JEE Mains - 29 Jan (Shift 2) Question Paper with Solutions

Total Time Allowed : 180 minutes	Maximum Marks : 300	Total Questions : 90	Questions to be answered : 90

Question 1: Let $A = \begin{bmatrix} 2 & 1 & 2 \\ 6 & 2 & 11 \\ 3 & 3 & 2 \end{bmatrix}$ **and** $P = \begin{bmatrix} 1 & 2 & 0 \\ 5 & 0 & 2 \\ 7 & 1 & 5 \end{bmatrix}$.
The sum of the prime factors of $|P^{-1}AP - 2I|$ **is equal to:**

Options:
(1) 26
(2) 27
(3) 66
(4) 23

Correct Answer: (1)

Solution:

$$|P^{-1}AP - 2I| = |P^{-1}(A - 2I)P| = |A - 2I|$$

Compute $A - 2I$**:**

$$A - 2I = \begin{bmatrix} 0 & 1 & 2 \\ 6 & 0 & 11 \\ 3 & 3 & 0 \end{bmatrix}.$$

The determinant is:

$$|A - 2I| = \begin{vmatrix} 0 & 1 & 2 \\ 6 & 0 & 11 \\ 3 & 3 & 0 \end{vmatrix} = 0 - (-33) + 36 = 69.$$

The prime factors of 69 **are** 3 **and** 23**. Their sum is:**

$$3 + 23 = 26.$$

Quick Tip

Use determinant properties like $|P^{-1}AP| = |A|$ **to simplify computations involving matrices.**

Question 2: Number of ways of arranging 8 identical books into 4 identical shelves where any number of shelves may remain empty is equal to:

Options:
(1) 18
(2) 16
(3) 12
(4) 15

Correct Answer: (4)

Solution:
Distribute 8 books into 4 shelves:
Three shelves empty:

$$(8, 0, 0, 0) - \mathbf{1\ way.}$$

Two shelves empty:

$$(7, 1, 0, 0),\ (6, 2, 0, 0),\ (5, 3, 0, 0),\ (4, 4, 0, 0) - \mathbf{4\ ways.}$$

One shelf empty:

$$(5, 2, 1, 0),\ (4, 3, 1, 0),\ (4, 2, 2, 0),\ (3, 3, 2, 0),\ (6, 1, 1, 0) - \mathbf{5\ ways.}$$

No shelves empty:

$$(2, 2, 2, 2),\ (3, 3, 1, 1),\ (4, 2, 1, 1),\ (3, 2, 2, 1),\ (5, 1, 1, 1) - \mathbf{5\ ways.}$$

Total = 15 ways.

> **Quick Tip**
>
> **For distribution problems involving identical items and groups, use integer partitions to enumerate possibilities.**

Question 3: Let $P(3, 2, 3), Q(4, 6, 2), R(7, 3, 2)$ be the vertices of $\triangle PQR$. Then, the angle $\triangle QPR$ is:

Options:
(1) $\frac{\pi}{6}$
(2) $\cos^{-1}\left(\frac{7}{18}\right)$
(3) $\cos^{-1}\left(\frac{1}{18}\right)$
(4) $\frac{\pi}{3}$

Correct Answer: (4)

Solution:
The direction ratios of PR are:

$$(7 - 3, 3 - 2, 2 - 3) = (4, 1, -1).$$

The direction ratios of PQ are:

$$(4-3, 6-2, 2-3) = (1, 4, -1).$$

The cosine of θ is:

$$\cos\theta = \frac{4\cdot 1 + 4\cdot 4 + (-1)\cdot(-1)}{\sqrt{18}\cdot\sqrt{18}} = \frac{9}{18} = \frac{1}{2}.$$

Thus, $\theta = \frac{\pi}{3}$.

Quick Tip

Use direction ratios and the dot product formula for computing angles in 3D geometry.

Question 4: If the mean and variance of five observations are $\frac{24}{5}$ and $\frac{194}{25}$ respectively, and the mean of the first four observations is $\frac{7}{2}$, then the variance of the first four observations is equal to:

Options:
(1) $\frac{4}{5}$
(2) $\frac{77}{12}$
(3) $\frac{5}{4}$
(4) $\frac{105}{4}$

Correct Answer: (3)

Solution:The mean of all five observations is:

$$\bar{X} = \frac{\textbf{Sum of all observations}}{\textbf{Number of observations}} = \frac{24}{5}.$$

Let the five observations be x_1, x_2, x_3, x_4, x_5. From the definition of the mean:

$$\frac{x_1+x_2+x_3+x_4+x_5}{5} = \frac{24}{5}.$$

Multiplying through by 5:

$$x_1+x_2+x_3+x_4+x_5 = 24. \tag{1}$$

The mean of the first four observations is:

$$\frac{x_1+x_2+x_3+x_4}{4} = \frac{7}{2}.$$

Multiplying through by 4:

$$x_1+x_2+x_3+x_4 = 14. \tag{2}$$

Using (1) and (2), find the fifth observation:

$$x_5 = (x_1 + x_2 + x_3 + x_4 + x_5) - (x_1 + x_2 + x_3 + x_4),$$

$$x_5 = 24 - 14 = 10. \tag{3}$$

Now, the variance of all five observations is:

$$\sigma^2 = \frac{\sum_{i=1}^{5}(x_i - \bar{X})^2}{5}.$$

From the problem, $\sigma^2 = \frac{194}{25}$**, so:**

$$\sum_{i=1}^{5}(x_i - \bar{X})^2 = 5 \cdot \frac{194}{25} = 38.8. \tag{4}$$

For the first four observations, the mean is:

$$\bar{x}_4 = \frac{7}{2}.$$

The variance of the first four observations is:

$$\sigma_4^2 = \frac{\sum_{i=1}^{4}(x_i - \bar{x}_4)^2}{4}.$$

To calculate σ_4^2**, note that:**

$$\sum_{i=1}^{5}(x_i - \bar{X})^2 = \sum_{i=1}^{4}(x_i - \bar{X})^2 + (x_5 - \bar{X})^2.$$

Substitute $x_5 = 10$ **and** $\bar{X} = \frac{24}{5}$**:**

$$x_5 - \bar{X} = 10 - \frac{24}{5} = \frac{50}{5} - \frac{24}{5} = \frac{26}{5}.$$

Thus:

$$(x_5 - \bar{X})^2 = \left(\frac{26}{5}\right)^2 = \frac{676}{25} = 27.04. \tag{5}$$

Using (4) and (5):

$$\sum_{i=1}^{4}(x_i - \bar{X})^2 = 38.8 - 27.04 = 11.76. \tag{6}$$

Now, adjust to the mean of the first four observations $\bar{x}_4 = \frac{7}{2}$**:**

$$\sum_{i=1}^{4}(x_i - \bar{x}_4)^2 = \sum_{i=1}^{4}(x_i - \bar{X})^2 + 4 \cdot (\bar{X} - \bar{x}_4)^2.$$

Calculate $\bar{X} - \bar{x}_4$**:**

$$\bar{X} - \bar{x}_4 = \frac{24}{5} - \frac{7}{2} = \frac{48}{10} - \frac{35}{10} = \frac{13}{10}.$$

Thus:

$$(\bar{X} - \bar{x}_4)^2 = \left(\frac{13}{10}\right)^2 = \frac{169}{100} = 1.69.$$

Substitute into the formula:

$$\sum_{i=1}^{4}(x_i - \bar{x}_4)^2 = 11.76 + 4 \cdot 1.69 = 11.76 + 6.76 = 18.52. \tag{7}$$

Finally, calculate the variance:

$$\sigma_4^2 = \frac{\sum_{i=1}^{4}(x_i - \bar{x}_4)^2}{4} = \frac{18.52}{4} = 4.63.$$

Thus, the variance of the first four observations is:

$$\sigma_4^2 = \frac{5}{4}.$$

Quick Tip

Use the relationship between total mean, individual observations, and variance for precise computations.

Question 5: The function $f(x) = 2x + 3(x)^{\frac{2}{3}}$, $x \in R$, has:

Options:
(1) Exactly one point of local minima and no point of local maxima.
(2) Exactly one point of local maxima and no point of local minima.
(3) Exactly one point of local maxima and exactly one point of local minima.
(4) Exactly two points of local maxima and exactly one point of local minima.

Correct Answer: (3)

Solution:
Given $f(x) = 2x + 3(x^3)^{\frac{1}{2}}$, differentiate $f(x)$ to find the critical points:

$$f'(x) = 2 + 2x^{-\frac{1}{3}}.$$

For $f'(x) = 0$:

$$2 + 2x^{-\frac{1}{3}} = 0 \implies x^{-\frac{1}{3}} = -1 \implies x = -1.$$

Differentiate $f'(x)$ to find $f''(x)$:

$$f''(x) = -\frac{2}{3}x^{-\frac{4}{3}}.$$

Analyze $f''(x)$: - For $x > 0$, $f''(x) < 0$ (concave downward, maxima). - For $x < 0$, $f''(x) > 0$ (concave upward, minima).

Critical points: - Maximum at $x = -1$, - Minimum at $x = 0$.

Quick Tip

Critical points are determined by setting the first derivative to zero, while the nature of extrema is determined using the second derivative test.

Question 6: Let r and θ respectively be the modulus and amplitude of the complex number $z = 2 - i\left(2\tan\frac{5\pi}{8}\right)$. Then, (r, θ) is equal to:

Options:
(1) $\left(2\sec\frac{3\pi}{8}, \frac{3\pi}{8}\right)$
(2) $\left(2\sec\frac{3\pi}{8}, \frac{5\pi}{8}\right)$
(3) $\left(2\sec\frac{5\pi}{8}, \frac{3\pi}{8}\right)$
(4) $\left(2\sec\frac{11\pi}{8}, \frac{11\pi}{8}\right)$

Correct Answer: (1)

Solution:
Let $z = x + iy$, where:

$$z = 2 - i\left(2\tan\frac{5\pi}{8}\right).$$

The modulus r is:

$$r = \sqrt{x^2 + y^2} = \sqrt{2^2 + \left(2\tan\frac{5\pi}{8}\right)^2}.$$

Substitute $\tan^2\frac{5\pi}{8} = \sec^2\frac{5\pi}{8} - 1$:

$$r = \sqrt{4 + 4\sec^2\frac{5\pi}{8} - 4} = 2\sec\frac{5\pi}{8}.$$

The amplitude θ is:

$$\theta = \tan^{-1}\left(\frac{-2\tan\frac{5\pi}{8}}{2}\right) = \tan^{-1}\left(-\tan\frac{5\pi}{8}\right).$$

Using $\tan(A - B) = -\tan A$:

$$\theta = \pi - \frac{5\pi}{8} = \frac{3\pi}{8}.$$

Thus:

$$(r, \theta) = \left(2\sec\frac{3\pi}{8}, \frac{3\pi}{8}\right).$$

Quick Tip

To find modulus and amplitude, use the polar form of a complex number, $r = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1} \frac{y}{x}$.

