

JEE Main 2023 April 6 Shift 2 Mathematics Question Paper

Time Allowed :1 Hour	Maximum Marks :100	Total Questions :30
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General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
 - (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and -1 mark for wrong answer.
 - (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and -1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

1. If $\gcd(m, n) = 1$ and $1^2 - 2^2 + 3^2 - 4^2 + \dots + (2021)^2 - (2022)^2 + (2023)^2 = 1012m^2n$, then $m^2 - n^2$ is equal to:

Options:

- (1) 180
- (2) 220
- (3) 200
- (4) 240

2. The area bounded by the curves $y = |x - 1| + |x - 2|$ and $y = 3$ is equal to:

Options:

- (1) 5
- (2) 4
- (3) 6
- (4) 3

3. For the system of equations $x + y + z = 6$, $x + 2y + \alpha z = 10$, $x + 3y + 5z = \beta$, which one of the following is NOT true:

Options:

- (1) System has a unique solution for $\alpha = 3$, $\beta \neq 14$.

- (2) System has a unique solution for $\alpha = -3, \beta = 14$.
 (3) System has no solution for $\alpha = 3, \beta = 24$.
 (4) System has infinitely many solutions for $\alpha = 3, \beta = 14$.
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4. Among the statements:

- (S1): $(p \Rightarrow q) \vee ((\sim p) \wedge q)$ is a tautology
- (S2): $(q \Rightarrow p) \Rightarrow ((\sim p) \wedge q)$ is a contradiction

Options:

1. Only (S2) is True
 2. Only (S1) is True
 3. Neither (S1) and (S2) is True
 4. Both (S1) and (S2) are True
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5. Evaluate the limit

$$\lim_{n \rightarrow \infty} \{(2^{\frac{1}{2}} - 2^{\frac{1}{3}})(2^{\frac{1}{2}} - 2^{\frac{1}{5}}) \dots (2^{\frac{1}{2}} - 2^{\frac{1}{2n+1}})\}$$

Options:

- (1) $\frac{1}{\sqrt{2}}$
 - (2) $\sqrt{2}$
 - (3) 1
 - (4) 0
-

6. Let P be a square matrix such that $P^2 = I - P$. For $\alpha, \beta, \gamma, \delta \in N$, if

$$P^\alpha + P^\beta = \gamma I - 29P \quad \text{and} \quad P^\alpha - P^\beta = \delta I - 13P$$

then $\alpha + \beta + \gamma - \delta$ is equal to :

Options:

- (1) 40
 - (2) 22
 - (3) 24
 - (4) 18
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7. A plane P contains the line of intersection of the planes $\vec{r} \cdot (i + j + k) = 6$ and $\vec{r} \cdot (2i + 3j + 4k) = -5$. If P passes through the point $(0, 2, -2)$, then the square of distance of the point $(12, 12, 18)$ from the plane P is:

Options:

- (1) 620
 - (2) 1240
 - (3) 310
 - (4) 155
-

8. Let $f(x)$ be a function satisfying $f(x) + f(\pi - x) = \pi^2$, for all $x \in R$. Then $\int_0^\pi f(x) \sin x \, dx$ is equal to:

Options:

- (1) $\frac{\pi^2}{2}$
 - (2) π^2
 - (3) $2\pi^2$
 - (4) $\frac{\pi^2}{4}$
-

9. If the coefficients of x^7 in $(ax^2 + \frac{1}{2bx})^{11}$ and x^7 in $(ax - \frac{1}{3bx^2})^{11}$ are equal, then:

Options:

- (1) $64ab = 243$
 - (2) $32ab = 729$
 - (3) $729ab = 32$
 - (4) $243ab = 64$
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10. If the tangents at the points P and Q on the circle $x^2 + y^2 - 2x + y = 5$ meet at the point $R(9/4, 2)$, then the area of the triangle PQR is:

Options:

- (1) $\frac{5}{4}$
 - (2) $\frac{13}{4}$
 - (3) $\frac{5}{8}$
 - (4) $\frac{13}{8}$
-

11. Three dice are rolled. If the probability of getting different numbers on the three dice is $\frac{p}{q}$, where p and q are co-prime, then $q - p$ is equal to:

Options:

- (1) 1
 - (2) 2
 - (3) 4
 - (4) 3
-

12. In a group of 100 persons, 75 speak English and 40 speak Hindi. Each person speaks at least one of the two languages. If the number of persons who speak only English is α and the number of persons who speak only Hindi is β , then the

eccentricity of the ellipse $25(\beta^2 x^2 + \alpha^2 y^2) = \alpha^2 \beta^2$ is:

Options:

- (1) $\frac{\sqrt{129}}{12}$
 - (2) $\frac{\sqrt{117}}{12}$
 - (3) $\frac{\sqrt{119}}{12}$
 - (4) $\frac{3\sqrt{15}}{12}$
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13. If the solution curve $f(x, y) = 0$ of the differential equation $(1 + \ln x) \frac{dx}{dy} - x \ln x = e^y, x > 0$, passes through the points $(1, 0)$ and $(\alpha, 2)$, then α^α is equal to:

Options:

- (1) $e^{\sqrt{2}e^2}$
 - (2) e^{e^2}
 - (3) $e^{2e^{\sqrt{2}}}$
 - (4) e^{2e^2}
-

14. Let the sets A and B denote the domain and range respectively of the function $f(x) = \frac{1}{\sqrt{[x]-x}}$, where $[x]$ denotes the smallest integer greater than or equal to x .

