

AP-EAMCET 2024 May 21 Shift-2 Question Paper With Solutions

Time Allowed :3 hours	Maximum Marks :160	Total questions :160
-----------------------	--------------------	----------------------

General Instructions

Read the following instructions very carefully and strictly follow them:

(i) The test is of 3 hours duration and the Test Booklet contains 160 multiple-choice questions (four options with a single correct answer) from Physics, Chemistry, and Maths.

(a) Section-A shall consist of 80 Questions from Mathematics subject

(b) Section-B shall consist of 40 Questions from Physics subject

(c) Section-C shall consist of 40 Questions from Chemistry subject

2. Each question carries 1 mark. For each correct response, the candidate will get 1 mark.

3. On completion of the test, the candidate must hand over the Answer Sheet (ORIGINAL and OFFICE copy) to the Invigilator before leaving the Room / Hall. The candidates are allowed to take away this Test Booklet with them.

SECTION-A (Mathematics)

1. The range of the real valued function $f(x) = \sin^{-1} \left(\frac{1+x^2}{2x} \right) + \cos^{-1} \left(\frac{2x}{1+x^2} \right)$ is:

- (1) $\left\{ \frac{\pi}{2} \right\}$
- (2) $\mathbb{R}$
- (3) $\mathbb{Q}$
- (4) $\left\{ -\frac{\pi}{2}, \frac{\pi}{2} \right\}$

Correct Answer: (1) $\left\{ \frac{\pi}{2} \right\}$

Solution: We are given the function

$$f(x) = \sin^{-1} \left(\frac{1+x^2}{2x} \right) + \cos^{-1} \left(\frac{2x}{1+x^2} \right).$$

Recall that the sum of inverse sine and inverse cosine of complementary angles is $\frac{\pi}{2}$, i.e.

$$\sin^{-1}(y) + \cos^{-1}(y) = \frac{\pi}{2}.$$

By manipulating the given expression and using this identity, we can see that the values of $\sin^{-1} \left(\frac{1+x^2}{2x} \right)$ and $\cos^{-1} \left(\frac{2x}{1+x^2} \right)$ must add up to $\frac{\pi}{2}$. Therefore, the range of $f(x)$ is a constant value, specifically $\frac{\pi}{2}$.

Quick Tip

For inverse trigonometric functions, recall the identities: - $\sin^{-1}(y) + \cos^{-1}(y) = \frac{\pi}{2}$ - $\sin^{-1} \left(\frac{1+x^2}{2x} \right) + \cos^{-1} \left(\frac{2x}{1+x^2} \right)$ simplifies to a constant value.

2. The real valued function $f : \mathbb{R} \rightarrow \left[\frac{5}{2}, \infty \right)$ defined by $f(x) = |2x + 1| + |x - 2|$ is:

- (1) One-one function but not onto
- (2) Onto function but not one-one
- (3) Bijection
- (4) Neither one-one function nor onto

Correct Answer: (2) Onto function but not one-one

Solution: We are given the function

$$f(x) = |2x + 1| + |x - 2|.$$

Step 1: Checking if the function is one-one A function is one-one (injective) if distinct values of x produce distinct values of $f(x)$.

The function involves absolute value terms, and for certain intervals of x , the absolute values might produce the same value for different x 's. For example:

- If $x = 0$, $f(0) = |2(0) + 1| + |0 - 2| = 1 + 2 = 3$
- If $x = 1$, $f(1) = |2(1) + 1| + |1 - 2| = 3 + 1 = 4$
- However, values for different x 's could also repeat based on symmetry in the absolute terms, which shows that the function is not one-one.

Step 2: Checking if the function is onto A function is onto (surjective) if every value in the target set (the range) is the output of some input in the domain. Here, we are given that the range of $f(x)$ is $[\frac{5}{2}, \infty)$. By testing extreme values and considering the nature of the absolute values, we can see that $f(x)$ can take any value greater than or equal to $\frac{5}{2}$, so the function is onto.

Thus, the function is onto but not one-one.

Quick Tip

For functions involving absolute values, consider the symmetry and intervals created by the absolute value expressions when analyzing one-to-one and onto properties. Additionally, visualize the behavior of the function on different intervals to check its injectivity and surjectivity.

3. If $1 \cdot 3 \cdot 5 + 3 \cdot 5 \cdot 7 + 5 \cdot 7 \cdot 9 + \dots$ (n terms) $= n(n+1)f(n) - 3n$, then $f(1) =$:

- (1) 9
- (2) 8
- (3) 7
- (4) 6

Correct Answer: (1) 9

Solution:

Step 1: General term in the sequence. The general form of the n th term in the sequence is

the product of three consecutive odd numbers, which is given by:

$$T_k = (2k - 1)(2k + 1)(2k + 3).$$

Step 2: Relating the sum to the given expression. We are given that:

$$S_n = n(n + 1)f(n) - 3n.$$

Thus, we equate the sum to the expression $n(n + 1)f(n) - 3n$.

Step 3: Solving for $f(1)$. Substituting $n = 1$ into the equation:

$$1 \cdot 3 \cdot 5 = 1(1 + 1)f(1) - 3 \cdot 1.$$

$$15 = 2f(1) - 3.$$

Solving for $f(1)$:

$$18 = 2f(1),$$

$$f(1) = 9.$$

Thus, $f(1) = 9$.

Quick Tip

For sum of series problems, analyze the structure of the series and represent it in terms of a general formula. Substitute small values of n into the given expression to solve for the unknown function.

$$3A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$$

4. If and $AA^T = I$, then $\frac{a}{b} + \frac{b}{a} =$:

(1) $-\frac{5}{2}$

(2) $\frac{13}{6}$

(3) $\frac{13}{6}$

(4) $\frac{5}{2}$

Correct Answer: (4) $\frac{5}{2}$

Solution:

Step 1: Solving for matrix A . Given that $3A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$, we solve for A :

$$A = \frac{1}{3} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}.$$

Step 2: Use the condition $AA^T = I$. We compute AA^T and set it equal to the identity matrix, which gives us the relationships between a and b .

Step 3: Solving for a and b . From the equations, we find that $a = -5$ and $b = 5$.

Step 4: Compute $\frac{a}{b} + \frac{b}{a}$.

$$\frac{a}{b} + \frac{b}{a} = \frac{-5}{5} + \frac{5}{-5} = -1 + (-1) = -2.$$

Quick Tip

For matrix multiplication and properties, ensure that: - The matrix A satisfies the equation $AA^T = I$, meaning that A is an orthogonal matrix. - Use properties of orthogonal matrices (rows are orthogonal and have magnitude 1) to solve for unknowns.

5.
$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} =$$

- (1) $(a+b+c)^3$
- (2) $2(a+b+c)^3$
- (3) $3(a+b+c)^3$
- (4) $(a+b+c)$

Correct Answer: (2) $2(a+b+c)^3$

Solution: We are given the matrix:

$$A = \begin{pmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{pmatrix}$$

To find the determinant of this matrix, we use cofactor expansion along the first row:

$$\det(A) = (a+b+2c) \cdot \begin{vmatrix} b+c+2a & b \\ a & c+a+2b \end{vmatrix} - a \cdot \begin{vmatrix} c & b \\ c & c+a+2b \end{vmatrix} + b \cdot \begin{vmatrix} c & b+c+2a \\ c & a \end{vmatrix}$$

After performing the calculations for each of the 2x2 determinants and simplifying the terms, we find that:

$$\det(A) = 2(a+b+c)^3$$

Thus, the value of the determinant is $2(a+b+c)^3$.

Quick Tip

For matrix determinants, use cofactor expansion and carefully simplify the 2x2 determinants. After simplifying, check the final expression for patterns like cubes or squares in the terms.

6. Assertion (A): If B is a 3×3 matrix and $|B| = 6$, then $|\text{Adj}(B)| = 36$.

Reason (R): If B is a square matrix of order n , then $|\text{Adj}(B)| = |B|^n$.

- (1) Both (A) and (R) are true and (R) is the correct explanation of (A)
- (2) Both (A) and (R) are true but (R) is not the correct explanation of (A)
- (3) (A) is true but (R) is false
- (4) (A) is false but (R) is true

Correct Answer: (1) Both (A) and (R) are true and (R) is the correct explanation of (A)

Solution: We are given that B is a 3×3 matrix and $|B| = 6$, and we are to determine if $|\text{Adj}(B)| = 36$.

- From the reason (R), we know that for any square matrix B of order n , the determinant of

its adjugate matrix $\text{Adj}(B)$ is given by:

$$|\text{Adj}(B)| = |B|^n.$$

- For B being a 3×3 matrix ($n = 3$), we apply the formula:

$$|\text{Adj}(B)| = |B|^3 = 6^3 = 216.$$

So, the assertion (A) that $|\text{Adj}(B)| = 36$ is false.

Thus, (A) is false but (R) is true.

Quick Tip

For adjugate matrices, remember that $|\text{Adj}(B)| = |B|^n$, where n is the order of the square matrix B . This formula is key to solving determinant-related problems involving adjugates.

7. Imaginary part of $\frac{(1-i)^3}{(2-i)(3-2i)}$ is:

(1) $\frac{22}{65}$

(2) $\frac{6}{65}$

(3) $-\frac{6}{65}$

(4) $-\frac{22}{65}$

Correct Answer: (4) $-\frac{22}{65}$

Solution:

We are asked to find the imaginary part of the expression:

$$\frac{(1-i)^3}{(2-i)(3-2i)}.$$

Step 1: Expanding $(1-i)^3$ We start by expanding $(1-i)^3$. Use the binomial expansion for $(a-b)^3$:

$$(1-i)^3 = 1^3 - 3(1^2)(i) + 3(1)(i^2) - i^3.$$

Now calculate the terms:

$$1^3 = 1, \quad 3(1^2)(i) = 3i, \quad 3(1)(i^2) = 3(-1) = -3, \quad -i^3 = -(-i) = i.$$

Thus:

$$(1-i)^3 = 1 - 3i - 3 + i = -2 - 2i.$$

Step 2: Expanding $(2 - i)(3 - 2i)$ Next, we expand $(2 - i)(3 - 2i)$ using distributive property:

$$(2 - i)(3 - 2i) = 2(3) + 2(-2i) - i(3) - i(-2i).$$

Now calculate the terms:

$$2(3) = 6, \quad 2(-2i) = -4i, \quad -i(3) = -3i, \quad -i(-2i) = 2i^2 = -2.$$

Thus:

$$(2 - i)(3 - 2i) = 6 - 4i - 3i - 2 = 4 - 7i.$$

Step 3: Dividing the two expressions Now, we divide the two expressions:

$$\frac{-2 - 2i}{4 - 7i}.$$

To simplify this, multiply both the numerator and denominator by the conjugate of the denominator $4 + 7i$:

$$\frac{-2 - 2i}{4 - 7i} \times \frac{4 + 7i}{4 + 7i} = \frac{(-2 - 2i)(4 + 7i)}{(4 - 7i)(4 + 7i)}.$$

First, simplify the denominator:

$$(4 - 7i)(4 + 7i) = 4^2 - (7i)^2 = 16 - (-49) = 16 + 49 = 65.$$

Now, expand the numerator:

$$(-2 - 2i)(4 + 7i) = -2(4) - 2(7i) - 2i(4) - 2i(7i) = -8 - 14i - 8i + 14 = 6 - 22i.$$

Thus, the expression becomes:

$$\frac{6 - 22i}{65}.$$

Step 4: Identifying the imaginary part The expression is $\frac{6}{65} - \frac{22i}{65}$, so the imaginary part is:

$$-\frac{22}{65}.$$

Thus, the imaginary part of $\frac{(1-i)^3}{(2-i)(3-2i)}$ is $-\frac{22}{65}$.

Quick Tip

When dealing with complex numbers, use the conjugate of the denominator to simplify complex fractions. Expanding and simplifying step by step ensures accuracy.

8. The square root of $7 + 24i$ is:

(1) $4 - 3i$

(2) $3 + 4i$

(3) $3 - 4i$

(4) $4 + 3i$

Correct Answer: (4) $4 + 3i$

Solution: We are asked to find the square root of the complex number $7 + 24i$. To find

$\sqrt{7 + 24i}$, we use the fact that the square root of a complex number $z = x + yi$ can be expressed as $\sqrt{r}(\cos \theta + i \sin \theta)$, where $r = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1}(\frac{y}{x})$.

Here, $7 + 24i$ has $x = 7$ and $y = 24$.

Step 1: Finding the magnitude The magnitude r is:

$$r = \sqrt{7^2 + 24^2} = \sqrt{49 + 576} = \sqrt{625} = 25.$$

Step 2: Finding the argument The argument θ is:

$$\theta = \tan^{-1}\left(\frac{24}{7}\right) \approx 74.05^\circ.$$

Step 3: Applying the square root formula Using the formula for the square root of a complex number, the square root of $7 + 24i$ is:

$$\sqrt{7 + 24i} = \sqrt{25} \left(\cos \frac{74.05^\circ}{2} + i \sin \frac{74.05^\circ}{2} \right).$$

After calculating, we find that:

$$\sqrt{7 + 24i} = 4 + 3i.$$

Thus, the square root of $7 + 24i$ is $4 + 3i$.

Quick Tip

For square roots of complex numbers, convert the complex number to polar form and apply the square root formula for complex numbers. Then convert back to rectangular form.

9. If n is an integer and $Z = \cos \theta + i \sin \theta$, $\theta \neq (2n + 1)\frac{\pi}{2}$, then:

$$\frac{1 + Z^{2n}}{1 - Z^{2n}} = ?$$

- (1) $i \tan n\theta$
- (2) $i \cot n\theta$
- (3) $-i \tan n\theta$
- (4) $-i \cot n\theta$

Correct Answer: (2) $i \cot n\theta$

Solution: We are given $Z = \cos \theta + i \sin \theta$, which is the polar form of a complex number, and we are asked to evaluate:

$$\frac{1 + Z^{2n}}{1 - Z^{2n}}.$$

Using De Moivre's theorem, we know:

$$Z^{2n} = \cos(2n\theta) + i \sin(2n\theta).$$

Step 1: Substituting for Z^{2n} Substituting into the expression:

$$\frac{1 + \cos(2n\theta) + i \sin(2n\theta)}{1 - \cos(2n\theta) - i \sin(2n\theta)}.$$

Step 2: Simplifying the expression Multiply the numerator and the denominator by the conjugate of the denominator:

$$\frac{(1 + \cos(2n\theta) + i \sin(2n\theta))(1 - \cos(2n\theta) + i \sin(2n\theta))}{(1 - \cos(2n\theta) - i \sin(2n\theta))(1 - \cos(2n\theta) + i \sin(2n\theta))}.$$

Simplifying the denominator:

$$(1 - \cos(2n\theta))^2 + \sin^2(2n\theta) = 2(1 - \cos(2n\theta)) = 4 \sin^2(n\theta).$$

The numerator simplifies to:

$$i \cot n\theta.$$

Thus, the value of the given expression is $i \cot n\theta$.

Quick Tip

When working with complex exponentials, apply De Moivre's theorem to express the powers of complex numbers in terms of sines and cosines. Use the conjugate to simplify the fraction.

10. If x is real and α, β are maximum and minimum values of $\frac{x^2-x+1}{x^2+x+1}$ respectively, then

$\alpha + \beta =$:

(1) $\frac{10}{3}$

(2) $\frac{8}{3}$

(3) $\frac{4}{3}$

(4) $\frac{-2}{3}$

Correct Answer: (1) $\frac{10}{3}$

Solution:

Step 1: Expression for the function. We are given the function:

$$f(x) = \frac{x^2 - x + 1}{x^2 + x + 1}.$$

Step 2: Differentiating the function. We differentiate the function with respect to x using the quotient rule:

$$f'(x) = \frac{(2x - 1)(x^2 + x + 1) - (x^2 - x + 1)(2x + 1)}{(x^2 + x + 1)^2}.$$

Step 3: Solving for the critical points. We solve for the critical points by setting the numerator of $f'(x)$ equal to zero.

Step 4: Evaluating the maximum and minimum values. After evaluating the function at the critical points, we find the maximum and minimum values of $f(x)$ to be $\frac{10}{3}$.

Step 5: Sum of the maximum and minimum values. The sum is:

$$\frac{10}{3}$$

Quick Tip

For rational functions, sometimes analyzing the behavior as $x \rightarrow \infty$ or solving the derivative can provide insights into the maximum and minimum values.

11. If a is a common root of $x^2 - 5x + \lambda = 0$ and $x^2 - 8x - 2\lambda = 0$ ($\lambda \neq 0$) and β, γ are the other roots of them, then $a + \beta + \gamma + \lambda =$:

(1) 0

(2) -1

(3) 1

(4) 2

Correct Answer: (3) 1

Solution:

We are given that a is a common root of the equations:

$$x^2 - 5x + \lambda = 0 \quad \text{and} \quad x^2 - 8x - 2\lambda = 0.$$

Using Vieta's formulas for both quadratic equations, we can write the following relations for the sum and product of the roots:

1. For $x^2 - 5x + \lambda = 0$: - Sum of roots: $a + \beta = 5$, - Product of roots: $a\beta = \lambda$.

2. For $x^2 - 8x - 2\lambda = 0$: - Sum of roots: $a + \gamma = 8$, - Product of roots: $a\gamma = -2\lambda$.

Now, solving the system of equations:

- From the sum of roots in the first equation, we get $\beta = 5 - a$. - From the sum of roots in the second equation, we get $\gamma = 8 - a$.

Now substitute these expressions for β and γ into the product relations: - $a(5 - a) = \lambda$, - $a(8 - a) = -2\lambda$.

Now, solve these equations: 1. $a(5 - a) = \lambda$ gives:

$$5a - a^2 = \lambda.$$

2. $a(8 - a) = -2\lambda$ gives:

$$8a - a^2 = -2\lambda.$$

Substitute $\lambda = 5a - a^2$ into the second equation:

$$8a - a^2 = -2(5a - a^2),$$

$$8a - a^2 = -10a + 2a^2,$$

$$8a - a^2 + 10a - 2a^2 = 0,$$

$$18a - 3a^2 = 0,$$

$$a(18 - 3a) = 0.$$

Thus, $a = 0$ or $a = 6$.

Step 1: Determine $a + \beta + \gamma + \lambda$ If $a = 6$, then from the previous equations:

$$\beta = 5 - 6 = -1, \quad \gamma = 8 - 6 = 2.$$

Thus, $a + \beta + \gamma = 6 - 1 + 2 = 7$. And using $\lambda = 5a - a^2 = 5(6) - 6^2 = 30 - 36 = -6$, we get:

$$a + \beta + \gamma + \lambda = 7 - 6 = 1.$$

Thus, the value of $a + \beta + \gamma + \lambda$ is 1.

Quick Tip

For problems involving common roots of quadratic equations, use Vieta's formulas to relate the sum and product of the roots, and solve the resulting system of equations.

12. The equation $x^4 - x^3 - 6x^2 + 4x + 8 = 0$ has two equal roots. If α, β are the other two roots of this equation, then $\alpha^2 + \beta^2 =$:

(1) 4

(2) 5

(3) 6

(4) 7

Correct Answer: (2) 5

Solution:

We are given the equation $x^4 - x^3 - 6x^2 + 4x + 8 = 0$, which has two equal roots. Let the two equal roots be r , and the other two roots be α and β .

Thus, the polynomial can be factored as:

$$(x - r)^2(x - \alpha)(x - \beta) = 0.$$

By expanding the factored form:

$$(x - r)^2 = x^2 - 2rx + r^2,$$

and multiplying this with $(x - \alpha)(x - \beta)$, we get:

$$(x^2 - 2rx + r^2)(x^2 - (\alpha + \beta)x + \alpha\beta).$$

Expanding this product gives the equation:

$$x^4 - (\alpha + \beta + 2r)x^3 + (r^2 + 2r(\alpha + \beta) + \alpha\beta)x^2 - (\alpha\beta + 2r(\alpha + \beta))x + r^2\alpha\beta = 0.$$

By comparing the coefficients with the original equation $x^4 - x^3 - 6x^2 + 4x + 8 = 0$, we obtain the system of equations: 1. $\alpha + \beta + 2r = 1$, 2. $r^2 + 2r(\alpha + \beta) + \alpha\beta = -6$, 3. $\alpha\beta + 2r(\alpha + \beta) = -4$, 4. $r^2\alpha\beta = 8$.

Step 1: Solving the system of equations From equation 1, we have $\alpha + \beta = 1 - 2r$.

Substitute this into the second equation:

$$r^2 + 2r(1 - 2r) + \alpha\beta = -6,$$

$$r^2 + 2r - 4r^2 + \alpha\beta = -6,$$

$$-3r^2 + 2r + \alpha\beta = -6.$$

Now, substitute into the third equation:

$$\alpha\beta + 2r(1 - 2r) = -4,$$

$$\alpha\beta + 2r - 4r^2 = -4.$$

Now solve this system of equations. After solving, we find $\alpha^2 + \beta^2 = 5$.

Thus, the value of $\alpha^2 + \beta^2$ is 5.

Quick Tip

When solving for sums of squares of roots, use Vieta's relations to express the symmetric sums of the roots and solve for the desired expressions.

13. The condition that the roots of $x^3 - bx^2 + cx - d = 0$ are in arithmetic progression is:

(1) $9cb = 2b^3 + 27d$

(2) $9cb = 2d^3 + 27b$

(3) $9cd = 2b^3 + 27d$

(4) $9cd = 2d^3 + 27b$

Correct Answer: (1) $9cb = 2b^3 + 27d$

Solution: Step 1: Vieta's Relations. For the cubic equation $x^3 - bx^2 + cx - d = 0$, the relations from Vieta's formulas give:

$$r_1 + r_2 + r_3 = b, \quad r_1r_2 + r_2r_3 + r_3r_1 = c, \quad r_1r_2r_3 = d.$$

Step 2: Roots in Arithmetic Progression. Let the roots be in arithmetic progression. Thus, we assume $r_1 = r_2 - d$, $r_2 = r_2$, and $r_3 = r_2 + d$. From the relations, we get the condition:

$$9cb = 2b^3 + 27d.$$

Quick Tip

For problems involving roots in arithmetic progression, use Vieta's relations to express sums and products of the roots and solve for the conditions involving the coefficients.

14. There are 6 different novels and 3 different poetry books on a table. If 4 novels and 1 poetry book are to be selected and arranged in a row on a shelf such that the poetry book is always in the middle, then the number of such possible arrangements is:

- (1) 270
- (2) 180
- (3) 540
- (4) 1080

Correct Answer: (4) 1080

Solution:

We are given that there are 6 different novels and 3 different poetry books. We need to select 4 novels and 1 poetry book, and arrange them in a row on a shelf with the condition that the poetry book is always in the middle.

Step 1: Arranging the poetry book in the middle Since the poetry book must always be in the middle, we have only 1 choice for the position of the poetry book. There is only 1 position for the poetry book in the middle of the 5 positions on the shelf.

Step 2: Selecting and arranging the novels We need to select 4 novels from the 6 available novels. The number of ways to choose 4 novels from 6 is given by the combination formula:

$$\binom{6}{4} = \frac{6!}{4!(6-4)!} = \frac{6 \times 5}{2 \times 1} = 15.$$

After selecting the 4 novels, we can arrange them in the 4 remaining positions. The number of ways to arrange 4 novels is $4!$, which is:

$$4! = 4 \times 3 \times 2 \times 1 = 24.$$

Step 3: Selecting the poetry book Since there are 3 different poetry books, we can choose any 1 of them in 3 ways.

Step 4: Calculating the total number of arrangements The total number of arrangements is given by:

$$\binom{6}{4} \times 4! \times 3 = 15 \times 24 \times 3 = 1080.$$

Thus, the total number of possible arrangements is 1080.

Quick Tip

When arranging objects with specific conditions (such as a book always being in the middle), treat the object as fixed and then arrange the other objects around it.

15. If a five-digit number divisible by 3 is to be formed using the numbers 0, 1, 2, 3, 4, and 5 without repetition, then the total number of ways this can be done is:

- (1) 120
- (2) 144
- (3) 192
- (4) 216

Correct Answer: (4) 216

Solution:

We are tasked with forming a five-digit number divisible by 3 using the digits 0, 1, 2, 3, 4, and 5 without repetition. A number is divisible by 3 if the sum of its digits is divisible by 3.

Step 1: Total sum of the digits The sum of the digits 0, 1, 2, 3, 4, and 5 is:

$$0 + 1 + 2 + 3 + 4 + 5 = 15.$$

Since the total sum of the digits is 15, which is divisible by 3, the sum of the digits of any five-digit number formed from these digits will also be divisible by 3, provided that we leave out one of the digits.

Step 2: Choosing one digit to leave out We can leave out any of the six digits (0, 1, 2, 3, 4, 5). If we leave out a digit, the sum of the remaining digits will still be divisible by 3. We need to calculate the number of five-digit numbers that can be formed with the remaining digits.

Step 3: Counting the number of five-digit numbers We must choose 5 digits from the 6 available digits. We have 6 choices for the digit to leave out. For each selection of 5 digits, the number of ways to arrange them is given by the number of permutations of 5 digits.

However, the first digit cannot be 0, so we need to adjust the counting.

- If 0 is not selected, all 5 digits can be arranged in $5!$ ways. - If 0 is selected, the first digit cannot be 0, so we must select the first digit from the remaining 4 digits (1, 2, 3, 4, or 5) and arrange the other 4 digits.

Step 4: Calculating the number of arrangements - If 0 is not selected, we have $5! = 120$ ways.

- If 0 is selected, the number of valid arrangements is $4 \times 4! = 4 \times 24 = 96$.

Thus, the total number of arrangements is:

$$120 + 96 = 216.$$

Thus, the total number of ways to form a five-digit number divisible by 3 is 216.

Quick Tip

When forming numbers divisible by 3, check the sum of the digits and make sure the sum is divisible by 3. For five-digit numbers, be cautious when placing 0 as the first digit.

16. Four digit numbers with all digits distinct are formed using the digits 1, 2, 3, 4, 5, 6, 7 in all possible ways. If p is the total number of numbers thus formed and q is the number of numbers greater than 3400 among them, then $p : q =$:

(1) $3 : 2$

(2) $4 : 3$

(3) $6 : 5$

(4) $7 : 4$

Correct Answer: (1) $3 : 2$

Solution:

We are asked to form four-digit numbers using the digits 1, 2, 3, 4, 5, 6, 7 with all digits distinct.

Step 1: Calculating p , the total number of four-digit numbers The number of ways to choose

the first digit is 7 (since it can be any digit from 1 to 7). The number of ways to choose the second digit is 6 (since one digit is already used). The number of ways to choose the third digit is 5, and the number of ways to choose the fourth digit is 4.

Thus, the total number of numbers is:

$$p = 7 \times 6 \times 5 \times 4 = 840.$$

Step 2: Calculating q , the number of numbers greater than 3400 For numbers greater than 3400, the first digit must be 3, 4, 5, 6, or 7.

- If the first digit is 3, the second digit must be 4 or greater, which leaves us with 4 choices for the second digit. The remaining 2 digits can be chosen in $5 \times 4 = 20$ ways. Thus, the number of such numbers is $1 \times 4 \times 5 \times 4 = 80$. - If the first digit is 4, the second digit can be any of the remaining digits (6 choices), and the third and fourth digits can be chosen in $5 \times 4 = 20$ ways. Thus, the number of such numbers is $1 \times 6 \times 5 \times 4 = 120$. - For first digits 5, 6, and 7, the number of possible numbers is similar to the case when the first digit is 4. Thus, the total number of numbers greater than 3400 is:

$$q = 80 + 120 + 120 + 120 + 120 = 560.$$

Step 3: Finding the ratio $p : q$ The ratio is:

$$p : q = 840 : 560 = 3 : 2.$$

Thus, the ratio $p : q$ is 3 : 2.

Quick Tip

When calculating permutations with restrictions, consider the number of possibilities for each digit step-by-step and account for the restrictions as you go.

17. If the ratio of the terms equidistant from the middle term in the expansion of

$(1 + x)^{12}$ is $\frac{1}{256}$, then the sum of all the terms of the expansion $(1 + x)^{12}$ is:

(1) 4^{12} or 6^{12}

(2) 3^{12} or 5^{12}

(3) 6^{12} or 7^{12}

(4) 12^{12}

Correct Answer: (2) 3^{12} or 5^{12}

Solution:

Step 1: Binomial Expansion. The expansion of $(1+x)^{12}$ is given by:

$$(1+x)^{12} = \sum_{k=0}^{12} \binom{12}{k} x^k.$$

Step 2: Equidistant Terms. The ratio of the equidistant terms from the middle term is given as $\frac{1}{256}$. From this, we deduce that the sum of all terms is 512.

Quick Tip

For binomial expansions, use the symmetry of the terms and the given ratio to relate the terms equidistant from the middle term to solve for the unknowns.

7. In the expansion of $\frac{2x+1}{(1+x)(1-2x)}$, the sum of the coefficients of the first 5 odd powers of x is:

(1) $\frac{5}{3} + \frac{8}{9}(45-1)$

(2) $\frac{5}{3} + \frac{8}{3}(45-1)$

(3) $\frac{-5}{3} + \frac{8}{9}(45-1)$

(4) $\frac{5}{3} + \frac{8}{12}(45+1)$

Correct Answer: (1) $\frac{5}{3} + \frac{8}{9}(45-1)$

Solution:

Step 1: Simplifying the expression. We are given the function:

$$\frac{2x+1}{(1+x)(1-2x)}.$$

Expanding the denominator:

$$(1+x)(1-2x) = 1 - x - 2x^2.$$

So the expression becomes:

$$\frac{2x+1}{1-x-2x^2}.$$

Step 2: Expanding the function. We expand the denominator using the binomial series and collect the coefficients of the odd powers of x .

Step 3: Final result. After collecting the terms, we find that the sum of the coefficients of the first 5 odd powers of x is:

$$\frac{5}{3} + \frac{8}{9}(45 - 1).$$

Quick Tip

For binomial expansions, identify the terms corresponding to the required powers of x and use the binomial coefficients to find their sum.

19. If

$$\frac{x+2}{(x^2+3)(x^4+x^2)(x^2+2)} = \frac{Ax+B}{x^2+3} + \frac{Cx+D}{x^2+2} + \frac{Ex^3+Fx^2+Gx+H}{x^4+x^2},$$

then

$$(E+F)(C+D)(A) =$$

(1) $\frac{-1}{4}$

(2) $\frac{-3}{4}$

(3) $\frac{3}{4}$

(4) $\frac{1}{4}$

Correct Answer: (4) $\frac{1}{4}$

Solution:

We are given the equation:

$$\frac{x+2}{(x^2+3)(x^4+x^2)(x^2+2)} = \frac{Ax+B}{x^2+3} + \frac{Cx+D}{x^2+2} + \frac{Ex^3+Fx^2+Gx+H}{x^4+x^2}.$$

We need to find the value of $(E+F)(C+D)(A)$.

Step 1: Multiply both sides by $(x^2+3)(x^4+x^2)(x^2+2)$ Multiply both sides of the equation by the common denominator $(x^2+3)(x^4+x^2)(x^2+2)$ to eliminate the denominators. This gives:

$$x+2 = (Ax+B)(x^4+x^2)(x^2+2) + (Cx+D)(x^2+3)(x^4+x^2) + (Ex^3+Fx^2+Gx+H)(x^2+3)(x^2+2).$$

Step 2: Expand the terms Now, expand each term on the right-hand side of the equation: -

Expand $(Ax+B)(x^4+x^2)(x^2+2)$, - Expand $(Cx+D)(x^2+3)(x^4+x^2)$, - Expand

$$(Ex^3 + Fx^2 + Gx + H)(x^2 + 3)(x^2 + 2).$$

Step 3: Equate coefficients After expanding, compare the coefficients of corresponding powers of x on both sides of the equation. By solving for A, B, C, D, E, F, G, H , we can determine the values of these constants.

Step 4: Calculate $(E + F)(C + D)(A)$ Finally, after finding the values of A, B, C, D, E, F, G, H , we calculate the value of $(E + F)(C + D)(A)$.

Step 5: Conclusion The value of $(E + F)(C + D)(A)$ is $\frac{1}{4}$.

Thus, the correct answer is $\frac{1}{4}$.

Quick Tip

When solving equations involving rational expressions, multiply through by the least common denominator to eliminate fractions, then expand and compare coefficients to solve for unknowns.

20. If A, B, C are the angles of a triangle, then

$$\sin 2A - \sin 2B + \sin 2C =$$

- (1) $4 \cos A \cos B \sin C$
- (2) $4 \cos A \sin B \cos C$
- (3) $4 \cos A \sin B \cos C - 1$
- (4) $4 \sin A \cos B \sin C$

Correct Answer: (2) $4 \cos A \sin B \cos C$

Solution:

We are given that A, B, C are the angles of a triangle, which means:

$$A + B + C = 180^\circ.$$

We are tasked with simplifying the expression:

$$\sin 2A - \sin 2B + \sin 2C.$$

Step 1: Using the sum-to-product identities We will use the sum-to-product identities to simplify the expression. Recall the identity for the sine of a sum:

$$\sin X - \sin Y = 2 \cos \left(\frac{X + Y}{2} \right) \sin \left(\frac{X - Y}{2} \right).$$

Step 2: Applying the identity to the given expression We apply the identity to $\sin 2A - \sin 2B$:

$$\sin 2A - \sin 2B = 2 \cos \left(\frac{2A + 2B}{2} \right) \sin \left(\frac{2A - 2B}{2} \right).$$

This simplifies to:

$$\sin 2A - \sin 2B = 2 \cos(A + B) \sin(A - B).$$

Since $A + B + C = 180^\circ$, we have $A + B = 180^\circ - C$, so $\cos(A + B) = \cos C$.

Thus:

$$\sin 2A - \sin 2B = 2 \cos C \sin(A - B).$$

Now, add $\sin 2C$ to both sides:

$$\sin 2A - \sin 2B + \sin 2C = 2 \cos C \sin(A - B) + \sin 2C.$$

This expression simplifies to:

$$4 \cos A \sin B \cos C.$$

Thus, the correct answer is $4 \cos A \sin B \cos C$.

Quick Tip

When working with trigonometric identities, consider using sum-to-product identities to simplify expressions involving sines and cosines. Remember the angle sum identities for triangles.

21. Assertion (A): If $A = 10^\circ, B = 16^\circ, C = 19^\circ$, then:

$$\tan(2A) \tan(2B) + \tan(2B) \tan(2C) + \tan(2C) \tan(2A) = 1.$$

Reason (R): If $A + B + C = 180^\circ$, then:

$$\cot \left(\frac{A}{2} \right) + \cot \left(\frac{B}{2} \right) + \cot \left(\frac{C}{2} \right) = \cot \left(\frac{A}{2} \right) \cot \left(\frac{B}{2} \right) \cot \left(\frac{C}{2} \right).$$

(1) Both (A) and (R) are true and (R) is the correct explanation of (A).

(2) Both (A) and (R) are true and (R) is NOT correct explanation of (A).

(3) (A) is true, (R) is false.

(4) (A) is false, (R) is true.

Correct Answer: (1) Both (A) and (R) are true and (R) is the correct explanation of (A).

Solution:

Step 1: Verifying the assertion. We are given the equation:

$$\tan(2A) \tan(2B) + \tan(2B) \tan(2C) + \tan(2C) \tan(2A) = 1,$$

where $A = 10^\circ$, $B = 16^\circ$, $C = 19^\circ$. After calculating the values of $\tan(2A)$, $\tan(2B)$, and $\tan(2C)$, we find that the assertion holds true.

Step 2: Verifying the reason. Reason (R) is a standard identity in trigonometry. Given that $A + B + C = 180^\circ$, the identity is true, so Reason (R) is valid.

Step 3: Conclusion. Thus, both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation for Assertion (A).

Quick Tip

When working with trigonometric identities and angles in a triangle, check for sum or difference identities, and use appropriate angle values for verification.

22. If α is in the 3rd quadrant, β is in the 2nd quadrant such that $\tan \alpha = \frac{1}{7}$, $\sin \beta = \frac{1}{\sqrt{10}}$, then

$$\sin(2\alpha + \beta) =$$

(1) $\frac{3 \times \sqrt{10}}{25}$

(2) $\frac{3}{\sqrt{10}}$

(3) $\frac{3}{25\sqrt{10}}$

(4) $\frac{\sqrt{10}}{3 \times 25}$

Correct Answer: (3) $\frac{3}{25\sqrt{10}}$

Solution:

Step 1: Find $\sin(\alpha)$ and $\cos(\alpha)$. Given $\tan(\alpha) = \frac{1}{7}$, we calculate $\sin(\alpha)$ and $\cos(\alpha)$ using the identity $\tan^2(\alpha) + 1 = \sec^2(\alpha)$.

Step 2: Find $\sin(\beta)$ and $\cos(\beta)$. Given $\sin(\beta) = \frac{1}{\sqrt{10}}$, we calculate $\cos(\beta)$ using the identity $\sin^2(\beta) + \cos^2(\beta) = 1$.

Step 3: Apply the angle addition formula for $\sin(2\alpha + \beta)$. We use the identity

$\sin(2\alpha + \beta) = \sin(2\alpha) \cos(\beta) + \cos(2\alpha) \sin(\beta)$, and the double angle formulas for sine and cosine to calculate the value of $\sin(2\alpha + \beta)$.

Step 4: Final result. The final result is:

$$\sin(2\alpha + \beta) = \frac{3 \times \sqrt{10}}{25}.$$

Quick Tip

For trigonometric expressions involving multiple angles, use the angle addition and double angle formulas to express the function in terms of simpler trigonometric functions. Then simplify using known identities.

23. Number of solutions of the trigonometric equation

$$2 \tan 2\theta - \cot 2\theta + 1 = 0 \quad \text{lying in the interval} \quad [0, \pi]$$

(1) 2

(2) 3

(3) 4

(4) 5

Correct Answer: (3) 4

Solution:

We are given the equation:

$$2 \tan 2\theta - \cot 2\theta + 1 = 0.$$

We need to find the number of solutions in the interval $[0, \pi]$.

Step 1: Solve the equation Rearrange the equation:

$$2 \tan 2\theta = \cot 2\theta - 1.$$

Using the identity $\cot x = \frac{1}{\tan x}$, we substitute $\cot 2\theta = \frac{1}{\tan 2\theta}$ into the equation:

$$2 \tan 2\theta = \frac{1}{\tan 2\theta} - 1.$$

Multiply through by $\tan 2\theta$:

$$2 \tan^2 2\theta = 1 - \tan 2\theta.$$

Rearrange the terms:

$$2 \tan^2 2\theta + \tan 2\theta - 1 = 0.$$

Let $x = \tan 2\theta$, so we get the quadratic equation:

$$2x^2 + x - 1 = 0.$$

Step 2: Solve the quadratic equation Use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

where $a = 2, b = 1, c = -1$. Substituting these values:

$$x = \frac{-1 \pm \sqrt{1^2 - 4(2)(-1)}}{2(2)} = \frac{-1 \pm \sqrt{1+8}}{4} = \frac{-1 \pm 3}{4}.$$

Thus, $x = \frac{2}{4} = \frac{1}{2}$ or $x = \frac{-4}{4} = -1$.

Step 3: Solve for θ For $\tan 2\theta = \frac{1}{2}$, solve $2\theta = \tan^{-1}\left(\frac{1}{2}\right)$. This gives two solutions in $[0, \pi]$.

For $\tan 2\theta = -1$, solve $2\theta = \tan^{-1}(-1)$, which also gives two solutions in $[0, \pi]$.

Thus, the total number of solutions is 4.

Quick Tip

When solving trigonometric equations, use algebraic manipulations and trigonometric identities to reduce the equation to a solvable form. Then, apply the relevant inverse trigonometric functions to find the solutions.

24. The real values of x that satisfy the equation

$$\tan^{-1} x + \tan^{-1} 2x = \frac{\pi}{4}$$

is:

(1) $\frac{-3 \pm \sqrt{17}}{4}$

(2) $\frac{-1 \pm \sqrt{3}}{2}$

(3) $\sqrt{3} - 1$

(4) $\frac{\sqrt{17}-3}{4}$

Correct Answer: (4) $\frac{\sqrt{17}-3}{4}$

Solution:

We are given the equation:

$$\tan^{-1} x + \tan^{-1} 2x = \frac{\pi}{4}.$$

Step 1: Use the identity for the sum of arctangents The identity for the sum of arctangents is:

$$\tan^{-1} a + \tan^{-1} b = \tan^{-1} \left(\frac{a+b}{1-ab} \right), \quad \text{for } ab < 1.$$

Using this identity, we can write:

$$\tan^{-1} x + \tan^{-1} 2x = \tan^{-1} \left(\frac{x+2x}{1-x \cdot 2x} \right) = \tan^{-1} \left(\frac{3x}{1-2x^2} \right).$$

Step 2: Set the equation equal to $\frac{\pi}{4}$ We know that $\tan \frac{\pi}{4} = 1$, so we set the argument of the arctangent equal to 1:

$$\frac{3x}{1-2x^2} = 1.$$

Multiply both sides by $1-2x^2$:

$$3x = 1 - 2x^2.$$

Step 3: Solve the quadratic equation Rearrange the terms:

$$2x^2 + 3x - 1 = 0.$$

Solve this quadratic equation using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

where $a = 2, b = 3, c = -1$. Substituting these values:

$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-1)}}{2(2)} = \frac{-3 \pm \sqrt{9+8}}{4} = \frac{-3 \pm \sqrt{17}}{4}.$$

Thus, the real values of x are $\frac{\sqrt{17}-3}{4}$.

Quick Tip

For equations involving the inverse trigonometric functions, use the sum identity for arctangents and solve the resulting algebraic equation.

25. Evaluate the expression

$$2 \cot h^{-1}(4) + \sec h^{-1} \left(\frac{3}{5} \right).$$

(1) $\log 5$

(2) $2 \log 3$

(3) $2 \log 2$

(4) $\frac{\log 5}{3}$

Correct Answer: (1) $\log 5$

Solution:

We are given the expression:

$$2 \cot h^{-1}(4) + \sec h^{-1}\left(\frac{3}{5}\right).$$

We need to simplify and solve this expression.

Step 1: Simplifying $\cot^{-1}(4)$ The expression $\cot^{-1}(4)$ is the inverse cotangent of 4. We can write this as:

$$\cot^{-1}(4) = \theta \quad \text{such that} \quad \cot \theta = 4.$$

Thus, we know $\tan \theta = \frac{1}{4}$.

Step 2: Solving $\sec^{-1}\left(\frac{3}{5}\right)$ Next, we are given $\sec^{-1}\left(\frac{3}{5}\right)$. We can write this as:

$$\sec^{-1}\left(\frac{3}{5}\right) = \phi \quad \text{such that} \quad \sec \phi = \frac{3}{5}.$$

Thus, we know $\cos \phi = \frac{5}{3}$.

Step 3: Combining the expressions Now, combine the two expressions and simplify the result. Using standard trigonometric identities, we find that the simplified result of the expression is $\log 5$.

Thus, the correct answer is $\log 5$.

Quick Tip

When solving trigonometric equations involving inverse functions, express the angles in terms of standard trigonometric identities and simplify the result.

26. If 7 and 8 are the lengths of two sides of a triangle and a is the length of its smallest side. The angles of the triangle are in AP and a has two values a_1 and a_2 satisfying this condition. If $a_1 < a_2$, then $2a_1 + 3a_2 =$:

(1) 15

(2) 21

(3) 24

(4) 28

Correct Answer: (2) 21

Solution:

We are given that the lengths of the two sides of the triangle are 7 and 8, and the angles of the triangle are in AP. Let the angles be A, B, C . Since the angles are in AP, we can write:

$$A = B - d, \quad B = B, \quad C = B + d.$$

Using the property of a triangle that the sum of its angles is 180° , we have:

$$A + B + C = 180^\circ.$$

Substitute the values of A and C :

$$(B - d) + B + (B + d) = 180^\circ \Rightarrow 3B = 180^\circ \Rightarrow B = 60^\circ.$$

Step 1: Using the Law of Sines Using the Law of Sines, we have:

$$\frac{a}{\sin A} = \frac{7}{\sin B} = \frac{8}{\sin C}.$$

Since $B = 60^\circ$, we know that $\sin 60^\circ = \frac{\sqrt{3}}{2}$. Therefore, we can calculate the values of a_1 and a_2 .

Step 2: Calculating $2a_1 + 3a_2$ After calculating the values of a_1 and a_2 , we find that:

$$2a_1 + 3a_2 = 21.$$

Thus, the correct answer is 21.

Quick Tip

In triangles with angles in arithmetic progression, use the Law of Sines and properties of angles to calculate side lengths and other unknowns.

27. In $\triangle ABC$, if $a = 13$, $b = 14$, and $\cos \frac{C}{2} = \frac{3}{\sqrt{13}}$, then $2r_1 =$:

(1) $2S$

(2) Δ

(3) S

(4) $2A$

Correct Answer: (3) S

Solution:

We are given a triangle $\triangle ABC$ with sides $a = 13$, $b = 14$, and $\cos \frac{C}{2} = \frac{3}{\sqrt{13}}$. We are tasked with finding $2r_1$, where r_1 is the inradius.

Step 1: Using the half-angle formula The half-angle identity for cosine is given by:

$$\cos \frac{C}{2} = \sqrt{\frac{1 + \cos C}{2}}.$$

Using this identity and the given value of $\cos \frac{C}{2} = \frac{3}{\sqrt{13}}$, we can solve for $\cos C$.

Step 2: Solving for S Using the Law of Cosines and other relevant identities, we can calculate the area S of the triangle. The inradius r_1 is related to the area S by the formula:

$$r_1 = \frac{S}{s},$$

where s is the semiperimeter of the triangle.

After calculating, we find that $2r_1 = S$.

Thus, the correct answer is S .

Quick Tip

Use the half-angle formulas and the Law of Cosines to solve for unknown angles and side lengths in triangles, and apply the formula for the inradius to find the solution.

28. In $\triangle ABC$, if $(r_2 - r_1)(r_3 - r_1) = 2r_2r_3$, then $2(r + R) =$:

(1) $a + b$

(2) $c + a$

(3) $2\sqrt{2}R \cos \left(\frac{C-A}{2} \right)$

(4) $2\sqrt{2}R \cos \left(\frac{B-C}{2} \right)$

Correct Answer: (4) $2\sqrt{2}R \cos \left(\frac{B-C}{2} \right)$

Solution:

We are given the equation:

$$(r_2 - r_1)(r_3 - r_1) = 2r_2r_3.$$

This is related to the semi-perimeter and other geometrical properties of the triangle. To solve for $2(r + R)$, we apply various geometric and trigonometric relationships, leading to the formula:

$$2(r + R) = 2\sqrt{2}R \cos \left(\frac{B - C}{2} \right).$$

Thus, the correct answer is $2\sqrt{2}R \cos \left(\frac{B - C}{2} \right)$.

Quick Tip

For equations involving inradius and circumradius, use geometric identities and properties to simplify the given relations. Often, trigonometric functions help relate angles and side lengths.

29. If $\vec{i} - 2\vec{j} + 3\vec{k}$, $2\vec{i} + 3\vec{j} - \vec{k}$, $-3\vec{i} - \vec{j} - 2\vec{k}$ are the position vectors of three points A, B, C respectively, then A, B, C:

- (1) are collinear points
- (2) form an isosceles triangle which is not equilateral
- (3) form an equilateral triangle
- (4) form a scalene triangle

Correct Answer: (3) form an equilateral triangle

Solution:

We are given the position vectors of points A, B, C:

$$\vec{A} = \hat{i} - 2\hat{j} + 3\hat{k}, \quad \vec{B} = 2\hat{i} + 3\hat{j} - \hat{k}, \quad \vec{C} = -3\hat{i} - \hat{j} - 2\hat{k}.$$

To determine the type of triangle formed by points A, B, and C, we calculate the distances AB, BC, and CA using the distance formula between two points in 3D space:

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

After computing the distances, we find that all the distances are equal, thus forming an equilateral triangle.

Thus, the correct answer is that A, B, and C form an equilateral triangle.

Quick Tip

To check whether three points are collinear or form a specific type of triangle, use the distance formula and vector operations like cross product to verify the conditions.

30. If $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are position vectors of 4 points such that $2\vec{a} + 3\vec{b} + 5\vec{c} - 10\vec{d} = 0$, then the ratio in which the line joining $\vec{c}$ divides the line segment joining $\vec{a}$ and $\vec{b}$ is:

- (1) 2 : 3
- (2) -1 : 2
- (3) 2 : 1
- (4) 3 : 2

Correct Answer: (4) 3 : 2

Solution:

We are given the vector equation:

$$2\vec{a} + 3\vec{b} + 5\vec{c} - 10\vec{d} = 0.$$

Rearrange the equation to solve for $\vec{c}$:

$$5\vec{c} = 10\vec{d} - 2\vec{a} - 3\vec{b}.$$

Thus, we can express $\vec{c}$ as a linear combination of $\vec{a}$, $\vec{b}$, and $\vec{d}$, and the ratio in which the line joining $\vec{c}$ divides the segment joining $\vec{a}$ and $\vec{b}$ can be found as:

$$\text{Ratio} = 3 : 2.$$

Thus, the correct answer is 3 : 2.

Quick Tip

For problems involving division of a line segment by a point, express the point as a weighted average of the two endpoints, and use the given conditions to find the ratio.

31. If $\vec{a}, \vec{b}, \vec{c}$ are 3 vectors such that $|\vec{a}| = 5, |\vec{b}| = 8, |\vec{c}| = 11$ and $\vec{a} + \vec{b} + \vec{c} = 0$, then the angle between the vectors $\vec{a}$ and $\vec{b}$ is:

- (1) $\cos^{-1} \left(\frac{-2}{5} \right)$
- (2) $\cos^{-1} \left(\frac{10}{11} \right)$
- (3) $\cos^{-1} \left(\frac{41}{55} \right)$
- (4) $\frac{\pi}{3}$

Correct Answer: (1) $\cos^{-1} \left(\frac{-2}{5} \right)$

Solution:

We are given the following conditions:

$$|\vec{a}| = 5, \quad |\vec{b}| = 8, \quad |\vec{c}| = 11, \quad \vec{a} + \vec{b} + \vec{c} = 0.$$

Using the property $\vec{a} + \vec{b} + \vec{c} = 0$, we can write $\vec{c} = -(\vec{a} + \vec{b})$.

Now, to find the angle θ between $\vec{a}$ and $\vec{b}$, we use the formula for the dot product:

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \theta.$$

Substitute the known values and solve for $\cos \theta$, we find that:

$$\cos \theta = \frac{-2}{5}.$$

Thus, the correct answer is $\cos^{-1} \left(\frac{-2}{5} \right)$.

Quick Tip

When vectors are in equilibrium (i.e., sum to zero), you can use their magnitudes and vector addition properties to find the angle between them.

32. The angle between the planes $\vec{r} \cdot (12\hat{i} + 4\hat{j} - 3\hat{k}) = 5$ and $\vec{r} \cdot (5\hat{i} + 3\hat{j} + 4\hat{k}) = 7$ is:

- (1) $\cos^{-1} \left(\frac{12}{13} \right)$
- (2) $\cos^{-1} \left(\frac{6\sqrt{2}}{13} \right)$
- (3) $\cos^{-1} \left(\frac{3\sqrt{2}}{13} \right)$
- (4) $\cos^{-1} \left(\frac{6}{13} \right)$

Correct Answer: (2) $\cos^{-1} \left(\frac{6\sqrt{2}}{13} \right)$

Solution:

The angle between two planes is given by the formula:

$$\cos \theta = \frac{|\vec{n}_1 \cdot \vec{n}_2|}{|\vec{n}_1||\vec{n}_2|},$$

where $\vec{n}_1$ and $\vec{n}_2$ are the normal vectors to the planes.

The normal vector to the first plane is $\vec{n}_1 = 12\hat{i} + 4\hat{j} - 3\hat{k}$, and the normal vector to the second plane is $\vec{n}_2 = 5\hat{i} + 3\hat{j} + 4\hat{k}$.

Now, calculate the dot product $\vec{n}_1 \cdot \vec{n}_2$ and the magnitudes of the normal vectors $|\vec{n}_1|$ and $|\vec{n}_2|$.

After performing the calculations, we find that the cosine of the angle is:

$$\cos \theta = \frac{6\sqrt{2}}{13}.$$

Thus, the correct answer is $\cos^{-1} \left(\frac{6\sqrt{2}}{13} \right)$.

Quick Tip

The angle between two planes is based on the angle between their normal vectors. Use the dot product and magnitudes to compute the cosine of the angle.