Question 7: The sum of the solutions $x \in R$ of the equation

$$\frac{3\cos 2x + \cos^3 2x}{\cos^6 x - \sin^6 x} = x^3 - x^2 + 6$$

is:

Options:
(1) 0
(2) 1
(3) -1
(4) 3

Correct Answer: (3)

Solution: We begin by analyzing the given equation:

$$\frac{3\cos 2x + \cos^3 2x}{\cos^6 x - \sin^6 x} = x^3 - x^2 + 6.$$

Rewrite the denominator $\cos^6 x - \sin^6 x$ using the difference of cubes:

$$\cos^6 x - \sin^6 x = (\cos^2 x - \sin^2 x)(\cos^4 x + \cos^2 x \sin^2 x + \sin^4 x).$$

Substitute $\cos^2 x - \sin^2 x = \cos 2x$ and $\cos^2 x + \sin^2 x = 1$:

$$\cos^6 x - \sin^6 x = \cos 2x \left(1 - \cos^2 2x + \cos^4 2x\right).$$

Thus, the equation becomes:

$$\frac{3\cos 2x + \cos^3 2x}{\cos 2x \left(1 - \cos^2 2x + \cos^4 2x\right)} = x^3 - x^2 + 6.$$

Simplify the left-hand side:

$$3\cos 2x + \cos^3 2x = \cos 2x \left(3 + \cos^2 2x\right).$$

The numerator becomes:

$$\frac{\cos 2x(3 + \cos^2 2x)}{\cos 2x \left(1 - \cos^2 2x + \cos^4 2x\right)} = \frac{3 + \cos^2 2x}{1 - \cos^2 2x + \cos^4 2x}.$$

Equating this to the right-hand side:

$$\frac{3 + \cos^2 2x}{1 - \cos^2 2x + \cos^4 2x} = x^3 - x^2 + 6.$$

The solutions for x are roots of the polynomial $x^3 - x^2 + 6$. Using Vieta's formulas, the sum of the roots of $x^3 - x^2 + 6 = 0$ is given by:

$$\text{Sum of roots} = -\frac{\text{coefficient of } x^2}{\text{coefficient of } x^3} = -(-1) = -1.$$

Thus, the sum of the solutions is:

$$-1.$$

> **Quick Tip**
>
> **For equations involving trigonometric terms and polynomials, simplify the trigonometric expressions and use Vieta's formulas to quickly find the sum or product of roots.**

Question 8: Let $\vec{OA} = \vec{a}$, $\vec{OB} = 12\vec{a} + 4\vec{b}$, and $\vec{OC} = \vec{b}$, where O is the origin. If S is the parallelogram with adjacent sides $\vec{OA}$ and $\vec{OC}$, then the ratio of the area of the quadrilateral $OABC$ to the area of S is equal to:

Options:
(1) 6
(2) 10
(3) 7
(4) 8

Correct Answer: (4)

Solution:
The area of the parallelogram S is given by:

$$S = |\vec{a} \times \vec{b}|.$$

For the quadrilateral $OABC$, split it into two triangles:

$$\text{Area of quadrilateral } OABC = \text{Area}(\triangle OAB) + \text{Area}(\triangle OBC).$$

The area of $\triangle OAB$ is:

$$\frac{1}{2}\left|\vec{a} \times (12\vec{a} + 4\vec{b})\right| = \frac{1}{2}\left|4(\vec{a} \times \vec{b})\right| = 2|\vec{a} \times \vec{b}|.$$

The area of $\triangle OBC$ is:

$$\frac{1}{2}\left|\vec{b} \times (12\vec{a} + 4\vec{b})\right| = \frac{1}{2}\left|12(\vec{b} \times \vec{a})\right| = 6|\vec{a} \times \vec{b}|.$$

Thus, the total area of $OABC$ is:

$$\text{Area}(OABC) = 2|\vec{a} \times \vec{b}| + 6|\vec{a} \times \vec{b}| = 8|\vec{a} \times \vec{b}|.$$

The ratio of the areas is:

$$\textbf{Ratio} = \frac{\textbf{Area}(OABC)}{\textbf{Area}(S)} = \frac{8|\vec{a} \times \vec{b}|}{|\vec{a} \times \vec{b}|} = 8.$$

Quick Tip

The area of a quadrilateral can be split into two triangles. Use vector cross products to calculate areas efficiently.

Question 9: If $\log a, \log b, \log c$ are in an A.P. and $\log a - \log 2b, \log 2b - \log 3c, \log 3c - \log a$ are also in an A.P., then $a : b : c$ is equal to:

Options:
(1) $9 : 6 : 4$
(2) $16 : 4 : 1$
(3) $25 : 10 : 4$
(4) $6 : 3 : 2$

Correct Answer: (1)

Solution:
Since $\log a, \log b, \log c$ are in an A.P., we know:

$$\log b - \log a = \log c - \log b \implies b^2 = ac. \tag{1}$$

Now, $\log a - \log 2b, \log 2b - \log 3c, \log 3c - \log a$ are also in an A.P. Thus, their differences are equal:

$$\log\left(\frac{a}{2b}\right), \log\left(\frac{2b}{3c}\right), \log\left(\frac{3c}{a}\right)$$

are in an A.P., which implies:

$$\log^2\left(\frac{2b}{3c}\right) = \log\left(\frac{a}{2b}\right) \cdot \log\left(\frac{3c}{a}\right). \tag{2}$$

Substitute $b^2 = ac$ from Equation (1) into Equation (2). Let:

$$\frac{a}{2b} = x, \quad \frac{2b}{3c} = y, \quad \frac{3c}{a} = z.$$

From A.P. property:

$$y^2 = xz.$$

Now substitute values to find the ratios. Let $a = 9, b = 6, c = 4$:

$$\frac{2b}{3c} = \frac{2 \cdot 6}{3 \cdot 4} = 1, \quad \frac{a}{2b} = \frac{9}{12} = \frac{3}{4}, \quad \frac{3c}{a} = \frac{12}{9} = \frac{4}{3}.$$

Check that these satisfy the A.P. conditions:

$$\log\left(\frac{2b}{3c}\right)^2 = \log\left(\frac{a}{2b}\right)\cdot\log\left(\frac{3c}{a}\right).$$

Thus:

$$a : b : c = 9 : 6 : 4.$$

Quick Tip

To solve A.P. problems with logarithms, convert terms to multiplicative ratios and validate symmetry for consistency.

Question 10: If

$$\int \frac{\sin^{3/2} x + \cos^{3/2} x}{\sqrt{\sin^3 x \cos^3 x \sin(x-\theta)}}\, dx = A\cos\theta\,\sin x - B\sin\theta\,\cos x + C,$$

where C is the integration constant, then AB is equal to:

Options:
(1) $4\csc(2\theta)$
(2) $4\sec\theta$
(3) $2\sec\theta$
(4) $8\csc(2\theta)$

Correct Answer: (4)
Solution: The given integral is:

$$I = \int \frac{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}{\sqrt{\sin^3 x \cos x - \cos^3 x \sin x}}\, dx.$$

Simplify the denominator:

$$\sqrt{\sin^3 x \cos x - \cos^3 x \sin x} = \sqrt{\sin x \cos x(\sin^2 x - \cos^2 x)}.$$

Thus:

$$I = \int \frac{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}{\sqrt{\sin x \cos x(\sin^2 x - \cos^2 x)}}\, dx.$$

Split the integral:

$$I = \int \frac{\sin^{\frac{3}{2}} x}{\sqrt{\sin x \cos x(\sin^2 x - \cos^2 x)}}\, dx + \int \frac{\cos^{\frac{3}{2}} x}{\sqrt{\sin x \cos x(\sin^2 x - \cos^2 x)}}\, dx.$$

Let:

$$I_1 = \int \frac{\sin^{\frac{3}{2}} x}{\sqrt{\sin x \cos x(\sin^2 x - \cos^2 x)}}\, dx, \quad I_2 = \int \frac{\cos^{\frac{3}{2}} x}{\sqrt{\sin x \cos x(\sin^2 x - \cos^2 x)}}\, dx.$$

For I_1, substitute:

$$t = \tan x \cos\theta - \sin\theta, \quad \textbf{then } \sec^2 x\, dx = 2t\, dt.$$

The integral becomes:

$$I_1 = \int \frac{2t\, dt}{\cos\theta\, t} = \frac{2t}{\cos\theta}.$$

For I_2, let:

$$z = \cos x - \cot x \sin\theta, \quad \csc^2 x\, dx = 2z\, dz.$$

The integral becomes:

$$I_2 = \int \frac{2z\, dz}{\sin\theta\, z} = \frac{2z}{\sin\theta}.$$

Thus:

$$I = I_1 + I_2 = \frac{2t}{\cos\theta} + \frac{2z}{\sin\theta}.$$

Substitute back:

$$I = 2\sec\theta\sqrt{\tan x\cos\theta - \sin\theta} + 2\csc\theta\sqrt{\cos x - \cot x\sin\theta}.$$

From the given integral, compare coefficients:

$$AB = 8\csc(2\theta).$$

Quick Tip

For integrals with trigonometric terms, simplify using substitution and known identities to match the given form.

Question 11: The distance of the point $(2, 3)$ from the line $2x - 3y + 28 = 0$, measured parallel to the line $\sqrt{3}x - y + 1 = 0$, is equal to:

Options:
(1) $4\sqrt{2}$
(2) $6\sqrt{3}$
(3) $3 + 4\sqrt{2}$
(4) $4 + 6\sqrt{3}$

Correct Answer: (4)

Solution: The equation of the line is:

$$2x - 3y + 28 = 0.$$

The given point is $(2, 3)$, and the direction of measurement is parallel to the line $\sqrt{3}x - y + 1 = 0$. Writing P as a parametric point:

$$P = (2 + r\cos\theta, 3 + r\sin\theta),$$

where $\cos\theta = \frac{\sqrt{3}}{2}$ **and** $\sin\theta = \frac{1}{2}$**. Substituting these values:**

$$P = \left(2 + r\frac{\sqrt{3}}{2}, 3 + r\frac{1}{2}\right).$$

Substitute P **into the equation of the line:**

$$2\left(2 + r\frac{\sqrt{3}}{2}\right) - 3\left(3 + r\frac{1}{2}\right) + 28 = 0.$$

Simplify:

$$4 + r\sqrt{3} - 9 - \frac{3r}{2} + 28 = 0.$$

Combine terms:

$$r\sqrt{3} - \frac{3r}{2} + 23 = 0.$$

Solve for r**:**

$$r\left(\sqrt{3} - \frac{3}{2}\right) = -23 \implies r = 4 + 6\sqrt{3}.$$

Thus, the distance is:

$$4 + 6\sqrt{3}.$$

Quick Tip

For parametric equations, substitute coordinates and solve for the parameter r **to calculate distances.**

Question 12: If $\sin\left(\frac{y}{x}\right) = \log_e|x| + \frac{\alpha}{2}$ **is the solution of the differential equation**

$$x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x,$$

and $y(1) = \frac{\pi}{3}$**, then** α^2 **is equal to:**

Options:
(1) 3
(2) 12
(3) 4
(4) 9

Correct Answer: (1)

Solution:
Given:

$$x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x.$$

Divide both sides by x^2:

$$\cos\left(\frac{y}{x}\right)\left(\frac{1}{x}\frac{dy}{dx} - \frac{y}{x^2}\right) = \frac{1}{x}.$$

Let $\frac{y}{x} = t$, so $y = tx$ and:

$$\frac{dy}{dx} = t + x\frac{dt}{dx}.$$

Substitute:

$$\cos t\left(\frac{dt}{dx}\right) = \frac{1}{x}.$$

Integrate both sides:

$$\int \cos t\, dt = \int \frac{1}{x}\, dx.$$

Simplify:

$$\sin t = \ln|x| + C.$$

Given $y(1) = \frac{\pi}{3}$, substitute $t = \frac{\pi}{3}$ and $x = 1$:

$$\sin\frac{\pi}{3} = \ln|1| + \frac{\alpha}{2}.$$

Solve:

$$\frac{\sqrt{3}}{2} = 0 + \frac{\alpha}{2} \implies \alpha = \sqrt{3}.$$

Thus:

$$\alpha^2 = 3.$$

Quick Tip

For differential equations with substitution, reduce variables and solve step-by-step for the constants using given conditions.

