

Notation

$\mathbb{N}$	The set of all natural numbers $\{1, 2, 3, \dots\}$
$\mathbb{Z}$	The set of all integers
$\mathbb{Q}$	The set of all rational numbers
$\mathbb{R}$	The set of all real numbers
S_n	The group of permutations of n distinct symbols
$\mathbb{Z}_n$	$\{0, 1, 2, \dots, n-1\}$ with addition and multiplication modulo n
ϕ	empty set
A^T	Transpose of A
i	$\sqrt{-1}$
$\hat{i}, \hat{j}, \hat{k}$	unit vectors having the directions of the positive x, y and z axes of a three dimensional rectangular coordinate system
∇	$\hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z}$
I_n	Identity matrix of order n
$\ln$	logarithm with base e

SECTION – A
MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 The sequence $\{s_n\}$ of real numbers given by

$$s_n = \frac{\sin \frac{\pi}{2}}{1 \cdot 2} + \frac{\sin \frac{\pi}{2^2}}{2 \cdot 3} + \dots + \frac{\sin \frac{\pi}{2^n}}{n \cdot (n+1)}$$

is

- (A) a divergent sequence
- (B) an oscillatory sequence
- (C) not a Cauchy sequence
- (D) a Cauchy sequence

Q.2 Let P be the vector space (over $\mathbb{R}$) of all polynomials of degree ≤ 3 with real coefficients. Consider the linear transformation $T: P \rightarrow P$ defined by

$$T(a_0 + a_1x + a_2x^2 + a_3x^3) = a_3 + a_2x + a_1x^2 + a_0x^3.$$

Then the matrix representation M of T with respect to the ordered basis $\{1, x, x^2, x^3\}$ satisfies

- (A) $M^2 + I_4 = 0$
- (B) $M^2 - I_4 = 0$
- (C) $M - I_4 = 0$
- (D) $M + I_4 = 0$

Q.3 Let $f: [-1, 1] \rightarrow \mathbb{R}$ be a continuous function. Then the integral

$$\int_0^\pi x f(\sin x) dx$$

is equivalent to

- (A) $\frac{\pi}{2} \int_0^\pi f(\sin x) dx$
- (B) $\frac{\pi}{2} \int_0^\pi f(\cos x) dx$
- (C) $\pi \int_0^\pi f(\cos x) dx$
- (D) $\pi \int_0^\pi f(\sin x) dx$

Q.4 Let σ be an element of the permutation group S_5 . Then the maximum possible order of σ is

- (A) 5
- (B) 6
- (C) 10
- (D) 15

Q.5 Let f be a strictly monotonic continuous real valued function defined on $[a, b]$ such that $f(a) < a$ and $f(b) > b$. Then which one of the following is TRUE?

- (A) There exists exactly one $c \in (a, b)$ such that $f(c) = c$
- (B) There exist exactly two points $c_1, c_2 \in (a, b)$ such that $f(c_i) = c_i$, $i = 1, 2$
- (C) There exists no $c \in (a, b)$ such that $f(c) = c$
- (D) There exist infinitely many points $c \in (a, b)$ such that $f(c) = c$

Q.6 The value of $\lim_{(x, y) \rightarrow (2, -2)} \frac{\sqrt{(x-y)}-2}{x-y-4}$ is

- (A) 0 (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$

Q.7 Let $\vec{r} = (x\hat{i} + y\hat{j} + z\hat{k})$ and $r = |\vec{r}|$. If $f(r) = \ln r$ and $g(r) = \frac{1}{r}$, $r \neq 0$, satisfy $2\nabla f + h(r)\nabla g = \vec{0}$, then $h(r)$ is