Then among the statements:

Options:

- (1) Only (S1) is true
 - (2) Neither (S1) nor (S2) is true
 - (3) Only (S2) is true
 - (4) Both (S1) and (S2) are true
-

15. Let $a \neq b$ be two real numbers. Then the number of elements in the set $X = \{z \in C : \operatorname{Re}(az^2 + bz) = a, \operatorname{Re}(bz^2 + az) = b\}$ is equal to:

Options:

- (1) 0
 - (2) 2
 - (3) 1
 - (4) 3
-

16. The sum of all values of α , for which the points whose position vectors are $\vec{i} - 2\vec{j} + 3\vec{k}$, $2\vec{i} - 3\vec{j} + 4\vec{k}$, $(\alpha + 1)\vec{i} + 2\vec{k}$ and $9\vec{i} + (\alpha - 8)\vec{j} + 6\vec{k}$ are coplanar, is equal to:

Options:

- (1) -2
- (2) 2
- (3) 6
- (4) 4

17. Let line L pass through the point $(0, 1, 2)$, intersect the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$, and be parallel to the plane $2x + y - 3z = 4$. Find the distance of the point $P(1, -9, 2)$ from line L.

- (1) 9
- (2) $\sqrt{54}$
- (3) $\sqrt{69}$
- (4) $\sqrt{74}$

18. All the letters of the word PUBLIC are written in all possible orders and these words are written as in a dictionary with serial numbers. Then the serial number of the word PUBLIC is:

Options:

- (1) 580
- (2) 578
- (3) 576
- (4) 582

19. Let the vectors $\vec{a}, \vec{b}, \vec{c}$ represent three coterminous edges of a parallelepiped of volume V . Then the volume of the parallelepiped, whose coterminous edges are represented by $\vec{a}, \vec{b} + \vec{c}$ and $\vec{a} + 2\vec{b} + 3\vec{c}$ is equal to:

Options:

- (1) $2V$
- (2) $6V$
- (3) $3V$
- (4) V

20. Among the statements:

(S1): $2023^{2022} - 1999^{2022}$ is divisible by 8

(S2): $13(13^n) - 11n - 13$ is divisible by 144 for infinitely many $n \in \mathbb{N}$

Options:

- (1) Only (S2) is correct
- (2) Only (S1) is correct
- (3) Both (S1) and (S2) are incorrect
- (4) Both (S1) and (S2) are correct

21. The value of $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$ is:

22. If $(20)^{19} + 2(21)(20)^{18} + 3(21)^2(20)^{17} + \dots + 20(21)^{19} = k(20)^{19}$, then k is equal to:

23. Let the eccentricity of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ be reciprocal to that of the hyperbola $2x^2 - 2y^2 = 1$. If the ellipse intersects the hyperbola at right angles, then the square of the length of the latus-rectum of the ellipse is:

24. For $\alpha, \beta, z \in C$ and $\lambda > 1$, if $\sqrt{\lambda - 1}$ is the radius of the circle $|z - \alpha|^2 + |z - \beta|^2 = 2\lambda$, then $|\alpha - \beta|$ is equal to:

25. Let a curve $y = f(x)$, $x \in (0, \infty)$ pass through the points $(1, \frac{3}{2})$ and $(a, \frac{1}{2})$. If the tangent at any point $R(b, f(b))$ to the given curve cuts the y-axis at the points $S(0, c)$ such that $bc = 3$, then $(PQ)^2$ is equal to:

26. If the lines

$$\frac{x-1}{2} = \frac{2-y}{-3} = \frac{z-3}{\alpha} \quad \text{and} \quad \frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{\beta}$$

intersect, then the magnitude of the minimum value of $8\alpha\beta$ is:

27. Let $f(x) = \frac{x}{1+x^{n1/n}}$, where $x \in R - \{-1\}$, and $n \in N, n > 2$. If $f^n(x) = f(f(\dots f(x)))$ (up to n times), then

$$\lim_{n \rightarrow \infty} \int_0^1 x^{n-2} \cdot f^n(x) dx \quad \text{is equal to:}$$

28. If the mean and variance of the frequency distribution are 9 and 15.08 respectively, then the value of $\alpha^2 + \beta^2 - \alpha\beta$ is:

x_i	2	4	6	8	10	12	14	16
f_i	4	4	α	15	8	β	4	5

29. The number of points where the curve $y = x^5 - 20x^3 + 50x + 2$ crosses the x-axis is:

30. The number of 4-letter words, with or without meaning, each consisting of 2 vowels and 2 consonants, which can be formed from the letters of the word "UNI-

VERSE” without repetition is: