B

2007 – GP

Test Paper Code : GP

Time : 3 Hours Maximum Marks : 300

INSTRUCTIONS

1. The question-cum-answer book has **56** pages and has **66** questions. Please ensure that the copy of the question-cum-answer book you have received contains all the questions.
2. Write your **Roll Number, Name** and **the name of the Test Centre** in the appropriate space provided on the right side.
3. Select **any two** Sections. Answer objective and subjective questions of the same two sections.
4. Write the answers to the objective questions against each Question No. in the **Answer Table for Objective Questions**, provided on Page No. **9**. Do not write anything else on this page.
5. Each objective question has **4 choices** for its answer: (A), (B), (C) and (D). Only **ONE** of them is the correct answer. There will be **negative marking** for wrong answers to objective questions. The following marking scheme for objective questions shall be used :
   (a) For each objective question, you will be awarded **3 (three)** marks if you have written only the correct answer.
   (b) In case you have not written any answer for a question, you will be awarded **0 (zero)** mark for that question.
   (c) In all other cases, you will be awarded **–1 (minus one)** mark for the question.
   (d) Negative marks for objective part(s) will be carried over to total marks.
6. Answer the subjective question only in the space provided after each question.
7. Do not write more than one answer for the same question. In case you attempt a subjective question more than once, please cancel the answer(s) you consider wrong. Otherwise, the answer appearing later only will be evaluated.
8. All answers must be written in blue/ black/blue-black ink only. Sketch pen, pencil or ink of any other colour should not be used.
9. All rough work should be done in the space provided and scored out finally.
10. No supplementary sheets will be provided to the candidates.
11. **Logarithmic Tables / Calculator of any kind / cellular phone / pager / electronic gadgets are not allowed.**
12. The question-cum-answer book must be returned in its entirety to the Invigilator before leaving the examination hall. Do not remove any page from this book.
13. Refer to useful data on the reverse.

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2007 – GP

READ THE INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

ROLL NUMBER						

Name :

Test Centre :

Do not write your Roll Number or Name anywhere else in this question-cum-answer book.

I have read all the instructions and shall abide by them.

.....

Signature of the Candidate

I have verified the information filled by the Candidate above.

.....

Signature of the Invigilator

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Useful Data

$\mathbb{R}$: The set of all real numbers.

$\mathbb{R}^2 = \{(x,y) : x \in \mathbb{R},\ y \in \mathbb{R}\}$.

$\mathbb{R}^3 = \{(x,y,z) : x \in \mathbb{R},\ y \in \mathbb{R},\ z \in \mathbb{R}\}$.

DO NOT WRITE ON THIS PAGE

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IMPORTANT NOTE FOR CANDIDATES

- **Geology Section : Q. Nos. 1–15 Objective Questions and Q. Nos. 46–52 Subjective Questions.**
- **Physics Section : Q. Nos. 16–30 Objective Questions and Q. Nos. 53–59 Subjective Questions.**
- **Mathematics Section : Q. Nos. 31–45 Objective Questions and Q. Nos. 60–66 Subjective questions.**
- **Select any *TWO* Sections.**
- **Attempt ALL objective and subjective questions of the selected *TWO* Sections.**
- **Questions 1-45 (objective questions) carry *three* marks each for correct answer and questions 46-66 (subjective questions) carry *fifteen* marks each. There will be negative marking for wrong answers to objective questions.**
- **Write the answers to the objective questions in the *Answer Table for Objective Questions* provided on page 9 only.**

GEOLOGY SECTION

1. Which one of the following statements, in relation to magmatic crystallization, is **CORRECT**?

(A) Phase boundary is a line on a phase diagram where only one phase is stable

(B) A sample that plots on liquidus will contain no crystals

(C) A sample will contain no liquid at temperature below solidus

(D) Latent head is released from a sample when it is converted from solid to liquid

2. A radioactive isotope has half-life of 6400 years. After how many half-lives 64000 atoms of this radio isotope will be reduced to 1000 atoms?

