are three unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ is:

(A) $-\frac{3}{2}$

(B) $\frac{3}{2}$

(C) 0

(D) -3

Correct Answer: (A) $-\frac{3}{2}$

Solution:

Step 1: Understanding the Concept:

The dot product of a vector with itself is its magnitude squared ($\vec{u}\cdot\vec{u} = |\vec{u}|^2$). We use the property of the square of a sum of vectors.

Key Formula or Approach:

Identity: $|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$.

Step 2: Detailed Explanation:

Given $\vec{a}, \vec{b}, \vec{c}$ are unit vectors, so $|\vec{a}| = 1, |\vec{b}| = 1, |\vec{c}| = 1$.

Also given $\vec{a}+\vec{b}+\vec{c} = \vec{0}$.

1. Square the given vector sum:

$$|\vec{a}+\vec{b}+\vec{c}|^2 = 0^2 = 0$$

2. Expand using the identity:

$$1^2 + 1^2 + 1^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$1 + 1 + 1 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$3 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

3. Solve for the required expression:

$$2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = -3$$

$$\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -\frac{3}{2}$$

Step 3: Final Answer:

The value of the expression is $-\frac{3}{2}$.

This matches Option (A).

Quick Tip

Quick Tip: For n unit vectors whose sum is zero, the sum of their pairwise dot products is always $-\frac{n}{2}$. Here $n = 3$, so $-3/2$.

---

67. If the sum of two roots of the cubic equation $x^3 - 5x^2 - 2x + 24 = 0$ is 2, then the roots of the equation are:

(A) 2, 3, 4

(B) 2, 3, 0

(C) 1, 3, 3

(D) 2,3,4

Correct Answer: (A) 2, 3, 4

Solution:

Step 1: Understanding the Concept:

For a cubic equation $ax^3 + bx^2 + cx + d = 0$, the sum of roots $\alpha + \beta + \gamma = -b/a$ and the product of roots $\alpha\beta\gamma = -d/a$.

Step 2: Detailed Explanation:

Let the roots be $\alpha, \beta,$ and γ.

Given equation: $x^3 - 5x^2 - 2x + 24 = 0$.

1. From Vieta's formulas:

Sum of roots: $\alpha + \beta + \gamma = -(-5)/1 = 5$.

Product of roots: $\alpha\beta\gamma = -24/1 = -24$.

2. Given that the sum of two roots is 2:

Let $\alpha + \beta = 2$.

Substitute this into the sum formula:

$$2 + \gamma = 5 \implies \gamma = 3$$

3. Find α and β:

Substitute $\gamma = 3$ into the product formula:

$$\alpha\beta(3) = -24 \implies \alpha\beta = -8$$

We have $\alpha + \beta = 2$ and $\alpha\beta = -8$.

The roots α, β are solutions to $t^2 - (\text{sum})t + (\text{product}) = 0$:

$$t^2 - 2t - 8 = 0 \implies (t-4)(t+2) = 0$$

So, the other roots are 4 and -2.

The complete set of roots is $\{-2, 3, 4\}$.

Step 3: Final Answer:

The roots are -2, 3, and 4.

This matches Option (A).

Quick Tip

Quick Tip: Use substitution! Test the options in the sum of roots.

Option (A): $-2+3+4=5$. (Correct)

Check sum of two roots: $-2+4=2$. (Correct)

Checking options is usually faster for polynomial roots questions.

---

68. If $A=\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}$, **then** $A^{-1}=$

(A) $-\frac{1}{2}\begin{pmatrix}4 & -2\\ -3 & 1\end{pmatrix}$

(B) $\frac{1}{2}\begin{pmatrix}4 & -2\\ -3 & 1\end{pmatrix}$

(C) $-\frac{1}{2}\begin{pmatrix}1 & -2\\ -3 & 4\end{pmatrix}$

(D) $\frac{1}{2}\begin{pmatrix}1 & -2\\ -3 & 4\end{pmatrix}$

Correct Answer: (A) $-\frac{1}{2}\begin{pmatrix}4 & -2\\ -3 & 1\end{pmatrix}$

Solution:

Step 1: Understanding the Concept:

The inverse of a 2×2 matrix is found by swapping diagonal elements, changing the signs of

off-diagonal elements, and dividing by the determinant.

Key Formula or Approach:

For $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $A^{-1} = \frac{1}{\det(A)} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$.

Step 2: Detailed Explanation:

Given $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$.

1. Calculate the determinant:

$$\det(A) = (1)(4) - (2)(3) = 4 - 6 = -2$$

2. Find the adjoint matrix (swap 1 and 4, negate 2 and 3):

$$\text{adj}(A) = \begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix}$$

3. Calculate the inverse:

$$A^{-1} = \frac{1}{-2} \begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix} = -\frac{1}{2} \begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix}$$

Step 3: Final Answer:

The inverse matrix is $-\frac{1}{2} \begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix}$.

This matches Option (A).

Quick Tip

Quick Tip: Always double check the sign of the determinant first. A common mistake is using $|ad + bc|$ instead of $ad - bc$. If the determinant is -2, look for options with $-1/2$ or $1/-2$.

69. The number of ways of arranging the letters of the word "EAPCET" is:

(A) 360

(B) 720

(C) 180

(D) 120

Correct Answer: (A) 360

Solution:

Step 1: Understanding the Concept:

The number of permutations of a set of objects where some objects are identical is found by dividing the total arrangements by the factorials of the counts of each repeated object.

Key Formula or Approach:

$$\text{Ways} = \frac{n!}{p_1!p_2!\dots}$$

where n is the total number of items and p_i are the frequencies of repeats.

Step 2: Detailed Explanation:

Word: "EAPCET".

1. Count the total number of letters: $n = 6$.

2. Identify repeated letters:

- E: occurs 2 times.

- A, P, C, T: each occurs 1 time.

3. Calculate arrangements:

$$\text{Ways} = \frac{6!}{2!}$$

$$\text{Ways} = \frac{720}{2} = 360$$

Step 3: Final Answer:

The number of ways to arrange the letters is 360.

This matches Option (A).

Quick Tip

Quick Tip: Standard word lengths and their factorials to remember: $5! = 120$, $6! = 720$, $7! = 5040$. Dividing by 2 for a single repeat is a very common scenario in EAPCET.

---

70. If ω is a complex cube root of unity, then $(1-\omega+\omega^2)^5+(1+\omega-\omega^2)^5 =$

(A) 32

(B) 32

(C) 64

(D) 64

Correct Answer: (A) 32

Solution:

Step 1: Understanding the Concept:

The complex cube roots of unity $(1, \omega, \omega^2)$ satisfy two main properties:

1. $1+\omega+\omega^2 = 0$.

2. $\omega^3 = 1$.

Step 2: Detailed Explanation:

We simplify the expressions inside the parentheses using the property $1+\omega+\omega^2 = 0$.

1. For the first term:

Since $1+\omega^2 = -\omega$:

$$1-\omega+\omega^2 = (1+\omega^2)-\omega = (-\omega)-\omega = -2\omega$$

2. For the second term:

Since $1+\omega=-\omega^2$:

$$1+\omega-\omega^2=(1+\omega)-\omega^2=(-\omega^2)-\omega^2=-2\omega^2$$

3. Substitute back into the expression:

$$(-2\omega)^5+(-2\omega^2)^5$$

$$-32\omega^5-32\omega^{10}$$

4. Use $\omega^3=1$ to reduce powers:
$\omega^5=\omega^3\cdot\omega^2=1\cdot\omega^2=\omega^2$.
$\omega^{10}=(\omega^3)^3\cdot\omega=1^3\cdot\omega=\omega$.

$$\text{Expression}=-32\omega^2-32\omega=-32(\omega^2+\omega)$$

5. Since $\omega^2+\omega=-1$:

$$\text{Expression}=-32(-1)=32$$

Step 3: Final Answer:
The value of the expression is 32.
This matches Option (A).

Quick Tip

Quick Tip: Always look for $1+\omega$ or $1+\omega^2$ pairs to substitute with the third root. This reduces the number of terms inside the powers immediately.

71. If the probability that a person suffers a bad reaction from an injection is 0.001, then the probability that out of 2000 individuals, exactly 3 will suffer a bad reaction is:

(A) $\frac{4}{3}e^{-2}$

(B) $4e^{-2}$

(C) $\frac{1}{3}e^{-2}$

(D) $2e^{-2}$

Correct Answer: (A) $\frac{4}{3}e^{-2}$

Solution:

Step 1: Understanding the Concept:

When the number of trials n is very large and the probability of success p is very small, the Binomial distribution can be approximated by the Poisson distribution.

This approximation is valid when $n \to \infty$ and $p \to 0$ such that the mean $\lambda = np$ remains constant.

Key Formula or Approach:

The Poisson probability mass function is given by:

$$P(X = r) = \frac{e^{-\lambda} \cdot \lambda^r}{r!}$$

where $\lambda = n \cdot p$ is the mean number of successes.

Step 2: Detailed Explanation:

Given values:

Number of individuals $n = 2000$.

Probability of reaction $p = 0.001$.

Number of affected individuals required $r = 3$.

1. Calculate the mean (λ):

$$\lambda = n \times p = 2000 \times 0.001 = 2$$

2. Substitute $\lambda = 2$ and $r = 3$ into the Poisson formula:

$$P(X = 3) = \frac{e^{-2} \cdot 2^3}{3!}$$

3. Simplify the expression:

$$P(X = 3) = \frac{e^{-2} \cdot 8}{3 \times 2 \times 1}$$

$$P(X = 3) = \frac{8e^{-2}}{6}$$

$$P(X = 3) = \frac{4}{3}e^{-2}$$

Step 3: Final Answer:

The probability is $\frac{4}{3}e^{-2}$.

This matches Option (A).

Quick Tip

Quick Tip: Always use the Poisson approximation when $n > 50$ and $np < 5$. It simplifies complex factorials involved in the Binomial formula into a much more manageable calculation.

---

72. The distance of the point (1, 2) from the line $3x + 4y - 32 = 0$ measured parallel to the line $x - y = 0$ is:

(A) $3\sqrt{2}$

(B) $2\sqrt{2}$

(C) $\sqrt{2}$

(D) $5\sqrt{2}$

Correct Answer: (A) $3\sqrt{2}$

Solution:

Step 1: Understanding the Concept:

When measuring distance from a point to a line along a specific direction, we use the parametric coordinates of a point.

Instead of the perpendicular distance, we find a point Q on the target line such that the segment PQ is parallel to a given direction.

Key Formula or Approach:

The coordinates of a point Q at a distance r from $P(x_1, y_1)$ in a direction θ are:

$$(x_1 + r\cos\theta, y_1 + r\sin\theta)$$

Step 2: Detailed Explanation:

1. Find the direction θ:

The distance is measured parallel to $x - y = 0$, which has a slope $m = 1$.

Since $\tan\theta = 1$, we have $\theta = 45°$.

Thus, $\cos\theta = \frac{1}{\sqrt{2}}$ and $\sin\theta = \frac{1}{\sqrt{2}}$.

2. Define point Q:

Let Q be the point on the line $3x + 4y - 32 = 0$. Its coordinates are:

$$Q = \left(1 + \frac{r}{\sqrt{2}}, 2 + \frac{r}{\sqrt{2}}\right)$$

3. Substitute Q into the line equation:

$$3\left(1 + \frac{r}{\sqrt{2}}\right) + 4\left(2 + \frac{r}{\sqrt{2}}\right) - 32 = 0$$

$$3 + \frac{3r}{\sqrt{2}} + 8 + \frac{4r}{\sqrt{2}} - 32 = 0$$

$$\frac{7r}{\sqrt{2}} + 11 - 32 = 0$$

$$\frac{7r}{\sqrt{2}} = 21$$

$$\frac{r}{\sqrt{2}} = 3 \implies r = 3\sqrt{2}$$

Step 3: Final Answer:

The required distance is $3\sqrt{2}$ units.

This matches Option (A).

Quick Tip

Quick Tip: For distance along a line, don't use the perpendicular distance formula $\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$. Use parametric coordinates. The "distance" r will naturally come out from the linear equation.

---

73. The length of the intercept made by the circle $x^2 + y^2 - 10x + 4y + 9 = 0$ on the x-axis is:

(A) 8

(B) 6

(C) 10

(D) 12

Correct Answer: (A) 8

Solution:

Step 1: Understanding the Concept:

The intercept made by a circle on the x-axis is the distance between the two points where the circle crosses the x-axis. At these points, the y-coordinate is zero.

Key Formula or Approach:

For a general circle $x^2 + y^2 + 2gx + 2fy + c = 0$, the length of the x-intercept is:

$$\text{Length} = 2\sqrt{g^2 - c}$$

Step 2: Detailed Explanation:

Given circle equation: $x^2 + y^2 - 10x + 4y + 9 = 0$.

1. Identify the parameters by comparing with the general form:

$2g = -10 \implies g = -5$.

$2f = 4 \implies f = 2$.

$c = 9$.

2. Apply the x-intercept formula:

$$\text{Length} = 2\sqrt{(-5)^2 - 9}$$

$$\text{Length} = 2\sqrt{25 - 9}$$

$$\text{Length} = 2\sqrt{16}$$

$$\text{Length} = 2 \times 4 = 8$$

Step 3: Final Answer:

The length of the intercept on the x-axis is 8 units.

This matches Option (A).

Quick Tip

Quick Tip: X-intercept: $2\sqrt{g^2-c}$.
Y-intercept: $2\sqrt{f^2-c}$.
If $g^2 < c$, the circle does not cut the x-axis. Always check the sign under the square root.

---

74. The equation of the tangent to the parabola $y^2 = 8x$ which is parallel to the line $2x-y+5=0$ is:

(A) $2x - y + 1 = 0$
(B) $2x - y - 1 = 0$
(C) $2x - y + 2 = 0$
(D) $2x - y - 2 = 0$

Correct Answer: (A) $2x - y + 1 = 0$

Solution:

Step 1: Understanding the Concept:
Two lines are parallel if they have the same slope. A tangent to a parabola must have a specific relationship between its slope, the parabola's constant, and its intercept.

Key Formula or Approach:
For a parabola $y^2 = 4ax$, the equation of a tangent with slope m is:

$$y = mx + \frac{a}{m}$$

Step 2: Detailed Explanation:
1. Analyze the parabola:
$y^2 = 8x \implies 4a = 8 \implies a = 2$.

2. Find the required slope:

The tangent is parallel to $2x - y + 5 = 0$.

Rearranging to $y = mx + c$ form: $y = 2x + 5$.

Therefore, the slope $m = 2$.

3. Form the tangent equation:

Substitute $a = 2$ and $m = 2$ into the tangent formula:

$$y = 2x + \frac{2}{2}$$

$$y = 2x + 1$$

4. Convert to general form:

$$2x - y + 1 = 0$$

Step 3: Final Answer:

The equation of the tangent is $2x - y + 1 = 0$.

This matches Option (A).

Quick Tip

Quick Tip: Tangency condition for parabola: $c = a/m$.

Tangency condition for circle: $c^2 = r^2(1 + m^2)$.

Tangency condition for ellipse: $c^2 = a^2m^2 + b^2$.

Memorizing these helps solve tangent problems without calculus.

---

75. The value of the limit $\lim_{x \to 0} \frac{e^{3x} - e^{-2x}}{\sin 4x}$ **is:**

(A) 5/4

(B) 1/4

(C) 3/4

(D) 1/2

Correct Answer: (A) 5/4

Solution:

Step 1: Understanding the Concept:

Direct substitution of $x = 0$ gives $\frac{e^0 - e^0}{\sin 0} = \frac{1-1}{0} = \frac{0}{0}$, which is an indeterminate form. We can resolve this using L'Hôpital's Rule or standard limits.

Key Formula or Approach:

Using L'Hôpital's Rule: $\lim \frac{f(x)}{g(x)} = \lim \frac{f'(x)}{g(x)'}$.

Standard limits: $\lim_{x\to 0} \frac{e^{ax}-1}{x} = a$ and $\lim_{x\to 0} \frac{\sin ax}{x} = a$.

Step 2: Detailed Explanation:

Applying L'Hôpital's Rule by differentiating the numerator and denominator:

1. Derivative of numerator: $\frac{d}{dx}(e^{3x} - e^{-2x}) = 3e^{3x} - (-2)e^{-2x} = 3e^{3x} + 2e^{-2x}$.
2. Derivative of denominator: $\frac{d}{dx}(\sin 4x) = 4\cos 4x$.
3. Evaluate the limit:

$$\lim_{x\to 0} \frac{3e^{3x} + 2e^{-2x}}{4\cos 4x}$$

$$= \frac{3e^0 + 2e^0}{4\cos 0}$$

$$= \frac{3(1) + 2(1)}{4(1)} = \frac{5}{4}$$

Step 3: Final Answer:

The limit evaluates to 5/4.

This matches Option (A).

Quick Tip

Quick Tip: Standard result: $\lim_{x\to 0}\frac{e^{ax}-e^{bx}}{\sin cx}=\frac{a-b}{c}$.
In this case: $a=3, b=-2, c=4$.
Result $=\frac{3-(-2)}{4}=\frac{5}{4}$.

---

76. If $y=\sqrt{\tan x+\sqrt{\tan x+\sqrt{\tan x+\ldots\infty}}}$, then $(2y-1)\frac{dy}{dx}=$

(A) $\sec^2 x$

(B) $-\sec^2 x$

(C) $\tan x$

(D) $\sec x\tan x$

Correct Answer: (A) $\sec^2 x$

Solution:

Step 1: Understanding the Concept:

This function is an infinite nested radical. Because the expression under the first square root is identical to the whole expression y, we can substitute y into itself to create a finite equation.

Key Formula or Approach:

1. Express the relation as $y=\sqrt{f(x)+y}$.
2. Square both sides to eliminate the square root.
3. Differentiate implicitly with respect to x.

Step 2: Detailed Explanation:

The given expression is:

$$y=\sqrt{\tan x+y}$$

1. Squaring both sides:

$$y^2 = \tan x + y$$

2. Differentiate both sides with respect to x:

$$\frac{d}{dx}(y^2) = \frac{d}{dx}(\tan x) + \frac{d}{dx}(y)$$

$$2y\frac{dy}{dx} = \sec^2 x + \frac{dy}{dx}$$

3. Rearrange to group the $\frac{dy}{dx}$ terms:

$$2y\frac{dy}{dx} - \frac{dy}{dx} = \sec^2 x$$

$$(2y-1)\frac{dy}{dx} = \sec^2 x$$

Step 3: Final Answer:

The value of the expression is $\sec^2 x$.

This matches Option (A).

Quick Tip

Quick Tip: General formula: If $y = \sqrt{f(x) + \sqrt{f(x) + \ldots}}$, then $\frac{dy}{dx} = \frac{f'(x)}{2y-1}$.
Here $f(x) = \tan x$, so $f'(x) = \sec^2 x$.