- (A) r (B) $\frac{1}{r}$ (C) $2r$ (D) $\frac{2}{r}$

Q.8 The nonzero value of n for which the differential equation

$$(3xy^2 + n^2x^2y)dx + (nx^3 + 3x^2y)dy = 0, \quad x \neq 0,$$

becomes exact is

- (A) -3 (B) -2 (C) 2 (D) 3

Q.9 One of the points which lies on the solution curve of the differential equation

$$(y - x)dx + (x + y)dy = 0,$$

with the given condition $y(0) = 1$, is

- (A) $(1, -2)$ (B) $(2, -1)$ (C) $(2, 1)$ (D) $(-1, 2)$

Q.10 Let S be a closed subset of $\mathbb{R}$, T a compact subset of $\mathbb{R}$ such that $S \cap T \neq \phi$. Then $S \cap T$ is

- (A) closed but not compact
(B) not closed
(C) compact
(D) neither closed nor compact

Q. 11 – Q. 30 carry two marks each.

Q.11 Let S be the series

$$\sum_{k=1}^{\infty} \frac{1}{(2k-1)2^{(2k-1)}}$$

and T be the series

$$\sum_{k=2}^{\infty} \left(\frac{3k-4}{3k+2} \right)^{\frac{(k+1)}{3}}$$

of real numbers. Then which one of the following is TRUE?

- (A) Both the series S and T are convergent
(B) S is convergent and T is divergent
(C) S is divergent and T is convergent
(D) Both the series S and T are divergent

Q.12 Let $\{a_n\}$ be a sequence of positive real numbers satisfying

$$\frac{4}{a_{n+1}} = \frac{3}{a_n} + \frac{a_n^3}{81}, \quad n \geq 1, \quad a_1 = 1.$$

Then all the terms of the sequence lie in

- (A) $\left[\frac{1}{2}, \frac{3}{2}\right]$ (B) $[0, 1]$ (C) $[1, 2]$ (D) $[1, 3]$

Q.13 The largest eigenvalue of the matrix $\begin{bmatrix} 1 & 4 & 16 \\ 4 & 16 & 1 \\ 16 & 1 & 4 \end{bmatrix}$ is

- (A) 16 (B) 21
(C) 48 (D) 64

Q.14 The value of the integral

$$\frac{(2n)!}{2^{2n} (n!)} \int_{-1}^1 (1 - x^2)^n dx, \quad n \in \mathbb{N}$$

is

- (A) $\frac{2}{(2n+1)!}$ (B) $\frac{2n}{(2n+1)!}$
(C) $\frac{2(n!)}{2n+1}$ (D) $\frac{(n+1)!}{2n+1}$

Q.15 If the triple integral over the region bounded by the planes

$$2x + y + z = 4, \quad x = 0, \quad y = 0, \quad z = 0$$

is given by

$$\int_0^2 \int_0^{\lambda(x)} \int_0^{\mu(x,y)} dz dy dx,$$

then the function $\lambda(x) - \mu(x, y)$ is

- (A) $x + y$ (B) $x - y$ (C) x (D) y

Q.16 The surface area of the portion of the plane $y + 2z = 2$ within the cylinder $x^2 + y^2 = 3$ is

- (A) $\frac{3\sqrt{5}}{2}\pi$ (B) $\frac{5\sqrt{5}}{2}\pi$ (C) $\frac{7\sqrt{5}}{2}\pi$ (D) $\frac{9\sqrt{5}}{2}\pi$

Q.17 Let $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ be defined by

$$f(x, y) = \begin{cases} \frac{xy^2}{x+y} & \text{if } x+y \neq 0 \\ 0 & \text{if } x+y = 0 \end{cases}.$$

Then the value of $\left(\frac{\partial^2 f}{\partial x \partial y} + \frac{\partial^2 f}{\partial y \partial x}\right)$ at the point $(0, 0)$ is

- (A) 0 (B) 1 (C) 2 (D) 4

Q.18 The function $f(x, y) = 3x^2y + 4y^3 - 3x^2 - 12y^2 + 1$ has a saddle point at

