JEE Main 2023 Jan 25 Shift 2 Mathematics Question Paper

Time Allowed : 1 Hour	**Maximum Marks :** 100	**Total Questions :** 30

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

SECTION-A

1. Let the function $f(x) = 2x^3 + (2p-7)x^2 + 3(2p-9)x - 6$ have a maxima for some value of $x < 0$ and a minima for some value of $x > 0$. Then, the set of all values of p is:

Options:
(1) $\left(\frac{9}{2}, \infty\right)$
(2) $\left(0, \frac{9}{2}\right)$
(3) $\left(-\infty, \frac{9}{2}\right)$
(4) $\left(-\frac{9}{2}, \frac{9}{2}\right)$

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2. Let z be a complex number such that $\frac{|z-2i|}{|z+i|} = 2$, $z \neq -i$. Then z lies on the circle of radius 2 and centre:

Options:
(1) $(2, 0)$
(2) $(0, 0)$
(3) $(0, 2)$

(4) $(0, -2)$

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3. If the function

$$f(x) = \begin{cases} (1 + |\cos x|)^{\lambda/|\cos x|}, & 0 < x < \frac{\pi}{2} \\ \mu, & x = \frac{\pi}{2} \\ e^{\frac{\cot 6x}{\cot 4x}}, & \frac{\pi}{2} < x < \pi \end{cases}$$

is continuous at $x = \frac{\pi}{2}$, then $9\lambda + 6\ln\mu + \mu^6 - e^{6\lambda}$ is equal to:
Options:
(1) 11
(2) 8
(3) $2e^4 + 8$
(4) 10

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4. Let $f(x) = 2x^n + \lambda$, where $\lambda \in R$ and $n \in N$. Given that $f(4) = 133$ and $f(5) = 255$, what is the sum of all the positive integer divisors of $f(3) - f(2)$?
Options:
(1) 61
(2) 60
(3) 58
(4) 59

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5. If four points with position vectors $\mathbf{a} = 3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$, $\mathbf{b} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$, $\mathbf{c} = -2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$, and $\mathbf{d} = 5\mathbf{i} - 2\alpha\mathbf{j} + 4\mathbf{k}$ are coplanar, then α is equal to:
Options:
(1) $\frac{73}{17}$
(2) $-\frac{107}{17}$
(3) $-\frac{73}{17}$
(4) $\frac{107}{17}$

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6. Let $A = \begin{bmatrix} 1/\sqrt{10} & 3/\sqrt{10} \\ -3/\sqrt{10} & 1/\sqrt{10} \end{bmatrix} /\sqrt{10}$ and $B = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix}$, where $i = \sqrt{-1}$. If $M = A^T BA$, then the inverse of the matrix $AM^{2023}A^T$ is:
Options:
(1) $\begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$

(2) $\begin{bmatrix} 1 & 0 \\ -2023i & 1 \end{bmatrix}$
(3) $\begin{bmatrix} 1 & 0 \\ 2023i & 1 \end{bmatrix}$
(4) $\begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$

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7. Let $\triangle$ and $\bigtriangledown \in [\wedge, \vee]$ be such that the expression $(p \rightarrow q) \wedge (p \vee q)$ is a tautology. Then:
Options:
(1) $\triangle = \wedge, \bigtriangledown = \vee$
(2) $\triangle = \vee, \bigtriangledown = \wedge$
(3) $\triangle = \vee, \bigtriangledown = \vee$
(4) $\triangle = \wedge, \bigtriangledown = \wedge$

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8. The number of numbers, strictly between 5000 and 10000, that can be formed using the digits 1, 3, 5, 7, 9 without repetition, is:
Options:
(1) 60
(2) 12
(3) 120
(4) 72

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9. The number of functions $f : \{1, 2, 3, 4\} \rightarrow \{a \in Z : |a| \leq 8\}$ satisfying $f(n)+1/nf(n+1)$ =1, for $n \in \{1, 2, 3\}$ is:
Options:
(1) 3
(2) 4
(3) 1
(4) 2

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10. The equations of two sides of a variable triangle are $x = 0$ and $y = 3$, and its third side is a tangent to the parabola $y^2 = 6x$. The locus of its circumcentre is:
Options:
(1) $4y^2 - 18y - 3x - 18 = 0$
(2) $4y^2 + 18y + 3x + 18 = 0$
(3) $4y^2 - 18y + 3x + 18 = 0$
(4) $4y^2 - 18y - 3x + 18 = 0$

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11. Let $f : R \to R$ be a function defined by $f(x) = \log_{\sqrt{m}}\left(\sqrt{2}(\sin x - \cos x) + m - 2\right)$, for some m, such that the range of f is [0, 2]. Then the value of m is:
Options:
(1) 5
(2) 3
(3) 2
(4) 4

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12. Let A, B, C be 3×3 matrices such that A is symmetric and B and C are skew-symmetric. Consider the statements:
(S1) $A^{13}B^{26} - B^{26}A^{13}$ **is symmetric.**
(S2) $A^{26}C^{13} - C^{13}A^{26}$ **is symmetric. Then:**
Options:
(1) Only S2 is true
(2) Only S1 is true
(3) Both S1 and S2 are false
(4) Both S1 and S2 are true