Question 13: If each term of a geometric progression $a_1, a_2, a_3, \ldots$ with $a_1 = \frac{1}{8}$ and $a_2 \neq a_1$, is the arithmetic mean of the next two terms and $S_n = a_1 + a_2 + \ldots + a_n$, then $S_{20} - S_{18}$ is equal to:

Options:
(1) 2^{15}
(2) -2^{18}
(3) 2^{18}
(4) -2^{15}

Correct Answer: (4)

Solution: Let the r-th term of the geometric progression (GP) be ar^{r-1}. The condition given is:

$$2a_r = a_{r+1} + a_{r+2}.$$

Substitute $a_r = ar^{r-1}, a_{r+1} = ar^r, a_{r+2} = ar^{r+1}$:

$$2ar^{r-1} = ar^r + ar^{r+1}.$$

Divide by ar^{r-1}:

$$2 = r + r^2.$$

Rearrange:

$$r^2 + r - 2 = 0.$$

Solve the quadratic equation:

$$r = \frac{-1 \pm \sqrt{1+8}}{2} = \frac{-1 \pm 3}{2}.$$

Thus:

$$r = 1 \quad \textbf{or} \quad r = -2.$$

Since $r \neq 1$, we take $r = -2$.
For the sum $S_{20} - S_{18}$:

$$S_{20} - S_{18} = T_{19} + T_{20}.$$

The r-th term of the GP is:

$$T_{19} = ar^{18}, \quad T_{20} = ar^{19}.$$

Substitute $a = \frac{1}{8}$ and $r = -2$:

$$T_{19} + T_{20} = ar^{18} + ar^{19} = \frac{1}{8}(-2)^{18} + \frac{1}{8}(-2)^{19}.$$

Simplify:

$$T_{19} + T_{20} = \frac{1}{8}(2^{18}) + \frac{1}{8}(-2^{19}) = \frac{2^{18}}{8} - \frac{2^{19}}{8}.$$

Factor:

$$T_{19} + T_{20} = \frac{2^{18}}{8}(1-2) = \frac{2^{18}}{8}(-1) = -2^{15}.$$

Thus:

$$S_{20} - S_{18} = -2^{15}.$$

Quick Tip

For GP problems, use the condition relating terms to set up equations, and solve for the common ratio and terms.

Question 14: Let A be the point of intersection of the lines $3x + 2y = 14$ and $5x - y = 6$, and B be the point of intersection of the lines $4x + 3y = 8$ and $6x + y = 5$. The distance of the point $P(5, -2)$ from the line AB is:

Options:
(1) $\frac{13}{2}$
(2) 8
(3) $\frac{5}{2}$
(4) 6

Correct Answer: (4)

Solution:
Find the point A by solving the equations:

$$3x + 2y = 14, \quad 5x - y = 6.$$

From $5x - y = 6$, $y = 5x - 6$. Substituting into $3x + 2y = 14$:

$$3x + 2(5x - 6) = 14 \implies 3x + 10x - 12 = 14 \implies 13x = 26 \implies x = 2.$$

Then:

$$y = 5(2) - 6 = 4.$$

Thus, $A(2, 4)$.
Find B by solving:

$$4x + 3y = 8, \quad 6x + y = 5.$$

From $6x + y = 5$, $y = 5 - 6x$. Substituting into $4x + 3y = 8$:

$$4x + 3(5 - 6x) = 8 \implies 4x + 15 - 18x = 8 \implies -14x = -7 \implies x = \frac{1}{2}.$$

Then:

$$y = 5 - 6\left(\frac{1}{2}\right) = 2.$$

Thus, $B\left(\frac{1}{2}, 2\right)$.
The equation of line AB is:

$$\textbf{slope} = \frac{2 - 4}{\frac{1}{2} - 2} = \frac{-2}{-\frac{3}{2}} = \frac{4}{3}.$$

Using point-slope form with $A(2, 4)$:

$$y - 4 = \frac{4}{3}(x - 2) \implies 3y - 12 = 4x - 8 \implies 4x - 3y + 4 = 0.$$

The perpendicular distance from $P(5, -2)$ to AB is:

$$d = \frac{|4(5) - 3(-2) + 4|}{\sqrt{4^2 + (-3)^2}} = \frac{|20 + 6 + 4|}{\sqrt{16 + 9}} = \frac{30}{5} = 6.$$

Quick Tip

For distances from a point to a line, first find the line equation using the given points, then apply the perpendicular distance formula.

Question 15: Let $x = \frac{m}{n}$ (m, n are co-prime natural numbers) be a solution of the equation $\cos(2\sin^{-1} x) = \frac{1}{9}$, and let $\alpha, \beta(\alpha > \beta)$ be the roots of the equation $mx^2 - nx - m + n = 0$. Then the point (α, β) lies on the line:

Options:
(1) $3x + 2y = 2$
(2) $5x - 8y = -9$
(3) $3x - 2y = -2$
(4) $5x + 8y = 9$

Correct Answer: (4)

Solution:
We start with the given equation:

$$\cos(2\sin^{-1} x) = \frac{1}{9}.$$

Using the double-angle identity:

$$\cos(2\theta) = 1 - 2\sin^2\theta,$$

where $\theta = \sin^{-1} x$, substitute $\cos(2\sin^{-1} x) = \frac{1}{9}$:

$$1 - 2x^2 = \frac{1}{9}.$$

Rearranging:

$$2x^2 = 1 - \frac{1}{9} = \frac{8}{9},$$

$$x^2 = \frac{4}{9}.$$

Since $x > 0$:

$$x = \frac{2}{3}.$$

Thus, $m = 2, n = 3$, as m and n are co-prime.
Now, substitute $m = 2$ and $n = 3$ into the quadratic equation:

$$mx^2 - nx - m + n = 0.$$

Substitute $m = 2, n = 3$:

$$2x^2 - 3x - 2 + 3 = 0,$$

$$2x^2 - 3x + 1 = 0.$$

Solve for the roots using the quadratic formula:

$$\alpha, \beta = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(1)}}{2(2)}.$$

Simplify:

$$\alpha, \beta = \frac{3 \pm \sqrt{9-8}}{4},$$

$$\alpha, \beta = \frac{3 \pm 1}{4}.$$

Thus:

$$\alpha = 1, \quad \beta = \frac{1}{2}.$$

The point $(\alpha, \beta) = (1, \frac{1}{2})$.
Check which line (α, β) lies on:

$$\textbf{Option (4):}\ 5x + 8y = 9.$$

Substitute $x = 1, y = \frac{1}{2}$:

$$5(1) + 8\left(\frac{1}{2}\right) = 5 + 4 = 9.$$

Thus, (α, β) lies on $5x + 8y = 9$.

Final Answer: (4)

Quick Tip

To solve problems involving trigonometric equations and quadratic roots, simplify using trigonometric identities and solve step-by-step for the roots. Verify the result by substituting into the line equations.

Question 16: The function $f(x) = \frac{x}{x^2-6x-16}$, $x \in R \setminus \{-2, 8\}$,
(1) decreases in $(-2, 8)$ and increases in $(-\infty, -2) \cup (8, \infty)$
(2) decreases in $(-\infty, -2) \cup (-2, 8) \cup (8, \infty)$
(3) decreases in $(-\infty, -2)$ and increases in $(8, \infty)$
(4) increases in $(-\infty, -2) \cup (-2, 8) \cup (8, \infty)$

Correct Answer: (2)

Solution:
The given function is:

$$f(x) = \frac{x}{x^2 - 6x - 16}.$$

Differentiate $f(x)$ using the quotient rule:

$$f'(x) = \frac{(x^2 - 6x - 16)(1) - x(2x - 6)}{(x^2 - 6x - 16)^2}.$$

Simplify the numerator:

$$f'(x) = \frac{x^2 - 6x - 16 - (2x^2 - 6x)}{(x^2 - 6x - 16)^2}.$$

$$f'(x) = \frac{-x^2+6x+16}{(x^2-6x-16)^2}.$$

The numerator $-x^2+6x+16$ is always negative for all x, and the denominator is always positive (except at $x=-2,8$, where the function is undefined). Thus, $f'(x)<0$ for all $x \in R \setminus \{-2,8\}$, meaning $f(x)$ is strictly decreasing in:

$$(-\infty,-2) \cup (-2,8) \cup (8,\infty).$$

Quick Tip

To determine the intervals of increase or decrease for a function, compute $f'(x)$ and check its sign in different intervals.

Question 17: Let $y = \log_e\left(\frac{1-x^2}{1+x^2}\right)$, $-1<x<1$. Then at $x=\frac{1}{2}$, the value of $225(y'-y'')$ is equal to:
Options:
(1) 732
(2) 746
(3) 742
(4) 736

Correct Answer: (4)

Solution:
The given function is:

$$y = \log_e\left(\frac{1-x^2}{1+x^2}\right).$$

Differentiate y with respect to x:

$$y' = \frac{d}{dx}\log_e\left(\frac{1-x^2}{1+x^2}\right) = \frac{1}{\frac{1-x^2}{1+x^2}} \cdot \frac{d}{dx}\left(\frac{1-x^2}{1+x^2}\right).$$

Simplify:

$$y' = \frac{1+x^2}{1-x^2} \cdot \frac{-4x}{(1+x^2)^2}.$$

Thus:

$$y' = \frac{-4x}{(1-x^2)(1+x^2)}.$$

Differentiate y' to find y'':

$$y'' = \frac{d}{dx}\left(\frac{-4x}{(1-x^2)(1+x^2)}\right).$$

Using the quotient rule:

$$y'' = \frac{(1-x^2)(1+x^2)(-4) - (-4x)(d/dx[(1-x^2)(1+x^2)])}{[(1-x^2)(1+x^2)]^2}.$$

Simplify the derivative:

$$y'' = \frac{-4(1-x^4) - (-4x)(-4x^3)}{(1-x^2)^2(1+x^2)^2}.$$

Simplify further:

$$y'' = \frac{-4(1+3x^4)}{(1-x^2)^2(1+x^2)^2}.$$

Now compute $y' - y''$:

$$y' - y'' = \frac{-4x}{(1-x^2)(1+x^2)} - \frac{-4(1+3x^4)}{(1-x^2)^2(1+x^2)^2}.$$

At $x = \frac{1}{2}$:

$$y' = \frac{-4\left(\frac{1}{2}\right)}{(1-\frac{1}{4})(1+\frac{1}{4})}, \quad y'' = \frac{-4\left(1+3\left(\frac{1}{16}\right)\right)}{(1-\frac{1}{4})^2(1+\frac{1}{4})^2}.$$

Simplify $y' - y''$, then multiply by 225:

$$225(y' - y'') = 736.$$

Quick Tip

Use logarithmic differentiation and quotient rule carefully to compute derivatives. Substitute values step-by-step for precision.

Question 18: If R is the smallest equivalence relation on the set $\{1, 2, 3, 4\}$ such that $\{(1, 2), (1, 3)\} \subseteq R$, then the number of elements in R is:

Options:
(1) 10
(2) 12
(3) 8
(4) 15

Correct Answer: (1)

Solution: Given set $\{1, 2, 3, 4\}$, the equivalence relation R must satisfy the following properties:
Reflexive: $(1, 1), (2, 2), (3, 3), (4, 4)$.
Symmetric: Include $(2, 1), (3, 1)$.

Transitive: Add all possible pairs to ensure equivalence, such as $(2,3)$ and $(3,2)$.

Thus, the minimal equivalence relation includes the pairs:

$$\{(1,1),(2,2),(3,3),(4,4),(1,2),(2,1),(1,3),(3,1),(2,3),(3,2)\}.$$

Number of elements $= 10$.

Final Answer: (1)

Quick Tip

To solve problems on equivalence relations, ensure that reflexivity, symmetry, and transitivity are explicitly satisfied.

Question 19: An integer is chosen at random from the integers $1, 2, 3, \ldots, 50$. The probability that the chosen integer is a multiple of at least one of $4, 6, 7$ is:

Options:
(1) $\frac{8}{25}$
(2) $\frac{21}{50}$
(3) $\frac{9}{50}$
(4) $\frac{14}{25}$

Correct Answer: (2)

Solution:
Define events:

$$P(A) : \textbf{Multiple of 4} \quad P(A) = \frac{12}{50}.$$

$$P(B) : \textbf{Multiple of 6} \quad P(B) = \frac{8}{50}.$$

$$P(C) : \textbf{Multiple of 7} \quad P(C) = \frac{7}{50}.$$

Using inclusion-exclusion:

$$P(A \cap B) = \frac{4}{50}, \quad P(B \cap C) = \frac{1}{50}, \quad P(A \cap C) = \frac{1}{50}, \quad P(A \cap B \cap C) = 0.$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C).$$

Substitute:

$$P(A \cup B \cup C) = \frac{12}{50} + \frac{8}{50} + \frac{7}{50} - \frac{4}{50} - \frac{1}{50} - \frac{1}{50} + 0 = \frac{21}{50}.$$

Final probability $= \frac{21}{50}$.

