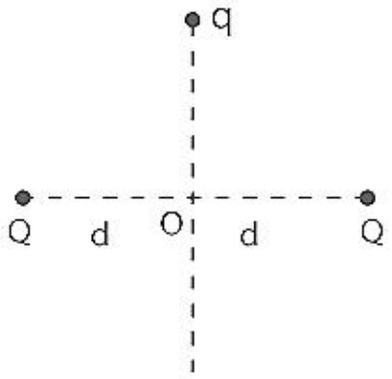
JEE-Main-29-06-2022-Shift-2 (Memory Based)

Physics

Question: Two point charge each of same magnitude 'Q' are placed as shown in figure determine distance from O at equatorial axis where force on q is maximum.



Options:

(a) $\frac{d}{\sqrt{3}}$

(b) $\frac{d}{\sqrt{2}}$

(c) $\frac{d}{\sqrt{5}}$

(d) $\frac{d}{\sqrt{7}}$

Answer: (b)

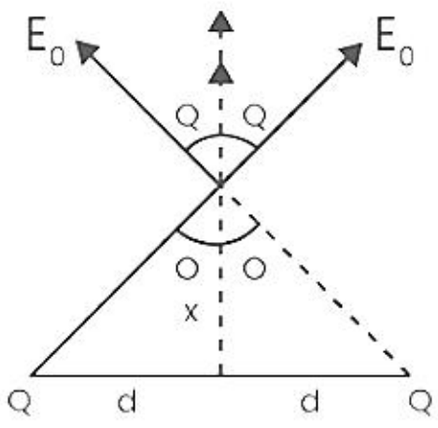
Solution:

Force on q is

$F = qE$

$$\Rightarrow F = q\frac{2kQx}{\left(x^2+d^2\right)^{\frac{3}{2}}}$$

for F = Fmax.



$$E = 2E_0\cos\theta$$

$$= \frac{2kQx}{\left(x^2+d^2\right)^{\frac{3}{2}}}$$

$$\Rightarrow \frac{d}{dx}\left[\frac{2x}{\left(x^2+d^2\right)^{\frac{3}{2}}}\right]=0 \Rightarrow x=\frac{d}{\sqrt{2}}$$

Question: A gas is expanded from volume 'V_1' to 'V_2' by three different process these three process are isothermal, adiabatic and isobaric. Work done by gas in isothermal is W_1 and that in adiabatic is W_2 and that in isobaric is W_3. Select the correct option?

Options:

(a) $W_1 > W_2 > W_3$

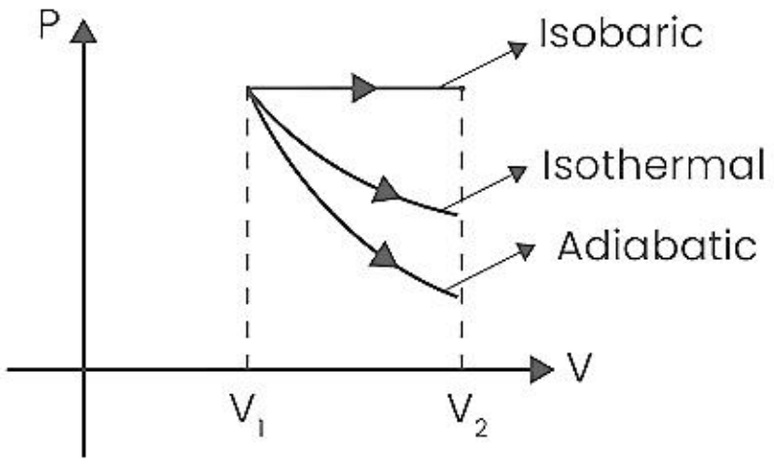
(b) $W_2 > W_1 > W_3$

(c) $W_1 = W_2 = W_3$

(d) $W_1 > W_2 = W_3$

Answer: (b)

Solution:



W= area under P-V curve

so according to graph

$W_3 > W_1 > W_2$

$V_1 \rightarrow V_2$ in three different ways

$W_1 \rightarrow$ Isothermal $W_2 \rightarrow$ Adiabatic

$W_3 \rightarrow$ Isochoric

$W_1 > W_2 > W_3$

$W_1 < W_2 < W_3$

Question: Time period of earth rotating in orbit is 7 hr. If radius is thrice then new time period of earth:

Options:

(a) 36 hr

(b) 30 hr

(c) 21 hr

(d) 28 hr

Answer: (a)

Solution:

$T^2 \times r^3$

$$\frac{7^2}{T^2} = \left(\frac{R}{3R}\right)^3$$

$$\frac{49}{T^2} = \frac{1}{27}$$

$$T^2 = 49 \times 27$$

$$T = 7 \times 3\sqrt{3} = 21 \times 1.732$$

$$= 35.7$$

Question: The Height of T.V. tower is 125 m if its range is doubled for signal, find the new height:

Options:

(a) 125 m

(b) 250 m

(c) 500 m

(d) 300 m

Answer: (c)

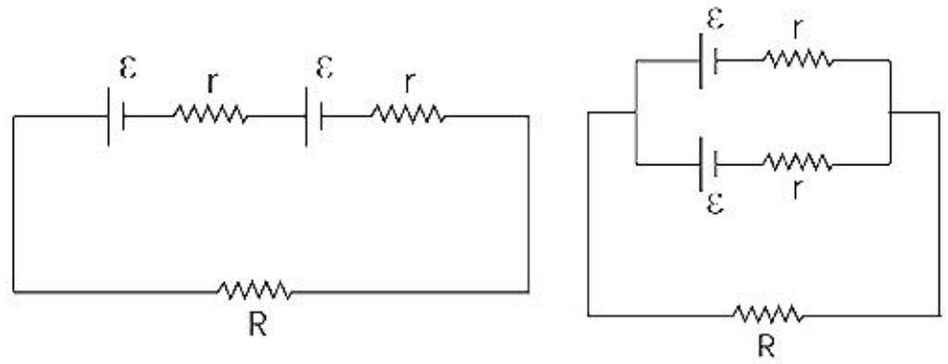
Solution:

$$d = \sqrt{2hR}$$

$$2d = \sqrt{2h^{'}R}$$

$$h^{'} = 4h = 4 \times 125 = 500\,\text{m}$$

Question: Two Identical cell give same current across R resistance when they are in series combination and when they are in parallel combination. Find internal resistance of cell?



Options:

(a) R

(b) 3R

(c) $\frac{R}{2}$

(d) 5R

Answer: (a)

Solution:

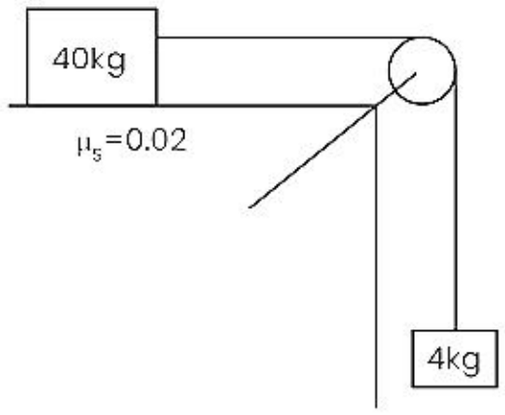
$$\frac{2\varepsilon}{R+2r} = \frac{\varepsilon}{R+\frac{r}{2}}$$

$$\Rightarrow 2R + r = R \div 2r$$

$$\Rightarrow r = R$$

$$r = R$$

Question: Find the acceleration of system shown.