(A) 2 (B) 4 (C) 6 (D) 8

3. Match the following metamorphic facies from **Group 1** to its characteristic minerals from **Group 2**.

	Group 1		**Group 2**
P.	Greenschist facies	1.	Pyroxene, Sillimanite
Q.	Amphibolite facies	2.	Chlorite, Garnet, Pyroxene
R.	Granulite facies	3.	Hornblende, Garnet, Quartz
		4.	Chlorite, Epidote, Quartz

Choose the correct answer from the following :

(A) P-4, Q-3, R-1 (B) P-2, Q-3, R-1

(C) P-3, Q-4, R-2 (D) P-1, Q-2, R-4

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4. In case of seismic waves, which one of the following is **TRUE**?
   (A) P (Primary) waves can not travel through the body of the earth
   (B) Rayleigh waves can travel through the body of the earth
   (C) S (Secondary) waves can not travel through the body of the earth
   (D) Love waves can not travel through the body of the earth

5. If in a fold the cleavage and bedding are parallel to each other on the limbs but crosses the bedding at high angles on the crest and trough, it is a / an
   (A) Cheveron fold
   (B) Isoclinal fold
   (C) Recumbent fold
   (D) Overturned fold

6. The acceleration due to gravity of earth (g)
   (A) Decreases from Equator to Pole
   (B) Increases from Equator to Pole
   (C) Does not vary from Equator to Pole
   (D) Is less down a mine than it is at the earth's surface

7. Which one of the following minerals is chemically and mechanically most stable?
   (A) Quartz
   (B) Olivine
   (C) Pyroxene
   (D) Orthoclase

8. A line joining points of equal atmospheric pressure is termed as
   (A) Isograd (B) Isobar (C) Isohyte (D) Isotherm

9. A coarse grained rock consisting of abundant ca-plagioclase, olivine and pyroxene is
   (A) Anorthosite
   (B) Ecologite
   (C) Gabbro
   (D) Dunite

10. Find the odd man out from the following :
   (A) Barchan
   (B) Yardang
   (C) Seif
   (D) Fjord

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11. Indicate the correct order (oldest to youngest) of the following volcanic episodes :

(A) Rajmahal – Malani – Panjal – Deccan

(B) Malani – Rajmahal – Deccan – Panjal

(C) Malani – Panjal – Rajmahal – Deccan

(D) Panjal – Rajmahal – Deccan – Malani

12. Which one of the following twins does **NOT** belong to monoclinic system?

(A) Manebach

(B) Baveno

(C) Carlsbad

(D) Dauphine

13. Indicate the correct order in terms of decreasing iron (wt. %) in the following iron minerals.

(A) Magnetite – Hematite – Goethite – Siderite

(B) Hematite – Magnetite – Goethite – Siderite

(C) Goethite – Siderite – Hematite – Magnetite

(D) Siderite – Goethite – Magnetite – Hematite

14. Match the sedimentary rocks in **Group 1** with their category **Group 2.**

	Group 1		**Group 2**
P.	Shale	1.	Chemical
Q.	Chert	2.	Clastic
R.	Sandstone	3.	Biogenic
S.	Limestone		

(A) P-1, Q-3, R-1, S-2

(B) P-3, Q-2, R-2, S-1

(C) P-2, Q-1, R-2, S-3

(D) P-2, Q-1, R-3, S-1

15. Find the odd man out from the following :

(A) Isogyres

(B) Extinction

(C) Melatope

(D) Isochromes

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PHYSICS SECTION

16. In a diatomic gas system, molecules are free to translate, rotate and vibrate. The average kinetic energy per molecule is

(A) $\frac{1}{2}kT$ (B) $\frac{3}{2}kT$ (C) $\frac{5}{2}kT$ (D) $\frac{7}{2}kT$

17. Moment of inertia of a uniform circular disk of radius R and mass M about the tangential axis parallel to its diameter is

(A) $\frac{MR^2}{4}$ (B) $\frac{MR^2}{2}$ (C) $\frac{5\,MR^2}{4}$ (D) $\frac{3\,MR^2}{2}$

18. A uniform electric field $\vec{E} = E_0 \hat{e}_z$ exists in a region of permittivity ε_1. A homogeneous sphere of radius a with permittivity $\varepsilon_2 (< \varepsilon_1)$ is embedded in the region. At the centre of the sphere

