

1. Let A be a 2×2 nonzero real matrix. Which of the following is true?

- (A) A has a nonzero eigenvalue.
- (B) A^2 has at least one positive entry.
- (C) $\text{trace}(A^2)$ is positive.
- (D) All entries of A^2 cannot be negative.

2. Let A be a 3×3 real matrix with zero diagonal entries. If $1 + i$ is an eigenvalue of A , the determinant of A equals

- (A) 4. (B) -4 . (C) 2. (D) -2 .

3. Let A be an $n \times n$ matrix and let b be an $n \times 1$ vector such that $Ax = b$ has a unique solution. Let A' denote the transpose of A . Then which of the following statements is **false**?

- (A) $A'x = 0$ has a unique solution.
- (B) $A'x = c$ has a unique solution for any non-zero c .
- (C) $Ax = c$ has a solution for any c .
- (D) $A^2x = c$ is inconsistent for some vector c .

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4. Let A and B be $n \times n$ matrices. Assuming all the inverses exist,

$$(A^{-1} - B^{-1})^{-1}$$

equals

- (A) $(I - AB^{-1})^{-1}B$.
- (B) $A(B - A)^{-1}B$.
- (C) $B(B - A)^{-1}A$.
- (D) $B(A - B)^{-1}A$.

5. Let f be a function defined on $(-\pi, \pi)$ as

$$f(x) = (|\sin x| + |\cos x|) \cdot \sin x.$$

Then f is differentiable at

- (A) all points.
- (B) all points except at $x = -\pi/2, \pi/2$.
- (C) all points except at $x = 0$.
- (D) all points except at $x = 0, -\pi/2, \pi/2$.

6. The equation of the tangent to the curve $y = \sin^2(\pi x^3/6)$ at $x = 1$ is

- (A) $y = \frac{1}{4} + \frac{\sqrt{3}\pi}{4}(x - 1)$.
- (B) $y = \frac{\sqrt{3}\pi}{4}x + \frac{1-\sqrt{3}\pi}{4}$.
- (C) $y = \frac{\sqrt{3}\pi}{4}x - \frac{1-\sqrt{3}\pi}{4}$.
- (D) $y = \frac{1}{4} - \frac{\sqrt{3}\pi}{4}(x - 1)$.

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7. Let f be a function defined from $(0, \infty)$ to $\mathbb{R}$ such that

$$\lim_{x \rightarrow \infty} f(x) = 1 \text{ and } f(x+1) = f(x) \text{ for all } x.$$

Then f is

- (A) continuous and bounded.
- (B) continuous but not necessarily bounded.
- (C) bounded but not necessarily continuous.
- (D) neither necessarily continuous nor necessarily bounded.

8. The value of $\lim_{x \rightarrow \infty} (\log x)^{1/x}$

- (A) is e . (B) is 0. (C) is 1. (D) does not exist.

9. The number of real solutions of the equation,

$$x^7 + 5x^5 + x^3 - 3x^2 + 3x - 7 = 0$$

is

- (A) 5. (B) 7. (C) 3. (D) 1.

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10. Let x be a real number. Then

$$\lim_{m \rightarrow \infty} \left(\lim_{n \rightarrow \infty} \cos^{2n}(m!\pi x) \right)$$

- (A) does not exist for any x .
- (B) exists for all x .
- (C) exists if and only if x is irrational.
- (D) exists if and only if x is rational.

11. Let $\{a_n\}_{n \geq 1}$ be a sequence such that $a_1 \leq a_2 \leq \cdots \leq a_n \leq \cdots$. Suppose the subsequence $\{a_{2n}\}_{n \geq 1}$ is bounded. Then

- (A) $\{a_{2n}\}_{n \geq 1}$ is always convergent but $\{a_{2n+1}\}_{n \geq 1}$ need not be convergent.
- (B) both $\{a_{2n}\}_{n \geq 1}$ and $\{a_{2n+1}\}_{n \geq 1}$ are always convergent and have the same limit.
- (C) $\{a_{3n}\}_{n \geq 1}$ is not necessarily convergent.
- (D) both $\{a_{2n}\}_{n \geq 1}$ and $\{a_{2n+1}\}_{n \geq 1}$ are always convergent but may have different limits.