---

77. The integral $\int \frac{dx}{x(x^4+1)} =$

(A) $\frac{1}{4}\ln\left|\frac{x^4}{x^4+1}\right| + C$

(B) $\frac{1}{4}\ln\left|\frac{x^4+1}{x^4}\right|+C$

(C) $\ln\left|\frac{x^4}{x^4+1}\right|+C$

(D) $\ln\left|\frac{x^4+1}{x^4}\right|+C$

Correct Answer: (A) $\frac{1}{4}\ln\left|\frac{x^4}{x^4+1}\right|+C$

Solution:

Step 1: Understanding the Concept:

To solve integrals of the form $\int \frac{dx}{x(x^n+1)}$, the most effective technique is to multiply and divide the integrand by x^{n-1} to facilitate a power substitution.

Step 2: Detailed Explanation:

1. Multiply numerator and denominator by x^3:

$$I=\int \frac{x^3}{x^4(x^4+1)}dx$$

2. Use substitution: Let $u=x^4 \implies du=4x^3dx \implies x^3dx=\frac{du}{4}$.

3. Rewrite the integral:

$$I=\frac{1}{4}\int \frac{du}{u(u+1)}$$

4. Resolve into partial fractions:

$$\frac{1}{u(u+1)}=\frac{(u+1)-u}{u(u+1)}=\frac{1}{u}-\frac{1}{u+1}$$

5. Integrate:

$$I=\frac{1}{4}\left[\int \frac{1}{u}du-\int \frac{1}{u+1}du\right]$$

$$I=\frac{1}{4}[\ln|u|-\ln|u+1|]+C$$

$$I = \frac{1}{4} \ln \left| \frac{u}{u+1} \right| + C$$

6. Substitute back $u = x^4$:

$$I = \frac{1}{4} \ln \left| \frac{x^4}{x^4+1} \right| + C$$

Step 3: Final Answer:

The result of the integration is $\frac{1}{4} \ln \left| \frac{x^4}{x^4+1} \right| + C$.

This matches Option (A).

Quick Tip

Quick Tip: Standard Formula: $\int \frac{dx}{x(x^n+1)} = \frac{1}{n} \ln \left| \frac{x^n}{x^n+1} \right| + C$.

In this problem, $n = 4$, giving the answer immediately.

---

78. The value of the definite integral $\int_0^{\pi/2} \ln(\tan x) dx$ is:

(A) 0

(B) $\pi/2$

(C) π

(D) $\ln 2$

Correct Answer: (A) 0

Solution:

Step 1: Understanding the Concept:

Using the property of definite integrals $\int_0^a f(x)dx = \int_0^a f(a-x)dx$, we can transform the trigonometric function into its co-function.

Step 2: Detailed Explanation:

Let $I = \int_0^{\pi/2} \ln(\tan x)dx$ – (1)

Applying the property $x \to \frac{\pi}{2} - x$:

$$I = \int_0^{\pi/2} \ln[\tan(\pi/2 - x)]dx$$

$$I = \int_0^{\pi/2} \ln(\cot x)dx$$

Since $\cot x = \frac{1}{\tan x}$:

$$I = \int_0^{\pi/2} \ln\left(\frac{1}{\tan x}\right)dx$$

Using log property $\ln(1/A) = -\ln A$:

$$I = -\int_0^{\pi/2} \ln(\tan x)dx$$

$$I = -I$$

$$2I = 0 \implies I = 0$$

Step 3: Final Answer:

The value of the definite integral is 0.

This matches Option (A).

Quick Tip

Quick Tip: Log-trigonometric integrals from 0 to $\pi/2$ involving reciprocal pairs (like tan and cot) always sum to zero or a constant related to $\ln 2$. Remember $\int_0^{\pi/2} \ln(\sin x)dx = -\frac{\pi}{2}\ln 2$.

---

79. The solution of the differential equation $x\frac{dy}{dx} + 2y = x^2$ is:

(A) $yx^2 = \frac{x^4}{4} + C$
(B) $yx = \frac{x^3}{3} + C$
(C) $yx^2 = \frac{x^3}{3} + C$
(D) $yx = \frac{x^4}{4} + C$

Correct Answer: (A) $yx^2 = \frac{x^4}{4} + C$

Solution:

Step 1: Understanding the Concept:

This is a first-order linear differential equation. To solve it, we must first put it into the standard form $\frac{dy}{dx} + P(x)y = Q(x)$ and then find the integrating factor.

Key Formula or Approach:

1. Standardize by dividing by x.
2. Integrating Factor I.F. $= e^{\int P(x)dx}$.
3. Solution: $y \cdot (\text{I.F.}) = \int Q(x) \cdot (\text{I.F.})dx + C$.

Step 2: Detailed Explanation:

Given: $x\frac{dy}{dx} + 2y = x^2$.

1. Divide by x:

$$\frac{dy}{dx} + \frac{2}{x}y = x$$

Here, $P(x) = \frac{2}{x}$ and $Q(x) = x$.

2. Find Integrating Factor:

$$\text{I.F.} = e^{\int \frac{2}{x}dx} = e^{2\ln x} = e^{\ln x^2} = x^2$$

3. Apply the solution formula:

$$y \cdot x^2 = \int (x) \cdot (x^2) dx + C$$

$$yx^2 = \int x^3 dx + C$$

4. Integrate the right side:

$$yx^2 = \frac{x^4}{4} + C$$

Step 3: Final Answer:

The general solution is $yx^2 = \frac{x^4}{4} + C$.

This matches Option (A).

Quick Tip

Quick Tip: Always ensure the equation is in standard form (leading coefficient of 1 for dy/dx) before calculating $P(x)$. Forgetting to divide by x would lead to an incorrect integrating factor.

---

81. If the dimensional formula of a physical quantity is $[M^1L^2T^{-2}]$, then the quantity is:

a)	Coefficient of Viscosity (η) స్నిగ్ధతాగుణకం (η)	(i)	$M^{-1}L^{3}T^{4}A^{2}$
b)	Young's Modulus (Y) యంగ్ గుణకం (Y)	(ii)	$ML^{-1}T^{-1}$
c)	Permittivity (ε) పెర్మిటివిటి (ε)	(iii)	$ML^{-1}T^{-2}$
d)	Universal Gravitational constant (G) విశ్వగురుత్వస్థిరాంకం (G)	(iv)	$M^{-1}L^{3}T^{-2}$

(A) Force

(B) Work

(C) Power

(D) Momentum

Correct Answer: (B) Work

Solution:

Step 1: Understanding the Concept:

Dimensional formulas represent physical quantities in terms of the fundamental dimensions: Mass (M), Length (L), and Time (T).

By analyzing the units of basic physical quantities, we can derive their dimensional formulas.

Step 2: Detailed Explanation:

Let us evaluate the dimensional formulas for the given options:

1. **Force**: Force is defined as Mass × Acceleration.

Units: $kg \cdot m/s^2$.

Dimensions: $[M^1][L^1T^{-2}] = [M^1L^1T^{-2}]$.

2. **Work**: Work is defined as Force × Displacement.

Units: $(kg \cdot m/s^2) \times m = kg \cdot m^2/s^2$.

Dimensions: $[M^1L^1T^{-2}][L^1] = [M^1L^2T^{-2}]$.

3. **Power**: Power is defined as Work / Time.

Dimensions: $[M^1L^2T^{-2}]/[T^1] = [M^1L^2T^{-3}]$.

4. **Momentum**: Momentum is defined as Mass × Velocity.

Dimensions: $[M^1][L^1T^{-1}] = [M^1L^1T^{-1}]$.

Comparing the derived formulas, the given dimensions $[M^1L^2T^{-2}]$ match Work.

Step 3: Final Answer:

The dimensional formula $[M^1L^2T^{-2}]$ represents Work.

This matches Option (B).

Quick Tip

Quick Tip: Work and Energy (Kinetic, Potential, etc.) always share the same dimensional formula $[ML^2T^{-2}]$. Memorizing this common set helps eliminate options quickly in dimensional analysis problems.

---

82. A projectile is launched from the ground with an initial velocity v at an angle θ to the horizontal. If its horizontal range is equal to its maximum height, then the value of $\tan\theta$ is:

(A) 4

(B) 2

(C) 1

(D) 1/4

Correct Answer: (A) 4

Solution:

Step 1: Understanding the Concept:

In projectile motion, the horizontal range (R) and maximum height (H) are determined by the initial velocity and the launch angle. The problem provides a specific condition where these two quantities are equal.

Key Formula or Approach:

Maximum Height: $H = \frac{v^2 \sin^2\theta}{2g}$.

Horizontal Range: $R = \frac{v^2 \sin 2\theta}{g} = \frac{2v^2 \sin\theta\cos\theta}{g}$.

Step 2: Detailed Explanation:

Given condition: $R = H$.

Substitute the formulas:

$$\frac{2v^2 \sin\theta \cos\theta}{g} = \frac{v^2 \sin^2\theta}{2g}$$

Cancel v^2 and g from both sides:

$$2\sin\theta\cos\theta = \frac{\sin^2\theta}{2}$$

Assuming $\theta \neq 0$, we can divide by $\sin\theta$:

$$2\cos\theta = \frac{\sin\theta}{2}$$

Multiply both sides by 2 and divide by $\cos\theta$:

$$4 = \frac{\sin\theta}{\cos\theta}$$

$$\tan\theta = 4$$

Step 3: Final Answer:

The value of $\tan\theta$ is 4.

This matches Option (A).

Quick Tip

Quick Tip: There is a direct shortcut relation between Range and Height: $\tan\theta = \frac{4H}{R}$.

If $H = R$, then $\tan\theta = 4$ immediately. This is a very frequent question in competitive exams.

83. A lift of mass M is accelerating upwards with an acceleration a. If the tension in the supporting cable is T_1 during upward acceleration and T_2 when it accelerates downwards with the same acceleration a, then the ratio T_1/T_2 is:

(A) $\frac{g+a}{g-a}$
(B) $\frac{g-a}{g+a}$
(C) $\frac{g^2+a^2}{g^2-a^2}$
(D) 1

Correct Answer: (A) $\frac{g+a}{g-a}$

Solution:

Step 1: Understanding the Concept:

The tension in the cable of a lift is equal to the "apparent weight" of the lift. This apparent weight changes depending on the direction of acceleration according to Newton's Second Law.

Step 2: Detailed Explanation:

1. **Case 1: Upward Acceleration (a):**

The net force is $T_1 - Mg = Ma$.

$$T_1 = M(g + a)$$

2. **Case 2: Downward Acceleration (a):**

The net force is $Mg - T_2 = Ma$.

$$T_2 = M(g - a)$$

3. **Calculate the ratio:**

$$\frac{T_1}{T_2} = \frac{M(g + a)}{M(g - a)}$$

$$\frac{T_1}{T_2} = \frac{g+a}{g-a}$$

Step 3: Final Answer:

The ratio of the tensions is $\frac{g+a}{g-a}$.

This matches Option (A).

Quick Tip

Quick Tip: Apparent gravity $g_{eff} = g \pm a$.

Upward: Add acceleration $(g + a)$.

Downward: Subtract acceleration $(g - a)$.

The ratio of tensions is simply the ratio of effective gravities.

---

84. An engine pumps water continuously through a hose pipe. If the water leaves the pipe with velocity v and m is the mass per unit length of the water in the pipe, then the rate at which kinetic energy is imparted to the water is:

(A) $\frac{1}{2}mv^3$

(B) $\frac{1}{2}mv^2$

(C) mv^3

(D) $\frac{3}{2}mv^2$

Correct Answer: (A) $\frac{1}{2}mv^3$

Solution:

Step 1: Understanding the Concept:

The rate at which kinetic energy is imparted is defined as Power (P). It is the derivative of Kinetic Energy (K) with respect to time.

Key Formula or Approach:

Kinetic Energy: $K = \frac{1}{2}Mv^2$.

Power: $P = \frac{dK}{dt}$.

Step 2: Detailed Explanation:

Given m is the mass per unit length (dm/dl).

The mass of water flowing out in time dt is $dM = m \times dl$.

Since $dl = v \times dt$ (distance = velocity × time):

$$dM = m(vdt)$$

The rate of mass flow is:

$$\frac{dM}{dt} = mv$$

The rate of imparting kinetic energy is:

$$P = \frac{d}{dt}\left(\frac{1}{2}Mv^2\right) = \frac{1}{2}v^2\frac{dM}{dt}$$

Substitute $\frac{dM}{dt} = mv$:

$$P = \frac{1}{2}v^2(mv) = \frac{1}{2}mv^3$$

Step 3: Final Answer:

The rate of imparting kinetic energy is $\frac{1}{2}mv^3$.

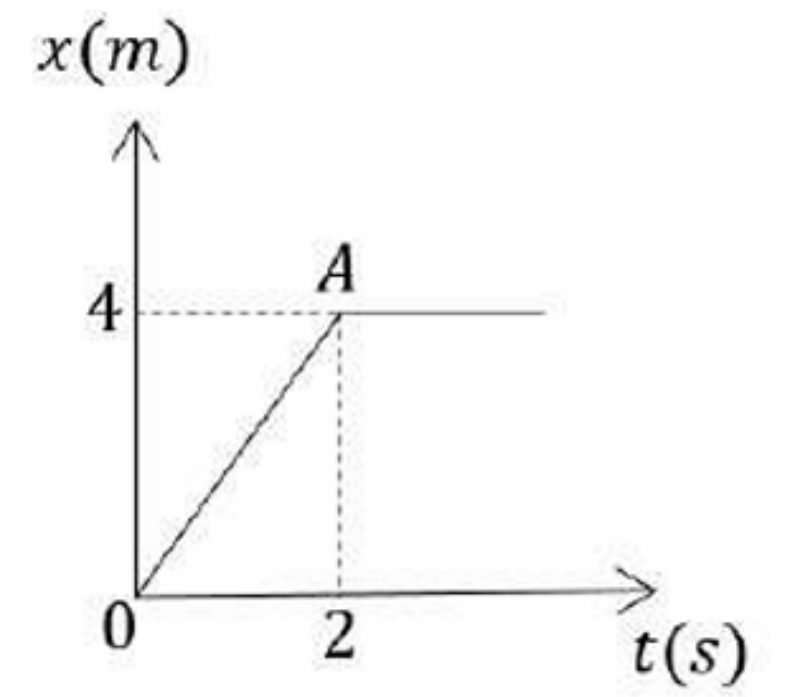
This matches Option (A).

Quick Tip

Quick Tip: Force exerted by a liquid jet is $F = v\frac{dm}{dt} = mv^2$.
Power is $F \times v = (mv^2) \times v = mv^3$.
However, the kinetic energy calculation involves a factor of $1/2$, leading to $\frac{1}{2}mv^3$. Be careful not to confuse force with the rate of energy change.

---

85. A thin circular ring of mass M and radius R is rotating about its central axis with a constant angular velocity ω. Two objects, each of mass m, are gently attached to the opposite ends of a diameter of the ring. The new angular velocity of the ring is:



(A) $\frac{M\omega}{M+2m}$
(B) $\frac{(M+2m)\omega}{M}$
(C) $\frac{M\omega}{M+m}$
(D) $\frac{(M-2m)\omega}{M+2m}$

Correct Answer: (A) $\frac{M\omega}{M+2m}$

Solution:

Step 1: Understanding the Concept:

Since the objects are "gently attached", no external torque acts on the system about the axis of rotation. Therefore, the total angular momentum of the system is conserved.

Key Formula or Approach:

Conservation of Angular Momentum: $I_1\omega_1 = I_2\omega_2$.

Moment of Inertia of a ring: $I = MR^2$.

Step 2: Detailed Explanation:

1. **Initial State**:

Initial Moment of Inertia $(I_1) = MR^2$.

Initial angular velocity $= \omega$.

Initial Angular Momentum $(L_1) = I_1\omega = MR^2\omega$.

2. **Final State**:

Two masses m are added at distance R from the center.

Final Moment of Inertia $(I_2) = I_{ring} + 2 \times I_{mass}$.

$$I_2 = MR^2 + 2(mR^2) = (M + 2m)R^2$$

Let final angular velocity be ω'.

3. **Apply Conservation**:

$$MR^2\omega = (M + 2m)R^2\omega'$$

Cancel R^2:

$$M\omega = (M + 2m)\omega'$$

$$\omega' = \frac{M\omega}{M + 2m}$$

Step 3: Final Answer:

The new angular velocity is $\frac{M\omega}{M+2m}$.

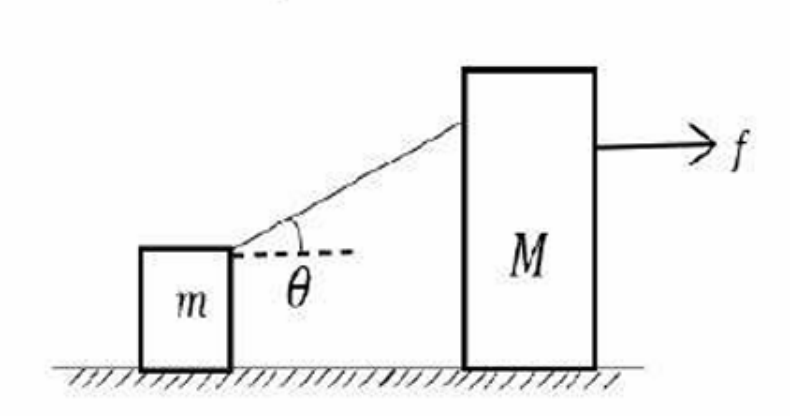
This matches Option (A).

Quick Tip

Quick Tip: Angular velocity is inversely proportional to the moment of inertia ($\omega \propto 1/I$). If you increase the mass (and thus I), the angular velocity must decrease. This rules out Option (B) immediately as it would mean the ring speeds up.

---

86. The time period of a simple pendulum is T in air. When the bob is completely immersed in a non-viscous liquid of density $\rho/10$ (where ρ is the density of the bob), the new time period of oscillation is:



(A) $T\sqrt{\frac{10}{9}}$
(B) $T\sqrt{\frac{9}{10}}$
(C) $T\sqrt{\frac{10}{11}}$
(D) $T\sqrt{\frac{11}{10}}$

Correct Answer: (A) $T\sqrt{\frac{10}{9}}$

Solution:

Step 1: Understanding the Concept:

When a pendulum bob is immersed in a liquid, it experiences an upward buoyant force (upthrust). This reduces the effective acceleration due to gravity acting on the bob.

Key Formula or Approach:

Time period: $T = 2\pi\sqrt{\frac{L}{g}}$.

Effective gravity in liquid: $g' = g\left(1 - \frac{\rho_L}{\rho_B}\right)$.

Step 2: Detailed Explanation:

1. Identify densities:

Density of bob $= \rho$.

Density of liquid $= \rho/10$.

2. Calculate effective gravity (g'):

$$g' = g\left(1 - \frac{\rho/10}{\rho}\right) = g\left(1 - \frac{1}{10}\right) = \frac{9g}{10}$$

3. Relate new time period (T') to original (T):

Since $T \propto 1/\sqrt{g}$:

$$\frac{T'}{T} = \sqrt{\frac{g}{g'}} = \sqrt{\frac{g}{9g/10}} = \sqrt{\frac{10}{9}}$$

$$T' = T\sqrt{\frac{10}{9}}$$

Step 3: Final Answer:

The new time period is $T\sqrt{\frac{10}{9}}$.

