

NEST – 2011



NATIONAL ENTRANCE SCREENING TEST (NEST – 2011)

Total Marks: 200

Time: 3 hours

General instructions

1. This question booklet contains 5 sections, with mark distribution as follows.

Section 1	General	50 marks	Compulsory
Section 2	Biology	50 marks	Choose any three sections
Section 3	Chemistry	50 marks	
Section 4	Mathematics	50 marks	
Section 5	Physics	50 marks	

$$\text{Total} = 50 + 3 \times 50 = 200 \text{ marks}$$

- Section 1 is a **General** section and is **compulsory**.
- Sections 2 to 5 are subject sections (**Biology, Chemistry, Mathematics** and **Physics**). **Choose any three.** That is, omit any one of the four subject sections.
- Carefully read and follow the instructions given in each section.
- Answers to the questions are to be marked in the Answer Sheet provided.
- Ensure that you have received Answer Sheet A.**
- Rough work should be done on the space provided at the end of the question paper. If necessary extra sheets will be provided.
- Return the Answer Sheet to the invigilator at the end of the examination.
- Calculators, log tables, cell phones, etc. are not permitted in the examination hall.

Instructions for writing on Answer Sheet

- Read and follow the instructions given on the Answer Sheet.
- Write your name, roll number and other required information with ball point pen in the appropriate boxes provided. Sign your name with ball point pen in the box provided.
- Your roll number (given on the admit card) must be entered correctly. If entered wrongly or not entered, the Answer Sheet WILL not be graded.**
- In the remaining part of Answer Sheet use **HB pencil only as instructed. Make sure that the bubbles are filled properly** (as shown in Answer Sheet).
- Each question has four options. Fill the appropriate bubble(s). Some questions (as specified in the question paper) have more than one correct option.
- Ensure that you are filling up the bubbles corresponding to correct sections.**
- Fill in the answers only when you are sure that you do not need to change the answer. As far as possible, avoid erasing the answer. In case you have to erase the answer, do so properly so that there is no black spot inside the bubble.

Section 1: GENERAL

Marks for Section 1: 50

This section contains 19 questions.

*For each question, **only one** of the four options is a correct answer. For questions 1.1 to 1.12, a correct answer will earn 3 marks. For questions 1.13 to 1.19, a correct answer will earn 2 marks. For this **GENERAL** section, a wrong answer or an unattempted question will earn 0 mark.*

Read the following passage carefully and answer questions 1.1 to 1.3.

Black holes are astronomical objects that exert such a strong gravitational force that no particle – not even light – can come out of its surface; hence they are invisible to external observers. If a black hole absorbs an in-falling material of total energy E , its mass increases by E/c^2 , in accordance with Einstein's well-known mass-energy equivalence relation. The total energy of the black hole and the in-falling material is thus conserved. This is like the first law of thermodynamics. It is also known that in any physical process, the surface area A of the black hole never decreases. Now, according to the second law of thermodynamics, the entropy of the universe never decreases. Thus, the surface area of a black hole is taken to be proportional to its entropy.

A black hole can be either a rotating one or a non-rotating one. The surface area of a rotating black hole is given by

$$A = 4\pi \left[\frac{2G^2 M^2}{c^4} + \frac{2GM}{c^2} \sqrt{\frac{G^2 M^2}{c^4} - \frac{L^2}{M^2 c^2}} \right]$$

where M is the mass of the black hole, c the speed of light, G the universal gravitational constant and L the angular momentum of the black hole about the axis of rotation. According to the second law of thermodynamics, energy extraction from a black hole is possible with maximum efficiency when the entropy of the black hole remains constant. The mechanism of this process is difficult but the process satisfies both the laws of thermodynamics.

1.1 Consider a non-rotating black hole B_1 and a rotating black hole B_2 , both having the same mass M . Choose the correct statement.

- (A) The entropy of B_2 is less than the entropy of B_1 .
- (B) The entropy of B_2 is more than the entropy of B_1 .
- (C) The entropy of B_2 is the same as that of B_1 .
- (D) The sum of the entropies of B_1 and B_2 will always remain constant.

1.2 Assume that the process of energy extraction from a black hole takes place with maximum efficiency. Choose the correct statement.

- (A) Both the mass and angular momentum of the black hole will remain constant in the process.
- (B) Both the mass and angular momentum of the black hole will decrease in the process.
- (C) Only the mass of the black hole will decrease by E/c^2 (where E is total energy extracted), while angular momentum of the black hole will remain constant.
- (D) Both the mass and surface area of the black hole will decrease in the process.

1.3 A rotating black hole of initial mass M_0 has angular momentum $L = GM_0^2/c$. After an energy extraction process it has turned into a non-rotating black hole. The maximum energy that can be extracted out of the process is

- (A) $\left(1 - \frac{1}{\sqrt{2}}\right) M_0 c^2$ (B) $M_0 c^2$ (C) $\frac{1}{2} M_0 c^2$ (D) zero

Read the following passage carefully and answer questions 1.4 to 1.6.

The phenomenon of twins is rare among human species. Statistically, there are about 40 twins in 1000 live births. Generally, twins are born when two separate eggs of a woman are fertilised by two sperms at the same time. These are called fraternal twins. In rarer cases, a single fertilised egg divides into two and the two parts develop into independent embryos. Such twins are called identical twins. The frequency of identical twins is about 3 per 1000 live births. Statistically, the genetic match between two fraternal twins is no different from that of any two siblings. The third type of twin is half-identical twin. In this case, a single egg divides into two before it is fertilised by two different sperms. In some very rare cases, identical twins are joined together and they are called Siamese twins. In any case, each fertilised egg gets 50% genetic material from father and 50% from mother.

Studies on identical twins are useful because they can throw light on whether a trait in a person can be attributed to genetics or environment. For example, some diseases are supposed to have genetic origin. In these cases, there would be strong correlation in observations made on identical twins whereas the correlation would be less in observations made on fraternal twins or siblings. However, the studies based on identical twins are only indicative of trends and not conclusive because of small sample sizes.

1.4 In the country of Raritania, 10,000 live births took place on April 1, which is its independence day. The government decided that a pair of randomly chosen children born on this day will receive full free education. The probability that the chosen pair will be a twin is about

- (A) 4×10^{-2} (B) 1.6×10^{-3} (C) 4×10^{-5} (D) 4×10^{-6}

- 1.5** In a study, three samples X, Y and Z of equal sizes are taken from a large eye hospital. X consists of pairs of identical twins, Y consists of pairs of fraternal twins and Z of sibling pairs who are not twins. P, Q, R and S are four diseases of the eye. Table below gives the number of pairs in each sample where both members test positive for a disease as indicated in the first column.

Diseases	X	Y	Z
P	14	9	12
Q	59	36	31
R	7	11	8
S	23	12	7

Based on the table, which of the diseases are most likely to have a genetic origin?

- (A) P & Q (B) R & S (C) Q & S (D) P & R

- 1.6** Which of the following characteristics need not be the same for identical twins?

- (A) Colour of eyes (B) Blood group
(C) Palm fingerprint pattern (D) Sex

- 1.7** 10 drops of a liquid haloalkane having density of 2 g/mL and molecular mass 150 g/mol have volume of 5 mL. The number of molecules in one drop of haloalkane is

- (A) 2.0×10^{21} (B) 4.0×10^{21} (C) 2.5×10^{23} (D) 6.0×10^{23}

- 1.8** An astronaut camps on the moon for a period of one month as per the earth's calendar. What would be the path of the earth seen by the astronaut in the lunar sky?

- (A) The earth remains approximately at a fixed altitude and direction.
(B) The earth completes one revolution parallel to the lunar horizon in one month.
(C) The earth completes one revolution from east (direction of rising sun) to west (direction of setting sun) in one month.
(D) The earth completes one circle around the Pole Star in one month but never goes below the horizon.

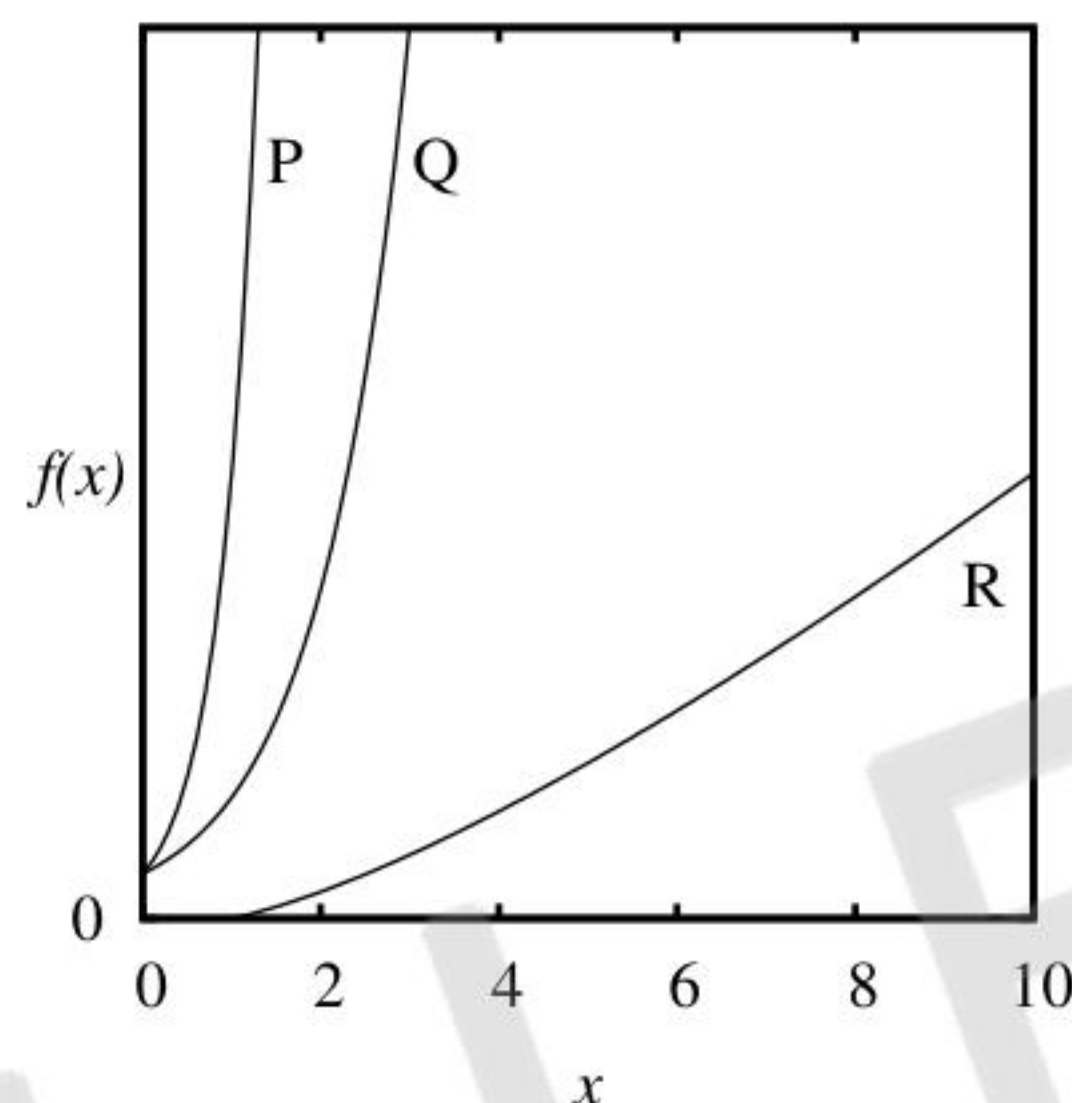
- 1.9** A pole fixed on the ground is leaning away from the vertical. When the Sun is directly overhead, the length of its shadow is 7.5 m. An observer standing 30 m away from the base of the pole in the direction of the shadow, measures the angle of elevation of the top of the pole to be 30° . Find the length of the pole. Ignore the height of the observer.

- (A) 7.5 m (B) 15 m (C) 22.5 m (D) 30 m

1.10 A number written as 213 in quadral system (number system with base 4) will be represented in hexal system (number system with base 6) as

- (A) 23 (B) 39 (C) 103 (D) 303

1.11 The figure shows the graphs of three curves labelled P, Q and R in the first quadrant, corresponding to the following functions $f(x)$: (i) e^x , (ii) 10^x and (iii) $x \log(x)$.



Choose the correct identification of the curves.