Options:

(a) $\frac{4}{3}\text{m/s}^2$

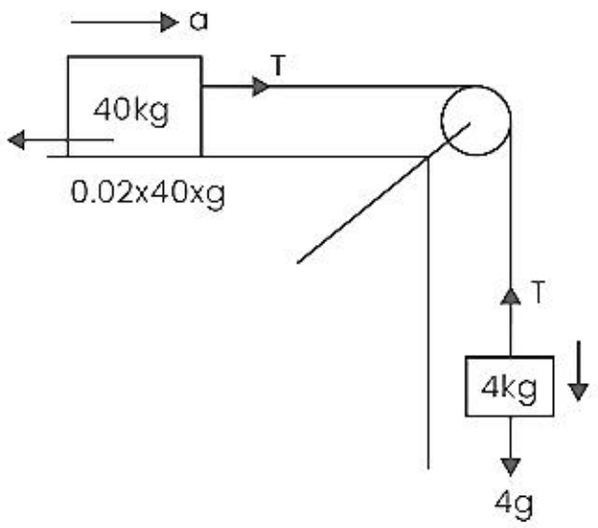
(b) $\frac{8}{12}\text{m/s}^2$

(c) $\frac{8}{9}\text{m/s}^2$

(d) $\frac{8}{11}\text{m/s}^2$

Answer: (d)

Solution:



$T - 8 = 40a$

$4g - T = 4a$

$32 = 44a$

$a = \frac{8}{11}\text{m/s}^2$

Question: A particle starts from rest along straight-line path with cost acceleration. If covers 10 m distance in first t, sec. Find distance covered by it in nest t sec:

Options:

(a) 20 m

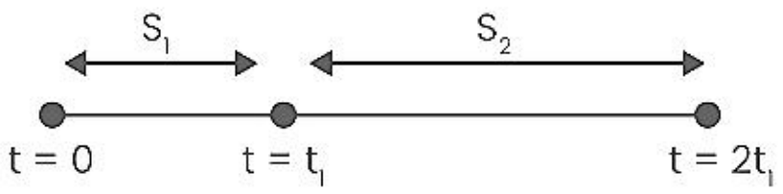
(b) 30 m

(c) 40 m

(d) 50 m

Answer: (b)

Solution:



$S_1 = \frac{1}{2}at_1^2$

$\Rightarrow S_1 + S_2 = \frac{1}{2}a(2t_1)^2 = \frac{1}{2}a4t_1^2$

$\Rightarrow S_1 : S_1 + S_2 = 1:4$

$\Rightarrow S_1 : S_2 = 1:3$

Here $S_1 = 10\,\text{m}$

So, $S_2 = 30\,\text{m}$

Question: Electric Potential varies as $V = 3x^2$ find electric field at the point having Co-ordinates (1, 0,3).

Options:

(a) $-6\,\text{V/m}$

(b) $-8\,\text{V/m}$

(c) $9\,\text{V/m}$

(d) $10\,\text{V/m}$

Answer: (a)

Solution:

We know that

$E = -\frac{dv}{dx}$

$\Rightarrow E = -\frac{d}{dx}(3x^2) = -6x$

at $(1,0,3), E = -6$

Question: If maximum possible range of a projectile is 100 m what will be the maximum possible height for same speed:

Options:

(a) 100 m

(b) 200 m

(c) 50 m

(d) 25 m

Answer: (a)

Solution:

$R_{\max} = u^2/\text{g} = 100\,\text{m}$

$H_{\max} = u^2/2g = 50m$

Question: Electric field of light is given by $E = 200\left[\sin\left(6\times10^{15}t\right) + \sin\left(9\times10^{15}\right)\right]$. It is incident on a metal surface of work function 2.5 eV find the maximum kinetic energy of emitted electrons:

Options:

(1) 3.4 eV

(2) 2.5 eV

(3) 3.8 eV

(4) 4 eV

Answer: (a)

Solution:

$$KE_{max} = E - \phi$$

$$= \frac{h\omega}{2\pi} - \phi$$

$$= \frac{4.14\times10^{-15}\times9\times10^{15}}{2\times3.14} - 2.5 = 5.9 - 2.5 = 3.4eV$$

Question: A block of mass M is released from rest from height of y. When it fall down by y. its kinetic energy is:

Options:

(a) $mg(y - y_0)$

(b) $1/2\,mgy_0$

(c) $mg\,y_0$

(d) mgy_{0^2}

Answer: (a)

Solution:

W by gravity $= mgh$

$h = y - y_0$

Question: In the decay process $^{182}_{74}D \xrightarrow[\alpha]{} D_1 \xrightarrow[\beta]{} D_2 \xrightarrow[\alpha]{} D_3 \xrightarrow[\gamma]{} D_4$ find the atomic number and mass number of element D_4 :

Options:

(a) 174, 171

(b) 176, 72

(c) 174, 70

(d) 176, 71

Answer: (a)

Solution:

$$^{182}_{74}D \xrightarrow[\alpha]{} {}^{178}_{72}D_1 \xrightarrow[\beta]{} {}^{178}_{73}D_2 \xrightarrow[\alpha]{} {}^{174}_{71}D_3 \xrightarrow[\gamma]{} {}^{174}_{71}D_4$$

Question: Equation of a simple pendulum is $\theta = \theta_0 \sin(\pi t + \phi)$. Find the length of pendulum:

Options:

(a) 1 m

(b) 2 m

(c) 0.5 m

(d) 4 m

Answer: (a)

Solution:

$\omega = \pi$

$$\sqrt{\frac{g}{\ell}} = \pi$$

$$g = \pi^2 \ell$$

$$\ell = \frac{g}{\pi^2} = 1\ m$$

Question: Statement-1 : Electric field changes the speed of change particle but magnetic field does not change the speed.

Statement-2: Charge particle travels perpendicular to electric field and parallel to magnetic field

Options:

(a) Statement-1 is false, statement-2 is true

(b) Both statement is true & statement 2 is not the correct explanation of statement-1.

(c) Statement-1 is true, statement-2 is false.

(d) Statement-1 is true, statement-2 is true and statement-2 is the correct explanation of statement -1

Answer: (c)

Solution:

Question: Moment of inertia of a rod about its and is I_1 Rod is bent into a ring and its moment of inertia about diameter is I_2 Find the $I_1 - I_2$.

Options:

(a) $m\ell^2\left[\frac{1}{3}+\frac{1}{4\pi^2}\right]$

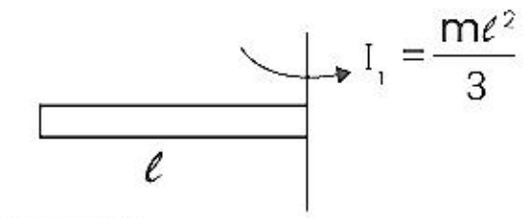
(b) $m\ell^2\left[\frac{1}{3}-\frac{1}{2\pi^2}\right]$

(c) $m\ell^2\left[\frac{1}{3}+\frac{1}{8\pi^2}\right]$

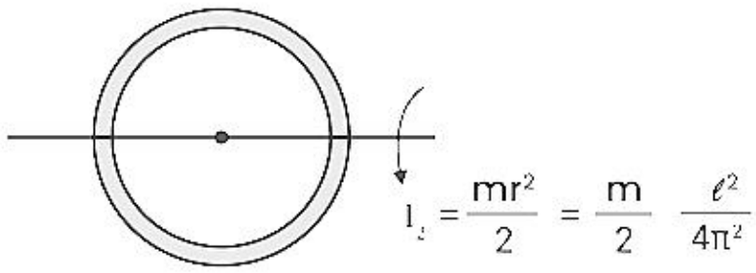
(d) $m\ell^2\left[\frac{1}{3}-\frac{1}{8\pi^2}\right]$

Answer: (d)

Solution:



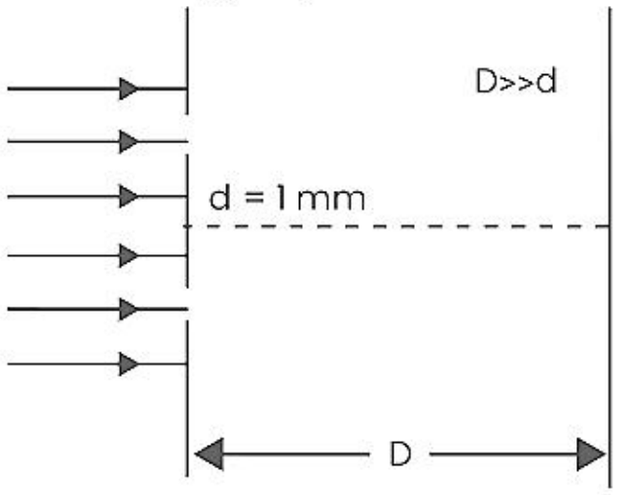
$\ell = 2\pi r$



$$I_2 = \frac{m\ell^2}{\delta\pi^2}$$

$$I_1 - I_2 = m\ell^2\left[\frac{1}{3} - \frac{1}{8\pi^2}\right]$$

Question: If in YDSE set up screen is shifted towards plane of slit by 0.3 metre then fringe width changes by 0.4 mm. Determine wavelength 'λ' of light.