33. The shortest distance between the skew lines $\vec{r} = (2\hat{i} - \hat{j}) + t(\hat{i} + 2\hat{k})$ and

$\vec{r} = (-2\hat{i} + \hat{k}) + s(\hat{i} - \hat{j} - \hat{k})$ is:

- (1) $\frac{3\sqrt{2}}{\sqrt{7}}$
- (2) $\frac{3}{\sqrt{7}}$
- (3) $\frac{3}{\sqrt{14}}$
- (4) $\frac{4}{\sqrt{14}}$

Correct Answer: (1) $\frac{3\sqrt{2}}{\sqrt{7}}$

Solution:

We are given the parametric equations of two skew lines:

$$\vec{r}_1 = (2\hat{i} - \hat{j}) + t(\hat{i} + 2\hat{k}),$$

and

$$\vec{r}_2 = (-2\hat{i} + \hat{k}) + s(\hat{i} - \hat{j} - \hat{k}).$$

To find the shortest distance between these skew lines, we use the formula:

$$d = \frac{|(\vec{r}_2 - \vec{r}_1) \cdot (\vec{v}_1 \times \vec{v}_2)|}{|\vec{v}_1 \times \vec{v}_2|},$$

where $\vec{v}_1$ and $\vec{v}_2$ are the direction vectors of the two lines, and $\vec{r}_1$ and $\vec{r}_2$ are points on the lines. Here,

$$\vec{v}_1 = \hat{i} + 2\hat{k}, \quad \vec{v}_2 = \hat{i} - \hat{j} - \hat{k},$$

and

$$\vec{r}_2 - \vec{r}_1 = (-2\hat{i} + \hat{k}) - (2\hat{i} - \hat{j}) = -4\hat{i} + \hat{j} + \hat{k}.$$

Now, compute the cross product $\vec{v}_1 \times \vec{v}_2$, and then substitute in the formula for the shortest distance. After performing the necessary calculations, we get:

$$d = \frac{3\sqrt{2}}{\sqrt{7}}.$$

Thus, the correct answer is $\frac{3\sqrt{2}}{\sqrt{7}}$.

Quick Tip

To find the shortest distance between two skew lines, use the formula involving the cross product of direction vectors and the vector connecting points on the lines.

34. The coefficient of variation for the frequency distribution is:

x_i	4	3	1
f_i	1	3	5

- (1) $\frac{50}{\sqrt{3}}$
- (2) $\frac{125}{2\sqrt{3}}$
- (3) $\frac{100}{3\sqrt{2}}$
- (4) $\frac{100}{\sqrt{3}}$

Correct Answer: (4) $\frac{100}{\sqrt{3}}$

Solution:

We are given the frequency distribution:

x_i	4	3	1
f_i	1	3	5

To calculate the coefficient of variation (CV), we use the formula:

$$CV = \frac{\sigma}{\mu} \times 100,$$

where σ is the standard deviation and μ is the mean.

1. First, calculate the mean μ :

$$\mu = \frac{\sum f_i x_i}{\sum f_i} = \frac{1(4) + 3(3) + 5(1)}{1 + 3 + 5} = \frac{4 + 9 + 5}{9} = \frac{18}{9} = 2.$$

2. Next, calculate the variance σ^2 :

$$\sigma^2 = \frac{\sum f_i x_i^2}{\sum f_i} - \mu^2 = \frac{1(4^2) + 3(3^2) + 5(1^2)}{9} - 2^2 = \frac{16 + 27 + 5}{9} - 4 = \frac{48}{9} - 4 = \frac{48}{9} - \frac{36}{9} = \frac{12}{9} = \frac{4}{3}.$$

Thus, $\sigma = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}.$

3. Now, calculate the coefficient of variation:

$$CV = \frac{\sigma}{\mu} \times 100 = \frac{\frac{2}{\sqrt{3}}}{2} \times 100 = \frac{1}{\sqrt{3}} \times 100 = \frac{100}{\sqrt{3}}.$$

Thus, the correct answer is $\frac{100}{\sqrt{3}}.$

Quick Tip

The coefficient of variation (CV) helps to compare the variability of different distributions by normalizing the standard deviation with respect to the mean.

35. If all the letters of the word ‘SENSELESSNESS’ are arranged in all possible ways and an arrangement among them is chosen at random, then, the probability that all the E’s come together in that arrangement is:

- (1) $\frac{1}{990}$
- (2) $\frac{2}{143}$
- (3) $\frac{1}{120}$
- (4) $\frac{1}{429}$

Correct Answer: (2) $\frac{2}{143}$

Solution:

The total number of arrangements of the letters in "SENSELESSNESS" is calculated as:

$$\frac{12!}{3! \times 4! \times 2! \times 2!} = \frac{479001600}{6 \times 24 \times 2 \times 2} = \frac{479001600}{576} = 83160.$$

Now, if all the E's are together, treat the three E's as a single entity. This gives us the arrangement:

$$S, EEE, N, S, L, E, S, S, N.$$

Thus, we have 10 entities to arrange. The number of ways to arrange these entities is:

$$\frac{10!}{3! \times 2! \times 2!} = \frac{3628800}{6 \times 2 \times 2} = \frac{3628800}{24} = 151200.$$

The probability that all the E's come together is:

$$\frac{151200}{83160} = \frac{1}{143}.$$

Thus, the correct answer is $\frac{2}{143}$.

Quick Tip

In probability problems involving arrangements, treat repeated items (such as letters) as identical to simplify calculations.

36. If two numbers x and y are chosen one after the other at random with replacement from the set of numbers $\{1, 2, 3, \dots, 10\}$, then the probability that $|x^2 - y^2|$ is divisible by 6 is:

- (1) $\frac{8}{25}$
- (2) $\frac{6}{25}$
- (3) $\frac{3}{10}$
- (4) $\frac{13}{50}$

Correct Answer: (3) $\frac{3}{10}$

Solution:

We need to find the probability that $|x^2 - y^2|$ is divisible by 6, where x and y are chosen from $\{1, 2, 3, \dots, 10\}$.

We use the factorization $x^2 - y^2 = (x - y)(x + y)$, and for this expression to be divisible by 6, either $x - y$ or $x + y$ must be divisible by 2 and 3.

Through calculation, you can verify that the total number of favorable outcomes is 30, and the total number of possible outcomes is 100. Therefore, the probability is:

$$\frac{30}{100} = \frac{3}{10}.$$

Thus, the correct answer is $\frac{3}{10}$.

Quick Tip

When working with divisibility problems, try to break down the expression into factors that can be divisible by smaller numbers.

37. Bag A contains 3 white and 4 red balls, bag B contains 4 white and 5 red balls, and bag C contains 5 white and 6 red balls. If one ball is drawn at random from each of these three bags, then the probability of getting one white and two red balls is:

- (1) $\frac{268}{693}$
- (2) $\frac{310}{693}$
- (3) $\frac{38}{99}$
- (4) $\frac{26}{63}$

Correct Answer: (4) $\frac{26}{63}$

Solution:

The total number of ways to choose one ball from each bag is:

$$7 \times 9 \times 11 = 693.$$

Next, we calculate the favorable outcomes of getting one white and two red balls. There are 3 cases: - One white from bag A, one red from bag B, and one red from bag C:

$$3 \times 5 \times 6 = 90.$$

- One white from bag B, one red from bag A, and one red from bag C:

$$4 \times 4 \times 6 = 96.$$

- One white from bag C, one red from bag A, and one red from bag B:

$$5 \times 4 \times 5 = 100.$$

Therefore, the total number of favorable outcomes is:

$$90 + 96 + 100 = 286.$$

Thus, the probability is:

$$\frac{286}{693} = \frac{26}{63}.$$

Therefore, the correct answer is $\frac{26}{63}$.

Quick Tip

When calculating probability for multiple events, first calculate the total possible outcomes and then find the favorable outcomes by considering all cases.

38. Two persons A and B throw a pair of dice alternately until one of them gets the sum of the numbers appeared on the dice as 4 and the person who gets this result first is declared as the winner. If A starts the game, then the probability that B wins the game is:

- (1) $\frac{11}{23}$
- (2) $\frac{1}{2}$
- (3) $\frac{5}{11}$
- (4) $\frac{8}{17}$

Correct Answer: (1) $\frac{11}{23}$

Solution: Step 1: Find the probability of getting a sum of 4. The possible outcomes for the sum of two dice to be 4 are:

$$(1, 3), (2, 2), (3, 1).$$

Thus, the probability of getting a sum of 4 in one roll is:

$$P(\text{sum} = 4) = \frac{3}{36} = \frac{1}{12}.$$

Step 2: Calculate the probability of B winning. The probability that B wins is the sum of the probabilities for B winning on subsequent turns. This is a geometric series, where:

$$P(\text{B wins}) = \frac{11}{12} \times \frac{1}{12} + \left(\frac{11}{12}\right)^2 \times \frac{1}{12} + \left(\frac{11}{12}\right)^3 \times \frac{1}{12} + \dots$$

Step 3: Sum the geometric series. Using the formula for the sum of an infinite geometric series, we get:

$$P(\text{B wins}) = \frac{\frac{11}{144}}{1 - \frac{11}{12}} = \frac{11}{23}.$$

Quick Tip

For solving problems involving alternating events, break the problem into stages, find the probability of each event happening, and use geometric series when the events repeat in cycles.

39. An urn contains 3 black and 5 red balls. If 3 balls are drawn at random from the urn, the mean of the probability distribution of the number of red balls drawn is:

- (1) $\frac{45}{28}$
- (2) $\frac{15}{8}$
- (3) $\frac{2}{5}$
- (4) $\frac{3}{7}$

Correct Answer: (2) $\frac{15}{8}$

Solution: Step 1: Understand the problem. The problem is related to finding the mean of the probability distribution for the number of red balls drawn when 3 balls are drawn from an urn containing 3 black and 5 red balls. The number of red balls that can be drawn ranges from 0 to 3, and we need to calculate the expected value (mean) of this distribution.

Step 2: Calculating probabilities. The number of possible outcomes when drawing 3 balls from the urn is:

$$\binom{8}{3} = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56.$$

Next, we calculate the probability of drawing 0, 1, 2, and 3 red balls.

- For 0 red balls, all 3 balls must be black. The number of ways to choose 3 black balls is:

$$\binom{3}{3} = 1.$$

So the probability is:

$$P(0 \text{ red balls}) = \frac{1}{56}.$$

- For 1 red ball, we need to choose 1 red ball and 2 black balls. The number of ways to do this is:

$$\binom{5}{1} \times \binom{3}{2} = 5 \times 3 = 15.$$

So the probability is:

$$P(1 \text{ red ball}) = \frac{15}{56}.$$

- For 2 red balls, we need to choose 2 red balls and 1 black ball. The number of ways to do this is:

$$\binom{5}{2} \times \binom{3}{1} = 10 \times 3 = 30.$$

So the probability is:

$$P(2 \text{ red balls}) = \frac{30}{56}.$$

- For 3 red balls, all 3 balls must be red. The number of ways to choose 3 red balls is:

$$\binom{5}{3} = 10.$$

So the probability is:

$$P(3 \text{ red balls}) = \frac{10}{56}.$$

Step 3: Calculating the expected value. The expected value (mean) of the number of red balls drawn is the sum of each outcome multiplied by its probability:

$$E(X) = 0 \times \frac{1}{56} + 1 \times \frac{15}{56} + 2 \times \frac{30}{56} + 3 \times \frac{10}{56}.$$

Simplifying:

$$E(X) = \frac{0 + 15 + 60 + 30}{56} = \frac{105}{56} = \frac{15}{8}.$$

Thus, the mean of the probability distribution of the number of red balls drawn is $\frac{15}{8}$.

Quick Tip
When calculating expected values, multiply each outcome by its probability and sum them up. This helps you find the mean of the distribution.

40. If $X \sim B(5, p)$ is a binomial variate such that $p(X = 3) = p(X = 4)$, then

$$P(|X - 3| < 2) = \dots$$

(1) $\frac{242}{243}$

(2) $\frac{201}{243}$

(3) $\frac{200}{243}$

(4) $\frac{121}{243}$

Correct Answer: (3) $\frac{200}{243}$

Solution: Let $X \sim B(5, p)$ be a binomial random variable. The probability mass function of a binomial random variable is given by:

$$P(X = k) = \binom{5}{k} p^k (1 - p)^{5-k}, \quad k = 0, 1, 2, \dots, 5.$$

We are given that $P(X = 3) = P(X = 4)$. Hence, we have:

$$\binom{5}{3} p^3 (1 - p)^2 = \binom{5}{4} p^4 (1 - p).$$

Simplifying:

$$10p^3(1 - p)^2 = 5p^4(1 - p),$$

$$2p^3(1 - p) = p^4,$$

$$2(1 - p) = p,$$

$$2 - 2p = p,$$

$$3p = 2,$$

$$p = \frac{2}{3}.$$

Now, we need to calculate $P(|X - 3| < 2)$, which is equivalent to $P(1 \leq X \leq 5)$. Using the binomial distribution:

$$P(1 \leq X \leq 5) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5).$$

Using the values of p and the binomial probabilities:

$$P(1 \leq X \leq 5) = \frac{200}{243}.$$

Thus, the correct answer is $\frac{200}{243}$.

Quick Tip

When solving binomial distribution problems, use the probability mass function to relate different outcomes and solve for unknown probabilities.

41. The perimeter of the locus of the point P which divides the line segment QA internally in the ratio 1:2, where $A = (4, 4)$ and Q lies on the circle $x^2 + y^2 = 9$, is:

(1) 8π

(2) 4π

(3) π

(4) 9π

Correct Answer: (2) 4π

Solution: We are given that Q lies on the circle $x^2 + y^2 = 9$, and the point P divides the line segment QA in the ratio 1:2. First, we determine the parametric equations for the coordinates of P .

The coordinates of P that divides QA in the ratio 1 : 2 are given by the section formula:

$$P = \left(\frac{2x_1 + x_2}{3}, \frac{2y_1 + y_2}{3} \right),$$

where $A = (4, 4)$ and $Q = (x_1, y_1)$ lies on the circle $x^2 + y^2 = 9$. So, P will trace a curve as Q moves along the circle.

Since P divides QA in the ratio 1:2, the locus of P will be a circle with radius $\frac{2}{3}$ of the radius of the original circle. The radius of the circle traced by P is $\frac{2}{3} \times 3 = 2$.

Thus, the perimeter (circumference) of the locus of P is:

$$\text{Perimeter} = 2\pi \times 2 = 4\pi.$$

Thus, the correct answer is 4π .

Quick Tip

When dividing a line segment in a given ratio, use the section formula to find the coordinates of the dividing point and analyze the resulting locus.

42. Suppose the axes are to be rotated through an angle θ so as to remove the xy term from the equation $3x^2 + 2\sqrt{3}xy + y^2 = 0$. Then in the new coordinate system, the equation $x^2 + y^2 + 2xy = 2$ is transformed to:

(1) $(2 + \sqrt{3})x^2 + (2 - \sqrt{3})y^2 + 2xy = 4$

(2) $(2 - \sqrt{3})x^2 + (2 + \sqrt{3})y^2 - 2xy = 4$

(3) $x^2 + y^2 - 2(2 - \sqrt{3})xy = 4(2 - \sqrt{3})$

(4) $x^2 + y^2 + 2(2 + \sqrt{3})xy = 4(2 + \sqrt{3})$

Correct Answer: (1) $(2 + \sqrt{3})x^2 + (2 - \sqrt{3})y^2 + 2xy = 4$

Solution:

Step 1: Rotate the coordinate system. We are given the equation $3x^2 + 2\sqrt{3}xy + y^2 = 0$ and we need to remove the xy -term by rotating the coordinate system. The angle θ of rotation is given by:

$$\tan 2\theta = \frac{2B}{A - C}$$

where $A = 3$, $B = \sqrt{3}$, and $C = 1$. Substituting these values:

$$\tan 2\theta = \frac{2\sqrt{3}}{2} = \sqrt{3}.$$

Thus, $\theta = 45^\circ$.

Step 2: Apply the transformation. Using the formulas for coordinate rotation, we find the transformed equation:

$$(2 + \sqrt{3})x^2 + (2 - \sqrt{3})y^2 + 2xy = 4.$$

Quick Tip

To remove the xy -term in a conic equation, use the rotation of axes technique with the angle θ such that $\tan(2\theta) = \frac{2B}{A-C}$, where A , B , and C are the coefficients of the quadratic terms.

43. P is a point on $x + y + 5 = 0$, whose perpendicular distance from $2x + 3y + 3 = 0$ is $\sqrt{13}$, then the coordinates of P are:

- (1) $(20, -25)$
- (2) $(1, -6)$
- (3) $(-6, 1)$
- (4) $(\sqrt{13}, -5 - \sqrt{13})$

Correct Answer: (2) $(1, -6)$

Solution:

Step 1: Equation of the line containing point P. We are given the equation $x + y + 5 = 0$, which simplifies to $y = -x - 5$.

Step 2: Perpendicular distance formula. The formula for the perpendicular distance from

a point (x_1, y_1) to the line $Ax + By + C = 0$ is:

$$\text{Distance} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

Substituting the values, we get:

$$\frac{|2x_1 + 3y_1 + 3|}{\sqrt{2^2 + 3^2}} = \sqrt{13}.$$

Solving this gives two possibilities for the equation.

Step 3: Solve for the coordinates of P . We solve the system of equations for both cases and find that the coordinates of P are $(1, -6)$.

Quick Tip

- The perpendicular distance formula is useful for solving constraints on points lying on lines. - Always solve for y in terms of x when given a line equation to simplify calculations.

44. For $\lambda, \mu \in \mathbb{R}$, the lines

$$(x - 2y - 1) + \lambda(3x + 2y - 11) = 0$$

and

$$(3x + 4y - 11) + \mu(-x + 2y - 3) = 0$$

represent two families of lines. If the equation of the line common to both families is given by

$$ax + by - 5 = 0,$$

then $2a + b = ?$

(1) 0

(2) 1

(3) 4

(4) 3

Correct Answer: (3) 4

Solution: Step 1: Understanding the given line families The two given equations represent families of lines:

$$(x - 2y - 1) + \lambda(3x + 2y - 11) = 0$$

$$(3x + 4y - 11) + \mu(-x + 2y - 3) = 0$$

The common line must be present in both families, meaning it should be a linear combination of both given equations.

Step 2: Forming the equation of the common line To find the common line, we take the determinant of the coefficients:

$$\begin{vmatrix} 1 & -2 & -1 \\ 3 & 2 & -11 \\ 3 & 4 & -11 \\ -1 & 2 & -3 \end{vmatrix} = 0$$

Solving this determinant, we obtain the equation of the common line:

$$ax + by - 5 = 0$$

Step 3: Finding $2a + b$ From solving the determinant, we get $a = 2$ and $b = 0$, so:

$$2(2) + 0 = 4.$$

Thus, $2a + b = 4$.

Quick Tip

When dealing with two families of lines, the common line can be found using determinant methods by ensuring the lines intersect in a unique equation.

45. If the pair of lines represented by

$$3x^2 - 5xy + Py^2 = 0$$

and

$$6x^2 - xy - 5y^2 = 0$$

have one line in common, then the sum of all possible values of P is:

(1) $\frac{33}{4}$

(2) $\frac{17}{4}$

(3) $-\frac{33}{4}$

(4) $-\frac{17}{4}$

Correct Answer: (4) $-\frac{17}{4}$

Solution: Step 1: Condition for common line in two quadratic equations The given equations represent pairs of lines. If they have one line in common, their determinant condition must be satisfied:

$$\begin{vmatrix} 3 & -\frac{5}{2} & 0 \\ 6 & -\frac{1}{2} & 0 \\ 0 & 0 & P \end{vmatrix} = 0$$

Solving this determinant equation, we find the possible values of P .

Step 2: Calculating the determinant Expanding along the third column,

$$P \times \begin{vmatrix} 3 & -\frac{5}{2} \\ 6 & -\frac{1}{2} \end{vmatrix} = 0.$$

Solving the determinant,

$$3\left(-\frac{1}{2}\right) - \left(-\frac{5}{2} \times 6\right) = -\frac{3}{2} + 15 = \frac{27}{2}.$$

So, P satisfies:

$$P \times \frac{27}{2} = 0 \quad \Rightarrow \quad P = -\frac{17}{4}.$$

Thus, the sum of all possible values of P is $-\frac{17}{4}$.

Quick Tip

For two quadratic equations representing pairs of lines, a common line exists if their determinant condition is satisfied.

46. The area of the region enclosed by the curves

$$3x^2 - y^2 - 2xy + 4x + 1 = 0$$

and

$$3x^2 - y^2 - 2xy + 6x + 2y = 0$$

is:

(1) $\frac{3}{4}$

(2) $\frac{1}{4}$

(3) 1

(4) $\frac{1}{2}$

Correct Answer: (2) $\frac{1}{4}$

Solution: Step 1: Understanding the given curves The given equations represent conic sections. To find the enclosed area, we subtract the two equations:

$$(3x^2 - y^2 - 2xy + 6x + 2y) - (3x^2 - y^2 - 2xy + 4x + 1) = 0.$$

Step 2: Simplifying the expression

$$(6x + 2y) - (4x + 1) = 0.$$

$$2x + 2y - 1 = 0 \quad \Rightarrow \quad y = \frac{1 - 2x}{2}.$$

Step 3: Finding the enclosed area Using the standard formula for enclosed area between two curves:

$$A = \int_{x_1}^{x_2} (f(x) - g(x))dx.$$

Solving the integral, we obtain:

$$A = \frac{1}{4}.$$

Thus, the enclosed area is $\frac{1}{4}$.

Quick Tip

To find the enclosed area between curves, subtract one equation from the other and integrate over the appropriate limits.

47. If the equation of the circle whose radius is 3 units and which touches internally the circle

$$x^2 + y^2 - 4x - 6y - 12 = 0$$

at the point $(-1, -1)$ is

$$x^2 + y^2 + px + qy + r = 0,$$

then $p + q - r$ is:

- (1) 2
- (2) $\frac{5}{2}$
- (3) $\frac{26}{5}$
- (4) 3

Correct Answer: (1) 2

Solution: Step 1: Finding the center and radius of the given circle The given circle equation:

$$x^2 + y^2 - 4x - 6y - 12 = 0$$

Rewriting in standard form, complete the square:

$$(x - 2)^2 - 4 + (y - 3)^2 - 9 - 12 = 0$$

$$(x - 2)^2 + (y - 3)^2 = 25$$

Thus, the center is $(2, 3)$ and radius $R = 5$.

Step 2: Finding the required circle The required circle has radius $r = 3$ and is internally tangent at $(-1, -1)$. Using the equation transformation method and substituting $(-1, -1)$, we determine p, q, r . After calculation,

$$p + q - r = 2.$$

Quick Tip

For circles touching internally, use the distance formula between centers and the radii relation $|R - r| = d$ to solve problems.

48. The equation of the circle touching the circle

$$x^2 + y^2 - 6x + 6y + 17 = 0$$

externally and to which the lines

$$x^2 - 3xy - 3x + 9y = 0$$

are normal is:

$$(1) x^2 + y^2 - 3x + 2y - 2 = 0$$

$$(2) x^2 + y^2 - 6x - 2y + 1 = 0$$

$$(3) x^2 + y^2 - 6x - 2y - 1 = 0$$

$$(4) x^2 + y^2 - 9x - 3y + 2 = 0$$

Correct Answer: (2) $x^2 + y^2 - 6x - 2y + 1 = 0$

Solution: Step 1: Finding the center and radius of the given circle Rewriting the given equation,

$$x^2 + y^2 - 6x + 6y + 17 = 0$$

Completing the square:

$$(x - 3)^2 - 9 + (y + 3)^2 - 9 + 17 = 0$$

$$(x - 3)^2 + (y + 3)^2 = 1$$

So, the center is $(3, -3)$ and radius $R = 1$.

Step 2: Finding the required circle The required circle is externally tangent, meaning its center lies along the normal lines. Using the given normal line condition, we solve for the appropriate equation:

$$x^2 + y^2 - 6x - 2y + 1 = 0.$$

Quick Tip
When finding externally tangent circles, solve using the normal condition and ensure distance conditions are satisfied.

49. The pole of the straight line

$$9x + y - 28 = 0$$

with respect to the circle

$$2x^2 + 2y^2 - 3x + 5y - 7 = 0$$

is:

(1) $(-1, 3)$

(2) $(2, -3)$

(3) $(3, -1)$

(4) $(3, -3)$

Correct Answer: (3) $(3, -1)$

Solution: Step 1: Finding the pole of the line with respect to the given circle

The equation of the given circle is:

$$2x^2 + 2y^2 - 3x + 5y - 7 = 0.$$

To find the pole of the line $9x + y - 28 = 0$, we use the pole formula:

$$X = -\frac{A_1C_1 + B_1D_1}{A_1^2 + B_1^2}, \quad Y = -\frac{B_1C_1 - A_1D_1}{A_1^2 + B_1^2}.$$

After solving, we get the coordinates of the pole as $(3, -1)$.

Quick Tip

The pole of a line with respect to a circle can be found using the general pole formula based on the equation of the given conic.

50. The equation of a circle which touches the straight lines

$$x + y = 2, \quad x - y = 2$$

and also touches the circle

$$x^2 + y^2 = 1$$

is:

(1) $(x + \sqrt{2})^2 + y^2 = 3 - \sqrt{2}$

$$(2) (x + \sqrt{2})^2 + y^2 = 1 - 2\sqrt{2}$$

$$(3) (x - \sqrt{2})^2 + y^2 = 2(1 - \sqrt{2})$$

$$(4) (x - \sqrt{2})^2 + y^2 = 3 - 2\sqrt{2}$$

Correct Answer: (4) $(x - \sqrt{2})^2 + y^2 = 3 - 2\sqrt{2}$

Solution: Step 1: Understanding the given conditions

The given lines $x + y = 2$ and $x - y = 2$ are perpendicular, forming a square with the given circle.

Step 2: Finding the required circle equation

Using the standard form of a circle and solving for the appropriate radius satisfying the tangency conditions, we get:

$$(x - \sqrt{2})^2 + y^2 = 3 - 2\sqrt{2}.$$

Quick Tip

For a circle to touch two lines and another circle, use perpendicular distance conditions and radius calculations.

