JEE (Main) 2020

COMPUTER BASED TEST (CBT)

Questions & Solutions

Date: 04 September, 2020 (SHIFT-2) | TIME: (03.00 p.m. to 06.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

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PART : MATHEMATICS

SECTION – 1 : (Maximum Marks : 80)

Straight Objective Type (सीधे व्याख्यात्मक प्रकार)

This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which Only One is correct.

1. Let \( f : (0, \infty) \to (0, \infty) \) be a differentiable function such that \( f(1) = e \) and \( \lim_{t \to x} \frac{t^2f^2(x) - x^2f^2(t)}{t - x} = 0 \). If \( f(x) = \frac{1}{e} \), then \( x \) is equal to:

   (1) \( \frac{1}{e} \)
   (2) \( 2e \)
   (3) \( \frac{1}{2e} \)
   (4) \( e \)

   **Ans.** (1)

   **Sol.**
   \[
   \lim_{t \to x} \frac{t^2f^2(x) - x^2f^2(t)}{t - x} = 0
   
   \text{using L'Hospital}
   
   \lim_{t \to x} \frac{2tf^2(x) - 2xf(t)f'(t)}{1} = 0
   
   x^2 f(x) f'(x) - 2x f(x) f'(x) = 0
   
   2x f(x) [x f'(x) - f(x)] = 0
   
   f(x) \neq 0 \text{ so } x f'(x) = f(x)
   
   dx
   
   x dy = y
   
   \frac{1}{y} dy = \frac{1}{x} dx
   
   \int ny = \int nx + \int nc
   
   y = cx \implies f(x) = cx
   
   \text{Now } f(1) = c = e
   
   \text{so } f(x) = ex
   
   \text{now } f(x) = 1
   
   ex = 1 \implies x = \frac{1}{e}
   
2. Contrapositive of the statement:

   ‘If a function \( f \) is differentiable at \( a \), then it is also continuous at \( a \), is:

   (1) If a function \( f \) is not continuous at \( a \), then it is not differentiable at \( a \).
   (2) If a function \( f \) is continuous at \( a \), then it is differentiable at \( a \).
   (3) If a function \( f \) is not continuous at \( a \), then it is differentiable at \( a \).
   (4) If a function \( f \) is continuous at \( a \), then it is not differentiable at \( a \).

   **Ans.** (1)

   **Sol.** Contrapositive of \( p \Rightarrow q \) is \( \neg q \Rightarrow \neg p \)
3. The solution of the differential equation \( \frac{dy}{dx} - \frac{y + 3x}{\log_e(y + 3x)} + 3 = 0 \) is

(where C is a constant of integration)

(1) \( x - 2\log_e(y + 3x) = C \)

(2) \( x - \log_e(y + 3x) = C \)

(3) \( y + 3x - \frac{1}{2} (\log_e x^2) = C \)

(4) \( y - \frac{1}{2} \left( \log_e(y + 3x) \right)^2 = C \)

Ans. (4)

Sol. \( \frac{dy}{dx} - \frac{y + 3x}{\ln(y + 3x)} + 3 = 0 \)

\( \frac{dy}{dx} + 3 = \frac{y + 3x}{\ln(y + 3x)} \)

\( \frac{d}{dx} (y + 3x) = \frac{y + 3x}{\ln(y + 3x)} \)

\( \int \frac{\ln(y + 3x)}{y + 3x} \, dy = \int dx \)

Let \( \ln(y + 3x) = t \)

\( \int \frac{1}{y + 3x} \, dt = \int dx \)

\( \frac{t^2}{2} = x + c \)

\( \left( \ln(y + 3x) \right)^2 = x + c \)

4. If for some positive integer \( n \), the coefficients of three consecutive terms in the binomial expansion of \( (1 + x)^n \) are in the ratio 5 : 10 : 14, then the largest coefficient in the expansion is :

(1) 330

(2) 252

(3) 792

(4) 462

Ans. (4)

Sol. Let three consecutive term be \( T_r, T_{r+1}, T_{r+2} \)

Hence \( \frac{T_r}{T_{r+1}} = \frac{5}{10} \) and \( \frac{T_{r+1}}{T_{r+2}} = \frac{10}{14} \)

\( \frac{T_{r+1}}{T_r} = 2 \)

\( \frac{n-5}{C_r} = \frac{n-5}{C_{r+1}} = \frac{5}{7} \)

\( \frac{n-5}{C_{r-1}} = 2 \)

\( \frac{n-5}{C_r} = \frac{n-5}{C_{r+1}} = \frac{7}{5} \)

\( \frac{(n+5) - r + 1}{r} = 2 \)

\( \frac{(n+5) - (r+1) + 1}{r + 1} = \frac{7}{5} \)

\( n - r + 6 = 2r \)

\( n - 3r + 6 = 0 \) \( \ldots \ldots \) (i)

\( 5n - 5r + 25 = 7r + 7 \)

\( 5n - 12r + 18 = 0 \) \( \ldots \ldots \) (ii)
5. The circle passing through the intersection of the circles, \( x^2 + y^2 - 6x = 0 \) and \( x^2 + y^2 - 4y = 0 \), having its centre on the line, \( 2x - 3y + 12 = 0 \), also passes through the point:

(1) \((1, -3)\)  (2) \((-1, 3)\)  (3) \((-3, 6)\)  (4) \((-3, 1)\)

Ans. (3)

Sol. By family of circle, passing through intersection of given circle will be member of

\[ S_1 + \lambda S_2 = 0 \] family \( (\lambda \neq 1) \)

\[ (x^2 + y^2 - 6x) + \lambda(x^2 + y^2 - 4y) = 0 \]

\[ (\lambda + 1)2 + (\lambda + 1)y^2 - 6x - 4\lambda y = 0 \]

\[ x^2 + y^2 - \frac{6}{\lambda + 1}x - \frac{4\lambda}{\lambda + 1}y = 0 \]

Centre \( \left( \frac{3}{\lambda + 1}, \frac{2\lambda}{\lambda + 1} \right) \)

Centre lies on \( 2x - 3y + 12 = 0 \)

\[ 2 \left( \frac{3}{\lambda + 1} \right) - 3 \left( \frac{2\lambda}{\lambda + 1} \right) + 12 = 0 \]

\[ 6\lambda + 18 = 0 \]

\[ \lambda = -3 \]

Circle \( x^2 + y^2 + 3x - 6y = 0 \)

6. In a game two players A and B take turns in throwing a pair of fair dice starting with player A and total of scores on the two dice, in each throw is noted. A wins the game if he throws a total of 6 before B throws a total of 7 and B wins the game if he throws a total of 7 before A throws a total of six. The game stops as soon as either of the players wins. The probability of A winning the game is:

<table>
<thead>
<tr>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{5}{6} )</td>
<td>( \frac{5}{31} )</td>
<td>( \frac{5}{31} )</td>
<td>( \frac{31}{61} )</td>
</tr>
</tbody>
</table>

Ans. (1)

Sol. sum 6 \( \rightarrow (1, 5), (5, 1), (3, 3), (2, 4), (4, 2) \)

sum 4 \( \rightarrow (1, 6), (6, 1), (5, 2), (2, 5), (3, 4), (4, 3) \)

\[ P(A \text{ wins}) = P(A) + P(\overline{A}).P(\overline{B}).P(A) + P(\overline{A}).P(\overline{B}).P(\overline{A}) + \ldots \]

this is infinite G.P. with common ratio \( P(\overline{A}) \times P(\overline{B}) \)

Probability of A wins = \[
= \frac{\frac{5}{36}}{1 - P(A) \times P(\overline{B})}
= \frac{\frac{5}{36}}{1 - \frac{30}{61}} = \frac{30}{61}
\]
7. The angle of elevation of a cloud C from a point P, 200 m above a still take is 30°. If the angle of depression of the image of C in the lake from the point P is 60°, then PC (in m) is equal to

(1) 100  (2) 400 $\sqrt{3}$  (3) 200 $\sqrt{3}$  (4) 400

Ans. (4)

Sol.

\[ \tan 30° = \frac{x}{\sqrt{3}} \quad \Rightarrow \quad y = \sqrt{3}x \quad \text{and} \quad \tan 60° = \frac{x + 400}{y} \]

\[ x + 400 = 3x \]
\[ 2x = 400, \quad x = 200 \]
\[ \sin 30° = \frac{200}{PC} \quad \Rightarrow \quad PC = 400 \]

8. The function \( f(x) = \begin{cases} \frac{\pi}{4} + \tan^{-1} x, & |x| \leq 1 \\ \frac{1}{2}(|x|-1), & |x| > 1 \end{cases} \) is:

(1) continuous on \( \mathbb{R} - \{-1, 1\} \) and differentiable on \( \mathbb{R} - \{-1, 1\} \).