12. Let $\{a_n\}_{n \geq 1}$ be a sequence of positive numbers such that $a_{n+1} \leq a_n$ for all n , and $\lim_{n \rightarrow \infty} a_n = a$. Let $p_n(x)$ be the polynomial

$$p_n(x) = x^2 + a_n x + 1,$$

and suppose $p_n(x)$ has no real roots for every n . Let α and β be the roots of the polynomial $p(x) = x^2 + ax + 1$. Then

- (A) $\alpha = \beta$, α and β are not real.
- (B) $\alpha = \beta$, α and β are real.
- (C) $\alpha \neq \beta$, α and β are real.
- (D) $\alpha \neq \beta$, α and β are not real.

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13. Consider the set of all functions from $\{1, 2, \dots, m\}$ to $\{1, 2, \dots, n\}$, where $n > m$. If a function is chosen from this set at random, what is the probability that it will be strictly increasing?

- (A) $\binom{n}{m}/m^n$. (B) $\binom{n}{m}/n^m$. (C) $\binom{m+n-1}{m}/m^n$. (D) $\binom{m+n-1}{m-1}/n^m$.

14. A flag is to be designed with 5 vertical stripes using some or all of the four colours: green, maroon, red and yellow. In how many ways can this be done so that no two adjacent stripes have the same colour?

- (A) 576. (B) 120. (C) 324. (D) 432.

15. Suppose $x_1, \dots, x_6$ are real numbers which satisfy

$$x_i = \prod_{j \neq i} x_j, \quad \text{for all } i = 1, \dots, 6.$$

How many choices of $(x_1, \dots, x_6)$ are possible?

- (A) Infinitely many. (B) 2. (C) 3. (D) 1.

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16. Suppose X is a random variable with $P(X > x) = 1/x^2$, for all $x > 1$.

The variance of $Y = 1/X^2$ is

- (A) $1/4$. (B) $1/12$. (C) 1 . (D) $1/2$.

17. Let $X \sim N(0, \sigma^2)$, where $\sigma > 0$, and

$$Y = \begin{cases} -1 & \text{if } X \leq -1, \\ X & \text{if } X \in (-1, 1), \\ 1 & \text{if } X \geq 1. \end{cases}$$

Which of the following statements is correct?

- (A) $\text{Var}(Y) = \text{Var}(X)$.
(B) $\text{Var}(Y) < \text{Var}(X)$.
(C) $\text{Var}(Y) > \text{Var}(X)$.
(D) $\text{Var}(Y) \geq \text{Var}(X)$ if $\sigma \geq 1$, and $\text{Var}(Y) < \text{Var}(X)$ if $\sigma < 1$.

18. If a fair coin is tossed 5 times, what is the probability of obtaining at least 3 consecutive heads?

- (A) $1/8$. (B) $5/16$. (C) $1/4$. (D) $3/16$.

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19. Let X and Y be random variables with mean λ . Define

$$Z = \begin{cases} \min(X, Y) & \text{with probability } \frac{1}{2}, \\ \max(X, Y) & \text{with probability } \frac{1}{2}. \end{cases}$$

What is $E(Z)$?

- (A) λ . (B) $4\lambda/3$. (C) λ^2 . (D) $\sqrt{3}\lambda/2$.

20. A finite population has $N(\geq 10)$ units marked $\{U_1, \dots, U_N\}$. The following sampling scheme was used to obtain a sample s . One unit is selected at random: if this is the i -th unit, then the sample is $s = \{U_{i-1}, U_i, U_{i+1}\}$, provided $i \notin \{1, N\}$. If $i = 1$ then $s = \{U_1, U_2\}$ and if $i = N$ then $s = \{U_{N-1}, U_N\}$. The probability of selecting U_2 in s is

- (A) $\frac{2}{N}$. (B) $\frac{3}{N}$. (C) $\frac{1}{(N-2)} + \frac{2}{N}$. (D) $\frac{3}{(N-2)}$.

21. Suppose $X_1, \dots, X_n$ are i.i.d. observations from a distribution assuming values $-1, 1$ and 0 with probabilities p, p and $1 - 2p$, respectively, where $0 < p < \frac{1}{2}$. Define $Z_n = \prod_{i=1}^n X_i$ and $a_n = P(Z_n = 1)$, $b_n = P(Z_n = -1)$, $c_n = P(Z_n = 0)$. Then as $n \rightarrow \infty$,

- (A) $a_n \rightarrow \frac{1}{4}, b_n \rightarrow \frac{1}{2}, c_n \rightarrow \frac{1}{4}$.
 (B) $a_n \rightarrow \frac{1}{3}, b_n \rightarrow \frac{1}{3}, c_n \rightarrow \frac{1}{3}$.
 (C) $a_n \rightarrow 0, b_n \rightarrow 0, c_n \rightarrow 1$.
 (D) $a_n \rightarrow p, b_n \rightarrow p, c_n \rightarrow 1 - 2p$.