- (A) $P \rightarrow (i), Q \rightarrow (ii), R \rightarrow (iii)$ (B) $P \rightarrow (ii), Q \rightarrow (i), R \rightarrow (iii)$
 (C) $P \rightarrow (ii), Q \rightarrow (iii), R \rightarrow (i)$ (D) $P \rightarrow (iii), Q \rightarrow (ii), R \rightarrow (i)$

1.12 There are five bulbs P, Q, R, S and T. The “on” and “off” states of the bulbs P, Q and R are represented by 1 and 0 respectively in the following table.

P	Q	R	S	T
1	1	1		
1	0	1		
0	1	1		
0	0	0		

The following information is given:

- S is on when at least one of P and Q is off .
- T is on when both R and S are on.

Choose the correct statement:

- (A) If S and T are on, Q must be on.
 (B) If P, Q and R are on, T must be on.
 (C) If T is on and P is off, R must be off.
 (D) If P, R, S and T are on, Q must be off.

For the rest of the questions in this section, each correct answer will earn 2 marks.

1.13 Which of the following is thought **NOT** to be associated with the Homo Erectus?

- (A) Use of fire
- (B) Hunting
- (C) Use of tools
- (D) Language with script

1.14 The priority-dispute regarding the discovery of calculus in mathematics involved:

- (A) Newton and Leibnitz
- (B) Fermat and Gauss
- (C) Newton and Gauss
- (D) Leibnitz and Fermat

1.15 The existence of dark energy is inferred by the observation of

- (A) rotation curves of galaxies.
- (B) accelerated expansion of the universe.
- (C) existence of microwave background radiation.
- (D) Hawking radiation from black holes.

1.16 When a person is bitten by a red ant, the burning sensation is due to

- (A) formic acid.
- (B) acetic acid.
- (C) carbonic acid.
- (D) ammonium acetate.

1.17 Taking into account the abundances of different elements in the country used for nuclear energy production, the Indian nuclear energy programme is likely to eventually rely on

- (A) Uranium-235
- (B) Uranium-238
- (C) Thorium-232
- (D) Plutonium-239

1.18 Which of the following is **NOT** a correct example of natural selection?

- (A) Emergence of vegetarianism in human society
- (B) Long necks of giraffes
- (C) Opposable thumb in great apes
- (D) Appearance of drug resistant bacterium

1.19 Which of the following is **NOT** an operating system?

- (A) Linux
- (B) MS Windows
- (C) MS Word
- (D) MS DOS

Section 2: Biology

Marks for Section 2: 50

This section contains 14 questions.

For questions 2.1 to 2.10 only one of the 4 options is correct. A correct answer will earn 3 marks, a wrong answer will earn (−1) mark, and an unattempted question will earn 0 mark.

2.1 In plants when one sperm fertilizes the egg and the other fertilizes the polar bodies, the process is referred to as double fertilization. This results in the formation of

- (A) diploid zygote and diploid endosperm.
- (B) diploid zygote and triploid endosperm.
- (C) diploid zygote and tetraploid endosperm.
- (D) tetraploid zygote and tetraploid endosperm.

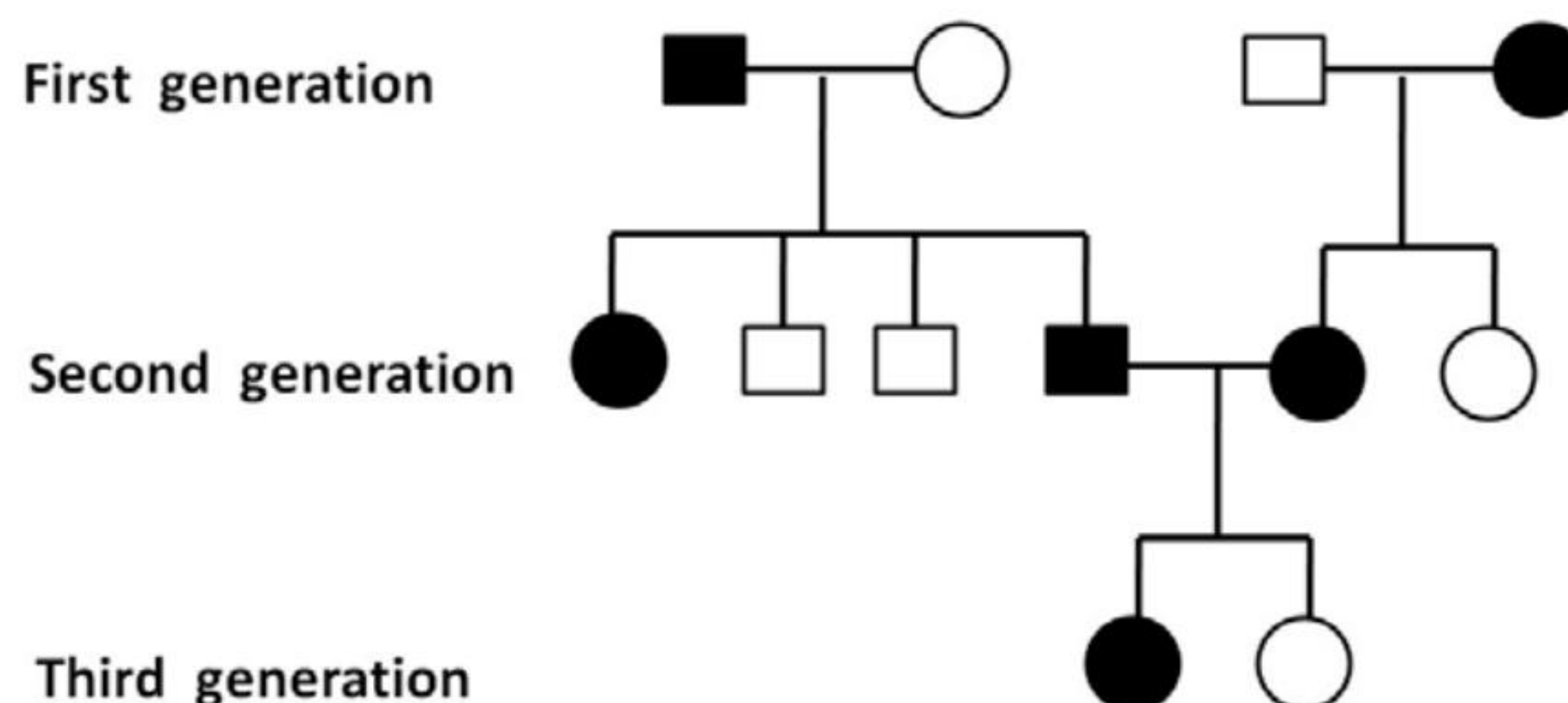
2.2 Sequence of the coding strand in a transcription unit is 5'-ACGGTTACAAGCT-3'. What would be the sequence of mRNA transcribed from this transcription unit?

- (A) 5'-UGCCAAUGUUCGU-3'
- (B) 3'-ACGGUUACAAGCU-5'
- (C) 3'-UCGAACAUUGGCA-5'
- (D) 5'-UCGGAAUCUUGCA-3'

2.3 What is the type of linkage between β - and γ - phosphates in nucleoside triphosphates?

- (A) Phosphoester
- (B) Phosphodiester
- (C) Phosphotriester
- (D) Phosphoanhydride

2.4 Following is a pedigree analysis of inheritance of a trait in a family:



The above inheritance pattern is due to a

- (A) dominant allele.
- (B) recessive allele.
- (C) co-dominant allele.
- (D) imprinted allele.

2.5 Whittaker had proposed a five-kingdom system for classification of living organisms, which are (i) Monera, (ii) Protista, (iii) Plantae, (iv) Fungi and (v) Animalia. However, based on molecular methods of taxonomy, this has now been revised to a three-domain system. These three domains are:

- (A) Monera, Protista and Eukarya.
- (B) Bacteria, Protista and Eukarya.
- (C) Bacteria, Archaea and Eukarya.
- (D) Bacteria, Plantae and Animalia.

2.6 In an individual X, the pituitary gland was found to function normally, while the adrenal glands were atrophied. In another individual Y, both the pituitary and adrenal glands were underdeveloped. If adrenocorticotrophic hormone (ACTH) is administered to these individuals as a remedial measure, it will be effective in

- (A) individual X alone.
- (B) individual Y alone.
- (C) both X and Y.
- (D) neither X nor Y.

2.7 The animal cloning technique adopted in the case of 'Dolly' the sheep involved the

- (A) transfer of a somatic cell nucleus into an enucleated oocyte.
- (B) transfer of the nucleus of an oocyte into an enucleated somatic cell.
- (C) transfer of a sperm nucleus into an oocyte.
- (D) fusion of somatic and oocyte nuclei.

2.8 Meiosis does not **always** result in the formation of four genetically identical haploid cells, because of

- (A) homologous pairing.
- (B) recombination.
- (C) non-disjunction.
- (D) anaphase separation.

2.9 Which statement is **NOT TRUE** for CAM plants?

- (A) CAM plants operate photosynthesis differently from C3 plants to prevent water-loss.
- (B) CAM plants open their stomata during the night and close them during the day.

- (C) Carbon dioxide incorporated into organic acids during the night is then used by CAM plants during the day for photosynthesis.
- (D) Calvin cycle does not operate in the CAM plants.

2.10 Which of the following statements is **NOT TRUE** for RNA ?

- (A) RNA can form high-energy ester bonds with amino acids and act as a bridge to convert genetic information into protein molecules.
- (B) RNA has a reactive 2'-OH group and acts as a biocatalyst as seen in the case of ribozyme.
- (C) RNA cannot serve as genetic material.
- (D) RNA is less stable than DNA.

For questions 2.11 to 2.14 one or more than one of the 4 options may be correct. Your answer is regarded correct only if you choose all the correct option(s) and no incorrect option(s). A correct answer will earn 5 marks, a wrong answer or an unattempted question will earn 0 mark.

2.11 Coleoptiles are the growing part of shoots in monocot plants. Peter Boysen-Jensen performed the famous experiment on phototropism of shoots in monocot plants. Later, Frits Went extended the experiments. Some of the essential observations of their experiments are listed below:

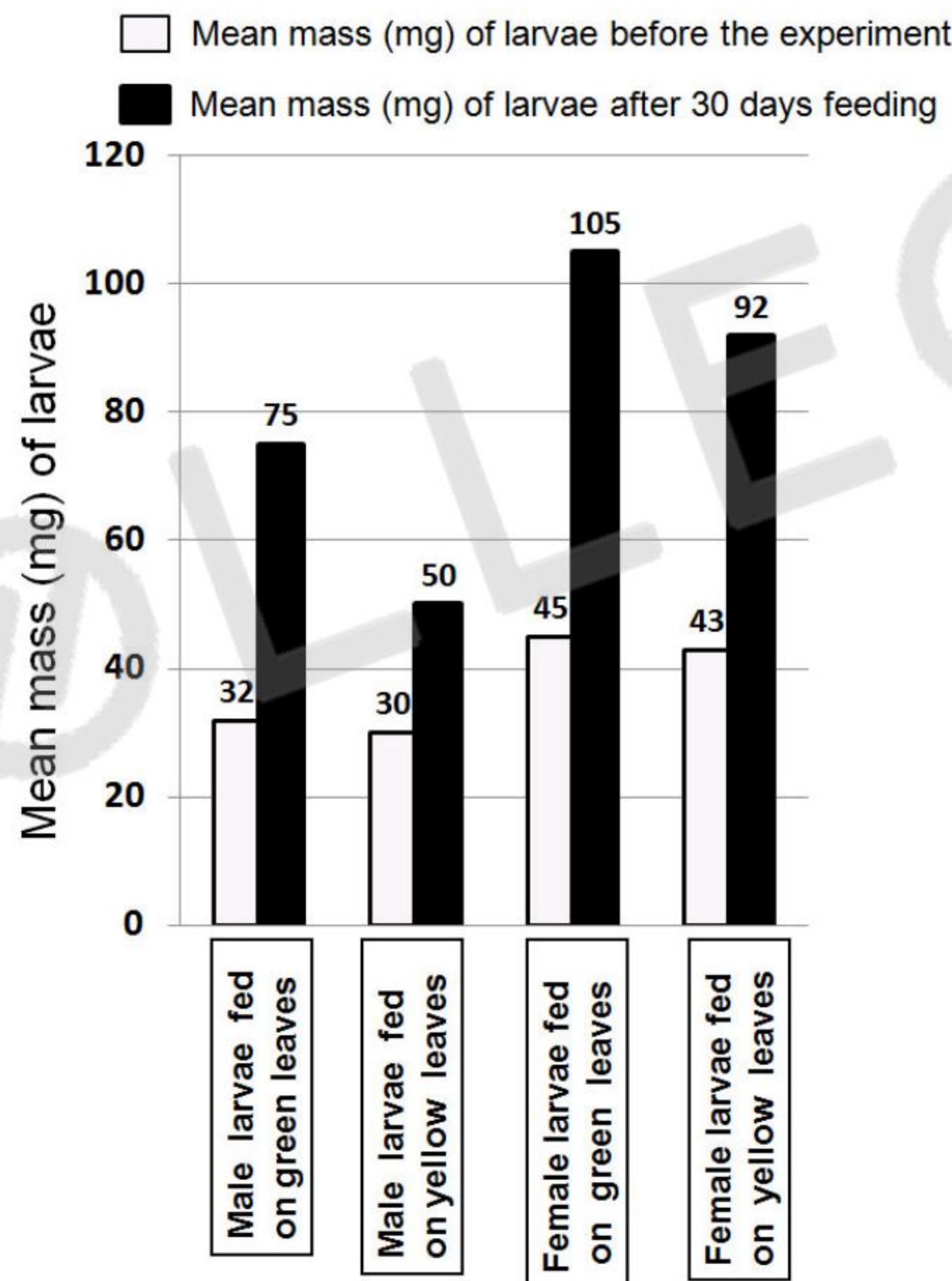
- I. Shoots of young rice plants bent in response to light.
- II. If the tip of a growing rice coleoptile was removed, the response of bending towards light was not seen.
- III. If a non-permeable substance was inserted between the tip and the rest of the shoot, then the response of bending towards light was not seen.
- IV. If an agar piece was inserted between the tip and the rest of the shoot, the bending response towards light was seen.
- V. If the agar pieces (which were inserted between the tip and the rest of the shoot and exposed to light as in item IV above) were collected and kept on decapitated coleoptiles in the dark, such plants showed bending of shoots in the dark.