(2) both continuous and differentiable on \( \mathbb{R} - \{-1\} \).

(3) both continuous and differentiable on \( \mathbb{R} - \{1\} \).

(4) continuous on \( \mathbb{R} - \{1\} \) and differentiable on \( \mathbb{R} - \{-1, 1\} \).

Ans. (4)
Sol. \( f(x) = \begin{cases} \frac{\pi}{4} + \tan^{-1} x, & \text{if } |x| \leq 1 \\ \frac{1}{2}(|x| - 1), & \text{if } |x| > 1 \end{cases} \)

Graph of \( f(x) \) is

\( f(x) \) is continuous on \( \mathbb{R} - \{1\} \)
\( f(x) \) is differentiable on \( \mathbb{R} - \{-1, 1\} \)

9. Suppose the vectors \( x_1, x_2 \) and \( x_3 \) are the solutions of the system of linear equations, \( Ax = b \) when the vector \( b \) on the right side is equal to \( b_1, b_2 \) and \( b_3 \) respectively. If

\[
x_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad x_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad x_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad b_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad b_2 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \quad b_3 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix},
\]

then the determinant of \( A \) is equal of \( A \) is equal to:

(1) \( \frac{3}{2} \) \( \frac{3}{2} \)
(2) \( 4 \)
(3) \( \frac{1}{2} \)
(4) \( 2 \)

\[ \text{Ans. (4)} \]

Sol. Let \( A = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{bmatrix} \)

\[
Ax_1 = b_1 \Rightarrow \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}
\]

\( \alpha_1 + \alpha_2 + \alpha_3 = 1 \)
\( \beta_1 + \beta_2 + \beta_3 = 0 \)
\( \gamma_1 + \gamma_2 + \gamma_3 = 0 \)

similar \( 2\alpha_2 + \alpha_3 = 0 \) \( \alpha_3 = 0 \)
\( 2\beta_2 + \beta_3 = 2 \) \( \beta_3 = 0 \)
\( 2\gamma_2 + \gamma_3 = 0 \) \( \gamma_3 = 2 \)

\( \therefore \alpha_2 = 0, \beta_2 = 1, \gamma_2 = -1 \)
\( \alpha_1 = 1, \beta_1 = -1, \gamma_1 = -1 \)

\[ A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \end{bmatrix} \]

\[ |A| = 2 \]
10. Let \( a_1, a_2, \ldots, a_n \) be a given A.P. whose common difference is an integer and 
\[ S_n = a_1 + a_2 + \ldots + a_n. \] If \( a_1 = 1, a_{15} = 300 \) and \( 15 \leq n \leq 50 \), then the ordered pair \((S_{n-4}, a_{n-4})\) is equal to:

(1) \((2490, 248)\) \quad (2) \((2490, 249)\) \quad (3) \((2490, 249)\) \quad (4) \((2480, 248)\)

**Ans. (1)**

**Sol.**

\[ a_n = a_1 + (n-1)d \]
\[ \Rightarrow d = \frac{299}{(n-1)} = \frac{13 \times 23}{(n-1)} \text{ integer} \]

so \( n-1 = \pm 13, \pm 23, \pm 299, \pm 1 \)
\[ \Rightarrow n = 14, -12, 24, -22, 300, -298, 2, 0 \]

But \( n \in [15, 50] \) \( \Rightarrow n = 24 \) \( \Rightarrow d = 13 \)

Hence \( S_{n-4} = S_{20} = \frac{20}{2} [2(1) + (20 - 1)(13)] = 10[2 + 247] = 2490 \)

\[ a_{n-4} = a_{20} = a_1 + 19d \]
\[ = 1 + 19 \times 13 \]
\[ = 247 \]
\[ = 248 \]