Options:

(a) $\frac{10^{-3}}{3} mm$

(b) $\frac{7}{3} \times 10^{-3} mm$

(c) $\frac{4}{3} \times 10^{-3} mm$

(d) $\frac{5}{3} \times 10^{-3} mm$

Answer: (c)

Solution:

$$\beta = \frac{\lambda D}{d}$$

$$\beta' = \frac{\lambda (D - 0.3)}{d}$$

$$\beta - \beta' = \frac{\lambda \times 0.3}{d}$$

$$0.4 = \frac{\lambda \times 0.3 \times 10^3}{1}$$

$$\lambda = \frac{4}{3} \times 10^{-3} mm$$

Question: Time taken by a capacitance to reduce its energy by half is t_1& time taken by the same capacitor to reduce its charge by $1/8^{th}$ is t_2. The value of t_1/t_2 will be

Options:

(a) 1/3

(b) 1/6

(c) 1/2

(d) 1/4

Answer: (b)

Solution:

$$q = Qe^{-\frac{t}{\tau}} \qquad U = \frac{q^2}{2C}$$

$$\frac{Q}{\sqrt{2}} = Qe^{-\frac{t_1}{\tau}}$$

$$t_1 = \tau \ell n \sqrt{2}$$

$$\frac{Q}{8} = Qe^{-\frac{t_2}{\tau}}$$

$$t_2 = \tau \ell n 8$$

$$\frac{t_1}{t_2} = \frac{\tau \ell n \sqrt{2}}{\tau \ell n 8} = \frac{\frac{1}{2} \tau \ell n 2}{3 \tau \ell n 2} = \frac{1}{6}$$

Question: Verneir constant of verneir scale = 0.1 mm on measuring diameter of shaft. Main scale reading = 1.7 cm. If main scale coincides with 5 division of verneir scale & zero error is -0.05 cm. Diameter of shaft in cm is:

Options:

(a) 1.80 cm

(b) 2.80 cm

(c) 4.80 cm

(d) 6.80 cm

Answer: (a)

Solution:

Reading = MSR + L.C × $V_{S.R + correction}$

Correction = - zero error = 0.05 cm

Reading = $1.7 + 0.1 \times 10^{-1}$ (5) + 0.05

= 1.7 + 0.05 + 0.05

⇒ 1.80 cm

Question: Two long wires are separated by 8 cm the magnetic field at the mid-point is 300 μT. Two wire carries current of same value which is:

Options:

(a) 30 A in opposite direction

(b) 30 A in same direction

(c) 60 A in same direction

(d) 60 A in opposite direction

Answer: (a)

Solution:

Current is opposite direction

$$B = \frac{2\mu_0 i}{2\pi 4cm} \Rightarrow 300\times10^{-6} = \frac{2\times2\times10^{-7}\times i}{4\times10^{-2}} \Rightarrow I = 30Amp$$

Question: If normal force exerted is 1/4th weight of box find acceleration of lift.

Options:

(a) 3g/4

(b) g/4

(c) g/2

(d) g

Answer: (a)

Solution:

$$mg - N = ma$$

$$\frac{mg - mg}{4} = mg$$

$$a = \frac{3g}{4}$$

Question: Half-life of a radioactive sample is 5 years. Find time taken to reduce the sample 6.25% of its initial value.

Options:

(a) 20 years

(b) 15 years

(c) 25 years

(d) 50 years

Answer: (a)

Solution:

Time taken in 50% is T_H

Time take in 25% is $2T_H$

Time take in 12.5% is $3T_H$

Time take in 6.25% is $4T_H$

So $4T_H = 4 \times 5 = 20$ years

Question: In resonance tube first resonance is obtain at 20 cm, then third resonance length will be: (frequency of source = 400 Hz, speed of sound in air = 336 m/s)

Options:

(a) 60 cm

(b) 104 cm

(c) 64 cm

(d) 100 cm

Answer: (b)

Solution:

$$\text{Wavelength of wave} \Rightarrow \lambda = \frac{V}{f} = \frac{336}{400} = 84\,cm$$

At first resonance

$$\frac{\lambda}{4} = \ell + e \Rightarrow \frac{84}{4} = 20 + e$$

$$\Rightarrow e = 1$$

So third resonance length

$$5\frac{\lambda}{4} = \ell_2 + e$$

$$5(21) = \ell_2 + 1$$

$$\ell_2 = 104\,cm$$

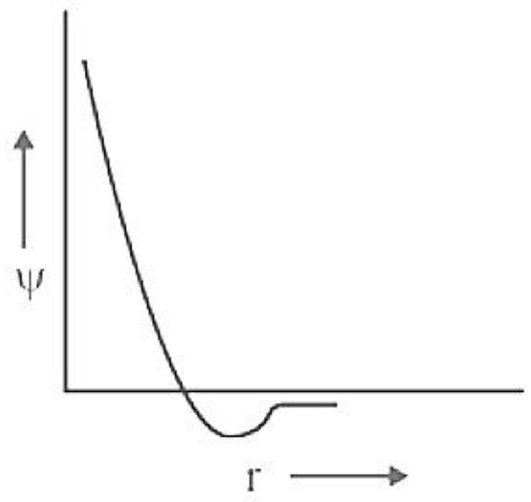
JEE-Main-29-06-2022-Shift-2 (Memory Based)

Chemistry

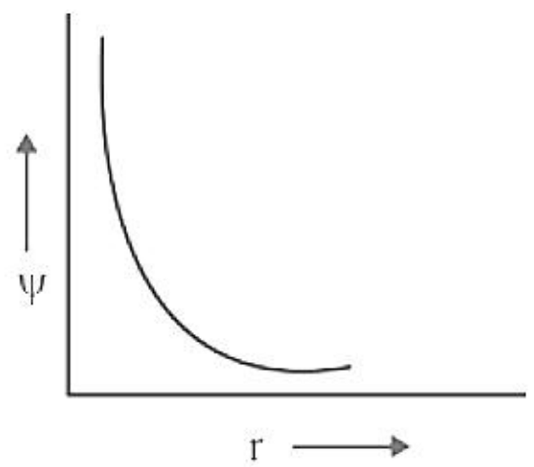
Question: Which of the following graph is correct for 2s orbital?