Which of the following statements is/are **CORRECT** for the above experiment?

- (A) The bending of shoot can be induced in another decapitated coleoptile plant after transferring the tip of light- exposed coleoptile.
- (B) The diffusion of a substance present in the tip is induced by light.

- (C) An agar piece not inserted in the coleoptile, when placed on a decapitated shoot, will cause its bending.
- (D) The substance that causes bending is stable and not destroyed by light.

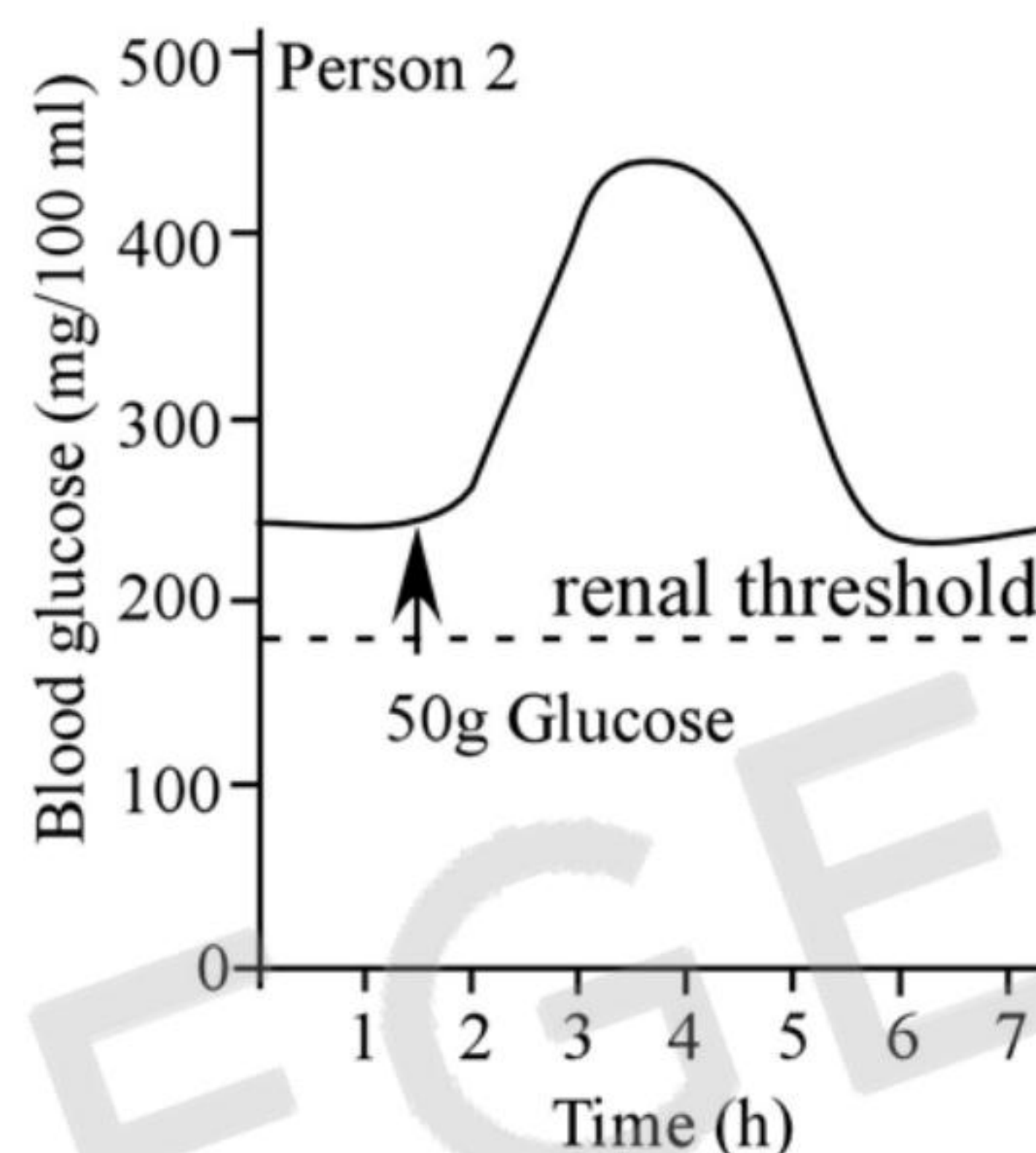
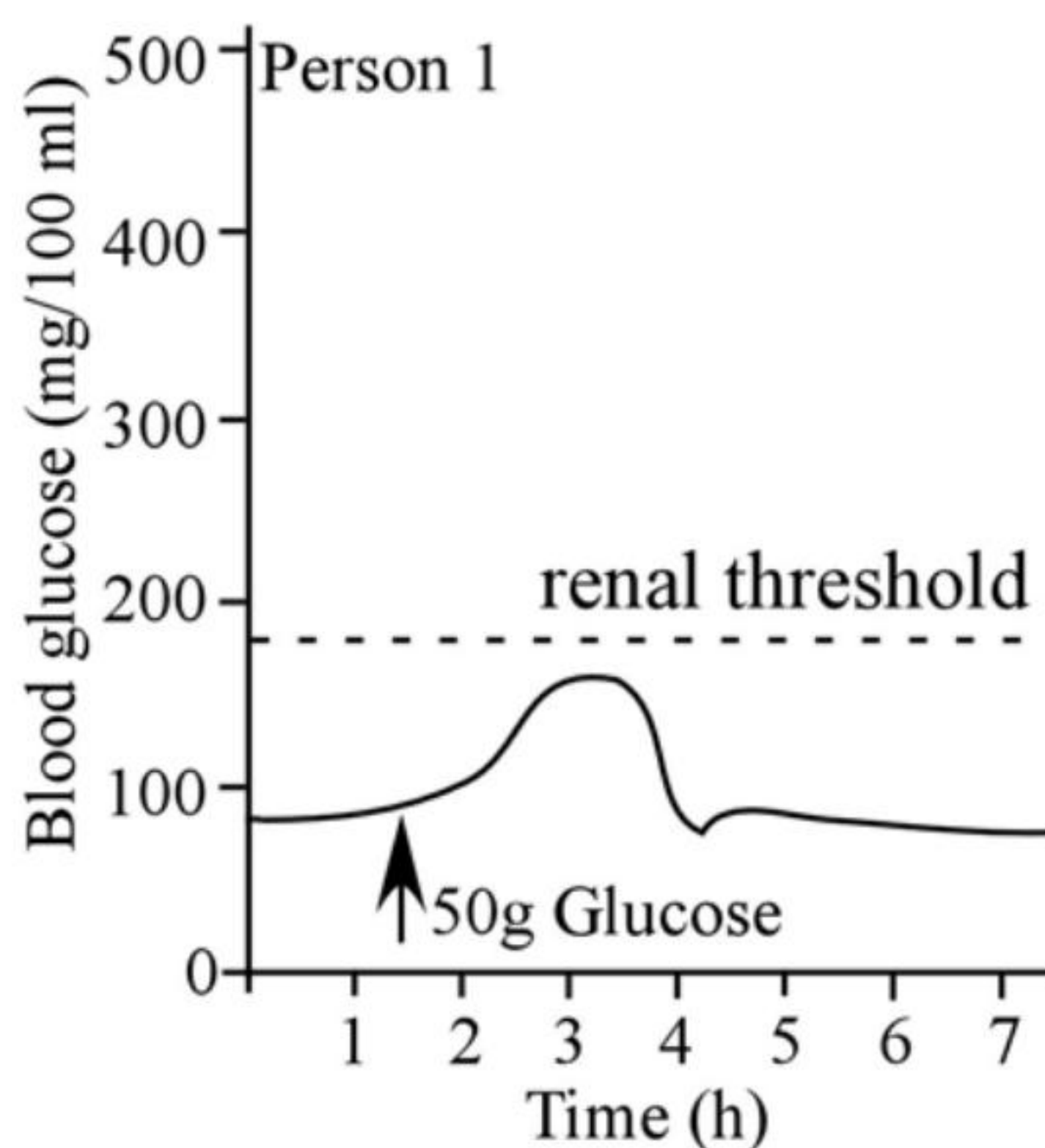
2.12 The larvae (caterpillars) of a species of insect that feed on leaves of a tree were used in this investigation. The larvae were fed for 30 days on either green leaves or yellow leaves. The bar-diagram below shows the mean mass of the larvae, after feeding them for 30 days.



Study the above chart and choose the **CORRECT** statement(s).

- (A) Female larvae, on an average, have higher mass than male larvae.
- (B) Yellow leaves may have some chemicals that contribute to the differential increase in mass between male and female larvae after feeding.
- (C) The increase in the mean mass of the larvae is governed by more than one factor.
- (D) The proportionate increase after feeding, the mean mass in female larvae is always more, compared to that of males.

2.13 The graphs show the levels of blood glucose in two persons, 1 and 2, before and after ingesting 50g of glucose.



Which of the statements below is/are **CORRECT**?

- (A) The low blood glucose levels in person 1 could be due to renal malfunction.
- (B) Insulin levels are likely to be low in person 2.
- (C) Glucose will be found in the urine of person 2.
- (D) Blood glucose in person 2 could be brought to normal levels by administration of glucagon.

2.14 Protein synthesis most often starts at AUG as a start codon and terminates at UAA, UAG and UGA as stop codons. Each codon specifies only one amino acid in all organisms. State which of the following statements is/are **CORRECT**.

- (A) The above information is consistent with the fact that the genetic code is universal.
- (B) Protein coding RNA from viruses, mammals or plants can be translated in bacteria.
- (C) In higher organisms, protein synthesis begins with methionine.
- (D) Use of more than one termination codon is more effective than a single termination codon in ensuring termination of protein synthesis.