This matches Option (A).

Quick Tip

Quick Tip: Upthrust reduces the restoring force, making the pendulum "lighter" and slower. Slower oscillation means the time period must increase. Since $\sqrt{10/9} > 1$, Option (A) is a logical choice.

---

87. The acceleration due to gravity at a height h above the Earth's surface is the same as that at a depth d below the surface. If both h and d are much smaller than the radius of Earth R,

then the relation between h and d is:

(A) $d = 2h$

(B) $h = 2d$

(C) $d = h$

(D) $d = 4h$

Correct Answer: (A) $d = 2h$

Solution:

Step 1: Understanding the Concept:

Acceleration due to gravity (g) decreases both as we go above the surface and as we go below it. For small changes ($h, d \ll R$), we use linear approximations for these variations.

Key Formula or Approach:

Gravity at height h: $g_h = g\left(1 - \frac{2h}{R}\right)$.

Gravity at depth d: $g_d = g\left(1 - \frac{d}{R}\right)$.

Step 2: Detailed Explanation:

The problem states that $g_h = g_d$.

Equating the two expressions:

$$g\left(1 - \frac{2h}{R}\right) = g\left(1 - \frac{d}{R}\right)$$

Cancel g from both sides:

$$1 - \frac{2h}{R} = 1 - \frac{d}{R}$$

Cancel 1 from both sides:

$$-\frac{2h}{R} = -\frac{d}{R}$$

Multiply by $-R$:

$$2h = d \implies d = 2h$$

Step 3: Final Answer:

The relation is $d = 2h$.

This matches Option (A).

Quick Tip

Quick Tip: Remember: "Height is twice as effective as depth in reducing g".

Therefore, to get the same reduction, the depth must be double the height.

---

88. Two copper wires of length L and $2L$ have radii r and $2r$ respectively. If they are subjected to the same tension force, the ratio of their extension ($\Delta L_1/\Delta L_2$) is:

(A) 2 : 1

(B) 1 : 2

(C) 1 : 1

(D) 4 : 1

Correct Answer: (A) 2 : 1

Solution:

Step 1: Understanding the Concept:

The extension produced in a wire depends on the applied force, its length, its area of cross-section, and the Young's modulus of the material.

Key Formula or Approach:

Young's Modulus: $Y = \frac{F/A}{\Delta L/L} \implies \Delta L = \frac{FL}{AY}$.

Area $A = \pi r^2$, so $\Delta L \propto \frac{L}{r^2}$ (for constant F and same material Y).

Step 2: Detailed Explanation:

1. **Wire 1**: Length $L_1 = L$, radius $r_1 = r$.

$$\Delta L_1 \propto \frac{L}{r^2}$$

2. **Wire 2**: Length $L_2 = 2L$, radius $r_2 = 2r$.

$$\Delta L_2 \propto \frac{2L}{(2r)^2} = \frac{2L}{4r^2} = \frac{1}{2}\left(\frac{L}{r^2}\right)$$

3. **Ratio**:

$$\frac{\Delta L_1}{\Delta L_2} = \frac{L/r^2}{\frac{1}{2}(L/r^2)} = 2$$

The ratio is $2:1$.

Step 3: Final Answer:

The ratio of their extensions is $2:1$.

This matches Option (A).

Quick Tip

Quick Tip: Doubling the length doubles the extension ($\times 2$).

Doubling the radius increases the area by 4, which divides the extension by 4 ($\times 1/4$).

Net factor $= 2 \times 1/4 = 1/2$.

So $\Delta L_2 = 0.5\Delta L_1$, which means $\Delta L_1/\Delta L_2 = 2$.

---

89. A Carnot engine operates between a heat source at $T_1 = 600$ K and a sink at $T_2 = 300$ K. If the engine absorbs 1000 J of heat from the source per cycle, the work done per cycle is:

(A) 500 J

(B) 1000 J

(C) 250 J

(D) 750 J

Correct Answer: (A) 500 J

Solution:

Step 1: Understanding the Concept:

A Carnot engine is an ideal heat engine. Its efficiency depends solely on the temperatures of the source and the sink. Efficiency is also the ratio of useful work output to heat input.

Key Formula or Approach:

Efficiency $\eta = 1 - \frac{T_{sink}}{T_{source}}$.

Also, $\eta = \frac{W}{Q_{absorbed}}$.

Step 2: Detailed Explanation:

1. **Calculate Efficiency**:

Given $T_1 = 600$ K and $T_2 = 300$ K.

$$\eta = 1 - \frac{300}{600} = 1 - 0.5 = 0.5 \text{ (or 50\%)}$$

2. **Calculate Work Done**:

Heat absorbed $Q_1 = 1000$ J.

$$\eta = \frac{W}{Q_1} \implies 0.5 = \frac{W}{1000}$$

$$W = 0.5 \times 1000 = 500 \text{ J}$$

Step 3: Final Answer:

The work done per cycle is 500 J.

This matches Option (A).

Quick Tip

Quick Tip: Observe the temperatures: the sink is at exactly half the temperature of the source. For any engine, if the sink temperature is half the source temperature, the efficiency is 50%. Half of the absorbed heat becomes work.

---

90. At what temperature is the root mean square (rms) speed of oxygen molecules (O_2) equal to that of helium molecules (He) at 27°C? (Given molar mass of $O_2 = 32$ g/mol, $He = 4$ g/mol)

(A) 2400 K

(B) 2127 K

(C) 2427°C

(D) 2127°C

Correct Answer: (A) 2400 K

Solution:

Step 1: Understanding the Concept:

The root mean square speed of a gas is determined by its temperature and its molar mass. For two different gases to have the same rms speed, the ratio of their absolute temperatures to their molar masses must be equal.

Key Formula or Approach:

$v_{rms} = \sqrt{\frac{3RT}{M}}$.

If $v_{rms}(O_2) = v_{rms}(He)$, then $\frac{T_{O_2}}{M_{O_2}} = \frac{T_{He}}{M_{He}}$.

Step 2: Detailed Explanation:

1. **Identify parameters**:

Helium: $T_2 = 27°C = 27 + 273 = 300$ K.

$M_{He} = 4$ g/mol.

Oxygen: $M_{O_2} = 32$ g/mol.

Let T_1 be the temperature of Oxygen in Kelvin.

2. **Equate the ratios**:

$$\frac{T_1}{32} = \frac{300}{4}$$

3. **Solve for T_1:**

$$T_1 = \frac{300}{4} \times 32$$

$$T_1 = 300 \times 8 = 2400 \text{ K}$$

Note: Since the answer is required in Kelvin based on the options, we keep it as 2400 K.

Step 3: Final Answer:

The temperature required is 2400 K.

This matches Option (A).

Quick Tip

Quick Tip: Oxygen is 8 times heavier than Helium (32/4 = 8). For their speeds to be equal, the temperature of Oxygen must be 8 times higher than that of Helium (in Kelvin). $300 \times 8 = 2400$ K.

---

91. The ratio of the stress to the strain within the elastic limit is called

(A) Modulus of Elasticity

(B) Poisson's ratio

(C) Plastic limit

(D) Yield point

Correct Answer: (A) Modulus of Elasticity

Solution:

Step 1: Understanding the Concept:

In material science, the behavior of a material under load is defined by the relationship between the internal restoring force per unit area (stress) and the resulting deformation per unit length (strain).

Step 2: Detailed Explanation:

According to Hooke's Law, for small deformations, the stress (σ) developed in a body is directly proportional to the strain (ϵ) produced in it, provided the material is within its elastic limit.
Mathematically, this is expressed as:

$$\text{Stress} \propto \text{Strain}$$

$$\text{Stress} = E \times \text{Strain}$$

Rearranging this gives:

$$E = \frac{\text{Stress}}{\text{Strain}}$$

The constant of proportionality E is known as the Modulus of Elasticity.
Depending on the type of stress and strain, it can be Young's modulus (tensile), Bulk modulus (volumetric), or Modulus of Rigidity (shear).
Other options like Poisson's ratio refer to the ratio of lateral strain to longitudinal strain, while Yield point and Plastic limit refer to specific points on the stress-strain curve where permanent deformation begins.

Step 3: Final Answer:

The ratio of stress to strain within the elastic limit is defined as the Modulus of Elasticity.
This matches Option (A).

Quick Tip

Quick Tip: Remember: Hooke's Law is only valid up to the proportional limit, which is very close to the elastic limit. Beyond this point, the ratio is no longer a constant (the Modulus of Elasticity).

---

92. A liquid rises to a height of 4 cm in a capillary tube of radius r. If another capillary tube of radius $r/2$ is dipped in the same liquid, the height of liquid rise will be

(A) 8 cm

(B) 2 cm

(C) 4 cm

(D) 16 cm

Correct Answer: (A) 8 cm

Solution:

Step 1: Understanding the Concept:

Capillary action describes the ability of a liquid to flow in narrow spaces without the assistance of external forces. The height to which a liquid rises in a capillary tube is governed by Jurin's Law.

Key Formula or Approach:

The height h of the liquid column is given by:

$$h = \frac{2T\cos\theta}{\rho g r}$$

For a given liquid and tube material (where T, θ, ρ, g are constant), we have:

$$h \propto \frac{1}{r} \text{ or } h_1 r_1 = h_2 r_2$$

Step 2: Detailed Explanation:

1. **Identify initial conditions**:

Height $h_1 = 4$ cm, Radius $r_1 = r$.

2. **Identify final conditions**:

Height $h_2 = ?$, Radius $r_2 = r/2$.

3. **Apply the inverse proportionality**:

$$h_1 r_1 = h_2 r_2$$

$$(4)(r) = h_2 \left(\frac{r}{2}\right)$$

$$4r = \frac{h_2 r}{2}$$

Multiply both sides by 2 and divide by r:

$$h_2 = 8 \text{ cm}$$

Step 3: Final Answer:

The liquid will rise to a height of 8 cm in the new tube.

This matches Option (A).

Quick Tip

Quick Tip: Jurin's Rule is simple: "Narrower the tube, higher the rise." Halving the radius always doubles the height, and doubling the radius always halves the height.

---

93. The rate of flow of a liquid through a capillary tube of radius r and length l under a pressure difference P is proportional to

(A) r^4/l

(B) r^2/l

(C) r/l

(D) r^3/l

Correct Answer: (A) r^4/l

Solution:

Step 1: Understanding the Concept:

The volume of liquid flowing through a tube per unit time (rate of flow) depends on the viscosity of the liquid, the pressure gradient, and the dimensions of the tube. This is described by Poiseuille's Law.

Key Formula or Approach:

According to Poiseuille's equation:

$$V = \frac{\pi P r^4}{8\eta l}$$

where V is the rate of flow, P is pressure difference, r is radius, η is viscosity, and l is length.

Step 2: Detailed Explanation:

From the equation, we can observe the following proportionalities:

1. The rate of flow is directly proportional to the pressure difference P.
2. The rate of flow is directly proportional to the fourth power of the radius r.
3. The rate of flow is inversely proportional to the length l of the tube.
4. The rate of flow is inversely proportional to the viscosity η.

Combining the dimensional factors (radius and length), we find:

$$V \propto \frac{r^4}{l}$$

This relationship indicates that even a small increase in radius significantly increases the flow rate because of the fourth power dependency.

Step 3: Final Answer:

The rate of flow is proportional to r^4/l.

This matches Option (A).

Quick Tip

Quick Tip: Remember "Radius is king" in fluid flow through tubes. If the radius is doubled, the flow rate increases 16 times (2^4). Length has only a linear effect.

---

94. A copper rod and an iron rod of the same length have their temperature raised by the same amount. If the coefficient of linear expansion of copper is greater than that of iron, then

(A) Copper expands more than iron
(B) Iron expands more than copper
(C) Both expand by the same amount
(D) Expansion depends on their masses

Correct Answer: (A) Copper expands more than iron

Solution:

Step 1: Understanding the Concept:

Thermal expansion is the tendency of matter to change its shape, area, and volume in response to a change in temperature. For solid rods, we specifically look at linear expansion.

Key Formula or Approach:

The change in length (ΔL) of a rod is given by:

$$\Delta L = L_0 \alpha \Delta T$$

where L_0 is the initial length, α is the coefficient of linear expansion, and ΔT is the change in temperature.

Step 2: Detailed Explanation:

Given:

Initial lengths are equal: $L_{Cu} = L_{Fe} = L_0$.

Temperature rise is the same: $\Delta T_{Cu} = \Delta T_{Fe} = \Delta T$.

Comparing the expansions:

$$\Delta L_{Cu} = L_0 \alpha_{Cu} \Delta T$$

$$\Delta L_{Fe} = L_0 \alpha_{Fe} \Delta T$$

The problem states that $\alpha_{Cu} > \alpha_{Fe}$.

Since L_0 and ΔT are positive constants and are equal for both rods, the rod with the higher coefficient of expansion will experience a greater change in length.

Therefore, $\Delta L_{Cu} > \Delta L_{Fe}$.

Step 3: Final Answer:

Copper rod expands more than the iron rod.

This matches Option (A).

Quick Tip

Quick Tip: Expansion is a property of the material (α). Always identify what is constant ($L, \Delta T$) and what varies to find the answer. Mass is irrelevant to linear expansion.

---

95. The specific heat capacity of a gas at constant pressure (C_p) and at constant volume (C_v) are related as

(A) $C_p - C_v = R$

(B) $C_v - C_p = R$

(C) $C_p / C_v = R$

(D) $C_p + C_v = R$

Correct Answer: (A) $C_p - C_v = R$

Solution:

Step 1: Understanding the Concept:

Specific heat capacities represent the amount of heat required to raise the temperature of one mole of a gas by one degree. There are two standard conditions: constant volume and constant pressure.

Step 2: Detailed Explanation:

When heat is added to a gas at constant volume (C_v), no work is done (since $\Delta V = 0$), so all the heat goes into increasing the internal energy.

When heat is added at constant pressure (C_p), the gas expands and does work ($P\Delta V$) against the surroundings. Therefore, more heat is required to achieve the same temperature rise at constant pressure than at constant volume.

For one mole of an ideal gas, using the First Law of Thermodynamics and the Ideal Gas Equation ($PV = RT$):

$$dQ = dU + dW$$

At constant volume: $C_v dT = dU$

At constant pressure: $C_p dT = dU + PdV$

Using $PV = RT$, at constant pressure $PdV = RdT$.

Substituting these into the constant pressure equation:

$$C_p dT = C_v dT + RdT$$

Dividing by dT:

$$C_p = C_v + R$$

$$C_p - C_v = R$$

This relationship is known as Mayer's Formula.

Step 3: Final Answer:

The relation between the molar heat capacities is $C_p - C_v = R$.

This matches Option (A).

Quick Tip

Quick Tip: Remember: C_p is always greater than C_v for any gas because work is done during expansion at constant pressure. This helps you remember that the difference $C_p - C_v$ must be a positive constant (R).

---

96. In an isothermal process, which of the following remains constant?

(A) Temperature

(B) Pressure

(C) Volume

(D) Heat

Correct Answer: (A) Temperature

Solution:

Step 1: Understanding the Concept:

Thermodynamic processes are classified based on the physical parameter that is kept constant during the change of state of a system.

Step 2: Detailed Explanation:

The word "Isothermal" comes from Greek "iso" (same) and "thermos" (heat/temperature).

1. **Isothermal process**: A process occurring at constant temperature ($T =$ constant). Any heat added to the system is converted into work to maintain temperature.
2. **Isobaric process**: A process occurring at constant pressure ($P =$ constant).
3. **Isochoric process**: A process occurring at constant volume ($V =$ constant).
4. **Adiabatic process**: A process where no heat is exchanged with the surroundings ($\Delta Q = 0$).

In an isothermal process, since T is constant, the change in internal energy (ΔU) for an ideal gas is zero.

Step 3: Final Answer:

In an isothermal process, the temperature remains constant throughout.

This matches Option (A).

Quick Tip

Quick Tip: Summary of "Iso" prefixes:

Isothermal $\rightarrow$ Temp (T) constant.

Isobaric $\rightarrow$ Pressure (P) constant.

Isochoric $\rightarrow$ Volume (V) constant.

---

97. The efficiency of a Carnot engine working between temperatures 127°C and 27°C is

(A) 25%

(B) 75%

(C) 50%

(D) 20%

Correct Answer: (A) 25%

Solution:

Step 1: Understanding the Concept:

The efficiency of a Carnot engine (the most efficient possible heat engine) depends only on the absolute temperatures of the heat source and the heat sink.

Key Formula or Approach:

Efficiency (η) is given by:

$$\eta = 1 - \frac{T_2}{T_1}$$

where T_1 is the temperature of the source (hot) and T_2 is the temperature of the sink (cold).

Temperatures must be in Kelvin.

Step 2: Detailed Explanation:

1. **Convert temperatures to Kelvin**:

Temperature of source $T_1 = 127^\circ\text{C} = 127 + 273 = 400$ K.

Temperature of sink $T_2 = 27^\circ\text{C} = 27 + 273 = 300$ K.

2. **Apply the efficiency formula**:

$$\eta = 1 - \frac{300}{400}$$

$$\eta = 1 - \frac{3}{4}$$

$$\eta = \frac{1}{4}$$

3. **Convert to percentage**:

$$\eta(\%) = \frac{1}{4} \times 100 = 25\%$$

Step 3: Final Answer:

The efficiency of the Carnot engine is 25%.

This matches Option (A).

Quick Tip

Quick Tip: Always convert Celsius to Kelvin first. If you use Celsius ($1 - 27/127$), you will get a completely different and incorrect answer. Add 273 to every temperature value.

98. The mean free path of a gas molecule is inversely proportional to

(A) Square of the molecular diameter

(B) Molecular diameter

(C) Temperature

(D) Square root of temperature

Correct Answer: (A) Square of the molecular diameter

Solution:

Step 1: Understanding the Concept:

The mean free path (λ) is the average distance traveled by a moving gas molecule between successive collisions with other molecules. It depends on the size of the molecules and their density.

Key Formula or Approach:

The expression for the mean free path is:

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$$

where d is the diameter of the molecule and n is the number of molecules per unit volume (number density).

Step 2: Detailed Explanation:

By examining the formula, we can determine the relationship between λ and various physical parameters:

1. **Molecular Diameter (d)**: Since d^2 is in the denominator, the mean free path is inversely proportional to the square of the molecular diameter ($\lambda \propto 1/d^2$).

2. **Number Density (n)**: The mean free path is inversely proportional to the number of molecules per unit volume.

3. **Temperature and Pressure**: For an ideal gas ($PV = NkT$), $n = N/V = P/kT$. Substituting this gives $\lambda = \frac{kT}{\sqrt{2}\pi d^2 P}$. Thus, λ is directly proportional to temperature if pressure is constant.

The question specifically asks for the inverse proportionality related to the molecule's physical dimensions.

Step 3: Final Answer:

The mean free path is inversely proportional to the square of the molecular diameter.

This matches Option (A).

Quick Tip

Quick Tip: Visualize it: If you double the diameter of a molecule, its "collision cross-section" increases 4 times, making a collision 4 times more likely, and thus reducing the path length between hits to 1/4th.

---

99. The time period of a simple pendulum of length l is T. If its length is increased to $4l$, its new time period will be

(A) $2T$

(B) $4T$

(C) $T/2$

(D) T

Correct Answer: (A) $2T$

Solution:

Step 1: Understanding the Concept:

The time period of a simple pendulum is the time it takes for one complete oscillation. It is independent of the mass of the bob and the amplitude (for small angles) but depends on the length of the string.

Key Formula or Approach:

The time period T is given by:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

where l is length and g is acceleration due to gravity.

This implies $T \propto \sqrt{l}$.

Step 2: Detailed Explanation:

1. **Initial Case**:

Time period $T = 2\pi\sqrt{\frac{l}{g}}$.

2. **Final Case**:

New length $l' = 4l$.

New time period $T' = 2\pi\sqrt{\frac{l'}{g}} = 2\pi\sqrt{\frac{4l}{g}}$.

3. **Calculate the relationship**:

$$T' = \sqrt{4}\left(2\pi\sqrt{\frac{l}{g}}\right)$$

$$T' = 2 \times T$$

The time period doubles when the length is quadrupled.

Step 3: Final Answer:

The new time period will be $2T$.

This matches Option (A).

Quick Tip

Quick Tip: Remember the square root relationship:

Length $\times 4 \rightarrow$ Time $\times 2$.

Length $\times 9 \rightarrow$ Time $\times 3$.

Length $\times 2 \rightarrow$ Time $\times 1.414$.

---

100. A transverse wave is represented by $y = A\sin(kx - \omega t)$. The maximum particle velocity is

(A) $A\omega$

(B) Ak

(C) ω/k

(D) $A\omega k$

Correct Answer: (A) $A\omega$

Solution:

Step 1: Understanding the Concept:

In a wave motion, it is vital to distinguish between wave velocity (how fast the disturbance travels) and particle velocity (how fast the medium particles vibrate about their mean position).

Key Formula or Approach:

The particle velocity v_p is the rate of change of displacement y with respect to time t:

$$v_p = \frac{dy}{dt}$$

Step 2: Detailed Explanation:

The given wave equation is:

$$y = A\sin(kx - \omega t)$$

Differentiate y with respect to t, keeping x constant:

$$v_p = \frac{d}{dt}[A\sin(kx - \omega t)]$$

Using the chain rule:

$$v_p = A\cos(kx - \omega t) \times \frac{d}{dt}(kx - \omega t)$$

$$v_p = A\cos(kx - \omega t) \times (-\omega)$$

$$v_p = -A\omega\cos(kx - \omega t)$$

The particle velocity is a cosine function, which varies over time.

The maximum value of the cosine function is 1 (or -1 for magnitude).

Therefore, the maximum particle velocity (v_{max}) is:

$$v_{max} = |-A\omega \times (1)| = A\omega$$

Step 3: Final Answer:

The maximum particle velocity is $A\omega$.

This matches Option (A).

Quick Tip

Quick Tip: Maximum velocity of a particle in SHM or waves is always (Amplitude × Angular Frequency). Angular frequency ω is always the coefficient of t.

---

101. An open organ pipe of length L resonates at its fundamental frequency. The wavelength of the sound wave produced is

(A) $2L$

(B) L

(C) $4L$

(D) $L/2$

Correct Answer: (A) $2L$

Solution:

Step 1: Understanding the Concept:

In an open organ pipe, both ends are open to the atmosphere. This means that the air at the ends can vibrate with maximum amplitude, forming antinodes in the standing wave pattern.

Key Formula or Approach:

For the fundamental mode (the simplest mode of vibration), there is one node in the middle and one antinode at each end of the pipe.

Step 2: Detailed Explanation:

The distance between two consecutive antinodes in a standing wave is equal to half a wavelength ($\lambda/2$).

In the fundamental mode of an open pipe:

The length of the pipe (L) is equal to the distance between these two terminal antinodes.

Therefore:

$$L = \frac{\lambda}{2}$$

Rearranging to solve for the wavelength λ:

$$\lambda = 2L$$

This means the sound wave produced has a wavelength exactly double the length of the pipe.

Step 3: Final Answer:

The wavelength produced at the fundamental frequency is $2L$.

This matches Option (A).

Quick Tip

Quick Tip: Standard cases for pipes:

Open Pipe: $\lambda = 2L$ (Fundamental).

Closed Pipe (one end): $\lambda = 4L$ (Fundamental).

Always remember that open ends must have antinodes, and closed ends must have nodes.

---

102. The apparent frequency of a siren increases as the source approaches a stationary observer.

This phenomenon is known as

(A) Doppler Effect

(B) Resonance

(C) Interference

(D) Diffraction

Correct Answer: (A) Doppler Effect

Solution:

Step 1: Understanding the Concept:

The frequency of a sound wave as perceived by an observer can differ from the actual frequency emitted by the source if there is relative motion between them.

Step 2: Detailed Explanation:

1. **Doppler Effect**: This phenomenon describes the change in the observed frequency of a wave due to the relative motion between the source and the observer.
2. **Approaching Source**: When a source (like a siren) moves towards a stationary observer, the sound waves are "bunched up" or compressed in front of the source. This effectively reduces the wavelength perceived by the observer.
3. **Frequency Relation**: Since the speed of sound remains constant in the medium, a reduction in perceived wavelength leads to an increase in perceived frequency ($v = f\lambda$).
4. **Other Options**:

- **Resonance** occurs when an external force's frequency matches a system's natural frequency.
- **Interference** is the superposition of two waves.
- **Diffraction** is the bending of waves around obstacles.

Step 3: Final Answer:

The phenomenon described is the Doppler Effect.

This matches Option (A).

Quick Tip

Quick Tip: Remember:

Source moving towards observer → Higher pitch (Higher frequency).

Source moving away from observer → Lower pitch (Lower frequency).

This is the same reason a race car sounds high-pitched as it approaches and low-pitched after it passes.

---

103. The electrostatic force between two charges q_1 and q_2 separated by a distance r in vacuum is F. If a dielectric medium of dielectric constant K is introduced between them, the new force is

(A) F/K

(B) KF

(C) F/K^2

(D) K^2F

Correct Answer: (A) F/K

Solution:

Step 1: Understanding the Concept:

Coulomb's Law quantifies the force between two stationary point charges. This force is influenced by the permittivity of the medium surrounding the charges.

Key Formula or Approach:

Force in vacuum: $F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2}$.

Force in a medium: $F_{med} = \frac{1}{4\pi\epsilon} \cdot \frac{q_1 q_2}{r^2}$.

Step 2: Detailed Explanation:

The permittivity of a medium (ϵ) is related to the permittivity of vacuum (ϵ_0) by the dielectric constant (K), also known as relative permittivity (ϵ_r):

$$\epsilon = K\epsilon_0$$

Substitute this into the formula for force in the medium:

$$F_{med} = \frac{1}{4\pi(K\epsilon_0)} \cdot \frac{q_1 q_2}{r^2}$$

We can factor out $1/K$:

$$F_{med} = \frac{1}{K}\left(\frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2}\right)$$

Since the term in the brackets is the force in vacuum (F):

$$F_{med} = \frac{F}{K}$$

Introduction of a dielectric medium reduces the net electrostatic force because the medium polarizes and creates an opposing electric field.

Step 3: Final Answer:
The new force is F/K.
This matches Option (A).

Quick Tip

Quick Tip: Always remember: $K > 1$ for all dielectric materials. Therefore, placing a dielectric between charges always decreases the force. This helps rule out options like KF or K^2F.

---

104. The electric potential at a distance r from a point charge q is proportional to

(A) $1/r$
(B) $1/r^2$
(C) r
(D) r^2

Correct Answer: (A) $1/r$

Solution:

Step 1: Understanding the Concept:

Electric potential (V) at a point is the work done per unit charge in bringing a positive test charge from infinity to that point in an electric field.

Key Formula or Approach:

The potential V due to a point charge q at a distance r is:

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

Step 2: Detailed Explanation:

From the mathematical expression $V = k\frac{q}{r}$, where k is a constant:

1. The potential depends directly on the magnitude of the charge q.
2. The potential depends inversely on the distance r.

Thus:

$$V \propto \frac{1}{r}$$

This is different from the electric field (E), which follows an inverse-square law ($E \propto 1/r^2$). Potential decreases more slowly with distance than the field strength does.

Step 3: Final Answer:

The electric potential is proportional to $1/r$.

This matches Option (A).

Quick Tip

Quick Tip: Potential $\propto 1/r$.

Electric Field $\propto 1/r^2$.

Be careful not to mix these up in the exam, as $1/r^2$ is usually provided as a distractor.

105. Three capacitors of capacitances $3\,\mu$F, $3\,\mu$F and $3\,\mu$F are connected in series. The equivalent capacitance is

(A) $1\,\mu$F

(B) $9\,\mu$F

(C) $3\,\mu$F

(D) $1.5\,\mu$F

Correct Answer: (A) $1\,\mu$F

Solution:

Step 1: Understanding the Concept:

In a series connection of capacitors, the charge on each capacitor is the same, but the total voltage is divided among them. The total equivalent capacitance is always less than the smallest individual capacitance.

Key Formula or Approach:

For capacitors in series:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$$

Step 2: Detailed Explanation:

Given: $C_1 = 3\,\mu$F, $C_2 = 3\,\mu$F, $C_3 = 3\,\mu$F.

Substitute the values into the series formula:

$$\frac{1}{C_{eq}} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$$

Adding the fractions:

$$\frac{1}{C_{eq}} = \frac{1+1+1}{3} = \frac{3}{3} = 1\,\mu\text{F}^{-1}$$

Taking the reciprocal to find C_{eq}:

$$C_{eq} = 1\,\mu\text{F}$$

Step 3: Final Answer:

The equivalent capacitance is $1\,\mu$F.

This matches Option (A).

Quick Tip

Quick Tip: Shortcut: For n identical capacitors of value C connected in series, the equivalent capacitance is $C_{eq} = C/n$.

Here $C = 3$ and $n = 3$, so $3/3 = 1\,\mu$F.

---

106. According to Ohm's law, the relation between the electric current I and the potential difference V across a conductor is

(A) $V \propto I$

(B) $V \propto I^2$

(C) $V^2 \propto I$

(D) $V \propto 1/I$

Correct Answer: (A) $V \propto I$

Solution:

Step 1: Understanding the Concept:

Ohm's Law describes the linear relationship between voltage and current in an ohmic conductor, provided physical conditions like temperature remain constant.

Step 2: Detailed Explanation:

Ohm's Law states that the potential difference (V) across the ends of a given metallic wire in

an electric circuit is directly proportional to the current (I) flowing through it.
Mathematically:

$$V \propto I$$

To remove the proportionality, we introduce a constant called Resistance (R):

$$V = IR$$

This linear relationship implies that if the voltage across a conductor is doubled, the current through it will also double. It holds true for most metals under standard conditions.

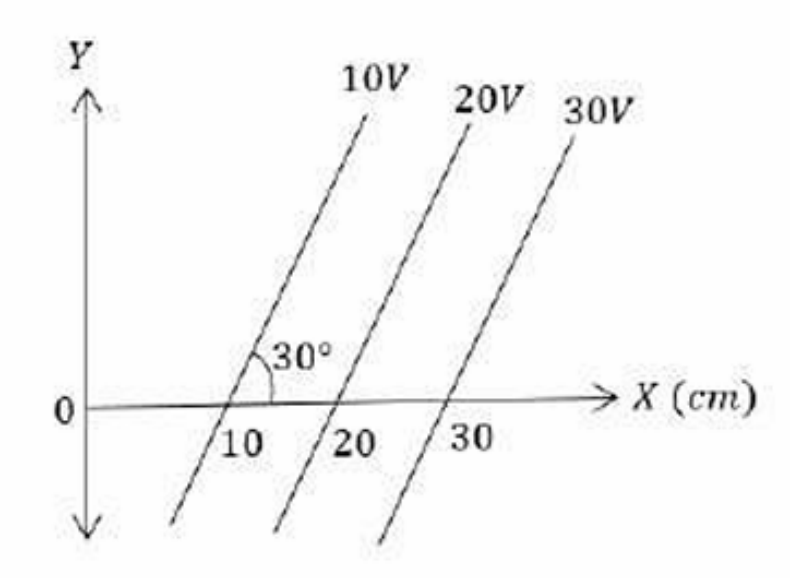
Step 3: Final Answer:
The correct relation is $V \propto I$.
This matches Option (A).

Quick Tip

Quick Tip: Ohm's Law results in a straight-line graph passing through the origin on a $V - I$ plot. The slope of this line represents the resistance R.

---

107. A wire of resistance R is stretched to double its original length. Its new resistance will be



(A) $4R$

(B) $2R$

(C) $R/2$

(D) $R/4$

Correct Answer: (A) $4R$

Solution:

Step 1: Understanding the Concept:

When a wire is stretched, its volume remains constant. This means as the length increases, the area of cross-section must decrease proportionally. Both of these changes affect the electrical resistance.

Key Formula or Approach:

Resistance $R = \rho \frac{L}{A}$.

Volume $V = AL =$ constant.

Step 2: Detailed Explanation:

Let original length be L and area be A.

Initial resistance $R = \rho \frac{L}{A}$.

1. New length $L' = 2L$.
2. Since volume $V = AL$ is constant, $A'L' = AL$.

$$A'(2L) = AL \implies A' = \frac{A}{2}$$

3. Calculate new resistance R':

$$R' = \rho \frac{L'}{A'} = \rho \frac{2L}{A/2}$$

$$R' = \rho \frac{4L}{A}$$

$$R' = 4\left(\rho \frac{L}{A}\right) = 4R$$

Stretching the wire increases the length and thins the conductor, both of which increase resistance.

Step 3: Final Answer:

The new resistance will be $4R$.

This matches Option (A).

Quick Tip

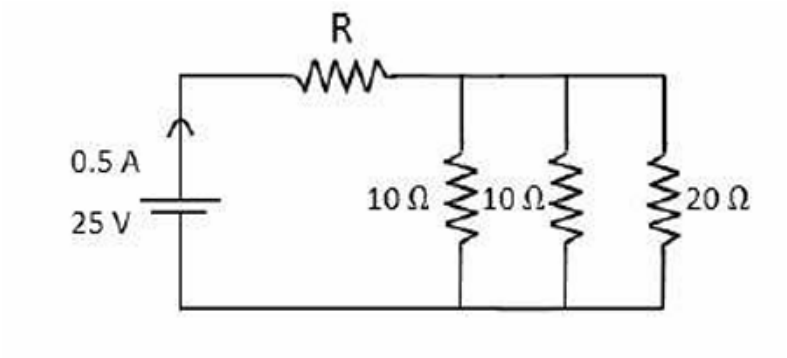
Quick Tip: Shortcut: When a wire is stretched n times its length, the new resistance is n^2R.

Here $n = 2$, so $R_{new} = 2^2R = 4R$.

This is much faster for multiple-choice questions!

---

108. In a Wheatstone bridge, the bridge is balanced when the ratio of the resistances in the four arms satisfies



(A) $P/Q = R/S$

(B) $P/R = S/Q$

(C) $PS = QR$

(D) $P + Q = R + S$

Correct Answer: (A) $P/Q = R/S$

Solution:

Step 1: Understanding the Concept:

A Wheatstone bridge consists of four resistors arranged in a diamond shape. It is used to find the value of an unknown resistance by balancing two legs of the circuit.

Step 2: Detailed Explanation:

The bridge is said to be "balanced" when the potential at the two mid-points of the parallel branches is the same.

When balanced:

1. No current flows through the galvanometer connected between these points.
2. This null deflection occurs when the potential drop across the first pair of resistors is proportional to the potential drop across the second pair.

Let the resistors be P, Q (in one path) and R, S (in the other path).

The balance condition is:

$$\frac{P}{Q} = \frac{R}{S}$$

Or alternatively:

$$\frac{P}{R} = \frac{Q}{S} \text{ or } PS = QR$$

While $PS = QR$ is mathematically equivalent, the most common standard form for the ratio of adjacent arms is $P/Q = R/S$.

Step 3: Final Answer:
The balance condition is $P/Q = R/S$.
This matches Option (A).

Quick Tip

Quick Tip: To avoid confusion, remember: "Top left / Bottom left = Top right / Bottom right" or "Adjacent ratio = Adjacent ratio". Balanced condition means the bridge acts like the galvanometer isn't even there.

---

109. The magnetic field at the centre of a circular coil of radius r carrying a current I is proportional to

(A) I/r
(B) Ir
(C) I/r^2
(D) Ir^2

Correct Answer: (A) I/r

Solution:
Step 1: Understanding the Concept:
A current-carrying conductor generates a magnetic field around it. For a loop or circular coil, the field lines concentrate at the center.

Key Formula or Approach:

According to the Biot-Savart Law, the magnetic field B at the center of a circular coil is:

$$B = \frac{\mu_0 I}{2r}$$

Step 2: Detailed Explanation:

In the formula $B = \frac{\mu_0 I}{2r}$:

1. μ_0 is the permeability of free space (a constant).

2. I is the current flowing through the coil.

3. r is the radius of the coil.

From this expression, we can clearly see the proportionalities:

- $B \propto I$ (Magnetic field increases with more current).

- $B \propto 1/r$ (Magnetic field decreases as the coil gets larger/wider).

Therefore:

$$B \propto \frac{I}{r}$$

Step 3: Final Answer:

The magnetic field is proportional to I/r.

This matches Option (A).

Quick Tip

Quick Tip: Field at center of coil $\propto 1/r$.

Field at center of a long solenoid $\propto I$ (independent of r).

Field due to straight wire $\propto 1/r$.

Field due to point charge $\propto 1/r^2$.

Distinguishing these helps solve proportionality questions instantly.

---

110. The force experienced by a charge q moving with velocity v in a magnetic field B is

maximum when the angle between v and B is

(A) 90°

(B) 0°

(C) 180°

(D) 45°

Correct Answer: (A) 90°

Solution:

Step 1: Understanding the Concept:

A moving charge in a magnetic field experiences a force known as the Lorentz Force. This force acts perpendicular to both the velocity of the charge and the magnetic field direction.

Key Formula or Approach:

The magnetic force vector is given by:

$$\vec{F} = q(\vec{v} \times \vec{B})$$

The magnitude of this force is:

$$F = qvB \sin\theta$$

where θ is the angle between the velocity vector $\vec{v}$ and the magnetic field vector $\vec{B}$.

Step 2: Detailed Explanation:

The force depends on the sine of the angle θ:

1. If $\theta = 0°$ or $180°$ (charge moving parallel or anti-parallel to the field), $\sin\theta = 0$, so $F = 0$.

2. If $\theta = 90°$ (charge moving perpendicular to the field), $\sin\theta = 1$.

Since 1 is the maximum possible value for the sine function, the force reaches its maximum value:

$$F_{max} = qvB$$

This maximum force causes the charge to move in a circular path.

