Physics

Question 1.
A proton, an electron, and an alpha particle have the same energies. Their de-Broglie wavelengths will be compared as:

1. $\lambda_e > \lambda_\alpha > \lambda_p$
2. $\lambda_\alpha < \lambda_p < \lambda_e$
3. $\lambda_p < \lambda_e < \lambda_\alpha$
4. $\lambda_p > \lambda_e > \lambda_\alpha$

Correct Answer: (2)
Solution:

The de-Broglie wavelength is given by:

$$\lambda_{DB} = \frac{h}{p} = \frac{h}{\sqrt{2mK}},$$

where m is the mass, K is the kinetic energy, and p is the momentum. Since the particles have the same energy K, the de-Broglie wavelength becomes:

$$\lambda_{DB} \propto \frac{1}{\sqrt{m}}.$$

Step 1: Compare masses - Electron (m_e): Lightest particle. - Proton (m_p): Heavier than an electron. - Alpha particle (m_α): Heaviest, $m_\alpha = 4m_p$.
Step 2: Compare wavelengths Since $\lambda_{DB} \propto \frac{1}{\sqrt{m}}$, the lighter the mass, the longer the wavelength:

$$\lambda_e > \lambda_p > \lambda_\alpha.$$

Final Answer: $\lambda_\alpha < \lambda_p < \lambda_e$.

> Quick Tip
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> 1. The de-Broglie wavelength is inversely proportional to the square root of the mass for particles with the same energy. 2. Lighter particles will have longer wavelengths, and heavier particles will have shorter wavelengths. 3. When comparing, rank particles by their masses to determine the order of wavelengths. 4. For particles of the same type, remember $m_\alpha = 4m_p$, making the alpha particle the heaviest.

Question 2.
A particle moving in a straight line covers half the distance with speed 6 m/s. The other half is covered in two equal time intervals with speeds 9 m/s and 15 m/s, respectively. The average speed of the particle during the motion is:

1. 8.8 m/s

2. $10\,\text{m/s}$
3. $9.2\,\text{m/s}$
4. $8\,\text{m/s}$

Correct Answer: (4)
Solution:

Step 1: Analyze the motion - Let the total distance covered by the particle be $2S$. - For the first half of the distance (S):

$$\text{Time taken} = t_1 = \frac{S}{6}.$$

- For the second half of the distance (S), it is covered in two equal time intervals (t, t): - In the first interval (t), speed = $9\,\text{m/s}$, so:

$$S_1 = 9t \quad \Longrightarrow \quad t = \frac{S_1}{9}.$$

- In the second interval (t), speed = $15\,\text{m/s}$, so:

$$S_2 = 15t.$$

Since $S_1 + S_2 = S$, solve for t:

$$9t + 15t = S \quad \Longrightarrow \quad t = \frac{S}{24}.$$

Step 2: Total time taken The total time is:

$$\text{Total time} = t_1 + 2t = \frac{S}{6} + 2 \cdot \frac{S}{24}.$$

Simplify:

$$\text{Total time} = \frac{S}{6} + \frac{S}{12} = \frac{2S}{12} + \frac{S}{12} = \frac{3S}{12} = \frac{S}{4}.$$

Step 3: Average speed The average speed is given by:

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{2S}{\frac{S}{4}}.$$

Simplify:

$$\text{Average speed} = \frac{2S \cdot 4}{S} = 8\,\text{m/s}.$$

Final Answer: $8\,\text{m/s}$.

Quick Tip

1. For average speed calculations, always divide total distance by total time. 2. Break down motion into segments and calculate time for each segment based on given speeds and distances. 3. For equal time intervals, ensure you sum distances to find total distance covered. 4. Simplify fractions carefully to avoid errors in final calculations.

Question 3.
A plane EM wave is propagating along the x-direction. It has a wavelength of $4\,\text{mm}$. If the electric field is in the y-direction with the maximum magnitude of $60\,\text{Vm}^{-1}$, the equation for the magnetic field is:

1. $B_z = 60\sin\left[\frac{\pi}{2}\left(x - 3\times10^8 t\right)\right]\hat{k}\,\text{T}$
2. $B_z = 2\times10^{-7}\sin\left[\frac{\pi}{2}\times10^3\left(x - 3\times10^8 t\right)\right]\hat{k}\,\text{T}$
3. $B_x = 60\sin\left[\frac{\pi}{2}\left(x - 3\times10^8 t\right)\right]\hat{i}\,\text{T}$
4. $B_z = 2\times10^{-7}\sin\left[\frac{\pi}{2}\left(x - 3\times10^8 t\right)\right]\hat{k}\,\text{T}$

Correct Answer: (2)
Solution:

Step 1: Relation between electric and magnetic fields The relationship between the electric field E and magnetic field B is:

$$E = cB,$$

where $c = 3\times10^8$ m/s.
Substitute $E = 60\,\text{Vm}^{-1}$:

$$60 = 3\times10^8\cdot B \implies B = \frac{60}{3\times10^8} = 2\times10^{-7}\,\text{T}.$$

Step 2: Calculate the frequency The wavelength is given as:

$$\lambda = 4\,\text{mm} = 4\times10^{-3}\,\text{m}.$$

The wave velocity c is related to the frequency f as:

$$c = f\lambda \implies f = \frac{c}{\lambda} = \frac{3\times10^8}{4\times10^{-3}} = \frac{3}{4}\times10^{11}\,\text{Hz}.$$

Step 3: Angular frequency The angular frequency ω is given by:

$$\omega = 2\pi f = 2\pi\cdot\frac{3}{4}\times10^{11} = \frac{3\pi}{2}\times10^{11}.$$

Thus:

$$\omega = \frac{\pi}{2}\times10^3.$$

Step 4: Determine the direction of the fields - The electric field is in the y-direction ($\hat{j}$). - The wave propagates in the x-direction ($\hat{i}$). - The magnetic field must be perpendicular to both $\hat{i}$ and $\hat{j}$, i.e., in the z-direction ($\hat{k}$).
Step 5: Equation of the magnetic field The magnetic field B_z is:

$$B_z = 2\times10^{-7}\sin\left[\frac{\pi}{2}\times10^3\left(x - 3\times10^8 t\right)\right]\hat{k}.$$

Final Answer: $B_z = 2\times10^{-7}\sin\left[\frac{\pi}{2}\times10^3\left(x - 3\times10^8 t\right)\right]\hat{k}\,\text{kT}$.

💡 Quick Tip

1. Use $E = cB$ to relate electric and magnetic fields. 2. The angular frequency ω is calculated as $2\pi f$, and $f = c/\lambda$. 3. Identify the propagation direction of the wave and ensure the electric and magnetic fields are perpendicular to it and to each other. 4. Carefully substitute values into the wave equation for the magnetic field.

Question 4.
Given below are two statements: **Statement (I):** When an object is placed at the centre of curvature of a concave lens, the image is formed at the centre of curvature of the lens on the other side.
Statement (II): Concave lens always forms a virtual and erect image.
In the light of the above statements, choose the correct answer from the options given below:

1. Statement I is false but Statement II is true.
2. Both Statement I and Statement II are false.
3. Statement I is true but Statement II is false.
4. Both Statement I and Statement II are true.

Correct Answer: (1)
Solution:

Statement I: A concave lens diverges light and does not have a centre of curvature for image formation in the context of lens theory. The centre of curvature is a concept applicable to mirrors, not lenses. Hence, Statement I is **false**.
Statement II: A concave lens always forms a virtual, erect, and diminished image irrespective of the position of the object. This is a fundamental property of concave lenses. Hence, Statement II is **true**.
Final Answer: (1) Statement I is false but Statement II is true.

💡 Quick Tip

1. Remember the fundamental difference between lenses and mirrors regarding the center of curvature. 2. A concave lens always forms a virtual, erect, and diminished image. 3. Analyze statements logically and compare them with established optical properties before making a conclusion.

Question 5.
A light-emitting diode (LED) is fabricated using GaAs semiconducting material whose band gap is $1.42\,\text{eV}$. The wavelength of light emitted from the LED is:

1. $650\,\text{nm}$
2. $1243\,\text{nm}$

3. $875\,\text{nm}$
4. $1400\,\text{nm}$

Correct Answer: (3)
Solution:

The wavelength of light emitted from an LED is related to its energy gap by the formula:

$$\lambda = \frac{1240}{E_g},$$

where:

- λ is the wavelength in nanometers (nm),
- E_g is the energy gap in electron volts (eV).

Substitute $E_g = 1.42\,\text{eV}$:

$$\lambda = \frac{1240}{1.42}.$$

Perform the calculation:

$$\lambda = 875\,\text{nm (approximately)}.$$

Final Answer: $875\,\text{nm}$.

Quick Tip

1. Use the formula $\lambda = \frac{1240}{E_g}$ to calculate the wavelength of light from the energy gap of a semiconductor. 2. Ensure the energy gap is in electron volts (eV) and the wavelength is calculated in nanometers (nm). 3. For LEDs, smaller energy gaps correspond to longer wavelengths (infrared), while larger gaps correspond to shorter wavelengths (visible/UV).

Question 6.
A sphere of relative density σ and diameter D has a concentric cavity of diameter d. The ratio of $\frac{D}{d}$, if it just floats on water in a tank, is:

1. $\left(\frac{\sigma}{\sigma-1}\right)^{\frac{1}{3}}$
2. $\left(\frac{\sigma+1}{\sigma-1}\right)^{\frac{1}{3}}$
3. $\left(\frac{\sigma-1}{\sigma}\right)^{\frac{1}{3}}$
4. $\left(\frac{\sigma-2}{\sigma+2}\right)^{\frac{1}{3}}$

Correct Answer: (1)
Solution:

Step 1: Weight of the sphere The weight of the sphere w is given by:

$$w = \frac{4}{3}\pi \left(\frac{D^3 - d^3}{8} \right) \sigma g,$$

where: - $D^3 - d^3$ accounts for the volume of the sphere minus the cavity, - σ is the relative density, - g is the acceleration due to gravity.

Step 2: Buoyant force The buoyant force F_b is given by:

$$F_b = \frac{4}{3}\pi \left(\frac{D^3}{8} \right) g,$$

where $\frac{D^3}{8}$ is the volume of displaced water.

Step 3: Equilibrium condition For the sphere to just float, the weight equals the buoyant force:

$$w = F_b.$$

Substitute expressions for w and F_b:

$$\frac{4}{3}\pi \left(\frac{D^3 - d^3}{8} \right) \sigma g = \frac{4}{3}\pi \left(\frac{D^3}{8} \right) g.$$

Cancel common terms:

$$\left(D^3 - d^3 \right) \sigma = D^3.$$

Simplify:

$$D^3 - d^3 = \frac{D^3}{\sigma}.$$

Step 4: Solve for $\frac{d}{D}$ Divide through by D^3:

$$1 - \frac{d^3}{D^3} = \frac{1}{\sigma}.$$

Rearrange:

$$\frac{d^3}{D^3} = 1 - \frac{1}{\sigma}.$$

Take the cube root:

$$\frac{d}{D} = \left(1 - \frac{1}{\sigma} \right)^{\frac{1}{3}}.$$

Invert to find $\frac{D}{d}$:

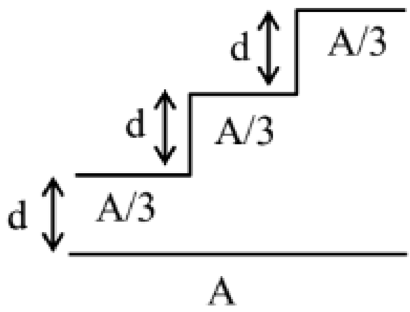
$$\frac{D}{d} = \left(\frac{\sigma}{\sigma - 1} \right)^{\frac{1}{3}}.$$

Final Answer: $\left(\frac{\sigma}{\sigma-1} \right)^{\frac{1}{3}}$.

Quick Tip

1. Use the principle of flotation: Weight of the object = Buoyant force. 2. Substitute expressions for the weight and buoyant force carefully. 3. Simplify equations step-by-step to isolate terms like $\frac{D}{d}$. 4. Take cube roots when dealing with volume ratios to find diameter ratios.

Question 7.
A capacitor is made of a flat plate of area A and a second plate having a stair-like structure as shown in the figure. If the area of each stair is $\frac{A}{3}$ and the height is d, the capacitance of the arrangement is:



1. $\frac{11\epsilon_0 A}{18d}$
2. $\frac{13\epsilon_0 A}{17d}$
3. $\frac{11\epsilon_0 A}{20d}$
4. $\frac{18\epsilon_0 A}{11d}$

Correct Answer: (1)
Solution:

Step 1: Capacitor arrangement The given system consists of three capacitors connected in parallel, each having:

$$\text{Area of overlap} = \frac{A}{3}.$$

The distances between the plates are $d, 2d,$ and $3d$, respectively.
Step 2: Capacitance of each section The capacitance of a parallel plate capacitor is given by:

$$C = \frac{\epsilon_0 A}{d}.$$

For the three sections: 1. $C_1 = \frac{\epsilon_0 \frac{A}{3}}{d} = \frac{\epsilon_0 A}{3d}$, 2. $C_2 = \frac{\epsilon_0 \frac{A}{3}}{2d} = \frac{\epsilon_0 A}{6d}$, 3. $C_3 = \frac{\epsilon_0 \frac{A}{3}}{3d} = \frac{\epsilon_0 A}{9d}$.
Step 3: Total capacitance Since the capacitors are in parallel, the equivalent capacitance is:

$$C_{\text{eq}} = C_1 + C_2 + C_3.$$

Substitute the values:

$$C_{\text{eq}} = \frac{\epsilon_0 A}{3d} + \frac{\epsilon_0 A}{6d} + \frac{\epsilon_0 A}{9d}.$$

Take the common denominator:

$$C_{\text{eq}} = \frac{\epsilon_0 A}{d}\left(\frac{1}{3} + \frac{1}{6} + \frac{1}{9}\right).$$

Simplify the fraction:

$$\frac{1}{3} + \frac{1}{6} + \frac{1}{9} = \frac{6+3+2}{18} = \frac{11}{18}.$$

Thus:

$$C_{\text{eq}} = \frac{\epsilon_0 A}{d} \cdot \frac{11}{18} = \frac{11\epsilon_0 A}{18d}.$$

Final Answer: $\frac{11\epsilon_0 A}{18d}$.

Quick Tip

1. For capacitors in parallel, the total capacitance is the sum of individual capacitances. 2. Use the formula $C = \frac{\epsilon_0 A}{d}$ for each capacitor and consider the effective area and separation. 3. Simplify fractions carefully when combining capacitances to avoid calculation errors. 4. Visualize the setup clearly to correctly identify parallel or series arrangements.

