PHYSICS

Q.1 If the screw on a screw-gauge is given six rotations, it moves by 3 mm on the main scale. If there are 50 divisions on the circular scale the least count of the screw gauge is-
(1) 0.001 cm (2) 0.02 mm (3) 0.01 cm (4) 0.001 mm
Ans. [1]
Sol. Pitch = \( \frac{3}{6} \) mm = 0.5 mm
Least count = \( \frac{0.5}{50} \) mm = 0.001 cm

Q.2 Radiation, with wavelength 6561 Å falls on a metal surface to produce photoelectrons. The electrons are made to enter a uniform magnetic field of 3 \( \times \) 10\(^{-4}\) T. If the radius of the largest circular path followed by the electrons is 10 mm, the work function of the metal is close to-
(1) 0.8 eV (2) 1.6 eV (3) 1.1 eV (4) 1.8 eV
Ans. [3]
Sol. \( h\nu = kE_{\text{max.}} \); \( h\nu = \frac{12400}{6561} = 1.9 \text{ eV} \)
\[
r = \frac{mv}{qB} ; p = mv = rqB
\]
\[
kE_{\text{max.}} = \frac{p^2}{2m} = \frac{q^2r^2B^2}{2m} J = \frac{(1.6 \times 10^{-19})^2(10 \times 10^{-3})^2(3 \times 10^{-4})^2}{2 \times 9 \times 10^{-31} \times 1.6 \times 10^{-19}} \text{ eV} \approx 0.8 \text{ eV}
\]
\[
\phi = h\nu - kE_{\text{max.}} = 1.9 - 0.8 = 1.1 \text{ eV}
\]

Q.3 In the given circuit diagram, a wire is joining points B and D. The current in this wire is-

\[
\begin{align*}
\text{A} & - \text{B} \quad 1\Omega \\
\text{B} & - \text{C} \quad 2\Omega \\
\text{D} & - \text{C} \quad 4\Omega \\
\text{D} & - \text{A} \quad 3\Omega \\
\text{20V} &
\end{align*}
\]
(1) 4 A (2) 0.4 A (3) 2 A (4) zero
Ans. [3]

### Sol.

\[ 20 - 4I - 6I = 0 \Rightarrow I = 2A \]

**Q.4** Two particles of equal mass \( m \) have respective initial velocities \( u\hat{i} \) and \( \left( \frac{\hat{i} + \hat{j}}{2} \right) \). They collide completely inelastically. The energy lost in the process is-

1. \( \frac{3}{4} mu^2 \)
2. \( \frac{1}{8} mu^2 \)
3. \( \frac{1}{3} mu^2 \)
4. \( \sqrt{\frac{2}{3}} mu^2 \)

**Ans.** [2]

**Sol.**

\[
mu\hat{i} + m\left( \frac{u\hat{i}}{2} + \frac{u\hat{j}}{2} \right) = 2m\tilde{v}
\]

\[
\tilde{v} = \frac{3u\hat{i}}{4} + \frac{u\hat{j}}{4}
\]

\[
|\tilde{v}|^2 = \frac{10u^2}{16}
\]

\[
\Delta KE = KE_f - KE_i
\]

\[
= \frac{1}{2}m \times \frac{10u^2}{16} - \frac{1}{2}mu^2 - \frac{1}{2}m \frac{u^2}{2}
\]

\[
\Delta KE = -\frac{mu^2}{8}
\]

**Q.5** A charged particle of mass 'm' and charge 'q' moving under the influence of uniform electric field \( E\hat{i} \) and a uniform magnetic field \( B\hat{k} \) follows a trajectory from point P to Q as shown in figure. The velocities at P and Q are respectively, \( \tilde{v} \) and \( 2\tilde{v}\hat{j} \). Then which of the following statements (A, B, C, D) are the correct?

(Trajectory shown in schematic and not to scale)
(A) \( E = \frac{3}{4} \left( \frac{mv^2}{qa} \right) \)

(B) Rate of work done by the electric field at P is \( \frac{3}{4} \left( \frac{mv^3}{a} \right) \)

(C) Rate of work done by both the fields at Q is zero

(D) The difference between the magnitude of angular momentum of the particle at P and Q is 2 \( mav \).


