

NG 17 (GROUP A)

PART AA — ENGINEERING MATHEMATICS

(Common to all candidates)

(Answer ALL questions)

1. The system of linear equations $4x + 3y = 7$, $2x + y = 6$ has

1. a unique solution
2. no solution
3. an infinite number of solutions
4. exactly two distinct solutions

2. Let $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$. The eigenvalues of $2A^{-1}$ are

1. $-\frac{1}{3}$ and -2
2. $\frac{1}{2}$ and $\frac{1}{3}$
3. -1 and -6
4. 3 and $\frac{1}{2}$

3. The quadratic form $Q(x, y) = 3x^2 + 2xy + 4y^2$ is

1. positive semidefinite
2. negative semidefinite
3. negative definite
4. positive definite

4. Let $u(x, y) = \log\left(\frac{x^2}{y}\right)$. The value of

$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ is equal to

1. $2u$
2. 1
3. 0
4. u

5. The particular integral of $\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + 2y = e^x \cos x$ is

1. $\frac{x^2 e^x \sin x}{2}$
2. $\frac{xe^x \sin x}{3}$
3. $\frac{xe^x \sin x}{2}$
4. $\frac{x^2 e^x \sin x}{3}$

6. By eliminating the constants 'a' and 'b' from $x^2 + y^2 + (z - a)^2 = b^2$, the partial differential equation is

1. $x^2 \frac{\partial z}{\partial y} - y^2 \frac{\partial z}{\partial x} = 0$
2. $x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y} = 0$
3. $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 0$
4. $x \frac{\partial z}{\partial y} - y \frac{\partial z}{\partial x} = 0$

7. If ϕ and ψ are scalar functions, then the value of $\nabla \cdot (\nabla \phi \times \nabla \psi)$ is

1. 1
2. 0
3. -1
4. 2

8. Let $\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$ and S be the surface of a unit sphere. By the Gauss divergent theorem, the value of $\iint_S \vec{F} \cdot \hat{n} dS$, where $\hat{n}$ is a unit outward normal to S , is

1. 2π
2. $\frac{4\pi}{3}$
3. 4π
4. $\frac{5\pi}{3}$

9. If $f(z) = u(x, y) + iv(x, y)$ is analytic in a domain D , then its component functions $u(x, y)$ and $v(x, y)$ are

1. harmonic in D
2. not harmonic in D
3. not satisfying the C-R equations in D
4. not differentially partially in D

10. The residue of $f(z) = \frac{ze^z}{(z-1)^3}$ is

1. 1
2. $\frac{3e}{2}$
3. $\frac{2e}{3}$
4. $\frac{e}{2}$

11. The Laurent expansion of $f(z) = \frac{1}{z(z-1)}$ valid for $|z| > 1$ is

1. $\frac{1}{z} \left(1 + \frac{1}{z} + \frac{1}{z^2} + \dots \right)$
2. $\frac{1}{z} \left(1 + \frac{1}{z} + \frac{1}{z^2} + \dots \right) - \frac{1}{z}$
3. $z \left(1 + \frac{1}{z} + \frac{1}{z^2} + \dots \right) - \frac{1}{z}$
4. $z \left(1 + \frac{1}{z} + \frac{1}{z^2} + \dots \right)$

12. Let $F(s) = \frac{1}{s(s^2+1)}$ be the Laplace transform of $f(t)$. By inverse Laplace transform, $f(t)$ is

1. $1 - \sin t$
2. $1 - \cos t$
3. $1 + \cos t$
4. $1 + \sin t$

13. The Fourier cosine transform of $f(x) = e^{-x}$, $x > 0$ is

1. $\sqrt{\frac{2}{\pi}} \left(\frac{1}{1+s^2} \right)$
2. $\sqrt{\frac{\pi}{2}} \left(\frac{1}{1+s^2} \right)$
3. $\sqrt{\frac{2}{\pi}} \left(\frac{s}{1+s^2} \right)$
4. $\sqrt{\frac{\pi}{2}} \left(\frac{s}{1+s^2} \right)$