(A) $\vec{\nabla} \bullet \vec{E} = 0$, $\vec{\nabla} \bullet \vec{D} = 0$
(B) $\vec{\nabla} \bullet \vec{E} = 0$, $\vec{\nabla} \bullet \vec{D} \neq 0$
(C) $\vec{\nabla} \bullet \vec{E} \neq 0$, $\vec{\nabla} \bullet \vec{D} = 0$
(D) $\vec{\nabla} \bullet \vec{E} \neq 0$, $\vec{\nabla} \bullet \vec{D} \neq 0$

19. The radiation emitted by a radioactive source of half life 3 hrs is 32 times higher than the safety limit. Minimum time for the radiation to be under the safety limit is

(A) 9 hrs (B) 12 hrs (C) 15 hrs (D) 18 hrs

20. Two long, parallel straight conducting wires carry the same current in the same direction. If the distance between them is halved and the current in both is doubled, the force per unit length between them will change by a factor of

(A) 1/8 (B) 8 (C) 4 (D) 1/4

21. In a reverse biased ideal p-n junction diode, with increase in the bias voltage the current will

(A) depend on temperature and saturate
(B) depend on temperature and increase linearly
(C) be independent of temperature and saturate
(D) be independent of temperature and increase linearly

22. A simple pendulum, suspended from the ceiling of a stationary cart, has a time period 2 seconds. When the cart accelerates in the horizontal direction with an acceleration of 10 m/s^2, the time period of the pendulum is (g = 10 ms $^{-2}$)

(A) $2^{1/2}$ seconds (B) $2^{3/2}$ seconds (C) $2^{1/4}$ seconds (D) $2^{3/4}$ seconds

23. In a simple cubic structure of lattice constant a, one plane among a set of parallel planes intercepts x, y and z-axis $2a$, a, and a respectively. The interplanar spacing is

(A) $a\sqrt{6}$ (B) a (C) $a/\sqrt{6}$ (D) $a/3$

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24. Parallel light is incident from air on a surface of glass plate at Brewster angle. Which one of the following statements is **CORRECT**?

(A) Incident and reflected rays are right angle to each other

(B) Incident and refracted rays are parallel to each other

(C) Refracted and reflected rays are at right angle to each other

(D) There is no refracted ray in the glass medium

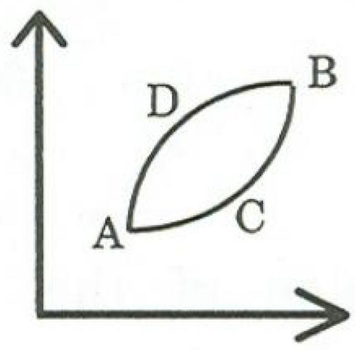
25. In the P-V diagram shown in the figure, the work done by the system of gases along the path ACB is

(A) smaller than the work done along ADB

(B) greater than the work done along ADB

(C) equal to the work done along ADB

(D) equal to the work done along BCA



26. The escape velocity from the earth is V_0. For a planet with radius three times and density twice that of the earth, the escape velocity will be

(A) $V_0\sqrt{2}$ (B) $3V_0\sqrt{2}$ (C) $2V_0\sqrt{2}$ (D) $V_0\sqrt{6}$

27. Which of the following phenomenon does **NOT** produce completely polarized light from unpolarized light?

(A) Absorption (B) Refraction (C) Scattering (D) Reflection

28. A transformer has 100 turns in primary and 200 turns in the secondary. If primary is connected to 220 V DC supply, the voltage across the secondary coil is

(A) 440 V (B) 220 V (C) 110 V (D) 0 V

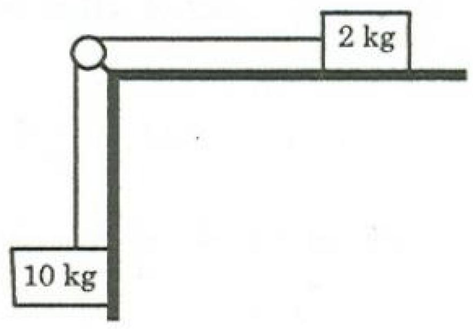
29. In the figure, the tension in the inelastic string is T when all surfaces are frictionless. If 2 kg block is glued on to the surface, the tension in the string will be

(A) zero

(B) greater than T

(C) less than T

(D) equal to T



30. Two metal wires A and B, having lengths l and $2l$ and radii R and $2R$ respectively are joined end to end along their axis. When one end of the system is fixed and other end is pulled with a constant force F, the elongation in both the wires is equal. The ratio of their Young's modulus $Y_A : Y_B$ is

