FINAL JEE–MAIN EXAMINATION – FEBRUARY, 2021

(Held On Friday 26th February, 2021) TIME : 9 : 00 AM to 12 : 00 NOON

MATHEMATICS — TEST PAPER WITH ANSWER

SECTION-A

1. If $\vec{a}$ and $\vec{b}$ are perpendicular, then $\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\vec{b}\right)\right)\right)$ is equal to

(1) $\vec{0}$ (2) $\frac{1}{2}|\vec{a}|^4\vec{b}$

(3) $\vec{a}\times\vec{b}$ (4) $|\vec{a}|^4\vec{b}$

Official Ans. by NTA (4)

2. A fair coin is tossed a fixed number of times. If the probability of getting 7 heads is equal to probability of getting 9 heads, then the probability of getting 2 heads is

(1) $\frac{15}{2^{13}}$ (2) $\frac{15}{2^{12}}$ (3) $\frac{15}{2^8}$ (4) $\frac{15}{2^{14}}$

Official Ans. by NTA (1)

3. Let A be a symmetric matrix of order 2 with integer entries. If the sum of the diagonal elements of A^2 is 1, then the possible number of such matrices is

(1) 4 (2) 1 (3) 6 (4) 12

Official Ans. by NTA (1)

4. In a increasing geometric series, the sum of the second and the sixth term is $\frac{25}{2}$ and the product of the third and fifth term is 25. Then, the sum of 4^{th}, 6^{th} and 8^{th} terms is equal to

(1) 30 (2) 26 (3) 35 (4) 32

Official Ans. by NTA (3)

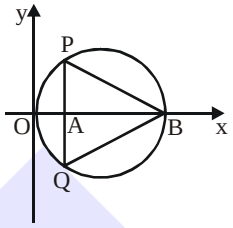
5. The value of $\sum_{n=1}^{100}\int_{n-1}^{n} e^{x-[x]}dx$, where [x] is the greatest integer $\le$ x, is

(1) 100(e – 1) (2) 100(1 – e)

(3) 100e (4) 100 (1 + e)

Official Ans. by NTA (1)

6. In the circle given below, let OA = 1 unit, OB = 13 unit and PQ $\perp$ OB. Then, the area of the triangle PQB (in square units) is



(1) $24\sqrt{2}$ (2) $24\sqrt{3}$

(3) $26\sqrt{3}$ (4) $26\sqrt{2}$

Official Ans. by NTA (2)

7. The sum of the infinite series $1+\frac{2}{3}+\frac{7}{3^2}+\frac{12}{3^3}+\frac{17}{3^4}+\frac{22}{3^5}+\ldots$ is equal to

(1) $\frac{13}{4}$ (2) $\frac{9}{4}$ (3) $\frac{15}{4}$ (4) $\frac{11}{4}$

Official Ans. by NTA (1)

8. The value of $\lim\limits_{h\to0} 2\left\{\frac{\sqrt{3}\sin\left(\frac{\pi}{6}+h\right)-\cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}h\left(\sqrt{3}\cosh-\sinh\right)}\right\}$ is

(1) $\frac{4}{3}$ (2) $\frac{2}{\sqrt{3}}$ (3) $\frac{3}{4}$ (4) $\frac{2}{3}$

Official Ans. by NTA (1)

9. The maximum value of the term independent of 't' in the expansion of $\left(tx^{\frac{1}{5}}+\frac{(1-x)^{\frac{1}{10}}}{t}\right)^{10}$ where $x \in (0,1)$ is

(1) $\frac{10!}{\sqrt{3}(5!)^2}$ (2) $\frac{2.10!}{3\sqrt{3}(5!)^2}$

(3) $\frac{2.10!}{3(5!)^2}$ (4) $\frac{10!}{3(5!)^2}$

Official Ans. by NTA (2)

10. The rate of growth of bacteria in a culture is proportional to the number of bacteris present and the bacteria count is 1000 at initial time t = 0. The number of bacteria is increased by 20% in 2 hours. If the population of bacteria is 2000 after $\frac{k}{\log_e\left(\frac{6}{5}\right)}$ hours, then $\left(\frac{k}{\log_e 2}\right)^2$ is equal to

(1) 4 (2) 8 (3) 2 (4) 16

Official Ans. by NTA (1)

11. If (1,5,35), (7,5,5), (1,λ,7) and (2λ,1,2) are coplanar, then the sum of all possible values of λ is

(1) $\frac{39}{5}$ (2) $-\frac{39}{5}$ (3) $\frac{44}{5}$ (4) $-\frac{44}{5}$

Official Ans. by NTA (3)

12. If $\frac{\sin^{-1}x}{a}=\frac{\cos^{-1}x}{b}=\frac{\tan^{-1}y}{c};0<x<1$, then the value of $\cos\left(\frac{\pi c}{a+b}\right)$ is

(1) $\frac{1-y^2}{y\sqrt{y}}$ (2) $1-y^2$

(3) $\frac{1-y^2}{1+y^2}$ (4) $\frac{1-y^2}{2y}$

Official Ans. by NTA (3)

13. The number of seven digit integers with sum of the digits equal to 10 and formed by using the digits 1,2 and 3 only is

(1) 42 (2) 82 (3) 77 (4) 35

Official Ans. by NTA (3)

14. Let f be any function defined on R and let it satisfy the condition :

$|f(x) - f(y)| \le |(x - y)^2|$, $\forall$ (x,y) $\in$ R

If $f(0) = 1$, then :

(1) $f(x)$ can take any value in R

(2) $f(x) < 0$, $\forall$ x $\in$ R

(3) $f(x) = 0$, $\forall$ x $\in$ R

(4) $f(x) > 0$, $\forall$ x $\in$ R

Official Ans. by NTA (4)

15. The maximum slope of the curve $y=\frac{1}{2}x^4-5x^3+18x^2-19x$ occurs at the point

(1) (2,2) (2) (0,0)

(3) (2,9) (4) $\left(3,\frac{21}{2}\right)$

Official Ans. by NTA (1)

16. The intersection of three lines x – y = 0, x + 2y = 3 and 2x + y = 6 is a

(1) Right angled triangle

(2) Equilateral triangle

(3) Isosceles triangle

(4) None of the above

Official Ans. by NTA (3)

17. Consider the three planes

P_1 : 3x + 15y + 21z = 9,

P_2 : x – 3y – z = 5, and

P_3 : 2x + 10y + 14z = 5

Then, which one of the following is true ?

(1) P_1 and P_2 are parallel

(2) P_1 and P_3 are parallel

(3) P_2 and P_3 are parallel

(4) P_1,P_2 and P_3 all are parallel

Official Ans. by NTA (2)

18. The value of $\begin{vmatrix}(a+1)(a+2) & a+2 & 1\\ (a+2)(a+3) & a+3 & 1\\ (a+3)(a+4) & a+4 & 1\end{vmatrix}$ is

(1) (a + 2) (a + 3) (a + 4)

(2) –2

(3) (a + 1) (a + 2) (a + 3)

(4) 0

Official Ans. by NTA (2)

19. The value of $\int_{-\pi/2}^{\pi/2}\frac{\cos^2 x}{1+3^x}dx$ is

(1) $\frac{\pi}{4}$ (2) 4π (3) $\frac{\pi}{2}$ (4) 2π

Official Ans. by NTA (1)

20. Let R = {(P,Q) | P and Q are at the same distance from the origin} be a relation, then the equivalence class of (1,–1) is the set :
(1) S = {(x,y) | $x^2 + y^2 = 4$}
(2) S = {(x,y) | $x^2 + y^2 = 1$}
(3) S = {(x,y) | $x^2 + y^2 = \sqrt{2}$ }
(4) S = {(x,y) | $x^2 + y^2 = 2$}
Official Ans. by NTA (4)

SECTION-B

1. The difference between degree and order of a differential equation that represents the family of curves given by $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right), a > 0$ is

Official Ans. by NTA (2)

2. The number of integral values of 'k' for which the equation 3sinx + 4 cosx = k + 1 has a solution, k ∈ R is
Official Ans. by NTA (11)

3. The number of solutions of the equation $\log_4(x - 1) = \log_2(x - 3)$ is
Official Ans. by NTA (1)

4. The sum of 162^{th} power of the roots of the equation $x^3 - 2x^2 + 2x - 1 = 0$ is
Official Ans. by NTA (3)

5. Let m,n ∈ N and gcd (2,n) = 1. If

$$30\binom{30}{0} + 29\binom{30}{1} + \ldots + 2\binom{30}{28} + 1\binom{30}{29} = n.2^m ,$$

then n + m is equal to

(Here $\binom{n}{k} = {}^nC_k$)

Official Ans. by NTA (45)

6. If y = y(x) is the solution of the equaiton

$$e^{\sin y}\cos y\frac{dy}{dx} + e^{\sin y}\cos x = \cos x, y(0) = 0 ;$$

then $1 + y\left(\frac{\pi}{6}\right) + \frac{\sqrt{3}}{2}y\left(\frac{\pi}{3}\right) + \frac{1}{\sqrt{2}}y\left(\frac{\pi}{4}\right)$ is equal to
Official Ans. by NTA (1)

7. Let (λ,2,1) be a point on the plane which passes through the ponit (4,–2,2). If the plane is perpendicular to the line joining the points (–2,–21,29) and (–1, –16, 23), then $\left(\frac{\lambda}{11}\right)^2 - \frac{4\lambda}{11} - 4$ is equal to
Official Ans. by NTA (8)

8. The area bounded by the lines y = ||x – 1| –2| is
Official Ans. by NTA (8)
Ans. By ALLEN (BONUS)

9. The value of the integral $\int_0^{\pi} |\sin 2x| dx$ is

Official Ans. by NTA (2)

10. If $\sqrt{3}(\cos^2 x) = (\sqrt{3} - 1)\cos x + 1$, the number of solutions of the given equation when $x \in \left[0, \frac{\pi}{2}\right]$ is

Official Ans. by NTA (1)