Step 3: Final Answer:

The force is maximum when the angle is 90°.

This matches Option (A).

Quick Tip

Quick Tip: Remember: Magnetic fields "don't like" to push charges moving along them. They only push when the charge crosses the field lines. Maximum push happens at a right angle.

---

111. The magnetic susceptibility is negative for:

(A) paramagnetic materials.

(B) ferromagnetic materials.

(C) superconducting materials.

(D) diamagnetic materials.

Correct Answer: (D) diamagnetic materials.

Solution:

Step 1: Understanding the Concept:

Magnetic susceptibility (χ_m) is a measure of how much a material will become magnetized in an applied magnetic field.

It is defined as the ratio of magnetization (M) to the applied magnetizing field intensity (H).

Step 2: Detailed Explanation:

1. **Diamagnetic materials**: These materials develop a weak induced magnetization in a direction opposite to the applied magnetic field.

Because the induced field opposes the external field, the ratio M/H is negative. Thus, for diamagnetic substances, $\chi_m < 0$.

2. **Paramagnetic materials**: These materials develop a weak magnetization in the same

direction as the applied field.

They have a small positive susceptibility ($\chi_m > 0$).

3. **Ferromagnetic materials**: These materials exhibit strong magnetization in the direction of the field.

They have very large positive susceptibility values.

4. **Superconductors**: While they exhibit perfect diamagnetism (Meissner effect), they are a special state of matter. In general physics classification, "negative susceptibility" is the defining characteristic of the diamagnetic family.

Step 3: Final Answer:

The magnetic susceptibility is negative for diamagnetic materials.

This matches Option (D).

Quick Tip

Quick Tip: Remember:

Diamagnetic: χ is small and negative.

Paramagnetic: χ is small and positive.

Ferromagnetic: χ is very large and positive.

---

112. The self-inductance of a solenoid of length l, area of cross-section A and number of turns N is proportional to

(A) N^2A/l

(B) NA/l

(C) N^2Al

(D) NAl

Correct Answer: (A) N^2A/l

Solution:

Step 1: Understanding the Concept:

Self-inductance (L) is the property of a coil by which it opposes the change in current flowing through it. For a solenoid, this property depends on its geometric dimensions and the number of wire turns.

Key Formula or Approach:

The formula for the self-inductance of a long solenoid is:

$$L = \frac{\mu_0 N^2 A}{l}$$

where μ_0 is permeability, N is total turns, A is area, and l is length.

Step 2: Detailed Explanation:

1. The magnetic field inside a solenoid is $B = \mu_0 nI$, where $n = N/l$.
2. Total magnetic flux through the solenoid is $\Phi = NBA = N(\mu_0 \frac{N}{l} I)A$.
3. By definition of self-inductance, $\Phi = LI$.
4. Equating the expressions:

$$LI = \frac{\mu_0 N^2 AI}{l}$$

$$L = \frac{\mu_0 N^2 A}{l}$$

From this result, we see that L is directly proportional to the square of the number of turns (N^2), directly proportional to the area (A), and inversely proportional to the length (l).

Step 3: Final Answer:

The self-inductance is proportional to $N^2 A/l$.

This matches Option (A).

Quick Tip

Quick Tip: Inductance always involves the "square" of the number of turns (N^2) because increasing turns increases both the field strength and the number of loops catching that field. If you see N with power 1, rule that option out immediately.

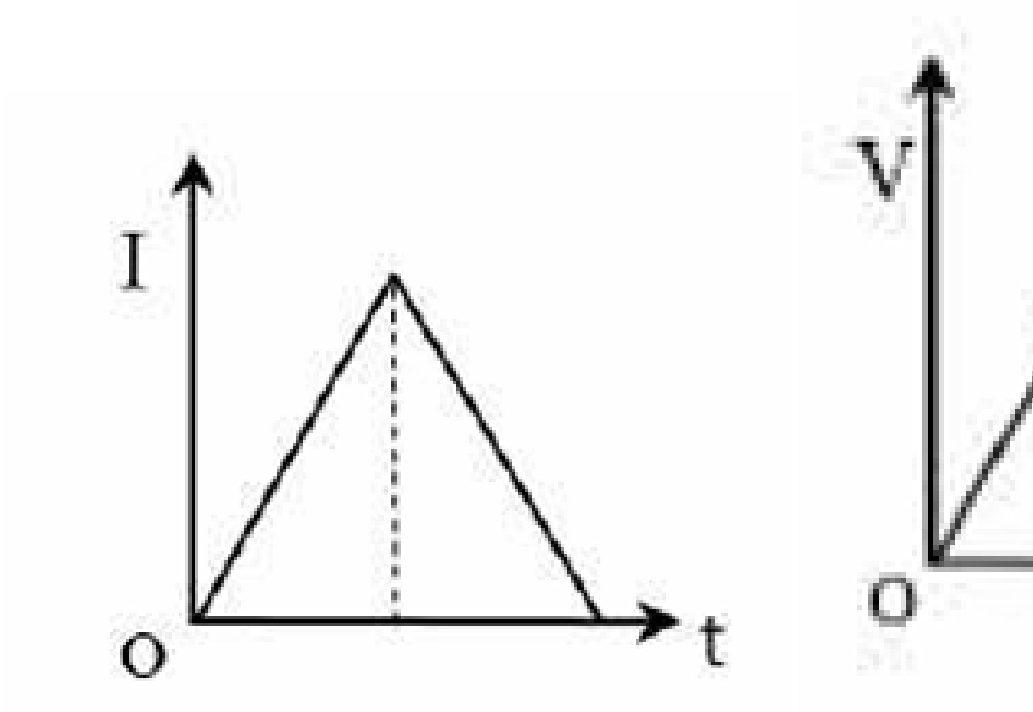
---

113. In an AC circuit containing only an inductor, the current

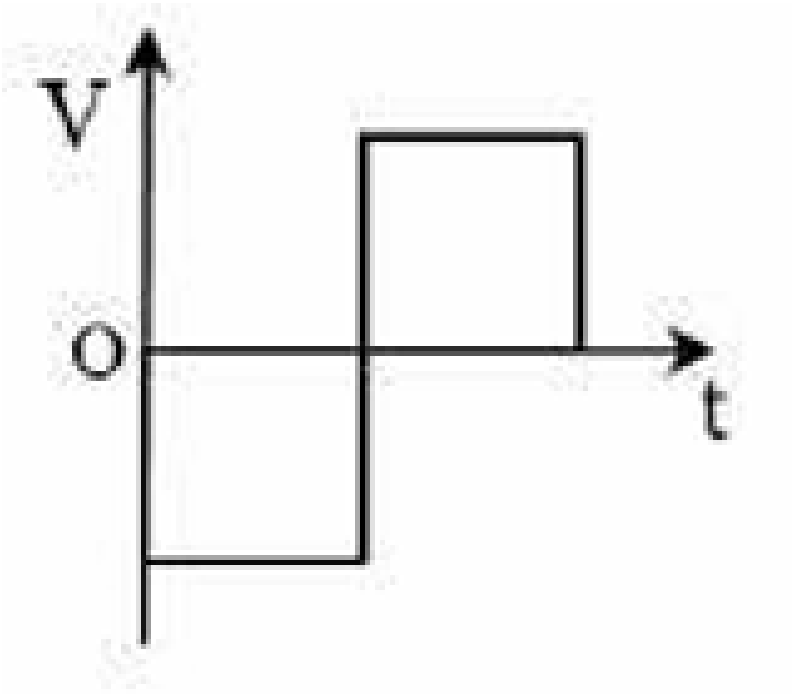
(A) Lags behind the voltage by $\pi/2$

(B) Leads the voltage by $\pi/2$

(C) Is in phase with the voltage



(D) Lags behind the voltage by π



Correct Answer: (A) Lags behind the voltage by $\pi/2$

Solution:

Step 1: Understanding the Concept:

In a purely inductive AC circuit, the opposition to the current is provided by the back EMF produced due to the changing magnetic flux. This causes a phase difference between the voltage and the current.

Key Formula or Approach:

Voltage across an inductor: $V = L\frac{di}{dt}$.

If current is $i = I_0 \sin(\omega t)$.

Step 2: Detailed Explanation:

1. Substitute the current into the voltage formula:

$$V = L\frac{d}{dt}[I_0 \sin(\omega t)]$$

$$V = L\omega I_0 \cos(\omega t)$$

2. Use the trigonometric identity $\cos\theta = \sin(\theta + \pi/2)$:

$$V = V_0 \sin(\omega t + \pi/2)$$

3. Compare the phase of current (ωt) and voltage ($\omega t + \pi/2$).
The voltage phase is higher than the current phase by $\pi/2$.
This means the current "lags" the voltage, or the voltage "leads" the current by 90 degrees.

Step 3: Final Answer:
The current lags behind the voltage by $\pi/2$ radians.
This matches Option (A).

Quick Tip

Quick Tip: Use the mnemonic **CIVIL**:
In a **C**apacitor, **I** (current) leads **V** (voltage).
V (voltage) leads **I** (current) in an **L** (inductor).

---

114. The velocity of electromagnetic waves in vacuum is given by

(A) $1/\sqrt{\mu_0\epsilon_0}$
(B) $\sqrt{\mu_0\epsilon_0}$
(C) $\mu_0\epsilon_0$
(D) $1/(\mu_0\epsilon_0)$

Correct Answer: (A) $1/\sqrt{\mu_0\epsilon_0}$

Solution:

Step 1: Understanding the Concept:
James Clerk Maxwell predicted that changing electric and magnetic fields propagate through space as waves. He derived the speed of these waves using the constants of electricity and

magnetism.

Step 2: Detailed Explanation:

From Maxwell's electromagnetic equations, the wave equation for electric and magnetic fields in a vacuum reveals that the speed of propagation (c) is linked to:

1. μ_0: Permeability of free space ($4\pi \times 10^{-7}\,\mathrm{T \cdot m/A}$).
2. ϵ_0: Permittivity of free space ($8.854 \times 10^{-12}\,\mathrm{C^2/N \cdot m^2}$).

The derived mathematical relationship is:

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

When these values are substituted into the formula, the result is approximately $3 \times 10^8\,\mathrm{m/s}$, which matches the measured speed of light. This was the first evidence that light is an electromagnetic wave.

Step 3: Final Answer:

The velocity is given by the expression $1/\sqrt{\mu_0 \epsilon_0}$.

This matches Option (A).

Quick Tip

Quick Tip: In a medium with refractive index n, the speed is $v = c/n = 1/\sqrt{\mu\epsilon}$. In vacuum, we use the "0" subscripts for both constants.

---

115. A convex lens of focal length 20 cm is placed in contact with a concave lens of focal length 40 cm. The power of the combination is

(A) +2.5 D

(B) -2.5 D

(C) +5 D

(D) -5 D

Correct Answer: (A) +2.5 D

Solution:

Step 1: Understanding the Concept:

The power of a lens is the reciprocal of its focal length measured in meters ($P = 1/f$). When two thin lenses are in contact, their total power is the algebraic sum of their individual powers.

Step 2: Detailed Explanation:

1. **Lens 1 (Convex)**: Focal length $f_1 = +20$ cm.

Convert to meters: $f_1 = 0.2$ m.

Power $P_1 = \frac{1}{0.2} = +5$ D.

2. **Lens 2 (Concave)**: Focal length $f_2 = -40$ cm. (Concave lenses have negative focal length).

Convert to meters: $f_2 = -0.4$ m.

Power $P_2 = \frac{1}{-0.4} = -2.5$ D.

3. **Combined Power**:

$$P = P_1 + P_2$$

$$P = 5 + (-2.5) = +2.5 \text{ D}$$

The positive sign indicates that the combination behaves like a converging (convex) lens.

Step 3: Final Answer:

The power of the combination is +2.5 D.

This matches Option (A).

Quick Tip

Quick Tip: Always convert cm to meters before calculating power. Also, strictly follow the sign convention: convex lenses are + and concave lenses are −.

116. In Young's double-slit experiment, the fringe width is proportional to

(A) λ

(B) $1/\lambda$

(C) d

(D) $1/D$

Correct Answer: (A) λ

Solution:

Step 1: Understanding the Concept:

Fringe width (β) is the distance between two consecutive bright or dark fringes on the screen in an interference pattern. Its value depends on the properties of the light and the geometry of the experimental setup.

Key Formula or Approach:

The fringe width formula is:

$$\beta = \frac{\lambda D}{d}$$

where λ is wavelength, D is distance to screen, and d is slit separation.

Step 2: Detailed Explanation:

From the formula $\beta = \frac{\lambda D}{d}$, we can observe the following relationships:

1. $\beta \propto \lambda$: Fringe width is directly proportional to the wavelength of light. Using red light (longer λ) produces wider fringes than blue light.
2. $\beta \propto D$: Fringe width is directly proportional to the distance of the screen from the slits.
3. $\beta \propto 1/d$: Fringe width is inversely proportional to the separation between the slits.

The question specifically asks what β is proportional to.

Step 3: Final Answer:

The fringe width is proportional to λ.

This matches Option (A).

Quick Tip

Quick Tip: If the entire YDSE setup is immersed in a liquid of refractive index μ, the wavelength decreases to λ/μ, and thus the fringe width also decreases to β/μ.

---

117. The work function of a metal depends on

(A) Nature of the metal

(B) Frequency of incident light

(C) Intensity of incident light

(D) Velocity of incident light

Correct Answer: (A) Nature of the metal

Solution:

Step 1: Understanding the Concept:

The work function (ϕ) is the minimum amount of energy required to just eject an electron from the surface of a metal. It is a fundamental property involved in the Photoelectric Effect.

Step 2: Detailed Explanation:

1. The work function is an intrinsic characteristic of the material itself. It depends on how tightly the electrons are bound to the metal's atomic lattice.

2. Different metals have different electronic structures, and thus require different energies to free an electron. For example, alkali metals have low work functions, while transition metals have higher ones.

3. **Analysis of other options**:

- Frequency (ν) affects the energy of the *incoming photons* ($E = h\nu$), but not the property of the metal itself.

- Intensity affects the *number* of photons (and thus the current), but not the per-photon interaction energy barrier.

- Velocity of light is a constant in a vacuum.

Step 3: Final Answer:

The work function depends solely on the nature of the metal (and its surface condition).

This matches Option (A).

Quick Tip

Quick Tip: Remember:

Work function → Material property.

Photon energy → Light property (Frequency).

Saturation current → Light property (Intensity).

---

118. According to Bohr's model, the angular momentum of an electron in a stable orbit is an integral multiple of

(A) $h/2\pi$

(B) h

(C) $2\pi/h$

(D) h/π

Correct Answer: (A) $h/2\pi$

Solution:

Step 1: Understanding the Concept:

Niels Bohr's second postulate introduced the quantization of angular momentum to explain why electrons only occupy specific "stable" orbits without radiating energy.

Key Formula or Approach:

The quantization condition is:

$$L = mvr = \frac{nh}{2\pi}$$

Step 2: Detailed Explanation:

1. Bohr suggested that the angular momentum (L) of an orbiting electron is quantized.
2. It can only take values that are integer multiples ($n = 1, 2, 3, \ldots$) of a fundamental unit.
3. This fundamental unit is $\hbar = \frac{h}{2\pi}$, where h is Planck's constant.
4. Therefore, the electron can only move in those orbits where its angular momentum is $\frac{h}{2\pi}, \frac{2h}{2\pi}, \frac{3h}{2\pi}$, etc.

Step 3: Final Answer:

The angular momentum is an integral multiple of $h/2\pi$.

This matches Option (A).

Quick Tip

Quick Tip: In modern physics, we often write $\frac{h}{2\pi}$ as $\hbar$ (pronounced "h-bar"). Keep an eye out for this symbol in more advanced versions of this question.

---

119. The half-life of a radioactive substance is 10 days. The decay constant is

(A) 0.0693 / day

(B) 0.693 / day

(C) 6.93 / day

(D) 0.00693 / day

Correct Answer: (A) 0.0693 / day

Solution:

Step 1: Understanding the Concept:

The rate of decay of a radioactive substance is proportional to the number of nuclei present. The time taken for half of the initial quantity to decay is called the half-life ($T_{1/2}$). The decay constant (λ) represents the probability of decay per unit time.

Key Formula or Approach:

The relationship is:

$$\lambda = \frac{\ln 2}{T_{1/2}} \approx \frac{0.693}{T_{1/2}}$$

Step 2: Detailed Explanation:

Given:

Half-life $T_{1/2} = 10$ days.

Using the formula:

$$\lambda = \frac{0.693}{T_{1/2}}$$

Substitute the value of half-life:

$$\lambda = \frac{0.693}{10}$$

$$\lambda = 0.0693\,\text{day}^{-1}$$

Step 3: Final Answer:

The decay constant is 0.0693 / day.

This matches Option (A).

Quick Tip

Quick Tip: Remember that $\ln 2$ is approximately 0.693. Division by multiples of 10 is common in these problems, so just shift the decimal point of 0.693 to the left once.

---

120. A p-n junction diode acts as a closed switch when it is

(A) Forward biased

(B) Reverse biased

(C) Unbiased

(D) Breakdown biased

Correct Answer: (A) Forward biased

Solution:

Step 1: Understanding the Concept:

A diode is a semiconductor device that typically allows current to flow in one direction only. This behavior makes it useful as a switch in electronic circuits.

Step 2: Detailed Explanation:

1. **Forward Bias**: When the positive terminal of the battery is connected to the p-type region and the negative terminal to the n-type region.

- The depletion layer width decreases.
- Resistance becomes very low (ideally zero).
- Current flows easily, acting like a **closed switch**.

2. **Reverse Bias**: When the positive terminal is connected to the n-type and negative to the p-type.

- The depletion layer width increases.
- Resistance becomes very high (ideally infinite).
- Practically no current flows, acting like an **open switch**.

Step 3: Final Answer:

The diode acts as a closed switch when it is forward biased.

This matches Option (A).

Quick Tip

Quick Tip: Closed switch = ON = Conducting = Forward Bias.

Open switch = OFF = Insulating = Reverse Bias.

---

121. The de Broglie wavelength of a particle of mass m moving with a velocity v is given by

(A) h/mv

(B) mv/h

(C) hm/v

(D) h/m^2v

Correct Answer: (A) h/mv

Solution:

Step 1: Understanding the Concept:

The de Broglie hypothesis states that every moving particle has an associated wave-like nature. The wavelength of this "matter wave" depends on the momentum of the particle.

Key Formula or Approach:

The de Broglie relation is given by:

$$\lambda = \frac{h}{p}$$

where λ is the wavelength, h is Planck's constant, and p is the linear momentum.

Step 2: Detailed Explanation:

Linear momentum (p) of a particle is the product of its mass (m) and its velocity (v). Mathematically:

$$p = mv$$

Substitute the expression for p into the de Broglie formula:

$$\lambda = \frac{h}{mv}$$

This equation shows that the wavelength is inversely proportional to both the mass and the velocity of the particle. Heavier or faster particles have shorter de Broglie wavelengths.