- (A) $(0, 0)$ (B) $(0, 2)$ (C) $(1, 1)$ (D) $(-2, 1)$

Q.19 Consider the vector field $\vec{F} = r^\beta(y\hat{i} - x\hat{j})$, where $\beta \in \mathbb{R}$, $\vec{r} = x\hat{i} + y\hat{j}$ and $r = |\vec{r}|$. If the absolute value of the line integral $\oint_C \vec{F} \cdot d\vec{r}$ along the closed curve $C: x^2 + y^2 = a^2$ (oriented counter clockwise) is 2π , then β is

- (A) -2 (B) -1 (C) 1 (D) 2

Q.20 Let S be the surface of the cone $z = \sqrt{x^2 + y^2}$ bounded by the planes $z = 0$ and $z = 3$. Further, let C be the closed curve forming the boundary of the surface S . A vector field $\vec{F}$ is such that $\nabla \times \vec{F} = -x\hat{i} - y\hat{j}$. The absolute value of the line integral $\oint_C \vec{F} \cdot d\vec{r}$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$, is

- (A) 0 (B) 9π (C) 15π (D) 18π

Q.21 Let $y(x)$ be the solution of the differential equation

$$\frac{d}{dx}\left(x \frac{dy}{dx}\right) = x; \quad y(1) = 0, \quad \frac{dy}{dx}\bigg|_{x=1} = 0.$$

Then $y(2)$ is

- (A) $\frac{3}{4} + \frac{1}{2} \ln 2$ (B) $\frac{3}{4} - \frac{1}{2} \ln 2$
(C) $\frac{3}{4} + \ln 2$ (D) $\frac{3}{4} - \ln 2$

Q.22 The general solution of the differential equation with constant coefficients

$$\frac{d^2 y}{dx^2} + b \frac{dy}{dx} + cy = 0$$

approaches zero as $x \rightarrow \infty$, if

- (A) b is negative and c is positive
(B) b is positive and c is negative
(C) both b and c are positive
(D) both b and c are negative

- Q.23 Let $S \subset \mathbb{R}$ and ∂S denote the set of points x in $\mathbb{R}$ such that every neighbourhood of x contains some points of S as well as some points of complement of S . Further, let $\bar{S}$ denote the closure of S . Then which one of the following is FALSE?

- (A) $\partial \mathbb{Q} = \mathbb{R}$
 (B) $\partial(\mathbb{R} \setminus T) = \partial T$, $T \subset \mathbb{R}$
 (C) $\partial(T \cup V) = \partial T \cup \partial V$, $T, V \subset \mathbb{R}$, $T \cap V \neq \emptyset$
 (D) $\partial T = \bar{T} \cap (\overline{\mathbb{R} \setminus T})$, $T \subset \mathbb{R}$

- Q.24 The sum of the series

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n^2 + n - 2}$$

is

- (A) $\frac{1}{3} \ln 2 - \frac{5}{18}$ (B) $\frac{1}{3} \ln 2 - \frac{5}{6}$ (C) $\frac{2}{3} \ln 2 - \frac{5}{18}$ (D) $\frac{2}{3} \ln 2 - \frac{5}{6}$

- Q.25 Let $f(x) = \frac{1}{1+|x|} + \frac{1}{1+|x-1|}$ for all $x \in [-1, 1]$. Then which one of the following is TRUE?

- (A) Maximum value of $f(x)$ is $\frac{3}{2}$
 (B) Minimum value of $f(x)$ is $\frac{1}{3}$
 (C) Maximum of $f(x)$ occurs at $x = \frac{1}{2}$
 (D) Minimum of $f(x)$ occurs at $x = 1$

- Q.26 The matrix $M = \begin{bmatrix} \cos \alpha & \sin \alpha \\ i \sin \alpha & i \cos \alpha \end{bmatrix}$ is a unitary matrix when α is

- (A) $(2n+1)\frac{\pi}{2}$, $n \in \mathbb{Z}$ (B) $(3n+1)\frac{\pi}{3}$, $n \in \mathbb{Z}$
 (C) $(4n+1)\frac{\pi}{4}$, $n \in \mathbb{Z}$ (D) $(5n+1)\frac{\pi}{5}$, $n \in \mathbb{Z}$

- Q.27 Let $M = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & \alpha \\ 2 & -\alpha & 0 \end{bmatrix}$, $\alpha \in \mathbb{R} \setminus \{0\}$ and $\mathbf{b}$ a non-zero vector such that $M\mathbf{x} = \mathbf{b}$ for some $\mathbf{x} \in \mathbb{R}^3$. Then the value of $\mathbf{x}^T \mathbf{b}$ is

- (A) $-\alpha$ (B) α (C) 0 (D) 1

- Q.28 The number of group homomorphisms from the cyclic group $\mathbb{Z}_4$ to the cyclic group $\mathbb{Z}_7$ is

- (A) 7 (B) 3 (C) 2 (D) 1

- Q.29 In the permutation group S_n ($n \geq 5$), if H is the smallest subgroup containing all the 3-cycles, then which one of the following is TRUE?

- (A) Order of H is 2
 (B) Index of H in S_n is 2
 (C) H is abelian
 (D) $H = S_n$

Q.30 Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined as

$$f(x) = \begin{cases} x(1 + x^\alpha \sin(\ln x^2)) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Then, at $x = 0$, the function f is

- (A) continuous and differentiable when $\alpha = 0$
- (B) continuous and differentiable when $\alpha > 0$
- (C) continuous and differentiable when $-1 < \alpha < 0$
- (D) continuous and differentiable when $\alpha < -1$

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 Let $\{s_n\}$ be a sequence of positive real numbers satisfying

$$2s_{n+1} = s_n^2 + \frac{3}{4}, \quad n \geq 1.$$

If α and β are the roots of the equation $x^2 - 2x + \frac{3}{4} = 0$ and $\alpha < s_1 < \beta$, then which of the following statement(s) is(are) TRUE ?

- (A) $\{s_n\}$ is monotonically decreasing
- (B) $\{s_n\}$ is monotonically increasing
- (C) $\lim_{n \rightarrow \infty} s_n = \alpha$
- (D) $\lim_{n \rightarrow \infty} s_n = \beta$

Q.32 The value(s) of the integral

$$\int_{-\pi}^{\pi} |x| \cos nx \, dx, \quad n \geq 1$$

is (are)

- (A) 0 when n is even
- (B) 0 when n is odd
- (C) $-\frac{4}{n^2}$ when n is even
- (D) $-\frac{4}{n^2}$ when n is odd

Q.33 Let $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ be defined by

$$f(x, y) = \begin{cases} \frac{xy}{|x|} & \text{if } x \neq 0 \\ 0 & \text{elsewhere} \end{cases}.$$

Then at the point $(0, 0)$, which of the following statement(s) is(are) TRUE ?

- (A) f is not continuous
- (B) f is continuous
- (C) f is differentiable
- (D) Both first order partial derivatives of f exist

Q.34 Consider the vector field $\vec{F} = x\hat{i} + y\hat{j}$ on an open connected set $S \subset \mathbb{R}^2$. Then which of the following statement(s) is(are) TRUE ?

- (A) Divergence of $\vec{F}$ is zero on S
- (B) The line integral of $\vec{F}$ is independent of path in S
- (C) $\vec{F}$ can be expressed as a gradient of a scalar function on S
- (D) The line integral of $\vec{F}$ is zero around any piecewise smooth closed path in S

Q.35 Consider the differential equation

$$\sin 2x \frac{dy}{dx} = 2y + 2 \cos x, \quad y\left(\frac{\pi}{4}\right) = 1 - \sqrt{2}.$$

Then which of the following statement(s) is(are) TRUE?

- (A) The solution is unbounded when $x \rightarrow 0$
- (B) The solution is unbounded when $x \rightarrow \frac{\pi}{2}$
- (C) The solution is bounded when $x \rightarrow 0$
- (D) The solution is bounded when $x \rightarrow \frac{\pi}{2}$

Q.36 Which of the following statement(s) is(are) TRUE?