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22. Suppose X_1, X_2 and X_3 are i.i.d. positive valued random variables. Define $Y_i = \frac{X_i}{X_1+X_2+X_3}$, $i = 1, 2, 3$. The correlation between Y_1 and Y_3 is

(A) 0. (B) $-1/6$. (C) $-1/3$. (D) $-1/2$.

23. Assume (y_i, x_i) satisfies the linear regression model,

$$y_i = \beta x_i + \epsilon_i, \quad \text{for } i = 1, \dots, n,$$

where, $\beta \in \mathbb{R}$ is unknown, $\{x_i : 1 \leq i \leq n\}$ are fixed constants and $\{\epsilon_i : 1 \leq i \leq n\}$ are i.i.d. errors with mean zero and variance $\sigma^2 \in (0, \infty)$. Let $\hat{\beta}$ be the least squares estimate of β and $\hat{y}_i = \hat{\beta}x_i$ be the predicted value of y_i . For each $n \geq 1$, define

$$a_n = \frac{1}{\sigma^2} \sum_{i=1}^n \text{Cov}(y_i, \hat{y}_i).$$

Then,

(A) $a_n = 1$. (B) $a_n \in (0, 1)$. (C) $a_n = n$. (D) $a_n = 0$.

24. Let X and Y be two random variables with $E(X|Y = y) = y^2$, where Y follows $N(\theta, \theta^2)$, with $\theta \in \mathbb{R}$. Then $E(X)$ equals

(A) θ . (B) θ^2 . (C) $2\theta^2$. (D) $\theta + \theta^2$.

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25. Suppose X is a random variable with finite variance. Define $X_1 = X$, $X_2 = \alpha X_1$, $X_3 = \alpha X_2, \dots, X_n = \alpha X_{n-1}$, for $0 < \alpha < 1$. Then $\text{Corr}(X_1, X_n)$ is

- (A) α^n . (B) 1. (C) 0. (D) α^{n-1} .

26. Let X be a random variable with $P(X = 2) = P(X = -2) = 1/6$ and $P(X = 1) = P(X = -1) = 1/3$. Define $Y = 6X^2 + 3$. Then

- (A) $\text{Var}(X - Y) < \text{Var}(X)$.
 (B) $\text{Var}(X - Y) < \text{Var}(X + Y)$.
 (C) $\text{Var}(X + Y) < \text{Var}(X)$.
 (D) $\text{Var}(X - Y) = \text{Var}(X + Y)$.

27. Suppose X is a random variable on $\{0, 1, 2, \dots\}$ with unknown p.m.f. $p(x)$. To test the hypothesis $H_0 : X \sim \text{Poisson}(1/2)$ against $H_1 : p(x) = 2^{-(x+1)}$ for all $x \in \{0, 1, 2, \dots\}$, we reject H_0 if $x > 2$. The probability of type-II error for this test is

- (A) $\frac{1}{4}$. (B) $1 - \frac{13}{8}e^{-1/2}$. (C) $1 - \frac{3}{2}e^{-1/2}$. (D) $\frac{7}{8}$.

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28. Let X be a random variable with

$$P_{\theta}(X = -1) = \frac{(1 - \theta)}{2}, \quad P_{\theta}(X = 0) = \frac{1}{2}, \quad \text{and} \quad P_{\theta}(X = 1) = \frac{\theta}{2}$$

for $0 < \theta < 1$. In a random sample of size 20, the observed frequencies of $-1, 0$ and 1 are 6, 4 and 10, respectively. The maximum likelihood estimate of θ is

- (A) $1/5$. (B) $4/5$. (C) $5/8$. (D) $1/4$.

29. Two judges evaluate n individuals, with (R_i, S_i) the ranks assigned to the i -th individual by the two judges. Suppose there are no ties and $S_i = R_i + 1$, for $i = 1, \dots, (n - 1)$, and $S_i = 1$ if $R_i = n$. If the Spearman's rank correlation between the two evaluations is 0, what is the value of n ?

- (A) 7. (B) 11. (C) 4. (D) 5.

30. Let $X_1, X_2, \dots$ be a sequence of i.i.d. random variables with variance 2. Then for all x ,

$$\lim_{n \rightarrow \infty} P \left(\frac{1}{\sqrt{n}} \sum_{i=1}^n (-1)^i X_i \leq x \right)$$

equals

- (A) $\Phi(x\sqrt{2})$. (B) $\Phi(x/\sqrt{2})$. (C) $\Phi(x)$. (D) $\Phi(2x)$.

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