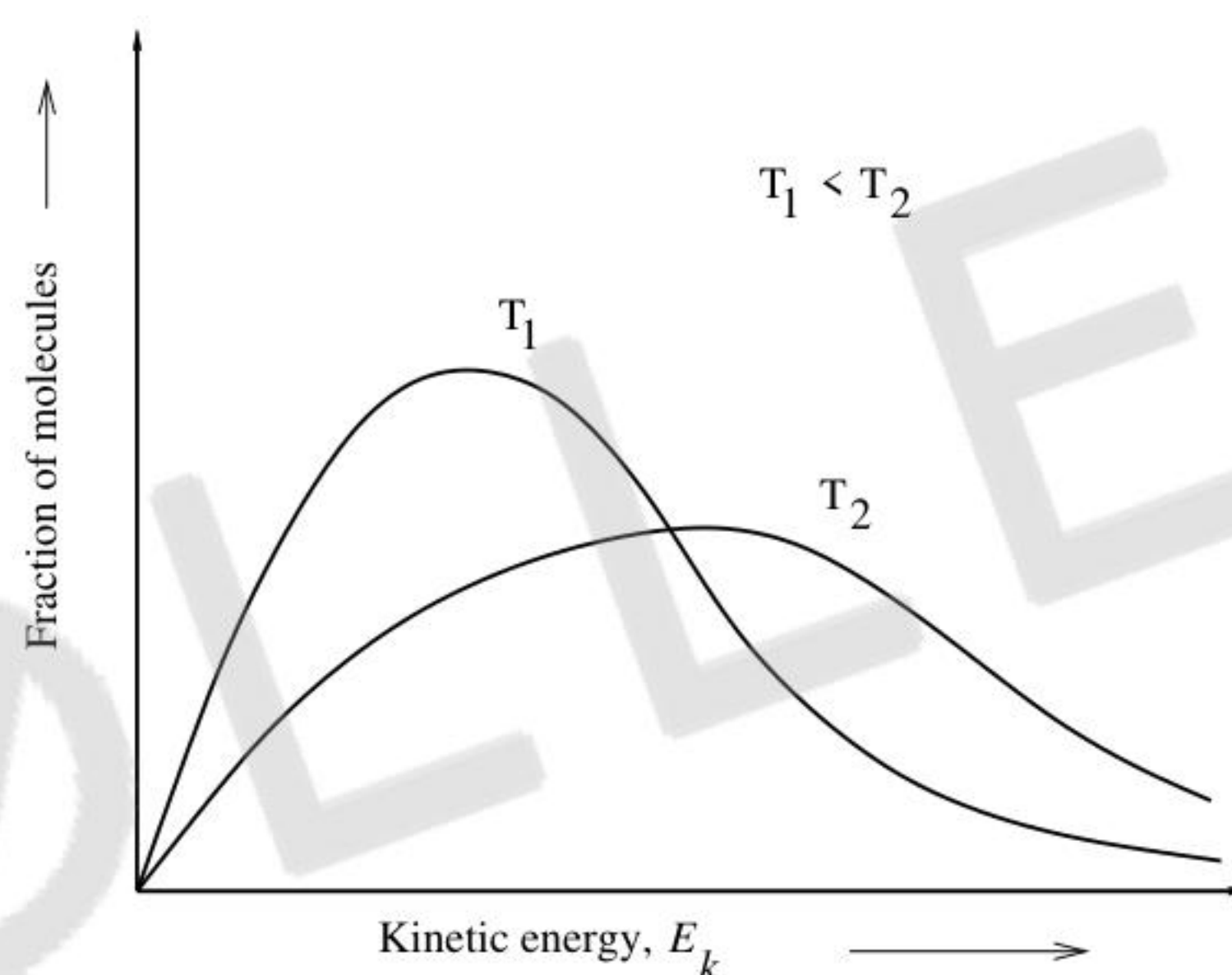
Section 3: Chemistry

Marks for Section 3: 50

This section contains 14 questions.

*For questions 3.1 to 3.10 **only one** of the 4 options is correct. A correct answer will earn 3 marks, a wrong answer will earn (−1) mark, and an unattempted question will earn 0 mark.*

3.1 Kinetic energy (E_k) of a molecule is related to its velocity (v) by the expression, $E_k = \frac{1}{2}mv^2$. E_k follows Boltzmann distribution as schematically given below.



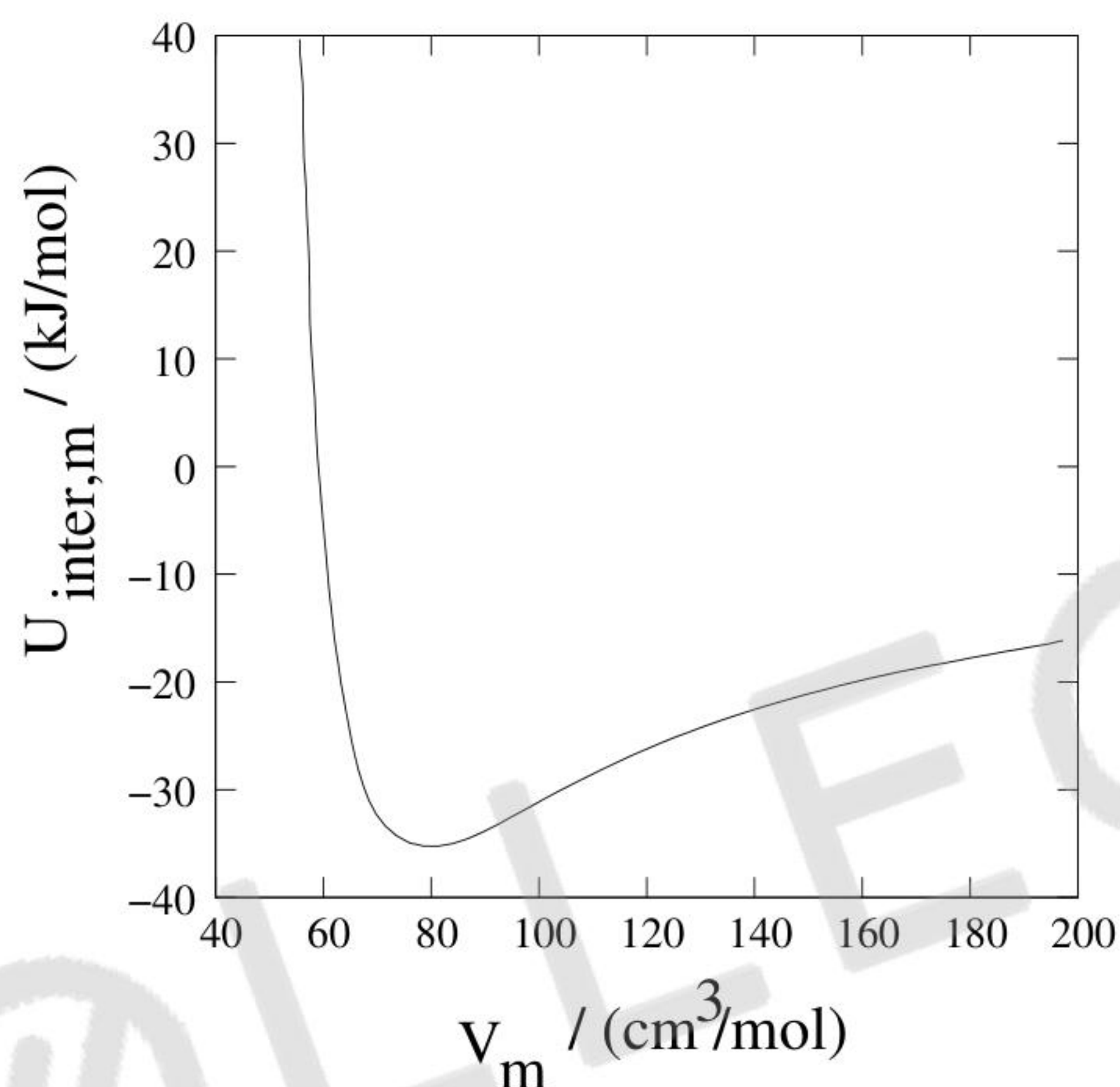
During a chemical reaction, only the reactant molecules that possess a minimum energy (E_a), called activation energy, give the products. For the reaction, $2\text{N}_2\text{O}_5 \rightarrow 4\text{NO}_2 + \text{O}_2$, the rate constant dependence on temperature is given by $k = Ae^{-E_a/RT}$. Choose the **INCORRECT** statement:

- (A) E_a may be altered in the presence of a catalyst.
- (B) E_a decreases with increase in temperature.
- (C) Fraction of molecules with energy higher than the activation energy increases with temperature.
- (D) Rate constant increases with increase in temperature.

3.2 The observed structure of xenon compounds may be explained on the basis of VSEPR theory. Choose the **INCORRECT** statement:

- (A) XeF_2 is linear with three lone pairs on the central atom.
- (B) XeF_4 is square planar with two lone pairs on the central atom.
- (C) XeOF_4 is square pyramidal with one lone pair on the central atom.
- (D) XeF_6 is octahedral with no lone pair on the central atom.

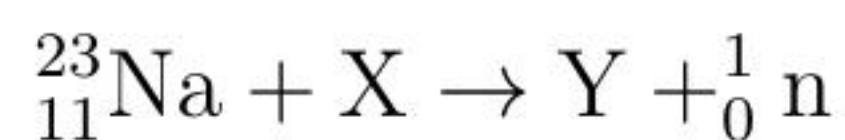
3.3 For a gas obeying van der Waals equation, the van der Waals molar intermolecular attraction ($U_{\text{inter,m}}$) is given by $U_{\text{inter,m}} = -a/V_m$, where, a is a positive constant and V_m is the molar volume. The plot of $U_{\text{inter,m}}$ versus V_m for diethyl ether ($(\text{C}_2\text{H}_5)_2\text{O}$) is shown below.



Given that V_m is $110 \text{ cm}^3/\text{mol}$ at 35°C and 1 atm, the ΔU_m and ΔH_m of vaporization of diethyl ether at 35°C respectively are

- | | |
|---|--|
| (A) 27 kJ/mol and 27 kJ/mol | (B) 27 kJ/mol and -27 kJ/mol |
| (C) -27 kJ/mol and -27 kJ/mol | (D) -27 kJ/mol and 27 kJ/mol |

3.4 In the nuclear reaction



X and Y are

- | | | | |
|--|---|--|--|
| (A) ${}_1^1\text{H}$ and ${}_{12}^{23}\text{Mg}$ | (B) ${}_2^4\text{He}$ and ${}_{13}^{27}\text{Al}$ | (C) ${}_1^1\text{H}$ and ${}_{12}^{24}\text{Mg}$ | (D) ${}_1^2\text{H}$ and ${}_{12}^{23}\text{Mg}$ |
|--|---|--|--|

3.5 The relation between the radius (r) of an atom and edge length (a) of the body centered cubic cell of closed pack structure is

- | | | | |
|---------------------|-------------------------------|-------------------------------|------------------------|
| (A) $r = \sqrt{2}a$ | (B) $r = \frac{\sqrt{3}}{2}a$ | (C) $r = \frac{\sqrt{3}}{4}a$ | (D) $r = \frac{1}{2}a$ |
|---------------------|-------------------------------|-------------------------------|------------------------|

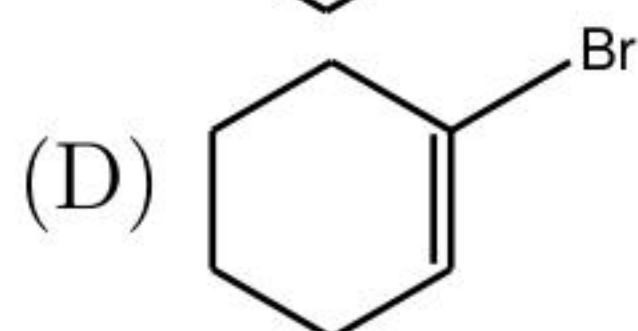
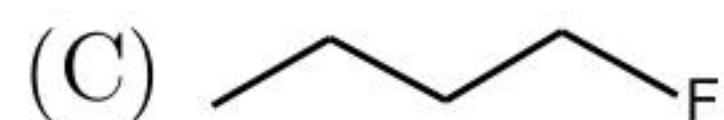
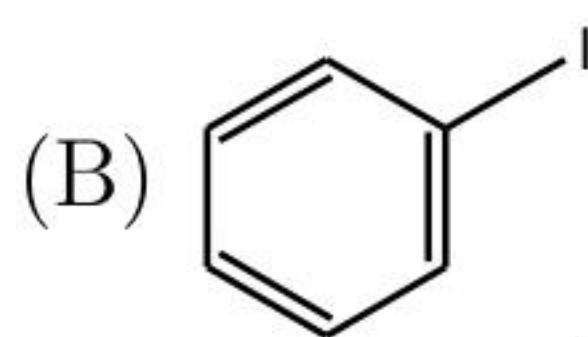
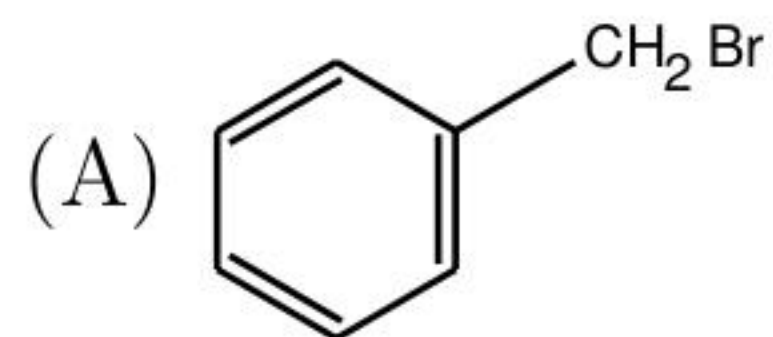
3.6 Lewis acidity of boron halides follow the order ($\approx$ meaning approximately equal to)