Ans.  [2]

Sol.  Work done by magnetic force = zero

Thus

\[ \Delta KE = \text{Work done by electric field} \]

\[ \frac{1}{2}m(2v)^2 - \frac{1}{2}mv^2 = qE \times 2a \]

\[ \Rightarrow E = \frac{3}{4} \frac{mv^2}{qa} \]

Rate of work done by electric field at P = \( qEv = \frac{3}{4} \frac{mv^3}{a} \)

Rate of work done by magnetic field at any point is zero since force \( \perp \) velocity, at Q \( \vec{E} \) is perpendicular to velocity thus rate of work done by it is zero.

Q.6  A body A of mass m is moving in a circular orbit of radius R about a planet. Another body B of mass \( \frac{m}{2} \) collides with A with a velocity which is half \( \left( \frac{\vec{v}}{2} \right) \) the instantaneous velocity \( \vec{v} \) of A. The collision is completely inelastic. Then, the combined body-

(1) starts moving in an elliptical orbit around the planet
(2) continues to move in a circular orbit
(3) falls vertically downwards towards the planet
(4) escapes from the planet's gravitational field

Ans.  [1]

Sol.  Orbital speed for orbital of A is \( v \)

After collision

\[ mv + \frac{m}{2} \times \frac{v}{2} = \frac{3m}{2}v' ; v' = \frac{5}{6}v \]

\[ \Rightarrow v' < v \Rightarrow \text{path will be elliptical with center of earth at the near focus.} \]
Q.7 A quantity \( f \) is given by \( f = \sqrt{\frac{hc^5}{G}} \) where \( c \) is speed of light, \( G \) universal gravitational constant of light and \( h \) is the Planck’s constant. Dimension of \( f \) is that of-

(1) area  (2) volume  (3) momentum  (4) energy

Ans. [4]

Sol. \( f = \sqrt{\frac{hc^5}{G}} \)

\[
[h] = [M^1 L^2 T^{-1}]
\]

\[
[c] = [L^1 T^{-1}]
\]

\[
[G] = [M^{-1} L^3 T^{-2}]
\]

\[
[f] = \sqrt{\frac{[M^1 L^2 T^{-1}][L^5 T^{-5}]}{[M^{-1} L^3 T^{-2}]}} = [M^1 L^2 T^{-3}]
\]

\( [f] \) is equal to dimension of energy.

Q.8 A long, straight wire of radius \( a \) carries a current distributed uniformly over its cross-section. The ratio of the magnetic fields due to wire at distance \( \frac{a}{3} \) and \( 2a \), respectively from the axis of the wire is-

(1) \( \frac{1}{2} \)  (2) 2  (3) \( \frac{2}{3} \)  (4) \( \frac{3}{2} \)

Ans. [3]

Sol.

Let current density be \( J \).

For point \( P \)

\[
\oint B \cdot d\ell = \mu_0 i
\]

\[
B_p 2\pi \frac{a}{3} = \mu_0 J \pi \frac{a^2}{9}
\]

For point \( Q \)

\[
B_Q 2\pi (2a) = \mu_0 J \pi a^2
\]

\[
B_p 2\pi \frac{a}{3} = \frac{\mu_0 J \pi a^2}{9}
\]

\[
\frac{B_p}{B_Q} = \frac{2}{3}
\]
Q.9  An electric dipole of moment \( \vec{p} = (-\hat{i} - 3\hat{j} + 2\hat{k}) \times 10^{-29} \text{C.m} \) is at the origin (0, 0, 0). The electric field due to this dipole at \( \vec{r} = +\hat{i} + 3\hat{j} + 5\hat{k} \) (note that \( \vec{r} \cdot \vec{p} = 0 \) is parallel to-

(1) \((+\hat{i} + 3\hat{j} - 2\hat{k})\)  
(2) \((-\hat{i} - 3\hat{j} - 2\hat{k})\)  
(3) \((-\hat{i} - 3\hat{j} + 2\hat{k})\)  
(4) \((-\hat{i} + 3\hat{j} - 2\hat{k})\)

Ans. [1]
Sol. Since \( \vec{r} \cdot \vec{p} = 0 \)  
\( \Rightarrow \) it is equatorial position  
\( \Rightarrow \vec{E} \) will be antiparallel to \( \vec{p} \)

Q.10  A particle moving with kinetic energy \( E \) has de Broglie wavelength \( \lambda \). If energy \( \Delta E \) is added to its energy, the wavelength become \( \lambda/2 \). Value of \( \Delta E \), is-

(1) \(3E\)  
(2) \(E\)  
(3) \(4E\)  
(4) \(2E\)