Step 3: Final Answer:

The de Broglie wavelength is h/mv.

This matches Option (A).

Quick Tip

Quick Tip: Remember that $\lambda \propto 1/p$. If the kinetic energy (K) is given instead of velocity, use the relation $p = \sqrt{2mK}$, which makes $\lambda = \frac{h}{\sqrt{2mK}}$.

---

122. Which of the following quantum numbers determines the shape of an orbital?

(A) Azimuthal quantum number

(B) Principal quantum number

(C) Magnetic quantum number

(D) Spin quantum number

Correct Answer: (A) Azimuthal quantum number

Solution:

Step 1: Understanding the Concept:

Quantum numbers are a set of four values used to describe the unique quantum state of an electron in an atom, including its energy, location, and spin properties.

Step 2: Detailed Explanation:

Each quantum number has a specific role:

1. **Principal Quantum Number (n)**: Determines the size and the main energy level (shell) of the orbital.

2. **Azimuthal Quantum Number (l)**: Also known as the orbital angular momentum quantum number. it defines the three-dimensional shape of the orbital (s, p, d, f).

- For $l = 0$, the shape is spherical (s).

- For $l = 1$, the shape is dumbbell (p).

- For $l = 2$, the shape is double-dumbbell (d).

3. **Magnetic Quantum Number (m_l)**: Determines the spatial orientation of the orbital in space.
4. **Spin Quantum Number (m_s)**: Describes the direction of the electron's spin ($+1/2$ or $-1/2$).

Step 3: Final Answer:
The Azimuthal quantum number determines the shape of the orbital.
This matches Option (A).

Quick Tip

Quick Tip: Size $\rightarrow n$
Shape $\rightarrow l$
Orientation $\rightarrow m_l$
Spin $\rightarrow m_s$
Remembering this "keyword" mapping helps solve quantum number questions in seconds.

---

123. The elements with atomic numbers 9, 17, 35, 53 belong to the family of

(A) Halogens
(B) Alkali metals
(C) Alkaline earth metals
(D) Noble gases

Correct Answer: (A) Halogens

Solution:
Step 1: Understanding the Concept:
Elements in the periodic table are grouped into families based on their valence electron configurations and chemical reactivity. Atomic numbers uniquely identify these elements.

Step 2: Detailed Explanation:
Let us identify the elements based on the given atomic numbers:

1. $Z = 9$: Fluorine (F)
2. $Z = 17$: Chlorine (Cl)
3. $Z = 35$: Bromine (Br)
4. $Z = 53$: Iodine (I)

These elements are all located in Group 17 (VIIA) of the periodic table. They all have 7 electrons in their outermost shell (ns^2np^5) and are highly reactive non-metals. This group is collectively known as the **Halogens**.

- Alkali metals are in Group 1 $(H, Li, Na \ldots)$.
- Alkaline earth metals are in Group 2 $(Be, Mg, Ca \ldots)$.
- Noble gases are in Group 18 $(He, Ne, Ar \ldots)$.

Step 3: Final Answer:

The elements belong to the Halogens family.

This matches Option (A).

Quick Tip

Quick Tip: Memorize the atomic numbers of the first few members of each group. Halogens are easily remembered as "Fluorine (9), then add 8 for Chlorine (17), then add 18 for Bromine (35), then add 18 for Iodine (53)".

---

124. Which of the following molecules has a linear shape?

(A) CO_2

(B) H_2O

(C) SO_2

(D) NH_3

Correct Answer: (A) CO_2

Solution:

Step 1: Understanding the Concept:

Molecular geometry is determined by the Valence Shell Electron Pair Repulsion (VSEPR) theory, which states that electron pairs around a central atom arrange themselves as far apart as possible to minimize repulsion.

Step 2: Detailed Explanation:

Let's analyze the geometry of each option:

1. **CO_2**: Carbon is the central atom with 4 valence electrons. It forms two double bonds with two Oxygen atoms. There are 2 bond pairs and 0 lone pairs around C. According to VSEPR, 2 electron pairs result in a **linear** shape with a bond angle of 180°.

2. **H_2O**: Oxygen has 6 valence electrons. It forms two single bonds with Hydrogen and has 2 lone pairs. This sp^3 arrangement with lone pairs results in a **Bent (V-shape)**.

3. **SO_2**: Sulfur has 6 valence electrons. It forms two double bonds and has 1 lone pair. This sp^2 arrangement results in a **Bent** shape.

4. **NH_3**: Nitrogen has 5 valence electrons. It forms three bonds and has 1 lone pair. This results in a **Trigonal Pyramidal** shape.

Step 3: Final Answer:

Carbon dioxide (CO_2) has a linear shape.

This matches Option (A).

Quick Tip

Quick Tip: A central atom with exactly two double bonds and no lone pairs (like CO_2, $BeCl_2$, or C_2H_2) will always be linear.

---

125. The hybridization of carbon in methane (CH_4) is

(A) sp^3

(B) sp^2

(C) sp

(D) dsp^2

Correct Answer: (A) sp^3

Solution:

Step 1: Understanding the Concept:

Hybridization involves the mixing of atomic orbitals to form new hybrid orbitals suitable for bonding. The type of hybridization depends on the number of sigma bonds and lone pairs (steric number) around the central atom.

Step 2: Detailed Explanation:

In methane (CH_4):

1. The central atom is Carbon ($Z = 6$). Its ground state configuration is $1s^2 2s^2 2p^2$.

2. In the excited state, one electron from the $2s$ orbital jumps to the $2p$ orbital, giving 4 unpaired electrons ($2s^1 2p^3$).

3. These four orbitals (one s and three p) mix to form four equivalent **sp^3** hybrid orbitals.

4. Each hybrid orbital overlaps with a $1s$ orbital of a Hydrogen atom to form 4 sigma (σ) bonds.

The molecule takes a tetrahedral geometry to minimize repulsion between these four bond pairs.

Step 3: Final Answer:

The hybridization of carbon in CH_4 is sp^3.

This matches Option (A).

Quick Tip

Quick Tip: Count the number of items (atoms + lone pairs) attached to the central atom:

4 items $\rightarrow sp^3$

3 items $\rightarrow sp^2$

2 items $\rightarrow sp$

In CH_4, there are 4 H-atoms and 0 lone pairs, so steric number = 4.

---

126. According to the kinetic theory of gases, the absolute temperature of a gas is directly proportional to

(A) Average kinetic energy of molecules

(B) Average velocity of molecules

(C) Average potential energy of molecules

(D) Volume of the gas

Correct Answer: (A) Average kinetic energy of molecules

Solution:

Step 1: Understanding the Concept:

The kinetic theory of gases models a gas as a large number of submicroscopic particles (atoms or molecules), all of which are in constant, random motion. Temperature is a macroscopic measure of this microscopic motion.

Key Formula or Approach:

The average kinetic energy ($\bar{K}$) of a gas molecule is given by:

$$\bar{K} = \frac{3}{2}k_B T$$

where k_B is the Boltzmann constant and T is the absolute temperature.

Step 2: Detailed Explanation:

1. The fundamental postulate of the kinetic theory is that the pressure and temperature of a gas arise from the motion of its molecules.
2. From the formula $\bar{K} = \frac{3}{2}k_B T$, it is evident that as the temperature (T) increases, the average kinetic energy of the molecules increases linearly.
3. Conversely, at absolute zero (0 K), the average kinetic energy of the molecules becomes zero (classically).
4. Absolute temperature is a direct measure of the *intensity* of the molecular motion. It is not directly proportional to velocity (it's proportional to velocity squared) or volume (that depends on pressure).

Step 3: Final Answer:

Absolute temperature is directly proportional to the average kinetic energy of the molecules. This matches Option (A).

Quick Tip

Quick Tip: Remember: $T \propto$ K.E. and $T \propto v_{rms}^2$. In exams, they often try to trick you by saying T is proportional to "velocity". Always check for the "average kinetic energy" option first.

---

127. The value of the gas constant R in J K^{-1} mol^{-1} is

(A) 8.314

(B) 0.0821

(C) 1.987

(D) 83.14

Correct Answer: (A) 8.314

Solution:

Step 1: Understanding the Concept:

The universal gas constant (R) is a physical constant featured in many fundamental equations in the physical sciences, such as the ideal gas law $PV = nRT$. Its numerical value depends on the units used for pressure, volume, and temperature.

Step 2: Detailed Explanation:

In the SI (International System of Units):

1. Pressure (P) is in Pascals (N/m^2).
2. Volume (V) is in cubic meters (m^3).
3. Temperature (T) is in Kelvin (K).

Under these conditions, R has the units of Joules per Kelvin per mole. The standard experimentally determined value is:

$$R = 8.314\,\text{J K}^{-1}\,\text{mol}^{-1}$$

Other values of R:

- $R = 0.0821\,\text{L} \cdot \text{atm K}^{-1}\,\text{mol}^{-1}$ (When volume is in Liters and pressure is in atm).

- $R \approx 2\,\text{cal K}^{-1}\,\text{mol}^{-1}$ (When energy is measured in calories).

Step 3: Final Answer:

The value in $\text{J K}^{-1}\,\text{mol}^{-1}$ is 8.314.

This matches Option (A).

Quick Tip

Quick Tip: Always check the units specified in the question! If the question used "L-atm", the answer would be 0.0821. If it asked for SI units, it is always 8.314.

---

128. For a spontaneous process at constant temperature and pressure, the change in Gibbs free energy (ΔG) is

(A) Negative

(B) Positive

(C) Zero

(D) Infinite

Correct Answer: (A) Negative

Solution:

Step 1: Understanding the Concept:

Gibbs free energy (G) is a thermodynamic potential that can be used to calculate the maximum of reversible work that may be performed by a thermodynamic system at a constant temperature and pressure. It is the definitive criterion for the spontaneity of a chemical reaction.

Key Formula or Approach:

The Gibbs-Helmholtz equation:

$$\Delta G = \Delta H - T\Delta S$$

Step 2: Detailed Explanation:

The Second Law of Thermodynamics dictates the direction of spontaneous change:

1. **Spontaneous Process**: A process that occurs on its own without external influence. For this to happen at constant T and P, the system must move to a state of lower free energy. Thus, $\Delta G < 0$ (**Negative**).
2. **Non-Spontaneous Process**: The process requires work input. $\Delta G > 0$ (**Positive**).
3. **Equilibrium**: The system is at its lowest energy state. $\Delta G = 0$.

Step 3: Final Answer:

For a spontaneous process, ΔG must be negative.

This matches Option (A).

Quick Tip

Quick Tip: Mnemonic: "G is for Go". If ΔG is negative, the reaction can "Go" (is spontaneous).

---

129. For the reaction $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$, the relation between K_p and K_c is

(A) $K_p = K_c(RT)^{-2}$

(B) $K_p = K_c(RT)^2$

(C) $K_p = K_c(RT)^{-1}$

(D) $K_p = K_c(RT)$

Correct Answer: (A) $K_p = K_c(RT)^{-2}$

Solution:

Step 1: Understanding the Concept:

The equilibrium constant can be expressed in terms of partial pressures (K_p) or molar concentrations (K_c). Their relationship depends on the change in the number of gaseous moles during the reaction.

Key Formula or Approach:

The general relationship is:

$$K_p = K_c(RT)^{\Delta n_g}$$

where $\Delta n_g = (\text{moles of gaseous products}) - (\text{moles of gaseous reactants})$.

Step 2: Detailed Explanation:

Given reaction: $1N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$.

1. Identify gaseous product moles: 2 (from $2NH_3$).
2. Identify gaseous reactant moles: $1 + 3 = 4$ (from $1N_2$ and $3H_2$).
3. Calculate Δn_g:

$$\Delta n_g = 2 - (1 + 3) = 2 - 4 = -2$$

4. Substitute into the formula:

$$K_p = K_c(RT)^{-2}$$

This means that for this specific reaction, K_c is numerically larger than K_p.

Step 3: Final Answer:

The relationship is $K_p = K_c(RT)^{-2}$.

This matches Option (A).

Quick Tip

Quick Tip: Always count only gaseous species! If a solid or liquid appears in the reaction, its coefficient is ignored in the Δn_g calculation.

---

130. Which of the following is a conjugate acid-base pair?

(A) NH_4^+ and NH_3

(B) HCl and $NaOH$

(C) H_2SO_4 and SO_4^{2-}

(D) HNO_3 and H_2O

Correct Answer: (A) NH_4^+ and NH_3

Solution:

Step 1: Understanding the Concept:

According to the Brønsted-Lowry theory, an acid is a proton (H^+) donor and a base is a proton acceptor. A conjugate acid-base pair consists of two species that differ by exactly one proton.

Step 2: Detailed Explanation:

1. **Acid** $\rightarrow$ **Conjugate Base +** H^+

2. **Base +** $H^+ \rightarrow$ **Conjugate Acid**

Let's analyze the pairs:

- **NH_4^+/NH_3**: $NH_4^+ \rightleftharpoons NH_3 + H^+$. They differ by exactly one H^+. Thus, they are a conjugate pair. (NH_4^+ is the acid, NH_3 is the base).

- **HCl/NaOH**: These are a strong acid and a strong base, but they are not a conjugate pair. The conjugate base of HCl is Cl^-.

- **H_2SO_4/SO_4^{2-}**: These differ by two protons ($2H^+$). A conjugate pair must differ by only one. The conjugate base of H_2SO_4 is HSO_4^-.

- **HNO_3/H_2O**: These are two different substances. The conjugate base of HNO_3 is NO_3^-.

Step 3: Final Answer:

The conjugate acid-base pair is NH_4^+ and NH_3.

This matches Option (A).

Quick Tip

Quick Tip: To find a conjugate pair, just add or subtract exactly one H atom and change the charge by exactly one unit. If there is a difference of more than one H, it's not a conjugate pair.

131. The pH of a 0.01 M NaOH solution is

(A) 12

(B) 2

(C) 10

(D) 7

Correct Answer: (A) 12

Solution:

Step 1: Understanding the Concept:

Sodium hydroxide ($NaOH$) is a strong base that undergoes complete dissociation in aqueous solution.

To find the pH, we first need to determine the concentration of hydroxide ions ($[OH^-]$), then calculate the pOH, and finally use the relationship between pH and pOH.

Key Formula or Approach:

1. $pOH = -\log_{10}[OH^-]$
2. $pH + pOH = 14$ (at 25°C)

Step 2: Detailed Explanation:

Given the concentration of $NaOH$ is 0.01 M.

Since $NaOH$ dissociates completely:

$$NaOH \rightarrow Na^+ + OH^-$$

The concentration of hydroxide ions is:

$$[OH^-] = 0.01\,\text{M} = 10^{-2}\,\text{M}$$

Now, calculate the pOH:

$$pOH = -\log_{10}(10^{-2})$$

$$pOH = 2$$

Finally, calculate the pH:

$$pH = 14 - pOH$$

$$pH = 14 - 2 = 12$$

The high pH value confirms that the solution is strongly basic.

Step 3: Final Answer:

The pH of the solution is 12.

This matches Option (A).

Quick Tip

Quick Tip: For strong bases, if the concentration is 10^{-n} M, the pOH is simply n.

Then, $pH = 14 - n$.

For 0.01 M (10^{-2}), $pH = 14 - 2 = 12$.

---

132. The oxidation number of sulphur in H_2SO_4 is

(A) +6

(B) +4

(C) -2

(D) 0

Correct Answer: (A) +6

Solution:

Step 1: Understanding the Concept:

The oxidation number is the hypothetical charge an atom would have if all bonds were ionic.

In a neutral molecule, the sum of the oxidation numbers of all atoms must equal zero.

Key Formula or Approach:

Use the standard oxidation numbers:

1. Hydrogen $(H) = +1$ (when bonded to non-metals).
2. Oxygen $(O) =$ -2 (in most compounds like sulfates).

Step 2: Detailed Explanation:

Let the oxidation number of Sulphur (S) be x.

The molecule is H_2SO_4.

Sum of oxidation numbers:

$$2(\text{Ox. No. of } H) + 1(\text{Ox. No. of } S) + 4(\text{Ox. No. of } O) = 0$$

Substitute the known values:

$$2(+1) + x + 4(-2) = 0$$

$$2 + x - 8 = 0$$

$$x - 6 = 0$$

$$x = +6$$

Sulphur is in its maximum oxidation state in sulfuric acid.

Step 3: Final Answer:

The oxidation number of sulphur in H_2SO_4 is +6.

This matches Option (A).

Quick Tip

Quick Tip: Sulphur belongs to Group 16. Its oxidation state can range from -2 to +6. Finding +6 in H_2SO_4 is consistent with it being in its highest possible state.

---

133. Temporary hardness of water is due to the presence of

(A) Calcium and magnesium bicarbonates

(B) Calcium and magnesium chlorides

(C) Calcium and magnesium sulphates

(D) Sodium carbonate

Correct Answer: (A) Calcium and magnesium bicarbonates

Solution:

Step 1: Understanding the Concept:

Hardness of water is caused by the presence of multivalent metal cations, primarily Ca^{2+} and Mg^{2+}. It is classified into two types: temporary and permanent.

Step 2: Detailed Explanation:

1. **Temporary Hardness**: This type of hardness is caused by the dissolved bicarbonates of calcium and magnesium, namely $Ca(HCO_3)_2$ and $Mg(HCO_3)_2$.
It is called "temporary" because it can be easily removed by simple boiling, which converts the soluble bicarbonates into insoluble carbonates.
2. **Permanent Hardness**: This is caused by the chlorides and sulphates of calcium and magnesium ($CaCl_2, MgCl_2, CaSO_4, MgSO_4$).
Simple boiling cannot remove permanent hardness; it requires chemical treatments like the ion-exchange or Clark's process.

Option (A) specifically lists the bicarbonates, which are the source of temporary hardness.

Step 3: Final Answer:

Temporary hardness is due to calcium and magnesium bicarbonates.

This matches Option (A).

Quick Tip

Quick Tip: Mnemonic: **B**icarbonates = **B**oiling (Temporary).

Chlorides and Sulphates = Permanent.

---

134. Which of the following alkali metals has the lowest melting point?

(A) Cs

(B) Li

(C) Na

(D) K

Correct Answer: (A) Cs

Solution:

Step 1: Understanding the Concept:

Melting point in metals depends on the strength of the metallic bond. Metallic bond strength depends on the number of valence electrons and the size of the atoms.

Step 2: Detailed Explanation:

1. **Group 1 Trend**: As we move down the group from Lithium (Li) to Cesium (Cs), the atomic size increases significantly.
2. **Metallic Bonding**: In alkali metals, there is only one valence electron per atom contributing to the "sea" of electrons.
3. **Effect of Size**: As the atom gets larger, the distance between the nucleus and the delocalized valence electrons increases. This weakens the electrostatic attraction that constitutes the metallic bond.

4. **Melting Point**: Since the metallic bond becomes weaker as we go down the group, the energy required to break the lattice decreases. Thus, the melting point decreases from Li to Cs.

Among the given options, Cesium (Cs) is the furthest down in the group and thus has the weakest metallic bond and the lowest melting point.

Step 3: Final Answer:

Cesium (Cs) has the lowest melting point among the given alkali metals.

This matches Option (A).

Quick Tip

Quick Tip: Remember the trend for Group 1: $Li > Na > K > Rb > Cs$.

Cesium's melting point is so low ($\approx 28.4^\circ$C) that it can almost melt in your hand on a warm day.

---

135. In diborane (B_2H_6), the number of bridge hydrogens is

(A) 2

(B) 4

(C) 6

(D) 0

Correct Answer: (A) 2

Solution:

Step 1: Understanding the Concept:

Diborane (B_2H_6) is an electron-deficient molecule. Its structure involves unique bonding called three-center two-electron (3c-2e) bonds.

Step 2: Detailed Explanation:

1. **Molecular Structure**: In B_2H_6, each Boron atom is sp^3 hybridized.

2. **Terminal Hydrogens**: There are four hydrogen atoms located at the ends of the molecule.

Each Boron forms two regular 2-center 2-electron covalent bonds with these terminal hydrogens.

3. **Bridge Hydrogens**: The remaining two hydrogen atoms are located in a plane perpendicular to the terminal hydrogens. These two atoms act as "bridges" between the two Boron atoms.

4. **Bonding**: These bridge hydrogens form two "banana bonds" or 3c-2e bonds, where one pair of electrons is shared among three atoms ($B-H-B$).

Thus, there are exactly 2 hydrogen atoms involved in bridging.

Step 3: Final Answer:

The number of bridge hydrogens in diborane is 2.

This matches Option (A).

Quick Tip

Quick Tip: Structure summary of B_2H_6:

4 Terminal Hydrogens (Regular bonds).

2 Bridge Hydrogens (3c-2e bonds).

---

136. Which allotrope of carbon is thermodynamically most stable?

(A) Graphite

(B) Diamond

(C) Fullerene

(D) Coal

Correct Answer: (A) Graphite

Solution:

Step 1: Understanding the Concept:

Thermodynamic stability of an allotrope is determined by its standard enthalpy of formation ($\Delta_f H^\circ$). The allotrope with the lowest energy (closest to zero or most negative relative to others) is the most stable.

Step 2: Detailed Explanation:

1. **Graphite**: In graphite, carbon atoms are arranged in hexagonal layers with sp^2 hybridization. This delocalized π-electron system contributes to high stability.

2. **Diamond**: In diamond, carbon is sp^3 hybridized in a rigid tetrahedral 3D lattice. While diamond is physically very hard, it has a slightly higher energy than graphite.

3. **Stability Data**: Experimentally, the $\Delta_f H^\circ$ of graphite is taken as 0.0 kJ/mol (standard state). The enthalpy of formation for diamond is $+1.9\,\text{kJ/mol}$.

4. **Conclusion**: Since graphite has the lower enthalpy value, it is the thermodynamically most stable form of carbon at standard temperature and pressure (STP). Given enough time (millions of years), diamond would theoretically convert to graphite.

Step 3: Final Answer:

Graphite is the thermodynamically most stable allotrope of carbon.

This matches Option (A).

Quick Tip

Quick Tip: Even though diamond is the "hardest" and often thought of as permanent, from a chemistry/energy perspective, Graphite is the "king" of stability.

---

137. The IUPAC name of $CH_3-CH_2-CH(CH_3)-CH_3$ is

(A) 2-Methylbutane

(B) Isopentane

(C) Pentane

(D) 3-Methylbutane

Correct Answer: (A) 2-Methylbutane

Solution:

Step 1: Understanding the Concept:

IUPAC nomenclature follows specific rules: find the longest continuous carbon chain, identify

substituents, and number the chain so that substituents get the lowest possible locant numbers.

Step 2: Detailed Explanation:

The structure is: $CH_3 - CH_2 - CH(CH_3) - CH_3$.

1. **Find the longest chain**: The longest continuous chain of carbons contains 4 carbon atoms. Thus, the parent alkane is **butane**.

2. **Identify substituents**: There is a methyl group ($-CH_3$) attached to the main chain.

3. **Number the chain**:

- If numbered from left to right: $C_1 - C_2 - C_3(CH_3) - C_4$. The methyl is at position 3.

- If numbered from right to left: $C_4 - C_3 - C_2(CH_3) - C_1$. The methyl is at position 2.

4. **Apply lowest locant rule**: 2 is lower than 3. Therefore, we use position 2.

Combining these parts, the name becomes **2-Methylbutane**.

Note: Isopentane is the common name, but the question asks for the IUPAC name.

Step 3: Final Answer:

The IUPAC name is 2-Methylbutane.

This matches Option (A).

Quick Tip

Quick Tip: Always start numbering from the end closer to the branch. "3-Methylbutane" and "2-Methylbutane" represent the same molecule, but the one with the smaller number (2) is correct per IUPAC rules.

---

138. Which of the following exhibits geometrical isomerism?

(A) 2-Butene

(B) 1-Butene

(C) Propene

(D) 2-Methylpropene

Correct Answer: (A) 2-Butene

Solution:

Step 1: Understanding the Concept:

Geometrical (cis-trans) isomerism occurs in alkenes due to restricted rotation around the carbon-carbon double bond ($C = C$).

Key Formula or Approach:

For a molecule to show geometrical isomerism, each carbon of the double bond must be attached to two **different** groups.

General form: $abC = Ccd$ (where $a \neq b$ and $c \neq d$).

Step 2: Detailed Explanation:

Let's check each option:

1. **2-Butene** ($CH_3 - CH = CH - CH_3$):

- Left Carbon is attached to H and CH_3. (Different)

- Right Carbon is attached to H and CH_3. (Different)

- It can exist as **cis-2-butene** and **trans-2-butene**. Thus, it shows geometrical isomerism.

2. **1-Butene** ($CH_2 = CH - CH_2 - CH_3$):

- The terminal carbon is attached to two H atoms. (Same). No isomerism.

3. **Propene** ($CH_2 = CH - CH_3$):

- The terminal carbon is attached to two H atoms. (Same). No isomerism.

4. **2-Methylpropene** ($(CH_3)_2C = CH_2$):

- Both carbons have identical groups (two methyls on one, two hydrogens on the other). No isomerism.

Step 3: Final Answer:

2-Butene exhibits geometrical isomerism.

This matches Option (A).

Quick Tip

Quick Tip: Terminal alkenes (where the double bond is at the end of the chain, like 1-butene or propene) almost never show geometrical isomerism because the $=CH_2$ group contains two identical Hydrogen atoms.

---

139. The reaction of benzene with methyl chloride in the presence of anhydrous $AlCl_3$ is called

(A) Friedel-Crafts alkylation

(B) Wurtz reaction

(C) Fittig reaction

(D) Friedel-Crafts acylation

Correct Answer: (A) Friedel-Crafts alkylation

Solution:

Step 1: Understanding the Concept:

Electrophilic aromatic substitution is a reaction where an atom attached to an aromatic ring (usually hydrogen) is replaced by an electrophile.

Step 2: Detailed Explanation:

1. **Reaction Components**: Benzene (C_6H_6) is the substrate. Methyl chloride (CH_3Cl) is the alkylating agent. Anhydrous $AlCl_3$ acts as a Lewis acid catalyst.
2. **Mechanism**: The catalyst $AlCl_3$ reacts with CH_3Cl to generate a methyl carbocation (CH_3^+), which acts as a strong electrophile.
3. **Product**: The methyl group substitutes a hydrogen on the benzene ring to form Toluene ($C_6H_5CH_3$).
4. **Classification**:

- Because an **alkyl** group is being added, it is called **Friedel-Crafts alkylation**.
- If an acyl group ($RCO-$) were added using an acid chloride, it would be Friedel-Crafts acylation.
- Wurtz and Fittig reactions involve sodium metal and aryl/alkyl halides, not benzene directly.

Step 3: Final Answer:

The reaction is Friedel-Crafts alkylation.

This matches Option (A).

Quick Tip

Quick Tip: Remember the catalyst: Friedel-Crafts reactions always require a Lewis acid catalyst like anhydrous $AlCl_3$ or $FeCl_3$ to generate the electrophile from the halide.

---

140. Which of the following gases causes greenhouse effect?

(A) CO_2
(B) N_2
(C) O_2
(D) Ar

Correct Answer: (A) CO_2

Solution:

Step 1: Understanding the Concept:

The greenhouse effect is a natural process where certain gases in the atmosphere trap infrared radiation (heat) emitted from the Earth's surface, preventing it from escaping into space. This maintains the Earth's temperature.

Step 2: Detailed Explanation:

1. **Greenhouse Gases (GHGs)**: To be a greenhouse gas, a molecule must be able to absorb infrared radiation. This usually requires a change in the molecule's dipole moment during vibration.

2. **Carbon Dioxide (CO_2)**: It is one of the primary greenhouse gases. Although it is a symmetric linear molecule, its vibrations (like asymmetric stretching and bending) create a temporary dipole, allowing it to absorb heat effectively.

3. **Analysis of other options**:

- **Nitrogen (N_2)** and **Oxygen (O_2)**: These are homonuclear diatomic molecules. They have no dipole moment and their vibrations do not change their dipole. They are transparent to

infrared radiation and do not cause the greenhouse effect.

- **Argon (Ar)**: Being a noble gas (monatomic), it has no vibrational modes to absorb infrared energy.

Step 3: Final Answer:

Carbon dioxide (CO_2) is the gas responsible for the greenhouse effect among the choices. This matches Option (A).

Quick Tip

Quick Tip: Major Greenhouse Gases: CO_2, CH_4, N_2O, H_2O (vapor), and O_3. Diatomic homonuclear gases like N_2, O_2, H_2 are never greenhouse gases.

---

141. In a face-centered cubic (fcc) lattice, the number of atoms per unit cell is

(A) 4

(B) 2

(C) 1

(D) 6

Correct Answer: (A) 4

Solution:

Step 1: Understanding the Concept:

A unit cell is the smallest repeating unit of a crystal lattice. In a face-centered cubic (fcc) arrangement, atoms are located at all eight corners and at the centers of all six faces of the cube.

Key Formula or Approach:

Total atoms (Z) = (Atoms at corners × Contribution) + (Atoms at faces × Contribution).

Step 2: Detailed Explanation:

1. **Corner Atoms**: There are 8 corners in a cube. Each corner atom is shared by 8 adjacent unit cells.

Contribution of corner atoms $= 8 \times \frac{1}{8} = 1$ atom.

2. **Face-Centered Atoms**: There are 6 faces in a cube. Each atom at the center of a face is shared by exactly 2 unit cells.

Contribution of face atoms $= 6 \times \frac{1}{2} = 3$ atoms.

3. **Total atoms per unit cell**:

$$Z = 1(\text{from corners}) + 3(\text{from faces}) = 4$$

This high number of atoms per unit cell contributes to the high packing efficiency (74%) of the fcc structure.

Step 3: Final Answer:

The number of atoms per unit cell in an fcc lattice is 4.

This matches Option (A).

Quick Tip

Quick Tip: Simple Cubic (sc): $Z = 1$.

Body-Centered Cubic (bcc): $Z = 2$.

Face-Centered Cubic (fcc): $Z = 4$.

Memorizing these "Z" values for cubic systems is essential for solid-state density calculations.

---

142. The osmotic pressure of a solution is given by the relation

(A) $\pi = CRT$

(B) $\pi = C/RT$

(C) $\pi = RT/C$

(D) $\pi = CR/T$

Correct Answer: (A) $\pi = CRT$

Solution:

Step 1: Understanding the Concept:

Osmotic pressure (π) is a colligative property defined as the pressure required to stop the flow of solvent through a semipermeable membrane into a solution. Van't Hoff observed that for dilute solutions, osmotic pressure follows laws similar to ideal gas laws.

Step 2: Detailed Explanation:

The Van't Hoff equation for osmotic pressure states that for a dilute solution, the osmotic pressure is directly proportional to the molar concentration (C) and the absolute temperature (T).

The mathematical relationship is:

$$\pi = CRT$$

Where:

- π = Osmotic pressure.
- C = Molarity of the solution (n/V).
- R = Universal gas constant.
- T = Temperature in Kelvin.

Since $C = n/V$, the equation can also be written as $\pi V = nRT$, which is identical in form to the ideal gas law.

Step 3: Final Answer:

The correct relation is $\pi = CRT$.

This matches Option (A).

Quick Tip

Quick Tip: Osmotic pressure is the only colligative property typically measured at room temperature. It is specifically useful for determining the molar mass of biomolecules like proteins and polymers because other properties like boiling point elevation could denature them.

143. The unit of rate constant for a first-order reaction is

(A) s^{-1}

(B) $mol\ L^{-1}s^{-1}$

(C) $L\ mol^{-1}s^{-1}$

(D) s

Correct Answer: (A) s^{-1}

Solution:

Step 1: Understanding the Concept:

The rate of a chemical reaction is proportional to the concentration of reactants raised to their stoichiometric powers. The rate constant (k) is the proportionality constant, and its units vary depending on the overall order of the reaction.

Key Formula or Approach:

General units for $k = (mol\ L^{-1})^{1-n}s^{-1}$, where n is the order of the reaction.

Step 2: Detailed Explanation:

For a first-order reaction ($n = 1$):

The rate law is:

$$\text{Rate} = k[A]^1$$

1. Unit of Rate $= mol\ L^{-1}s^{-1}$.
2. Unit of Concentration $[A] = mol\ L^{-1}$.
3. Substitute units into the rate law:

$$(mol\ L^{-1}s^{-1}) = k \cdot (mol\ L^{-1})$$

$$k = \frac{mol\ L^{-1}s^{-1}}{mol\ L^{-1}}$$

$$k = s^{-1} \text{ (or } time^{-1})$$

This unit shows that for first-order reactions, the rate constant is independent of concentration.

Step 3: Final Answer:

The unit of rate constant for a first-order reaction is s^{-1}.

This matches Option (A).

Quick Tip

Quick Tip: Mnemonic:

Zero order: Units of rate ($mol\, L^{-1}s^{-1}$).

First order: Just $time^{-1}$.

Second order: Reciprocal of zero order ($L\, mol^{-1}s^{-1}$).

---

144. According to Faraday's first law of electrolysis, the mass of substance deposited (w) is proportional to

(A) Quantity of electricity (Q)

(B) Current (I) only

(C) Time (t) only

(D) Resistance (R)

Correct Answer: (A) Quantity of electricity (Q)

Solution:

Step 1: Understanding the Concept:

Electrolysis involves the decomposition of an electrolyte by passing an electric current. Michael Faraday established the quantitative relationship between the amount of electricity passed and the mass of material liberated at the electrodes.

Key Formula or Approach:

The law states: $w \propto Q$.

Step 2: Detailed Explanation:

1. Faraday's first law of electrolysis states that the mass of any substance (w) deposited or liberated at any electrode during electrolysis is directly proportional to the quantity of electricity (Q) passed through the electrolyte.
2. The quantity of electricity (Q) is the product of current (I) and time (t): $Q = I \times t$.
3. Therefore:

$$w \propto Q$$

$$w = ZQ = ZIt$$

Where Z is the electrochemical equivalent of the substance.

- Option (B) and (C) are only partial components of Q. The mass depends on both, meaning it depends on the total charge Q.

Step 3: Final Answer:

The mass deposited is proportional to the quantity of electricity (Q).

This matches Option (A).

Quick Tip

Quick Tip: Faraday's 1st Law: $w \propto Q$.

Faraday's 2nd Law: $w \propto E$ (Equivalent weight) for the same Q.

Remember that 1 Faraday ($96500\,C$) deposits exactly 1 equivalent weight of a substance.

---

145. The process of adsorption is always

(A) Exothermic

(B) Endothermic

(C) Isothermal only

(D) Non-spontaneous

Correct Answer: (A) Exothermic

Solution:

Step 1: Understanding the Concept:

Adsorption is a surface phenomenon where particles (adsorbate) accumulate on the surface of a solid or liquid (adsorbent). To understand the heat change, we examine the thermodynamics of the process.

Step 2: Detailed Explanation:

1. **Entropy change (ΔS):** During adsorption, the movement of adsorbate molecules is restricted as they settle on the surface. This leads to a decrease in randomness, so ΔS is negative ($\Delta S < 0$).
2. **Spontaneity (ΔG):** For adsorption to occur spontaneously, the Gibbs free energy change must be negative ($\Delta G < 0$).
3. **Thermodynamic relation:** We know that $\Delta G = \Delta H - T\Delta S$.
4. Since ΔS is negative, the term $-T\Delta S$ becomes positive. For ΔG to be negative overall, the enthalpy change (ΔH) **must** be negative and its magnitude must be larger than $T\Delta S$.
5. A negative ΔH indicates that energy is released during the process. Therefore, adsorption is always an **exothermic** process.

Step 3: Final Answer:

The process of adsorption is always exothermic.

This matches Option (A).

Quick Tip

Quick Tip: Bonding releases energy. In adsorption, new residual force interactions are satisfied between the adsorbent and adsorbate, which naturally leads to a release of heat.

146. The principal ore of aluminium is

(A) Bauxite

(B) Haematite

(C) Galena

(D) Magnetite

Correct Answer: (A) Bauxite

Solution:

Step 1: Understanding the Concept:

Ores are naturally occurring solid materials from which a metal or valuable mineral can be profitably extracted. Each major metal has one or more primary ores found in the Earth's crust.

Step 2: Detailed Explanation:

Let's analyze the ores mentioned in the options:

1. **Bauxite** ($Al_2O_3 \cdot 2H_2O$): This is the primary and most important ore used for the industrial extraction of Aluminium via the Bayer process and Hall-Héroult process.
2. **Haematite** (Fe_2O_3): This is a major ore of Iron.
3. **Galena** (PbS): This is the principal ore of Lead.
4. **Magnetite** (Fe_3O_4): This is another important ore of Iron, known for its magnetic properties.

Clearly, Bauxite is the only ore listed belonging to Aluminium.

Step 3: Final Answer:

The principal ore of aluminium is bauxite.

This matches Option (A).

Quick Tip

Quick Tip: Remember:

Al → Bauxite.

Fe → Haematite/Magnetite.

Pb → Galena.

Cu → Copper Pyrites.

Zn → Zinc Blende (Calamine).

---

147. The formula of phosphine gas is

(A) PH_3

(B) P_2H_4

(C) H_3PO_3

(D) H_3PO_4

Correct Answer: (A) PH_3

Solution:

Step 1: Understanding the Concept:

Phosphine is the simplest hydride of Phosphorus, belonging to Group 15 of the periodic table. It is a colorless, flammable, and highly toxic gas with a characteristic smell of rotten fish.

Step 2: Detailed Explanation:

1. **Group 15 Hydrides**: The general formula for Group 15 hydrides is MH_3. Examples include NH_3 (Ammonia) and PH_3 (**Phosphine**).

2. **Other options**:

- P_2H_4 is Diphosphane (liquid).

- H_3PO_3 is Phosphorous acid (an oxyacid).

- H_3PO_4 is Phosphoric acid (an oxyacid).

3. **Geometry**: In PH_3, the phosphorus atom is sp^3 hybridized with one lone pair, giving it a trigonal pyramidal shape.

Step 3: Final Answer:

The formula of phosphine gas is PH_3.

This matches Option (A).

Quick Tip

Quick Tip: Don't confuse Phosphine (PH_3) with Phosgene ($COCl_2$). Both are toxic gases but they are chemically very different.

---

148. Which of the following transition elements exhibits the highest oxidation state?

(A) Mn

(B) Fe

(C) Cr

(D) Ti

Correct Answer: (A) Mn

Solution:

Step 1: Understanding the Concept:

The oxidation state of a transition metal depends on the number of electrons it can lose or share from its s and d subshells. Elements in the middle of the transition series generally show the widest range of oxidation states.

Step 2: Detailed Explanation:

Let's look at the maximum oxidation states for the 3d series elements provided:

1. **Titanium (Ti)**: Outer config $3d^2 4s^2$. Maximum O.S. = **+4**.
2. **Chromium (Cr)**: Outer config $3d^5 4s^1$. Maximum O.S. = **+6** (e.g., in $K_2Cr_2O_7$).
3. **Manganese (Mn)**: Outer config $3d^5 4s^2$. Maximum O.S. = **+7** (e.g., in $KMnO_4$). It uses all 7 valence electrons for bonding.
4. **Iron (Fe)**: Outer config $3d^6 4s^2$. Although it has 8 valence electrons, the max O.S. commonly seen is +2 and +3. High states like +6 are very rare and unstable.

Manganese has the maximum number of unpaired electrons available in its d and s orbitals among these options.

Step 3: Final Answer:

Manganese (Mn) exhibits the highest oxidation state (+7).

This matches Option (A).

Quick Tip

Quick Tip: In the 3d series, the maximum oxidation state increases up to Manganese (+7) and then decreases. For the entire periodic table, the highest states are +8 (shown by Osmium and Ruthenium).

---

149. According to Werner's theory of coordination compounds, the secondary valency represents

(A) Coordination number

(B) Oxidation state

(C) Charge on the complex

(D) Ionic character

Correct Answer: (A) Coordination number

Solution:

Step 1: Understanding the Concept:

Alfred Werner was the first to propose a theory for coordination compounds. He suggested that metal atoms in these complexes exhibit two types of valencies: primary and secondary.

Step 2: Detailed Explanation:

According to Werner's postulates:

1. **Primary Valency**:

- It is ionisable.

- It corresponds to the **oxidation state** of the metal.

- It is satisfied by negative ions.

2. **Secondary Valency**:

- It is non-ionisable.

- It corresponds to the **coordination number** (the number of ligands directly bonded to the metal).
- It is fixed for a particular metal.
- It is directional, determining the spatial arrangement (geometry) of the complex.

Step 3: Final Answer:

The secondary valency represents the coordination number.

This matches Option (A).

Quick Tip

Quick Tip: Mnemonic:

P.O. → **P**rimary = **O**xidation state.

S.C. → **S**econdary = **C**oordination number.

Secondary valencies are represented by solid lines in structural diagrams.

---

150. The coordination number of cobalt in $[Co(NH_3)_6]^{3+}$ is

(A) 6

(B) 4

(C) 3

(D) 2

Correct Answer: (A) 6

Solution:

Step 1: Understanding the Concept:

The coordination number (CN) of a metal ion in a complex is defined as the total number of ligand donor atoms to which the metal is directly bonded.

Step 2: Detailed Explanation:

1. Identify the complex: $[Co(NH_3)_6]^{3+}$.
2. The central metal atom is Cobalt (Co).

3. The ligands are ammonia molecules (NH_3).
4. Ammonia is a **monodentate** ligand, meaning each molecule donates exactly one pair of electrons from its nitrogen atom to form one coordinate bond.
5. There are 6 ammonia molecules attached to the Cobalt ion.
6. Total number of coordinate bonds $= 6 \times 1 = 6$.

Therefore, the coordination number is 6. This complex has an octahedral geometry.

Step 3: Final Answer:

The coordination number of cobalt is 6.

This matches Option (A).

Quick Tip

Quick Tip: Always check the denticity of the ligand! If the ligand was oxalate ($C_2O_4^{2-}$), which is bidentate, 3 ligands would result in a coordination number of 6. For monodentate ligands like NH_3, H_2O, Cl^-, the number of ligands equals the CN.

---

151. The monomers of Nylon-6,6 are

(A) Adipic acid and hexamethylenediamine

(B) Caprolactam

(C) Ethylene glycol and terephthalic acid

(D) Styrene and butadiene

Correct Answer: (A) Adipic acid and hexamethylenediamine

Solution:

Step 1: Understanding the Concept:

Nylon-6,6 is a synthetic polyamide fiber produced via a condensation polymerization reaction. The name "6,6" refers to the number of carbon atoms present in each of the two monomers used in the synthesis.

Step 2: Detailed Explanation:

1. **Monomer 1: Adipic Acid**: This is a dicarboxylic acid with the formula $HOOC(CH_2)_4COOH$. It contains 6 carbon atoms.

2. **Monomer 2: Hexamethylenediamine**: This is a diamine with the formula $NH_2(CH_2)_6NH_2$. It also contains 6 carbon atoms.

3. **Polymerization**: During the reaction, a molecule of water is eliminated as the amino group ($-NH_2$) of the diamine reacts with the carboxyl group ($-COOH$) of the acid to form an amide linkage ($-CONH-$).

4. **Other Options**:

- Caprolactam is the monomer for Nylon-6.
- Ethylene glycol and terephthalic acid form Terylene (Dacron).
- Styrene and butadiene form Buna-S rubber.

Step 3: Final Answer:

The monomers for Nylon-6,6 are Adipic acid and hexamethylenediamine.

This matches Option (A).

Quick Tip

Quick Tip: To remember Nylon-6,6: Think "Six and Six". Count the carbons in the names: Adipic (6) and Hexamethylene (6). Nylon-6 (one number) comes from one monomer (Caprolactam).

---

152. Which of the following is a natural polymer?

(A) Cellulose

(B) Nylon-6

(C) PVC

(D) Teflon

Correct Answer: (A) Cellulose

Solution:

Step 1: Understanding the Concept:

Polymers are classified based on their source into natural, semi-synthetic, and synthetic polymers.

Step 2: Detailed Explanation:

1. **Natural Polymers**: These are found in nature, mostly in plants and animals. Examples include starch, cellulose, proteins, and natural rubber.
2. **Cellulose**: It is a complex carbohydrate (polysaccharide) consisting of thousands of β-D-glucose units. It is the primary structural component of the cell wall of green plants.
3. **Synthetic Polymers**: These are man-made polymers synthesized in labs.

- **Nylon-6**: A synthetic polyamide.
- **PVC (Polyvinyl Chloride)**: A synthetic addition polymer used in pipes.
- **Teflon (PTFE)**: A synthetic fluoropolymer used in non-stick cookware.

Step 3: Final Answer:

Cellulose is the natural polymer among the given options.

This matches Option (A).

Quick Tip

Quick Tip: Natural polymers are basically "bio-polymers". If it grows in a plant or is part of an animal (like DNA or Silk), it's natural. If it sounds like a chemical brand name (PVC, Teflon), it's synthetic.

---

153. Glucose on reduction with HI and red phosphorus gives

(A) n-Hexane

(B) Gluconic acid

(C) Sorbitol

(D) Saccharic acid

Correct Answer: (A) n-Hexane

Solution:

Step 1: Understanding the Concept:

Hydrogen iodide (HI) in the presence of red phosphorus is a very powerful reducing agent used in organic chemistry to reduce oxygen-containing functional groups (like alcohols, aldehydes, and ketones) completely to alkanes.

Step 2: Detailed Explanation:

1. Glucose is a polyhydroxy aldehyde with a six-carbon chain ($C_6H_{12}O_6$).
2. When glucose is heated with HI and red phosphorus for a prolonged period, all the hydroxyl ($-OH$) groups and the aldehyde group ($-CHO$) are reduced.
3. The oxygen atoms are removed, and the carbons are saturated with hydrogen.
4. This reaction proves that the six carbon atoms in glucose are linked in a straight, unbranched chain.
5. The resulting product is the straight-chain alkane with six carbons, which is **n-hexane**.

- Gluconic acid and Saccharic acid are products of oxidation, not reduction.
- Sorbitol is the product of mild reduction (e.g., $NaBH_4$).

Step 3: Final Answer:

The reduction of glucose with HI/P yields n-hexane.

This matches Option (A).

Quick Tip

Quick Tip: This reaction is a classic structural proof. If HI/P gives n-hexane, it means the parent skeleton of the sugar is a straight chain of 6 carbons.

---

154. Which of the following vitamins is water-soluble?

(A) Vitamin C

(B) Vitamin A

(C) Vitamin D

(D) Vitamin K

Correct Answer: (A) Vitamin C

Solution:

Step 1: Understanding the Concept:

Vitamins are essential organic compounds classified into two groups based on their solubility: fat-soluble and water-soluble.

Step 2: Detailed Explanation:

1. **Fat-soluble Vitamins**: These are soluble in fats and oils but insoluble in water. They are stored in the liver and adipose tissues. This group includes **Vitamins A, D, E, and K**.
2. **Water-soluble Vitamins**: These dissolve in water and are not stored in the body to a significant extent (except B_{12}); they must be supplied regularly in the diet. This group includes the **B-complex vitamins** and **Vitamin C** (ascorbic acid).
3. Since Vitamin C is water-soluble, excess amounts are excreted in urine, making it non-toxic but requiring daily intake.

Step 3: Final Answer:

Among the options, Vitamin C is the water-soluble vitamin.

This matches Option (A).

Quick Tip

Quick Tip: Mnemonic: **ADEK** are fat-soluble. Any vitamin not in "ADEK" is water-soluble. (Specifically B and C).

---

155. Aspirin is chemically known as

(A) Acetylsalicylic acid

(B) Methyl salicylate

(C) Salicylic acid

(D) Ethyl salicylate

Correct Answer: (A) Acetylsalicylic acid

Solution:

Step 1: Understanding the Concept:

Aspirin is one of the most widely used analgesic, antipyretic, and anti-inflammatory drugs. It is a derivative of salicylic acid.

Step 2: Detailed Explanation:

1. **Synthesis**: Aspirin is prepared by the acetylation of the phenolic hydroxyl group of salicylic acid using acetic anhydride in the presence of an acid catalyst.
2. **Reaction**:

Salicylic acid + Acetic anhydride $\xrightarrow{H^+}$ Acetylsalicylic acid + Acetic acid.

3. In this reaction, the $-OH$ group of salicylic acid is converted into an $-OCOCH_3$ (acetyl) group.
4. The chemical name for the resulting ester is **Acetylsalicylic acid**.

- Methyl salicylate is known as "Oil of Wintergreen".
- Salicylic acid is the precursor.

Step 3: Final Answer:

The chemical name of aspirin is Acetylsalicylic acid.

This matches Option (A).

Quick Tip

Quick Tip: Aspirin has an "Acetyl" group attached to the oxygen of "Salicylic" acid. Hence the name: Acetyl-Salicylic Acid.

---

156. The main constituent of dettol is

(A) Chloroxylenol and terpineol

(B) Bithionol

(C) Iodine

(D) Phenol

Correct Answer: (A) Chloroxylenol and terpineol

Solution:

Step 1: Understanding the Concept:

Antiseptics are chemical substances which prevent the growth of microorganisms or kill them on living tissues. Dettol is a common household antiseptic liquid.

Step 2: Detailed Explanation:

1. Dettol is a mixture of two main active chemical components.
2. **Chloroxylenol**: This is the primary antiseptic and disinfectant agent. It has the chemical formula C_8H_9ClO.
3. **Terpineol**: This is a naturally occurring terpene alcohol that acts as a solvent and provides the characteristic pine-like smell of Dettol.
4. **Other Options**:

- Bithionol is added to soaps to impart antiseptic properties.
- Tincture of iodine is a 2-3% solution of iodine in alcohol-water.
- 0.2% Phenol is an antiseptic, but 1% Phenol is a disinfectant.

Step 3: Final Answer:

The main constituents of Dettol are chloroxylenol and terpineol.

This matches Option (A).

Quick Tip

Quick Tip: This is a high-frequency fact-based question. Memorize the pair "Chloroxylenol + Terpineol" as a single unit for Dettol.

---

157. The conversion of alkyl halide to alcohol by aqueous KOH is an example of

(A) Nucleophilic substitution

(B) Electrophilic addition

(C) Nucleophilic addition

(D) Electrophilic substitution

Correct Answer: (A) Nucleophilic substitution

Solution:

Step 1: Understanding the Concept:

Substitution reactions involve the replacement of one atom or group in a molecule by another. If the attacking species is a nucleophile (electron-rich), the reaction is a nucleophilic substitution.

Step 2: Detailed Explanation:

1. **Reaction**: $R-X+KOH(aq) \rightarrow R-OH+KX$.
2. **Mechanism**: In aqueous solution, KOH dissociates into K^+ and OH^-.
3. The hydroxide ion (OH^-) is a strong **nucleophile** because it has a lone pair and a negative charge.
4. The alkyl halide ($R-X$) has a polar bond where Carbon is partially positive (δ^+) and the Halogen (X) is partially negative (δ^-).
5. The OH^- attacks the electrophilic carbon and displaces the halide ion (X^-).
6. Since a nucleophile replaces a leaving group, it is a **nucleophilic substitution** reaction (specifically S_N1 or S_N2).

Step 3: Final Answer:

The reaction is an example of nucleophilic substitution.

This matches Option (A).

Quick Tip

Quick Tip: Remember: Alkyl Halide + **Aqueous** KOH → Alcohol (Substitution).

Alkyl Halide + **Alcoholic** KOH → Alkene (Elimination).

The medium (aq vs alc) changes the type of reaction!

---

158. Which of the following organic compounds will give Lucas test immediately?

(A) 2-Methylpropan-2-ol

(B) Propan-1-ol

(C) Propan-2-ol

(D) Ethanol

Correct Answer: (A) 2-Methylpropan-2-ol

Solution:

Step 1: Understanding the Concept:

The Lucas test is used to distinguish between primary (1°), secondary (2°), and tertiary (3°) alcohols. The reagent used is a mixture of concentrated HCl and anhydrous $ZnCl_2$.

Step 2: Detailed Explanation:

The Lucas reagent reacts with alcohols to form alkyl chlorides, which are insoluble and appear as turbidity (cloudiness). The rate of reaction depends on the stability of the carbocation intermediate:

1. **Tertiary (3°) Alcohols**: React **immediately** at room temperature to produce turbidity.
2. **Secondary (2°) Alcohols**: React within **5 to 10 minutes**.
3. **Primary (1°) Alcohols**: Do not react at room temperature; turbidity appears only on heating.

Analyze the options:

- (A) **2-Methylpropan-2-ol**: Structure is $(CH_3)_3C-OH$. This is a **tertiary** alcohol.
- (B) Propan-1-ol: $CH_3CH_2CH_2OH$. Primary.
- (C) Propan-2-ol: $CH_3CH(OH)CH_3$. Secondary.
- (D) Ethanol: CH_3CH_2OH. Primary.

Step 3: Final Answer:

2-Methylpropan-2-ol gives the Lucas test immediately because it is a tertiary alcohol.

This matches Option (A).

Quick Tip

Quick Tip: Basics of Lucas Test:

3° → Immediate.

2° → 5 mins.

1° → Heat required.

159. The reaction of an aldehyde with Tollens' reagent gives

(A) Silver mirror

(B) Red precipitate

(C) Yellow precipitate

(D) Blue solution

Correct Answer: (A) Silver mirror

Solution:

Step 1: Understanding the Concept:

Tollens' test is a qualitative chemical test used to distinguish between aldehydes and ketones. Tollens' reagent is an ammoniacal solution of silver nitrate, containing the complex ion $[Ag(NH_3)_2]^+$.

Step 2: Detailed Explanation:

1. Aldehydes are easily oxidized to carboxylic acids, even by mild oxidizing agents like Tollens' reagent.

2. During the reaction, the aldehyde is oxidized to a carboxylate ion, while the silver ions (Ag^+) in the reagent are **reduced** to metallic silver (Ag).

3. The reaction can be represented as:

$RCHO + 2[Ag(NH_3)_2]^+ + 3OH^- \rightarrow RCOO^- + 2Ag \downarrow + 4NH_3 + 2H_2O$.

4. The metallic silver deposits on the inner wall of the test tube, forming a bright, shining **silver mirror**.

5. Ketones (except α-hydroxy ketones) do not react with Tollens' reagent because they are difficult to oxidize.

- A red precipitate is formed with Fehling's solution (for aldehydes).

Step 3: Final Answer:

The reaction of an aldehyde with Tollens' reagent results in a silver mirror.

This matches Option (A).

Quick Tip

Quick Tip: Tollens' Test = Silver Mirror (Aldehydes).
Fehling's Test = Red Precipitate (Aliphatic Aldehydes).
Both tests distinguish aldehydes from ketones.

---

160. Which of the following is the strongest base in aqueous solution?

(A) Dimethylamine
(B) Methylamine
(C) Trimethylamine
(D) Aniline

Correct Answer: (A) Dimethylamine

Solution:

Step 1: Understanding the Concept:
The basic strength of amines in aqueous solution is determined by a combination of three factors: the inductive effect ($+I$), the solvation effect (hydration), and steric hindrance.

Step 2: Detailed Explanation:
In the case of methyl-substituted amines in water, the order of basicity is unique:
1. **Inductive Effect**: Alkyl groups are electron-donating ($+I$), increasing electron density on Nitrogen. This suggests $3^\circ > 2^\circ > 1^\circ$.
2. **Solvation Effect**: The conjugate acid (ammonium ion) is stabilized by hydrogen bonding with water. Smaller ions with more hydrogens are better solvated. This suggests $1^\circ > 2^\circ > 3^\circ$.
3. **Steric Effect**: Large alkyl groups hinder the approach of protons.
4. **Net Effect (Methyl series)**: For methyl amines in water, the combined effect results in the order: **Secondary (2°) > Primary (1°) > Tertiary (3°)**.
- Dimethylamine ($(CH_3)_2NH$) is 2°.
- Methylamine (CH_3NH_2) is 1°.
- Trimethylamine ($(CH_3)_3N$) is 3°.
- Aniline ($C_6H_5NH_2$) is extremely weak because the lone pair is delocalized into the benzene ring.

Step 3: Final Answer:

The strongest base in aqueous solution is Dimethylamine.

This matches Option (A).

Quick Tip

Quick Tip: Basicity Order for Methyl amines in water: **2-1-3**.

Basicity Order for Ethyl amines in water: **2-3-1**.

In both cases, the **secondary** amine is always the strongest in aqueous medium.