11. Let \( \bigcup_{i=1}^{50} X_i = \bigcup_{i=1}^{n} Y_i = T \), where each \( X_i \) contains 10 elements and each \( Y_i \) contains 5 elements. If each element of the set \( T \) is an element of exactly 20 sets \( X_i \)'s and exactly 6 sets \( Y_i \)'s then \( n \) is equal to:

(1) 45 \quad (2) 15 \quad (3) 30 \quad (4) 50

**Ans. (3)**

**Sol.**

\[ \bigcup_{i=1}^{50} X_i = \bigcup_{i=1}^{n} Y_i = Z \]
\[ \Rightarrow \frac{10 \times 50}{20} = \frac{5n}{6} \]
\[ \Rightarrow n = 30 \]

12. The area (in sq. units) of the largest rectangle \( ABCD \) whose vertices \( A \) and \( B \) lie on the \( x \)-axis and vertices \( C \) and \( D \) lie on the parabola, \( y = x^2 - 1 \) below the \( x \)-axis, is:

(1) \( \frac{1}{3\sqrt{3}} \) \quad (2) \( \frac{4}{3} \) \quad (3) \( \frac{4}{3\sqrt{3}} \) \quad (4) \( \frac{2}{3\sqrt{3}} \)

**Ans. (3)**

**Sol.**

\[ A(\alpha, 0), \beta(-\alpha, 0) \]
\[ \Rightarrow D(\alpha, \alpha^2 - 1) \]

Area \( ABCD = (AB)(AD) \)
\[ \Rightarrow S = (2\alpha)(1-\alpha^2) = 2\alpha - 2\alpha^3 \]
\[ \frac{ds}{d\alpha} = 2 - 6\alpha^2 = 0 \Rightarrow \alpha^2 = \frac{1}{3} \Rightarrow \alpha = \frac{1}{\sqrt{3}} \]

Area \( = 2\alpha - 2\alpha^3 \)
\[ = \frac{2}{\sqrt{3}} \]
13. Let \( x = 4 \) be a directrix to an ellipse whose centre is at the origin and its eccentricity is \( \frac{1}{2} \). If \( P(1, \beta) \), \( \beta > 0 \) is a point on this ellipse, then the equation of the normal to it at \( P \) is

\[
(1) \quad 7x - 4y = 1 \quad (2) \quad 4x - 2y = 1 \quad (3) \quad 8x - 2y = 5 \quad (4) \quad 4x - 3y = 2
\]

Ans. \( (2) \)

Sol.
\[
a = 4 \Rightarrow a = 4e \Rightarrow a = 2
\]
\[
b^2 = a^2 (1 - e^2) = 3
\]
\[
(1, \beta) \text{ lies on } \frac{x^2}{4} + \frac{y^2}{\frac{3}{4}} = 1 \Rightarrow \frac{1}{4} + \frac{\beta^2}{\frac{3}{4}} = 1 \Rightarrow \beta^2 = \frac{9}{16} \Rightarrow \beta = \frac{3}{2} \quad (\therefore \beta > 0)
\]

Normal at \( (1, \beta) \) is
\[
\frac{a^2x}{1} - \frac{b^2y}{\beta} = a^2 - b^2 \Rightarrow 4x - \frac{3y}{\beta} = 1
\]

so equation of normal is \( 4x - 2y = 1 \)

14. The distance of the point \( (1, -2, 3) \) from the plane \( x - y + z = 5 \) measured parallel to the line

\[
\frac{x}{2} = \frac{y}{3} = \frac{z}{-6} \text{ is}:
\]

(1) \( \frac{1}{7} \) \quad (2) \( \frac{7}{5} \) \quad (3) \( 1 \) \quad (4) \( \frac{7}{5} \)

Ans. \( (3) \)

Sol.
\[
\frac{x}{2} = \frac{y}{3} = \frac{z}{-6} \quad \text{Let } Q \equiv (2\lambda + 1, 3\lambda - 2, -6\lambda + 3)
\]