Ans. [1]
Sol. \( \lambda = \frac{h}{\sqrt{2mE}} \)

\( \lambda' = \frac{h}{\sqrt{2mE'}} \)

\( \lambda' = \frac{\lambda}{2} \Rightarrow E' = 4E \)

\( \Delta E = 4E - E = 3E \)

Q.11  The aperture diameter of a telescope is 5 m. The separation between the moon and the earth is \( 4 \times 10^5 \text{ km} \). With light of wavelength of 5500 Å, the minimum separation between objects on the surface of moon, so that they are just resolved, is close to-

(1) \(20 \text{ m}\)  
(2) \(200 \text{ m}\)  
(3) \(60 \text{ m}\)  
(4) \(600 \text{ m}\)

Ans. [3]
Sol. \( \theta = 1.22 \frac{\lambda}{a} \)

\( d_{\text{min}} = 0D \)

\( = \frac{1.22\lambda D}{a} \)

\( \lambda = 5500 \times 10^{-10} \)

\( D = 4 \times 10^8 \text{ m} \)

\( a = 5\text{ m} \)

\( d_{\text{min}} = \frac{1.22 \times 5500 \times 10^{-10} \times 4 \times 10^8}{5} = 53.6 \text{ m} \)

Correct answer = 60 m
Q.12 Three solid spheres each of mass m and diameter d are struck together such that the lines connecting the centres form an equilateral triangle of side of length d. The ratio \( \frac{I_0}{I_A} \) of moment of inertia \( I_0 \) of the system about an axis passing the centroid and about center of any of the spheres \( I_A \) and perpendicular to the plane of the triangle is-

\[
\begin{align*}
1. & \quad \frac{13}{15} \\
2. & \quad \frac{15}{13} \\
3. & \quad \frac{23}{13} \\
4. & \quad \frac{13}{23}
\end{align*}
\]

Ans. [4]

Sol.

Distance; \( AB = d \)

\[ OA = \frac{d}{\sqrt{3}} \]

\[
I_0 = 3 \left[ \frac{2}{5}m \left( \frac{d}{2} \right)^2 + m \left( \frac{d}{\sqrt{3}} \right)^2 \right] = \frac{13md^2}{10}
\]

\[
I_A = \frac{2m}{5} \left( \frac{d}{2} \right)^2 + 2 \left[ \frac{2}{5}m \left( \frac{d}{2} \right)^2 + m(d)^2 \right] = \frac{23md^2}{10}
\]

\[
\frac{I_0}{I_A} = \frac{13}{23}
\]

Q.13 Consider two identical diatomic gases A and B at some temperature \( T \). Molecules of the gas A are rigid, and have a mass \( m \). Molecules of the gas B have an additional vibrational mode, and have a mass \( \frac{m}{4} \). The ratio of the specific heats \( (C_V^A \text{ and } C_V^B) \) of gas A and B, respectively is-

\[
\begin{align*}
1. & \quad 5 : 9 \\
2. & \quad 7 : 9 \\
3. & \quad 3 : 5 \\
4. & \quad 5 : 7
\end{align*}
\]

Ans. [4]

Sol.

\[
\begin{align*}
f_A &= 5 \\
f_B &= 7
\end{align*}
\]

\[
\frac{C_V^A}{C_V} = \frac{5R}{2} \quad \frac{C_V^B}{C_V} = \frac{7R}{2}
\]

Ratio= \frac{5}{7}
Q.14 A vessel of depth 2h is half filled with a liquid of refractive index \(2\sqrt{2}\) and the upper half with another liquid of refractive index \(\sqrt{2}\). The liquids are immiscible. The apparent depth of the inner surface of the bottom of vessel will be-

(1) \(\frac{3}{4}h\sqrt{2}\)  
(2) \(\frac{h}{2\sqrt{2}(\sqrt{2} + 1)}\)  
(3) \(\frac{h}{3\sqrt{2}}\)  
(4) \(\frac{h}{\sqrt{2}}\)

Ans. [1]

Sol.

\[
d = \frac{h}{2\sqrt{2}} + \frac{h}{\sqrt{2}} = \frac{3h}{2\sqrt{2}}
\]

\[
d = \frac{3\sqrt{2}h}{4}
\]

Q.15 Which of the following is an equivalent cyclic process corresponding to the thermodynamics cyclic given in the figure? Where, 1 \(\rightarrow\) 2 is adiabatic. (Graphs are schematic and are not to scale)