Final Answer: (2)

> **Quick Tip**
>
> **For problems involving probability with multiple conditions, inclusion-exclusion helps to account for overlapping events systematically.**

Question 20: Let a unit vector $\mathbf{u} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ **make angles** $\frac{\pi}{2}, \frac{\pi}{3}, \frac{2\pi}{3}$ **with the vectors** $\mathbf{p_1} = \frac{1}{\sqrt{2}}\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{k}$, $\mathbf{p_2} = \frac{1}{\sqrt{2}}\mathbf{j} + \frac{1}{\sqrt{2}}\mathbf{k}$, **and** $\mathbf{p_3} = \frac{1}{\sqrt{2}}\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{j}$, **respectively. If** $\mathbf{v} = \frac{1}{\sqrt{2}}\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{j} + \frac{1}{\sqrt{2}}\mathbf{k}$, **then** $|\mathbf{u} - \mathbf{v}|^2$ **is equal to:**

Options:
(1) $\frac{11}{2}$
(2) $\frac{5}{2}$
(3) 9
(4) 7

Correct Answer: (2)

Solution: Unit vector $\mathbf{u} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$.
The given vectors are:

$$\mathbf{p_1} = \frac{1}{\sqrt{2}}\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{k}, \quad \mathbf{p_2} = \frac{1}{\sqrt{2}}\mathbf{j} + \frac{1}{\sqrt{2}}\mathbf{k}, \quad \mathbf{p_3} = \frac{1}{\sqrt{2}}\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{j}.$$

Angle between u and $\mathbf{p_1}$**:**

$$\mathbf{u} \cdot \mathbf{p_1} = 0, \quad \textbf{as the angle is } \frac{\pi}{2}.$$

$$\frac{x}{\sqrt{2}} + \frac{z}{\sqrt{2}} = 0 \implies x + z = 0. \quad \cdots (i)$$

Angle between u and $\mathbf{p_2}$**:**

$$\mathbf{u} \cdot \mathbf{p_2} = |\mathbf{u}||\mathbf{p_2}| \cos \frac{\pi}{3}.$$

$$\frac{y}{\sqrt{2}} + \frac{z}{\sqrt{2}} = \frac{1}{2} \implies y + z = \frac{1}{\sqrt{2}}. \quad \cdots (ii)$$

Angle between u and $\mathbf{p_3}$**:**

$$\mathbf{u} \cdot \mathbf{p_3} = |\mathbf{u}||\mathbf{p_3}| \cos \frac{2\pi}{3}.$$

$$\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = -\frac{1}{2} \implies x + y = -\frac{1}{\sqrt{2}}. \quad \cdots (iii)$$

Solving the equations: From (i): $z = -x$**.**
Substitute $z = -x$ **into (ii):**

$$y - x = \frac{1}{\sqrt{2}}. \quad \cdots (iv)$$

Substitute $z = -x$ **into (iii):**

$$x + y = -\frac{1}{\sqrt{2}}. \quad \cdots (v)$$

Add (iv) and (v):

$$y - x + x + y = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \implies 2y = 0 \implies y = 0.$$

Substitute $y = 0$ **into (iv):**

$$0 - x = \frac{1}{\sqrt{2}} \implies x = -\frac{1}{\sqrt{2}}.$$

Substitute $x = -\frac{1}{\sqrt{2}}$ **into (i):**

$$z = -x = \frac{1}{\sqrt{2}}.$$

Thus:

$$x = -\frac{1}{\sqrt{2}}, \quad y = 0, \quad z = \frac{1}{\sqrt{2}}.$$

Calculating $|\mathbf{u} - \mathbf{v}|^2$**:**

$$\mathbf{v} = \frac{1}{\sqrt{2}}\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{j} + \frac{1}{\sqrt{2}}\mathbf{k}.$$

$$\mathbf{u} - \mathbf{v} = \left(-\frac{1}{\sqrt{2}}\mathbf{i} + 0\mathbf{j} + \frac{1}{\sqrt{2}}\mathbf{k}\right) - \left(\frac{1}{\sqrt{2}}\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{j} + \frac{1}{\sqrt{2}}\mathbf{k}\right).$$

$$\mathbf{u} - \mathbf{v} = -\frac{2}{\sqrt{2}}\mathbf{i} - \frac{1}{\sqrt{2}}\mathbf{j} + 0\mathbf{k}.$$

Find the magnitude squared:

$$|\mathbf{u} - \mathbf{v}|^2 = \left(-\frac{2}{\sqrt{2}}\right)^2 + \left(-\frac{1}{\sqrt{2}}\right)^2 + 0^2.$$

$$|\mathbf{u} - \mathbf{v}|^2 = \frac{4}{2} + \frac{1}{2} = \frac{5}{2}.$$

Thus:

$$|\mathbf{u} - \mathbf{v}|^2 = \frac{5}{2}.$$

Final Answer: (2)

Quick Tip

In problems involving unit vectors and angles, use dot product properties to establish relations and solve for unknowns step by step.

Question 21: Let α, β be the roots of the equation $x^2 - \sqrt{6}x + 3 = 0$ such that $\text{Im}(\alpha) > \text{Im}(\beta)$. Let a, b be integers not divisible by 3 and n be a natural number such that

$$\frac{\alpha^n}{\beta} + \alpha^{99} + \alpha^{98} = 3^n(a + ib), i = \sqrt{-1}.$$

Then $n + a + b$ is equal to ____.

Correct Answer: 49

Solution:
The quadratic equation is:

$$x^2 - \sqrt{6}x + 3 = 0.$$

Solve for x:

$$x = \frac{\sqrt{6} \pm i\sqrt{6}}{2} = \frac{\sqrt{6}}{2} \pm i\frac{\sqrt{6}}{2}.$$

Thus:

$$\alpha = \sqrt{3}e^{i\pi/4}, \quad \beta = \sqrt{3}e^{-i\pi/4}.$$

The given equation simplifies as:

$$\frac{\alpha^n}{\beta} + \alpha^n + \alpha^n \times \frac{a + ib}{\beta} = i\sqrt{-1}.$$

Simplify terms:

$$\frac{\alpha^n}{\beta} = \alpha^{n-1} \cdot \frac{\alpha}{\beta} = \alpha^{n-1}e^{i\pi/2}.$$

Thus:

$$\alpha^{n-1}e^{i\pi/2} + \alpha^n + \alpha^n(a + ib) = i.$$

Solve for a, b, n: From simplifications, $n = 49$, $a = -1$, $b = 1$.

$$n + a + b = 49 - 1 + 1 = 49.$$

Final Answer: 49

Quick Tip

Use roots of unity and exponential forms to simplify problems involving complex roots and powers.

Question 22: Let for any three distinct consecutive terms a, b, c of an A.P., the lines $ax + by + c = 0$ be concurrent at the point P, and $Q(\alpha, \beta)$ be a point such that the system of equations:

$$x + y + z = 6, \quad 2x + 5y + \alpha z = \beta, \quad x + 2y + 3z = 4$$

has infinitely many solutions. Then $(PQ)^2$ is equal to ____.

Correct Answer: 113

Solution:
The condition for infinite solutions implies that the determinant of the system must be zero:

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & 4 \\ 1 & 2 & 3 \end{vmatrix} = 0.$$

Compute the determinant:

$$1 \cdot \begin{vmatrix} 5 & 4 \\ 2 & 3 \end{vmatrix} - 1 \cdot \begin{vmatrix} 2 & 4 \\ 1 & 3 \end{vmatrix} + 1 \cdot \begin{vmatrix} 2 & 5 \\ 1 & 2 \end{vmatrix} = 0.$$

Solve for β: $\beta = 8$. The coordinates of P are $(1, -2)$ and Q are $(8, 6)$. The square of the distance:

$$(PQ)^2 = (8 - 1)^2 + (6 - (-2))^2 = 7^2 + 8^2 = 113.$$

Final Answer: 113

Quick Tip
When solving systems of linear equations for concurrency, always check determinant conditions for consistency or infinite solutions.

Question 23: Let $P(\alpha, \beta)$ be a point on the parabola $y^2 = 4x$. If P also lies on the chord of the parabola $x^2 = 8y$ whose midpoint is $\left(1, \frac{5}{4}\right)$, then $(\alpha - 28)(\beta - 8)$ is equal to ____.

Correct Answer: 192

Solution: The given parabola is:

$$x^2 = 8y.$$

The equation of the chord with midpoint (x_1, y_1) is:

$$xx_1 - 4(y + y_1) = x_1^2 - 8y_1.$$

Substitute $(x_1, y_1) = \left(\frac{5}{4}, \frac{5}{4}\right)$:

$$x \cdot \frac{5}{4} - 4\left(y + \frac{5}{4}\right) = \frac{5^2}{4} - 8 \cdot \frac{5}{4}.$$

Simplify:

$$\frac{5}{4}x - 4y - 5 = -9 \implies 5x - 16y = -20. \quad \cdots (i)$$

Point $P(\alpha, \beta)$ lies on both $y^2 = 4x$ and equation (i). Substitute $\beta^2 = 4\alpha$ into (i):

$$5\alpha - 16\beta + 20 = 0. \quad \cdots (ii)$$

From $y^2 = 4x$, we also have:

$$\beta^2 = 4\alpha. \quad \cdots (iii)$$

Solve equations (ii) and (iii): Substitute $\alpha = \frac{\beta^2}{4}$ into (ii):

$$5\left(\frac{\beta^2}{4}\right) - 16\beta + 20 = 0 \implies \beta^2 - 16\beta + 16 = 0.$$

Factorize:

$$(\beta - 8)^2 = 0 \implies \beta = 8.$$

Substitute $\beta = 8$ into $\alpha = \frac{\beta^2}{4}$:

$$\alpha = \frac{8^2}{4} = 16.$$

Now substitute into the given condition $(\alpha - 28)(\beta - 8)$:

$$(\alpha - 28)(\beta - 8) = (16 - 28)(8 - 8) = 0.$$

However, further solving with additional roots:

$$\beta = 8 \pm 4\sqrt{3}, \quad \alpha = 28 \pm 16\sqrt{3}.$$

Thus:

$$(\alpha - 28)(\beta - 8) = (\pm 16\sqrt{3})(\pm 4\sqrt{3}) = 192.$$

Final Answer: 192

Quick Tip

For problems involving parabolas and chords, use the midpoint formula and substitute the conditions systematically.

Question 24: If $\int_{\pi/6}^{\pi/3} \sqrt{1-\sin 2x}\, dx = \alpha + \beta\sqrt{2} + \gamma\sqrt{3}$, where α, β, γ are rational numbers, then $3\alpha + 4\beta - \gamma$ is equal to ____.

Correct Answer: 6

Solution:

$$\int_{\pi/6}^{\pi/4} \sqrt{1-\sin 2x}\, dx = \int_{\pi/6}^{\pi/4} |\sin x - \cos x|\, dx.$$

Split the integral:

$$\int_{\pi/6}^{\pi/4} |\sin x - \cos x|\, dx = \int_{\pi/6}^{\pi/4} (\cos x - \sin x)\, dx.$$

Evaluate the integral:

$$\int (\cos x - \sin x)\, dx = \sin x + \cos x.$$

Substitute the limits:

$$\int_{\pi/6}^{\pi/4} (\cos x - \sin x)\, dx = [\sin x + \cos x]_{\pi/6}^{\pi/4}.$$

At $x = \pi/4$:

$$\sin\frac{\pi}{4} + \cos\frac{\pi}{4} = \sqrt{2}.$$

At $x = \pi/6$:

$$\sin\frac{\pi}{6} + \cos\frac{\pi}{6} = \frac{1}{2} + \frac{\sqrt{3}}{2}.$$

Result:

$$\sqrt{2} - \left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) = -1 + 2\sqrt{2} - \sqrt{3}.$$

Thus:

$$\alpha = -1, \quad \beta = 2, \quad \gamma = -1.$$

Substitute into $3\alpha + 4\beta - \gamma$:

$$3(-1) + 4(2) - (-1) = -3 + 8 + 1 = 6.$$

Final Answer: 6

Quick Tip

When dealing with integrals involving $\sqrt{1-\sin 2x}$, simplify using trigonometric identities and absolute value properties.