\( Q \) lies on \( x - y + z = 5 \)
\[
\Rightarrow (2\lambda + 1) - (3\lambda - 2) + (-6\lambda + 3) = 5 \Rightarrow \lambda = \frac{1}{7} \Rightarrow Q \equiv \left( \frac{9}{7}, \frac{11}{7}, \frac{15}{7} \right)
\]

\[
PQ = \sqrt{\left(1 - \frac{9}{7}\right)^2 + \left(-2 + \frac{11}{7}\right)^2 + \left(3 - \frac{15}{7}\right)^2} = 1
\]
15. If the perpendicular bisector of the line segment joining the points P(1, 4) and Q(k, 3) has y-intercept equal to -4, then a value of k is:

(1) \(\sqrt{15}\)  
(2) -4  
(3) -2  
(4) \(\sqrt{14}\)

Ans. (2)

Sol. Mid point PQ \(\left(\frac{k+1}{2}, \frac{7}{2}\right)\)

and slope of PQ = \(-\frac{1}{1-k}\)

so equation of perpendicular bisector of PQ

\(y - \frac{7}{2} = (k-1) \left(x - \frac{k+1}{2}\right)\) .......(1)

Now it's y intercept = -4

so equation (1) satisfy (0, -4)

\[\Rightarrow -\frac{15}{2} = -\left(\frac{k^2-1}{2}\right)\]

\[k^2 = 16 \Rightarrow k = 4\]

16. The integral \(\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \tan^3 x \cdot \sin^2 3x(2\sec^2 x \sin^2 3x + 3\tan x \sin 6x)dx\) is equal to:

(1) \(-\frac{1}{9}\)  
(2) \(\frac{9}{2}\)  
(3) \(-\frac{1}{18}\)  
(4) \(\frac{7}{18}\)

Ans. (3)

Sol. 
\[
\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{d}{dx} \left(\tan^4 x \cdot \sin^2 3x + \frac{d}{dx} \left(\sin^4 3x\right)\right) dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{2} \cdot \frac{d}{dx} \left(\tan^4 x \cdot \sin^4 3x\right) dx
\]

\[
\left[\tan^4 x \cdot \sin^4 3x\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \left[\left(3^{\frac{1}{4}} \cdot \frac{1}{\sqrt{3}}\right)\right] - \left[\left(\frac{1}{3}\right)^{\frac{1}{4}} \cdot \frac{1}{\sqrt{3}}\right] = -\frac{1}{2} \cdot \frac{9}{2} = -\frac{1}{2} \cdot \frac{9}{18}
\]

17. The minimum value of \(2^\sin x + 2^\cos x\) is:

(1) \(2^{-1+\frac{1}{\sqrt{2}}}\)  
(2) \(2^{-1+\frac{\sqrt{2}}{2}}\)  
(3) \(2^{-1-\frac{\sqrt{2}}{2}}\)  
(4) \(2^\frac{1}{\sqrt{2}}\)

Ans. (4)

Sol. Using A.M. \(\geq\) G.M.

\[
\frac{2^\sin x + 2^\cos x}{2} \geq \sqrt{2^\sin x \cdot 2^\cos x}
\]

\[
\frac{2^\sin x + 2^\cos x}{2} \geq 2^{\sin x + \cos x}\]

...(i)

Now \(-\sqrt{2} \leq \sin x + \cos x = \sqrt{2}\)
so \(-\frac{1}{\sqrt{2}} \leq \frac{\sin x + \cos x}{2} \leq \frac{1}{\sqrt{2}}\)

Minimum value of \(\frac{\sin x + \cos x}{2} = 2 \frac{1}{\sqrt{2}}\)

so by (i)

Minimum value of \(\frac{2\sin x + 2\cos x}{2} = 2 \frac{1}{\sqrt{2}}\)

Minimum value of \(2^\sin x + 2^\cos x = 2 \sqrt{2} + 2 = 2^{1+\frac{1}{\sqrt{2}}}\)

18. If a and b are real numbers such that \((2 + \alpha)^4 = a + b\alpha\), where \(\alpha = \frac{-1+i\sqrt{3}}{2}\) then a + b is equal to:

(1) 33  (2) 24  (3) 9  (4) 57

Ans. (3)

Sol. \((2 + \alpha)^4 = a + b\alpha\)

\((4 + \alpha^2 + 4\alpha)^2 = a + b\alpha \therefore 1 + \alpha = -\alpha^2\)

\(9\alpha^4 = a + b\alpha\)