Ans. [4]

For process 1 \(\rightarrow\) 2 \(TV^{-1}\) = const.
For process 2 \(\rightarrow\) 3 \(\frac{V}{T}\) = const.
For process 3 \(\rightarrow\) 1 \(V =\) const.
Thus correct ans. is option 4
Q.16 Consider a force \( \vec{F} = -x\hat{i} + y\hat{j} \). The work done by this force in moving a particle from point A(1, 0) to B(0, 1) along the line segment is (all quantities are in SI units)

\[
W = \int_{A}^{B} \vec{F} \cdot d\vec{r} = \int_{0}^{1} -x\,dx + \int_{0}^{1} y\,dy
\]

\[
= \left[ -\frac{x^2}{2} \right]_{0}^{1} + \left[ \frac{y^2}{2} \right]_{0}^{1}
\]

\[W = 1 \text{ J}\]

Ans. \[\boxed{2}\]

Q.17 Consider a sphere of radius \( R \) which carries a uniform charge density \( \rho \). If a sphere of radius \( \frac{R}{2} \) is carved out of it, as shown, the ratio \( \frac{|\vec{E}_A|}{|\vec{E}_B|} \) of magnitude of electric field \( \vec{E}_A \) and \( \vec{E}_B \), respectively, at points A and B due to the remaining portion is-

\[
\frac{|\vec{E}_A|}{|\vec{E}_B|} = \frac{\rho R/2}{\rho R/6} = \frac{3}{2}
\]

Ans. \[\boxed{3}\]
\[ E_B = \frac{E \left( \frac{\rho}{3} \pi R^3 \right)}{R^2} - \frac{k \left( \frac{\rho}{3} \pi \frac{R^3}{8} \right)}{\left( \frac{3R}{2} \right)^2} = \frac{17\rho R}{54\varepsilon_0} \]

\[ \frac{E_A}{E_B} = \frac{\rho R / 6\varepsilon_0}{17 \rho R / 54 \varepsilon_0} = \frac{9}{17} \]

Correct ans. is \(\frac{18}{34}\)

**Q.18** Three harmonic waves having equal frequency \(\nu\) and same intensity \(I_0\), have phase angles, 0, \(\frac{\pi}{4}\) and \(-\frac{\pi}{4}\) respectively. When they are superimposed the intensity of the resultant wave is close to-

(1) 3\(I_0\)  
(2) 5.8\(I_0\)  
(3) \(I_0\)  
(4) 0.2\(I_0\)

Ans. [2]

\[ \vec{E}_1 = E_0 \hat{j} \cos(\omega t - kx) \text{ and } \vec{E}_2 = E_0 \hat{k} \cos(\omega t - ky) \]

At \(t = 0\), a particle of charge \(q\) is at origin with a velocity \(\vec{v} = 0.8\hat{c}\) \((c \text{ is the speed of light in vacuum})\). The instantaneous force experienced by the particle is-

(1) \(E_0q(0.4\hat{i} - 3\hat{j} + 0.8\hat{k})\)  
(2) \(E_0q(-0.8\hat{i} + \hat{j} + \hat{k})\)  
(3) \(E_0q(0.8\hat{i} - \hat{j} + 0.4\hat{k})\)  
(4) \(E_0q(0.8\hat{i} + \hat{j} + 0.2\hat{k})\)

Ans. [4]

at \(x = 0, t = 0\)

\[ \vec{E} = E_0 \hat{j} + E_0 \hat{k} \]

\[ \vec{B} = \frac{E_0}{c} \hat{k} + \frac{E_0}{c} \hat{i} \]

Using \(B_0 = \frac{E_0}{c}\)

\[ \hat{E} \times \hat{B} = \hat{c} \]

\[ \vec{F} = q[\vec{E} + (\vec{v} \times \vec{B})] \]

\[ \vec{F} = qE_0[0.8\hat{i} + \hat{j} + 0.2\hat{k}] \]
Q.20 Water flows in a horizontal tube (see figure). The pressure of water changes by 700 Nm\(^{-2}\) between A and B where the area of cross section are 40 cm\(^2\) and 20 cm\(^2\), respectively. Find the rate of flow of water through the tube. (density of water = 1000 kgm\(^{-3}\))

![Diagram of a horizontal tube with pressure drops from A to B](Fig)

(1) 1810 cm\(^3\)/s  
(2) 2720 cm\(^3\)/s  
(3) 3020 cm\(^3\)/s  
(4) 2420 cm\(^3\)/s

**Ans. [2]**

\[
P_A + \frac{1}{2} \rho V_A^2 = P_B + \frac{1}{2} \rho V_B^2 \quad \{h_A = h_B\}
\]

\[
P_A - P_B = \frac{1}{2} \rho \left( V_B^2 - V_A^2 \right) \quad ; \quad A_1 V_1 = A_2 V_2 \Rightarrow 2V_A = V_B
\]

\[
700 = \frac{1}{2} \times 10^3 \left[ 3V_A^2 \right]
\]

\[
V_A = \frac{14}{30} \Rightarrow V_A = 2720 \text{ cm}^3/\text{s}
\]

\[
\text{Flow rate} = AV = 40 \times 10^{-4} \times \sqrt{\frac{14}{30}} \approx 2720 \text{ cm}^3/\text{s}
\]

Q.21 The distance \(x\) covered by a particle in one dimensional motion varies with time \(t\) as \(x^2 = at^2 + 2bt + c\). If the acceleration of the particle depends on \(x\) as \(x^{-n}\), where \(n\) is an integer, the value of \(n\) is ________

**Ans. [3.00]**

\[
x^2 = at^2 + 2bt + c \quad .... \text{(i)}
\]

\[
2xv = 2at + 2b \quad .... \text{(ii)}
\]

\[
\Rightarrow v = \frac{at + b}{x}
\]

Differentiating (ii) w.r.t. time

\[
v + x a' = a
\]

\[
a' = \frac{a - v^2}{x}
\]

\[
a' = \frac{a - (at + b)^2}{x^2}
\]

\[
a' = \frac{1}{x^3}
\]

\[
n = 3
\]
Q.22 A body of mass \( m = 10 \text{ kg} \) is attached to one end of a wire of length 0.3 m. The maximum angular speed (in \( \text{rad s}^{-1} \)) with which it can be rotated about its other end in space station is (Breaking stress of wire = \( 4.8 \times 10^7 \text{ Nm}^{-2} \) and area of cross-section of the wire = \( 10^{-2} \text{ cm}^2 \)) is-

Ans. \([4.00]\)

\[
T = m\omega^2 \ell \quad ; \quad \sigma = \frac{T}{A}
\]

\[
\sigma = \frac{m\omega^2 \ell}{A}
\]

\[
\omega^2 = \frac{4.8 \times 10^7 \times 10^{-2} \times 10^{-4}}{10 \times 0.3}
\]

\[
\omega^2 = 16
\]

\[
\omega = 4 \text{ rad/s}
\]

Q.23 In a fluorescent lamp choke (a small transformer) 100 V of reverse voltage is produced with the choke current changes uniformly from 0.25 A to 0 in a duration of 0.025 ms. The self-inductance of the choke (in mH) is estimated to be ________

Ans. \([10.00]\)

\[
L \frac{di}{dt} = V
\]

\[
L \times \frac{0.25}{0.025 \times 10^{-3}} = 100
\]

\[
\Rightarrow L = 10^{-5} \text{ H}
\]

\[
\Rightarrow L = 10 \text{ mH}
\]

Q.24 One end of a straight uniform 1 m long bar is pivoted on horizontal table. It is released from rest when it makes an angle 30º from the horizontal (see figure). Its angular speed when it hits the table is given as \( \sqrt{n} \text{ s}^{-1} \), where \( n \) is an integer. The value of \( n \) is ________

Ans. \([15.00]\)

\[
\ell = 1 \text{ m}
\]

\[
30^\circ
\]
By energy conservation

\[ mgh = \frac{1}{2} I \omega^2 \]

\[ mg \frac{1}{2} \sin 30^\circ = \frac{1}{2} m \ell \omega^2 \]

\[ \omega^2 = 15 \Rightarrow \omega = \sqrt{15} \text{ rad/sec.} \]

\[ n = 15 \]

Q.25 Both the diodes used in the circuit shown are assumed to be ideal and have negligible resistance when these are forward biased. Built in potential in each diode is 0.7 V. For the input voltages shown in the figure, the voltage (in Volts) at point A is __________

\[ V_{in} = 12.7 \text{ V} \]

Ans. [12.00]

\[ V_A = 12.7 - 0.7 \]
\[ V_A = 12 \text{ V} \]