(A) 2 : 1 (B) 4 : 1 (C) 1 : 2 (D) 1 : 4

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MATHEMATICS SECTION

31. If $\nu(x,y)=\beta xy(x^2-y^2)$ is a conjugate harmonic function of $u(x,y)=x^4-\alpha x^2y^2+y^4$ then the value of (α, β) is equal to

(A) (6, 4) (B) (6, 2) (C) (4, 6) (D) (4, 4)

32. The directions along which there is no change in the value of the function $f(x,y)=e^{(x^2+xy)}$ at the point $(3, -2)$ are equal to

(A) $-0.6\,\hat{i}-0.8\,\hat{j},\ 0.6\,\hat{i}+0.8\,\hat{j}$ (B) $0.6\,\hat{i}-0.8\,\hat{j},\ -0.6\,\hat{i}+0.8\,\hat{j}$

(C) $-0.6\,\hat{i}-0.8\,\hat{j},\ 0.6\,\hat{i}-0.8\,\hat{j}$ (D) $0.6\,\hat{i}+0.8\,\hat{j},\ -0.6\,\hat{i}+0.8\,\hat{j}$

33. The value of the integral $\int_{(0,0)}^{(1,1)}\left[\left(3x^2+4xy+y^2\right)dx+\left(2x^2+2xy\right)dy\right]$ along the path $x^4+y^4=2xy$ is equal to

(A) 0 (B) 1 (C) 2 (D) 4

34. Let the data given in the following table be such that the third divided difference is a constant.

x	1	2	3	4	5
$f(x)$	-3	0	15	k	105

Then the value of k is equal to

(A) 81 (B) 51 (C) 48 (D) 40

35. Let $x\geq 1$ and $0<\left|\frac{dy}{dx}\right|<\infty$ at $x=1$. If $y\frac{dy}{dx}=\ln x$ then the value of y^2 at $x=2$ is equal to

(A) $4\ln 2+2$ (B) $2\ln 2+2$ (C) $4\ln 2-2$ (D) $2\ln 2-1$

36. Let $T:\mathbb{R}^2\to\mathbb{R}^3$ be a linear transformation such that

$$T\begin{pmatrix}-1\\2\end{pmatrix}=\begin{pmatrix}1\\-1\\1\end{pmatrix} \text{ and } T\begin{pmatrix}2\\-1\end{pmatrix}=\begin{pmatrix}-1\\1\\-1\end{pmatrix}$$

Then the value of $T\begin{pmatrix}2\\1\end{pmatrix}$ is equal to

(A) $\frac{1}{3}\begin{pmatrix}1\\1\\-1\end{pmatrix}$ (B) $\frac{1}{3}\begin{pmatrix}-1\\1\\1\end{pmatrix}$ (C) $\frac{1}{3}\begin{pmatrix}1\\-1\\1\end{pmatrix}$ (D) $\frac{1}{3}\begin{pmatrix}-1\\1\\-1\end{pmatrix}$

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37. If the moment generating function of a random variable X is $\left(\frac{2}{3}+\frac{1}{3}e^t\right)^5$ then $Var(X)$ is equal to

(A) 35/9 (B) 20/9 (C) 5/3 (D) 10/9

38. Let $\{x_k\}$ be a sequence obtained by using the iterative scheme

$$x_{k+1} = 0.45 + (0.1 + 0.2\,c)\,x_k - 0.8\,c\,x_k^3.$$

The value of c for which $\{x_k\}$ converges quadratically to 0.5 is equal to

(A) $-1/8$ (B) $-1/2$ (C) 1/6 (D) 1/4

39. Let

$$f(x) = \begin{cases} \frac{4-x}{16}, & -2 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

be the probability distribution function of a random variable X. If $Z = X^2$ and $P\{Z \le \alpha\} = \frac{1}{8}$, then the value of α is equal to

(A) 1/16 (B) 1/8 (C) 1/4 (D) 1/2

40. Let $\mathbb{R}$ be the set of real numbers. Which one of the following statements **CAN NOT** hold?

(A) Arbitrary union of open sets in $\mathbb{R}$ is an open set in $\mathbb{R}$

(B) Arbitrary intersection of open sets in $\mathbb{R}$ is an open set in $\mathbb{R}$

(C) Finite union of closed sets in $\mathbb{R}$ is a closed set in $\mathbb{R}$

(D) Finite intersection of open sets in $\mathbb{R}$ is an open set in $\mathbb{R}$

41. If the volume of the parallelopiped generated by the three vectors $\vec{\mathbf{a}} = 2\alpha\hat{i} + 2\hat{j} + \alpha\hat{k}$, $\vec{\mathbf{b}} = 2\hat{i} + 4\hat{j} + \hat{k}$ and $\vec{\mathbf{c}} = \beta\hat{i} + \beta\hat{j} - \hat{k}$ is equal to 4 then α and β are related as

(A) $\alpha = \beta/4$ (B) $\alpha = \beta/2$ (C) $\alpha = 2\beta$ (D) $\alpha = 4\beta$

42. The value of the integral $\oint_C \left(e^{z^2/2}/z^3\right)dz$, where $C = \{z : |z| = 1\}$, is equal to

(A) 0 (B) $0.5\,\pi i$ (C) πi (D) $2\pi i$

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43. Volume of the solid generated by revolving the region bounded by the lines $x=0$, $y=1$ and the curve $y=\sqrt{x}$ about the line $y=1$ is equal to

(A) $\pi/6$ (B) $\pi/2$ (C) $5\pi/2$ (D) $3\pi/2$

44. Consider the function $f:\mathbb{R}^2\to\mathbb{R}$ defined by

$$f(x,y)=\begin{cases}1, & x\geq 0\\ x+y, & x<0.\end{cases}$$

Then f is

(A) continuous on $\mathbb{R}^2$

(B) discontinuous only at one point on the non-negative y-axis

(C) continuous only at one point on the y-axis

(D) continuous only on the set $\{(x,y)\in\mathbb{R}^2 : x\neq 0\}$

45. Let $\{x_n\}$, $\{a_n\}$ and $\{b_n\}$ be three sequences whose nth terms are related by $a_n=2\left(\frac{x_n}{3}\right)$ and $b_n=\left(\frac{2}{a_n}\right)^2$. If f is a function generated by the sequence $\{x_n\}$, where $x_{n+1}=a_n+b_n$, then the number of real roots of the equation $f(x)=0$ is equal to

(A) 0 (B) 1 (C) 2 (D) 3

Space for rough work

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QUESTION-CUM-ANSWER BOOK CODE **B**

Answer Table for Objective Questions

Write the Code of your chosen answer only in the 'Answer' Column against each Question No. Do not write anything else on this page.

Question No.	Answer	Do not write in this column	Question No.	Answer	Do not write in this column
01			31		
02			32		
03			33		
04			34		
05			35		
06			36		
07			37		
08			38		
09			39		
10			40		
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FOR EVALUATION ONLY

Correct		**Marks**	**(+)**
Incorrect		**Marks**	**(–)**
Total Marks in Q. Nos. 1 - 15			**( )**

Correct		**Marks**	**(+)**
Incorrect		**Marks**	**(–)**
Total Marks in Q. Nos. 16 - 30			**( )**

Correct		**Marks**	**(+)**
Incorrect		**Marks**	**(–)**
Total Marks in Q. Nos. 31 - 45			**( )**

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GEOLOGY SECTION

46. (a) Briefly describe how seismic waves can be used to infer that the outer core of the Earth is liquid while the inner core is solid. **(9)**

(b) Why is the composition of the core of the Earth thought to be largely metallic iron? **(6)**

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47. (a) Define Isomorphism and Polymorphism. **(6)**

(b) What are polymorphs? Give names of **two minerals** each of the polymorphs of **any two** – $CaCO_3$, C, $AlSiO_5$ and FeS_2. **(9)**

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48. (a) Define, giving figures, (i) dendritic, (ii) rectangular and (iii) radial drainage pattern and how does each form? **(9)**

(b) Define cirque, valley and piedmont glacier. **(6)**

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49. (a) Define briefly (i) an overturned fold (ii) a recumbent fold (iii) a nappe. **(9)**

(b) What is a transform fault? What are triple junctions and how are they classified? **(6)**

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50. (a) Briefly describe the main features of seafloor spreading. **(6)**

(b) Briefly explain what critical test proved the seafloor does move? **(9)**

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51. (a) Name (i) six main physical properties of minerals and (ii) six crystal systems in which they crystallize?