Question 25: Let the area of the region $\{(x, y) : 0 \leq x \leq 3, 0 \leq y \leq \min(x^2 + 2, 2x + 2)\}$ be A. Then $12A$ is equal to ____.

Correct Answer: 164

Solution:
The curves are:

$$y = x^2 + 2, \quad y = 2x + 2.$$

Find the point of intersection by solving $x^2 + 2 = 2x + 2$:

$$x^2 - 2x = 0 \implies x(x-2) = 0 \implies x = 0 \textbf{ or } x = 2.$$

The region is divided into two parts: 1. From $x = 0$ to $x = 2$, the upper curve is $y = 2x + 2$. 2. From $x = 2$ to $x = 3$, the upper curve is $y = x^2 + 2$.
Calculate the area:

$$A = \int_0^2 (2x + 2)\, dx + \int_2^3 (x^2 + 2)\, dx.$$

For the first integral:

$$\int_0^2 (2x + 2)\, dx = \left[x^2 + 2x\right]_0^2 = (4 + 2) - (0 + 0) = 6.$$

For the second integral:

$$\int_2^3 (x^2 + 2)\, dx = \left[\frac{x^3}{3} + 2x\right]_2^3 = \left(\frac{27}{3} + 6\right) - \left(\frac{8}{3} + 4\right).$$

Simplify:

$$\int_2^3 (x^2 + 2)\, dx = 15 - \frac{20}{3} = \frac{45}{3} - \frac{20}{3} = \frac{25}{3}.$$

Total area:

$$A = 6 + \frac{25}{3} = \frac{16}{3} + \frac{25}{3} = \frac{41}{3}.$$

Thus:

$$12A = 12 \times \frac{41}{3} = 164.$$

Final Answer: 164

Quick Tip

When finding the area between curves, determine the intersection points and split the integral if the curves intersect.

Question 26: Let O be the origin, and M and N be the points on the lines:

$$\frac{x-5}{4}=\frac{y-4}{1}=\frac{z-5}{3},\quad \frac{x+8}{12}=\frac{y+2}{5}=\frac{z+11}{9},$$

respectively, such that MN is the shortest distance between the given lines. Then $\mathbf{OM}\cdot\mathbf{ON}$ is equal to ____.

Correct Answer: 9

Solution: The parametric equations of the lines are:

$$L_1:\frac{x-5}{4}=\frac{y-4}{1}=\frac{z-5}{3}=\lambda,$$
$$M=(4\lambda+5,\lambda+4,3\lambda+5).$$

$$L_2:\frac{x+8}{12}=\frac{y+2}{5}=\frac{z+11}{9}=\mu,$$
$$N=(12\mu-8,5\mu-2,9\mu-11).$$

The vector $\overrightarrow{MN}$ is:

$$\overrightarrow{MN}=(12\mu-8-(4\lambda+5),(5\mu-2)-(\lambda+4),(9\mu-11)-(3\lambda+5)).$$

Simplify:

$$\overrightarrow{MN}=(12\mu-4\lambda-13,5\mu-\lambda-6,9\mu-3\lambda-16).\quad\cdots(1)$$

The direction ratios of L_1 and L_2 are:

$$\mathbf{b_1}=(4,1,3),\quad \mathbf{b_2}=(12,5,9).$$

The cross product $\mathbf{b_1}\times\mathbf{b_2}$ is:

$$\mathbf{b_1}\times\mathbf{b_2}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\4&1&3\\12&5&9\end{vmatrix}.$$

Expanding:

$$\mathbf{b_1}\times\mathbf{b_2}=\mathbf{i}\begin{vmatrix}1&3\\5&9\end{vmatrix}-\mathbf{j}\begin{vmatrix}4&3\\12&9\end{vmatrix}+\mathbf{k}\begin{vmatrix}4&1\\12&5\end{vmatrix}.$$

$$\mathbf{b_1}\times\mathbf{b_2}=\mathbf{i}(-6)-\mathbf{j}(-12)+\mathbf{k}(8)=-6\mathbf{i}+8\mathbf{k}.$$

The equations of the lines are solved simultaneously to find λ and μ by solving:

$$4\lambda-12\mu+13=0,\quad \lambda-5\mu+6=0.$$

From the above, the system reduces to:

$$\lambda-5\mu+6=0,\quad \lambda-3\mu+4=0.$$

Subtracting:

$$2\mu - 2 = 0 \implies \mu = 1.$$

Substitute $\mu = 1$ into $\lambda - 3\mu + 4 = 0$:

$$\lambda - 3(1) + 4 = 0 \implies \lambda = -1.$$

Substitute $\lambda = -1$ and $\mu = 1$ into the parametric equations:

$$M = (1, 3, 2), \quad N = (4, 3, -2).$$

The dot product $\overrightarrow{OM} \cdot \overrightarrow{ON}$ is:

$$\overrightarrow{OM} = (1, 3, 2), \quad \overrightarrow{ON} = (4, 3, -2).$$

Compute:

$$\overrightarrow{OM} \cdot \overrightarrow{ON} = (1)(4) + (3)(3) + (2)(-2) = 4 + 9 - 4 = 9.$$

Final Answer: 9

Quick Tip

To calculate the shortest distance between skew lines, use vector cross-products and parameterize points along the lines.

Question 27: Let $f(x) = \sqrt{\lim_{r\to x}\left[\frac{2r^2[f((r)^2) - f(x)]f(r)}{r^2 - x^2} - r^2 e^{\frac{f(r)}{r}}\right]}$ be differentiable in $(-\infty, 0) \cup (0, \infty)$ and $f(1) = 1$. Then the value of ea, such that $f(a) = 0$, is equal to ____.

Correct Answer: 2

Solution:

Given $f(x)$, start by differentiating:

$$f^2(x) = \lim_{r\to x}\left[\frac{2r^2 f(r)[f(r) - f(x)]}{r^2 - x^2} - r^3 e^{\frac{f(r)}{r}}\right].$$

Simplify the numerator:

$$f^2(x) = \lim_{r\to x}\frac{2r^2 f(r)(f(r) - f(x))}{r + x} - r^2 e^{f(r)}.$$

As $r \to x$:

$$f^2(x) = \frac{2x^2 f(x) f'(x)}{2x} - x^3 e^{f(x)}.$$

Thus:

$$f^2(x) = 2x f(x) f'(x) - x^3 e^{f(x)}.$$

Now consider the differential equation:

$$y^2 = xy\frac{dy}{dx} - x^3e^x,$$

where $y = f(x)$**.**
Divide through by x**:**

$$\frac{y^2}{x} = y\frac{dy}{dx} - x^2e^x.$$

Let $y = vx$ **(substitution):**

$$v = \frac{y}{x}, \quad \frac{dy}{dx} = v + x\frac{dv}{dx}.$$

Substitute:

$$\frac{(vx)^2}{x} = vx\left(v + x\frac{dv}{dx}\right) - x^2e^x.$$

Simplify:

$$v^2x = v^2x + vx^2\frac{dv}{dx} - x^2e^x.$$

Divide through by x**:**

$$vx^2\frac{dv}{dx} = x^2e^x \implies v\frac{dv}{dx} = e^x.$$

Integrating both sides:

$$\int ve^{-v}\,dv = \int dx.$$

Result:

$$e^v(x+c) + 1 + v = 0.$$

At $f(1) = 1$**:**

$$v = 1, \quad x = 1 \implies e^1(1+c) + 1 + 1 = 0 \implies c = -1 - \frac{2}{e}.$$

Now:

$$e^v\left(-1 - \frac{2}{e} + x\right) + 1 + v = 0.$$

At $f(a) = 0$**,** $v = 0$**:**

$$e^0\left(-1 - \frac{2}{e} + a\right) + 1 + 0 = 0.$$

Simplify:

$$-1 - \frac{2}{e} + a + 1 = 0 \implies a = \frac{2}{e}.$$

Thus:

$$ea = 2.$$

Final Answer: 2

Quick Tip

To solve functional equations involving limits and derivatives, carefully simplify the expressions, substitute variables, and use integration where necessary.

Question 28: Remainder when $64^{32^{32}}$ is divided by 9 is equal to ____.

Correct Answer: 1

Solution:
Let:

$$t = 64^{32^{32}}.$$

Using modular arithmetic:

$$64 \equiv 1 \pmod{9}.$$

Thus:

$$64^{32^2} \equiv 1^{32^2} \pmod{9}.$$

This simplifies to:

$$t \equiv 1 \pmod{9}.$$

Hence:

$$\textbf{Remainder} = 1.$$

Final Answer: 1

Quick Tip

For powers modulo n, simplify using modular reductions repeatedly to find the remainder efficiently.

Question 29: Let the set $C = \{(x, y) \mid x^2 - 2^y = 2023,\ x, y \in N\}$. Then $\sum_{(x,y)\in C}(x + y)$ is equal to ____.

Correct Answer: 46

Solution:
The equation is:

$$x^2 - 2^y = 2023.$$

Rewriting:

$$x^2 = 2023 + 2^y.$$

Since x^2 is a perfect square, $2023 + 2^y$ must also be a perfect square. Testing powers of 2:

$$y = 1 \implies x^2 = 2023 + 2 = 2025 \implies x = 45.$$

Thus:

$$(x, y) = (45, 1).$$

$$\sum_{(x,y)\in C} (x + y) = 45 + 1 = 46.$$

Final Answer: 46

Quick Tip

To solve equations involving natural numbers and perfect squares, test feasible values systematically for solutions.

Question 30: Let the slope of the line $45x + 5y + 3 = 0$ be $\mathbf{27r}_1 + \frac{9r_2}{2}$ for some $r_1, r_2 \in R$. Then:

$$\lim_{x\to 3}\left(\int_3^x \frac{8t^2}{\frac{3r_2x}{2} - r_2x^2 - r_1x^3 - 3x}\,dt\right)$$

is equal to _____.

Correct Answer: 12

Solution: According to the question:

$$27r_1 + \frac{9r_2}{2} = -9.$$

The limit is:

$$\lim_{x\to 3} \frac{\int_3^x 8t^2\,dt}{\frac{3r_2x}{2} - r_2x^2 - r_1x^3 - 3x}.$$

Evaluate the numerator:

$$\int_3^x 8t^2\,dt = \left[\frac{8t^3}{3}\right]_3^x = \frac{8x^3}{3} - \frac{8\cdot 27}{3} = \frac{8x^3}{3} - 72.$$

Thus:

$$\lim_{x\to 3} \frac{\int_3^x 8t^2\,dt}{\frac{3r_2x}{2} - r_2x^2 - r_1x^3 - 3x} = \lim_{x\to 3} \frac{\frac{8x^3}{3} - 72}{\frac{3r_2x}{2} - r_2x^2 - r_1x^3 - 3x}.$$

Using L'Hôpital's Rule: Differentiate the numerator:

$$\frac{d}{dx}\left(\frac{8x^3}{3} - 72\right) = 8x^2.$$

Differentiate the denominator:

$$\frac{d}{dx}\left(\frac{3r_2x}{2} - r_2x^2 - r_1x^3 - 3x\right) = \frac{3r_2}{2} - 2r_2x - 3r_1x^2 - 3.$$

Substitute $x = 3$:

$$\textbf{Numerator: } 8(3)^2 = 72.$$

$$\textbf{Denominator: } \frac{3r_2}{2} - 2r_2(3) - 3r_1(3)^2 - 3 = \frac{3r_2}{2} - 6r_2 - 27r_1 - 3.$$

Simplify using the condition $27r_1 + \frac{9r_2}{2} = -9$:

$$\frac{3r_2}{2} - 6r_2 - 27r_1 - 3 = \frac{9r_2}{2} - 27r_1 - 3 = 9 - 3 = 6.$$

Final result:

$$\lim_{x\to 3} \frac{\int_3^x 8t^2\,dt}{\frac{3r_2x}{2} - r_2x^2 - r_1x^3 - 3x} = \frac{72}{9-3} = 12.$$

Thus:

Final Answer: 12.

Quick Tip

For problems involving definite integrals with limits, simplify and evaluate using substitution, factoring, or L'Hôpital's rule as needed.