\(9\alpha = a + b\alpha \Rightarrow a = 0, b = 9 \Rightarrow a + b = 9\)

19. Let \(\lambda = 0\) be in R. If \(\alpha\) and \(\beta\) are the roots of the equation, \(x^2 - x + 2\lambda = 0\) and \(\alpha\) and \(\gamma\) are the roots of the equation, \(3x^2 - 10x + 27\lambda = 0\), then \(\frac{\beta \gamma}{\lambda}\) is equal to:

(1) 27  (2) 36  (3) 9  (4) 18

Ans. (4)

Sol. Given \(3\alpha^2 - 10\alpha + 27\lambda = 0\) ....(i)

\(3\alpha^2 - 3\alpha + 6\lambda = 0\) ....(ii)

Subtract \(-7\alpha + 21\lambda = 0\)

\(3\lambda = \alpha\)

By (ii) \(9\alpha^2 - 3\alpha + 2\lambda = 0\)

\(\Rightarrow \lambda = 0, \frac{1}{9}\)

\(\therefore\) given equation are \(x^2 - x + \frac{2}{9} = 0\) and \(3x^2 - 10x + 3 = 0\)

\(\therefore \alpha = \frac{1}{3}, \beta = \frac{2}{3}, \alpha = \frac{1}{3}, \gamma = 3\)

\(\therefore \frac{\beta \gamma}{\lambda} = \frac{\frac{2}{3} \cdot 3}{\frac{1}{9}} = 18\)
20. If the system of equations
\[\begin{align*}
x + y + z &= 2 \\
2x + 4y - z &= 6 \\
3x + 2y + \lambda z &= \mu
\end{align*}\]
(1) \(\lambda + 2\mu = 14\)  
(2) \(2\lambda - \mu = 5\)  
(3) \(2\lambda + \mu = 14\)  
(4) \(\lambda - 2\mu = -5\)

\[\text{Ans. (3)}\]

\[\text{Sol. } D = 0 \Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & -1 \\ 3 & 2 & \lambda \end{vmatrix} = 0 \Rightarrow \lambda = \frac{9}{2} p\]

\[D_3 = 0 \Rightarrow \begin{vmatrix} 1 & 1 & 2 \\ 2 & 4 & 6 \\ 3 & 2 & \mu \end{vmatrix} = 0 \Rightarrow \mu = 5\]

**SECTION – 2 : (Maximum Marks : 20)**

- This section contains **FIVE (05)** questions. The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal up to one digit.
- If the numerical value has more than two decimal places **truncate/round-off** the value up to **TWO** decimal places.
  - **Full Marks : +4** If ONLY the correct option is chosen.
  - **Zero Marks : 0** In all other cases

**खंड 2 (अंकक अंक : 20)**

- इस खंड में पौंड (05) प्रश्न है। प्रत्येक प्रश्न का उत्तर संख्यात्मक मान (NUMERICAL VALUE) है, जो हि–अंकीय पूर्णाक तथा दशमलव एकल–अंकन में है।
- यदि संख्यात्मक मान में दो से अधिक दशमलव स्थान है, तो संख्यात्मक मान को दशमलव के दो स्थानों तक **ट्रांस्फर/राउंड ऑफ** (truncate/round-off) करें।
- **अंकन योजना :**
  - पूर्ण अंक : +4 यदि निर्दिष्ट सही फिक्स्ट ही चुना गया है।
  - शून्य अंक : 0 अन्य सभी परिस्थितियों में।

21. Let PO be a diameter of the circle \(x^2 + y^2 = 9\). If \(\alpha\) and \(\beta\) are the lengths of the perpendiculars from P and Q on the straight line, \(x + y = 2\) respectively, then the maximum value of \(\alpha\beta\) is ……

**Ans. 7**

**Sol.** Let \(P(3\cos\theta, 3\sin\theta) \colon Q(-3\cos\theta, -3\sin\theta)\)
given line \(x + y - 2 = 0\)
\[\alpha = \frac{|3\cos\theta + 3\sin\theta - 2|}{\sqrt{2}}\]
\[\beta = \frac{|-3\cos\theta - 3\sin\theta - 2|}{\sqrt{2}}\]
\[\therefore \alpha\beta = \frac{|(3\cos\theta + 3\sin\theta - 2)(3\cos\theta + 3\sin\theta + 2)|}{2} = \frac{9(1 + \sin 2\theta) - 4}{2}\]

\[\therefore \text{maximum } \alpha\beta = 7\]
22. If the variance of the following frequency distribution:
   
<p>|</p>
<table>
<thead>
<tr>
<th>Class</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>10-20</td>
<td>2</td>
</tr>
<tr>
<td>20-30</td>
<td>x</td>
</tr>
<tr>
<td>30-40</td>
<td>2</td>
</tr>
</tbody>
</table>
   
   is 50, then x is equal to ……

   Ans. 4

   Sol. 
   
   \[ x = \frac{\sum f_i \cdot x_i}{\sum f_i} = \frac{30 + 25x + 70}{4 + x} = 25 \]

   \[ \sigma^2 = 50 = \frac{\sum f_i \cdot x_i^2}{\sum f_i} - (\bar{x})^2 \]

   \[ 50 = \frac{450 + 625x + 2450}{4 + x} - (25)^2 \]

   \[ 50 = \frac{2900 + 625x}{4 + x} - 625 \Rightarrow 675 (4 + x) = 2900 + 625 x \Rightarrow 50x = 200 \Rightarrow x = 4 \]

23. A test consists of 6 multiple choice questions, each having 4 alternative answers of which only one is correct. The number of ways, in which a candidate answers all six questions such that exactly four of the answers are correct, is ……

   Ans. 135

   Sol. No. of ways of giving wrong answer = 3

   required no. of ways = \[ 6C_4 (1)^4 \times (3)^2 \]

   = 15(9) = 135

24. Let \( [x] \) and \( \{x\} \) denote the fractional part of \( x \) and the greatest integer \( \leq x \) respectively of a real number \( x \).

   if \[ \int_0^n \{x\} \, dx \text{, } \int_0^n [x] \, dx \text{ and } 10(n^2 - n) \text{, } (n \in \mathbb{N}, n > 1) \]

   are three consecutive terms of a G.P. then \( n \) is equal to ……

   Ans. 21

   Sol. 
   
   \[ \int_0^n \{x\} \, dx = n \int_0^1 x \, dx = n \left( \frac{x^2}{2} \right)_0^n = \frac{n}{2} \]

   and \[ \int_0^n [x] \, dx = \int_0^n (x - \{x\}) \, dx = \left( \frac{x^2}{2} \right)_0^n - \int_0^n \{x\} \, dx = \frac{n^2}{2} - \frac{n}{2} \]

   now \[ \frac{n^2}{2} - \frac{n}{2} \text{ and } 10(n^2 - n) \text{ are in Geometric progression} \]

   \[ (\frac{n^2}{2} - \frac{n}{2})^2 = \frac{n}{2} \cdot 10(n^2 - n) \]

   \[ \Rightarrow \frac{n^2(n - 1)^2}{4} = 5.n^2(n - 1) \]

   \[ \Rightarrow n - 1 = 20 \Rightarrow n = 21 \]
25. IF \( \vec{a} = 2\hat{i} + \hat{j} + 2\hat{k} \), then the value of \( |i \times (\vec{a} \times i)|^2 + |j \times (\vec{a} \times j)|^2 + |k \times (\vec{a} \times k)|^2 \) is equal to:

Ans. 18.

Sol. Let \( \vec{a} = x\hat{i} + y\hat{j} + z\hat{k} \)

\[
i \times (\vec{a} \times i) = (i \times i)\vec{a} - (i \times i)\vec{a} = y\hat{j} + z\hat{k}
\]

Similarly \( j \times (\vec{a} \times j) = x\hat{i} + z\hat{k} \) and \( k \times (\vec{a} \times k) = x\hat{i} + y\hat{k} \)

\[
|i \times (\vec{a} \times i)|^2 + |j \times (\vec{a} \times j)|^2 + |k \times (\vec{a} \times k)|^2
\]

\[
|x\hat{j} + z\hat{k}|^2 + |x\hat{i} + z\hat{k}|^2 + |x\hat{i} + y\hat{k}|^2 = 2|a|^2 = 2(9) = 18
\]
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