(iii) What is the Mohs scale of hardness? How does one determine the hardness of a mineral? **(9)**

(b) Plagioclase Feldspar contains variable amounts of Na and Ca in addition to Al, Si, and O. The Na end-member has the formula $NaAlSi_3O_8$. Because the size of Na^{+1} and Ca^{+2} ions are similar Ca^{+2} can substitute for Na^{+1}. Would the formula of Ca end-member be $CaAlSi_3O_8$? Explain. **(6)**

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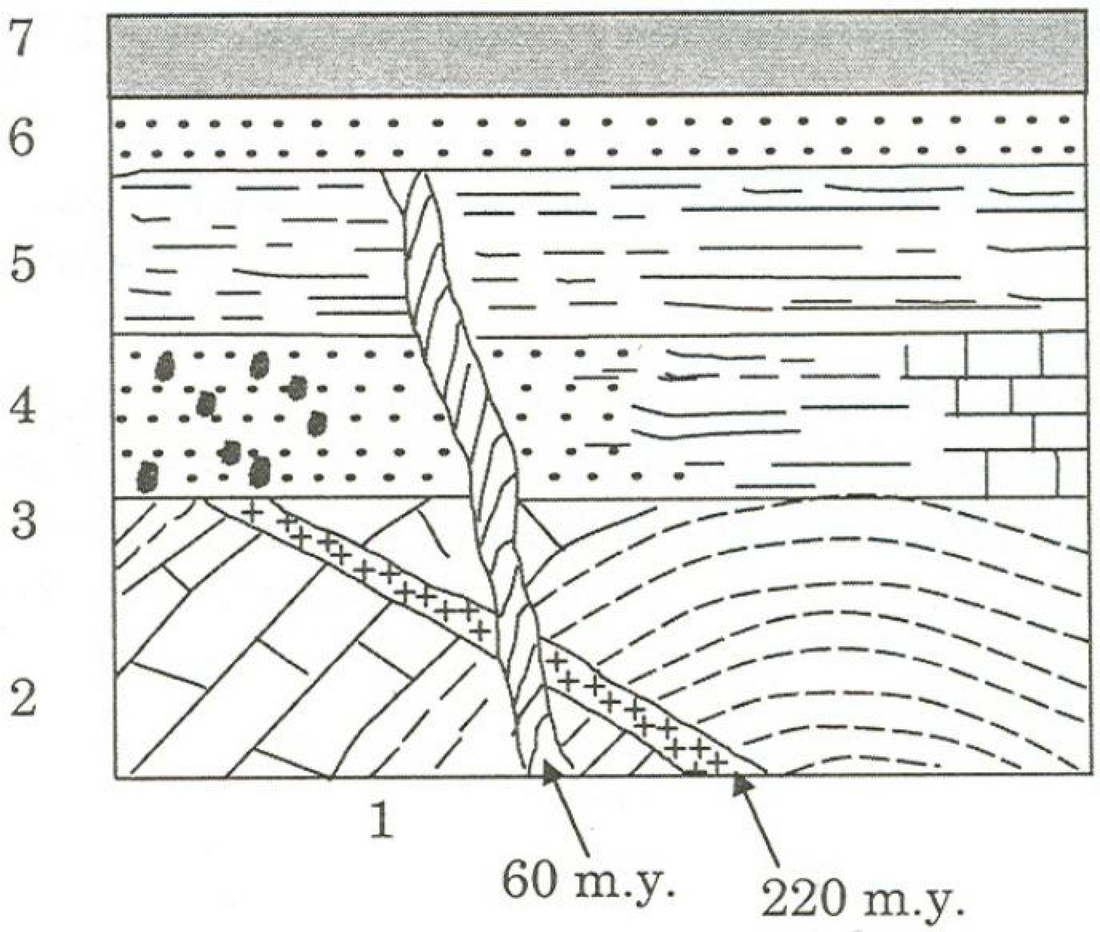
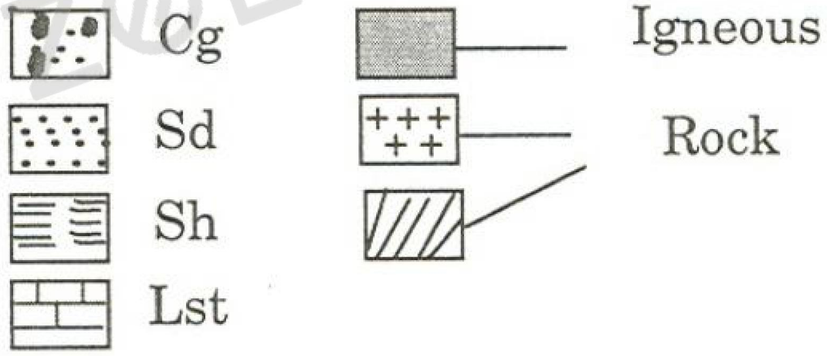
52.