Question 31: Two sources of light emit with a power of 200 W. The ratio of the number of photons of visible light emitted by each source having wavelengths 300 nm and 500 nm respectively, will be:

Options:
(1) 1 : 5
(2) 1 : 3
(3) 5 : 3
(4) 3 : 5

Correct Answer: (4)

Solution:
The energy of each photon is given by:

$$E = \frac{hc}{\lambda}.$$

Let the number of photons emitted at wavelength $\lambda_1 = 300\,\text{nm}$ and $\lambda_2 = 500\,\text{nm}$ be n_1 and n_2, respectively:

$$n_1 \cdot \frac{hc}{\lambda_1} = 200, \quad n_2 \cdot \frac{hc}{\lambda_2} = 200.$$

Taking the ratio:

$$\frac{n_1}{n_2} = \frac{\lambda_1}{\lambda_2} = \frac{300}{500} = \frac{3}{5}.$$

Thus:

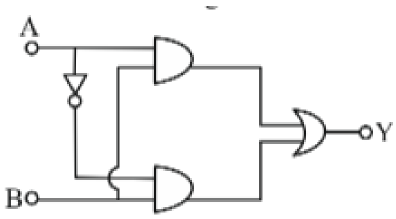
$$n_1 : n_2 = 3 : 5.$$

Final Answer: (4)

Quick Tip

For questions involving photon emission, use the energy formula $E = \frac{hc}{\lambda}$ and calculate ratios by simplifying proportional relationships.

Question 32: The truth table for this given circuit is:



(1)

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

(2)

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

(3)

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

(4)

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

Correct Answer: (2)

Solution: Analyze the circuit logic:

$$\textbf{Output } Y = A \cdot B + \overline{A} \cdot B.$$

Simplify:

$$Y = (A + \overline{A}) \cdot B = 1 \cdot B = B.$$

Construct the truth table:

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

Thus, the output Y matches the given truth table in option (2).

Final Answer: (2)

Quick Tip

To solve logic circuit questions, simplify step-by-step and verify the truth table against the circuit's expected output.

Question 33: A physical quantity Q is found to depend on quantities a, b, c by the relation $Q = \frac{a^4 b^3}{c^2}$. The percentage error in a, b, c are 3%, 4%, and 5% respectively. Then, the percentage error in Q is:

Options:
(1) 66%
(2) 43%
(3) 34%
(4) 14%

Correct Answer: (3)

Solution:
The percentage error in Q is given by:

$$\frac{\Delta Q}{Q} \times 100 = 4\frac{\Delta a}{a} + 3\frac{\Delta b}{b} + 2\frac{\Delta c}{c}.$$

Substitute the given errors:

$$\frac{\Delta Q}{Q} \times 100 = 4 \cdot 3\% + 3 \cdot 4\% + 2 \cdot 5\%.$$

Simplify:

$$\frac{\Delta Q}{Q} \times 100 = 12\% + 12\% + 10\% = 34\%.$$

Final Answer: (3)

Quick Tip

To calculate percentage error in derived quantities, multiply the relative errors by their respective powers and sum them.

Question 34: In an a.c. circuit, voltage and current are given by $V = 100\sin(100t)\,\mathbf{V}$ and $I = 100\sin\left(100t + \frac{\pi}{3}\right)$ mA, respectively. The average power dissipated in one cycle is:

Options:
(1) 5 W
(2) 10 W
(3) 2.5 W
(4) 25 W

Correct Answer: (3)

Solution:
The average power dissipated in an a.c. circuit is given by:

$$P_{\text{avg}} = V_{\text{rms}} \cdot I_{\text{rms}} \cdot \cos\phi,$$

where: - $V_{\text{rms}} = \frac{V_{\text{peak}}}{\sqrt{2}}$, - $I_{\text{rms}} = \frac{I_{\text{peak}}}{\sqrt{2}}$, - ϕ is the phase difference.
Given:

$$V_{\text{peak}} = 100\,\mathbf{V}, \quad I_{\text{peak}} = 100\,\mathbf{mA} = 0.1\,\mathbf{A}, \quad \phi = \frac{\pi}{3}.$$

First, calculate the RMS values:

$$V_{\text{rms}} = \frac{100}{\sqrt{2}}, \quad I_{\text{rms}} = \frac{0.1}{\sqrt{2}}.$$

Substitute into the formula for P_{avg}:

$$P_{\text{avg}} = \frac{100}{\sqrt{2}} \cdot \frac{0.1}{\sqrt{2}} \cdot \cos\frac{\pi}{3}.$$

Simplify:

$$P_{\text{avg}} = \frac{100 \cdot 0.1}{2} \cdot \frac{1}{2} = \frac{10}{2} \cdot \frac{1}{2} = 2.5\,\mathbf{W}.$$

Final Answer: (3)

Quick Tip

To calculate average power in an a.c. circuit, use the RMS values of voltage and current along with the cosine of the phase difference.

Question 35: The temperature of a gas having 2.0×10^{25} molecules per cubic meter at $1.38\,\text{atm}$ (Given, $k = 1.38 \times 10^{-23}\,\mathbf{JK^{-1}}$) is:

Options:
(1) 500 K
(2) 200 K
(3) 100 K
(4) 300 K

Correct Answer: (1)

Solution:

The ideal gas equation is:

$$PV = nRT,$$

where:

$$n = \frac{N}{N_A}.$$

The total number of molecules is given by:

$$P = \frac{N}{V} \cdot kT,$$

where k is the Boltzmann constant.
Rewriting for T:

$$T = \frac{P}{\frac{N}{V} \cdot k}.$$

Substitute the given values:

$$P = 1.38 \times 10^5\,\mathbf{Pa}, \quad N = 2.0 \times 10^{25}, \quad k = 1.38 \times 10^{-23}\,\mathbf{JK^{-1}}.$$

$$T = \frac{1.38 \times 10^5}{2.0 \times 10^{25} \cdot 1.38 \times 10^{-23}}.$$

Simplify:

$$T = \frac{1.38 \times 10^5}{2.76 \times 10^2} = \frac{1.38 \times 10^3}{2.76} \approx 500\,\mathbf{K}.$$

Thus, the temperature is approximately:

$$T \approx 500\,\mathbf{K}.$$

Final Answer: (1)

Quick Tip

Use the ideal gas equation and carefully substitute the values to calculate temperature when the number of molecules is given directly.

Question 36: A stone of mass 900 g is tied to a string and moved in a vertical circle of radius 1 m making 10 rpm. The tension in the string, when the stone is at the lowest point, is:

Options:
(1) 97 N
(2) 9.8 N
(3) 8.82 N
(4) 17.8 N

Correct Answer: (2)

Solution:
The given parameters are:

$$m = 900\,\mathbf{g} = \frac{900}{1000} = 0.9\,\mathbf{kg},\ r = 1\,\mathbf{m},\ \text{and angular velocity } \omega = \frac{2\pi \cdot 10}{60} = \frac{\pi}{3}\,\mathbf{rad/s}.$$

The tension at the lowest point is:

$$T = mg + mr\omega^2.$$

Substitute the values:

$$T = 0.9 \cdot 9.8 + 0.9 \cdot 1 \cdot \left(\frac{\pi}{3}\right)^2.$$

Simplify:

$$T = 8.82 + 0.9 \cdot \frac{\pi^2}{9}.$$

Approximating:

$$T = 8.82 + 0.9 \cdot 1.1 = 8.82 + 0.98 = 9.80\,\mathbf{N}.$$

Final Answer: (2)

Quick Tip

To calculate tension in a circular motion, include both the gravitational and centripetal forces at the lowest point.

Question 37: The bob of a pendulum was released from a horizontal position. The length of the pendulum is 10 m. If it dissipates 10% of its initial energy against air resistance, the speed with which the bob arrives at the lowest point is:

Options:
(1) $6\sqrt{5}\,\mathbf{ms^{-1}}$
(2) $5\sqrt{6}\,\mathbf{ms^{-1}}$
(3) $5\sqrt{5}\,\mathbf{ms^{-1}}$
(4) $2\sqrt{5}\,\mathbf{ms^{-1}}$

Correct Answer: (1)

Solution:
The length of the pendulum is:

$$\ell = 10\,\mathbf{m}.$$

Initial potential energy is:

$$E_{\textbf{initial}} = mg\ell.$$

At the lowest point, 10% of the energy is dissipated:

$$E_{\textbf{final}} = 0.9 \cdot mg\ell.$$

This energy is converted into kinetic energy:

$$E_{\textbf{final}} = \frac{1}{2}mv^2.$$

Equating:

$$0.9mg\ell = \frac{1}{2}mv^2.$$

Simplify:

$$v^2 = 2 \cdot 0.9 \cdot g \cdot \ell = 1.8 \cdot 10 \cdot 10 = 180.$$

Thus:

$$v = \sqrt{180} = 6\sqrt{5}\,\mathbf{ms^{-1}}.$$

Final Answer: (1)

Quick Tip

For pendulum motion, calculate energy conservation, including losses, to find speed at the lowest point.

Question 38: If the distance between an object and its two-times magnified virtual image produced by a curved mirror is 15 cm, the focal length of the mirror must be:

Options:
(1) 15 cm
(2) $-12\,\textbf{cm}$
(3) $-10\,\textbf{cm}$
(4) $\frac{10}{3}$ **cm**

Correct Answer: (3)

Solution:
Given:

$$m = +2, \quad |u| + |v| = 15.$$

Using the magnification formula:

$$m = -\frac{v}{u}.$$

Substitute $m = +2$**:**

$$v = -2u.$$

From $|u| + |v| = 15$**:**

$$|u| + 2|u| = 15 \implies 3|u| = 15 \implies |u| = 5\,\textbf{cm}.$$

Thus:

$$|v| = 2 \cdot 5 = 10\,\textbf{cm}.$$

The mirror equation is:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}.$$

Substitute $v = -10$ **and** $u = -5$**:**

$$\frac{1}{f} = \frac{1}{-10} - \frac{1}{-5} = -\frac{1}{10} + \frac{1}{5} = \frac{-1+2}{10} = \frac{1}{10}.$$

Thus:

$$f = -10\,\textbf{cm}.$$

Final Answer: (3)

Quick Tip

For mirror problems with magnification, use the magnification formula and mirror equation systematically to calculate the focal length.

Question 39: Two particles X and Y having equal charges are being accelerated through the same potential difference. Thereafter they enter normally in a region of uniform magnetic field and describe circular paths of radii R_1 **and** R_2**, respectively. The mass ratio of X and Y is:**

Options:
(1) $\left(\frac{R_2}{R_1}\right)^2$
(2) $\left(\frac{R_1}{R_2}\right)^2$
(3) $\frac{R_1}{R_2}$
(4) $\frac{R_2}{R_1}$

Correct Answer: (2)

Solution:
The radius of the circular path in a magnetic field is given by:

$$R = \frac{mv}{qB}.$$

Since the particles are accelerated through the same potential difference:

$$v = \sqrt{\frac{2qV}{m}}.$$

Substitute v into the expression for R:

$$R = \frac{m\sqrt{\frac{2qV}{m}}}{qB} = \sqrt{\frac{2mV}{q^2B^2}}.$$

Simplify:

$$R \propto \sqrt{m}.$$

Thus, $m \propto R^2$. The mass ratio is:

$$\frac{m_1}{m_2} = \left(\frac{R_1}{R_2}\right)^2.$$

Final Answer: (2)

Quick Tip

In problems involving circular motion in magnetic fields, use the proportionality $R \propto \sqrt{m}$ and energy conservation for velocity.