- (A) There exists a connected set in $\mathbb{R}$ which is not compact
- (B) Arbitrary union of closed intervals in $\mathbb{R}$ need not be compact
- (C) Arbitrary union of closed intervals in $\mathbb{R}$ is always closed
- (D) Every bounded infinite subset V of $\mathbb{R}$ has a limit point in V itself

Q.37 Let $P(x) = \left(\frac{5}{13}\right)^x + \left(\frac{12}{13}\right)^x - 1$ for all $x \in \mathbb{R}$. Then which of the following statement(s) is(are) TRUE?

- (A) The equation $P(x) = 0$ has exactly one solution in $\mathbb{R}$
- (B) $P(x)$ is strictly increasing for all $x \in \mathbb{R}$
- (C) The equation $P(x) = 0$ has exactly two solutions in $\mathbb{R}$
- (D) $P(x)$ is strictly decreasing for all $x \in \mathbb{R}$

Q.38 Let G be a finite group and $o(G)$ denotes its order. Then which of the following statement(s) is(are) TRUE?

- (A) G is abelian if $o(G) = pq$ where p and q are distinct primes
- (B) G is abelian if every non identity element of G is of order 2
- (C) G is abelian if the quotient group $\frac{G}{Z(G)}$ is cyclic, where $Z(G)$ is the center of G
- (D) G is abelian if $o(G) = p^3$, where p is prime

Q.39 Consider the set $V = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \mid \alpha x + \beta y + z = \gamma, \alpha, \beta, \gamma \in \mathbb{R} \right\}$. For which of the following choice(s) the set V becomes a two dimensional subspace of $\mathbb{R}^3$ over $\mathbb{R}$?

- (A) $\alpha = 0, \beta = 1, \gamma = 0$
- (B) $\alpha = 0, \beta = 1, \gamma = 1$
- (C) $\alpha = 1, \beta = 0, \gamma = 0$
- (D) $\alpha = 1, \beta = 1, \gamma = 0$

Q.40 Let $S = \left\{ \frac{1}{3^n} + \frac{1}{7^m} \mid n, m \in \mathbb{N} \right\}$. Then which of the following statement(s) is(are) TRUE?

- (A) S is closed
- (B) S is not open
- (C) S is connected
- (D) 0 is a limit point of S

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 Let $\{s_n\}$ be a sequence of real numbers given by

$$s_n = 2^{(-1)^n} \left(1 - \frac{1}{n} \right) \sin \frac{n\pi}{2}, \quad n \in \mathbb{N}.$$

Then the least upper bound of the sequence $\{s_n\}$ is _____

Q.42 Let $\{s_k\}$ be a sequence of real numbers, where

$$s_k = k^{\alpha/k}, \quad k \geq 1, \quad \alpha > 0.$$

Then

$$\lim_{n \rightarrow \infty} (s_1 s_2 \dots s_n)^{1/n}$$

is _____

- Q.43 Let $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3$ be a non-zero vector and $A = \frac{\mathbf{x} \mathbf{x}^T}{\mathbf{x}^T \mathbf{x}}$. Then the dimension of the vector space $\left\{ \mathbf{y} \in \mathbb{R}^3 \mid A\mathbf{y} = \mathbf{0} \right\}$ over $\mathbb{R}$ is _____

- Q.44 Let f be a real valued function defined by

$$f(x, y) = 2 \ln \left(x^2 y^2 e^{\frac{y}{x}} \right), \quad x > 0, y > 0.$$

Then the value of $x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y}$ at any point (x, y) , where $x > 0, y > 0$, is _____

- Q.45 Let $\vec{F} = \sqrt{x} \hat{i} + (x + y^3) \hat{j}$ be a vector field for all (x, y) with $x \geq 0$ and $\vec{r} = x \hat{i} + y \hat{j}$. Then the value of the line integral $\int_C \vec{F} \cdot d\vec{r}$ from $(0, 0)$ to $(1, 1)$ along the path $C: x = t^2, y = t^3, 0 \leq t \leq 1$ is _____

- Q.46 If $f: (-1, \infty) \rightarrow \mathbb{R}$ defined by $f(x) = \frac{x}{1+x}$ is expressed as

$$f(x) = \frac{2}{3} + \frac{1}{9} (x - 2) + \frac{c(x - 2)^2}{(1 + \xi)^3},$$

where ξ lies between 2 and x , then the value of c is _____

- Q.47 Let $y_1(x)$, $y_2(x)$ and $y_3(x)$ be linearly independent solutions of the differential equation

$$\frac{d^3 y}{dx^3} - 6 \frac{d^2 y}{dx^2} + 11 \frac{dy}{dx} - 6y = 0.$$

If the Wronskian $W(y_1, y_2, y_3)$ is of the form ke^{bx} for some constant k , then the value of b is _____

- Q.48 The radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-4)^n}{n(n+1)} (x+2)^{2n} \text{ is } \underline{\hspace{2cm}}$$

Q.49 Let $f: (0, \infty) \rightarrow \mathbb{R}$ be a continuous function such that

$$\int_0^x f(t) dt = -2 + \frac{x^2}{2} + 4x \sin 2x + 2 \cos 2x.$$

Then the value of $\frac{1}{\pi} f\left(\frac{\pi}{4}\right)$ is _____

Q.50 Let G be a cyclic group of order 12. Then the number of non-isomorphic subgroups of G is _____

Q. 51 – Q. 60 carry two marks each.

Q.51 The value of $\lim_{n \rightarrow \infty} \left(8n - \frac{1}{n}\right)^{\frac{(-1)^n}{n^2}}$ is equal to _____

Q.52 Let R be the region enclosed by $x^2 + 4y^2 \geq 1$ and $x^2 + y^2 \leq 1$. Then the value of

$$\iint_R |xy| dx dy \quad \text{is} \quad \underline{\hspace{2cm}}$$

Q.53 Let

$$M = \begin{bmatrix} \alpha & 1 & 1 \\ 1 & \beta & 1 \\ 1 & 1 & \gamma \end{bmatrix}, \quad \alpha\beta\gamma = 1, \quad \alpha, \beta, \gamma \in \mathbb{R} \quad \text{and} \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3.$$

Then $M\mathbf{x} = \mathbf{0}$ has infinitely many solutions if $\text{trace}(M)$ is _____

Q.54 Let C be the boundary of the region enclosed by $y = x^2$, $y = x + 2$, and $x = 0$. Then the value of the line integral

$$\oint_C (xy - y^2) dx - x^3 dy,$$

where C is traversed in the counter clockwise direction, is _____

- Q.55 Let S be the closed surface forming the boundary of the region V bounded by $x^2 + y^2 = 3$, $z = 0$, $z = 6$. A vector field $\vec{F}$ is defined over V with $\nabla \cdot \vec{F} = 2y + z + 1$. Then the value of

$$\frac{1}{\pi} \iint_S \vec{F} \cdot \hat{n} \, ds,$$

where $\hat{n}$ is the unit outward drawn normal to the surface S , is _____,

- Q.56 Let $y(x)$ be the solution of the differential equation

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0, \quad y(0) = 1, \quad \left. \frac{dy}{dx} \right|_{x=0} = -1.$$

Then $y(x)$ attains its maximum value at $x =$ _____

- Q.57 The value of the double integral

$$\int_0^\pi \int_0^x \frac{\sin y}{\pi - y} \, dy \, dx$$

is _____

- Q.58 Let H denote the group of all 2×2 invertible matrices over $\mathbb{Z}_5$ under usual matrix multiplication. Then the order of the matrix $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ in H is _____

- Q.59 Let $A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 5 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 3 & 1 \end{bmatrix}$, $N(A)$ the null space of A and $R(B)$ the range space of B . Then the dimension of $N(A) \cap R(B)$ over $\mathbb{R}$ is _____

- Q.60 The maximum value of $f(x, y) = x^2 + 2y^2$ subject to the constraint $y - x^2 + 1 = 0$ is _____

END OF THE QUESTION PAPER