In the figure above, a diagrammatic cross-section showing relationship of various sedimentary and igneous rock units is given. Note the given ages of the two igneous intrusive rocks. Cg-conglomerate, Sd-sandstone, Sh-shale and Lst-limestone. With reference to this figure answer the following questions :

(i) Arrange the formations 1 to7 from oldest to youngest.

(ii) What can be inferred about the ages of the sedimentary formations 1-7 based on the ages of the two intrusive igneous rocks?

(iii) When did the folding occur in the area?

(iv) What are the surfaces between formations 3-4 and 5-6 called? **(15)**

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PHYSICS SECTION

53. Consider a solid sphere of mass M and radius R with uniform mass distribution. Find out the gravitational field strength at a distance r away from the centre of the sphere for

(a) $0 < r < R$ and $r > R$ **(9)**

(b) Sketch the gravitational field strength as a function of r. **(6)**

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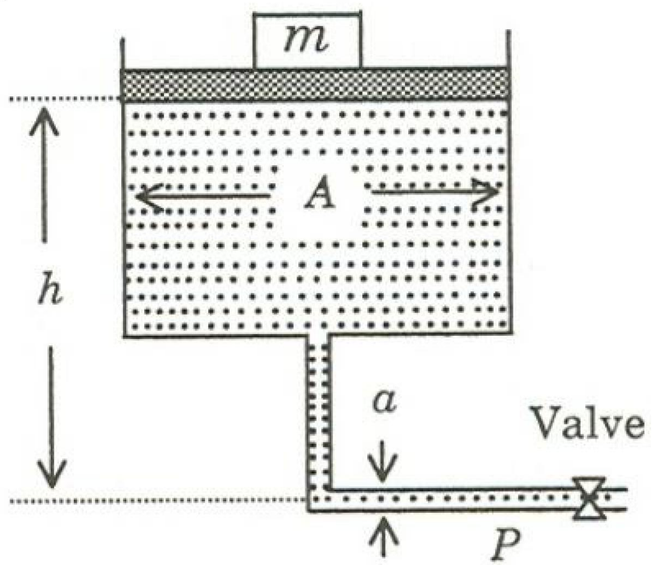
54. A water supply tower of cross sectional area A with a load of m on the water surface is drained out by a narrow pipe of a cross sectional area $a\,(<<A)$ through a valve, a distance h below the water level. Calculate pressure and velocity at point P when

(a) Valve is closed **(6)**

(b) Valve is open. **(9)**



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55. X-rays of 4 Å wavelength falls on electron cloud and gets scattered. Determine maximum change in

(a) kinetic energy of electron **(9)**

(b) velocity of electron. **(6)**

$$\left(\text{use } \frac{h}{m_e c} = 0.024\,\text{Å},\ h = 6.6 \times 10^{-34} \text{ Joule - sec, and } c = 3 \times 10^{8} \text{ m/sec} \right)$$

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56. A thin rectangular polarizer P_1 with x-axis as pass axis is kept at the origin. Another identical polarizer P_2 with z-axis as the pass axis is kept parallel to P_1 at $y=d$. A Half Wave Plate (HWP) is introduced between P_1 and P_2 with its optic-axis making 45° with the pass axis of P_1. An unpolarized light propagating along y-axis is incident normally on P_1 from left hand side. What would be the state of polarization after P_1, half wave plate, and P_2? **(15)**

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57. Consider two isolated vessels A and B. Each contains N molecules of a perfect monatomic gas at pressure P and temperature T_A and T_B respectively. The vessels are brought into thermal contact, keeping the pressure of the gas constant at P. Find the change in the entropy of the system at the equilibrium and show that the change in the entropy is positive. **(15)**

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58. A n-type silicon rod of length 0.7 cm has a cross sectional area of 0.1 cm^2. A DC bias voltage of 35 V across the rod results in a 5.6 ampere DC current. Under this DC bias, an electric pulse applied at one end takes 10 μs to traverse the length of silicon rod.