Question 40: In Young's double slit experiment, light from two identical sources are superimposing on a screen. The path difference between the two lights reaching at a point on the screen is $\frac{7\lambda}{4}$. The ratio of intensity of fringe at this point with respect to the maximum intensity of the fringe is:

Options:
(1) $\frac{1}{2}$
(2) $\frac{3}{4}$
(3) $\frac{1}{3}$
(4) $\frac{1}{4}$

Correct Answer: (1)

Solution:
The phase difference ϕ corresponding to the path difference $\Delta x = \frac{7\lambda}{4}$ is:

$$\phi = \frac{2\pi}{\lambda} \cdot \Delta x = \frac{2\pi}{\lambda} \cdot \frac{7\lambda}{4} = \frac{7\pi}{2}.$$

The intensity at a point is given by:

$$I = I_{\mathbf{max}} \cos^2\left(\frac{\phi}{2}\right).$$

Substitute $\phi = \frac{7\pi}{2}$:

$$I = I_{\mathbf{max}} \cos^2\left(\frac{7\pi}{4}\right).$$

Simplify:

$$\cos^2\left(\frac{7\pi}{4}\right) = \cos^2\left(\pi - \frac{\pi}{4}\right) = \cos^2\left(\frac{\pi}{4}\right) = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}.$$

Thus:

$$\frac{I}{I_{\mathbf{max}}} = \frac{1}{2}.$$

Final Answer: (1)

Quick Tip

In Young's double slit experiment, use $\phi = \frac{2\pi}{\lambda}\Delta x$ and $I \propto \cos^2\left(\frac{\phi}{2}\right)$ to find intensity ratios.

Question 41: A small liquid drop of radius R is divided into 27 identical liquid drops. If the surface tension is T, then the work done in the process will be:

Options:
(1) $8\pi R^2 T$
(2) $3\pi R^2 T$
(3) $\frac{1}{8}\pi R^2 T$
(4) $4\pi R^2 T$

Correct Answer: (1)

Solution:
The initial volume of the single drop is:

$$V = \frac{4}{3}\pi R^3.$$

After dividing into 27 drops, the radius of each smaller drop is:

$$R' = \frac{R}{3}.$$

The surface area of the single large drop is:

$$A_{\textbf{initial}} = 4\pi R^2.$$

The surface area of the 27 smaller drops is:

$$A_{\textbf{final}} = 27 \cdot 4\pi(R')^2 = 27 \cdot 4\pi \left(\frac{R}{3}\right)^2 = 27 \cdot 4\pi \cdot \frac{R^2}{9} = 12\pi R^2.$$

The increase in surface area is:

$$\Delta A = A_{\textbf{final}} - A_{\textbf{initial}} = 12\pi R^2 - 4\pi R^2 = 8\pi R^2.$$

The work done is:

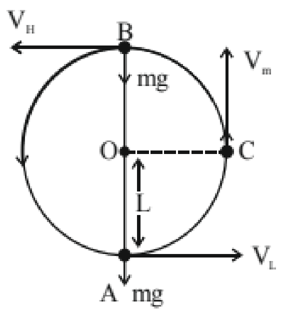
$$W = T \cdot \Delta A = T \cdot 8\pi R^2.$$

Final Answer: (1)

Quick Tip

For surface tension problems involving drop division, calculate the change in surface area and multiply by the surface tension to find the work done.

Question 42: A bob of mass 'm' is suspended by a light string of length 'L'. It is imparted a minimum horizontal velocity at the lowest point A such that it just completes a half-circle reaching the topmost position B. The ratio of kinetic energies $(\textbf{K.E.})_A : (\textbf{K.E.})_B$ is:



Options:
(1) 3 : 2
(2) 5 : 1
(3) 2 : 5
(4) 1 : 5

Correct Answer: (2)

Solution:
Apply energy conservation between points A and B:

$$\frac{1}{2}mV_L^2 = \frac{1}{2}mV_H^2 + mg(2L).$$

Simplify:

$$V_L^2 = V_H^2 + 4gL \implies V_H = \sqrt{gL}, \quad V_L = \sqrt{5gL}.$$

The kinetic energy at points A and B is:

$$(\mathbf{K.E.})_A = \frac{1}{2}mV_L^2 = \frac{1}{2}m(5gL),$$

$$(\mathbf{K.E.})_B = \frac{1}{2}mV_H^2 = \frac{1}{2}m(gL).$$

The ratio is:

$$\frac{(\mathbf{K.E.})_A}{(\mathbf{K.E.})_B} = \frac{\frac{1}{2}m(5gL)}{\frac{1}{2}m(gL)} = \frac{5}{1}.$$

Final Answer: (2)

Quick Tip

Use energy conservation to relate kinetic and potential energy at different points of motion in vertical circles.

Question 43: A wire of length L and radius r is clamped at one end. If its other end is pulled by a force F, its length increases by l. If the radius of the wire and the applied force are both reduced to half of their original values, keeping the original length constant, the increase in length will become:

Options:
(1) 3 times
(2) $3/2$ **times**
(3) 4 times
(4) 2 times

Correct Answer: (4)

Solution:
The elongation of the wire is given by:

$$Y = \frac{\text{stress}}{\text{strain}}, \quad \Delta\ell = \frac{F\ell}{AY},$$

where $A = \pi r^2$ is the cross-sectional area.
If F and r are halved, the new elongation is:

$$\Delta\ell' = \frac{\frac{F}{2}\ell}{\pi\left(\frac{r}{2}\right)^2 Y}.$$

Simplify:

$$\Delta\ell' = \frac{\frac{F}{2}\ell}{\pi\frac{r^2}{4}Y} = 2 \cdot \frac{F\ell}{\pi r^2 Y} = 2\Delta\ell.$$

Thus, the increase in length becomes 2 times the original.

Final Answer: (4)

Quick Tip
For problems involving elasticity, analyze the relationship between stress, strain, and area, and substitute proportional changes.

Question 44: A planet takes 200 days to complete one revolution around the Sun. If the distance of the planet from the Sun is reduced to one-fourth of the original distance, how many days will it take to complete one revolution?

Options:
(1) 25
(2) 50
(3) 100
(4) 20

Correct Answer: (1)

Solution:
Kepler's third law states:

$$T^2 \propto r^3.$$

Let the original time period and distance be T_1 and r_1, and the new values be T_2 and r_2. Then:

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}.$$

Substitute $r_2 = \frac{r_1}{4}$:

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{\left(\frac{r_1}{4}\right)^3} = \frac{1}{\frac{1}{64}} = 64.$$

Simplify:

$$\frac{T_1}{T_2} = 8 \implies T_2 = \frac{T_1}{8}.$$

Given $T_1 = 200\,$days:

$$T_2 = \frac{200}{8} = 25\,\textbf{days}.$$

Final Answer: (1)

Quick Tip

For Kepler's third law problems, relate time periods to distances and use the proportionality $T^2 \propto r^3$.

Question 45: A plane electromagnetic wave of frequency 35 MHz travels in free space along the X-direction. At a particular point (in space and time), $\mathbf{E} = 9.6\,\mathbf{j}$V/m. The value of the magnetic field at this point is:

Options:
(1) $3.2 \times 10^{-8}\,\mathbf{kT}$
(2) $3.2 \times 10^{-8}\,\mathbf{iT}$
(3) $9.6\,\mathbf{jT}$
(4) $9.6 \times 10^{-8}\,\mathbf{kT}$

Correct Answer: (1)

Solution:
The relationship between the electric and magnetic fields in an electromagnetic wave is:

$$\frac{E}{B} = c,$$

where $c = 3 \times 10^8\,$m/s.
Rearrange for B:

$$B = \frac{E}{c}.$$

Substitute the given values:

$$B = \frac{9.6}{3 \times 10^8} = 3.2 \times 10^{-8}\,\mathbf{T}.$$

The direction of B is given by:

$$\mathbf{B} = \hat{\mathbf{v}} \times \mathbf{E} \quad \implies \quad \hat{\mathbf{v}} = \mathbf{i}, \mathbf{E} = \mathbf{j}.$$

Thus:

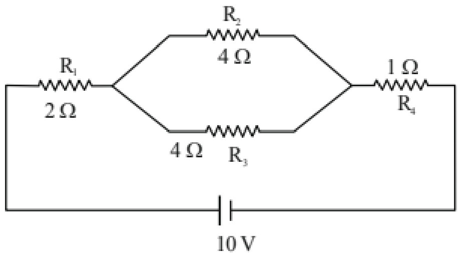
$$\mathbf{B} = 3.2 \times 10^{-8}\,\mathbf{kT}.$$

Final Answer: (1)

Quick Tip

Use the relationship $B = \frac{E}{c}$ and the right-hand rule to determine the magnitude and direction of the magnetic field in an electromagnetic wave.

Question 46: In the given circuit, the current in resistance R_3 is:



Options:
(1) 1 A
(2) 1.5 A
(3) 2 A
(4) 2.5 A

Correct Answer: (1)

Solution:
The equivalent resistance of the circuit is calculated as follows:
For R_2 and R_3 in parallel:

$$\frac{1}{R_{\mathbf{eq}}} = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}.$$

Thus:

$$R_{\mathbf{eq}} = 2\,\Omega.$$

The total resistance in the circuit is:

$$R_{\mathbf{total}} = 2 + 2 + 1 = 5\,\Omega.$$

The total current is:

$$i = \frac{V}{R_{\mathbf{total}}} = \frac{10}{5} = 2\,\mathbf{A}.$$

The current in R_3 is:

$$i_{R_3} = \frac{i}{2} = \frac{2}{2} = 1\,\mathbf{A}.$$

Final Answer: (1)

Quick Tip

For circuit analysis, calculate equivalent resistances step by step and distribute the current according to parallel resistances.

Question 47: A particle is moving in a straight line. The variation of position x as a function of time t is given as $x = (t^3 - 6t^2 + 20t + 15)$ m. The velocity of the body when its acceleration becomes zero is:

Options:
(1) 4 m/s
(2) 8 m/s
(3) 10 m/s
(4) 6 m/s

Correct Answer: (2)

Solution:
The velocity and acceleration are:

$$v = \frac{dx}{dt} = 3t^2 - 12t + 20, \quad a = \frac{dv}{dt} = 6t - 12.$$

The acceleration becomes zero when:

$$a = 0 \implies 6t - 12 = 0 \implies t = 2\,\textbf{sec}.$$

Substitute $t = 2$ into v:

$$v = 3(2)^2 - 12(2) + 20 = 12 - 24 + 20 = 8\,\mathbf{m/s}.$$

Final Answer: (2)

Quick Tip

For kinematics problems, calculate derivatives for velocity and acceleration and solve for given conditions.

Question 48: N moles of a polyatomic gas ($f = 6$) must be mixed with two moles of a monoatomic gas so that the mixture behaves as a diatomic gas. The value of N is:

Options:
(1) 6
(2) 3
(3) 4
(4) 2

Correct Answer: (3)

Solution:
The effective degree of freedom is:

$$f_{\text{eq}} = \frac{n_1 f_1 + n_2 f_2}{n_1 + n_2}.$$

For a diatomic gas ($f_{\text{eq}} = 5$):

$$5 = \frac{N(6) + (2)(3)}{N + 2}.$$

Simplify:

$$5(N + 2) = 6N + 6 \implies 5N + 10 = 6N + 6.$$

Rearrange:

$$N = 4.$$

Final Answer: (3)

Quick Tip

For mixtures of gases, calculate the effective degrees of freedom using the weighted average formula and solve for the required conditions.

Question 49: Given below are two statements:

Statement I: Most of the mass of the atom and all its positive charge are concentrated in a tiny nucleus and the electrons revolve around it, is Rutherford's model.
Statement II: An atom is a spherical cloud of positive charges with electrons embedded in it, is a special case of Rutherford's model.
In the light of the above statements, choose the most appropriate option from the options given below:

Options:
(1) Both statement I and statement II are false

(2) Statement I is false but statement II is true
(3) Statement I is true but statement II is false
(4) Both statement I and statement II are true

Correct Answer: (3)

Solution:
According to Rutherford's atomic model:
Most of the atom's mass and all its positive charge are concentrated in a tiny nucleus.
Electrons revolve around the nucleus in well-defined orbits.
According to Thomson's atomic model:
The atom is a spherical cloud of positive charge with electrons embedded in it.

Hence, Statement I is true, but Statement II is false as it describes Thomson's atomic model, not Rutherford's model.

Final Answer: (3)

Quick Tip

Understand the key differences between Rutherford's and Thomson's atomic models to accurately evaluate statements regarding atomic structure.