Options:

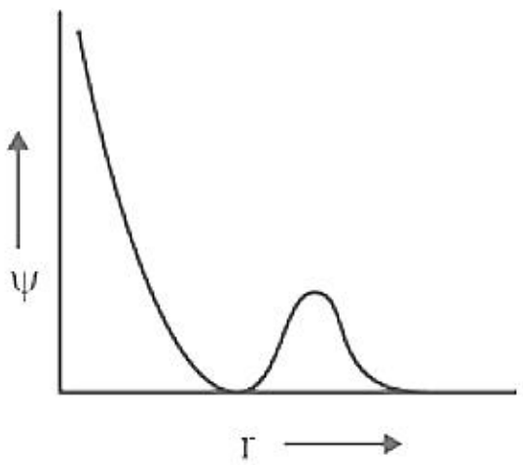
(a)



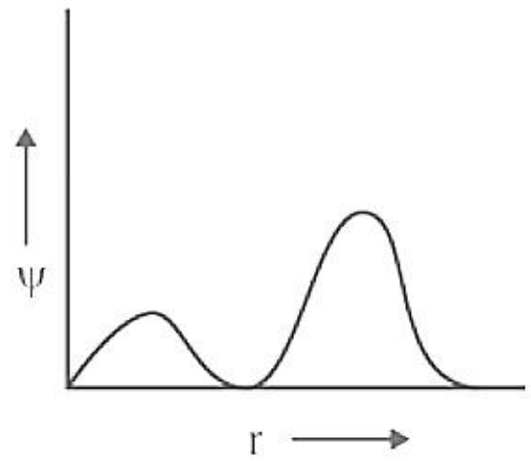
(b)



(c)



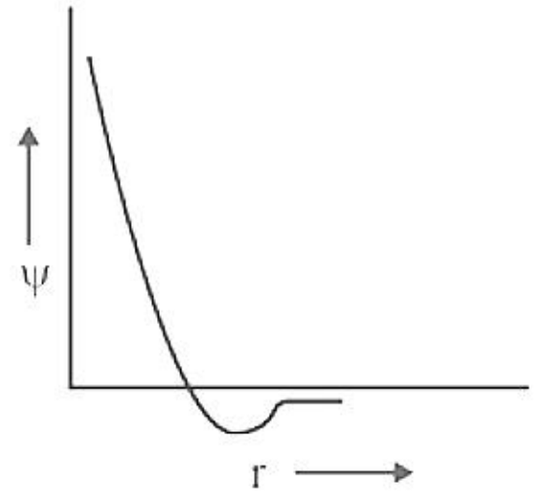
(d)



Answer: (a)

Solution: 2s orbital has n – l = 2 – 1 = 1 node

Correct graph of wavefunction and radius for 2s orbital is



Question: Match the following.

Column-I	**Column-II**
(i) Siderite	(A) $ZnCO_3$
(ii) Malachite	(B) ZnS
(iii) Sphalerite	(C) $Cu(OH)_2CuCO_3$
(iv) Calamine	(D) $FeCO_3$

Options:

(a) i → C; ii → A; iii → D; iv → B

(b) i → D; ii → C; iii → A; iv → B

(c) i → C; ii → B; iii → A; iv → D

(d) i → D; ii → C; iii → B; iv → A

Answer: (d)

Solution:

(i) Siderite ⇒ $FeCO_3$

(ii) Malachite ⇒ $Cu(OH)_2CuCO_3$

(iii) Sphalerite ⇒ ZnS

(iv) Calamine $\Rightarrow ZnCO_3$

Question: The spin only magnetic moment of the compound $[MnCl_6]^{4-}$ is

Options:

(a) 4.89

(b) 5.91

(c) 2.83

(d) 1.73

Answer: (b)

Solution:

$[MnCl_6]^4$

Mn^{2+}: $[Ar]\,3d^5$

Number of unpaired electrons = 5

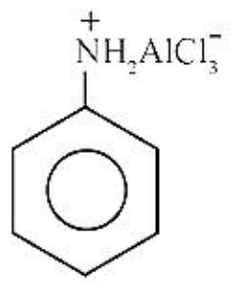
Magnetic moment of the complex

$$\mu = \sqrt{n(n+2)} = \sqrt{35} = 5.91 \text{ B.M.}$$

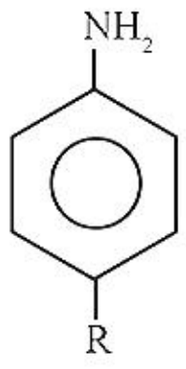
Question: Which of the following product is formed when Friedel craft reaction of aniline takes place?

Options:

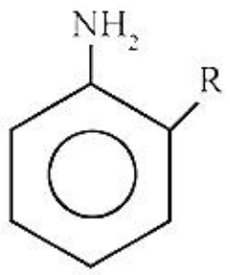
(a)



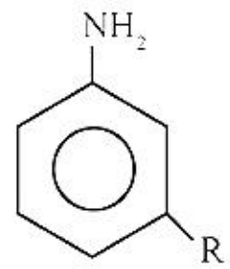
(b)



(c)



(d)



Answer: (a)

Solution: Aniline acts as Lewis base and reacts with aluminium chloride to form salt. Due to this, nitrogen of aniline acquires positive charge and acts as strong deactivating group.

Question: Which of the following species have carbonate ion?

Options:

(a) Washing Soda

(b) Caustic Soda

(c) Baking Soda

(d) All of the above

Answer: (a)

Solution:

Washing Soda $\Rightarrow Na_2CO_3 \cdot 10H_2O$

Caustic Soda $\Rightarrow NaOH$

Baking Soda $\Rightarrow NaHCO_3$

Question: Dichlorodiphenyltrichloroethane act as:

Options:

(a) Antiseptic

(b) Disinfectant

(c) Pesticide

(d) Water softner

Answer: (c)

Solution: DDT (dichloro-diphenyl-trichloroethane) is used as pesticide for insect control

Question: Consider the following calculation:

$$\frac{0.002858 \times 0.112}{0.5702} = X$$

What is X ?

Options:

(a) 0.00056

(b) 0.000561

(c) 0.000563

(d) 0.0005

Answer: (b)

Solution:

Number of significant fig in 0.002858 = 4

Number of significant fig in 0.112 = 3

Number of significant fig in 0.5702 = 4

Answer should be in 3 significant figures

$$\frac{0.002858 \times 0.112}{0.5702} = 0.000561$$

Question: Which of the following structure of protein does not change its structure on heating?

Options:

(a) Primary

(b) Secondary

(c) Quaternary

(d) Tertiary

Answer: (a)

Solution: Primary structure of protein is not affected by heat.

Question: CH_4, NH_4^+, BH_4^- which statement is true about them

Options:

(a) They are isoelectronic species

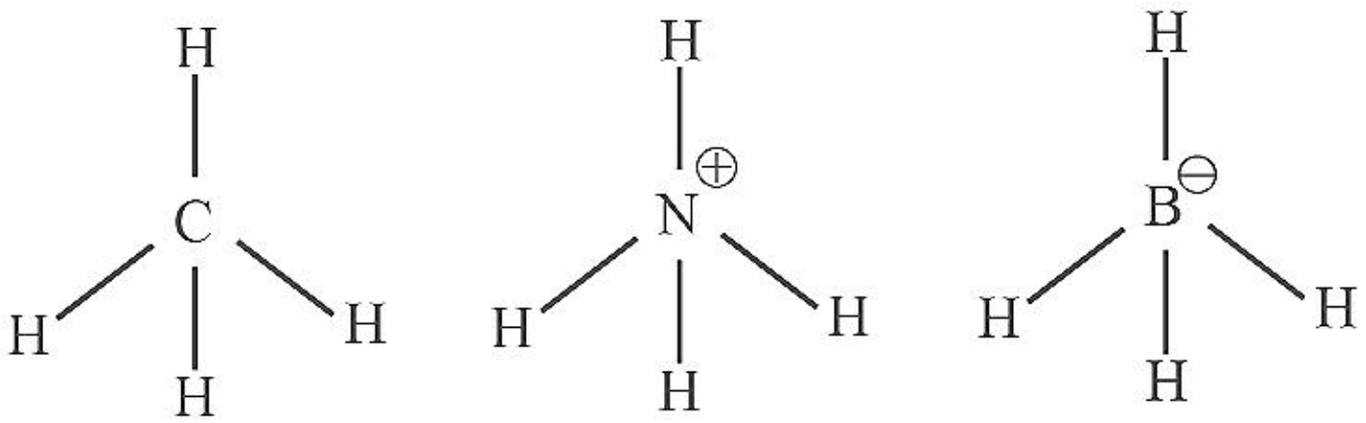
(b) 2 of them are isoelectronic and all tetrahedral structure