Question 8.
A light, unstretchable string passing over a smooth light pulley connects two blocks of masses m_1 and m_2. If the acceleration of the system is $\frac{g}{8}$, then the ratio of the masses $\frac{m_2}{m_1}$ is:

1. $9 : 7$
2. $4 : 3$
3. $5 : 3$
4. $8 : 1$

Correct Answer: (1)
Solution:

Step 1: Equation for acceleration The acceleration of the system is given by:

$$a_{\text{sys}} = \frac{(m_2 - m_1)}{m_1 + m_2} \cdot g.$$

Substitute $a_{\text{sys}} = \frac{g}{8}$:

$$\frac{(m_2 - m_1)}{m_1 + m_2} \cdot g = \frac{g}{8}.$$

Cancel g from both sides:

$$\frac{(m_2 - m_1)}{m_1 + m_2} = \frac{1}{8}.$$

Step 2: Solve for $\frac{m_2}{m_1}$ Rearrange the equation:

$$8(m_2 - m_1) = m_1 + m_2.$$

Simplify:

$$8m_2 - 8m_1 = m_1 + m_2.$$

Combine like terms:

$$8m_2 - m_2 = 8m_1 + m_1.$$

$$7m_2 = 9m_1.$$

Take the ratio:

$$\frac{m_2}{m_1} = \frac{9}{7}.$$

Final Answer: $\frac{m_2}{m_1} = 9 : 7.$

> Quick Tip
>
> 1. For pulley systems, use the equation $a_{\text{sys}} = \frac{(m_2 - m_1)}{m_1 + m_2} g$ to relate acceleration and masses. 2. Substitute the given acceleration and simplify step-by-step to isolate the mass ratio. 3. Check for proper cancellation of terms like g when working with ratios.

Question 9.
The dimensional formula of latent heat is:

1. $[M^0 LT^{-2}]$
2. $[MLT^{-2}]$
3. $[M^0 L^2 T^{-2}]$
4. $[ML^2 T^2]$

Correct Answer: (3)
Solution:

Latent heat is the energy absorbed or released during a phase change per unit mass. Hence, it is specific energy:

$$\text{Latent Heat} = \frac{\text{Energy}}{\text{Mass}}.$$

The dimensional formula of energy is:

$$\text{Energy} = [ML^2 T^{-2}].$$

Divide by mass:

$$\text{Latent Heat} = \frac{[ML^2 T^{-2}]}{[M]} = [M^0 L^2 T^{-2}].$$

Final Answer: $[M^0 L^2 T^{-2}]$.

Quick Tip

1. Remember that latent heat is energy per unit mass: $L = \frac{\text{Energy}}{\text{Mass}}$. 2. The dimensional formula of energy is $[ML^2T^{-2}]$, and dividing it by mass $[M]$ gives $[M^0L^2T^{-2}]$. 3. Any quantity per unit mass typically removes the M-dimension, leaving M^0. 4. Always verify the dimensional consistency by checking the physical relationship between quantities. 5. Familiarize yourself with common dimensional formulas for energy, work, force, and power for quick derivations.

Question 10.
The volume of an ideal gas ($\gamma = 1.5$) is changed adiabatically from 5 litres to 4 litres. The ratio of initial pressure to final pressure is:

1. $\frac{4}{5}$
2. $\frac{16}{25}$
3. $\frac{8}{5\sqrt{5}}$
4. $\frac{2}{\sqrt{5}}$

Correct Answer: (3)
Solution:

For an adiabatic process, the relation between pressure and volume is:

$$P_iV_i^\gamma = P_fV_f^\gamma,$$

where $\gamma = 1.5$.
Substitute the given volumes ($V_i = 5\,\text{litres}, V_f = 4\,\text{litres}$):

$$P_i(5)^{1.5} = P_f(4)^{1.5}.$$

Rearranging for $\frac{P_i}{P_f}$:

$$\frac{P_i}{P_f} = \frac{(4)^{1.5}}{(5)^{1.5}}.$$

Simplify:

$$\frac{P_i}{P_f} = \left(\frac{4}{5}\right)^{1.5}.$$

Write 1.5 as $\frac{3}{2}$:

$$\frac{P_i}{P_f} = \left(\frac{4}{5}\right)^{\frac{3}{2}} = \left(\frac{4}{5}\right)^{1} \cdot \left(\frac{4}{5}\right)^{\frac{1}{2}}.$$

Simplify each term:

$$\frac{P_i}{P_f} = \frac{4}{5} \cdot \sqrt{\frac{4}{5}} = \frac{4}{5} \cdot \frac{2}{\sqrt{5}} = \frac{8}{5\sqrt{5}}.$$

Final Answer: $\frac{8}{5\sqrt{5}}$.

> 💡 Quick Tip
>
> 1. Use the adiabatic relation $P_iV_i^\gamma = P_fV_f^\gamma$ to connect initial and final states. 2. Convert powers like 1.5 into $\frac{3}{2}$ for easier simplification. 3. Express the ratio $\left(\frac{V_f}{V_i}\right)^\gamma$ as $\left(\frac{4}{5}\right)^{1.5}$ and split into $\left(\frac{4}{5}\right)^1 \cdot \left(\frac{4}{5}\right)^{\frac{1}{2}}$ for clarity. 4. Always simplify square roots and fractions step by step to avoid errors in final calculations. 5. Keep track of significant digits, especially when expressing results in terms of surds.

Question 11.
The energy equivalent of 1 g of substance is:

1. 11.2×10^{24} MeV
2. 5.6×10^{12} MeV
3. 5.6 eV
4. 5.6×10^{26} MeV

Correct Answer: (4)
Solution:

The energy equivalent of a mass is given by Einstein's equation:

$$E = mc^2,$$

where:

- $m = 1\,\text{g} = 10^{-3}\,\text{kg}$,
- $c = 3 \times 10^8$ m/s.

Step 1: Substitute values into $E = mc^2$

$$E = (10^{-3}) \cdot (3 \times 10^8)^2\,\text{J}.$$

Simplify:

$$E = (10^{-3}) \cdot (9 \times 10^{16})\,\text{J}.$$

$$E = 9 \times 10^{13}\,\text{J}.$$

Step 2: Convert energy to electron volts (eV) Using the conversion $1\,\text{J} = 6.241 \times 10^{18}$ eV:

$$E = (9 \times 10^{13}) \cdot (6.241 \times 10^{18})\,\text{eV}.$$

$$E = 56.169 \times 10^{31}\,\text{eV}.$$

Convert to MeV ($1\,\text{MeV} = 10^6\,\text{eV}$):

$$E = 56.169 \times 10^{25}\,\text{MeV}.$$

Approximate:

$$E \approx 5.6 \times 10^{26}\,\text{MeV}.$$

Final Answer: $5.6 \times 10^{26}\,\text{MeV}$.

> Quick Tip
>
> 1. Always use Einstein's relation $E = mc^2$ for mass-energy equivalence. 2. Convert mass into SI units (kg) before substitution. 3. Use the conversion factor $1\,\text{J} = 6.241 \times 10^{18}\,\text{eV}$ for energy. 4. Remember $1\,\text{MeV} = 10^6\,\text{eV}$ to simplify units. 5. Perform calculations step-by-step to avoid errors with large powers of 10.

Question 12.
An astronaut takes a ball of mass m from Earth to space. He throws the ball into a circular orbit about Earth at an altitude of $318.5\,\text{km}$. From Earth's surface to the orbit, the change in total mechanical energy of the ball is $x\frac{GM_em}{21R_e}$. The value of x is:

1. 11
2. 9
3. 12
4. 10

Correct Answer: (1)
Solution:

Step 1: Data and formula for mechanical energy The total mechanical energy at the Earth's surface is:

$$T.E_i = -\frac{GM_em}{R_e},$$

where:

- $R_e = 6370\,\text{km}$ (radius of Earth),
- G is the gravitational constant,
- M_e is the mass of Earth.

The altitude of the orbit is given as $h = 318.5\,\text{km}$. Approximate:

$$h \approx \frac{R_e}{20}.$$

The total mechanical energy in the orbit is:

$$T.E_f = -\frac{GM_em}{2(R_e + h)}.$$

Step 2: Substitute $h \approx \frac{R_e}{20}$

$$T.E_f = -\frac{GM_e m}{2\left(R_e + \frac{R_e}{20}\right)} = -\frac{GM_e m}{2\left(\frac{21R_e}{20}\right)}.$$

Simplify:

$$T.E_f = -\frac{10GM_e m}{21R_e}.$$

Step 3: Change in mechanical energy The change in total mechanical energy is:

$$\Delta E = T.E_f - T.E_i.$$

Substitute:

$$\Delta E = \left(-\frac{10GM_e m}{21R_e}\right) - \left(-\frac{GM_e m}{R_e}\right).$$

Simplify:

$$\Delta E = -\frac{10GM_e m}{21R_e} + \frac{21GM_e m}{21R_e}.$$

$$\Delta E = \frac{11GM_e m}{21R_e}.$$

Thus, $x = 11$.
Final Answer: $x = 11$.

Quick Tip

1. Use the formula for total mechanical energy: $T.E = -\frac{GM_e m}{2r}$ for an orbit and $T.E = -\frac{GM_e m}{R_e}$ at the surface. 2. When approximating small altitudes, express h as a fraction of R_e for easier calculations. 3. Always compute the change in energy as $T.E_f - T.E_i$, and simplify step by step. 4. Keep track of signs to ensure proper subtraction between energy terms.

Question 13.
Given below are two statements:

- **Statement I:** When currents vary with time, Newton's third law is valid only if momentum carried by the electromagnetic field is taken into account.
- **Statement II:** Ampere's circuital law does not depend on Biot-Savart's law.

In the light of the above statements, choose the correct answer from the options given below:

1. Both Statement I and Statement II are false.
2. Statement I is true but Statement II is false.
3. Statement I is false but Statement II is true.
4. Both Statement I and Statement II are true.

Correct Answer: (2)
Solution:

Explanation: 1. Statement I: Newton's third law states that for every action, there is an equal and opposite reaction. When currents vary with time, the electromagnetic fields also carry momentum. For Newton's third law to hold in such cases, this momentum must be included in the analysis. Hence, Statement I is **true**.
2. *tatement II: Ampere's circuital law is an independent law that relates the magnetic field around a closed loop to the electric current passing through the loop. It does not depend on Biot-Savart's law, which describes the magnetic field generated by a current element. Hence, Statement II is **false**.
Conclusion: Statement I is true but Statement II is false.
Final Answer: (2)

Quick Tip

1. Newton's third law in electromagnetic systems must consider the momentum carried by fields during time-varying currents. 2. Ampere's circuital law is fundamental and does not rely on Biot-Savart's law; they are independent concepts. 3. Carefully analyze the dependency of one law on another to avoid confusion in theoretical physics problems.

Question 14.
A particle of mass m moves on a straight line with its velocity increasing with distance according to the equation $v = \alpha\sqrt{x}$, where α is a constant. The total work done by all the forces applied on the particle during its displacement from $x = 0$ to $x = d$, will be:

1. $\frac{m}{2\alpha^2 d}$
2. $\frac{md}{2\alpha^2}$
3. $\frac{m\alpha^2 d}{2}$
4. $2m\alpha^2 d$

Correct Answer: (3)
Solution:

Step 1: Velocity equation The velocity of the particle is given as:

$$v = \alpha\sqrt{x}.$$

At $x = 0$, the velocity is:

$$v = 0.$$

At $x = d$, the velocity becomes:

$$v = \alpha\sqrt{d}.$$

Step 2: Work-Energy Theorem The work done by all forces is equal to the change in kinetic energy:

$$W.D = K_f - K_i,$$

where:

$$K = \frac{1}{2}mv^2.$$

Substitute the velocities:

$$W.D = \frac{1}{2}m(\alpha\sqrt{d})^2 - \frac{1}{2}m(0)^2.$$

Simplify:

$$W.D = \frac{1}{2}m(\alpha^2 d) - 0.$$

$$W.D = \frac{m\alpha^2 d}{2}.$$

Final Answer: $\frac{m\alpha^2 d}{2}$.

> Quick Tip
>
> 1. Use the Work-Energy Theorem: Work done equals the change in kinetic energy. 2. Ensure the velocity function is correctly substituted into the kinetic energy formula. 3. Remember to evaluate initial and final velocities to compute the change in energy accurately. 4. Simplify expressions step-by-step to avoid errors in squaring terms or constants.

Question 15.
A galvanometer has a coil of resistance $200\,\Omega$ with a full-scale deflection at $20\,\mu$A. The value of resistance to be added to use it as an ammeter of range $0{-}20$ mA is:

1. $0.40\,\Omega$
2. $0.20\,\Omega$
3. $0.50\,\Omega$
4. $0.10\,\Omega$

Correct Answer: (2)
Solution:

Step 1: Formula for shunt resistance The shunt resistance R_s is given by:

$$R_s = \frac{I_g R_g}{I - I_g},$$

where:

- $I_g = 20\,\mu\text{A} = 20 \times 10^{-6}$ A (full-scale deflection current),

- $R_g = 200\,\Omega$ (resistance of the galvanometer),
- $I = 20\,\text{mA} = 20 \times 10^{-3}\,\text{A}$ (ammeter range).

Step 2: Substitute the values

$$R_s = \frac{(20 \times 10^{-6}) \cdot 200}{(20 \times 10^{-3}) - (20 \times 10^{-6})}.$$

Simplify the numerator:

$$(20 \times 10^{-6}) \cdot 200 = 4 \times 10^{-3}.$$

Simplify the denominator:

$$(20 \times 10^{-3}) - (20 \times 10^{-6}) = 19.98 \times 10^{-3}.$$

$$R_s = \frac{4 \times 10^{-3}}{19.98 \times 10^{-3}} \approx 0.20\,\Omega.$$

Final Answer: $0.20\,\Omega$.

> Quick Tip
>
> 1. Use the formula for shunt resistance $R_s = \frac{I_g R_g}{I - I_g}$ for converting a galvanometer to an ammeter. 2. Convert all currents to amperes before substitution. 3. Carefully handle small differences like $I - I_g$ to avoid calculation errors. 4. Ensure units of R_s are consistent with the resistance of the galvanometer.

Question 16.