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13. Let y = $y(t)$ be a solution of the differential equation $\frac{dy}{dt} + \alpha y = \gamma e^{-\beta t}$, where $\alpha, \beta, \gamma > 0$. If $\lim_{t\to\infty} y(t)$, then:
Options:
(1) 0
(2) does not exist
(3) 1
(4) -1

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14. Evaluate $\sum_{k=0}^{6} {}^{(51-k)}C_3$:
Options:
(1) ${}^{51}C_4 - {}^{45}C_4$
(2) ${}^{51}C_3 - {}^{45}C_3$
(3) ${}^{52}C_4 - {}^{45}C_4$
(4) ${}^{52}C_3 - {}^{45}C_3$

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15. The shortest distance between the lines $x + 1 = 2y = -12z$ and $x = y + 2 = 6z - 6$ is:
Options:
(1) 2
(2) 3
(3) $\frac{5}{2}$
(4) $\frac{3}{2}$

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16. Let N be the sum of the numbers appeared when two fair dice are rolled and let the probability that $N-2, \sqrt{3N}, N+2$ are in geometric progression be $\frac{k}{48}$. Then the value of k is:

Options:
(1) 2
(2) 4
(3) 16
(4) 8

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17. The integral $16\int_1^2 \frac{dx}{x^3(x^2+2)^2}$ is equal to:

Options:
(1) $\frac{11}{6}+\ln 4$
(2) $\frac{11}{12}+\ln 4$
(3) $\frac{11}{12}-\ln 4$
(4) $\frac{11}{6}-\ln 4$

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18. Let T and C respectively be the transverse and conjugate axes of the hyperbola $16x^2-y^2+64x+4y+44=0$. Then the area of the region above the parabola $x^2=y+4$, below the transverse axis T and on the right of the conjugate axis C is:

Options:
(1) $4\sqrt{6}+\frac{44}{3}$
(2) $4\sqrt{6}+\frac{28}{3}$
(3) $4\sqrt{6}-\frac{44}{3}$
(4) $4\sqrt{6}-\frac{28}{3}$

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19. Let $\vec{a}=-\mathbf{i}-\mathbf{j}+\mathbf{k}$, $\vec{a}\cdot\vec{b}=1$ and $\vec{a}\times\mathbf{b}=\mathbf{i}-\mathbf{j}$. Then $\vec{a}-6\vec{b}$ is equal to:

Options:
(1) $3(\mathbf{i}-\mathbf{j}-\mathbf{k})$
(2) $3(\mathbf{i}+\mathbf{j}+\mathbf{k})$
(3) $3(\mathbf{i}-\mathbf{j}+\mathbf{k})$
(4) $3(\mathbf{i}+\mathbf{j}-\mathbf{k})$

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20. The foot of the perpendicular from the point $(2,0,5)$ on the line $\frac{x+1}{2}=\frac{y-1}{5}=\frac{z+1}{-1}$ is (α,β,γ). Then, which of the following is NOT correct?

Options:
(1) $\frac{\alpha\beta}{\gamma}=\frac{4}{15}$
(2) $\frac{\alpha}{\beta}=-8$

(3) $\frac{\beta}{\gamma} = -5$
(4) $\frac{\gamma}{\alpha} = \frac{5}{8}$

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21. For the two positive numbers a, b, if a, b and $\frac{1}{18}$ are in a geometric progression, while $\frac{1}{a}, 10, \frac{1}{b}$ are in an arithmetic progression, then $16a + 12b$ is equal to:

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22. Points $P(-3, 2), Q(9, 10)$, and $R(\alpha, 4)$ lie on a circle C with PR as its diameter. The tangents to C at Q and R intersect at point S. If S lies on the line $2x - ky = 1$, then k is equal to:

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23. Let $a \in R$ and let α, β be the roots of the equation $x^2 + 60^{\frac{1}{4}}x + a = 0$. If $\alpha^4 + \beta^4 = -30$, then the product of all possible values of a is:

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24. Suppose Anil's mother wants to give 5 whole fruits to Anil from a basket of 7 red apples, 5 white apples, and 8 oranges. If in the selected 5 fruits, at least 2 oranges, at least one red apple, and at least one white apple must be given, then the number of ways Anil's mother can offer 5 fruits to Anil is:

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25. If m and n respectively are the numbers of positive and negative values of θ in the interval $[-\pi, \pi]$ that satisfy the equation $\cos 2\theta \cdot \cos\frac{\theta}{2} = \cos 3\theta \cdot \cos\frac{9\theta}{2}$, then mn is equal to:

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26. If $\int_{\frac{1}{3}}^{3} |\log_e x|\, dx = \frac{m}{n}\log_e\left(\frac{n^2}{e}\right)$, where m and n are coprime natural numbers, then $m^2 + n^2 - 5$ is equal to ____.

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27. The remainder when $(2023)^{2023}$ is divided by 35 is:

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28. If the shortest distance between the line joining the points $(1, 2, 3)$ and $(2, 3, 4)$, and the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-2}{0}$ is α, then $28\alpha^2$ is equal to:

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29. 25% of the population are smokers. A smoker has 27 times more chances to develop lung cancer than a non-smoker. If a person is diagnosed with lung cancer, and the probability that this person is a smoker is $\frac{k}{10}$, then the value of k is:

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30. A triangle is formed by the X-axis, Y-axis, and the line $3x + 4y = 60$. Then the

number of points $P(a, b)$, where a is an integer and b is a multiple of a, which lie strictly inside the triangle, is: