Q.1 Three point particles of masses 1.0 kg, 1.5 kg and 2.5 kg are placed at three corners of a right angle triangle of sides 4.0 cm, 3.0 cm and 5.0 cm as shown in the figure. The center of mass of the system is at a point -

(1) 1.5 cm right and 1.2 cm above 1 kg mass
(2) 2.0 cm right and 0.9 cm above 1 kg mass
(3) 0.9 cm right and 2.0 cm above 1 kg mass
(4) 0.6 cm right and 2.0 cm above 1 kg mass

Ans. [3]

Sol.

\[ x_{cm} = \frac{2.5 \times 0 + 1.5 \times 3 + 1 \times 0}{2.5 + 1 + 1.5} \]
\[ y_{cm} = \frac{2.5 \times 4 + 1 \times 0 + 1.5 \times 0}{5} \]
\[ x_{cm} = 0.9 \text{ cm} \]
\[ y_{cm} = 2.0 \text{ cm} \]
Q.2. Visible light of wavelength $6000 \times 10^{-8} \text{ cm}$ falls normally on a single slit and produces a diffraction pattern. It is found that the second diffraction minimum is at $60^\circ$ from the central maximum. If the first minimum is produced at $\theta_1$, then $\theta_1$ is close to -

(1) $45^\circ$  (2) $20^\circ$  (3) $30^\circ$  (4) $25^\circ$

Ans. [4]

Sol. For $2^{\text{nd}}$ minima
$$d \sin \theta = 2\lambda,$$
$$d \sin 60^\circ = 2\lambda,$$
$$\Rightarrow \frac{\lambda}{d} = \frac{\sqrt{3}}{4}.$$

For $1^{\text{st}}$ minima
$$\sin \theta = \frac{\lambda}{d} = \frac{\sqrt{3}}{4},$$
$$\theta \approx 25^\circ$$

Q.3. The time period of revolution of electron in its ground state orbit in a hydrogen atom is $1.6 \times 10^{-16} \text{ s}$. The frequency of revolution of the electron in its first excited state (in $\text{s}^{-1}$) is -

(1) $1.6 \times 10^{14}$  (2) $5.6 \times 10^{12}$  (3) $6.2 \times 10^{15}$  (4) $7.8 \times 10^{14}$

Ans. [4]

Sol.
$$\frac{2\pi r}{v} \propto \frac{n^2}{z/n}$$
$$\frac{r}{v} \propto \frac{n^3}{z^2}$$
$$\frac{1}{f} \propto \frac{n^3}{z^2}$$
$$f \propto \frac{z^2}{n^3}$$
$$f_2 = \frac{1}{8} f_1$$
$$= \frac{1}{8} \times \frac{1}{1.6 \times 10^{-16}}$$
$$f_2 = 7.8 \times 10^{14}$$

Q.4. A litre of dry air at STP expands adiabatically to volume of 3 litres. If $\gamma = 1.40$, the work done by air is :

(1) $100.8 \text{ J}$  (2) $48 \text{ J}$  (3) $90.5 \text{ J}$  (4) $60.7 \text{ J}$

Ans. [3]
Sol. \[ P_fV_f = P_iV_i \]

\[ P_f = \frac{P_i}{3} = \frac{10^5}{3^{1.4}} \]

\[ \Delta w = \frac{P_iV_f - P_iV_i}{1 - \gamma} \]

\[ = \frac{10^5 \times 3 \times 10^{-3} - 10^5 \times 10^{-3}}{1 - 1.4} \]

\[ \Delta w \approx 90.5 \text{ J} \]

Q.5 A long solenoid of radius R carries a time \((t)\) dependent current \(I(t) = I_0(1 - t)\). A ring of radius 2R is placed coaxially near its middle. During the time interval \(0 \leq t \leq 1\), the induced current \((I_R)\) and the induced EMF \((V_R)\) in the ring change as -

1. Direction of \(I_R\) remains unchanged and \(V_R\) is maximum at \(t = 0.5\)
2. At \(t = 0.25\) direction of \(I_R\) reverses and \(V_R\) is maximum
3. Direction of \(I_R\) remains unchanged and \(V_R\) is zero at \(t = 0.25\)
4. At \(t = 0.5\) direction of \(I_R\) reverses and \(V_R\) is zero

Ans. [4]

Sol. \[ I = I_0 t(1 - t) \]

\[ = I_0 t - I_0 t^2 \]

\[ \phi = \mu_0 n IA \]

\[ V_R = -\frac{d\phi}{dt} = \mu_0 n\text{A} (I_0 - 2I_0 t) \]

\[ V_R = 0 \text{ at } t = 0.5 \]

\[ I_R = \frac{V_R}{\text{Resistance}} = \frac{\mu_0 n\text{A}(I_0 - 2I_0 t)}{\text{Resistance}} \]

after \( t = 0.5 \) \(I_R\), reverses its direction

Q.6 Which of the following gives a reversible operation?

1. ![Diagram](1)
2. ![Diagram](2)
3. ![Diagram](3)
4. ![Diagram](4)

Ans. [4]

Sol.

\[
\begin{array}{c}
A \\
\hline
0 & 1 \\
1 & 0 \\
\end{array}
\]

It will behave as a NOT gate. Hence gives reversible operation.
Q.7

As shown in the figure, a bob of mass \( m \) is tied by massless string whose other end portion is wound on a fly wheel (disc) of radius \( r \) and mass \( m \). When released from rest the bob starts falling vertically. When it has covered a distance of \( h \), the angular speed of the wheel will be -

(1) \( r \sqrt{\frac{3}{2gh}} \)
(2) \( \frac{1}{r} \sqrt{\frac{4gh}{3}} \)
(3) \( r \sqrt{\frac{3}{4gh}} \)
(4) \( \frac{1}{r} \sqrt{\frac{2gh}{3}} \)

Ans. [2]

Sol.

Disc (m, r)

Energy conservation

\[ mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \]  

\[ \omega = \frac{v}{r} \]

Solving (1) & (2), we get

\[ \omega = \frac{1}{r} \sqrt{\frac{4gh}{3}} \]

Q.8 Two infinite planes each with uniform surface charge density \(+\sigma\) are kept in such a way that the angle between them is 30º. The electric field in the region shown between them is given by -

(1) \( \frac{\sigma}{2\varepsilon_0} \left[ (1 + \sqrt{3})\hat{y} + \frac{\hat{x}}{2} \right] \)
(2) \( \frac{\sigma}{\varepsilon_0} \left[ \frac{1 + \sqrt{3}}{2} \hat{y} + \frac{\hat{x}}{2} \right] \)
(3) \( \frac{\sigma}{2\varepsilon_0} \left[ (1 + \sqrt{3})\hat{y} - \frac{\hat{x}}{2} \right] \)
(4) \( \frac{\sigma}{\varepsilon_0} \left[ \frac{1 - \sqrt{3}}{2} \hat{y} - \frac{\hat{x}}{2} \right] \)
**Q.9** A polarizer-analyser set is adjusted such that the intensity of light coming out of the analyser is just 10% of the original intensity. Assuming that the polarizer-analyser set does not absorb any light, the angle by which the analyser need to be rotated further to reduce the output intensity to be zero, is -

(1) 90°  
(2) 45°  
(3) 18.4°  
(4) 71.6°

**Ans.** [3]

**Sol.**

\[ I = I_0 \cos^2 \theta \]

\[ \frac{I_0}{10} = I_0 \cos^2 \theta \]

\[ \cos \theta = \frac{1}{\sqrt{10}} \]

\[ \theta \approx 71.6^\circ \]

Hence, further 18.4° rotation reduce output intensity to be zero.
Q.10

A parallel plate capacitor has plates of area A separated by distance 'd' between them. It is filled with a dielectric which has a dielectric constant that varies as \( k(x) = K(1 + \alpha x) \) where 'x' is the distance measured from one of the plates. If \((\alpha d) << 1\), the total capacitance of the system is best given by the expression:

\[
\begin{align*}
(1) & \quad \frac{AK}{d} (1 + \alpha d) \\
(2) & \quad \frac{A}{d} \frac{\varepsilon_0 K}{1 + \frac{\alpha^2 d^2}{2}} \\
(3) & \quad \frac{AK}{d} \left( 1 + \frac{\alpha d}{2} \right) \\
(4) & \quad \frac{A}{d} \frac{\varepsilon_0 K}{1 + \left( \frac{\alpha d}{2} \right)^2}
\end{align*}
\]

Ans. [3]

Sol.

\[ k(x) = K(1 + \alpha x) \]

\[ \begin{align*}
\text{d}C_1 &= \frac{K}{\varepsilon_0} \frac{A}{dx} \\
\text{d}C_1 &= \frac{K(1 + \alpha x)A}{\varepsilon_0} \frac{dx}{dx} \\
\frac{1}{C_{eq}} &= \int \frac{1}{dC_1} \\
&= \int_0^d \frac{dx}{K\alpha(1 + \alpha x)} \\
&= \frac{1}{\varepsilon_0 K\alpha} \ln(1 + \alpha d)
\end{align*} \]

Using expansion of \( \ln(1 + x) \) keeping \( x << 1 \)

\[ \frac{1}{C_{eq}} = \frac{1}{\varepsilon_0 K\alpha} \left( \alpha d - \frac{\alpha^2 d^2}{2} \right) \]

\[ C_{eq} = \frac{\varepsilon_0 KA}{d} \left( 1 - \frac{\alpha d}{2} \right)^{-1} = \frac{\varepsilon_0 KA}{d} \left( 1 + \frac{\alpha d}{2} \right) \]
Q.11 A satellite of mass \( m \) is launched vertically upwards with an initial speed \( u \) from the surface of the earth. After it reaches height \( R(R = \text{radius of the earth}) \), it ejects a rocket of mass \( \frac{m}{10} \) so that subsequently the satellite moves in a circular orbit. The kinetic energy of the rocket is \((G \text{ is the gravitational constant; } M \text{ is the mass of the earth})\):

\[
\begin{align*}
(1) & \quad 5m \left( u^2 - \frac{119 GM}{200 R} \right) \\
(2) & \quad \frac{m}{20} \left( u - \frac{2GM}{3R} \right)^2 \\
(3) & \quad \frac{3m}{8} \left( u + \frac{5GM}{6R} \right)^2 \\
(4) & \quad \frac{m}{20} \left( u^2 + \frac{113 GM}{200 R} \right)
\end{align*}
\]

Ans. \[1\]

Sol.

Using energy conservation

\[
\frac{1}{2}mu^2 - \frac{GmM}{R} = - \frac{GmM}{2R} + \frac{1}{2}mv^2
\]

\[
\frac{u^2 - 2GM}{R} = - \frac{GM}{R} + v^2
\]

\[
\frac{u^2}{R} = v^2
\]

Again after ejecting rocket

\[
\vec{P}_i = \vec{P}_f
\]

\[
\frac{v}{m} = \sqrt{\frac{GM}{2R}}
\]

\[
\frac{9m}{10} \sqrt{\frac{GM}{2R}} = \frac{m}{10} v_2 \quad (\vec{p} \text{ conservation along tangent})
\]

\[
V_2 = 9 \sqrt{\frac{GM}{2R}}
\]

\[
v = \frac{m}{10} v_i \quad (\vec{p} \text{ conservation along radius})
\]

\[
v_1 = 10 v
\]

K.E. of rocket

\[
= \frac{1}{2} \frac{m}{10} (v_1^2 + v_2^2)
\]

\[
= \frac{1}{2} \frac{m}{10} \left( 100 \left( u^2 - \frac{GM}{k} \right) + 81 \frac{GM}{2R} \right)
\]

\[
= \frac{m}{20} \left( 100u^2 - \frac{100GM}{R} + 81GM \right)
\]

K.E. = \(5m \left( u^2 - \frac{119 GM}{200 R} \right)\)
Q.12 A LCR circuit behaves like a damped harmonic oscillator. Comparing it with a physical spring-mass damped oscillator having damping constant 'b', the correct equivalence would be -

(1) \( L \leftrightarrow k, C \leftrightarrow b, R \leftrightarrow m \)

(2) \( L \leftrightarrow m, C \leftrightarrow k, R \leftrightarrow b \)

(3) \( L \leftrightarrow m, C \leftrightarrow \frac{1}{k}, R \leftrightarrow b \)

(4) \( L \leftrightarrow \frac{1}{b}, C \leftrightarrow \frac{1}{m}, R \leftrightarrow \frac{1}{k} \)

Ans. [3]

Sol. Spring mass damped oscillator

\[
\frac{d^2x}{dt^2} + \frac{b}{m} \frac{dx}{dt} + \frac{k}{m} x = 0 \quad \text{...(1)}
\]

For LC oscillations

\[
L \left( \frac{di}{dt} + iR + \frac{q}{C} \right) = 0
\]

\[
\frac{d^2q}{dt^2} + \frac{R}{L} \frac{dq}{dt} + \frac{q}{LC} = 0 \quad \text{...(2)}
\]

Comparing (1) & (2)

\( b \leftrightarrow R \)

\( L \leftrightarrow m \)

\( \frac{1}{C} \leftrightarrow K \)

Q.13 Consider a circular coil of wire carrying constant current I, forming a magnetic dipole. The magnetic flux through an infinite plane that contains the circular coil and excluding the circular coil area is given by \( \phi_i \). The magnetic flux through the area of the circular coil area is given by \( \phi_0 \). Which of the following option is correct?

(1) \( \phi_i < \phi_0 \)  (2) \( \phi_i > \phi_0 \)  (3) \( \phi_i = \phi_0 \)  (4) \( \phi_i = -\phi_0 \)

Ans. [4]

Sol.

As, magnetic field lines forms a closed loop, hence each line from circular area will pass through outer area in opposite direction hence \( \phi_i = -\phi_0 \).
Q.14 The current $I_1$ (in A) flowing through 1$\Omega$ resistor in the following circuit is –

![Image of the circuit]

(1) 0.2  (2) 0.4  (3) 0.25  (4) 0.5

Ans. [1]

Sol.

$\frac{V}{R_{eq}} = I \left( \frac{2}{5} + \frac{1}{2} \right)$

$I = 0.9$ A

$i_1 = \frac{1}{4.5} \times 2$

$i = \frac{i_1}{2}$

$i = \frac{I}{4.5} = 0.2$ A
Q.15  Speed of a transverse wave on a straight wire (mass 6.0 g, length 60 cm and area of cross-section 1.0 mm$^2$) is 90 ms$^{-1}$. If the Young's modulus of wire is $16 \times 10^{11}$ Nm$^{-2}$, the extension of wire over its natural length is -

(1) 0.01 mm   (2) 0.04 mm   (3) 0.02 mm   (4) 0.03 mm

Ans.  [4]

Sol.  
\[ v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{TA}{A\mu}} \]

Stress = \[ \frac{T}{A} = Y \times \text{Strain} = \frac{Y\Delta L}{L} \]

\[ \Rightarrow v = \sqrt{\frac{Y\Delta LA}{L m / L}} \left( \because \mu = \frac{m}{L} \right) \]

\[ v = \sqrt{\frac{Y\Delta LA}{m}} \]

\[ \Delta L = \frac{v^2 m}{YA} \]

\[ = \frac{90 \times 90 \times 6 \times 10^{-3}}{16 \times 10^{11} \times 1 \times 10^{-6}} \]

\[ \Delta L \approx 0.03 \text{ mm} \]

Q.16  If the magnetic field in a plane electromagnetic wave is given by \[ \vec{B} = 3 \times 10^{-8} \sin (1.6 \times 10^{3} x + 48 \times 10^{10} t) \hat{j} \text{ T} \]
then what will be expression for electric field ?

(1) \[ \vec{E} = [9\sin(1.6 \times 10^{3} x + 48 \times 10^{10} t)] \hat{k} \text{ V/m} \]

(2) \[ \vec{E} = [60 \sin (1.6 \times 10^{3} x + 48 \times 10^{10} t)] \hat{k} \text{ V/m} \]

(3) \[ \vec{E} = [3 \times 10^{-8} \sin (1.6 \times 10^{3} x + 48 \times 10^{10} t)] \hat{i} \text{ V/m} \]

(4) \[ \vec{E} = [3 \times 10^{-8} \sin(1.6 \times 10^{3} x + 48 \times 10^{10} t)] \hat{j} \text{ V/m} \]

Ans.  [1]

Sol.  
\[ E_0 = B_0 c \]

\[ = (3 \times 10^{-8}) (3 \times 10^8) \sin (ot + kx) \]

\[ E_0 = 9 \sin (1.6 \times 10^{3} x + 48 \times 10^{10} t) \]

Q.17  If we need a magnification of 375 from a compound microscope of tube length 150 mm and an objective of focal length 5 mm, the focal length of the eye-piece, should be close to -

(1) 12 mm   (2) 33 mm   (3) 22 mm   (4) 2 mm

Ans.  [3]

Sol.  
M.P. = 375

\[ f_0 = 5 \text{ mm} \]

Case-I

If final image is at infinity
M.P. = $\frac{1}{f_0} \left( \frac{D}{f_e} \right)$

$375 = \frac{150}{5} \left( \frac{250}{f_e} \right)$

$f_e = 20$

Near point adjustment

M.P. = $\frac{L_0}{f_0} \left( 1 + \frac{D}{f_e} \right)$

$375 = \frac{150}{5} \left( 1 + \frac{250}{f_e} \right)$

$f_e \approx 22 \text{ mm}$

Q.18 A 60 HP electric motor lifts an elevator having a maximum total load capacity of 2000 kg. If the frictional force on the elevator is 4000 N, the speed of the elevator at full load is close to: (1 HP = 746 W, g = 10 ms$^{-2}$)

(1) 1.7 ms$^{-1}$
(2) 1.9 ms$^{-1}$
(3) 2.0 ms$^{-1}$
(4) 1.5 ms$^{-1}$

Ans. [2]

Sol.

\[ P = Fv + Mgv \]

Applied power = $4000 \times v + 20000 \times v$

$60 \times 246 = 4000 \times v + 20000 \times v$

\[ v = 1.865 \text{ m/s} \]

\[ v \approx 1.9 \text{ m/s} \]

Q.19 The radius of gyration of a uniform rod of length $\ell$, about an axis passing through a point $\frac{\ell}{4}$ away from the centre of the rod, and perpendicular to it, is :

(1) $\frac{3}{8} \ell$
(2) $\frac{1}{8} \ell$
(3) $\frac{\ell}{48}$
(4) $\frac{1}{4} \ell$

Ans. [3]

Sol.

Radius of gyration

\[ I = \frac{M\ell^2}{12} + M \left( \frac{\ell}{4} \right)^2 \]

\[ I = \frac{7}{48} M\ell^2 \]

\[ Mk^2 = \frac{7}{48} M\ell^2 \]

\[ k = \ell \cdot \frac{7}{48} \]
Q.20 Two moles of an ideal gas with $\frac{C_p}{C_v} = \frac{5}{3}$ are mixed with 3 moles of another ideal gas with $\frac{C_p}{C_v} = \frac{4}{3}$. The value of $\frac{C_p}{C_v}$ for the mixture is -

(1) 1.50  
(2) 1.42  
(3) 1.47  
(4) 1.45

Ans. [2]

Sol. 

\[
\gamma_{\text{mix}} = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 C_{v_1} + n_2 C_{v_2}}
\]

\[
C_p = \left( \frac{\gamma}{\gamma - 1} \right) R
\]

\[
C_v = \frac{R}{\gamma - 1}
\]

\[
\gamma_{\text{mix}} = \frac{n_1 \gamma_1}{\gamma_1 - 1} + \frac{n_2 \gamma_2}{\gamma_2 - 1}
\]

\[
\frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}
\]

$n_1 = 2, \gamma_1 = \frac{5}{3}, n_2 = 3, \gamma_2 = \frac{4}{3}$

We get

\[
\gamma_{\text{mix}} = 1.42
\]

Q.21 A beam of electromagnetic radiation of intensity $6.4 \times 10^{-5}$ W/cm² is comprised of wavelength, $\lambda = 310$ nm. It falls normally on a metal (work function $\phi = 2$eV) of surface area of 1 cm². If one in $10^3$ photons ejects an electron, total number of electrons ejected in 1 s is $10^x$. (hc = 1240 eVnm, 1eV = $1.6 \times 10^{-19}$ J), then x is ............... .

Ans. [11]

Sol. Energy of photon = $\frac{1240}{310}$ = 4eV

Energy is greater than work function

No. of photons = $\frac{\text{Intensity}}{\text{Energy of one photon}}$

= $\frac{6.4 \times 10^{-5}}{4 \times 1.6 \times 10^{-19}} = 10^{14}$

Total no. of electron = $\frac{10^{14}}{10^3} = 10^{11}$
Q.22 A loop ABCDEFA of straight edges has six corner points A(0, 0, 0), B(5, 0, 0), C(5, 5, 0), D(0, 5, 0), E(0, 5, 5) and F(0, 0, 5). The magnetic field in this region is \( \mathbf{B} = (3 \mathbf{i} + 4 \mathbf{k}) \) T. The quantity of flux through the loop ABCDEFA (in Wb) is ..................

**Ans.** [175]

**Sol.**

\[
\mathbf{B} = (3 \mathbf{i} + 4 \mathbf{k}) \text{ T}
\]

Area of plane = 25
\[
\phi = (3 \times 25) + 4 \times 25 = 175 \text{ weber}
\]

Q.23 A non-isotropic solid metal cube has coefficients of linear expansion as: \( 5 \times 10^{-5}/^\circ \text{C} \) along the x-axis and \( 5 \times 10^{-6}/^\circ \text{C} \) along the y and the z-axis. If the coefficient of volume expansion of the solid is \( \gamma \times 10^{-6}/^\circ \text{C} \) then the value of \( \gamma \) is ..................

**Ans.** [60]

**Sol.**

For non-isotropic
\[
\gamma = \alpha_x + \alpha_y + \alpha_z
\]
\[
= 5 \times 10^{-5} + 5 \times 10^{-6} + 5 \times 10^{-6}
\]
\[
= 60 \times 10^{-6}
\]
Comparing, we get
\[
\gamma = 60
\]

Q.24 A Carnot engine operates between two reservoirs of temperatures 900 K and 300 K. The engine performs 1200 J of work per cycle. The heat energy (in J) delivered by the engine to the low temperature reservoir, in a cycle, is ..............

**Ans.** [600]

**Sol.**

\[
\eta = 1 - \frac{T_L}{T_H}
\]
\[
= 1 - \frac{300}{900}
\]
\[
= \frac{2}{3}
\]
\[
\frac{w}{Q_s} = \frac{2}{3}
\]
\[
Q_s = \frac{3}{2} \times 1200
\]
\[
Q_s = 1800 \text{ J}
\]
Rejected = \( Q_s - w = 1800 - 1200 = 600 \text{ J} \)
A particle (m = 1 kg) slides down a frictionless track (AOC) starting from rest at a point A (height 2 m). After reaching C, the particle continues to move freely in air as a projectile. When it reaching its highest point P (height 1 m), the kinetic energy of the particle (in J) is: (Figure drawn is schematic and not to scale; take $g = 10 \text{ ms}^{-2}$) ..................

$$\text{K.E.} = \Delta \text{P.E.} = mgh = 1 \times 10 \times 1 = 10 \text{ Joule}.$$