A heavy iron bar, of weight W, is having its one end on the ground and the other on the shoulder of a person. The bar makes an angle θ with the horizontal. The weight experienced by the person is:

1. $\frac{W}{2}$
2. W
3. $W \cos\theta$
4. $W \sin\theta$

Correct Answer: (1)
Solution:

Step 1: Concept of center of mass The weight of the bar is uniformly distributed. Since the bar is uniform, the center of mass lies at the midpoint of the bar.
The total weight W of the bar is supported by two points:

- One end is on the ground.
- The other end is on the shoulder of the person.

Step 2: Distribution of weight In a symmetric situation like this, the total weight is evenly distributed between the two points of contact.

$$\text{Weight experienced by the person} = \frac{W}{2}.$$

Final Answer: $\frac{W}{2}$.

Quick Tip

1. For uniform objects, the weight is equally distributed between the two points of support. 2. The angle θ with the horizontal does not affect the division of weight in this scenario. 3. Always consider the center of mass when dealing with distributed forces. 4. Symmetry simplifies the problem, dividing the total weight equally.

Question 17.

One main scale division of a vernier caliper is equal to m units. If n^{th} division of the main scale coincides with $(n+1)^{th}$ division of the vernier scale, the least count of the vernier caliper is:

1. $\frac{n}{n+1}$
2. $\frac{m}{n+1}$
3. $\frac{1}{n+1}$
4. $\frac{m}{n(n+1)}$

Correct Answer: (2)
Solution:

Step 1: Relationship between main scale and vernier scale Given that:

$$n\,\text{MSD} = (n+1)\,\text{VSD}.$$

From this:

$$1\,\text{VSD} = \frac{n}{n+1}\,\text{MSD}.$$

Step 2: Least count formula The least count (L.C.) of a vernier caliper is given by:

$$\text{L.C.} = 1\,\text{MSD} - 1\,\text{VSD}.$$

Substitute $1\,\text{VSD}$ from Step 1:

$$\text{L.C.} = m - m\left(\frac{n}{n+1}\right).$$

Simplify:

$$\text{L.C.} = m\left[1 - \frac{n}{n+1}\right].$$

$$\text{L.C.} = m\left(\frac{n+1-n}{n+1}\right).$$

$$\text{L.C.} = \frac{m}{n+1}.$$

Final Answer: $\frac{m}{n+1}$.

Quick Tip

1. The least count of a vernier caliper is the difference between one main scale division and one vernier scale division. 2. Always express 1 VSD in terms of MSD for simplification. 3. Carefully simplify fractional terms to avoid algebraic errors. 4. For problems involving scales, clarity in understanding the relationship between divisions is key.

Question 18.
A bulb and a capacitor are connected in series across an AC supply. A dielectric is then placed between the plates of the capacitor. The glow of the bulb:

1. increases
2. remains same
3. becomes zero
4. decreases

Correct Answer: (1)
Solution:

Step 1: Understanding the circuit The capacitor is in series with the bulb in an AC circuit. The impedance Z of the capacitor is given by:

$$Z_C = \frac{1}{\omega C},$$

where:

- ω is the angular frequency of the AC supply,
- C is the capacitance of the capacitor.

Step 2: Effect of placing a dielectric Placing a dielectric between the plates of the capacitor increases the capacitance C, as:

$$C' = \kappa C,$$

where $\kappa > 1$ is the dielectric constant.
Step 3: Impedance of the capacitor Since $Z_C \propto \frac{1}{C}$, increasing C reduces the capacitive impedance Z_C.

Step 4: Impact on the bulb The total impedance of the circuit decreases, leading to an increase in the current through the circuit. As the current increases, the glow of the bulb increases.

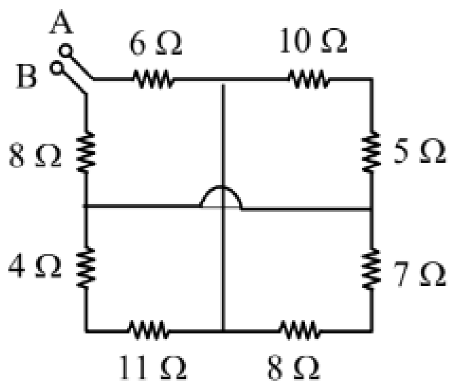
Final Answer: The glow of the bulb **increases**.

Quick Tip

1. In an AC circuit, the impedance of a capacitor decreases when the capacitance increases. 2. Placing a dielectric between the plates increases the capacitance. 3. A decrease in total impedance increases the current, enhancing the glow of the bulb. 4. Always analyze the effect of circuit changes on impedance and current flow.

Question 19.

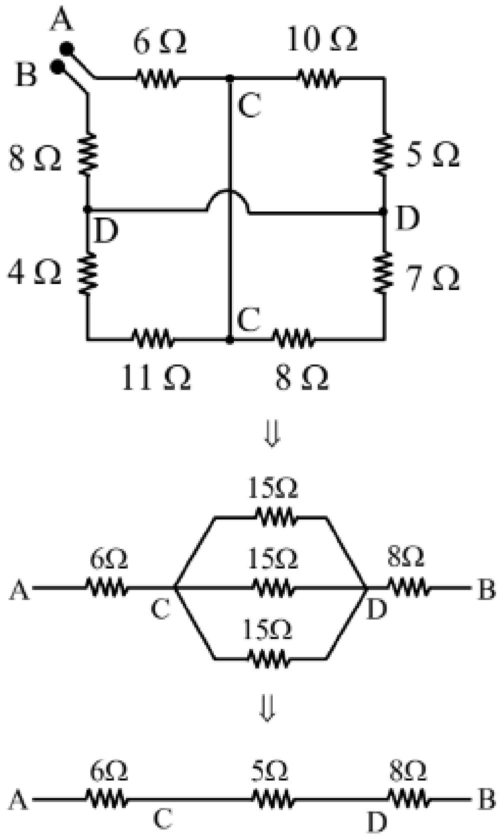
The equivalent resistance between A and B is:



1. $18\,\Omega$
2. $25\,\Omega$
3. $27\,\Omega$
4. $19\,\Omega$

Correct Answer: (4)

Solution:



$\Rightarrow R_{eq} = 6\Omega + 5\Omega + 8\Omega = 19\Omega$

Final Answer: $19\,\Omega$.

> 💡 Quick Tip
>
> 1. Simplify complex resistor networks step-by-step by reducing parallel and series connections. 2. Use the formulas:
>
> $$R_{\text{parallel}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}, \quad R_{\text{series}} = R_1 + R_2.$$
>
> 3. Keep track of intermediate results to avoid mistakes. 4. Verify results by re-checking combinations of resistors.