(c) All are isoelectronic and tetrahedral structure

(d) All are isoelectronic and 2 are tetrahedral

Answer: (c)

Solution:



All these are tetrahedral and are isoelectronic. (10 electrons)

Question: The shape and number of lone pairs present in one molecule of BrF_3 are respectively

Options:

(a) Tetrahedral, five

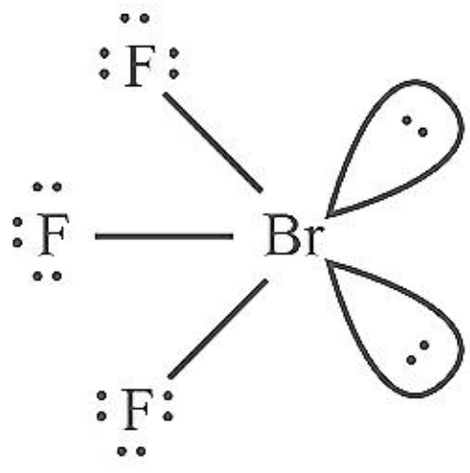
(b) T-shape, two

(c) Trigonal planar, zero

(d) T-shape, eleven

Answer: (d)

Solution:



Question: Common Monomer of Bakelite and Novolac is

Options:

(a) Phenol and formaldehyde

(b) Caprolactum

(c) Ethene and phenol

(d) Formaldehyde and neoprene

Answer: (a)

Solution: Monomers for both the polymers are phenol and formaldehyde.

Question: What is formed by the mixture of Chloroxylenol and terpineol?

Options:

(a) Disinfectant

(b) Antibiotic

(c) Antiseptic

(d) Antacid

Answer: (c)

Solution: Mixture of chloroxylenol and terpineol is known as Dettol. It acts as an antiseptic

Question: Find the volume in cm^3 at standard temperature and pressure. Given 16 g of Hydrogen and 128 g of oxygen and value of R is 0.0821 L atm mol^{-1} K^{-1}

Answer: 269000.00

Solution:

$$\text{Number of moles of hydrogen} = \frac{16}{2} = 8 \text{ moles}$$

$$\text{Number of moles of oxygen} = \frac{128}{32} = 4 \text{ moles}$$

Total number of moles = 12

$$PV = nRT$$

$$V = \frac{nRT}{P} = \frac{12 \times 0.0821 \times 2.23}{1} = 268.9\,L = 269\,L = 269000\,cm^3$$

Question: Half life of a radioactive decay is 5 years. The time required to fall the rate of decay to 6.25 %(in years) is

Answer: 20.00

Solution:

$$N = N_o\left(\frac{1}{2}\right)^{1/T} \quad ....(1)$$

N = 6.25 % of No

$$\therefore N = N_o \times \frac{6.25}{100}$$

$$\text{or}\ \frac{N}{N_o} = \frac{1}{16}$$

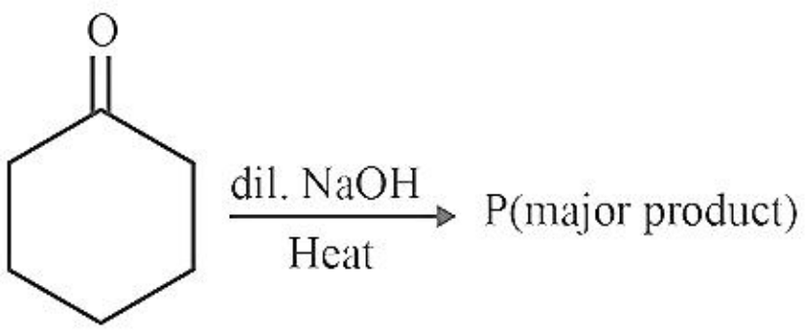
From eq (1)

$$\left(\frac{1}{16}\right) = \left(\frac{1}{2}\right)^{t/5}$$

$$\left(\frac{1}{2}\right)^{4} = \left(\frac{1}{2}\right)^{t/5}$$

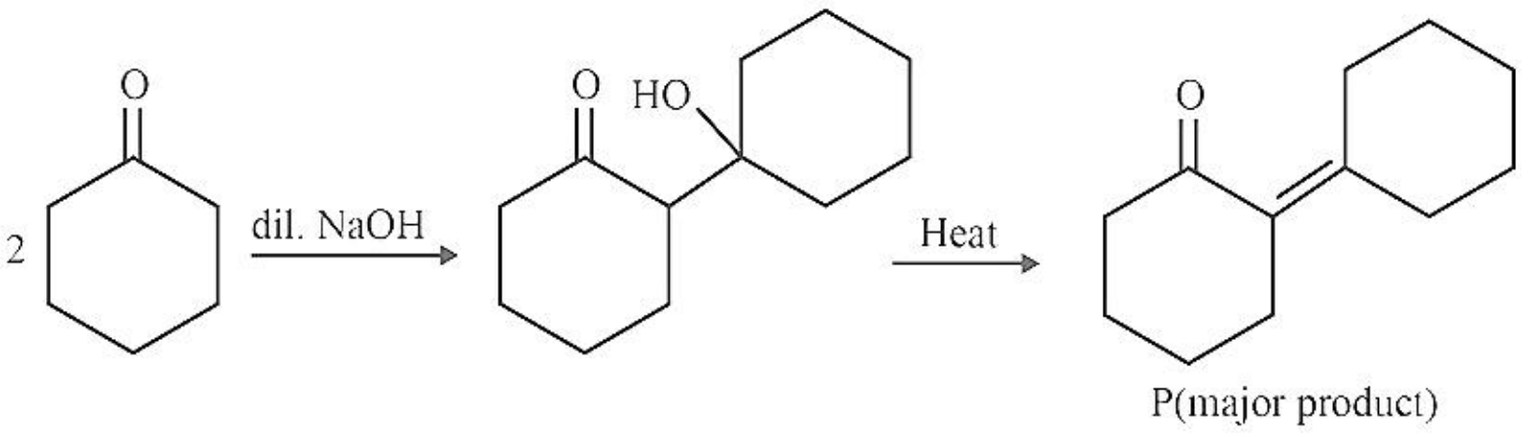
t = 20 years

Question: Calculate the number of π bonds in product P.



Answer: 2.00

Solution:



Number of π bond = 2

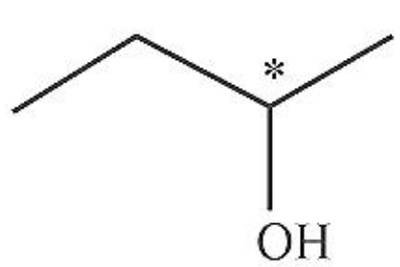
Question: How many chiral alcohols have molecular formula $C_4H_{10}O$ (including stereoisomers)?

Answer: 2.00

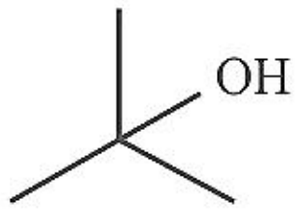
Solution: Molecular formula = $C_4H_{10}O$

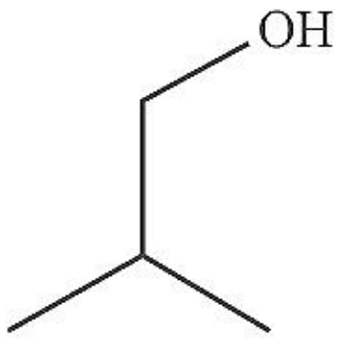
$$\text{Degree of unsaturation} = (C + 1) = \frac{H - X - N}{2} = 5 - \frac{10}{2} = 0$$

Possible alcohols are



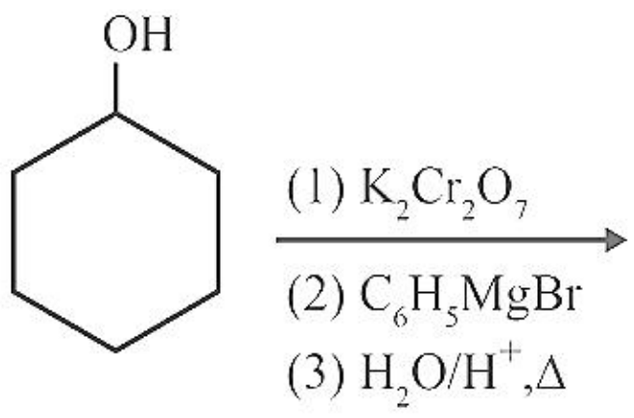
(Optically active)





Hence only one optically active alcohol so number of chiral alcohols is 2

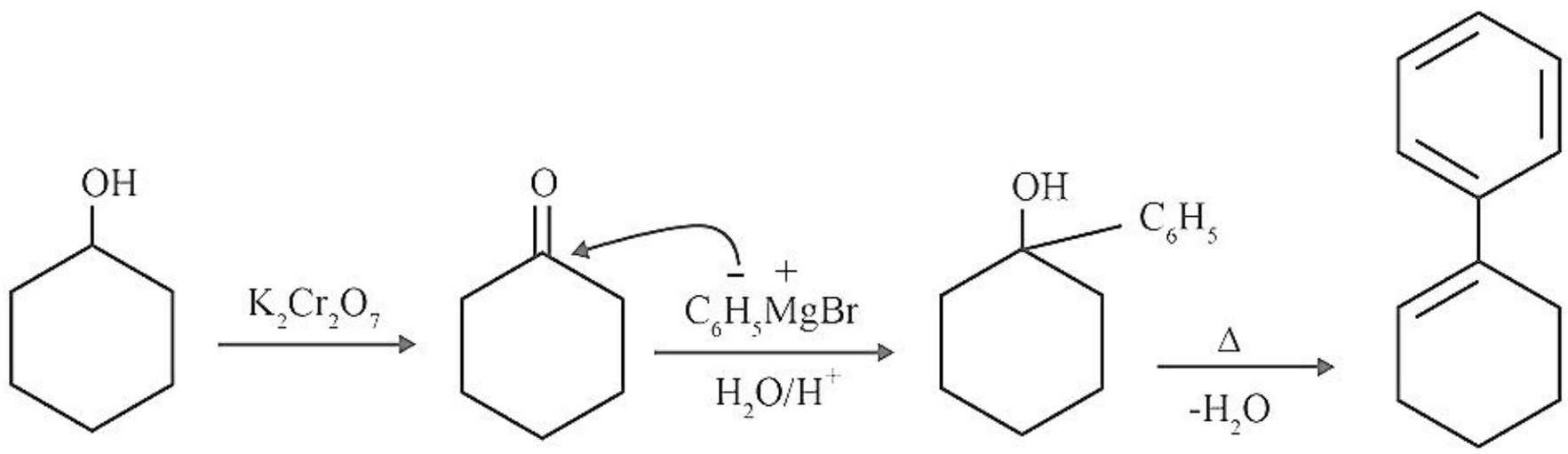
Question:



Number of sp^2 hybridised carbon are

Answer: 8.00

Solution:



Number of sp^2 hybridised carbon are 8

JEE-Main-29-06-2022-Shift-2 (Memory Based)

MATHEMATICS

Question: $\lim\limits_{x\to 1}\dfrac{\left(x^2-1\right)\sin^2\left(\pi x\right)}{x^4-2x^3+2x-1}$ is equal to

Options:

(a) $2\pi^2$

(b) π^2

(c) $3\pi^2$

(d) $\dfrac{\pi^2}{2}$

Answer: (b)

Solution:

Given, $\lim\limits_{x\to 1}\dfrac{\left(x^2-1\right)\sin^2\left(\pi x\right)}{x^4-2x^3+2x-1}$

$$=\lim_{x\to 1}\frac{\left(x^2-1\right)\sin^2\left(\pi x\right)}{x^4-2x^3-x^2+x^2+2x-1}$$

$$=\lim_{x\to 1}\frac{\left(x^2-1\right)\sin^2\left(\pi x\right)}{\left(x^2-1\right)\left(x^2-2x+1\right)}$$

$$=\lim_{h\to 0}\frac{\sin^2\left(\pi\left(1+h\right)\right)}{\left(1+h-1\right)^2}=\lim_{h\to 0}\frac{\sin^2\pi h}{h^2}$$

$$=\lim_{h\to 0}\frac{\sin^2\pi h}{h^2}$$

$$=\lim_{h\to 0}\left(\frac{\sin\pi h}{\pi h}\right)\cdot\pi^2=\pi^2$$

Question: If the line $\dfrac{x-2}{3}=\dfrac{y-2}{4}=\dfrac{z+6}{2}$ intersects the plane $2x+4y+3z=0$ at point P. Find the distance OP (where O is origin $\left(0,0,0\right)$).

Options:

(a) $\dfrac{\sqrt{8096}}{7}$

(b) $\dfrac{\sqrt{9053}}{14}$

(c) $\frac{\sqrt{7084}}{7}$

(d) $\frac{\sqrt{9017}}{14}$

Answer: (b)

Solution:

Let a point on line be $(3\lambda+2, 4\pi+2, 2\lambda-6)$

Now, $2(3\lambda+2)+4(4\lambda+2)+3(2\lambda-6)=0$

$28\lambda = 6$

$\Rightarrow \lambda = \frac{3}{14}$

Point is: $\left(\frac{37}{14}, \frac{40}{14}, -\frac{78}{14}\right)$

$$OP = \sqrt{\left(\frac{37}{14}\right)^2 + \left(\frac{40}{14}\right)^2 + \left(\frac{-78}{14}\right)^2}$$

$$= \frac{\sqrt{9053}}{14}$$

Question: $3, 6, 9, \ldots$ upto 78 terms

$5, 9, 13, \ldots$ upto 59 terms.

Find the sum of common terms between them.

Options:

(a) 2223

(b) 1785

(c) 1805

(d) 2025

Answer: (a)

Solution:

$3, 6, 9, \ldots$ upto 78 terms

$\Rightarrow t_{78} = 3 + 77 \times 3 = 234$

$5, 9, 13, \ldots$ upto 59 terms

$\Rightarrow t_{59} = 5 + 58 \times 4 = 237$

Common difference of common terms $= \text{LCM}\{3, 4\} = 12$

$9, 21, 33, \ldots, 225$

$225 = 9 + (n-1)12$

$\Rightarrow n = 19$

$S = \frac{n}{2}[a+l] = \frac{19}{2}[9+225] = 2223$

Question: If $S = 1 + \frac{5}{6} + \frac{10}{6^2} + \frac{16}{6^3} + \ldots$ then find S.

Options:

(a) $\frac{16}{216}$

(b) $\frac{301}{125}$

(c) $\frac{25}{216}$

(d) $\frac{276}{125}$

Answer: (d)

Solution:

Given,

$$S = 1 + \frac{5}{6} + \frac{10}{6^2} + \frac{16}{6^3} + \ldots$$

$$\frac{S}{6} = \quad \frac{1}{6} + \frac{5}{6^2} + \frac{10}{6^3} + \ldots.$$

$$\frac{5S}{6} = 1 + \frac{4}{6} + \frac{5}{6^2} + \frac{6}{6^3} + \ldots$$

$$\frac{5S}{6} - 1 = \frac{4}{6} + \frac{5}{6^2} + \frac{6}{6^3} + \ldots$$

$$\frac{1}{6}\left(\frac{5S}{6} - 1\right) = \frac{4}{6^2} + \frac{5}{6^3} + \ldots$$

$$\frac{5}{6}\left(\frac{5S}{6} - 1\right) = \frac{4}{6} + \frac{1}{6^2} + \frac{1}{6^3} + \ldots$$

$$\frac{5}{6}\left(\frac{5S}{6} - 1\right) = \frac{4}{5} + \frac{\left(\frac{1}{36}\right)}{1 - \frac{1}{6}}$$

$$\Rightarrow \frac{5S}{6} - 1 = \frac{21}{25}$$

$$\Rightarrow \frac{5S}{6} = \frac{46}{25}$$

$$\Rightarrow S = \frac{46}{25} \times \frac{6}{5}$$

$$\Rightarrow S = \frac{276}{125}$$

Question: Let f be a continuous function in $[0,1]$ such that $f(x)=x+\int_0^1 (x-t)f(t)dt$, then which of the following points does not lie on the curve $y=f(x)$?

Options:

(a) $\left(\frac{1}{2},\frac{5}{13}\right)$

(b) $\left(\frac{1}{3},\frac{2}{13}\right)$

(c) $\left(\frac{2}{9},0\right)$

(d) $\left(\frac{1}{6},\frac{1}{13}\right)$

Answer: (d)

Solution:

$\because f(x)=x\left(1+\int_0^1 f(t)dt\right)-\int_0^1 t\cdot f(t)dt$

Let $a=1+\int_0^1 f(t)dt$ and $b=\int_0^1 t\cdot f(t)dt$

$\Rightarrow f(x)=ax-b$

$a=1+\int_0^1 f(t)dt$

$\Rightarrow \frac{a}{2}=1-b \quad ....(1)$

$b=\int_0^1 t\cdot(at-b)dt$

$\Rightarrow \frac{3b}{2}=\frac{a}{3} \quad ....(2)$

From (1) and (2), we get

$a=\frac{8}{13}$ and $b=\frac{4}{13}$

$\Rightarrow f(x)=\frac{18x-4}{13}$

Clearly $\left(\frac{1}{6},\frac{1}{13}\right)$ does not lie on the curve $y=f(x)$

Question: Find the probability that a relation $\{x,y\}\rightarrow\{x,y\}$ is symmetric as well as transitive.

Options:

(a) $\frac{1}{4}$

(b) $\frac{3}{8}$

(c) $\frac{5}{16}$

(d) $\frac{1}{8}$

Answer: (c)

Solution:

$\{x, y\} \times \{x, y\} = \{(x, x), (x, y), (y, x), (y, y)\}$

Number of possible relations $= 2^4 = 16$

The relations which are symmetric as well as transitive are

$\phi, \{x, x\}, \{y, y\}, \{(x, x), (y, y)\}$ and $\{(x, x), (x, y), (y, x), (y, y)\}$

$\therefore$ Required probability $= \frac{5}{16}$

Question: The height of a pole is $20\ m$. If the angle of elevation of the tower from the top of the pole is $30°$ and the same from bottom of the tower is $60°$, then the height of the tower is

Options:

(a) $20\sqrt{3}\ m$

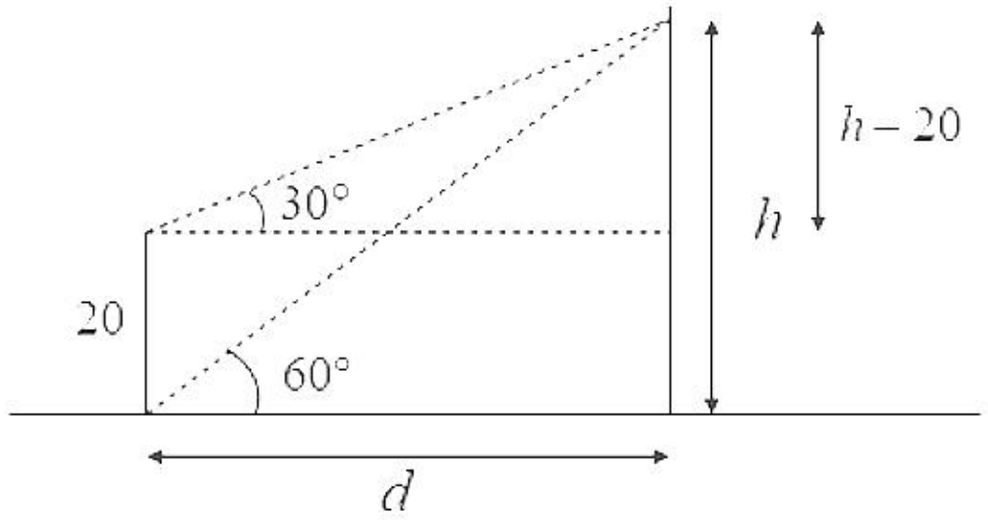
(b) $\left(20 + 10\sqrt{3}\right) m$

(c) $30\ m$

(d) $\left(10 + 20\sqrt{3}\right) m$

Answer: (c)

Solution:



$\tan 30° = \frac{h - 20}{d}$

$$\tan 60^\circ = \frac{h}{d}$$
$$\Rightarrow \frac{1}{3} = \frac{h-20}{h}$$
$$\Rightarrow h = 30\ m$$

Question: If $\sin x = \cos^2 x$, then the number of solutions in $x \in (0,10)$ are ____.

Answer: 4.00

Solution:

Given, $\sin x = \cos^2 x$

$$\Rightarrow \sin^2 x + \sin x - 1 = 0$$
$$\Rightarrow \sin x = \left(\frac{-1 \pm \sqrt{5}}{2}\right)$$
$$\Rightarrow \sin x = \frac{\left(\sqrt{5}-1\right)}{2}$$
$$\Rightarrow \text{ Number of solutions in } (0,10) = 4$$

Question: In the expansion of $\left(2x^{\frac{1}{5}} - \frac{1}{x^{\frac{1}{5}}}\right)15$, coefficients of x^{-1} and x^{-3} are m and n respectively. If $m \cdot n^2 = {}^{15}C_r \cdot 2^r$, then r is equal to _____.

Answer: 5.00

Solution:

$$T_{r+1} = {}^{15}C_r \left(2x^{\frac{1}{5}}\right)^{15-r} \left(\frac{-1}{x^{\frac{1}{5}}}\right)^r$$

For term having x^{-1}

$$\frac{15-r}{5} - \frac{r}{5} = -1$$
$$\Rightarrow r = 10$$
$$\Rightarrow m = {}^{15}C_{10} \cdot 2^5 \cdot (-1)^{10}$$

For term having x^{-3}

$$\frac{15-2r}{5} = -3$$
$$\Rightarrow r = 15$$
$$\Rightarrow n = {}^{15}C_{15} \cdot 2^0 \cdot (-1)^{15} = -1$$
$$\therefore mn^2 = {}^{15}C_{10} \cdot 2^5 \cdot (-1)^2 = {}^{15}C_r \cdot 2^r$$

$\Rightarrow {}^{15}C_5 \cdot 2^5 = {}^{15}C_r \cdot 2^r$

$\Rightarrow r = 5$

Question: If $A = \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}$, then the modulus of sum of all elements of matrix B which is satisfying $B = I - {}^5C_1\, adj(A) + {}^5C_2 \left(adj(A)\right)^2 - {}^5C_3 \left(adj(A)\right)^3 + {}^5C_4 \left(adj(A)\right)^4 - {}^5C_5 \left(adj(A)\right)^5$ is ____.

Answer: 7.00

Solution:

Given, $B = I - {}^5C_1\, adj(A) + {}^5C_2 \left(adj(A)\right)^2 - {}^5C_3 \left(adj(A)\right)^3 + {}^5C_4 \left(adj(A)\right)^4 - {}^5C_5 \left(adj(A)\right)^5$

$\Rightarrow B = \left(I - adj(A)\right)^5$

$A = \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}$

$\Rightarrow adj(A) = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$

$B = \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix}^5$

$-B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^5 = \begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}$

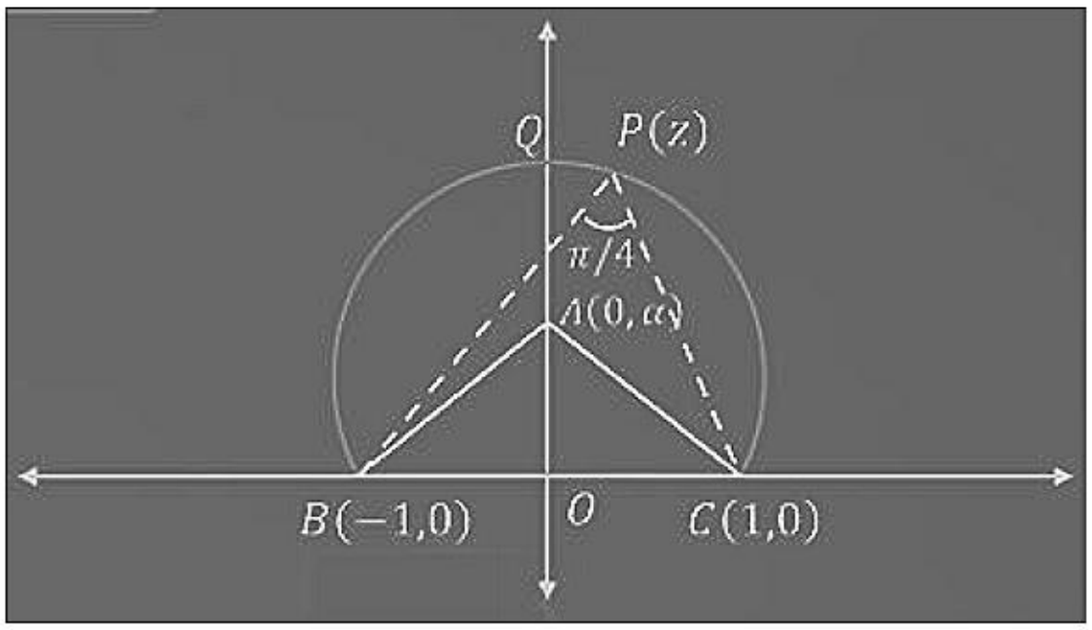
$B = \begin{bmatrix} -1 & -5 \\ 0 & -1 \end{bmatrix}$

Sum of elements $= -1 - 5 - 1 = -7$

Question: The number of complex numbers z such that $|z| = 3$ and $\arg(z-1) - \arg(z+1) = \frac{\pi}{4}$ are ____.

Answer: 0.00

Solution:



Given, $\arg(z-1)-\arg(z+1)=\frac{\pi}{4}$

$\Rightarrow \arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{4}$

$\Rightarrow z$ is on major arc of a circle having BC as chord as shown in figure

$\angle OAC=\angle BPC=\frac{\pi}{4}$

$\Rightarrow OA=OC=1=\alpha$

Radius $=AC=\sqrt{2}$

$OQ=\alpha+\text{Radius}=1+\sqrt{2}$

$\Rightarrow Q=\left(0,1+\sqrt{2}\right)$

$|z|=3$ represents a circle of radius 3 and centre at $z=0$

Then both circles do not intersect.

Hence, no common point.

Question: Number of four digit numbers in which first three digit number is divisible by last digit i.e., fourth digit is _____.

Answer: 2545.00

Solution:

If the last digit is d then there are $\left[\frac{900}{d}\right]$ possibilities for first three digits.

Total number of 4 digit numbers $=\sum_{d=1}^{9}\left[\frac{900}{d}\right]$

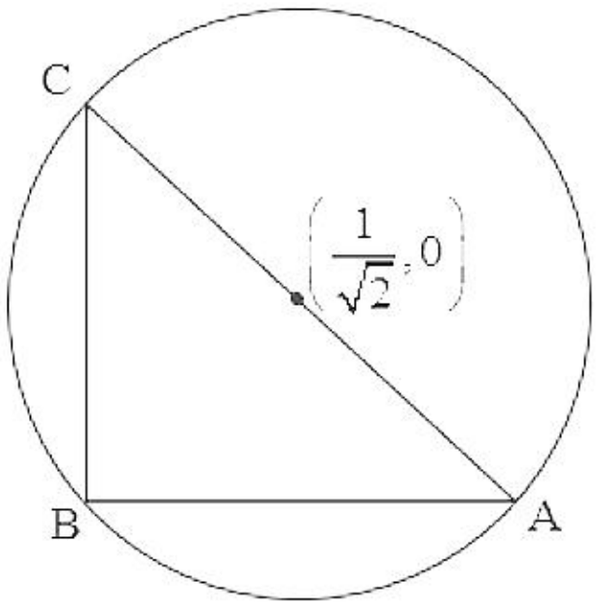
$=900+450+300+225+180+150+128+112+100$

$=2545$

Question: ΔABC is inscribed in a circle $x^2-\sqrt{2}x+y^2=0$ where $\angle ABC=\frac{\pi}{2}$, then the maximum area of triangle ABC is _____.

Answer: 0.5

Solution:



$$\text{Centre} \equiv \left(\frac{1}{\sqrt{2}},0\right),\ r=\frac{1}{\sqrt{2}}$$

$$AC = \text{diameter} = \sqrt{2}$$

$$ar\left(\Delta ABC\right)=\frac{1}{2}\times AB\times BC$$

$$=\frac{1}{2}\times\sqrt{2}\cos\theta\times\left(\sqrt{2}\sin\theta\right)$$

$$=\frac{1}{2}\times\sin 2\theta$$

$$\therefore ar\left(\Delta ABC\right)_{\max}=\frac{1}{2}=0.5$$

Question: $M=\begin{bmatrix}0 & -\alpha\\ \alpha & 0\end{bmatrix}$. If $\left(I-M^2\right)N=-21$ & $N=\sum_{k=1}^{49}M^{2k}$, then $\alpha^2=$ ___.

Answer: 1.00

Solution:

$$M=\begin{bmatrix}0 & -\alpha\\ \alpha & 0\end{bmatrix}$$

$$\therefore M^2=\begin{bmatrix}0 & -\alpha\\ \alpha & 0\end{bmatrix}\begin{bmatrix}0 & -\alpha\\ \alpha & 0\end{bmatrix}=\begin{bmatrix}-\alpha^2 & 0\\ 0 & -\alpha^2\end{bmatrix}=-\alpha^2 I$$

$$\text{So, } N=\sum_{k=1}^{49}\left(M^2\right)^k=I\left(-\alpha^2+\alpha^4-\alpha^6+\ldots 49 \text{ terms}\right)=-\frac{\alpha^2\left(1+\alpha^{98}\right)}{1+\alpha^2}$$

$$\text{Now, } \left(I-M^2\right)N=\left(1+\alpha^2\right)\left[-\frac{\alpha^2\left(1+\alpha^{98}\right)}{1+\alpha^2}\right]I=-2I$$

$$\Rightarrow \alpha^2\left(1+\alpha^{98}\right)=2$$

$$\Rightarrow \alpha^2=1$$