- | | |
|---|---|
| (A) $\text{BF}_3 \approx \text{BBr}_3 \approx \text{BCl}_3$ | (B) $\text{BF}_3 < \text{BCl}_3 < \text{BBr}_3$ |
| (C) $\text{BCl}_3 \approx \text{BBr}_3 < \text{BF}_3$ | (D) $\text{BBr}_3 < \text{BCl}_3 < \text{BF}_3$ |

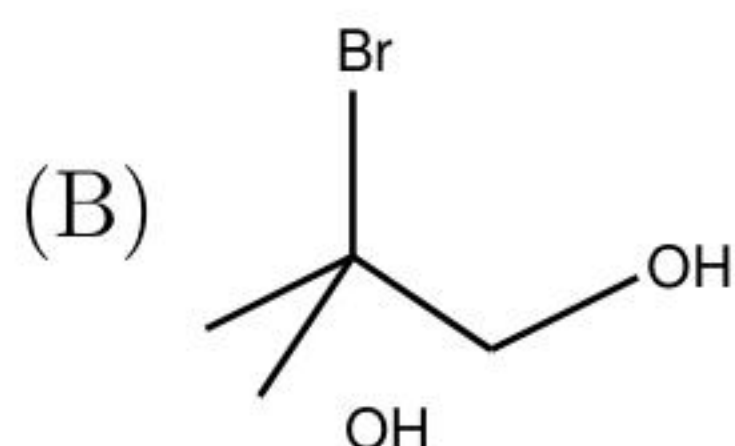
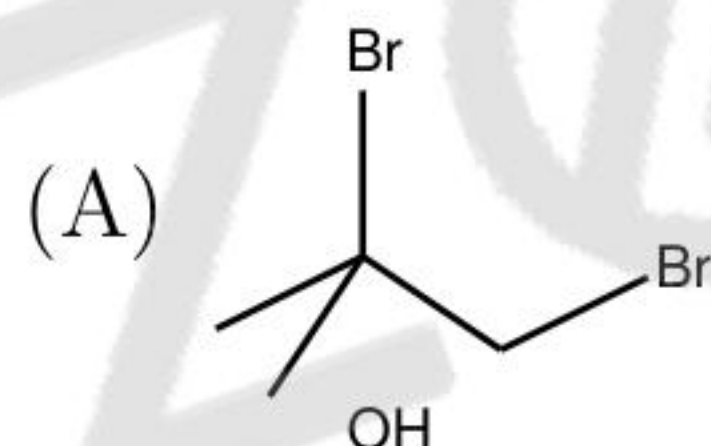
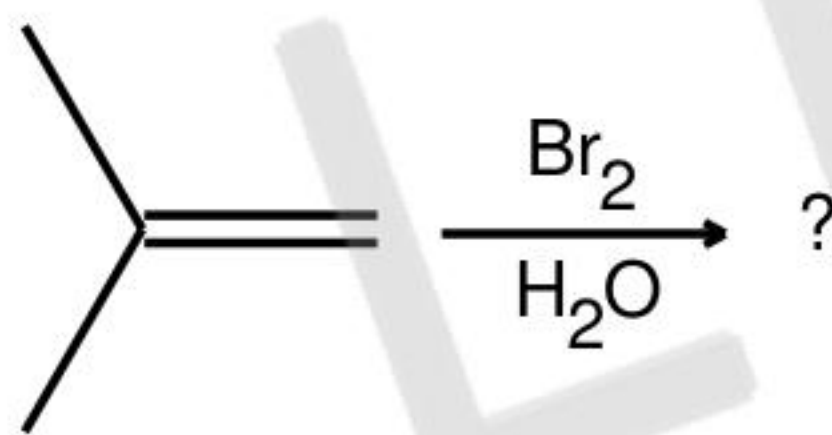
3.7 Among Mn^{2+} , V^{2+} , Ni^{2+} , and Ti^{2+} , the cation having the highest hydration energy to form aqua complex $[\text{M}(\text{H}_2\text{O})_6]^{2+}$ is

- (A) Mn^{2+} (B) V^{2+} (C) Ni^{2+} (D) Ti^{2+}

3.8 The compound which would react fastest with ammonia is



3.9 The main product of the following reaction is

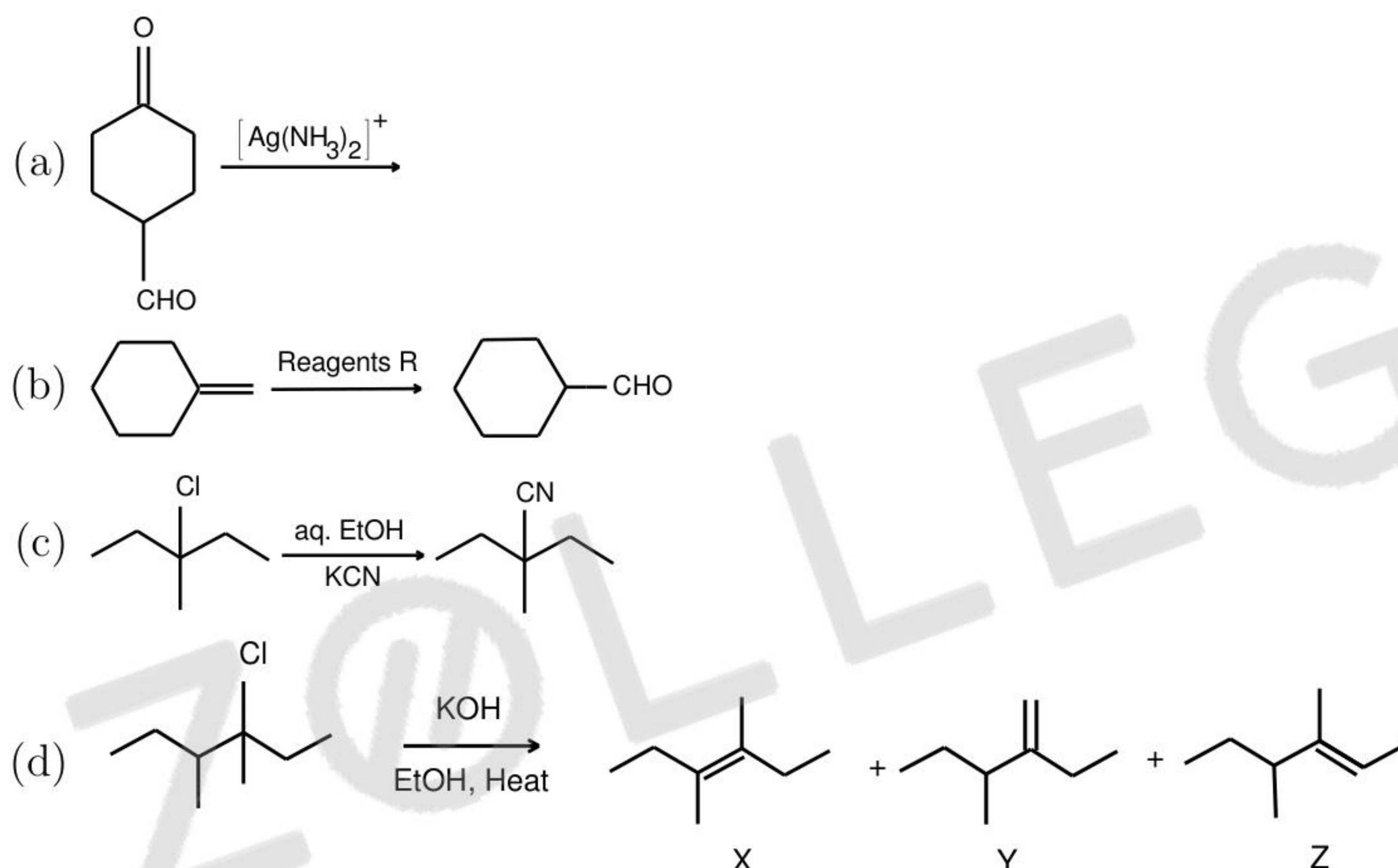


3.10 The IUPAC name of the compound $\text{CH}_3\text{CH}=\text{CH}-\text{COOCH}_3$ is

- (A) pent-3-en-2-one (B) butenyl ethanoate
(C) methyl butanoate (D) methyl 2-butenate

For questions 3.11 to 3.14 **one or more than one** of the 4 options may be correct. Your answer is regarded correct **only if you choose all the correct option(s) and no incorrect option(s)**. A correct answer will earn 5 marks, a wrong answer or an unattempted question will earn 0 mark.

3.11 Consider the following reactions



Choose the correct statement(s).

- (A) The product of reaction (a) is cyclohexan-1,4-dione.
- (B) The reagents R in reaction (b) are in the following order:
(i) BH_3 (ii) $\text{H}_2\text{O}_2/\text{OH}^-$ (iii) PCC.
- (C) Reaction (c) is an $\text{S}_{\text{N}}1$ type reaction.
- (D) The major product of reaction (d) is X.

3.12 The standard equilibrium constant K_P° is positive and follows the relation

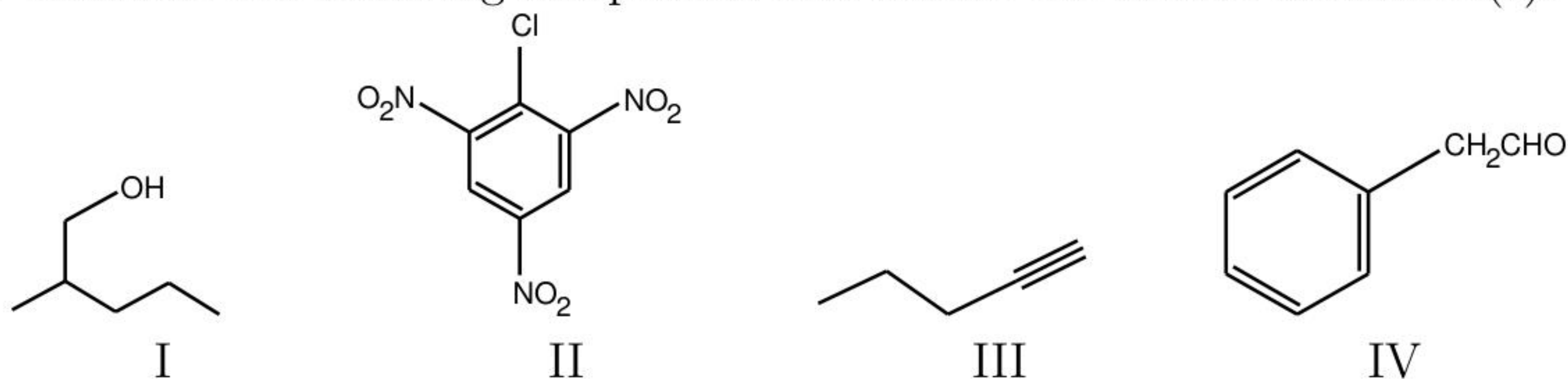
$$K_P^\circ = e^{-\Delta G^\circ/RT}.$$

Choose the correct statement(s).

- (A) $\frac{d(\ln K_P^\circ)}{dT} = \frac{\Delta H^\circ}{RT^2}$ and the sign of ΔH° determines whether K_P° increases or decreases as T increases.
- (B) Only the sign of ΔS° determines the temperature dependence of K_P° .
- (C) At low T, equilibrium position is determined by internal energy change (ΔU°).

(D) At high T , equilibrium position is determined by the entropy change (ΔS°).

3.13 Consider the following compounds and choose the correct statement(s).



- (A) Optically active I reacts with HCl/ZnCl_2 to form a racemate of the corresponding alkyl chloride.
- (B) II on warming with water gives 2,4,6-trinitrophenol.
- (C) III on reaction with sodium metal forms a sodium salt.
- (D) IV on treatment with conc. KOH gives a mixture of potassium salt of a carboxylic acid and an alcohol.

3.14 Aluminium and copper are two important metals required on a large scale. Aluminium is extracted from its oxide ore bauxite through separation of alumina (Bayer process) followed by electrolysis of alumina. Copper is extracted from its sulphide ore copper pyrites. The concentrated ore is heated in a reverberatory furnace with silica and the copper matte obtained is heated in a Bessemer furnace with a blast of air to obtain copper. Choose the correct statement(s).

- (A) Copper is formed due to autoredution of copper oxide and copper sulphide.
- (B) Aluminium is difficult to obtain by chemical reduction due to its strong electropositive nature and high affinity with oxygen.
- (C) In the Bayer process sodium carbonate can replace sodium hydroxide.
- (D) Copper can be extracted from a copper oxide ore by heating with coke.

Section 4: Mathematics

Marks for Section 4: 50

This section contains 14 questions.

For questions 4.1 to 4.10 only one of the 4 options is correct. A correct answer will earn 3 marks, a wrong answer will earn (-1) mark, and an unattempted question will earn 0 mark.

- 4.1 Let $X = \{1, 2, 3, \dots, 25\}$. If a student selects a function randomly from the set of all functions from X onto X , then what is the probability that the selected function maps prime numbers to prime numbers ?

(A) $\frac{9!}{25!}$ (B) $\frac{16!}{25!}$ (C) $1/\binom{25}{9}$ (D) $9!/\binom{25}{9}$

- 4.2 A pole is fixed vertically on a plane. An ant while walking on the plane, observes that its angle of elevation to the top of the pole is 45° . Then it walks 1 m straight towards the pole, turns left and then moves 2 m further. If it finds the angle of elevation is again 45° , then the height of the pole is

(A) 2.5 m. (B) 2 m. (C) $\sqrt{2}$ m. (D) $\frac{\sqrt{3}}{2}$ m.

- 4.3 The interval in which the function

$$f(x) = \log_{\frac{1}{2}}(x^2 - 2x - 8)$$

is monotonically increasing, is

(A) $(4, +\infty)$. (B) $(-\infty, -2)$. (C) $(-\infty, 4)$. (D) $(-2, +\infty)$.

- 4.4 The number of non-congruent triangles with sides of length a, b, c where a, b, c belong to $\{0, 1, 2, \dots, 9\}$; so that the 3 digit number $10^2a + 10b + c$ lies between 200 and 300, is

(A) 24. (B) 16. (C) 8. (D) 4.

- 4.5 Let $\{a_n\}$ be a sequence of numbers satisfying the relation $(3 - a_{n+1})(6 + a_n) = 18$ for all $n \geq 0$ and $a_0 = 3$. Then

$$\lim_{n \rightarrow \infty} \frac{1}{2^{n+2}} \sum_{j=0}^n \frac{1}{a_j}$$

is

(A) $\frac{1}{18}$. (B) $\frac{1}{6}$. (C) $\frac{1}{4}$. (D) $\frac{1}{3}$.

4.6 Define a binary operation $\star$ on the set of all 3-dimensional vectors by

$$\vec{u} \star \vec{v} = (\vec{u} \cdot \vec{v})(\vec{u} \times \vec{v}).$$

Suppose $\vec{a}$, $\vec{b}$ and $\vec{c}$ are any three non-coplanar vectors satisfying $\vec{a} \cdot \vec{b} \times \vec{c} \neq 0$. If $\vec{a} \cdot (\vec{b} \star \vec{c}) = (\vec{a} \cdot \vec{b}) \star \vec{c}$, then

- (A) $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c}$.
 (B) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c}$.
 (C) $|\vec{a} \cdot \vec{b}| = |\vec{a} \cdot \vec{c}|$.
 (D) $|\vec{a} \cdot \vec{b}| = |\vec{b} \cdot \vec{c}|$.

4.7 The interval $(-1, 1)$ is the range of

- (A) $\frac{x}{1+|x|}$.
 (B) $\cos x - \sin x$.
 (C) $x^4 + 4x^3 + 1$.
 (D) $\frac{|x|}{1+x}$, where $x \neq -1$.

4.8 The value of the integral $\int_0^1 e^{x^2} dx$ lies in the interval

- (A) (e, e^2) .
 (B) (e^2, ∞) .
 (C) $(1, e)$.
 (D) $(1/e, 1)$.

4.9 Let P and A be 3×3 real matrices such that $PAP^t = -A^t$, where P^t denotes the transpose of P . Then the determinant of P is

- (A) -1 .
 (B) 1 .
 (C) any positive real number.
 (D) any real number.

4.10 Which of the following function is differentiable everywhere ?

- (A) $\begin{cases} \frac{|x-a|}{x-a}, & \text{if } x \neq a \\ 1, & \text{if } x = a. \end{cases}$
 (B) $(x-a)|x-a|$.
 (C) $(x-a) + |x-a|$.
 (D) $|x-a|^{1/3}$.

For questions 4.11 to 4.14 one or more than one of the 4 options may be correct. Your answer is regarded correct **only if you choose all the correct option(s) and no incorrect option(s)**. A correct answer will earn 5 marks, a wrong answer or an unattempted question will earn 0 mark.

4.11 For a function $f(x)$, let $\frac{d^i(f(x))}{dx^i} \Big|_{x=a}$ denotes its i -th derivative with respect to x evaluated at $x = a$. Consider the matrix $A = (a_{i,j})_{n \times m}$ of $n(> 1)$ rows and $m(> 1)$ columns,

with

$$a_{i,j} = \left(\frac{d^i x^n}{dx^i} \Big|_{x=1} \right) / \left(\frac{d^j y^m}{dy^j} \Big|_{y=1} \right).$$

Let $p_i = \max\{a_{i,j} : j = 1, 2, \dots, m\}$, $q_j = \min\{a_{i,j} : i = 1, 2, \dots, n\}$,
 $p = \min\{p_i : i = 1, 2, \dots, n\}$ and $q = \max\{q_j : j = 1, 2, \dots, m\}$. Then

- (A) $p > q$.
- (B) $a_{1,1} < a_{2,2} < \dots < a_{m,m}$, if $n > m$.
- (C) $a_{n,m} = a_{n-1,m-1} = \dots = a_{n-\ell+1,m-\ell+1}$, where $\ell = \min\{n, m\}$.
- (D) A is a symmetric matrix, if $m = n$.

4.12 Consider the equation $x^2 + bx + c = 0$, where b, c are integers. Suppose it has an integer solution p . Then

- (A) p is odd, if c is odd.
- (B) c is odd, if p is odd.
- (C) $b + c$ is odd, if p is odd.
- (D) b is even, if $b + c$ is even.

4.13 Let O be the origin and let A, C be the intersection points of the ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$ with the x -axis and B, D be with y -axis such that A, B, C, D are in anti-clockwise direction. Let K, L be the foci of the ellipse such that K lies between A and O , and L between O and C . Then

- (A) DL is the median of the triangle DOC .
- (B) triangle DKL is equilateral.
- (C) $\angle KDC = 90^\circ$.
- (D) DL bisects $\angle KDC$.

4.14 If ω is a complex cube root of unity, then

- (A) ω is a zero of $x^2 + x + 1$.
- (B) $\pm\omega^2$ are zeros of $x^4 + x + 1$.
- (C) $\pm\omega^2$ are the only common zeros of $x^{12} - 1$ and $x^4 + x^2 + 1$.
- (D) $\pm\omega$ and $\pm\omega^2$ are the only common zeros of $x^{12} - 1$ and $x^4 + x^2 + 1$.

Section 5: Physics

Marks for Section 5: 50

This section contains 14 questions.

For questions 5.1 to 5.10 only one of the 4 options is correct. A correct answer will earn 3 marks, a wrong answer will earn (-1) mark, and an unattempted question will earn 0 mark.

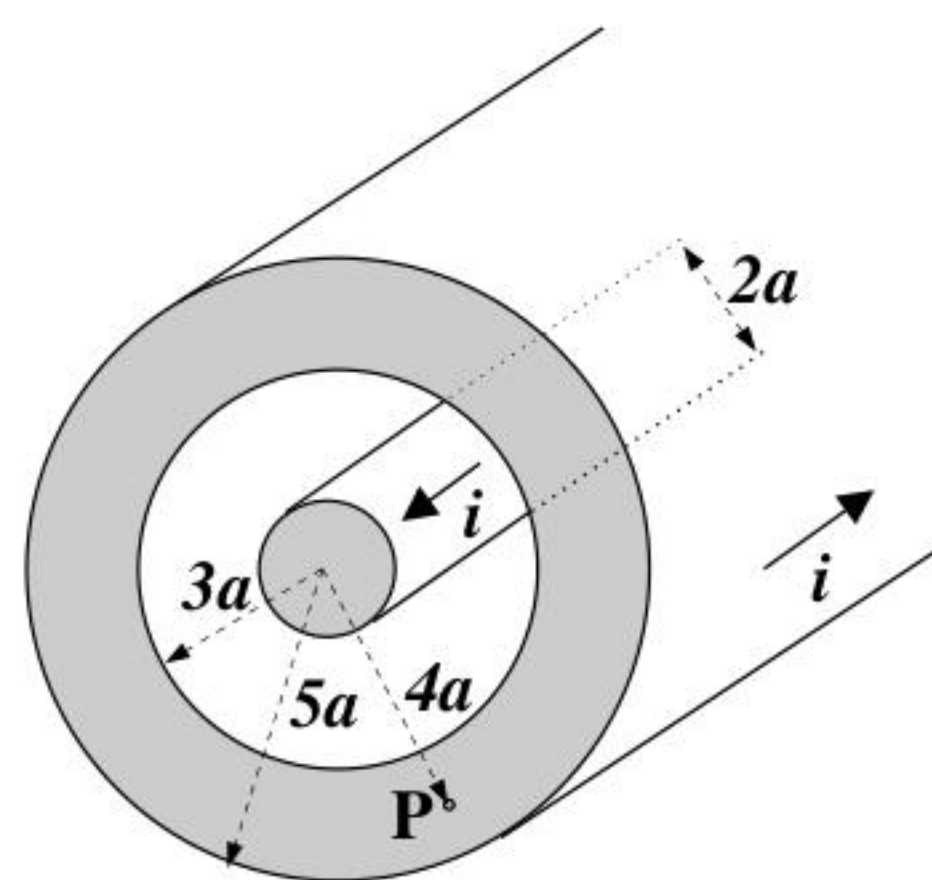
- 5.1 The distance between the objective and eyepiece of a compound microscope is 0.12 m and their focal lengths are the same. When an object is placed 0.025 m in front of the objective, the image formed by the eyepiece is at infinity. The focal length of the lenses is

(A) 0.02 m (B) 0.03 m (C) 0.10 m (D) 0.12 m

- 5.2 A wheel is spinning about its axis with constant angular acceleration. With time, the linear acceleration of a point on its rim

(A) remains constant in magnitude and the direction becomes increasingly radial.
(B) remains constant in magnitude and the direction becomes increasingly tangential.
(C) increases in magnitude and the direction becomes increasingly tangential.
(D) increases in magnitude and the direction becomes increasingly radial.

- 5.3 The inner wire of a straight coaxial cable has radius a while the outer wire has radii $3a$ and $5a$. The cable carries equal and opposite currents of magnitude i (distributed uniformly across the cross-section) on the inner and outer wires (see figure).