Question 20.
A sample of 1 mole gas at temperature T is adiabatically expanded to double its volume. If the adiabatic constant for the gas is $\gamma = \frac{3}{2}$, then the work done by the gas in the process is:

1. $RT\left[2 - \sqrt{2}\right]$
2. $\frac{R}{T}\left[2 - \sqrt{2}\right]$

3. $RT\left[2+\sqrt{2}\right]$
4. $\frac{T}{R}\left[2+\sqrt{2}\right]$

Correct Answer: (1)
Solution:

Step 1: Adiabatic condition The adiabatic condition is given by:

$$TV^{\gamma-1} = \text{constant.}$$

For the initial and final states:

$$T(V)^{\frac{3}{2}-1} = T_f(2V)^{\frac{3}{2}-1}.$$

Simplify:

$$TV^{\frac{1}{2}} = T_f(2V)^{\frac{1}{2}},$$

$$T\sqrt{V} = T_f\sqrt{2V}.$$

Cancel $\sqrt{V}$:

$$T = T_f\sqrt{2}.$$

Solve for T_f:

$$T_f = \frac{T}{\sqrt{2}}.$$

Step 2: Work done in adiabatic expansion The work done in an adiabatic process is given by:

$$W.D. = \frac{nR}{1-\gamma}\left[T_f - T\right].$$

Substitute $T_f = \frac{T}{\sqrt{2}}$, $\gamma = \frac{3}{2}$, and $n = 1$:

$$W.D. = \frac{R}{1-\frac{3}{2}}\left[\frac{T}{\sqrt{2}} - T\right].$$

Simplify:

$$W.D. = \frac{R}{-\frac{1}{2}}\left[\frac{T}{\sqrt{2}} - T\right],$$

$$W.D. = -2R\left[\frac{T}{\sqrt{2}} - T\right].$$

Factorize:

$$W.D. = 2RT\left[1 - \frac{1}{\sqrt{2}}\right].$$

Simplify further:

$$W.D. = RT\left[2 - \sqrt{2}\right].$$

Final Answer: $\mathbf{RT\left[2-\sqrt{2}\right]}$.

Quick Tip

1. For adiabatic processes, use $TV^{\gamma-1} =$ constant to relate initial and final states. 2. The work done in an adiabatic process is:

$$W = \frac{nR}{1-\gamma}\left(T_f - T_i\right).$$

3. Simplify step-by-step and substitute values carefully. 4. For $\gamma = \frac{3}{2}$, ensure proper handling of fractions during calculations.

Question 21.
If $\vec{a}$ and $\vec{b}$ make an angle $\cos^{-1}\left(\frac{5}{9}\right)$ with each other, then $|\vec{a}+\vec{b}| = \sqrt{2}|\vec{a}-\vec{b}|$ for $|\vec{a}| = n|\vec{b}|$. The integer value of n is:
Correct Answer: (3)
Solution:

$$\cos\theta = \frac{5}{9}$$

$$\frac{\vec{a}\cdot\vec{b}}{ab} = \frac{5}{9} \tag{1}$$

Using $|\vec{a}+\vec{b}|^2 = \sqrt{2}|\vec{a}-\vec{b}|^2$:

$$a^2 + b^2 + 2a\cdot b = 2(a^2 + b^2 - 2a\cdot b)$$

Simplify:

$$a^2 + b^2 + 2a\cdot b = 2a^2 + 2b^2 - 4a\cdot b$$

$$6(\vec{a}\cdot\vec{b}) = a^2 + b^2 \tag{2}$$

Substitute $\vec{a}\cdot\vec{b} = ab\cdot\frac{5}{9}$ from (1):

$$6\cdot\frac{5}{9}ab = a^2 + b^2$$

$$\frac{10}{3}ab = a^2 + b^2$$

Assume $a = nb$:

$$\frac{10}{3}nb^2 = n^2b^2 + b^2$$

Divide through by b^2:

$$\frac{10}{3}n = n^2 + 1$$

Rearrange:

$$3n^2 - 10n + 3 = 0$$

Solve using the quadratic formula:

$$n = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(3)(3)}}{2(3)}$$

$$n = \frac{10 \pm \sqrt{100 - 36}}{6}$$
$$n = \frac{10 \pm \sqrt{64}}{6}$$
$$n = \frac{10 \pm 8}{6}$$
$$n = \frac{18}{6} = 3 \quad \text{(only positive integer value).}$$

Final Answer: $\mathbf{n = 3}$.

💡 Quick Tip

1. Use the dot product and vector magnitude formulas for solving relationships between vectors. 2. Substitute the given trigonometric values into equations and simplify step by step. 3. For $|\vec{a} + \vec{b}|$ and $|\vec{a} - \vec{b}|$, expand their squared forms and use algebra to solve for unknowns.

Question 22. At the centre of a half-ring of radius $R = 10\,\text{cm}$ and linear charge density $4\,\text{nC/m}$, the potential is $x\pi V$. The value of x is _____.
Correct Answer: (36)

Solution:
The potential at the center of a half-ring is given by:

$$V = \frac{KQ}{R}$$

where:

$$Q = \lambda \pi R$$

Substituting:

$$V = \frac{K\lambda\pi R}{R}$$
$$V = K\lambda\pi$$

Given:

$$K = 9 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2, \quad \lambda = 4 \times 10^{-9}\,\text{C/m}$$
$$V = 9 \times 10^9 \cdot 4 \times 10^{-9} \cdot \pi$$
$$V = 36\pi\,\text{V}$$

Thus, $x = 36$.
Final Answer: $x = 36$.

Quick Tip

For the potential of a charged ring, always use $V = \frac{KQ}{R}$.
For a half-ring, the charge is $Q = \lambda\pi R$ since the arc length is πR.
Substitute K (Coulomb's constant) as $9 \times 10^9\,\text{N·m}^2/\text{C}^2$ and carefully handle powers of 10 for charge density.
Ensure unit consistency: radius R in meters, charge density λ in C/m.
Simplify step-by-step to avoid missing factors like π.

Question 23.
A star has 100% helium composition. It starts to convert three ^{4}He into ^{12}C via the triple alpha process as:

$$^4\text{He} +^4 \text{He} +^4 \text{He} \rightarrow^{12} \text{C} + Q.$$

The mass of the star is 2.0×10^{32} kg and it generates energy at the rate of 5.808×10^{30} W. The rate of converting these ^{4}He to ^{12}C is $n \times 10^{42}\,\text{s}^{-1}$, where n is .... [Take, mass of ^{4}He $= 4.0026\,\text{u}$, mass of ^{12}C $= 12\,\text{u}$].
Correct Answer: (5)

Solution:

The given triple alpha process is:

$$^4\text{He} +^4 \text{He} +^4 \text{He} \rightarrow^{12} \text{C} + Q$$

The power generated by the star is given as:

$$\text{Power} = \frac{N}{t}Q,$$

where N/t is the number of reactions per second.
The energy released per reaction (Q) is:

$$Q = (3m_{\text{He}} - m_{\text{C}})\,c^2$$

Substituting the given values:

$$Q = (3 \times 4.0026 - 12) \times \left(3 \times 10^8\right)^2$$

$$Q = 7.266\,\text{MeV}$$

Convert Q into joules:

$$Q = 7.266 \times 1.602 \times 10^{-13}\,\text{J}$$

$$Q = 1.163 \times 10^{-12}\,\text{J}$$

Rearranging the power equation:

$$\frac{N}{t} = \frac{\text{Power}}{Q}$$

Substitute the values:

$$\frac{N}{t} = \frac{5.808 \times 10^{30}}{1.163 \times 10^{-12}}$$

Simplify:

$$\frac{N}{t} = 5 \times 10^{42}\,\text{s}^{-1}$$

Final Answer: The rate of conversion of ^{4}He to ^{12}C is:

$$n = 5\,(\text{where } \frac{N}{t} = n \times 10^{42}\,\text{s}^{-1}).$$

Quick Tip

1. Calculate the mass defect Δm for the reaction. 2. Use $Q = \Delta m \cdot c^2$ to find energy released per reaction. 3. Relate power and reaction rate with Power $= \frac{N}{t} \cdot Q$. 4. Multiply the reaction rate by the number of ^{4}He nuclei involved to find the total conversion rate. 5. Double-check units for mass (u), energy (MeV), and power (W).

Question 24.
In a Young's double-slit experiment, the intensity at a point is $\frac{1}{4}$ of the maximum intensity. The minimum distance of the point from the central maximum is .... μm. (Given: $\lambda = 600\,\text{nm}$, $d = 1.0\,\text{mm}$, $D = 1.0\,\text{m}$)
Correct Answer: (200)
Solution:

Step 1: Intensity relation The intensity at a point in the interference pattern is given by:

$$I = I_0 \cos^2\left(\frac{\Delta\phi}{2}\right).$$

Given:

$$I = \frac{I_0}{4}.$$

Substitute:

$$\frac{I_0}{4} = I_0 \cos^2\left(\frac{\Delta\phi}{2}\right).$$

Simplify:

$$\cos^2\left(\frac{\Delta\phi}{2}\right) = \frac{1}{4}.$$

Taking square root:

$$\cos\left(\frac{\Delta\phi}{2}\right) = \frac{1}{2}.$$

This implies:

$$\frac{\Delta\phi}{2} = \frac{\pi}{3}.$$

So:

$$\Delta\phi = \frac{2\pi}{3}.$$

Step 2: Path difference relation The phase difference $\Delta\phi$ is related to the path difference by:

$$\Delta\phi = \frac{2\pi}{\lambda} \cdot \frac{y\,d}{D}.$$

Substitute $\Delta\phi = \frac{2\pi}{3}$:

$$\frac{2\pi}{3} = \frac{2\pi}{\lambda} \cdot \frac{y\,d}{D}.$$

Cancel 2π:

$$\frac{1}{3} = \frac{y\,d}{\lambda\,D}.$$

Rearrange to solve for y:

$$y = \frac{\lambda\,D}{3\,d}.$$

Step 3: Substitution of values Substitute $\lambda = 600\,\text{nm} = 600 \times 10^{-9}\,\text{m}$, $D = 1.0\,\text{m}$, and $d = 1.0\,\text{mm} = 1.0 \times 10^{-3}\,\text{m}$:

$$y = \frac{600 \times 10^{-9} \cdot 1.0}{3 \cdot 1.0 \times 10^{-3}}.$$

Simplify:

$$y = \frac{600 \times 10^{-9}}{3 \times 10^{-3}} = \frac{600}{3} \times 10^{-6} = 200 \times 10^{-6}\,\text{m}.$$

Convert to μm:

$$y = 200\,\mu\text{m}.$$

Final Answer: 200 μm.

💡 Quick Tip

1. The intensity in a double-slit experiment is related to the phase difference using $I = I_0 \cos^2\left(\frac{\Delta\phi}{2}\right)$. 2. The phase difference $\Delta\phi$ is linked to the path difference by $\Delta\phi = \frac{2\pi}{\lambda} \cdot \frac{y\,d}{D}$. 3. Carefully substitute values in standard units (e.g., λ in meters, d in meters). 4. Simplify step-by-step to avoid errors in unit conversions.

Question 25.

A string is wrapped around the rim of a wheel of moment of inertia $0.40\,\text{kgm}^2$ and radius $10\,\text{cm}$. The wheel is free to rotate about its axis. Initially, the wheel is at rest. The string is now pulled by a force of $40\,\text{N}$. The angular velocity of the wheel after $10\,\text{s}$ is x rad/s, where x is ....

Correct Answer: (100)
Solution:

Step 1: Torque and angular acceleration The torque (τ) acting on the wheel is:

$$\tau = F \cdot R.$$

Substitute $F = 40\,\text{N}$ and $R = 0.10\,\text{m}$:

$$\tau = 40 \cdot 0.1 = 4\,\text{Nm}.$$

From the rotational dynamics equation:

$$\tau = I \cdot \alpha,$$

where $I = 0.40\,\text{kgm}^2$ is the moment of inertia and α is the angular acceleration. Solving for α:

$$\alpha = \frac{\tau}{I} = \frac{4}{0.4} = 10\,\text{rad/s}^2.$$

Step 2: Angular velocity after 10 seconds Using the kinematic relation for angular motion:

$$\omega_f = \omega_i + \alpha \cdot t.$$

Here, the initial angular velocity $\omega_i = 0$, $\alpha = 10\,\text{rad/s}^2$, and $t = 10\,\text{s}$. Substituting these values:

$$\omega_f = 0 + 10 \cdot 10 = 100\,\text{rad/s}.$$

Final Answer: $\mathbf{100}\,\text{rad/s}$.

Quick Tip

1. Torque is given by $\tau = F \cdot R$. Ensure correct units for force and radius. 2. Relate torque to angular acceleration using $\tau = I \cdot \alpha$. 3. Use angular kinematics ($\omega_f = \omega_i + \alpha \cdot t$) to find the final angular velocity. 4. Always check units for consistency when calculating torque, moment of inertia, and angular velocity.

Question 26.
A square loop of edge length 2 m carrying a current of 2 A is placed with its edges parallel to the x-y-axis. A magnetic field is passing through the x-y-plane and is expressed as:

$$\vec{B} = B_0(1 + 4x)\hat{k},$$

where $B_0 = 5\,\text{T}$. The net magnetic force experienced by the loop is .... N.
Correct Answer: (160)
Solution:

Step 1: Magnetic force on a current-carrying conductor The magnetic force on a straight conductor in a magnetic field is given by:

$$\vec{F} = I \int (\vec{dl} \times \vec{B}),$$

where I is the current, $\vec{dl}$ is the element of the wire, and $\vec{B}$ is the magnetic field.
Step 2: Magnetic field variation The magnetic field is given as:

$$\vec{B} = B_0(1 + 4x)\hat{k}.$$

Since the loop is aligned parallel to the x-y-plane, $\vec{B}$ varies with x, causing forces on the opposite sides of the loop to not cancel.
Step 3: Net force on the loop For the two vertical sides of the loop (along the y-direction):

$$F_{\text{vertical}} = IL\Delta B,$$

where ΔB is the difference in the magnetic field at $x = 2\,\text{m}$ and $x = 0\,\text{m}$.

$$\Delta B = B_0(1 + 4 \cdot 2) - B_0(1 + 4 \cdot 0),$$
$$\Delta B = 5 \times (1 + 8) - 5 \times (1 + 0),$$
$$\Delta B = 45 - 5 = 40\,\text{T}.$$

Substitute $I = 2\,\text{A}$, $L = 2\,\text{m}$, and $\Delta B = 40\,\text{T}$:

$$F_{\text{vertical}} = 2 \cdot 2 \cdot 40 = 160\,\text{N}.$$

For the horizontal sides of the loop (along the x-direction), the magnetic forces cancel out because the magnetic field is uniform along the y-axis.
Step 4: Final result The net magnetic force on the loop is:

$$\boxed{160\,\text{N}}.$$

Quick Tip

1. Use $\Delta B = B(x_2) - B(x_1)$ to find the magnetic field difference along non-uniform directions. 2. Net magnetic force is calculated only for non-canceling components. 3. Horizontal forces cancel out when the field is uniform along the horizontal direction. 4. Ensure consistent units for current, length, and magnetic field.

Question 27.
Two persons pull a wire towards themselves. Each person exerts a force of $200\,\text{N}$ on the wire. The Young's modulus of the material of the wire is 1×10^{11} N/m^2. The original length of the wire is $2\,\text{m}$, and the area of the cross-section is $2\,\text{cm}^2$. The wire will extend in length by .... μm.
Correct Answer: (20)
Solution:

Step 1: Relation between stress and strain Young's modulus is given by:

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\frac{F}{A}}{\frac{\Delta \ell}{\ell}}.$$

Rearranging for $\Delta\ell$:

$$\Delta\ell = \frac{F\ell}{AY}.$$

Step 2: Substitute given values - Force, $F = 200\,\text{N}$, - Original length, $\ell = 2\,\text{m}$, - Area of cross-section, $A = 2\,\text{cm}^2 = 2 \times 10^{-4}\,\text{m}^2$, - Young's modulus, $Y = 1 \times 10^{11}\,\text{N/m}^2$. Substitute into the formula:

$$\Delta\ell = \frac{200 \cdot 2}{2 \times 10^{-4} \cdot 10^{11}}.$$

Step 3: Simplify the expression

$$\Delta\ell = \frac{400}{2 \times 10^{7}} = 2 \times 10^{-5}\,\text{m}.$$

Convert to micrometers (μm):

$$\Delta\ell = 20\,\mu\text{m}.$$

Final Answer: 20 μm.

> Quick Tip
>
> 1. Use the relation $\Delta\ell = \frac{F\ell}{AY}$ for elongation in a wire. 2. Ensure units are consistent: convert cm^2 to m^2 for area calculations. 3. Convert the result to the required units (e.g., μm). 4. Double-check input values for accuracy in calculations.

Question 28.
When a coil is connected across a 20 V DC supply, it draws a current of 5 A. When it is connected across a 20 V, 50 Hz AC supply, it draws a current of 4 A. The self-inductance of the coil is ...... mH. ($\pi = 3$)
Correct Answer: (10)
Solution:

Case 1: DC Circuit When the coil is connected to a DC supply, the inductive reactance is 0, so:

$$I = \frac{V}{R} \implies R = \frac{20}{5} = 4\,\Omega.$$

Case 2: AC Circuit When the coil is connected to an AC supply:

$$I = \frac{V}{Z} \implies Z = \frac{20}{4} = 5\,\Omega,$$

where Z is the impedance of the coil, given by:

$$Z = \sqrt{R^2 + X_L^2}.$$

Substitute $R = 4\,\Omega$:

$$5 = \sqrt{4^2 + X_L^2} \implies X_L^2 = 25 - 16 = 9 \implies X_L = 3\,\Omega.$$

The inductive reactance X_L is related to the self-inductance L by:

$$X_L = 2\pi f L \implies L = \frac{X_L}{2\pi f}.$$

Substitute $X_L = 3\ \Omega$, $f = 50$ Hz, and $\pi = 3$:

$$L = \frac{3}{2 \cdot 3 \cdot 50} = \frac{3}{300} = 0.01\ \text{H} = 10\ \text{mH}.$$

Final Answer: **10** mH.

Quick Tip

1. In DC circuits, the inductive reactance $X_L = 0$, so only the resistance R contributes to the impedance. 2. In AC circuits, the total impedance $Z = \sqrt{R^2 + X_L^2}$. 3. Inductive reactance is given by $X_L = 2\pi f L$, where f is the frequency. 4. Always use consistent units: convert millihenries to henries when necessary.

Question 29.
The position, velocity, and acceleration of a particle executing simple harmonic motion are found to have magnitudes of 4 m, 2 ms^{-1}, and 16 ms^{-2} at a certain instant. The amplitude of the motion is $\sqrt{x}$ m, where x is ....
Correct Answer: (17)
Solution:

The given data is:

$$x = 4\,\text{m}, \quad v = 2\,\text{m/s}, \quad a = 16\,\text{m/s}^2$$

For a particle in Simple Harmonic Motion (SHM), the equations for position, velocity, and acceleration are:

$$x = A\cos\omega t, \quad v = A\omega\sin\omega t, \quad a = -A\omega^2\cos\omega t$$

—

Step 1: Using the relation between acceleration and position
The acceleration is given by:

$$a = -\omega^2 x$$

Substitute $a = 16\,\text{m/s}^2$ and $x = 4\,\text{m}$:

$$16 = \omega^2 \cdot 4 \quad \implies \quad \omega^2 = 4$$

Thus:

$$\omega = 2\,\text{rad/s}$$

—

Step 2: Using the relation between velocity and amplitude

The velocity equation in SHM is:

$$v^2 = \omega^2 \left(A^2 - x^2\right)$$

Substitute $v = 2\,\text{m/s}, \omega = 2\,\text{rad/s}, x = 4\,\text{m}$:

$$2^2 = 2^2 \left(A^2 - 4^2\right)$$

$$4 = 4\left(A^2 - 16\right) \quad \Longrightarrow \quad A^2 - 16 = 1$$

$$A^2 = 17$$

—

Step 3: Amplitude

The amplitude of the motion is:

$$A = \sqrt{17}\,\text{m}$$

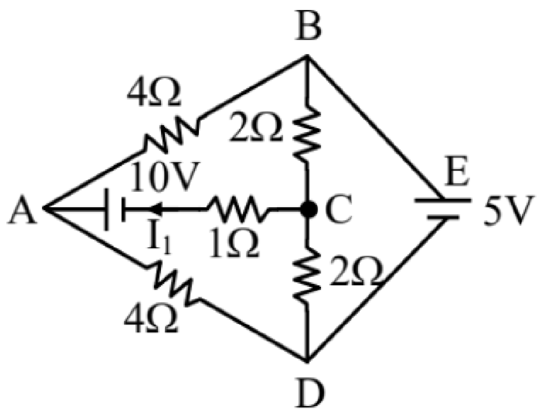
Thus, $x = 17$.

Quick Tip

1. Use $|a| = \omega^2 x$ to find the angular frequency ω. 2. Use $v = \omega\sqrt{A^2 - x^2}$ to relate velocity, amplitude, and displacement. 3. Always ensure consistent units for all quantities. 4. For SHM problems, solving step-by-step reduces errors and ensures clarity.

Question 30.

The current flowing through the $1\,\Omega$ resistor is $\frac{n}{10}$. The value of n is ....



Correct Answer: (25)

Solution:

Let the potentials at points A, B, and C be $x, y,$ and $0,$ respectively.

Applying Kirchhoff's Current Law (KCL) at node B:

$$\frac{y-5}{2} + \frac{y-0}{2} + \frac{y-x+10}{1} = 0$$

$$\Rightarrow 4y - 2x + 15 = 0 \quad \text{(i)}$$

Applying KCL at node A:

$$\frac{x-5}{4} + \frac{x-0}{4} + \frac{x-10-y}{1} = 0$$

$$\Rightarrow 6x - 4y - 45 = 0 \quad \text{(ii)}$$

Solving equations (i) and (ii):

$$\text{From (i): } y = \frac{15}{4}x - \frac{15}{4}$$

$$\text{Substituting in (ii): } x = \frac{15}{2}, \, y = 0$$

The current through the $1\,\Omega$ resistor is:

$$i = \frac{y - x + 10}{1} = \frac{0 - 7.5 + 10}{1} = 2.5\,\text{A}.$$

Therefore:

$$i = \frac{n}{10}, \quad n = 25.$$

Final Answer: $\mathbf{n = 25}$.

Quick Tip

1. Use Kirchhoff's Current Law (KCL) at all junctions to form equations. 2. Label potentials at key points for clarity. 3. Solve the simultaneous equations systematically. 4. Substitute values back to verify the solution.