Determine

(a) mobility of the charge carriers **(6)**

(b) carrier concentration. **(9)**

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59. A long, straight, cylindrical conductor of radius a carries a uniform current I. This conductor has a cylindrical hole along its length at a distance d from the centre with its axis parallel to the axis of the conductor. Calculate the magnetic field inside the hole. **(15)**

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MATHEMATICS SECTION

60. (a) Solve : $\frac{x^2+1}{y^2}\frac{dy}{dx}-5\left(x^2-1\right)=\frac{4x}{y}$. **(9)**

(b) Discuss the convergence of the series

$$\sum_{n=1}^{\infty}\frac{\ln n}{1+2+\cdots+n}.$$ **(6)**

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61. (a) Perform two iterations of Newton-Raphson method to estimate the point on the parabola $y=x^2$ closest to the point $(2, 1)$, taking the initial approximation $x_0=1$. **(9)**

(b) Find the eigenvalues of A^4, where $A=\begin{pmatrix} 1 & 0 & -1 \\ 9 & 4 & 1 \\ 3 & 1 & 1 \end{pmatrix}$. **(6)**

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62. (a) Find the circulation of the fied

$$\vec{F} = -x^2 y\hat{i} + xy^2\hat{j} + \left(y^3 - x^3\right)\hat{k}$$

around the curve C, where C is the intersection of the sphere $x^2 + y^2 + z^2 = 25$ and the plane $z = 3$. The orientation of the curve C is counterclockwise when viewed from above. **(9)**

(b) For $n = 1, 2, \cdots$ and $x \in [0,1]$, let $g_n(x) = x / \left(1 + n\,x^2\right)$. Given that $x = 1/\sqrt{n}$ is a point of maxima of the function $g_n(x)$ on $[0, 1]$, discuss the uniform convergence of the series

$$\sum_{n=1}^{\infty} \frac{x}{n\left(1 + n\,x^2\right)} \text{ on } [0,1].$$ **(6)**

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63. Consider the following system of linear equations

$$\begin{aligned} 3x+y+2z &= -1 \\ x+2y-z &= a \\ x+z &= -1 \\ 2x+by-z &= c. \end{aligned}$$

Determine the values of a, b and c for which the above system has (i) no solution, (ii) infinitely many solutions, and (iii) a unique solution. **(15)**

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64. (a) A die is thrown 120 times independently and the following data is observed.

Number on die	1	2	3	4	5	6
Frequency	k	20	20	20	20	$40-k$

Determine the values of k for which the hypothesis that the die is unbiased be rejected at the 0.025 significance level. **(9)**

[Given : $\chi^2_5(0.975)=12.83$, $\chi^2_5(0.025)=0.83$, $\chi^2_6(0.975)=14.45$ and $\chi^2_6(0.025)=1.24$.]

(b) Let X and Y be the continuous random variables with joint probability density function

$$f(x,y)=\begin{cases}\dfrac{x+y}{5}, & 0<x<1, \quad 1<y<3\\ 0, & \text{otherwise.}\end{cases}$$

Find $P\{(X+Y)>3\}$. **(6)**

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65. Apply the Residue theorem for complex variable to evaluate the integral

$$\int_{-\infty}^{\infty} \frac{x \sin x}{(x^2+16)(x+2)} dx. \qquad \textbf{(15)}$$

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66. Find the mass of a metallic block of varying density $\rho(x,y,z)=\rho_0+xyz$, bounded by the planes $x=0$, $y=0$, $z=0$ and the sphere $x^2+y^2+z^2=a^2$ lying in the octant $x\geq 0$, $y\geq 0$ and $z\geq 0$. Here ρ_0 is the constant density of the metallic block at $(0,0,0)$. **(15)**

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