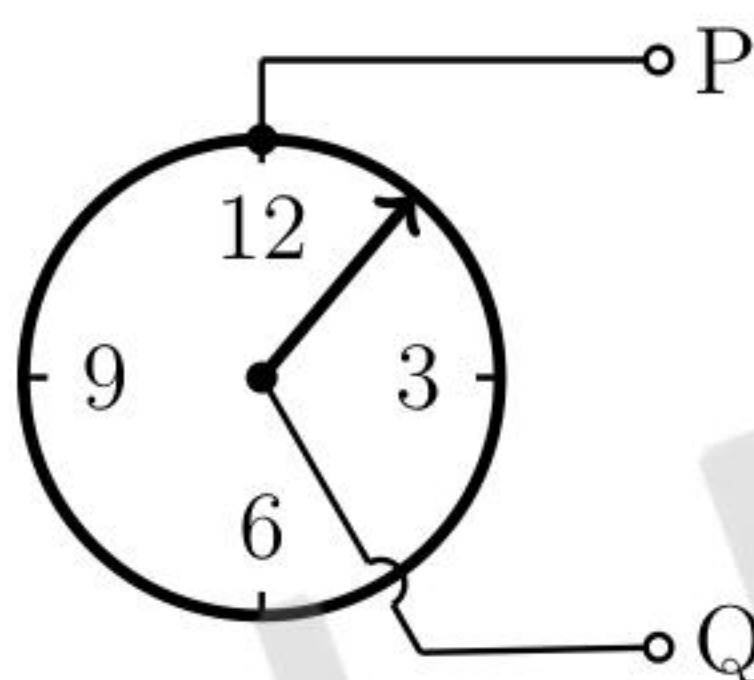
The magnitude of the magnetic field at point P inside the cable at a distance $4a$ from the axis is

(A) $\frac{9\mu_0 i}{128\pi a}$ (B) $\frac{7\mu_0 i}{128\pi a}$ (C) $\frac{23\mu_0 i}{128\pi a}$ (D) $\frac{\mu_0 i}{32\pi a}$

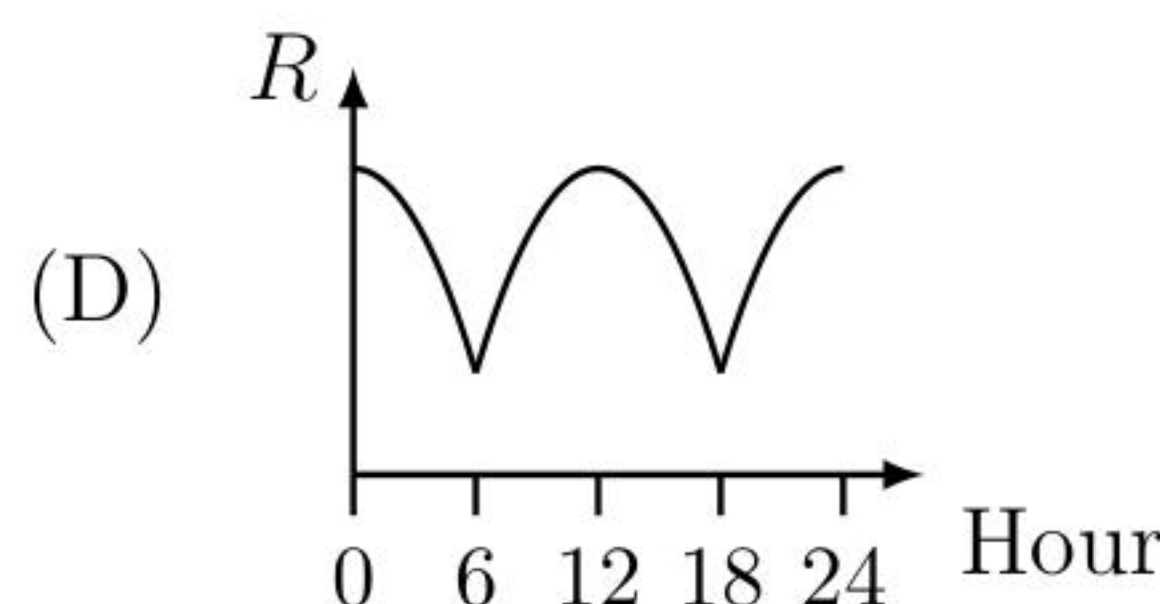
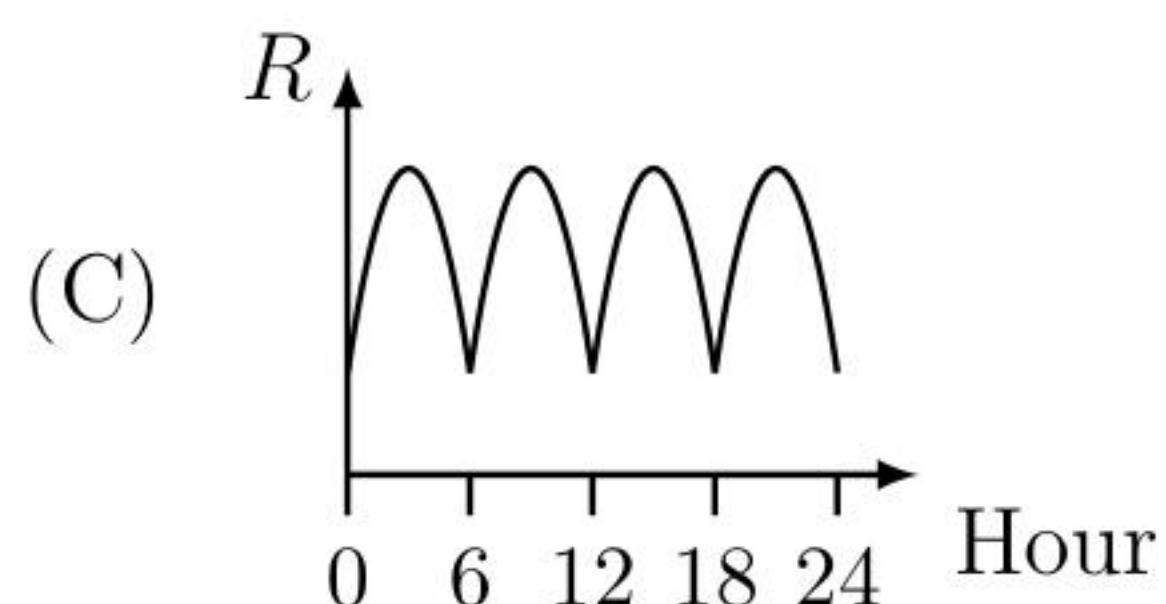
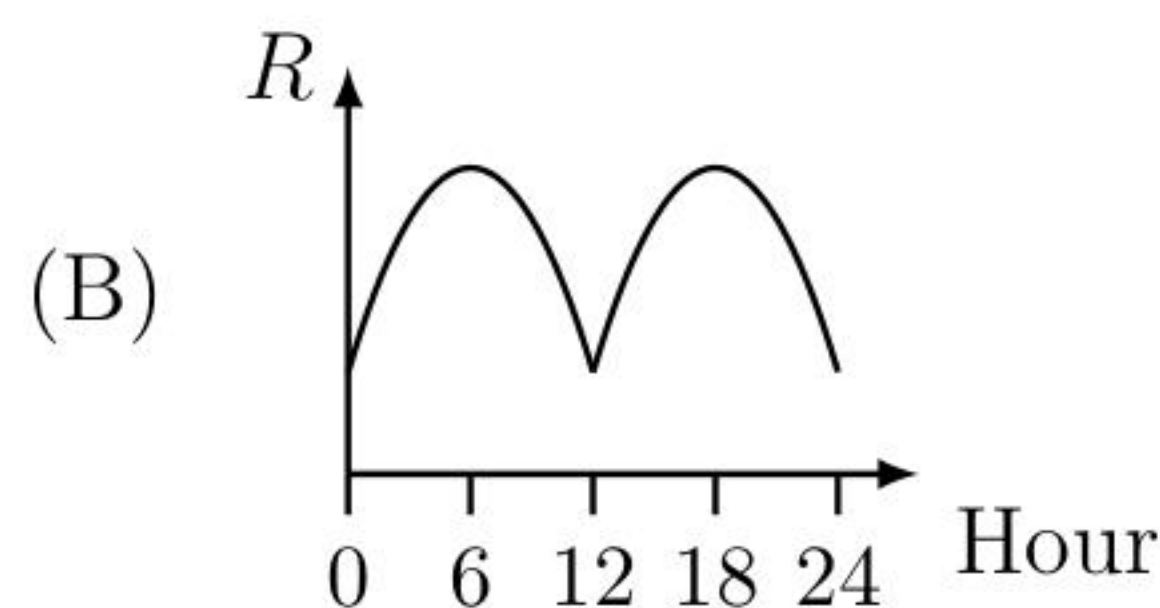
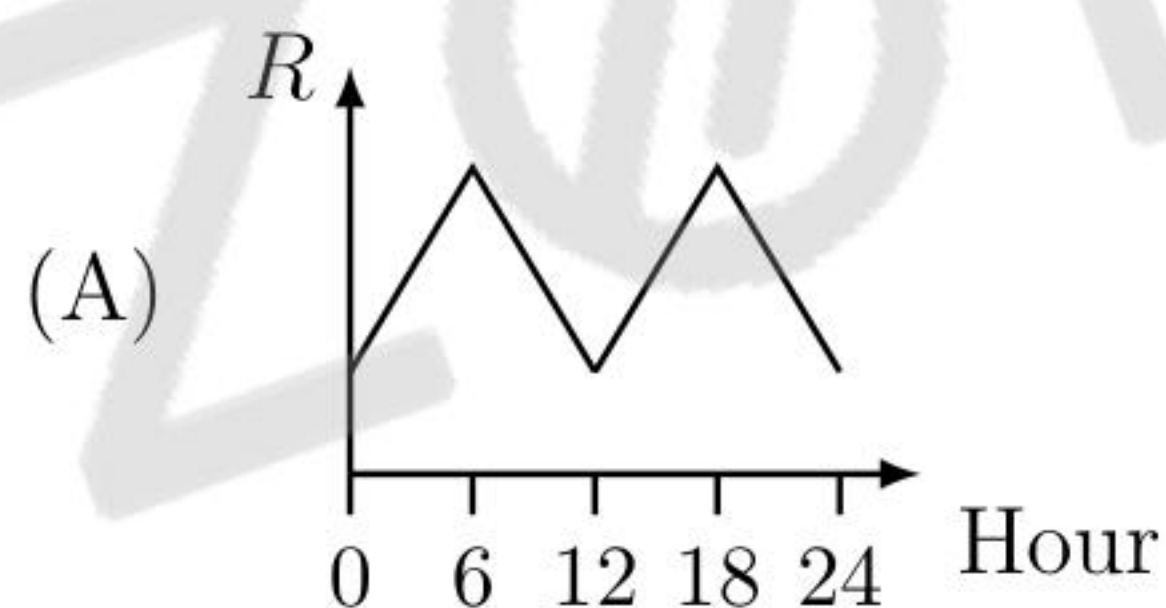
5.4 A tube of length 1 m, closed at one end, resonates at a frequency of 249 Hz. The speed of sound in air is 332 ms^{-1} . The total number of possible frequencies for resonance within the range 1 kHz to 2 kHz is

- (A) 4 (B) 5 (C) 6 (D) 7

5.5 A clock having a circular metal rim of uniform resistivity is mounted on a non-conducting dial. The hand showing the hour is also made of the same metal. The clock hand makes contact with the rim as it moves. A circuit node P is fixed to the metal rim at the 12 o'clock position as shown. Another node Q is connected to the central pivot.



Which one of the following graphs describes the equivalent resistance R , between the nodes P and Q, as a function of time over a 24 hour cycle, starting from midnight?



5.6 A spherical dewdrop, when deformed slightly, oscillates about its equilibrium shape. The physically relevant variables to describe the frequency of this oscillation f are its mass M , radius R and a third variable Q , where Q could be either the universal gravitational constant G or the surface tension S . One must decide the relevant variable Q and employ dimensional analysis to express f as

$$f = kM^x R^y Q^z,$$

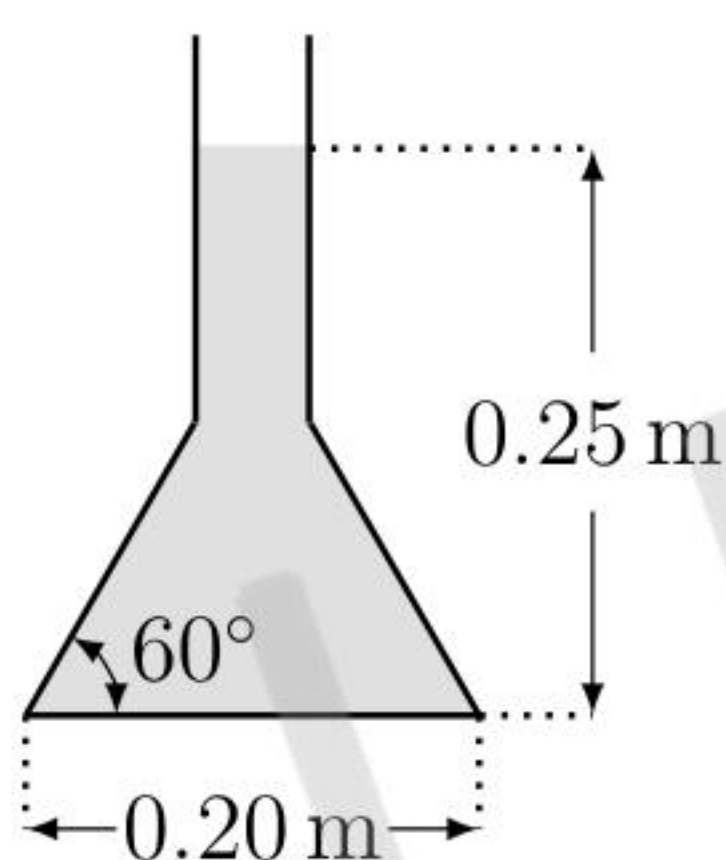
where k is a dimensionless constant. Then

- (A) $x = -1/2$, $y = 0$ and $z = 1/2$ (B) $x = 1/2$, $y = -3/2$ and $z = 1/2$
 (C) $x = 1/2$, $y = 0$ and $z = -1/2$ (D) $x = -1/2$, $y = -3/2$ and $z = 1/2$

5.7 A solid aluminium cube of side 0.1 m is heated for 2 minutes in a furnace at temperature 1000 K such that the cube's temperature increases by 5 K. A larger aluminium cube of side 0.2 m is put in the furnace at temperature 2000 K for 1 minute. Assuming that the furnace behaves like a blackbody, the increase in temperature of the larger cube will be

- (A) 2.5 K. (B) 5 K. (C) 10 K. (D) 20 K.

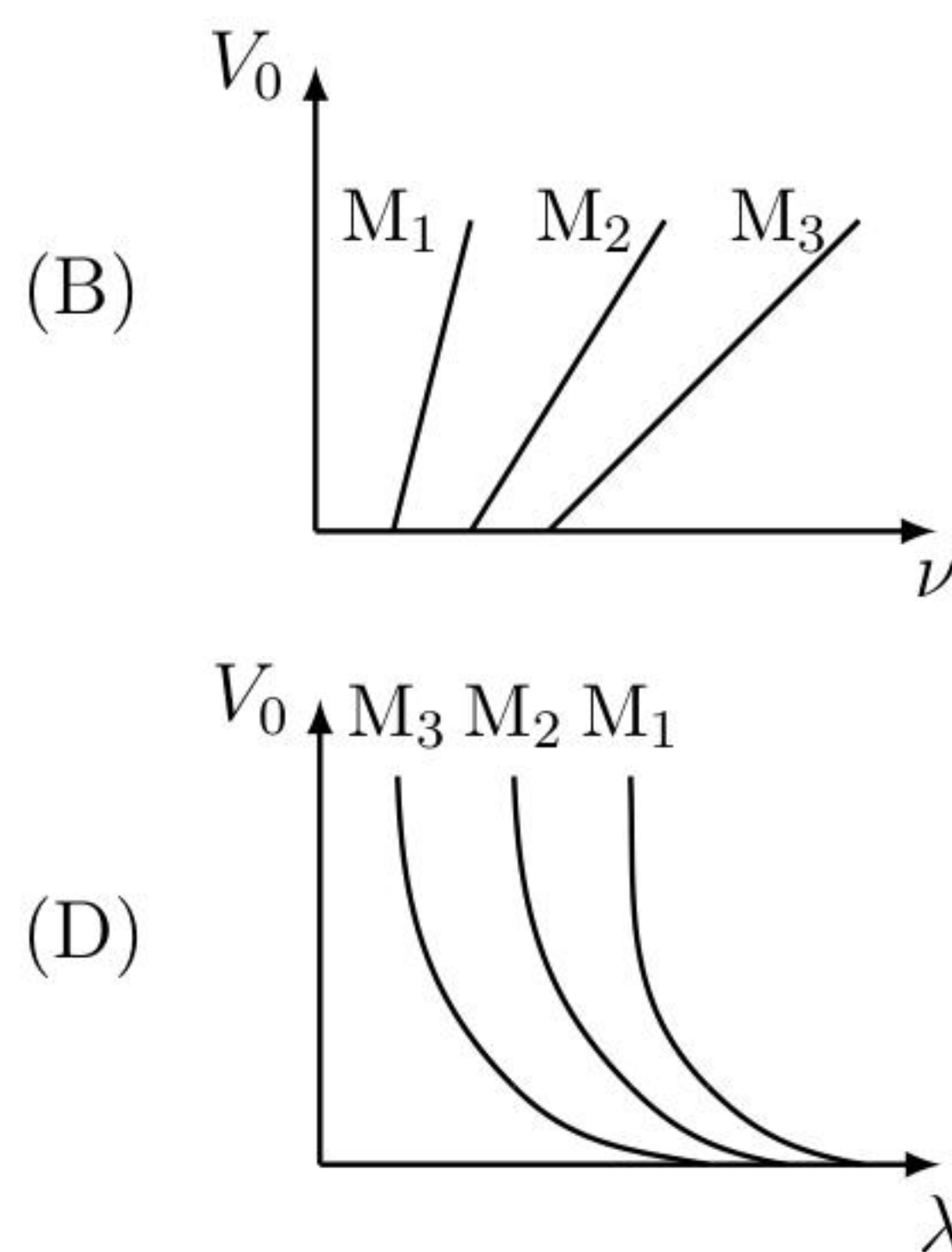
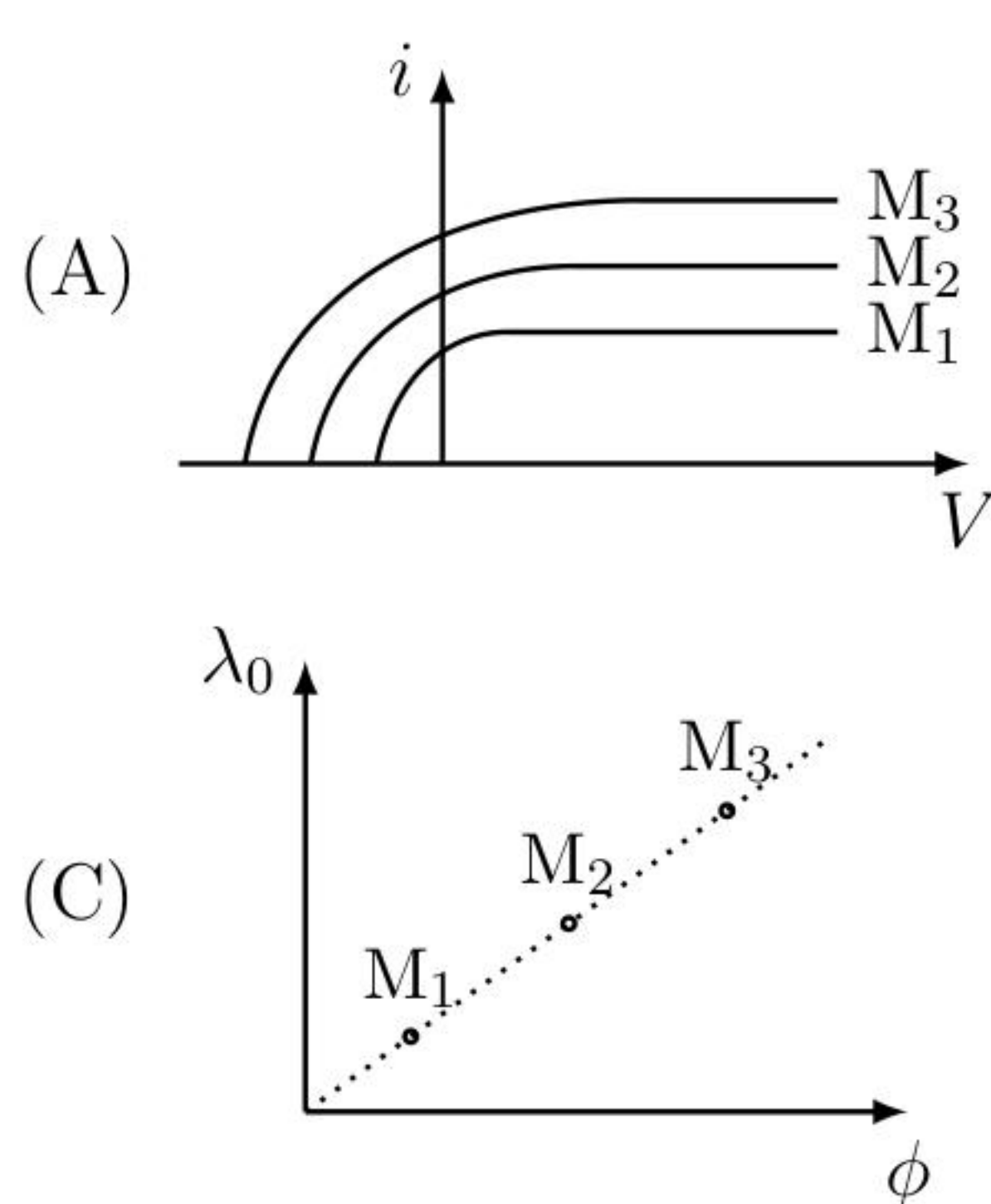
5.8 The figure shows the cross-section of a conical flask having a circular base of diameter 0.20 m. The slanted surface of the cone makes an angle of 60° with the base. When 2.0 litres ($1 \text{ litre} = 10^{-3} \text{ m}^3$) of water is poured in the flask, the level of water is 0.25 m above the base.



What is the magnitude of the total vertical force exerted by the water on the slanted surface of the flask? (Take acceleration due to gravity $g = 10 \text{ ms}^{-2}$ and ignore atmospheric pressure.)

- (A) 0 N (B) 20.0 N (C) 58.5 N (D) 78.5 N

5.9 In a photoelectric setup, three metals M_1 , M_2 and M_3 with work functions ϕ_1 , ϕ_2 and ϕ_3 respectively ($0 < \phi_1 < \phi_2 < \phi_3$), are illuminated with light of frequency ν and wavelength λ . The currents i and voltages V are registered. The stopping potential and cutoff wavelength are denoted by V_0 and λ_0 , respectively. Which plot below could be a possible outcome of the experiment?



5.10 A satellite GeoSAT is in a circular geostationary orbit of radius R_G , above a point P on the equator. Another satellite ComSAT is in a lower circular orbit of radius $0.81R_G$. At 7 P.M on January 1, ComSAT is sighted directly above P. On which day among the following can ComSAT be sighted directly above P between 7 P.M. and 8 P.M.?

- (A) January 3 (B) January 9 (C) January 15 (D) January 21

For questions 5.11 to 5.14 one or more than one of the 4 options may be correct. Your answer is regarded correct only if you choose all the correct option(s) and no incorrect option(s). A correct answer will earn 5 marks, a wrong answer or an unattempted question will earn 0 mark.

5.11 A long horizontal wire PQ of mass m and carrying a steady current i_1 is free to move in the vertical plane. It is in equilibrium at a height d above another horizontal long wire RS of mass m and carrying a current i_2 . The wire RS is fixed and not free to move. Which of the following statements is/are correct?

- (A) The magnetic force on the lower wire is in the vertically downward direction.
 (B) The magnitude of the magnetic force per unit length on the upper wire is $\frac{\mu_0 i_1 i_2}{2\pi d}$
 (C) If the upper wire as a whole is displaced slightly upward, it will perform simple harmonic motion with time period $T = 2\pi\sqrt{d/g}$.
 (D) If the current in the upper wire is reversed, the wire will no longer be in equilibrium.

5.12 The electron in the Li^{2+} ion is replaced by a meson which has the same charge as the electron. Such an ion is called a Li^{2+} mesic ion. Let E_B be the binding energy of the hydrogen atom, and r_B the Bohr radius. You may take the mass of a meson to be 200 times the mass of electron. Which of the following statements is/are true for the Li^{2+} mesic ion?

- (A) The binding energy is $1800E_B$.
 (B) The radius of the meson in the ground state is $r_B/1800$.
 (C) The angular momentum in the ground state is $200h/2\pi$, where h is Planck's constant.
 (D) The Balmer series will be in the infrared wavelengths.

5.13 A particle of mass m that moves along the x -axis has potential energy $V(x) = a + bx^2$, where a and b are positive constants. Its initial velocity is v_0 at $x = 0$. It will execute simple harmonic motion. Then

- (A) the acceleration of the particle at its extreme position is zero.
- (B) at its extreme position the particle will have total energy $a + mv_0^2/2$.
- (C) its time period will be determined by the values of b and m .
- (D) the equilibrium position of the particle is determined by a .

5.14 An ideal gas initially occupies volume V_0 at pressure P_0 . Its volume and pressure are changed to $2V_0$ and $P_0/2$ through a certain process which is represented by a straight line on P - V diagram. Which of the following statements is/are correct?

- (A) The process is represented by a parabola on the V - T diagram, where T represents its temperature.
- (B) The net work done by the gas in this process is $P_0V_0/4$.
- (C) The internal energy of the gas remains constant throughout the process.
- (D) The net heat supplied to the gas in this process is more than the heat needed in an isothermal process from (V_0, P_0) to $(2V_0, P_0/2)$.