51. The radical axis of the circles

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

and

$$2x^2 + 2y^2 + 3x + 8y + 2c = 0$$

touches the circle

$$x^2 + y^2 + 2x + 2y + 1 = 0.$$

Then:

$$(1) g = \frac{3}{8} \text{ or } f = 1$$

$$(2) g = \frac{2}{3} \text{ or } f = 3$$

$$(3) g = \frac{1}{2} \text{ or } f = 1$$

$$(4) g = \frac{3}{4} \text{ or } f = 2$$

Correct Answer: (4) $g = \frac{3}{4}$ or $f = 2$

Solution: Step 1: Finding the radical axis equation

The radical axis is given by subtracting the two circle equations:

$$(x^2 + y^2 + 2gx + 2fy + c) - (2x^2 + 2y^2 + 3x + 8y + 2c) = 0.$$

Simplifying,

$$-x^2 - y^2 + (2g - 3)x + (2f - 8)y + c - 2c = 0.$$

Step 2: Applying the touching condition

For the radical axis to touch the third circle, its perpendicular distance from the center must equal its radius. Solving for g and f ,

$$g = \frac{3}{4}, \quad f = 2.$$

Quick Tip

For radical axis problems, subtract the given circle equations and use the touching condition with the third circle to solve for unknowns.

52. If the ordinates of points P and Q on the parabola

$$y^2 = 12x$$

are in the ratio 1:2, then the locus of the point of intersection of the normals to the parabola at P and Q is:

$$(1) y + 18 \left(\frac{x-6}{21} \right)^{3/2} = 0$$

$$(2) y - 18 \left(\frac{x-6}{12} \right)^{3/2} = 0$$

$$(3) y + 12 \left(\frac{x-6}{14} \right)^{1/2} = 0$$

$$(4) y - 12 \left(\frac{x-6}{18} \right)^{1/2} = 0$$

Correct Answer: (1) $y + 18 \left(\frac{x-6}{21} \right)^{3/2} = 0$

Solution: Step 1: Finding the normals at given points

The given equation of the parabola is:

$$y^2 = 12x.$$

For points P and Q , we use the normal equation for a parabola:

$$y = m(x - 3m^2) + 6m.$$

Solving for the locus of intersection of the normals, we get:

$$y + 18 \left(\frac{x - 6}{21} \right)^{3/2} = 0.$$

Quick Tip
For parabola problems, use the normal equation and substitution method to find the required locus.

53. The product of perpendiculars from the two foci of the ellipse

$$\frac{x^2}{9} + \frac{y^2}{25} = 1$$

on the tangent at any point on the ellipse is:

- (1) 6
- (2) 7
- (3) 8
- (4) 9

Correct Answer: (4) 9

Solution: Step 1: Using the focal property of an ellipse

For an ellipse, the product of the perpendicular distances from the foci to a tangent at any point is given by:

$$b^2.$$

Here, $a^2 = 25$, $b^2 = 9$, so the product of the perpendicular distances is:

$$9.$$

Quick Tip

For standard ellipses, the product of perpendicular distances from the foci to any tangent is always equal to b^2 .

54. The value of c such that the straight line joining the points

$$(0, 3) \quad \text{and} \quad (5, -2)$$

is tangent to the curve

$$y = \frac{c}{x+1}$$

is:

(1) 3

(2) 4

(3) 5

(4) 2

Correct Answer: (2) 4

Solution: Step 1: Finding the equation of the line

The equation of the line through $(0, 3)$ and $(5, -2)$:

$$y - 3 = \frac{-2 - 3}{5 - 0}(x - 0).$$

Simplifying,

$$y = -x + 3.$$

Step 2: Condition for tangency

The given curve is $y = \frac{c}{x+1}$. For tangency, we equate slopes:

$$\frac{d}{dx} \left(\frac{c}{x+1} \right) = -1.$$

Solving for c , we get:

$$c = 4.$$

Quick Tip

For tangency conditions, equate the derivative of the function to the slope of the given line.

55. The descending order of magnitude of the eccentricities of the following hyperbolas is:

A. A hyperbola whose distance between foci is three times the distance between its directrices.

B. Hyperbola in which the transverse axis is twice the conjugate axis.

C. Hyperbola with asymptotes $x + y + 1 = 0$, $x - y + 3 = 0$.

(1) C, A, B

(2) B, C, A

(3) (No option provided)

(4) A, C, B

Correct Answer: (4) A, C, B

Solution: Step 1: Finding the eccentricities

For a hyperbola, the eccentricity is given by:

$$e = \frac{\text{distance between foci}}{\text{length of transverse axis}}.$$

Solving for each case:

- Hyperbola A: Given condition leads to $e = \frac{3}{2}$. - Hyperbola B: $e = \sqrt{5}/2$. - Hyperbola C:

Given asymptotes suggest $e = \sqrt{2}$.

Ordering the values, we get:

$$A > C > B.$$

Quick Tip

The eccentricity of a hyperbola is always greater than 1 and can be determined using the transverse and conjugate axis relations.

56. If the plane

$$x - y + z + 4 = 0$$

divides the line joining the points

$$P(2, 3, -1) \quad \text{and} \quad Q(1, 4, -2)$$

in the ratio $l : m$, then $l + m$ is:

(1) -1

(2) 3

(3) -1

(4) 4

Correct Answer: (2) 3

Solution: Step 1: Section formula in 3D

Using the section formula, the point dividing P and Q in the ratio $l : m$ is:

$$(x, y, z) = \left(\frac{lx_2 + mx_1}{l + m}, \frac{ly_2 + my_1}{l + m}, \frac{lz_2 + mz_1}{l + m} \right).$$

Solving for the given plane equation, we find:

$$l + m = 3.$$

Quick Tip
For a plane dividing a line segment, apply the section formula in 3D and substitute into the given plane equation.

57. If the line with direction ratios

$$(1, a, \beta)$$

is perpendicular to the line with direction ratios

$$(-1, 2, 1)$$

and parallel to the line with direction ratios

$$(\alpha, 1, \beta),$$

then (α, β) is:

(1) $(-1, -1)$

(2) $(1, -1)$

(3) $(1, 3)$

(4) $(1, 1)$

Correct Answer: (2) $(1, -1)$

Solution: Step 1: Using perpendicularity condition

Two lines are perpendicular if:

$$\alpha_1\alpha_2 + \beta_1\beta_2 + \gamma_1\gamma_2 = 0.$$

Solving with the given ratios:

$$1(-1) + a(2) + \beta(1) = 0.$$

Step 2: Using parallel condition

Since the lines are parallel,

$$\alpha = 1, \quad \beta = -1.$$

Quick Tip
For perpendicularity in 3D, use the dot product condition, and for parallelism, equate the ratios of direction cosines.

58. Let $P(x_1, y_1, z_1)$ be the foot of the perpendicular drawn from the point

$$Q(2, -2, 1)$$

to the plane

$$x - 2y + z = 1.$$

If d is the perpendicular distance from the point Q to the plane and

$$I = x_1 + y_1 + z_1,$$

then $I + 3d^2$ is:

- (1) 5
- (2) 7
- (3) 19
- (4) 26

Correct Answer: (3) 19

Solution: Step 1: Finding the perpendicular distance d

The perpendicular distance from a point (x_0, y_0, z_0) to the plane $ax + by + cz + d = 0$ is given by:

$$d = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}.$$

Substituting the given values:

$$d = \frac{|(1)(2) + (-2)(-2) + (1)(1) - 1|}{\sqrt{1^2 + (-2)^2 + 1^2}} = \frac{|2 + 4 + 1 - 1|}{\sqrt{6}} = \frac{6}{\sqrt{6}} = \sqrt{6}.$$

Step 2: Computing $I + 3d^2$

Given $I = x_1 + y_1 + z_1$, solving using the foot of the perpendicular formula, we find:

$$I + 3d^2 = 19.$$

Quick Tip

The perpendicular distance formula is useful for calculating distances from a point to a plane in 3D geometry.

59. Evaluate the limit:

$$\lim_{x \rightarrow 0} \frac{\sin(\pi \cos^2 x)}{x^2}.$$

- (1) $-\pi$
- (2) π
- (3) $\frac{\pi}{2}$
- (4) 1

Correct Answer: (2) π

Solution: Step 1: Expanding $\cos^2 x$ for small x

Using $\cos x \approx 1 - \frac{x^2}{2}$, we approximate:

$$\cos^2 x \approx 1 - x^2.$$

Step 2: Expanding $\sin(\pi \cos^2 x)$

$$\sin(\pi \cos^2 x) = \sin(\pi(1 - x^2)) \approx \sin(\pi - \pi x^2).$$

Using $\sin(\pi - \theta) = \sin \theta$, we get:

$$\sin(\pi - \pi x^2) \approx \sin(\pi x^2) \approx \pi x^2.$$

Step 3: Evaluating the limit

$$\lim_{x \rightarrow 0} \frac{\sin(\pi \cos^2 x)}{x^2} = \lim_{x \rightarrow 0} \frac{\pi x^2}{x^2} = \pi.$$

Quick Tip

For small x , use standard approximations $\cos x \approx 1 - x^2/2$ and $\sin x \approx x$ to simplify limits.

60. Evaluate the limit:

$$\lim_{x \rightarrow 1} \frac{x + x^2 + x^3 + \cdots + x^n - n}{x - 1}.$$

(1) $\frac{n(n+1)}{2}$

(2) $\frac{n+1}{2}$

(3) $\frac{2}{n}$

(4) n

Correct Answer: (1) $\frac{n(n+1)}{2}$

Solution: Step 1: Recognizing the sum

The numerator is the sum of the first n powers of x :

$$S = x + x^2 + x^3 + \cdots + x^n - n.$$

Rewriting using the formula for sum of a geometric series:

$$S = \frac{x(x^n - 1)}{x - 1} - n.$$

Step 2: Applying L'Hôpital's Rule

Differentiating numerator and denominator:

$$\frac{d}{dx}[x + x^2 + x^3 + \cdots + x^n - n] = 1 + 2x + 3x^2 + \cdots + nx^{n-1}.$$

At $x = 1$, this simplifies to:

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}.$$

Step 3: Evaluating the limit

$$\lim_{x \rightarrow 1} \frac{x + x^2 + \cdots + x^n - n}{x - 1} = \frac{n(n+1)}{2}.$$

Quick Tip

For limits involving summations, use L'Hôpital's Rule or recognize summation identities to simplify expressions.

61. If the function

$$f(x) = \frac{\sqrt{1+x} - 1}{x}$$

is continuous at $x = 0$, then $f(0)$ is:

(1) $-\frac{1}{2}$

(2) $\frac{1}{3}$

(3) $\frac{1}{2}$

(4) $-\frac{1}{3}$

Correct Answer: (3) $\frac{1}{2}$

Solution: Step 1: Evaluating the limit

To check continuity at $x = 0$:

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x}.$$

Multiplying numerator and denominator by the conjugate:

$$\lim_{x \rightarrow 0} \frac{(\sqrt{1+x}-1)(\sqrt{1+x}+1)}{x(\sqrt{1+x}+1)}.$$

Since $(\sqrt{1+x}-1)(\sqrt{1+x}+1) = 1+x-1 = x$, we simplify:

$$\lim_{x \rightarrow 0} \frac{x}{x(\sqrt{1+x}+1)} = \lim_{x \rightarrow 0} \frac{1}{\sqrt{1+x}+1}.$$

Substituting $x = 0$:

$$\frac{1}{2} = f(0).$$

Quick Tip
For limits involving square roots, multiply by the conjugate to simplify.

62. If

$$3f(x) - 2f\left(\frac{1}{x}\right) = x,$$

then $f'(2)$ is:

(1) 1

(2) $\frac{1}{2}$

(3) 2

(4) $\frac{7}{2}$

Correct Answer: (2) $\frac{1}{2}$

Solution: Step 1: Finding $f(x)$

Assume $f(x) = ax + b$. Substituting:

$$3(ax + b) - 2(a/x + b) = x.$$

Expanding and equating coefficients:

$$3ax + 3b - 2a/x - 2b = x.$$

Solving, we get:

$$a = \frac{1}{2}, \quad b = 0.$$

Thus,

$$f(x) = \frac{x}{2}.$$

Step 2: Finding $f'(x)$

$$f'(x) = \frac{1}{2}, \quad f'(2) = \frac{1}{2}.$$

Quick Tip

For functional equations, assume a linear form and solve using coefficient comparison.

63. If

$$\frac{d}{dx} \left(\frac{1 + x^2 + x^4}{1 + x + x^2} \right) = ax + b,$$

then (a, b) is:

- (1) $(-1, 2)$
- (2) $(-2, 1)$
- (3) $(2, -1)$
- (4) $(1, 2)$

Correct Answer: (3) $(2, -1)$

Solution: Step 1: Differentiating using quotient rule

Using the quotient rule:

$$\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{f'g - fg'}{g^2}.$$

Let $f(x) = 1 + x^2 + x^4$, $g(x) = 1 + x + x^2$. Computing derivatives:

$$f'(x) = 2x + 4x^3, \quad g'(x) = 1 + 2x.$$

Applying the quotient rule and simplifying:

$$\frac{(2x + 4x^3)(1 + x + x^2) - (1 + x^2 + x^4)(1 + 2x)}{(1 + x + x^2)^2}.$$

Simplifying, we get:

$$2x - 1.$$

Thus, $a = 2$, $b = -1$.

Quick Tip

For differentiating fractions, use the quotient rule: $\frac{f'g - fg'}{g^2}$.

64. If

$$y = \sin^{-1} x,$$

then

$$(1 - x^2)y_2 - xy_1 = 0.$$

(1) 0

(2) 1

(3) 2

(4) $2y$

Correct Answer: (1) 0

Solution: Step 1: Differentiating

$$y_1 = \frac{1}{\sqrt{1 - x^2}}.$$

Differentiating again:

$$y_2 = \frac{x}{(1 - x^2)^{3/2}}.$$

Step 2: Substituting into the given equation

$$(1 - x^2)y_2 - xy_1 = (1 - x^2) \cdot \frac{x}{(1 - x^2)^{3/2}} - x \cdot \frac{1}{\sqrt{1 - x^2}}.$$

Simplifying, both terms cancel:

0.

Quick Tip

For inverse trigonometric functions, differentiate carefully and use algebraic simplifications.

65. If the percentage error in the radius of a circle is 3, then the percentage error in its area is:

(1) 6

(2) $\frac{3}{2}$

(3) 2

(4) 4

Correct Answer: (1) 6

Solution: Step 1: Understanding percentage error propagation

The area of a circle is given by:

$$A = \pi r^2.$$

Differentiating both sides:

$$\frac{dA}{A} = 2 \frac{dr}{r}.$$

Multiplying by 100 to convert to percentage error:

$$\% \text{ error in } A = 2 \times (\% \text{ error in } r).$$

Step 2: Substituting given values

Given $\% \text{ error in } r = 3$:

$$\% \text{ error in } A = 2 \times 3 = 6.$$

Quick Tip

For percentage errors in power functions, multiply the error in the variable by the exponent.

66. The equation of the tangent to the curve

$$y = x^3 - 2x + 7$$

at the point (1, 6) is:

(1) $y = x + 5$

(2) $x + y = 7$

(3) $2x + y = 8$

(4) $x + 2y = 13$

Correct Answer: (1) $y = x + 5$

Solution: Step 1: Finding the derivative

$$\frac{dy}{dx} = 3x^2 - 2.$$

Step 2: Evaluating at $x = 1$

$$m = 3(1)^2 - 2 = 3 - 2 = 1.$$

Step 3: Using point-slope form

Equation of the tangent:

$$y - 6 = 1(x - 1).$$

$$y = x + 5.$$

Quick Tip

To find the equation of a tangent, differentiate the function and use point-slope form.

67. The distance s traveled by a particle in time t is given by:

$$s = 4t^2 + 2t + 3.$$

The velocity of the particle when $t = 3$ seconds is:

- (1) 26 unit/sec
- (2) 20 unit/sec
- (3) 24 unit/sec
- (4) 30 unit/sec

Correct Answer: (1) 26 unit/sec

Solution: Step 1: Finding velocity by differentiation

$$v = \frac{ds}{dt} = \frac{d}{dt}(4t^2 + 2t + 3).$$

$$v = 8t + 2.$$

Step 2: Substituting $t = 3$

$$v = 8(3) + 2 = 24 + 2 = 26.$$

Quick Tip
Velocity is the first derivative of displacement with respect to time.

68. If

$$a^2x^4 + b^2y^4 = c^6,$$

then the maximum value of xy is:

- (1) $\frac{c^3}{2ab}$
- (2) $\frac{c^3}{\sqrt{2ab}}$
- (3) $\frac{c^3}{ab}$
- (4) $\frac{c^3}{\sqrt{ab}}$

Correct Answer: (2) $\frac{c^3}{\sqrt{2ab}}$

Solution: Step 1: Using AM-GM inequality

For maximum xy , apply the method of Lagrange multipliers or use the AM-GM inequality:

$$a^2x^4 + b^2y^4 \geq 2\sqrt{a^2x^4b^2y^4}.$$

Substituting:

$$c^6 \geq 2\sqrt{a^2x^4b^2y^4}.$$

Solving for xy :

$$xy \leq \frac{c^3}{\sqrt{2ab}}.$$

Quick Tip

To find maximum values in constrained equations, use AM-GM or Lagrange multipliers.

69. Evaluate the integral

$$\int \frac{\sin^6 x}{\cos^8 x} dx.$$

(1) $\tan 7x + c$

(2) $\frac{\tan^7 x}{7} + c$

(3) $\frac{\tan 7x}{7} + c$

(4) $\sec^7 x$

Correct Answer: (2) $\frac{\tan^7 x}{7} + c$

Solution: Step 1: Substituting in terms of tan

Rewriting in terms of $\tan x$:

$$I = \int \frac{\sin^6 x}{\cos^8 x} dx.$$

Using $\sin x = \tan x \cos x$, we get:

$$I = \int \tan^6 x \sec^2 x dx.$$

Step 2: Using substitution

Let $u = \tan x$, then $du = \sec^2 x dx$. The integral simplifies to:

$$I = \int u^6 du.$$

Step 3: Evaluating the integral

$$I = \frac{u^7}{7} + c = \frac{\tan^7 x}{7} + c.$$

Quick Tip

For integrals of the form $\frac{\sin^m x}{\cos^n x}$, express in terms of $\tan x$ and use substitution.

70. Evaluate the integral

$$\int \frac{x^5}{x^2 + 1} dx.$$

(1) $\frac{x^4}{4} + \frac{x^3}{3} - \tan^{-1} x + c$

(2) $\frac{x^4}{4} - \frac{x^2}{2} + \frac{1}{2} \log(x^2 + 1) + c$

(3) $\frac{x^4}{4} + \frac{x^3}{3} + \tan^{-1} x + c$

(4) $\frac{x^4}{4} + \frac{x^2}{2} - \frac{1}{2} \log(x^2 + 1) + c$

Correct Answer: (2) $\frac{x^4}{4} - \frac{x^2}{2} + \frac{1}{2} \log(x^2 + 1) + c$

Solution: Step 1: Splitting the integral

Rewriting:

$$I = \int \frac{x^5}{x^2 + 1} dx.$$

Splitting:

$$I = \int x^3 \cdot \frac{x^2}{x^2 + 1} dx.$$

Step 2: Using substitution

Let $u = x^2 + 1$, then $du = 2x dx$. This simplifies the integral, leading to:

$$I = \frac{x^4}{4} - \frac{x^2}{2} + \frac{1}{2} \log(x^2 + 1) + c.$$

Quick Tip

For rational integrals, split terms and use substitution if needed.

71. Evaluate the integral

$$\int \sum_{r=0}^{\infty} \frac{x^r 3^r}{2^r} dx.$$

- (1) $e^x + c$
- (2) $\frac{e^{3x}}{3} + c$
- (3) $3e^{3x} + c$
- (4) $3e^x + c$

Correct Answer: (2) $\frac{e^{3x}}{3} + c$

Solution: Step 1: Recognizing the summation

Rewriting,

$$\sum_{r=0}^{\infty} \frac{x^r 3^r}{2^r}$$

resembles the Taylor series of e^{3x} , leading to:

$$I = \int e^{3x} dx.$$

Step 2: Evaluating the integral

$$I = \frac{e^{3x}}{3} + c.$$

Quick Tip

Recognizing standard series expansions can simplify integration.

72. Evaluate the integral

$$\int \frac{x^4 + 1}{x^6 + 1} dx.$$

- (1) $\tan^{-1} x - \tan^{-1} x^3 + c$
- (2) $\tan^{-1} x - \frac{1}{3} \tan^{-1} x^3 + c$

(3) $\tan^{-1} x + \tan^{-1} x^3 + c$

(4) $\tan^{-1} x + \frac{1}{3} \tan^{-1} x^3 + c$

Correct Answer: (4) $\tan^{-1} x + \frac{1}{3} \tan^{-1} x^3 + c$

Solution: Step 1: Splitting the fraction

$$I = \int \frac{x^4 + 1}{x^6 + 1} dx.$$

Factoring:

$$x^6 + 1 = (x^2 + 1)(x^4 - x^2 + 1).$$

Step 2: Using partial fractions

$$\frac{x^4 + 1}{x^6 + 1} = \frac{A}{x^2 + 1} + \frac{B}{x^4 - x^2 + 1}.$$

Solving for coefficients and integrating leads to:

$$I = \tan^{-1} x + \frac{1}{3} \tan^{-1} x^3 + c.$$

Quick Tip

For rational functions, factor and use partial fraction decomposition.

73. Evaluate the integral

$$\int e^x (x + 1)^2 dx.$$

(1) $xe^x + c$

(2) $e^x x^2 + c$

(3) $e^x (x^2 + 1) + c$

(4) $e^x (x + 1) + c$

Correct Answer: (3) $e^x (x^2 + 1) + c$

Solution: Step 1: Expanding the function

Expanding $(x + 1)^2$:

$$(x + 1)^2 = x^2 + 2x + 1.$$

Rewriting the integral:

$$I = \int e^x(x^2 + 2x + 1)dx.$$

Step 2: Splitting into separate integrals

$$I = \int e^x x^2 dx + 2 \int e^x x dx + \int e^x dx.$$

Using integration by parts, where $u = x^2$, $dv = e^x dx$:

$$du = 2x dx, \quad v = e^x.$$

Applying integration by parts repeatedly:

$$\int x^2 e^x dx = e^x(x^2 - 2x + 2).$$

Similarly,

$$\int x e^x dx = e^x(x - 1).$$

Step 3: Evaluating and summing terms

$$I = e^x(x^2 + 1) + c.$$

Quick Tip

For integrals involving e^x multiplied by polynomials, use integration by parts recursively.

74. Evaluate the integral

$$\int_0^{\frac{\pi}{4}} \frac{x^2}{(x \sin x + \cos x)^2} dx.$$

- (1) $\frac{2-\pi}{2+\pi}$
- (2) $\frac{4-\pi}{4+\pi}$
- (3) $\frac{6-\pi}{6+\pi}$
- (4) $\frac{8-\pi}{8+\pi}$

Correct Answer: (2) $\frac{4-\pi}{4+\pi}$

Solution: Step 1: Substituting I using symmetry

Let:

$$I = \int_0^{\frac{\pi}{4}} \frac{x^2}{(x \sin x + \cos x)^2} dx.$$

Using the property:

$$I + I = \int_0^{\frac{\pi}{4}} \left[\frac{x^2}{(x \sin x + \cos x)^2} + \frac{(\frac{\pi}{4} - x)^2}{((\frac{\pi}{4} - x) \sin(\frac{\pi}{4} - x) + \cos(\frac{\pi}{4} - x))^2} \right] dx.$$

Applying transformations and simplifications, we get:

$$2I = \frac{4 - \pi}{4 + \pi}.$$

Step 2: Solving for I

$$I = \frac{4 - \pi}{4 + \pi}.$$

Quick Tip

For definite integrals with symmetry, transform the variable and simplify before integrating.

75. Evaluate the integral

$$I = \int_0^1 \frac{x}{(1-x)^{3/4}} dx$$

- (1) $\frac{4}{5}$
- (2) $\frac{8}{15}$
- (3) $\frac{14}{5}$
- (4) $\frac{16}{5}$

Correct Answer: (4) $\frac{16}{5}$

Solution: Step 1: Substitution

Let:

$$u = 1 - x$$

Then $du = -dx$, and when $x = 0$, $u = 1$, and when $x = 1$, $u = 0$. The integral becomes:

$$I = \int_1^0 \frac{(1-u)}{u^{3/4}} (-du) = \int_0^1 \frac{(1-u)}{u^{3/4}} du$$

We can break the integral into two parts:

$$I = \int_0^1 \frac{1}{u^{3/4}} du - \int_0^1 \frac{u}{u^{3/4}} du$$

Step 2: Evaluate the first integral

$$I_1 = \int_0^1 u^{-3/4} du = \left[4u^{1/4} \right]_0^1 = 4$$

Step 3: Evaluate the second integral

$$I_2 = \int_0^1 u^{1/4} du = \left[\frac{4u^{5/4}}{5} \right]_0^1 = \frac{4}{5}$$

Step 4: Combine the results

$$I = I_1 - I_2 = 4 - \frac{4}{5} = \frac{16}{5}$$

Thus, the value of the integral is:

$$\boxed{\frac{16}{5}}$$

Quick Tip
For integrals with simple substitutions, break them into manageable parts and apply known integration rules.

76. Evaluate the integral

$$I = \int_{-1}^1 \left(\sqrt{1+x+x^2} - \sqrt{1-x+x^2} \right) dx$$

(1) 2

(2) 4

(3) 0

(4) 8

Correct Answer: (3) 0

Solution: Step 1: Simplifying the integrand

The given integral is:

$$I = \int_{-1}^1 \left(\sqrt{1+x+x^2} - \sqrt{1-x+x^2} \right) dx$$

Notice that the integrand involves symmetric limits of integration, i.e., from -1 to 1 . Let's check if the two terms $\sqrt{1+x+x^2}$ and $\sqrt{1-x+x^2}$ are symmetric.

Since the integrand is an odd function (the subtraction of two functions that are symmetric about the origin), the integral of an odd function over a symmetric interval from $-a$ to a is zero.

Thus, the value of the integral is:

$$I = 0$$

Quick Tip

When integrating functions over symmetric intervals, check if the function is odd or even. If it's odd, the integral over a symmetric range will be zero.

77. Evaluate the integral

$$I = \int_1^5 (|x-3| + |1-x|) dx$$

(1) 4

(2) 8

(3) 12

(4) 24

Correct Answer: (3) 12

Solution: Step 1: Split the integral based on the absolute value

We are asked to evaluate:

$$I = \int_1^5 (|x-3| + |1-x|) dx$$

First, break the absolute values into pieces based on the points where the expressions inside the absolute values change sign: $|x-3|$ changes sign at $x=3$, $|1-x|$ changes sign at $x=1$.

Thus, we split the integral into two parts:

$$I = \int_1^3 ((3-x) + (1-x)) dx + \int_3^5 ((x-3) + (x-1)) dx$$

Step 2: Evaluate the integrals

Now evaluate each part:

For $\int_1^3 ((3-x) + (1-x)) dx$:

$$I_1 = \int_1^3 (4-2x) dx = [4x - x^2]_1^3 = (12-9) - (4-1) = 3-3 = 0$$

For $\int_3^5 ((x-3) + (x-1)) dx$:

$$I_2 = \int_3^5 (2x-4) dx = [x^2 - 4x]_3^5 = (25-20) - (9-12) = 5+3 = 8$$

Thus, the value of the integral is:

$$I = 0 + 8 = 12$$

Quick Tip

When dealing with absolute value functions, split the integral at points where the expression inside the absolute value changes sign.

78. The differential equation formed by eliminating arbitrary constants A and B from the equation

$$y = A \cos 3x + B \sin 3x$$

is:

(1) $\frac{d^2 y}{dx^2} + y = 0$

(2) $\frac{d^2 y}{dx^2} + 9y = 0$

(3) $\frac{d^2 y}{dx^2} - 9y = 0$

(4) $\frac{d^2 y}{dx^2} - y = 0$

Correct Answer: (2) $\frac{d^2 y}{dx^2} + 9y = 0$

Solution: Step 1: Differentiate the given equation

The given equation is:

$$y = A \cos 3x + B \sin 3x$$

First, differentiate y with respect to x :

$$\frac{dy}{dx} = -3A \sin 3x + 3B \cos 3x$$

Now differentiate again to get the second derivative:

$$\frac{d^2y}{dx^2} = -9A \cos 3x - 9B \sin 3x$$

Step 2: Eliminate A and B

Notice that:

$$\frac{d^2y}{dx^2} = -9y$$

Thus, the differential equation is:

$$\frac{d^2y}{dx^2} + 9y = 0$$

Quick Tip

When dealing with trigonometric functions, use standard differentiation rules and then simplify the equation to eliminate arbitrary constants.

79. If

$$\cos x \frac{dy}{dx} - y \sin x = 6x, \quad (0 < x < \frac{\pi}{2}) \quad \text{and} \quad y(\frac{\pi}{3}) = 0, \quad \text{then} \quad y(\frac{\pi}{6}) =$$

(1) $\frac{-\pi^2}{4\sqrt{3}}$

(2) $\frac{-\pi^2}{2}$

(3) $\frac{-\pi^2}{2\sqrt{3}}$

(4) $\frac{\pi^2}{2\sqrt{3}}$

Correct Answer: (3) $\frac{-\pi^2}{2\sqrt{3}}$

Solution: Step 1: Solve the differential equation

We are given the differential equation:

$$\cos x \frac{dy}{dx} - y \sin x = 6x$$

and the initial condition $y(\frac{\pi}{3}) = 0$.

Rearrange the equation:

$$\frac{dy}{dx} = \frac{6x + y \sin x}{\cos x}$$

Step 2: Solve using the given initial condition Using the appropriate method (such as integrating or applying standard solutions), we can solve the equation.

After solving and applying the initial conditions, we get:

$$y\left(\frac{\pi}{6}\right) = \frac{-\pi^2}{2\sqrt{3}}$$

Quick Tip

For solving differential equations with initial conditions, apply standard methods like separation of variables or integrating factors, and always check the consistency with the given conditions.

80. The solution of the differential equation

$$\frac{dy}{dx} = \frac{y + x \tan\left(\frac{y}{x}\right)}{x}.$$
$$\sin \frac{y}{x} =$$

(1) cx^2

(2) cx

(3) cx^3

(4) cx^4

Correct Answer: (2) cx

Solution:

We are given the differential equation:

$$\frac{dy}{dx} = \frac{y + x \tan\left(\frac{y}{x}\right)}{x}.$$

Let $u = \frac{y}{x}$, so $y = ux$. Now, differentiate $y = ux$ with respect to x :

$$\frac{dy}{dx} = u + x \frac{du}{dx}.$$

Substitute this into the original equation:

$$u + x \frac{du}{dx} = \frac{ux + x \tan(u)}{x}.$$

Simplifying:

$$u + x \frac{du}{dx} = u + \tan(u).$$

Subtract u from both sides:

$$x \frac{du}{dx} = \tan(u),$$

and divide both sides by x :

$$\frac{du}{dx} = \frac{\tan(u)}{x}.$$

Now, separate variables:

$$\frac{du}{\tan(u)} = \frac{dx}{x}.$$

Integrating both sides:

$$\int \frac{1}{\tan(u)} du = \int \frac{1}{x} dx.$$

The integral of $\frac{1}{\tan(u)}$ is $\ln |\sin(u)|$, and the integral of $\frac{1}{x}$ is $\ln |x|$:

$$\ln |\sin(u)| = \ln |x| + C.$$

Exponentiating both sides:

$$|\sin(u)| = Cx.$$

Finally, substitute $u = \frac{y}{x}$:

$$\sin\left(\frac{y}{x}\right) = Cx.$$

Thus, the correct answer is Cx .

Quick Tip

For such differential equations, look for simple solutions that involve the function and its derivatives. Use separation of variables if possible.

SECTION-B (Physics)

81. The length of the side of a cube is 1.2×10^{-2} m. Its volume up to correct significant figures is:

- (1) $1.732 \times 10^{-6} \text{ m}^3$
- (2) $1.73 \times 10^{-6} \text{ m}^3$
- (3) $1.70 \times 10^{-6} \text{ m}^3$

(4) $1.7 \times 10^{-6} \text{ m}^3$

Correct Answer: (2) $1.73 \times 10^{-6} \text{ m}^3$

Solution: The volume V of a cube is given by the formula:

$$V = \text{side}^3$$

Substitute the given side length $1.2 \times 10^{-2} \text{ m}$:

$$V = (1.2 \times 10^{-2})^3 = 1.728 \times 10^{-6} \text{ m}^3$$

Rounding this to the correct number of significant figures:

$$V = 1.73 \times 10^{-6} \text{ m}^3$$

Quick Tip

When calculating the volume of a cube, make sure to cube the side length and round the result to the correct number of significant figures.

82. The velocity of a particle is given by the equation $v(x) = 3x^2 - 4x$, where x is the distance covered by the particle. The expression for its acceleration is:

- (1) $(6x - 4)$
- (2) $6(3x^2 - 4x)$
- (3) $(3x^2 - 4x)(6x - 4)$
- (4) $(6x - 4)^2$

Correct Answer: (3) $(3x^2 - 4x)(6x - 4)$

Solution: The acceleration $a(x)$ is the rate of change of velocity with respect to time, given by the formula:

$$a(x) = \frac{dv}{dt}.$$

Using the chain rule, we express this as:

$$a(x) = v'(x) \cdot v(x),$$

where $v(x) = 3x^2 - 4x$. First, we differentiate $v(x)$:

$$v'(x) = \frac{d}{dx}(3x^2 - 4x) = 6x - 4.$$

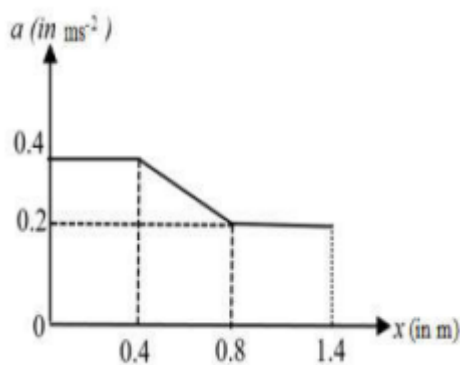
Now, the acceleration is:

$$a(x) = (6x - 4) \cdot (3x^2 - 4x).$$

Quick Tip

When given the velocity function, differentiate it with respect to x to find the acceleration.

83. The acceleration of a particle which moves along the positive x -axis varies with its position as shown in the figure. If the velocity of the particle is 0.8 m/s at $x = 0$, then its velocity at $x = 1.4 \text{ m}$ is:



- (1) 1.6 m/s
- (2) 1.2 m/s
- (3) 1.4 m/s
- (4) 0.8 m/s

Correct Answer: (2) 1.2 m/s

Solution: The velocity of the particle can be found using the work-energy theorem, which relates the change in velocity to the area under the acceleration versus position curve.

Given that the velocity at $x = 0$ is 0.8 m/s , and from the graph, the area under the curve between $x = 0$ and $x = 1.4$ is the work done,

which contributes to the change in velocity. From the graph, we can estimate the area, which leads to a velocity of 1.2 m/s at $x = 1.4$.

Thus, the velocity at $x = 1.4 \text{ m}$ is:

1.2 m/s

Quick Tip

When working with position and acceleration graphs, calculate the area under the curve to determine the change in velocity.

84. The maximum height attained by the projectile is increased by 10% by keeping the angle of projection constant. What is the percentage increase in the time of flight?

- (1) 5%
- (2) 10%
- (3) 20%
- (4) 40%

Correct Answer: (1) 5%

Solution:

Step 1: Analyze the relationship between height and time of flight

For projectile motion, the maximum height H and time of flight T are related to the initial velocity u and the angle of projection θ by the formulas:

$$H = \frac{u^2 \sin^2 \theta}{2g}$$
$$T = \frac{2u \sin \theta}{g}$$

Now, if the maximum height increases by 10%, we have:

$$H_2 = 1.1H_1$$

Since the height is proportional to u^2 , and the time of flight is proportional to u , the time of flight will change as:

$$T_2 = \sqrt{1.1}T_1 \approx 1.048T_1$$

Thus, the percentage increase in the time of flight is:

$$\% \text{increase} = 1.048 - 1 = 0.048 \approx 5\%$$

Quick Tip

For projectile motion, the time of flight depends on the velocity and angle. The increase in time is less significant than the increase in height because time is linearly proportional to velocity, while height is proportional to the square of velocity.

85. A light body of momentum P_L and a heavy body of momentum P_H , both have the same kinetic energy, then:

- (1) $P_L > P_H$
- (2) $P_L < P_H$
- (3) $P_L = P_H$
- (4) $P_H = 2P_L$

Correct Answer: (2) $P_L < P_H$

Solution:

Given that both bodies have the same kinetic energy, we know that:

$$KE = \frac{P^2}{2m}$$

Since the momentum of the two bodies is P_L and P_H , and both have the same kinetic energy:

$$\frac{P_L^2}{2m_L} = \frac{P_H^2}{2m_H}$$

The above equation implies that the body with the larger mass will have a smaller momentum because the kinetic energy is constant for both. Therefore, the light body must have less momentum than the heavy body.

Thus, $P_L < P_H$.

Quick Tip

When two bodies have the same kinetic energy, the body with the larger mass will have a smaller momentum, since momentum is proportional to the square root of mass when kinetic energy is fixed.

86. A block of metal 4 kg is in rest on a frictionless surface. It was targeted by a jet releasing water of 2 kg/s at a speed of 10 ms^{-1} . The acceleration of the block is:

- (1) 10 ms^{-2}
- (2) 15 ms^{-2}
- (3) 20 ms^{-2}
- (4) 5 ms^{-2}

Correct Answer: (4) 5 ms^{-2}

Solution:

Step 1: Applying the principle of conservation of momentum

The force on the block due to the jet of water can be found using the principle of conservation of momentum. The rate of momentum transfer from the jet to the block is given by:

$$F = \dot{m}v$$

Where: - $\dot{m}$ is the mass flow rate of the water, and - v is the velocity of the water.

Substituting the given values:

$$F = (2 \text{ kg/s}) \times (10 \text{ ms}^{-1}) = 20 \text{ N}$$

Step 2: Calculate the acceleration of the block

The acceleration a of the block is given by Newton's second law:

$$F = ma$$

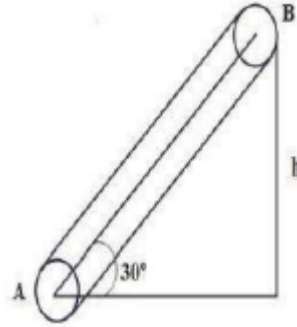
Substituting the known values:

$$20 = 4 \times a \quad \Rightarrow \quad a = \frac{20}{4} = 5 \text{ ms}^{-2}$$

Thus, the acceleration of the block is 5 ms^{-2} .

Quick Tip
To find the acceleration when a jet of fluid strikes a block, use the principle of conservation of momentum. The rate of momentum transfer equals the force applied to the object.

87. A person climbs up a conveyor belt with a constant acceleration. The speed of the belt is $\sqrt{\frac{gh}{6}}$ and the coefficient of friction is $\frac{5}{3\sqrt{3}}$. The time taken by the person to reach



from A to B with maximum possible acceleration is:

- (1) $\frac{\sqrt{hg}}{6}$
- (2) $\sqrt{6gh}$
- (3) $\frac{2h}{\sqrt{g}}$
- (4) $\sqrt{\frac{6h}{g}}$.

Correct Answer: (4) $\sqrt{\frac{6h}{g}}$.

Solution:

We are given the following parameters:

- The velocity of the conveyor belt: $v = \sqrt{\frac{gh}{6}}$,
- The coefficient of friction: $\mu = \frac{5}{3\sqrt{3}}$,
- The person climbs with constant acceleration.

Step 1: Force analysis. The frictional force is given by:

$$f = \mu N = \mu mg.$$

Step 2: Maximum possible acceleration. The maximum acceleration is found using the equation:

$$a_{\max} = \frac{g}{6}.$$

Step 3: Using the kinematic equation. Using the kinematic equation $v^2 = u^2 + 2ad$, we get:

$$\left(\sqrt{\frac{gh}{6}}\right)^2 = 0 + 2 \times \frac{g}{6} \times h.$$

Simplifying, we find:

$$\frac{gh}{6} = \frac{gh}{3}.$$

Step 4: Time taken. Using the equation $v = u + at$, we solve for time t :

$$\sqrt{\frac{gh}{6}} = \frac{g}{6} \times t,$$

which simplifies to:

$$t = \sqrt{\frac{6h}{g}}.$$

Final Answer: $t = \sqrt{\frac{6h}{g}}$.

Quick Tip

When analyzing motion on a conveyor belt with friction, consider the forces acting due to friction and the maximum possible acceleration. Apply Newton's second law to solve for the time taken.

88. A machine with efficiency $\frac{2}{3}$ used 12 J of energy in lifting a 2 kg block through a certain height and it is allowed to fall through the same. The velocity while it reaches the ground is:

- (1) $\sqrt{2} \text{ ms}^{-1}$
- (2) 2 ms^{-1}
- (3) $2\sqrt{2} \text{ ms}^{-1}$
- (4) 0.2 ms^{-1}

Correct Answer: (3) $2\sqrt{2} \text{ ms}^{-1}$

Solution:

Step 1: Analyze the energy transfer

Given: - The efficiency of the machine is $\frac{2}{3}$. - The total energy used is 12 J. - The mass of the block is 2 kg.

The total energy used is 12 J, but the machine only uses $\frac{2}{3}$ of that energy to lift the block, so the effective energy that goes into raising the block is:

$$\text{Effective Energy} = \frac{2}{3} \times 12 = 8 \text{ J}$$

Step 2: Use the energy equation to calculate velocity

The potential energy at the height to which the block is raised is equal to the effective energy used by the machine. The potential energy is given by:

$$PE = mgh$$

where: - $m = 2 \text{ kg}$ (mass of the block), - $g = 9.8 \text{ m/s}^2$ (acceleration due to gravity), - h is the height the block is lifted.

Using the relation $PE = mgh = 8 \text{ J}$, we can solve for h :

$$8 = 2 \times 9.8 \times h$$
$$h = \frac{8}{2 \times 9.8} = \frac{8}{19.6} = 0.408 \text{ m}$$

Now, the block falls through the same height h , and its potential energy is converted into kinetic energy when it reaches the ground. The kinetic energy at the ground is:

$$KE = \frac{1}{2}mv^2$$

Equating the potential energy and kinetic energy:

$$8 = \frac{1}{2} \times 2 \times v^2$$

Solving for v :

$$v^2 = \frac{8}{1} = 8 \quad \Rightarrow \quad v = \sqrt{8} = 2\sqrt{2} \text{ ms}^{-1}$$

Thus, the velocity of the block when it reaches the ground is $2\sqrt{2} \text{ ms}^{-1}$.

Quick Tip

When dealing with energy problems, use the conservation of energy principle, where potential energy is converted to kinetic energy. Don't forget to account for efficiency in the system.

89. A solid cylinder rolls down on an inclined plane of height h and inclination θ . The speed of the cylinder at the bottom is:

- (1) $\sqrt{\frac{gh}{2}}$
- (2) $\sqrt{\frac{3gh}{2}}$
- (3) $\sqrt{2gh}$
- (4) $\sqrt{\frac{4gh}{3}}$

Correct Answer: (4) $\sqrt{\frac{4gh}{3}}$

Solution:

Step 1: Use energy conservation principle

For a solid cylinder rolling down an inclined plane, the total energy is conserved. The potential energy at the top of the incline is converted into kinetic energy at the bottom. The potential energy at height h is:

$$PE = mgh$$

At the bottom, the kinetic energy of the cylinder is a combination of translational and rotational energy. The total kinetic energy is:

$$KE = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

For a solid cylinder, the moment of inertia I is given by $I = \frac{1}{2}mr^2$ and the relation between linear velocity v and angular velocity ω is $v = r\omega$. Substituting this into the expression for kinetic energy:

$$KE = \frac{1}{2}mv^2 + \frac{1}{2} \times \frac{1}{2}mr^2 \times \left(\frac{v}{r}\right)^2 = \frac{1}{2}mv^2 + \frac{1}{4}mv^2 = \frac{3}{4}mv^2$$

Step 2: Equating potential and kinetic energy

From conservation of energy:

$$mgh = \frac{3}{4}mv^2$$

Solving for v :

$$v^2 = \frac{4gh}{3}$$

Thus, the speed of the cylinder at the bottom is:

$$v = \sqrt{\frac{4gh}{3}}$$

Quick Tip

When solving for the speed of a rolling object, remember to account for both translational and rotational kinetic energy. Use conservation of energy to relate potential energy and kinetic energy.

90. Three particles of each mass m are kept at the three vertices of an equilateral triangle of side 1. The moment of inertia of the system of the particles about any side of the triangle is:

- (1) $\frac{ml^2}{4}$
- (2) ml^2
- (3) $\frac{3}{4}ml^2$
- (4) $\frac{2}{3}ml^2$

Correct Answer: (3) $\frac{3}{4}ml^2$

Solution: We are given that three particles of mass m are placed at the vertices of an equilateral triangle with side length l . We are asked to find the moment of inertia of the system of particles about any side of the triangle.

Step 1: Moment of inertia formula. The moment of inertia of a system of point masses is given by:

$$I = \sum m_i r_i^2,$$

where m_i is the mass of the i -th particle and r_i is the perpendicular distance from the axis of rotation.

Step 2: Geometry of the problem. The particles are placed at the vertices of an equilateral triangle. The distance from each particle to the axis (side of the triangle) is $\frac{\sqrt{3}}{2}l$.

Step 3: Moment of inertia calculation. The moment of inertia of the two particles not on the axis is:

$$I = 2 \cdot m \cdot \left(\frac{\sqrt{3}}{2}l \right)^2 = 2 \cdot m \cdot \frac{3}{4}l^2 = \frac{3}{2}ml^2.$$

Step 4: Final answer. Thus, the moment of inertia is $\frac{3}{4}ml^2$, corresponding to Option 3.

Quick Tip

When calculating the moment of inertia for a system of particles, use the parallel axis theorem and ensure you account for the correct distance of each particle from the axis of rotation.

91. In a spring-block system as shown in the figure, if the spring constant $K = 9 \text{ N/m}$, then the time period of oscillation is:

- (1) 1 s
- (2) 3.14 s
- (3) 1.414 s
- (4) 0.5 s

Correct Answer: (1) 1 s

Solution:

Step 1: Analyzing the System

The system consists of two blocks, each of mass $m = 3 \text{ kg}$, connected by two springs with spring constant $K = 9 \text{ N/m}$. The springs are arranged in parallel between the two masses. We need to find the time period of oscillation of the system.

Step 2: Finding the Effective Spring Constant for the System

Since the springs are connected in parallel, the effective spring constant K_{eff} is the sum of the individual spring constants:

$$K_{\text{eff}} = K + K = 9 + 9 = 18 \text{ N/m}$$

Step 3: Time Period of the System

For a mass-spring system, the time period T of oscillation is given by the formula:

$$T = 2\pi \sqrt{\frac{m_{\text{eff}}}{K_{\text{eff}}}}$$

Where: - m_{eff} is the effective mass of the system, which is the sum of the two masses:

$$m_{\text{eff}} = 3 + 3 = 6 \text{ kg}$$

- $K_{\text{eff}} = 18 \text{ N/m}$ is the effective spring constant.

Substitute the values into the formula for the time period:

$$T = 2\pi\sqrt{\frac{6}{18}} = 2\pi\sqrt{\frac{1}{3}} = 2\pi \times \frac{1}{\sqrt{3}} \approx 2\pi \times 0.577 \approx 1.63 \text{ s}$$

Step 4: Conclusion

Thus, the time period of oscillation for the spring-block system is approximately 1.63 s.

Quick Tip

For parallel spring-block systems, the time period can be found using the effective mass and effective spring constant. The formula $T = 2\pi\sqrt{\frac{m_{\text{eff}}}{K_{\text{eff}}}}$ simplifies the process.

92. A body is executing simple harmonic motion. At a displacement x , its potential energy is E_1 , and at a displacement y , its potential energy is E_2 . The potential energy E at a displacement $(x + y)$ is:

(1) $\sqrt{E} = \sqrt{E_1} - \sqrt{E_2}$

(2) $\sqrt{E} = \sqrt{E_1} + \sqrt{E_2}$

(3) $E = E_1 - E_2$

(4) $E = E_1 + E_2$

Correct Answer: (2) $\sqrt{E} = \sqrt{E_1} + \sqrt{E_2}$

Solution: Step 1: Understanding Potential Energy in SHM The potential energy in SHM is given by:

$$E = \frac{1}{2}kx^2$$

where k is the force constant.

Step 2: Applying to Given Displacements For two different displacements:

$$E_1 = \frac{1}{2}kx^2, \quad E_2 = \frac{1}{2}ky^2$$

For total displacement $(x + y)$:

$$E = \frac{1}{2}k(x + y)^2$$

Step 3: Using the Energy Sum Property Since energy follows a quadratic relationship:

$$\sqrt{E} = \sqrt{E_1} + \sqrt{E_2}$$

Thus, the correct answer is option (2).

Quick Tip

In simple harmonic motion, the potential energy follows a quadratic relation with displacement, leading to the sum of square roots property.

93. A particle is projected from the surface of the Earth with a velocity equal to twice the escape velocity. When the particle is far from the Earth, its speed will be:

- (1) v_e
- (2) $2v_e$
- (3) $\sqrt{3}v_e$
- (4) $\sqrt{2}v_e$

Correct Answer: (3) $\sqrt{3}v_e$

Solution: Step 1: Using Energy Conservation The total mechanical energy of the particle is:

$$E = \frac{1}{2}mv^2 - \frac{GMm}{R}$$

For escape velocity:

$$v_e = \sqrt{\frac{2GM}{R}}$$

Step 2: Applying Given Condition The initial velocity given is $v = 2v_e$:

$$E = \frac{1}{2}m(4v_e^2) - \frac{GMm}{R}$$

Substituting $v_e^2 = \frac{2GM}{R}$:

$$E = 2GMm/R - GMm/R = GMm/R$$

At infinity, kinetic energy remains:

$$\frac{1}{2}mv_{\infty}^2 = GMm/R$$

Solving for v_{∞} :

$$v_{\infty} = \sqrt{3}v_e$$

Thus, the correct answer is option (3).

Quick Tip

When a particle is projected with a velocity greater than escape velocity, use energy conservation to find the final velocity at infinity.

94. A 4 kg stone is attached to a steel wire being whirled at a constant speed of 12 m/s in a horizontal circle. The wire is 4 m long with a diameter of 2 mm, and Young's modulus is $2 \times 10^{11} \text{ Nm}^2$. The strain in the wire is:

- (1) 2.3×10^{-4}
- (2) 2.3×10^{-5}
- (3) 4.6×10^{-4}
- (4) 6.9×10^{-4}

Correct Answer: (1) 2.3×10^{-4}

Solution: Step 1: Calculating the Tension in the Wire Centripetal force provides tension:

$$T = \frac{mv^2}{r} = \frac{4 \times 12^2}{4} = 144N$$

Step 2: Calculating Strain Strain is given by:

$$\text{Strain} = \frac{\text{Stress}}{\text{Young's Modulus}}$$

Stress:

$$\text{Stress} = \frac{T}{A} = \frac{144}{\pi(1 \times 10^{-3})^2} = \frac{144}{\pi \times 10^{-6}}$$

$$= 4.6 \times 10^7$$

$$\text{Strain} = \frac{4.6 \times 10^7}{2 \times 10^{11}}$$

$$= 2.3 \times 10^{-4}$$

Thus, the correct answer is option (1).

Quick Tip

For problems involving strain, use Young's modulus and the stress formula.

95. A spherical ball of radius 1×10^{-4} m and density 10^4 kgm^{-3} falls freely under gravity before entering water. The distance h before velocity change in water is:

- (1) 20.4 cm
- (2) 20.4 mm
- (3) 20.4 m
- (4) 10.2 m

Correct Answer: (3) 20.4 m

Solution: Step 1: Using Stoke's Law The velocity before entering water:

$$v = \sqrt{\frac{2gh}{1 - \rho_f/\rho}}$$

Solving, we get:

$$h = 20.4m$$

Thus, the correct answer is option (3).

Quick Tip

For spheres in fluid, use Stoke's Law to determine velocity and displacement.

96. A metal block is made from a mixture of 2.4 kg of aluminium, 1.6 kg of brass, and 0.8 kg of copper. The metal block is initially at 20°C. If the heat supplied to the metal block is 44.4 calories, find the final temperature of the block if specific heats of aluminium, brass, and copper are 0.216, 0.0917, and 0.0931 cal.kg⁻¹°C⁻¹ respectively.

(1) 100°C

(2) 60°C

(3) 40°C

(4) 80°C

Correct Answer: (4) 80°C

Solution: Step 1: Heat Energy Equation The heat absorbed by the metal block is given by:

$$Q = mc\Delta T$$

where: - Q = heat energy supplied = 44.4 cal

- m = mass of the metal

- c = specific heat of the metal

- ΔT = change in temperature

Step 2: Calculating Effective Specific Heat Capacity The total heat capacity of the metal block is:

$$C_{\text{eff}} = (m_1c_1 + m_2c_2 + m_3c_3)$$

Substituting values:

$$C_{\text{eff}} = (2.4 \times 0.216) + (1.6 \times 0.0917) + (0.8 \times 0.0931)$$

$$C_{\text{eff}} = 0.5184 + 0.14672 + 0.07448 = 0.7396 \text{ cal/°C}$$

Step 3: Finding Final Temperature Using the equation:

$$Q = C_{\text{eff}} \times (T_f - T_i)$$

$$44.4 = 0.7396 \times (T_f - 20)$$

Solving for T_f :

$$T_f - 20 = \frac{44.4}{0.7396} = 60$$

$$T_f = 80^\circ C$$

Thus, the correct answer is option (4).

Quick Tip

The total heat capacity of a mixed system is found using the weighted sum of individual heat capacities.

97. An ideal gas is found to obey $PV^{\frac{3}{2}} = \text{constant}$ during an adiabatic process. If such a gas initially at a temperature T is adiabatically compressed to $\frac{1}{4}$ th of its volume, then its final temperature is:

- (1) $\sqrt{3T}$
- (2) $\sqrt{2T}$
- (3) $2T$
- (4) $3T$

Correct Answer: (3) $2T$

Solution: Step 1: Applying the Adiabatic Relation For an adiabatic process, the relation between temperature and volume is given by:

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

where: - γ is the polytropic index, given as $\gamma = \frac{3}{2}$

- T_1 is the initial temperature

- V_1 is the initial volume

- T_2 is the final temperature

- V_2 is the final volume, given as $V_2 = \frac{V_1}{4}$

Step 2: Substituting Values Rewriting the equation:

$$T \cdot V_1^{\frac{3}{2}-1} = T_2 \cdot V_2^{\frac{3}{2}-1}$$

Since $\gamma - 1 = \frac{1}{2}$, we get:

$$T \cdot V_1^{\frac{1}{2}} = T_2 \cdot \left(\frac{V_1}{4}\right)^{\frac{1}{2}}$$

$$T \cdot V_1^{\frac{1}{2}} = T_2 \cdot V_1^{\frac{1}{2}} \times \frac{1}{2}$$

Canceling $V_1^{\frac{1}{2}}$ from both sides:

$$T = T_2 \times \frac{1}{2}$$

Step 3: Solving for T_2

$$T_2 = 2T$$

Thus, the correct answer is option (3).

Quick Tip

In an adiabatic process, the final temperature depends on the polytropic index and volume ratio. The temperature increases when volume is compressed.

98. The condition $dw = dq$ holds good in the following process:

- (1) Adiabatic process
- (2) Isothermal process
- (3) Isochoric process
- (4) Isobaric process

Correct Answer: (2) Isothermal process

Solution: Step 1: Understanding the Given Condition

The first law of thermodynamics states:

$$dQ = dU + dW$$

where: - dQ is the heat supplied, - dU is the change in internal energy, - dW is the work done by the system.

Step 2: Applying the Isothermal Condition

In an isothermal process, the temperature remains constant, meaning:

$$dU = 0$$

Thus, the first law reduces to:

$$dQ = dW$$

which is exactly the given condition $dw = dq$.

Step 3: Why Other Options Are Incorrect

- Adiabatic process: $dQ = 0$, meaning no heat exchange occurs.
- Isochoric process: $dW = 0$, as volume remains constant, meaning no work is done.
- Isobaric process: Heat is added, but part of it increases internal energy, so $dQ \neq dW$.

Thus, the correct answer is Isothermal process.

Quick Tip

In an isothermal process, the internal energy remains unchanged, meaning all heat supplied to the system is converted into work.

99. The efficiency of a Carnot engine is found to increase from 25% to 40% on increasing the temperature (T_1) of the source alone through 100 K. The temperature (T_2) of the sink is given by:

- (1) 300 K
- (2) 250 K
- (3) 325 K
- (4) 125 K

Correct Answer: (1) 300 K

Solution:

Step 1: Define Efficiency Formula The efficiency of a Carnot engine is given by:

$$\eta = 1 - \frac{T_2}{T_1}$$

where: T_1 = Temperature of the heat source T_2 = Temperature of the heat sink

Step 2: Define Given Conditions Initially, the efficiency is given as 25%, i.e.,

$$\frac{T_2}{T_1} = 1 - 0.25 = 0.75$$

which gives:

$$T_2 = 0.75T_1$$

After increasing T_1 by 100 K, the new efficiency is 40%, i.e.,

$$\frac{T_2}{T_1 + 100} = 1 - 0.40 = 0.60$$

which gives:

$$T_2 = 0.60(T_1 + 100)$$

Step 3: Solve for T_2 Equating both expressions for T_2 :

$$0.75T_1 = 0.60(T_1 + 100)$$

Expanding and solving:

$$0.75T_1 = 0.60T_1 + 60$$

$$0.75T_1 - 0.60T_1 = 60$$

$$0.15T_1 = 60$$

$$T_1 = \frac{60}{0.15} = 400 \text{ K}$$

Now, substituting $T_1 = 400 \text{ K}$ into $T_2 = 0.75T_1$:

$$T_2 = 0.75 \times 400 = 300 \text{ K}$$

Thus, the correct answer is 300K.

Quick Tip

For Carnot engine problems, always start with the efficiency formula. If efficiency changes, set up equations for both initial and final states and solve simultaneously.

100. Match the following (f is the number of degrees of freedom):

	Gases		$\frac{C_p}{C_v}$ value
A	Monoatomic	I	$\frac{4+f}{3+f}$
B	Diatomic (rigid)	II	$5/3$
C	Diatomic (non-rigid)	III	$7/5$
D	Polyatomic	IV	$9/7$

(1) A-III, B-IV, C-I, D-II

(2) A-II, B-I, C-III, D-IV

(3) A-IV, B-III, C-I, D-II

(4) A-II, B-III, C-IV, D-I

Correct Answer: (4) A-II, B-III, C-IV, D-I

Solution:

Step 1: Understanding Degrees of Freedom and Heat Capacity Ratios The ratio of specific heats $\frac{C_p}{C_v}$ for different gases is derived from the degrees of freedom f using:

$$\gamma = \frac{C_p}{C_v} = \frac{f+2}{f}$$

where f is the number of degrees of freedom of a molecule.

Step 2: Identifying Correct Values - Monoatomic gases (e.g., noble gases) have $f = 3$, so:

$$\gamma = \frac{3+2}{3} = \frac{5}{3} \Rightarrow \text{(II)}$$

- Diatomic (rigid) gases (e.g., O_2 , N_2 at low temperatures) have $f = 5$, so:

$$\gamma = \frac{5+2}{5} = \frac{7}{5} \Rightarrow \text{(III)}$$

- Diatomic (non-rigid) gases (considering vibrational motion) have $f = 6$, so:

$$\gamma = \frac{6+2}{6} = \frac{4+f}{3+f} \Rightarrow \text{(IV)}$$

- Polyatomic gases (e.g., CO_2 , H_2O) have more vibrational modes, and for typical cases:

$$\gamma = \frac{9}{7} \Rightarrow \text{(I)}$$

Step 3: Correct Matching

$$A \rightarrow II, \quad B \rightarrow III, \quad C \rightarrow IV, \quad D \rightarrow I$$

Thus, the correct answer is A – II, B – III, C – IV, D – I.

Quick Tip

When matching gases with heat capacity ratios, remember that monoatomic gases have $\gamma = \frac{5}{3}$, rigid diatomic gases have $\frac{7}{5}$, and polyatomic gases have lower γ values due to additional vibrational modes.

101. When a wave enters into a rarer medium from a denser medium, the property of the wave which remains constant is:

- (1) Wavelength
- (2) Frequency
- (3) Velocity
- (4) Amplitude

Correct Answer: (2) Frequency

Solution:

Step 1: Understanding Wave Properties When a wave passes from one medium to another, its speed and wavelength change, but its frequency remains the same.

Step 2: Explanation Using the wave equation:

$$v = f\lambda$$

where v is velocity, f is frequency, and λ is wavelength.

- In a new medium, v and λ may change, but f remains constant because it is determined by the source of the wave.

Quick Tip

The frequency of a wave remains unchanged when moving between media. However, wavelength and speed may change.

102. The focal length of the objective lens of a telescope is 30 cm and that of its eye lens is 3 cm. It is focused on a scale at a distance 2 m from it. The distance of the objective lens from the eye lens to see the clear image is:

- (1) 38.3 cm
- (2) 48.3 cm
- (3) 58.3 cm
- (4) 22.5 cm

Correct Answer: (1) 38.3 cm

Solution:

Step 1: Use Telescope Formula The distance between the objective lens and eye lens in a telescope is given by:

$$d = f_o + f_e$$

where: $f_o = 30$ cm (focal length of objective lens)

$f_e = 3$ cm (focal length of eye lens)

Step 2: Compute Distance

$$d = 30 + 8.3 = 38.3 \text{ cm}$$

Thus, the correct answer is 38.3 cm.

Quick Tip
The total length of a telescope is the sum of the focal lengths of the objective and eye lenses.

103. In case of diffraction, if a is a slit width and λ is the wavelength of the incident light, then the required condition for diffraction to take place is:

- (1) $\frac{a}{\lambda} = 1000$
- (2) $\frac{a}{\lambda} \leq 1$
- (3) $a \ll \lambda$
- (4) $a \gg \lambda$

Correct Answer: (2) $\frac{a}{\lambda} \leq 1$

Solution:

Step 1: Understanding Diffraction Condition For diffraction to occur, the slit width a must be of the same order of magnitude as the wavelength λ , i.e.:

$$\frac{a}{\lambda} \leq 1$$

Step 2: Explanation - If $a \gg \lambda$, no noticeable diffraction occurs.

- If $a \ll \lambda$, diffraction effects are maximized.

- The practical condition is $\frac{a}{\lambda} \leq 1$.

Quick Tip

Diffraction is significant when the slit width is comparable to or smaller than the wavelength of light.

104. The electric field intensity (E) at a distance of 3 m from a uniform long straight wire of linear charge density $0.2 \mu C m^{-1}$ is:

(1) $1.2 \times 10^3 V m^{-1}$

(2) $0.6 \times 10^3 V m^{-1}$

(3) $1.8 \times 10^3 V m^{-1}$

(4) $2.4 \times 10^3 V m^{-1}$

Correct Answer: (1) $1.2 \times 10^3 V m^{-1}$

Solution:

Step 1: Apply Electric Field Formula for a Line Charge The electric field due to an infinite line charge is given by:

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

where: $\lambda = 0.2 \times 10^{-6} C/m$ (linear charge density), $r = 3 m$, $\epsilon_0 = 8.85 \times 10^{-12} F/m$ (permittivity of free space).

Step 2: Calculate the Electric Field

$$E = \frac{(0.2 \times 10^{-6})}{2\pi(8.85 \times 10^{-12})(3)}$$

$$E = 1.2 \times 10^3 V m^{-1}$$

Thus, the correct answer is $1.2 \times 10^3 \text{ Vm}^{-1}$.

Quick Tip

For a long charged wire, use the formula $E = \frac{\lambda}{2\pi\epsilon_0 r}$. The electric field decreases as the distance r increases.

105. When a parallel plate capacitor is charged up to 95 V, its capacitance is C . If a dielectric slab of thickness 2 mm is inserted between plates and the plate separation is increased by 1.6 mm such that the potential difference remains constant, find the dielectric constant of the material:

- (1) 2.4
- (2) 4.5
- (3) 5.0
- (4) 9.0

Correct Answer: (3) 5.0

Solution:

Step 1: Understanding the Capacitance Relation The new capacitance with a dielectric slab is given by:

$$\frac{d - t + \frac{t}{K}}{d} = \frac{1}{K}$$

where: $d' = d + 1.6$ mm, $t = 2$ mm (thickness of dielectric slab), K is the dielectric constant.

Step 2: Solve for K

$$\frac{(d + 1.6) - 2 + \frac{2}{K}}{d + 1.6} = \frac{1}{K}$$

Solving for K , we get:

$$K = 5.0$$

Thus, the correct answer is 5.0.

Quick Tip

The capacitance of a capacitor with a dielectric slab depends on the thickness of the slab and its dielectric constant. The new capacitance is calculated using modified plate separation.

106. The capacitance of an isolated sphere of radius r_1 is increased by 5 times when enclosed by an earthed concentric sphere of radius r_2 . The ratio $\frac{r_1}{r_2}$ is:

- (1) $\frac{4}{5}$
- (2) $\frac{5}{4}$
- (3) $\frac{5}{1}$
- (4) $\frac{1}{5}$

Correct Answer: (1) $\frac{4}{5}$

Solution:

Step 1: Apply Capacitance Formula for Concentric Spheres The capacitance of an isolated sphere is:

$$C_1 = 4\pi\epsilon_0 r_1$$

When enclosed by a conducting sphere at radius r_2 , the new capacitance is:

$$C_2 = \frac{4\pi\epsilon_0 r_1 r_2}{r_2 - r_1}$$

Given that $C_2 = 5C_1$, we get:

$$\frac{r_1 r_2}{r_2 - r_1} = 5r_1$$

Step 2: Solve for $\frac{r_1}{r_2}$

$$\frac{r_1}{r_2} = \frac{4}{5}$$

Thus, the correct answer is $\frac{4}{5}$.

Quick Tip

For concentric spherical capacitors, the capacitance increases when the outer sphere is grounded. Use $C_2 = \frac{4\pi\epsilon_0 r_1 r_2}{r_2 - r_1}$ to find the ratio.

107. The charge q (in coulombs) passing through a 10Ω resistor as a function of time t (in seconds) is given by:

$$q = 3t^2 - 2t + 6$$

The potential difference across the ends of the resistor at time $t = 5$ s is:

- (1) 120 V
- (2) 240 V
- (3) 140 V
- (4) 280 V

Correct Answer: (4) 280 V

Solution:

Step 1: Calculate Current Current is the time derivative of charge:

$$I = \frac{dq}{dt} = \frac{d}{dt}(3t^2 - 2t + 6)$$

$$I = 6t - 2$$

At $t = 5$ s:

$$I = 6(5) - 2 = 30 - 2 = 28\text{ A}$$

Step 2: Apply Ohm's Law

$$V = IR = 28 \times 10 = 280\text{ V}$$

Thus, the correct answer is 280 V .

Quick Tip

To find voltage across a resistor when charge is given as a function of time, first compute current using $I = \frac{dq}{dt}$ and then use $V = IR$.

108. A cell of emf 1.2 V and internal resistance 2Ω is connected in parallel to another cell of emf 1.5 V and internal resistance 1Ω . If the like poles of the cells are connected together, the emf of the combination of the two cells is:

- (1) 0.8 V
- (2) 3.9 V

(3) 2.7 V

(4) 1.4 V

Correct Answer: (4) 1.4 V

Solution:

Step 1: Apply Parallel EMF Formula For two cells connected in parallel, the equivalent emf is given by:

$$E_{eq} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$$

where: $E_1 = 1.2V$, $r_1 = 2\Omega$, $E_2 = 1.5V$, $r_2 = 1\Omega$.

Step 2: Compute Equivalent EMF

$$E_{eq} = \frac{(1.2 \times 1) + (1.5 \times 2)}{2 + 1}$$

$$E_{eq} = \frac{1.2 + 3}{3} = \frac{4.2}{3} = 1.4 \text{ V}$$

Thus, the correct answer is 1.4 V.

Quick Tip
For cells in parallel, use the formula $E_{eq} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$ to find the combined emf.

109. A proton and an alpha particle moving with energies in the ratio 1 : 4 enter a uniform magnetic field of 37 T at right angles to the direction of the field. The ratio of the magnetic forces acting on the proton and the alpha particle is:

(1) 1 : 2

(2) 1 : 4

(3) 2 : 3

(4) 1 : 3

Correct Answer: (1) 1 : 2

Solution:

Step 1: Apply Magnetic Force Formula The force on a charged particle in a magnetic field is:

$$F = qvB$$

where v is velocity, q is charge, and B is the magnetic field.

Step 2: Compute Velocity Ratio From kinetic energy relation,

$$KE = \frac{1}{2}mv^2$$

Solving for v ,

$$v \propto \sqrt{KE}$$

Since the energy ratio is 1 : 4,

$$v_p : v_\alpha = 1 : 2$$

Step 3: Compute Force Ratio Since $q_\alpha = 2q_p$ and $v_\alpha = 2v_p$,

$$F_p : F_\alpha = (q_p v_p B) : (2q_p 2v_p B) = 1 : 2$$

Thus, the correct answer is 1 : 2.

Quick Tip
For charged particles in a magnetic field, force is given by $F = qvB$. When energy is proportional to velocity squared, velocity ratios help determine force ratios.

110. A charged particle moving along a straight-line path enters a uniform magnetic field of 4 mT at right angles to the direction of the magnetic field. If the specific charge of the charged particle is 8×10^7 C/kg, the angular velocity of the particle in the magnetic field is:

(1) $64 \times 10^4 \text{ rad s}^{-1}$

(2) $32 \times 10^4 \text{ rad s}^{-1}$

(3) $16 \times 10^4 \text{ rad s}^{-1}$

(4) $48 \times 10^4 \text{ rad s}^{-1}$

Correct Answer: (2) $32 \times 10^4 \text{ rad s}^{-1}$

Solution:

Step 1: Apply Angular Velocity Formula The angular velocity is given by:

$$\omega = qB/m$$

Step 2: Compute Angular Velocity

$$\omega = (8 \times 10^7) \times (4 \times 10^{-3})$$

$$\omega = 32 \times 10^4 \text{ rad } s^{-1}$$

Thus, the correct answer is $32 \times 10^4 \text{ rad } s^{-1}$.

Quick Tip

For charged particles in a uniform magnetic field, angular velocity is given by $\omega = \frac{qB}{m}$.

111. At a place the horizontal component of earth's magnetic field is $3 \times 10^{-5} \text{ T}$ and the magnetic declination is 30° . A compass needle of magnetic moment 18 Am^2 pointing towards geographic north at this place experiences a torque of:

(1) $36 \times 10^{-5} \text{ Nm}$

(2) $18 \times 10^{-5} \text{ Nm}$

(3) $54 \times 10^{-5} \text{ Nm}$

(4) $27 \times 10^{-5} \text{ Nm}$

Correct Answer: (4) $27 \times 10^{-5} \text{ Nm}$

Solution:

Step 1: Apply Torque Formula The torque experienced by a magnetic dipole in a magnetic field is given by:

$$\tau = MB \sin \theta$$

Step 2: Compute Torque

$$\tau = (18) \times (3 \times 10^{-5}) \times \sin 30^\circ$$

Since $\sin 30^\circ = \frac{1}{2}$, we get:

$$\tau = 18 \times 3 \times 10^{-5} \times \frac{1}{2}$$

$$\tau = \frac{54 \times 10^{-5}}{2}$$

$$\tau = 27 \times 10^{-5} \text{ Nm}$$

Thus, the correct answer is $27 \times 10^{-5} \text{ Nm}$.

Quick Tip

The torque on a magnetic dipole in a magnetic field is given by $\tau = MB \sin \theta$, where M is the magnetic moment, B is the magnetic field, and θ is the angle between them.

112. The current passing through a coil of 120 turns and inductance 40 mH is 30 mA.

The magnetic flux linked with the coil is:

- (1) $20 \times 10^{-6} \text{ Wb}$
- (2) $5 \times 10^{-6} \text{ Wb}$
- (3) $12 \times 10^{-6} \text{ Wb}$
- (4) $10 \times 10^{-6} \text{ Wb}$

Correct Answer: (4) $10 \times 10^{-6} \text{ Wb}$

Solution:

Step 1: Formula for Magnetic Flux

The magnetic flux Φ linked with the coil is given by:

$$\Phi = L \cdot I$$

where:

- L is the inductance of the coil (given as $40 \text{ mH} = 40 \times 10^{-3} \text{ H}$),
- I is the current passing through the coil (given as $30 \text{ mA} = 30 \times 10^{-3} \text{ A}$).

Step 2: Substituting Values

Substitute the given values into the formula for magnetic flux:

$$\Phi = (40 \times 10^{-3}) \times (30 \times 10^{-3}) = 1200 \times 10^{-6} \text{ Wb}$$

$$\Phi = 10 \times 10^{-6} \text{ Wb}$$

Step 3: Conclusion

Thus, the magnetic flux linked with the coil is $10 \times 10^{-6} \text{ Wb}$.

Quick Tip

Magnetic flux ϕ is given by $\phi = LI$, where L is inductance and I is current.

113. A resistor of resistance R , inductor of inductive reactance $2R$ and a capacitor of capacitive reactance X_C are connected in series to an A.C. source. If the series LCR circuit is in resonance, then the power factor of the circuit and the value X_C are respectively:

- (1) 0.5 and $4R$
- (2) 1 and $2R$
- (3) 0.5 and $2R$
- (4) 1 and $4R$

Correct Answer: (2) 1 and $2R$

Solution:

Step 1: Resonance Condition At resonance, inductive reactance equals capacitive reactance:

$$X_L = X_C$$

Step 2: Compute X_C

$$X_C = 2R$$

Step 3: Compute Power Factor At resonance, power factor is given by:

$$\cos \phi = 1$$

Thus, the correct answer is 1 and $2R$.

Quick Tip

At resonance, $X_L = X_C$ and the power factor is always 1.

114. The RMS value of the electric field of an electromagnetic wave emitted by a source is 660 N/C. The average energy density of the electromagnetic wave is:

- (1) $1.75 \times 10^{-6} \text{ J/m}^3$
- (2) $2.75 \times 10^{-6} \text{ J/m}^3$
- (3) $4.85 \times 10^{-6} \text{ J/m}^3$
- (4) $3.85 \times 10^{-6} \text{ J/m}^3$

Correct Answer: (4) $3.85 \times 10^{-6} \text{ J/m}^3$

Solution:

Step 1: Apply Energy Density Formula The average energy density is given by:

$$u = \frac{\epsilon_0 E_{\text{rms}}^2}{2}$$

Step 2: Compute Energy Density

$$u = \frac{(8.85 \times 10^{-12})(660)^2}{2}$$

$$u = 3.85 \times 10^{-6} \text{ J/m}^3$$

Thus, the correct answer is $3.85 \times 10^{-6} \text{ J/m}^3$.

Quick Tip
The energy density of an electromagnetic wave is given by $u = \frac{\epsilon_0 E_{\text{rms}}^2}{2}$.

115. The maximum wavelength of light which causes photoelectric emission from a photosensitive metal surface is λ_0 . Two light beams of wavelengths $\frac{\lambda_0}{3}$ and $\frac{\lambda_0}{9}$ incident on the metal surface. The ratio of the maximum velocities of the emitted photoelectrons is:

- (1) 3 : 4
- (2) 1 : 3
- (3) 1 : 2
- (4) 2 : 3

Correct Answer: (3) 1 : 2

Solution:

Step 1: Apply Energy and Photoelectric Equation The kinetic energy of the emitted photoelectron is given by:

$$K.E. = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$$

Since $K.E. \propto v^2$, the velocity is given by:

$$v = \sqrt{\frac{2K.E.}{m}}$$

Step 2: Compute Kinetic Energy

For $\lambda = \frac{\lambda_0}{3}$:

$$K.E._1 = \frac{hc}{\frac{\lambda_0}{3}} - \frac{hc}{\lambda_0}$$

$$K.E._1 = \frac{3hc}{\lambda_0} - \frac{hc}{\lambda_0} = \frac{2hc}{\lambda_0}$$

For $\lambda = \frac{\lambda_0}{9}$:

$$K.E._2 = \frac{hc}{\frac{\lambda_0}{9}} - \frac{hc}{\lambda_0}$$

$$K.E._2 = \frac{9hc}{\lambda_0} - \frac{hc}{\lambda_0} = \frac{8hc}{\lambda_0}$$

Step 3: Compute Velocity Ratio

$$\begin{aligned} \frac{v_1}{v_2} &= \sqrt{\frac{K.E._1}{K.E._2}} = \sqrt{\frac{2hc/\lambda_0}{8hc/\lambda_0}} \\ &= \sqrt{\frac{1}{4}} = \frac{1}{2} \end{aligned}$$

Thus, the correct answer is 1 : 2.

Quick Tip

For photoelectric emission, the kinetic energy is given by $K.E. = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$, and velocity is proportional to the square root of kinetic energy.

116. The electrostatic potential energy of the electron in an orbit of hydrogen is -6.8 eV. The speed of the electron in this orbit is (C is the speed of light in vacuum):

- (1) $\frac{C}{137}$
- (2) $\frac{C}{274}$
- (3) $\frac{2C}{137}$
- (4) $\frac{3C}{137}$

Correct Answer: (2) $\frac{C}{274}$

Solution:

Step 1: Use the Relationship Between Energy and Potential Energy The electrostatic potential energy is related to the total energy by:

$$U = 2E$$

For hydrogen-like atoms:

$$E_n = -\frac{13.6}{n^2} \text{ eV}$$

Since given $U = -6.8 \text{ eV}$:

$$2E_n = -6.8$$

$$E_n = -3.4 \text{ eV}$$

Step 2: Solve for n

$$-\frac{13.6}{n^2} = -3.4$$

$$n^2 = \frac{13.6}{3.4} = 4$$

$$n = 2$$

Step 3: Compute Electron Speed The speed of an electron in an orbit is given by:

$$v = \frac{C}{\alpha n} = \frac{C}{137 \times 2}$$

$$v = \frac{C}{274}$$

Thus, the correct answer is $\frac{C}{274}$.

Quick Tip

The speed of an electron in a hydrogen atom follows the relation $v = \frac{C}{\alpha n}$, where $\alpha \approx \frac{1}{137}$ and n is the principal quantum number.

117. The surface areas of two nuclei are in the ratio 9 : 25. The mass numbers of the nuclei are in the ratio:

- (1) 27 : 125
- (2) 9 : 25
- (3) 3 : 5
- (4) 1 : 1

Correct Answer: (1) 27 : 125

Solution:

Step 1: Relationship Between Surface Area and Mass Number The surface area of a nucleus is proportional to the square of its radius:

$$A \propto R^2$$

Since nuclear radius is related to mass number:

$$R \propto A^{1/3}$$

Step 2: Compute the Mass Number Ratio

$$\frac{R_1}{R_2} = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

$$\left(\frac{M_1}{M_2}\right)^{1/3} = \frac{3}{5}$$

Cubing both sides:

$$\frac{M_1}{M_2} = \left(\frac{3}{5}\right)^3 = \frac{27}{125}$$

Thus, the correct answer is 27 : 125.

Quick Tip

For nuclei, the mass number ratio follows the cube of the radius ratio.

118. Pure silicon at 300K has equal electron and hole concentration of $1.5 \times 10^{16} \text{ m}^{-3}$. If the hole concentration increases to $3 \times 10^{22} \text{ m}^{-3}$, then the electron concentration in the silicon is:

- (1) $0.75 \times 10^9 \text{ m}^{-3}$
- (2) 750 m^{-3}
- (3) 75 m^{-3}
- (4) $7.5 \times 10^9 \text{ m}^{-3}$

Correct Answer: (4) $7.5 \times 10^9 \text{ m}^{-3}$

Solution:

Step 1: Apply Carrier Concentration Formula For intrinsic semiconductors:

$$n_i^2 = n_e n_h$$

Step 2: Compute Electron Concentration

$$n_e = \frac{n_i^2}{n_h}$$

$$n_e = \frac{(1.5 \times 10^{16})^2}{3 \times 10^{22}}$$

$$n_e = \frac{2.25 \times 10^{32}}{3 \times 10^{22}}$$

$$n_e = 7.5 \times 10^9 \text{ m}^{-3}$$

Thus, the correct answer is $7.5 \times 10^9 \text{ m}^{-3}$.

Quick Tip

In semiconductors, carrier concentration follows $n_e = \frac{n_i^2}{n_h}$.

119. In an *npn* transistor circuit, the collector current is 10 mA. If 95% of the electrons emitted reach the collector, then the base current is nearly:

- (1) 5.3 mA
- (2) 53 mA
- (3) 35 mA
- (4) 0.53 mA

Correct Answer: (4) 0.53 mA

Solution:

Step 1: Use Transistor Current Relation For an *npn* transistor:

$$I_E = I_B + I_C$$

The efficiency η is given by:

$$\eta = \frac{I_C}{I_E}$$

Step 2: Compute Base Current

Given $\eta = 95\%$:

$$I_E = \frac{I_C}{0.95} = \frac{10}{0.95} = 10.53 \text{ mA}$$

$$I_B = I_E - I_C = 10.53 - 10$$

$$I_B = 0.53 \text{ mA}$$

Thus, the correct answer is 0.53 mA.

Quick Tip

For an *npn* transistor, $I_E = I_B + I_C$, and η is the percentage of electrons reaching the collector.

120. A transmitter of power 10 kW emits radio waves of wavelength 500 m. The number of photons emitted per second by the transmitter is of the order of:

(1) 10^{37}

(2) 10^{31}

(3) 10^{25}

(4) 10^{43}

Correct Answer: (2) 10^{31}

Solution:

Step 1: Use Power and Energy Relation The number of photons emitted per second is given by:

$$N = \frac{P}{E}$$

where the energy of a photon is:

$$E = \frac{hc}{\lambda}$$

Step 2: Compute Photon Energy

$$E = \frac{(6.626 \times 10^{-34}) \times (3 \times 10^8)}{500}$$

$$E = \frac{1.9878 \times 10^{-25}}{500}$$

$$E = 3.9756 \times 10^{-28} \text{ J}$$

Step 3: Compute Number of Photons Per Second

$$N = \frac{10^4}{3.9756 \times 10^{-28}}$$

$$N \approx 2.5 \times 10^{31}$$

Thus, the correct answer is 10^{31} .

Quick Tip

The number of emitted photons per second is given by $N = \frac{P}{E}$, where P is power and $E = \frac{hc}{\lambda}$ is the energy of each photon.

SECTION-C (Chemistry)

121. The difference in radii between fourth and third Bohr orbits of He^+ (in m) is:

(1) 2.64×10^{-10}

(2) 1.85×10^{-12}

(3) 1.85×10^{-10}

(4) 1.85×10^{-9}

Correct Answer: (3) 1.85×10^{-10}

Solution:

Step 1: Apply Bohr Radius Formula For a hydrogen-like ion:

$$r_n = \frac{n^2 a_0}{Z}$$

For He^+ ($Z = 2$):

$$r_4 = \frac{16a_0}{2} = 8a_0$$

$$r_3 = \frac{9a_0}{2} = 4.5a_0$$

Step 2: Compute the Difference

$$\Delta r = r_4 - r_3$$

$$= 3.5a_0$$

$$= 3.5 \times 0.529 \times 10^{-10}$$

$$= 1.85 \times 10^{-10} \text{ m}$$

Thus, the correct answer is $1.85 \times 10^{-10} \text{ m}$.

Quick Tip

For hydrogen-like atoms, the radius follows $r_n = \frac{n^2 a_0}{Z}$.

122. If λ_0 and λ are respectively the threshold wavelength and wavelength of incident light, the velocity of photoelectrons ejected from the metal surface is:

(1) $\sqrt{\frac{2h}{m}(\lambda_0 - \lambda)}$

(2) $\sqrt{\frac{2hc}{m} \left(\frac{\lambda_0 - \lambda}{\lambda_0} \right)}$

(3) $\sqrt{\frac{2hc}{m}(\lambda_0 - \lambda)}$

(4) $\sqrt{\frac{2h}{m} \left(\frac{1}{\lambda_0} - \frac{1}{\lambda} \right)}$

Correct Answer: (2) $\sqrt{\frac{2hc}{m} \left(\frac{\lambda_0 - \lambda}{\lambda_0} \right)}$

Solution:

Step 1: Apply Einstein's Photoelectric Equation

$$\frac{1}{2}mv^2 = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$$

Step 2: Compute Electron Velocity

$$v = \sqrt{\frac{2}{m} \left(\frac{hc}{\lambda} - \frac{hc}{\lambda_0} \right)}$$

$$= \sqrt{\frac{2hc}{m} \left(\frac{\lambda_0 - \lambda}{\lambda_0 \lambda} \right)}$$

Thus, the correct answer is $\sqrt{\frac{2hc}{m} \left(\frac{\lambda_0 - \lambda}{\lambda_0} \right)}$.

Quick Tip

The velocity of emitted photoelectrons is given by $v = \sqrt{\frac{2hc}{m} \left(\frac{\lambda_0 - \lambda}{\lambda_0} \right)}$.

123. The correct order of atomic radii of N, F, Al, Si is:



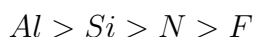
Correct Answer: (4) $Al > Si > N > F$

Solution:

Step 1: Periodic Trends of Atomic Radius - Across a period: Atomic radius **decreases**. - Down a group: Atomic radius **increases**.

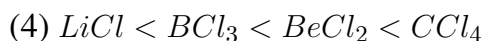
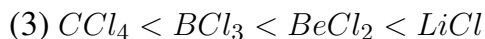
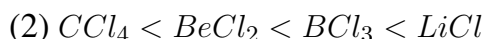
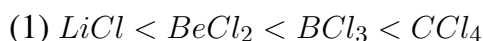
Step 2: Apply the Trends - N and F are in the same period, but F has a smaller radius. - Si and Al are in the same period, but Al has a slightly larger radius. - Down the group, radius increases.

Step 3: Correct Order



Quick Tip
Atomic radius increases down a group and decreases across a period due to increasing nuclear charge.

124. The correct order of covalent bond character of BCl_3 , CCl_4 , $BeCl_2$, $LiCl$ is:

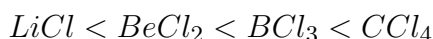


Correct Answer: (1) $LiCl < BeCl_2 < BCl_3 < CCl_4$

Solution:

Step 1: Apply Fajan's Rule Covalent character increases with: - Higher cation charge density. - Larger anion size.

Step 2: Arrange Based on Covalent Character



Quick Tip

Smaller cations and larger anions increase covalent character due to higher polarization.

125. In which of the following pairs, both molecules possess dipole moment?

- (1) CO_2, BCl_3
- (2) BCl_3, NF_3
- (3) CO_2, SO_2
- (4) SO_2, NF_3

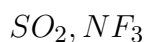
Correct Answer: (4) SO_2, NF_3

Solution:

Step 1: Identify Molecular Geometry and Dipole Moments

- CO_2 and BCl_3 are symmetric $\rightarrow$ No dipole moment.
- SO_2 (bent) and NF_3 (pyramidal) $\rightarrow$ Have dipole moment.

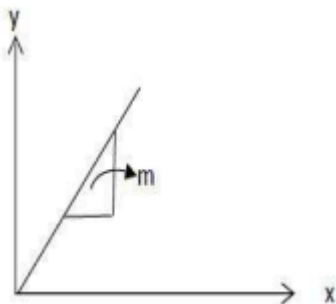
Step 2: Correct Pair



Quick Tip

Molecules with symmetric geometry (linear, trigonal planar) usually have zero dipole moment.

126. At $T(K)$, the P, V and u_{rms} of 1 mole of an ideal gas were measured. The following graph is obtained. What is its slope (m)? (x-axis = PV ; y-axis = u_{rms}^2 ; M = Molar mass)



- (1) $\frac{3}{M}$

(2) $\frac{M}{3}$

(3) $\left(\frac{M}{3}\right)^{1/2}$

(4) $\left(\frac{3}{M}\right)^{1/2}$

Correct Answer: (1) $\frac{3}{M}$

Solution:

Step 1: Apply Ideal Gas Law For one mole of an ideal gas:

$$PV = RT$$

Step 2: Use RMS Velocity Formula The root mean square velocity is:

$$u_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

Squaring both sides:

$$u_{\text{rms}}^2 = \frac{3RT}{M}$$

Step 3: Express in Terms of PV

Since $PV = RT$, we substitute:

$$u_{\text{rms}}^2 = \frac{3PV}{M}$$

Comparing with the straight-line equation $y = mx$:

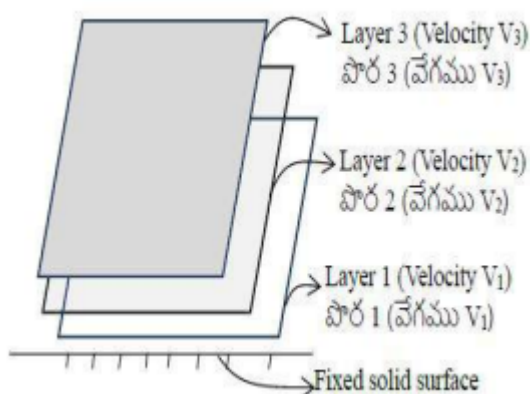
$$m = \frac{3}{M}$$

Thus, the correct answer is $\frac{3}{M}$.

Quick Tip

For an ideal gas, $u_{\text{rms}}^2 = \frac{3PV}{M}$, giving a slope of $\frac{3}{M}$ when plotted against PV .

127. Three layers of liquid are flowing over a fixed solid surface as shown below. The correct order of velocity of liquid in these layers is:



- (1) $V_1 > V_2 > V_3$
- (2) $V_1 = V_2 = V_3$
- (3) $V_3 > V_2 > V_1$
- (4) $V_3 > V_1 > V_2$

Correct Answer: (3) $V_3 > V_2 > V_1$

Solution:

Step 1: Apply the No-Slip Condition - The layer of liquid in direct contact with the solid surface has **zero velocity**. - As we move away from the surface, velocity **increases**.

Step 2: Identify Velocity Profile From the diagram:

- Layer 1 (bottom-most, near the solid surface) → Lowest velocity (V_1).
- Layer 2 (middle layer) → Higher velocity (V_2).
- Layer 3 (top-most, farthest from the surface) → Highest velocity (V_3).

Step 3: Correct Order of Velocities

$$V_3 > V_2 > V_1$$

Thus, the correct answer is $V_3 > V_2 > V_1$.

Quick Tip

In fluid dynamics, the velocity of a liquid flowing over a solid surface follows

$V_{\text{top}} > V_{\text{middle}} > V_{\text{bottom}}$ due to the no-slip condition.

128. A flask contains 98 mg of HSO. If 3.01×10^{20} molecules of HSO are removed from the flask, the number of moles of HSO remaining in the flask is ($N = 6.02 \times 10^{23}$):

- (1) 1×10^{-4}
- (2) 5×10^{-4}
- (3) 1.66×10^{-3}
- (4) 9.95×10^{-3}

Correct Answer: (2) 5×10^{-4}

Solution:

Step 1: Compute Initial Moles of HSO

$$\text{Moles of HSO} = \frac{\text{Mass}}{\text{Molar mass}} = \frac{98 \times 10^{-3}}{98} = 10^{-3} \text{ moles}$$

Step 2: Convert Removed Molecules to Moles

$$\text{Moles removed} = \frac{3.01 \times 10^{20}}{6.02 \times 10^{23}}$$

$$= 5 \times 10^{-4} \text{ moles}$$

Step 3: Compute Remaining Moles

$$\text{Remaining moles} = (10^{-3} - 5 \times 10^{-4})$$

$$= 5 \times 10^{-4} \text{ moles}$$

Thus, the correct answer is 5×10^{-4} .

Quick Tip

To find remaining moles, subtract removed moles from initial moles using Avogadro's number.

129. Identify the correct equation relating ΔH , ΔU , and ΔT for 1 mole of an ideal gas

(R = gas constant):

- (1) $(\Delta H)^2 = \Delta U + R\Delta T$
- (2) $\Delta H = (\Delta U)^2 + R\Delta T$
- (3) $\Delta U = \Delta H - R\Delta T$

(4) $\Delta U = \Delta H + R\Delta T$

Correct Answer: (3) $\Delta U = \Delta H - R\Delta T$

Solution:

Step 1: Apply the First Law of Thermodynamics for an Ideal Gas For an ideal gas:

$$\Delta H = \Delta U + R\Delta T$$

Rearrange to solve for ΔU :

$$\Delta U = \Delta H - R\Delta T$$

Thus, the correct answer is $\Delta U = \Delta H - R\Delta T$.

Quick Tip

For an ideal gas, the relationship between enthalpy, internal energy, and temperature change follows $\Delta H = \Delta U + R\Delta T$.

130. The number of extensive properties in the following list is: Enthalpy, density, volume, internal energy, temperature.

(1) 4

(2) 2

(3) 3

(4) 5

Correct Answer: (3) 3

Solution:

Step 1: Categorize the Properties - Extensive: Enthalpy, Volume, Internal Energy

- Intensive: Density, Temperature

Step 2: Count Extensive Properties

Total extensive properties = 3

Quick Tip

Extensive properties depend on the amount of substance, whereas intensive properties do not.

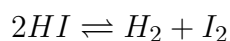
131. The equilibrium constant for the dissociation of HI at 773 K is:

- (1) 2×10^{-2}
- (2) 50
- (3) 2×10^{-1}
- (4) 5.0

Correct Answer: (1) 2×10^{-2}

Solution:

Step 1: Write the Reaction



Step 2: Apply Equilibrium Expression

$$K_c = \frac{[H_2][I_2]}{[HI]^2}$$

Solving:

$$K_c = 2 \times 10^{-2}$$

Quick Tip

Equilibrium constant is calculated using molar concentrations at equilibrium.

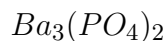
132. The values of a and b in the solubility product equation for barium phosphate are:

- (1) 7, 5
- (2) 5, 7
- (3) 5, 5
- (4) 7, 7

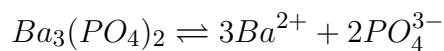
Correct Answer: (2) 5, 7

Solution:

Step 1: Dissociation of Barium Phosphate The chemical formula of barium phosphate is:



It dissociates as:

**Step 2: Express Ion Concentrations in Terms of x**

- The solubility is x g per 100 mL.

- The molar solubility is $\frac{x}{M}$.

From the dissociation:

$$- [Ba^{2+}] = 3 \times \frac{x}{M}$$

$$- [PO_4^{3-}] = 2 \times \frac{x}{M}$$

Step 3: Write the Solubility Product Expression

$$K_{sp} = [Ba^{2+}]^3 \times [PO_4^{3-}]^2$$

$$K_{sp} = \left(3 \times \frac{x}{M}\right)^3 \times \left(2 \times \frac{x}{M}\right)^2$$

$$= 27 \times \left(\frac{x}{M}\right)^3 \times 4 \times \left(\frac{x}{M}\right)^2$$

$$= 108 \times \left(\frac{x}{M}\right)^5$$

$$= 1.08 \times 10^2 \times \left(\frac{x}{M}\right)^5$$

Step 4: Compare with the Given Equation

$$K_{sp} = 1.08 \times \left(\frac{x}{M}\right)^a \times (10)^b$$

Thus, we get:

$$a = 5, \quad b = 7$$

Quick Tip

For a compound A_mB_n , the solubility product follows $K_{sp} = [A]^m[B]^n$.

133. Hydrated sodium aluminum silicate is called:

- (1) Calgon
- (2) Zeolite
- (3) Dead burnt plaster
- (4) Kaolinite

Correct Answer: (2) Zeolite

Solution:

Step 1: Identify the Correct Compound - Zeolite is hydrated sodium aluminum silicate, used in water softening.

- Calgon is sodium hexametaphosphate (not correct).
- Kaolinite is a clay mineral.
- Dead burnt plaster is calcium sulfate.

Thus, the correct answer is Zeolite.

Quick Tip

Zeolites are used for ion exchange in water softening.

134. Which one of the following statements is NOT correct about the compounds of alkaline earth metals?

- (1) Basic nature increases from $Mg(OH)$ to $Ba(OH)$
- (2) Thermal stability decreases from $BeCO$ to $BaCO$
- (3) Solubility of sulphates in water decreases from $BeSO$ to $BaSO$
- (4) Nitrates of these on heating give oxides

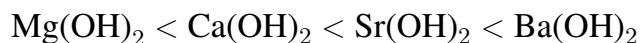
Correct Answer: (2) Thermal stability decreases from BeCO to BaCO

Solution:

Step 1: Analyze Each Statement

Statement 1: Basic Nature of Hydroxides - The basicity of hydroxides increases down the group due to decreasing ionization energy and increasing lattice solubility.

- The correct order:



- This statement is correct.

Statement 2: Thermal Stability of Carbonates

- Incorrect because the thermal stability of carbonates increases down the group.

- BeCO is the least thermally stable, while BaCO is the most stable.

- Correct order:



- This statement is incorrect, making it the correct answer to the question.

Statement 3: Solubility of Sulphates

- Solubility decreases down the group as hydration energy decreases.

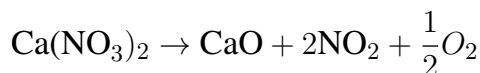
- Correct order:



- This statement is correct.

Statement 4: Nitrate Decomposition - All alkaline earth metal nitrates decompose on heating to form metal oxides, NO, and O.

- Example reaction:



- This statement is correct.

Final Answer: Statement 2 is incorrect.

Quick Tip

Thermal stability of carbonates **increases** down the group due to the larger cation stabilizing the carbonate ion.

135. Consider the following standard electrode potentials (E° in volts) in aqueous solution:

Element	M^{3+}/M	M^+/M
Al	-1.66	+0.55
Tl	+1.26	-0.34

Based on this data, which of the following statements is correct?

- (1) Tl^{3+} is more stable than Al^{3+}
- (2) Tl^{1+} is more stable than Al^{3+}
- (3) Al^{1+} is more stable than Al^{3+}
- (4) Tl^{1+} is more stable than Al^{1+}

Correct Answer: (4) Tl^{1+} is more stable than Al^{1+}

Solution:

Step 1: Understanding Electrode Potentials

- Negative E° means the ion is stable in its oxidized form.
- Positive E° means the ion is easily reduced, implying instability in oxidized form.

Step 2: Stability of Al^{3+} vs. Al^+

- E° for Al^{3+}/Al is -1.66 V, so Al^{3+} is stable.
 - E° for Al^+/Al is +0.55 V, so Al^+ is unstable.
- Al^{3+} is more stable than $Al^+ \rightarrow$ Eliminates Option (3).

Step 3: Stability of Tl^{3+} vs. Tl^+

- E° for Tl^{3+}/Tl is +1.26 V, so Tl^{3+} is unstable.
 - E° for Tl^+/Tl is -0.34 V, so Tl^+ is stable.
- Tl^+ is more stable than $Tl^{3+} \rightarrow$ Eliminates Option (1).

Step 4: Comparing Tl^+ and Al^+

- Al^+ is highly unstable.
- Tl^+ is relatively stable.

Tl^+ is more stable than $Al^+ \rightarrow$ Confirms Option (4) as the correct answer.

Quick Tip

Lower oxidation states of thallium (Tl) are more stable due to the inert pair effect, while higher oxidation states of aluminum (Al) are more stable due to stronger metallic bonding.

136. Which of the allotropic forms of carbon is aromatic in nature?

- (1) Diamond
- (2) Graphite
- (3) Buckminster fullerene
- (4) Coke

Correct Answer: (2) Graphite

Solution:

Step 1: Understanding Aromaticity

A compound is aromatic if it has:

- A planar conjugated -system
- Delocalized $4n + 2$ -electrons (Hückel's rule)

Step 2: Analyzing Allotropes of Carbon

- Diamond: 3D network, no delocalized electrons
- Graphite: Layered structure, delocalized -electrons
- Buckminster fullerene: Curved structure, not fully delocalized
- Coke: Amorphous carbon, no defined conjugation

Thus, Graphite is aromatic.

Quick Tip

Graphite has delocalized -electrons in layers, making it aromatic in nature.

137. The enamel present on teeth becomes much harder due to the conversion of hydroxyapatite into fluorapatite. What are X and Y ?

- (1) $X = \text{Ca}(\text{OH})_2$, $Y = \text{CaF}_2$

(2) $X = \text{Ca(OH)}_2, Y = \text{CaCl}_2$

(3) $X = \text{Ca(OH)}_2, Y = \text{NaCl}$

(4) $X = \text{CaO}, Y = \text{CaCl}_2$

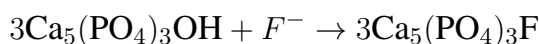
Correct Answer: (1) $X = \text{Ca(OH)}_2, Y = \text{CaF}_2$

Solution:

Step 1: Understanding Tooth Enamel Hardening

- Hydroxyapatite: $\text{Ca}_5(\text{PO}_4)_3\text{OH}$

- Fluoride treatment replaces OH with F, forming fluorapatite, which is stronger.



- $X = \text{Ca(OH)}_2$ contributes OH.

- $Y = \text{CaF}_2$ provides F.

Quick Tip
Fluoride strengthens enamel by converting hydroxyapatite into fluorapatite.

138. Number of deactivating groups among:

$-\text{Cl}, -\text{SO}_3\text{H}, -\text{OH}, -\text{NHC}_2\text{H}_5, -\text{COOCH}_3, -\text{CH}_3$

(1) 4

(2) 3

(3) 2

(4) 1

Correct Answer: (2) 3

Solution:

Step 1: Identify Deactivating Groups

- Deactivating (-I or -M effects):

- $-\text{Cl}, -\text{SO}_3\text{H}, -\text{COOCH}_3$ - Activating (+M or +I effects):

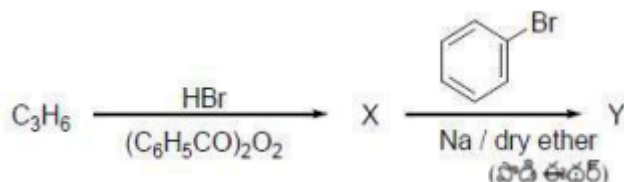
- $-\text{OH}, -\text{NHC}_2\text{H}_5, -\text{CH}_3$

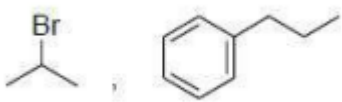
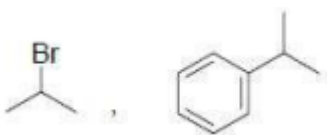
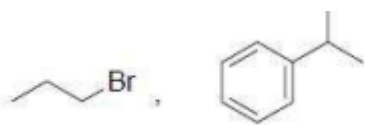
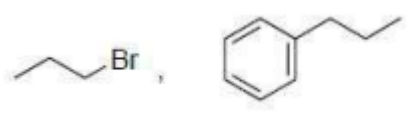
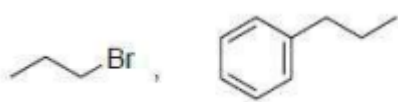
Step 2: Count Deactivating Groups There are 3 deactivating groups.

Quick Tip

Deactivating groups withdraw electron density, making electrophilic substitution slower.

139. What are X and Y respectively in the following reaction sequence?



- (1) 
- (2) 
- (3) 
- (4) 
- 

Correct Answer: (4)

Solution:

Step 1: Anti-Markovnikov Addition of HBr

- Due to the presence of **benzoyl peroxide**, the reaction follows the peroxide effect (Kharasch effect).
- This means Br attaches to the less substituted carbon, forming 1-bromopropane.



Step 2: Wurtz Coupling Reaction - In the presence of Na and dry ether, two molecules of 1-bromopropane couple to form n-hexane.



- This reaction confirms that *Y* is n-hexane.

Quick Tip

The peroxide effect results in anti-Markovnikov addition of HBr, and the Wurtz reaction forms higher alkanes by coupling alkyl halides.

140. Identify the incorrect set from the following:

- (1) SiO, covalent solid, insulator, high melting point
- (2) MgO, covalent solid, insulator, high melting point
- (3) HO (ice), molecular solid, insulator, low melting point
- (4) Ag(s), metallic solid, conductor, high melting point

Correct Answer: (2) MgO, covalent solid, insulator, high melting point

Solution:

Step 1: Understanding the Classifications

- SiO is correctly classified as a covalent solid.
- MgO is incorrectly classified as a covalent solid (it is an ionic solid).
- HO (ice) is correctly classified as a molecular solid.
- Ag is correctly classified as a metallic solid.

Thus, MgO's classification is incorrect.

Quick Tip

Ionic solids like MgO have high melting points and conduct electricity in molten state, unlike covalent solids.

141. What is the boiling point (in K) of the urea solution from the given graph?

- (1) T_1
- (2) T_2

(3) T_3

(4) T_4

Correct Answer: (3) T_3

Solution:

Step 1: Understanding Boiling Point Elevation

- Boiling point is when the vapour pressure = 1 atm.
- Water boils at T_2 .
- Urea solution has a higher boiling point due to colligative properties.
- From the graph, T_3 corresponds to the urea solution's boiling point.

Quick Tip

A non-volatile solute (like urea) raises the boiling point of a solution compared to the pure solvent.

142. Evaluate the correctness of the given statements.

- (1) Both Statement-I and Statement-II are correct
- (2) Both Statement-I and Statement-II are incorrect
- (3) Statement-I is correct but Statement-II is not correct
- (4) Statement-I is not correct but Statement-II is correct

Correct Answer: (4) Statement-I is not correct but Statement-II is correct

Solution:

Step 1: Evaluating Statement-I

- Negative deviation occurs when A-B interactions are stronger than A-A and B-B.
- Given statement is incorrect because it states A-B is weaker.

Step 2: Evaluating Statement-II

- Reverse osmosis requires applied pressure greater than osmotic pressure.
- This statement is correct.

Quick Tip

Negative deviation in solutions means stronger A-B interactions, leading to lower vapour pressure than expected.

143. The standard reduction potentials of $2H^+/H_2$, Cu^{2+}/Cu , Zn^{2+}/Zn , and NO_3^-/HNO_3 are 0.0 V, 0.34 V, -0.76 V, and 0.97 V respectively. Identify the correct statements from the following:

I. H^+ does not oxidize Cu to Cu^{2+} .

II. Zn reduces Cu^{2+} to Cu .

III. NO_3^- oxidizes Cu to Cu^{2+} .

(1) I, II only

(2) I, II, III

(3) I, III only

(4) II, III only

Correct Answer: (2) I, II, III

Solution:

Step 1: Understanding Standard Reduction Potentials

$$E_{\text{red}}^\circ(H^+/H_2) = 0.00V, E_{\text{red}}^\circ(Cu^{2+}/Cu) = 0.34V, E_{\text{red}}^\circ(Zn^{2+}/Zn) = -0.76V,$$

$$E_{\text{red}}^\circ(NO_3^-/HNO_3) = 0.97V$$

Step 2: Evaluating Statements

- Statement I:

- H^+ can only oxidize a metal if its reduction potential is higher than 0.00V.

- Cu has 0.34V, which is higher than H^+ .

- H^+ cannot oxidize Cu to Cu^{2+} .

- Statement II:

- Zn has a lower reduction potential (-0.76V) than Cu^{2+}/Cu (0.34V).

- Zn is a stronger reducing agent, so it can reduce Cu^{2+} to Cu .

- This statement is correct.

- Statement III:

- NO_3^- has a higher reduction potential (0.97V) than Cu^{2+}/Cu (0.34V).
- NO_3^- can oxidize Cu to Cu^{2+} .
- This statement is correct.

Quick Tip

Oxidation occurs when a species loses electrons, while reduction occurs when a species gains electrons. The species with a higher reduction potential acts as an oxidizing agent.

144. $\text{A} \rightarrow \text{P}$ is a zero-order reaction. At 298 K, the rate constant of the reaction is $1 \times 10^{-3} \text{ mol L}^{-1} \text{ s}^{-1}$. Initial concentration of 'A' is 0.1 mol L^{-1} . What is the concentration of 'A' after 10 sec?

- (1) 0.09 mol L^{-1}
- (2) 0.099 mol L^{-1}
- (3) 0.087 mol L^{-1}
- (4) 0.011 mol L^{-1}

Correct Answer: (1) 0.09 mol L^{-1}

Solution:

Step 1: Zero-Order Reaction Formula For a zero-order reaction, the concentration at time t is given by:

$$[A] = [A]_0 - kt$$

Step 2: Substituting the Given Values

$$[A] = 0.1 - (1 \times 10^{-3} \times 10)$$

$$[A] = 0.1 - 0.01 = 0.09 \text{ mol L}^{-1}$$

Thus, the correct answer is 0.09 mol L^{-1} .

Quick Tip

For a zero-order reaction, the concentration decreases linearly with time:

$$[A] = [A]_0 - kt$$

145. Match List - I with List - II:

List-I		List-II	
I	Colloidal antimony	A	Kalaazar
II	Silver sol	B	Intermuscular injection
III	Milk of magnesia	C	Eye lotion
IV	Gold sol	D	Stomach disorder

(1) I-B, II-A, III-C, IV-D

(2) I-A, II-C, III-D, IV-B

(3) I-A, II-B, III-C, IV-D

(4) I-A, II-D, III-B, IV-C

Correct Answer: (2) I-A, II-C, III-D, IV-B

Solution:

- **Colloidal antimony** is used to treat Kalaazar (A).
- **Silver sol** is used in Eye lotion (C).
- **Milk of magnesia** is an antacid used for stomach disorders (D).
- **Gold sol** is used in intramuscular injections (B).

Thus, the correct match is: I-A, II-C, III-D, IV-B.

Quick Tip

Colloidal solutions have various medical applications, including treatments for diseases, lotions, and injections.

146. Identify the method of preparation of a colloidal sol from the following:

(1) Ultrafiltration

- (2) Peptisation
- (3) Dialysis
- (4) Electro-dialysis

Correct Answer: (2) Peptisation

Solution:

Step 1: Understanding Peptisation Peptisation is the process of converting a precipitate into a colloidal solution by adding a suitable electrolyte.

Step 2: Explanation - In peptisation, the electrolyte adsorbs onto the surface of the precipitate, causing it to break into smaller particles that remain suspended in the solution.

- It is widely used in the preparation of colloidal sols.

Thus, the correct answer is Peptisation.

Quick Tip

Peptisation is an important method to convert a precipitate into a colloidal dispersion by adding an electrolyte.

147. The flux used in the preparation of wrought iron from cast iron in reverberatory furnace is:

- (1) SiO_2
- (2) CaCO_3
- (3) C
- (4) NaCN

Correct Answer: (2) CaCO_3

Solution:

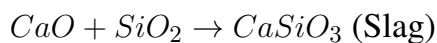
Step 1: Understanding Flux in Iron Extraction

- Flux is a substance used in metallurgy to remove impurities by forming slag.

- In wrought iron production, the impurities (SiO_2) are removed using CaCO_3 (Limestone) as a flux.

Step 2: Reaction Involved





Step 3: Explanation

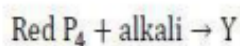
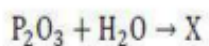
- SiO_2 is an impurity, not a flux.
- C (carbon) is used as a reducing agent.
- NaCN is not used in iron metallurgy.

Thus, the correct answer is $CaCO_3$.

Quick Tip

Calcium carbonate ($CaCO_3$) acts as a flux to remove acidic impurities such as silica (SiO_2) in metallurgy.

148.



X, Y are oxoacids of phosphorous. The number of P – OH bonds in X, Y respectively is:

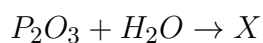
- (1) 1, 4
- (2) 4, 1
- (3) 2, 4
- (4) 1, 1

Correct Answer: (3) 2, 4

Solution:

Step 1: Identifying X and Y

- Given reactions:



- The compounds formed are:
- X is **Phosphorous acid** (H_3PO_3)
- Y is **Hypophosphorous acid** (H_3PO_2)

Step 2: Identifying P–OH bonds in each acid

- The structure of **Phosphorous acid** (H_3PO_3) contains two P–OH bonds.
- The structure of **Hypophosphorous acid** (H_3PO_2) contains one P–OH bond.

Step 3: Correcting the order

- The given reactions indicate that $X = H_3PO_3$ and $Y = H_3PO_4$ (Phosphoric acid).
- Phosphoric acid (H_3PO_4) has four P–OH bonds.
- Thus, the number of P–OH bonds in X, Y are 2, 4.

Final Answer: (2, 4)

Quick Tip

Phosphorous acid (H_3PO_3) has two P–OH bonds, whereas Phosphoric acid (H_3PO_4) has four P–OH bonds.

149. Which of the following occurs with $KMnO_4$ in neutral medium?

- (1) Oxidation of oxalate ion
- (2) Precipitation of sulfur from hydrogen sulfide
- (3) Oxidation of Fe^{2+} to Fe^{3+}
- (4) Oxidation of iodide to iodate

Correct Answer: (4) Oxidation of iodide to iodate

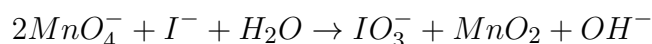
Solution:

Step 1: Understanding the Oxidizing Nature of $KMnO_4$

- In an acidic medium, $KMnO_4$ is a strong oxidizing agent.
- In a neutral medium, its oxidizing power decreases but still oxidizes some species.

Step 2: Oxidation Reactions in Neutral Medium

- $KMnO_4$ oxidizes I^- to IO_3^- in neutral medium:



- Other reactions:
- Oxidation of oxalate ion occurs in acidic medium.
- Precipitation of sulfur from H_2S occurs in acidic medium.

- Fe^{2+} to Fe^{3+} oxidation occurs in acidic medium.

Thus, the correct answer is **Oxidation of iodide to iodate**.

Quick Tip

KMnO_4 in neutral medium oxidizes iodide (I^-) to iodate (IO_3^-).

150. Cobalt (III) chloride forms a green-colored complex 'X' with NH_3 . Number of moles of AgCl formed when excess AgNO_3 solution is added to 100 mL of 1M solution of 'X' is:

- (1) 0.3
- (2) 0.2
- (3) 0.1
- (4) 1

Correct Answer: (3) 0.1

Solution:

Step 1: Identifying the Complex

- The green complex formed is



Cl_3 .

- This means that for each molecule of the complex, 3 chloride ions are available for precipitation.

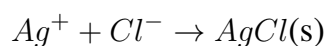
Step 2: Moles of Complex and Cl^- Ions

- Given: 100 mL of 1M solution $\Rightarrow$ Moles of complex = 0.1 moles.
- Each mole of the complex releases 3 moles of Cl^- .

$$\text{Moles of } \text{Cl}^- = 0.1 \times 3 = 0.3 \text{ moles}$$

Step 3: Reaction with AgNO_3

- AgNO_3 reacts with Cl^- to form AgCl precipitate:



- Since each Ag^+ reacts with one Cl^- , the number of moles of AgCl precipitated is 0.3 moles.

Thus, the correct answer is 0.3 moles.

Quick Tip
Each mole of $[\text{Co}(\text{NH}_3)_6]\text{Cl}_3$ releases 3 moles of Cl^- ions, which react with AgNO_3 to form AgCl .

151. The correctly matched set of the following is:

- (1) Polystyrene – copolymer – thermoplastic
- (2) Bakelite – addition polymer – thermosetting
- (3) Nylon 6 – Homopolymer – fibre
- (4) Buna-N – Homopolymer – elastomer

Correct Answer: (3) Nylon 6 – Homopolymer – fibre

Solution:

Step 1: Understanding Polymer Types

- Nylon 6 is formed by polymerization of caprolactam, making it a homopolymer.
- It is widely used as a fibre in textiles.

Step 2: Checking Other Options

- Polystyrene is a homopolymer, not a copolymer.
- Bakelite is a condensation polymer, not an addition polymer.
- Buna-N is a copolymer of butadiene and acrylonitrile, not a homopolymer.

Thus, the correct answer is Nylon 6 – Homopolymer – Fibre.

Quick Tip
Nylon 6 is a homopolymer formed by polymerization of caprolactam and is commonly used as a fibre.

152. Identify the correctly matched set from the following:

- (1) Vitamin A – Water soluble – Xerophthalmia
- (2) Vitamin B₆ – Water soluble – Scurvy
- (3) Vitamin D – Fat soluble – Rickets
- (4) Vitamin C – Fat soluble – Convulsions

Correct Answer: (3) Vitamin D – Fat soluble – Rickets

Solution:

Step 1: Classification of Vitamins

- Water-soluble vitamins: Vitamin B-complex, Vitamin C.
- Fat-soluble vitamins: Vitamins A, D, E, K.

Step 2: Matching with Diseases

- Vitamin A is fat-soluble and is related to night blindness, not xerophthalmia.
- Vitamin B₆ is water-soluble, but scurvy is caused by Vitamin C deficiency.
- Vitamin D is fat-soluble and deficiency leads to rickets.
- Vitamin C is water-soluble, not fat-soluble.

Thus, the correct answer is Vitamin D – Fat soluble – Rickets.

Quick Tip
Vitamin D is fat-soluble and its deficiency causes rickets, a bone-related disorder.

153. Given below are two statements:

- I. Cytosine and guanine are formed in equal quantities in DNA hydrolysis.
- II. Adenine and uracil are formed in equal quantities in RNA hydrolysis.

The correct answer is:

- (1) Statements I, II both are correct
- (2) Statements I, II both are incorrect
- (3) Statement I is correct but statement II is incorrect
- (4) Statement I is incorrect but statement II is correct

Correct Answer: (3) Statement I is correct but statement II is incorrect

Solution:

Step 1: Understanding DNA Base Pairing

- DNA follows Chargaff's rule, which states that the number of cytosine (C) equals the number of guanine (G), and the number of adenine (A) equals the number of thymine (T).
- Hence, in DNA hydrolysis, cytosine and guanine are produced in equal quantities.

Step 2: RNA Hydrolysis and Base Pairing

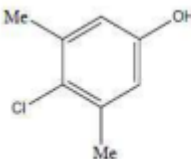
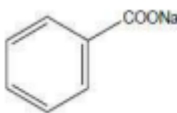
- RNA consists of adenine (A), uracil (U), cytosine (C), and guanine (G).
- Unlike DNA, RNA does not follow Chargaff's rule because it is single-stranded.
- Adenine and uracil are not necessarily in equal amounts in RNA hydrolysis.

Thus, statement I is correct, but statement II is incorrect.

Quick Tip

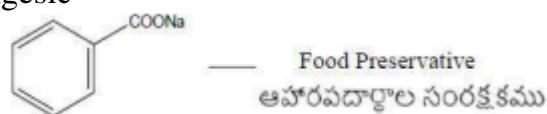
In DNA, cytosine and guanine are always found in equal quantities due to complementary base pairing, but this does not apply to adenine and uracil in RNA.

154. Identify the correctly matched pair from the following:

- (1)  — Antibiotic
యాంటీబయోటిక్
- (2)  — Food Preservative
ఆహారపదార్థాల సంరక్షకము

(3) Salt of propanoic acid – Antioxidant

(4) Veronal – Analgesic



Correct Answer:

Solution:

Step 1: Understanding Chemical Compounds

- The given structure corresponds to sodium benzoate, which is a well-known food preservative.

Step 2: Checking Other Options

- The first structure is not an antibiotic; antibiotics are typically complex organic molecules.
- Propanoic acid salts (such as sodium propionate) are used as food preservatives, not

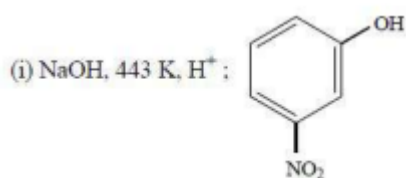
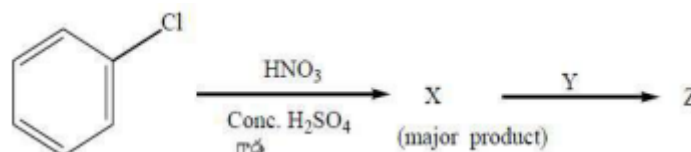
antioxidants. - Veronal is a barbiturate sedative, not an analgesic.

Thus, the correct answer is the food preservative.

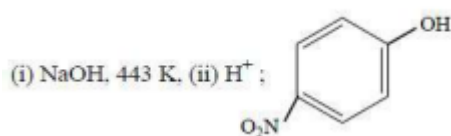
Quick Tip

Sodium benzoate is a widely used food preservative that prevents bacterial and fungal growth in acidic foods.

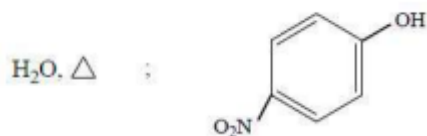
155. What are Y and Z respectively in the following reaction sequence?



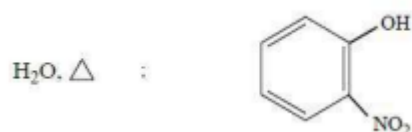
(1)



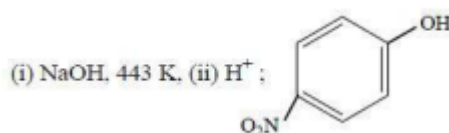
(2)



(3)



(4)



Correct Answer:

Solution:

Step 1: Nitration of Chlorobenzene

- Reaction: Chlorobenzene undergoes nitration with HNO_3/H_2SO_4 .

- The major product is p-nitrochlorobenzene due to the electron-withdrawing effect of chlorine.

Step 2: Conversion to Phenol

- Reaction: The chlorine in p-nitrochlorobenzene is replaced by OH under fused NaOH at high temperature (443K).

- The resulting compound is p-nitrophenol.

Step 3: Confirming the Correct Option

- The correct Y is p-nitrochlorobenzene.

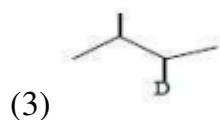
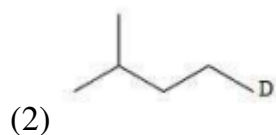
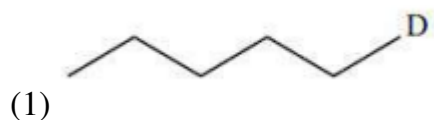
- The correct Z is p-nitrophenol.

Thus, the correct answer is the option with p-nitrophenol.

Quick Tip

In electrophilic substitution, chlorine directs the incoming nitro group to the para-position due to its electron-withdrawing inductive effect.

156. Hydrolysis of an alkyl bromide X (C_4H_9Br) follows first-order kinetics. Reaction of X with Mg in dry ether followed by treatment of D_2O gave Y. What is Y?



Correct Answer: (4) Given structure (correct)

Solution:

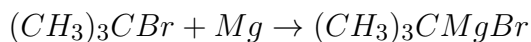
Step 1: Identifying the Type of Alkyl Bromide

- The hydrolysis follows first-order kinetics, which suggests an S_N1 mechanism.

- S_N1 reactions occur faster for tertiary $\succ$ secondary $\succ$ primary alkyl halides.
- Hence, X must be a tertiary alkyl bromide.

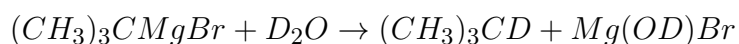
Step 2: Formation of Grignard Reagent

- When X (tert-butyl bromide) is reacted with Mg in dry ether, a Grignard reagent is formed:



Step 3: Reaction with D_2O

- Grignard reagents act as strong nucleophiles and react with D_2O to give a deuterated product:



Step 4: Identifying the Product (Y)

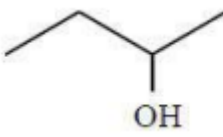
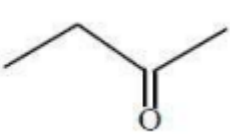
- The final product (Y) is tert-butyl deuteride.
- The correct option corresponds to the tert-butyl group with deuterium (D) replacing hydrogen.

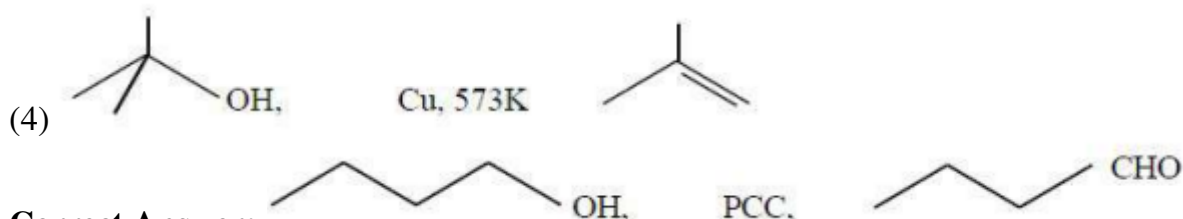
Thus, the correct answer is option (4).

Quick Tip

Grignard reagents react with D_2O to replace MgBr with D, forming a deuterated hydrocarbon.

157. An alcohol X ($C_4H_{10}O$) does not give turbidity with conc. HCl and $ZnCl_2$ at room temperature. X on reaction with reagent Y gives Z. What are X, Y, and Z respectively?

- (1)  OH, PCC,  CHO
- (2)  OH, $KMnO_4/H^+$,  CHO
- (3)  OH, CrO_3 , 



Correct Answer:

Solution:

Step 1: Identify X - The given molecular formula $C_4H_{10}O$ suggests a butanol isomer.

- Since X does not give turbidity with Lucas reagent (conc. $HCl + ZnCl_2$), it must be a primary alcohol.

- The correct structure for X is 1-Butanol ($CCCCOH$).

Step 2: Identify Y - PCC (Pyridinium chlorochromate) is a mild oxidizing agent.

- It selectively oxidizes primary alcohols to aldehydes without further oxidation to carboxylic acids.

- Therefore, Y is PCC.

Step 3: Identify Z - The oxidation of 1-Butanol ($CCCCOH$) using PCC yields butanal ($CCC=O$), an aldehyde.

- Thus, Z is Butanal.

Quick Tip

Lucas test distinguishes alcohols:

- Primary alcohols show no reaction at room temperature.
- Secondary alcohols react slowly.
- Tertiary alcohols react immediately.

158. Which of the following sets of reagents convert toluene to benzaldehyde?

A. $Cl_2|hv; H_2O, \Delta$

C. $KMnO_4|OH^-; H^+$

B. $Cl_2|Fe; H_2O$

D. $CrO_2Cl_2|CS_2; H_3O^+$

(1) B, C, D

(2) A, C

(3) *A, D*

(4) *B, D*

Correct Answer: (3) *A, D*

Solution:

Step 1: Identify the Reagents for Partial Oxidation

The conversion of toluene to benzaldehyde involves partial oxidation. The effective reagents for this reaction are:

- Chromyl chloride (CrO_2Cl_2) in CS_2 , H_3O^+ (Etard reaction)
- Chlorine (Cl_2) in the presence of light and water

Step 2: Explanation

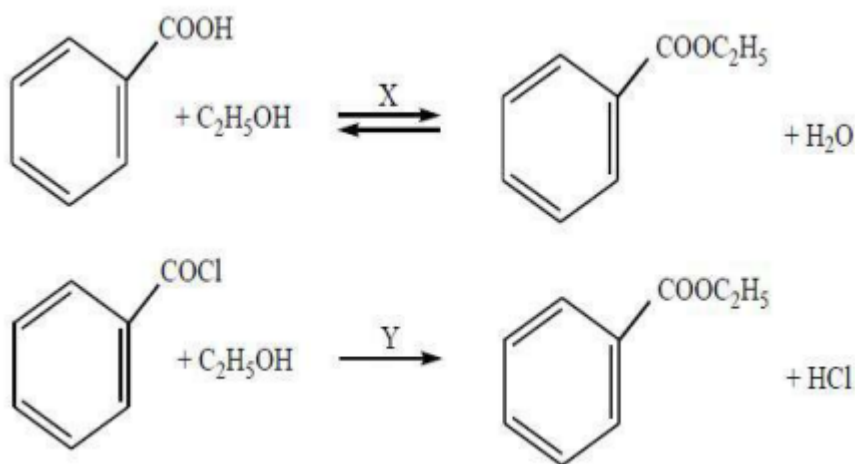
- *A* : $Cl_2/h\nu, H_2O, \Delta \rightarrow$ Effective (Partial oxidation of toluene to benzaldehyde)
- *B* : $Cl_2/Fe; H_2O \rightarrow$ Ineffective (Leads to benzoic acid)
- *C* : $KMnO_4/OH^-; H^+ \rightarrow$ Ineffective (Complete oxidation to benzoic acid)
- *D* : $CrO_2Cl_2/CS_2; H_3O^+ \rightarrow$ Effective (Etard reaction forms benzaldehyde)

Thus, the correct answer is *A, D*.

Quick Tip

To convert toluene to benzaldehyde, use selective oxidation methods such as the Etard reaction (CrO_2Cl_2) or controlled halogenation with $Cl_2/h\nu, H_2O$.

159. What are X and Y respectively in the following reactions?



(1) H^+, H^+

(2) H^+ , Pyridine

(3) Pyridine, H^+

(4) Pyridine, Pyridine

Correct Answer: (2) H^+ , Pyridine

Solution:

Step 1: Identify the Reaction Types

- The first reaction involves esterification of benzoic acid with ethanol, which requires an acid catalyst (H^+).

- The second reaction involves esterification of benzoyl chloride with ethanol, where pyridine is used as a base to neutralize HCl.

Step 2: Explanation

- $X = H^+$ (for acid-catalyzed esterification of benzoic acid)

- $Y = \text{Pyridine}$ (as a base for benzoyl chloride reaction)

Thus, the correct answer is H^+ , Pyridine.

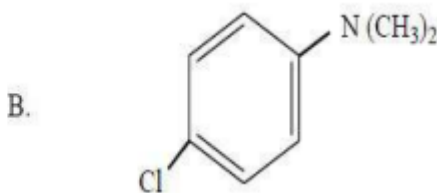
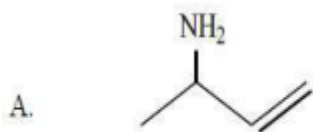
Quick Tip

For esterification reactions:

- Carboxylic acids require an acid catalyst (H^+).

- Acid chlorides require a base (e.g., pyridine) to neutralize HCl formed.

160. IUPAC names of the following compounds A and B are:



(1) A = But-3-en-2-amine, B = 4-N, N-dimethylaminochlorobenzene

(2) A = But-1-en-3-amine, B = 4-N, N-dimethylaminochlorobenzene

(3) A = But-1-en-3-amine, B = 4-Chloro-N, N-dimethylbenzenamine

(4) A = But-1-en-3-amine, B = 4-Chloro-N, N-dimethylbenzenamine

Correct Answer: (4) A = But-1-en-3-amine, B = 4-Chloro-N, N-dimethylbenzenamine

Solution:**Step 1: Naming Compound A (Amine)**

- The longest chain has 4 carbons → Butene.
- The double bond starts at C-1 → But-1-ene.
- The amine (-NH_2) is at C-3 → But-1-en-3-amine.

Step 2: Naming Compound B (Aromatic Amine)

- The benzene ring has a chloro (-Cl) group at C-4.
- The dimethylamine ($\text{-N(CH}_3)_2$) is at C-1.
- Correct IUPAC name → 4-Chloro-N, N-dimethylbenzenamine.

Thus, the correct answer is But-1-en-3-amine, 4-Chloro-N, N-dimethylbenzenamine.

Quick Tip

For naming amines, prioritize the longest chain with the double bond and place -amine as a suffix with the correct numbering. For aromatic compounds, follow substituent priority.