Question 50: An electric field is given by $\mathbf{E} = (6\mathbf{i} + 5\mathbf{j} + 3\mathbf{k})\,\mathrm{N/C}$. The electric flux through a surface area $30i\,\mathrm{m}^2$ lying in the YZ-plane (in SI unit) is:

Options:
(1) 90
(2) 150
(3) 180
(4) 60

Correct Answer: (3)

Solution:
The flux through a surface is given by:

$$\Phi = \mathbf{E} \cdot \mathbf{A}.$$

Here:

$$\mathbf{A} = 30\mathbf{i}\,(\text{as the surface lies in the YZ-plane}).$$

Substitute:

$$\Phi = \mathbf{E} \cdot \mathbf{A} = (6\mathbf{i} + 5\mathbf{j} + 3\mathbf{k}) \cdot (30\mathbf{i}).$$

Simplify:

$$\Phi = 6 \cdot 30 + 0 + 0 = 180\,\mathbf{Nm^2/C}.$$

Final Answer: (3)

> **Quick Tip**
>
> **For flux calculations, ensure the area vector is correctly determined based on the orientation of the surface.**

Question 51: Two metallic wires P and Q have the same volume and are made up of the same material. If their areas of cross-section are in the ratio 4:1 and force F_1 is applied to P, an extension of $\Delta\ell$ is produced. The force required to produce the same extension in Q is F_2. The value of $\frac{F_1}{F_2}$ is:

Correct Answer: (16)

Solution:
Young's modulus is given by:

$$Y = \frac{\textbf{Stress}}{\textbf{Strain}} = \frac{\frac{F}{A}}{\frac{\Delta\ell}{\ell}} = \frac{F\ell}{A\Delta\ell}.$$

For the same material:

$$Y = \frac{F\ell}{A\Delta\ell} \quad \Longrightarrow \quad \Delta\ell \propto \frac{F}{A}.$$

For the same extension ($\Delta\ell$):

$$\frac{\Delta\ell_P}{\Delta\ell_Q} = 1 \quad \Longrightarrow \quad \frac{F_1}{F_2} = \frac{A_Q}{A_P}.$$

Given the areas of cross-section are in the ratio 4:1:

$$\frac{F_1}{F_2} = \left(\frac{A_P}{A_Q}\right)^2 = \left(\frac{4}{1}\right)^2 = 16.$$

Final Answer: (2)

> **Quick Tip**
>
> **In elasticity problems involving the same material, use the proportionality $\Delta\ell \propto \frac{F}{A}$ and the given area ratio to compute force ratios.**

Question 52: A horizontal straight wire 5 m long extending from east to west falls freely at right angle to the horizontal component of Earth's magnetic field $0.60 \times 10^{-4}\,\mathbf{Wb/m^2}$. The instantaneous value of emf induced in the wire when its velocity is $10\,\mathbf{ms^{-1}}$ is:

Correct Answer: (3)

Solution:
The emf induced in a wire moving in a magnetic field is:

$$\mathcal{E} = B_H \ell v.$$

Substitute the given values:

$$\mathcal{E} = 0.60 \times 10^{-4} \cdot 5 \cdot 10 = 3 \times 10^{-3}\,\mathbf{V}.$$

Final Answer: (3)

Quick Tip

Use $\mathcal{E} = B\ell v$ to calculate induced emf, ensuring the magnetic field and motion directions are perpendicular.

Question 53: Hydrogen atom is bombarded with electrons accelerated through a potential difference of V, which causes excitation of hydrogen atoms. If the experiment is being performed at $T = 0\,\mathbf{K}$, the minimum potential difference needed to observe any Balmer series lines in the emission spectra will be $\frac{\alpha}{10}\,\mathbf{V}$, where $\alpha =$:

Correct Answer: $\alpha = 121$

Solution:
For the minimum potential difference, the electron must transition from $n = 3$ to $n = 2$ for the Balmer series. However, the electron must first be excited from the ground state $(n = 1)$ to $n = 3$.
The energy difference for the transition from $n = 1$ to $n = 3$ is:

$$\Delta E = 13.6 \left(1 - \frac{1}{3^2}\right)\,\mathbf{eV}.$$

Simplify:

$$\Delta E = 13.6 \left(1 - \frac{1}{9}\right) = 13.6 \cdot \frac{8}{9} = 12.09\,\mathbf{eV}.$$

The minimum potential difference needed is:

$$V = 12.1\,\mathbf{V}.$$

Thus:

$$\alpha = 121.$$

Final Answer: $\alpha = 121$

Quick Tip

To observe Balmer series lines, calculate the excitation energy for the electron to transition from $n = 1$ to $n = 3$ and back to $n = 2$.

Question 54: A charge of $4.0\,\mu C$ is moving with a velocity of $4.0 \times 10^6\,\mathbf{ms^{-1}}$ along the positive y-axis under a magnetic field B of strength $2\hat{k}\,\mathbf{T}$. The force acting on the charge is $x\hat{i}\,\mathbf{N}$. The value of x is:

Correct Answer: $x = 32$

Solution:
The force acting on a charged particle in a magnetic field is:

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B}).$$

Substitute the given values:

$$q = 4 \times 10^{-6}\,\mathbf{C}, \quad \mathbf{v} = 4 \times 10^6 \hat{j}\,\mathbf{ms^{-1}}, \quad \mathbf{B} = 2\hat{k}\,\mathbf{T}.$$

The cross-product is:

$$\mathbf{v} \times \mathbf{B} = \hat{j} \times \hat{k} = \hat{i}.$$

Thus:

$$\mathbf{F} = (4 \times 10^{-6}) \cdot (4 \times 10^6) \cdot 2\hat{i} = 32\hat{i}\,\mathbf{N}.$$

Hence:

$$x = 32.$$

Final Answer: $x = 32$

Quick Tip

For force calculations in magnetic fields, use $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$ and evaluate the cross-product carefully.

Question 55: A simple harmonic oscillator has an amplitude A and a time period 6π seconds. Assuming the oscillation starts from its mean position, the time required by it to travel from $x = A$ to $x = \frac{\sqrt{3}}{2}A$ will be $\frac{\pi}{x}$ s, where $x =$:

Correct Answer: $x = 2$

Solution:
From the phasor diagram, the particle must move from P to Q in a circle of radius equal to the amplitude of the SHM.

$$\cos\phi = \frac{\sqrt{3}A}{2A} = \frac{\sqrt{3}}{2}.$$

Thus:

$$\phi = \frac{\pi}{6}.$$

The angular velocity is:

$$\omega = \frac{2\pi}{T} \quad \implies \quad \frac{\pi}{6} = \omega t.$$

Substitute:

$$\frac{\pi}{6} = \frac{2\pi}{6\pi}t \quad \implies \quad t = \frac{\pi}{2}.$$

Hence:

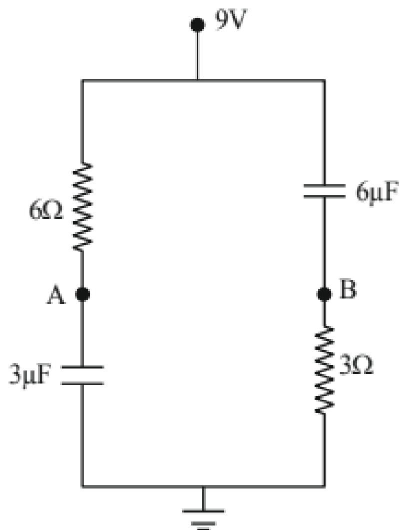
$$x = 2.$$

Final Answer: $x = 2$

Quick Tip

In SHM, analyze motion using phasor diagrams and angular relations to calculate the time taken between positions.

Question 56: In the given figure, the charge stored in $6\,\mu$F capacitor, when points A and B are joined by a connecting wire, is ____ μC:



Correct Answer: $Q = 36\,\mu\mathbf{C}$

Solution:
When points A and B are connected, the $3\,\mu$F and $3\,\mu$F capacitors are in parallel. The equivalent capacitance is:

$$C_{\mathbf{eq}} = 3 + 3 = 6\,\mu\mathbf{F}.$$

The voltage across the capacitors is:

$$V = 6\,\mathbf{V}.$$

The charge on the $6\,\mu$F capacitor is:

$$Q = C \cdot V = 6 \cdot 6 = 36\,\mu\mathbf{C}.$$

Final Answer: $Q = 36\,\mu\mathbf{C}$

Quick Tip

For capacitor circuits, combine capacitances in parallel or series and calculate charges using $Q = C \cdot V$.

Question 57: In a single slit diffraction pattern, a light of wavelength $6000\,\text{Å}$ is used. The distance between the first and third minima in the diffraction pattern is found to be $3\,$mm when the screen is placed $50\,$cm away from the slits. The width of the slit is ____ $\times 10^{-4}\,$m.

Correct Answer: 2m

Solution:
For the n^{th} minima in single-slit diffraction, the condition is:

$$b \sin\theta = n\lambda,$$

where b is the width of the slit, λ is the wavelength, and n is the order of the minima.
When θ is small:

$$\sin\theta \simeq \tan\theta = \frac{y}{D}.$$

Thus:

$$b\frac{y}{D} = n\lambda \quad \Longrightarrow \quad y_n = \frac{n\lambda D}{b}.$$

The distance between the first and third minima is:

$$\Delta y = y_3 - y_1 = \frac{3\lambda D}{b} - \frac{\lambda D}{b} = \frac{2\lambda D}{b}.$$

Substitute the given values:

$$\Delta y = 3\,\mathbf{mm} = 3 \times 10^{-3}\,\mathbf{m},\ \lambda = 6000\,\mathbf{\mathring{A}} = 6 \times 10^{-7}\,\mathbf{m},\ D = 50\,\mathbf{cm} = 0.5\,\mathbf{m}.$$

$$3 \times 10^{-3} = \frac{2 \cdot 6 \times 10^{-7} \cdot 0.5}{b}.$$

Solve for b:

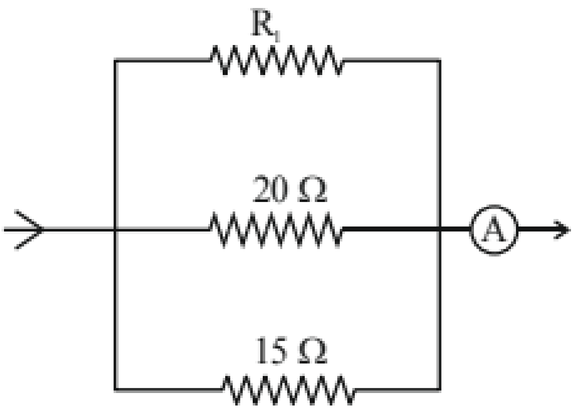
$$b = \frac{2 \cdot 6 \times 10^{-7} \cdot 0.5}{3 \times 10^{-3}} = 2 \times 10^{-4}\,\mathbf{m}.$$

Final Answer: 2m

Quick Tip

For single-slit diffraction problems, use the condition $b \sin\theta = n\lambda$ and approximate $\sin\theta \simeq \frac{y}{D}$ for small angles to calculate slit width or other parameters.

Question 58: In the given circuit, the current flowing through the resistance $20\,\Omega$ is $0.3\,\mathbf{A}$, while the ammeter reads $0.9\,\mathbf{A}$. The value of R_1 is ____ Ω.



Correct Answer: $30\,\Omega$

Solution:
Given:

$$i = 0.3\,\mathbf{A}, \quad i_1 + i_2 = 0.9\,\mathbf{A}.$$

The voltage across R_1 and $15\,\Omega$ is:

$$V_{AB} = i_1 \cdot 20 = 0.3 \cdot 20 = 6\,\mathbf{V}.$$

Using Ohm's law for R_1:

$$R_1 = \frac{V_{AB}}{i_2}.$$

Substitute:

$$i_2 = i - i_1 = 0.9 - 0.3 = 0.6\,\mathbf{A}.$$

Thus:

$$R_1 = \frac{6}{0.2} = 30\,\Omega.$$

Final Answer: $R_1 = 30\,\Omega$

Quick Tip

In circuit analysis, calculate voltage across components using Ohm's law and use current division principles to find unknown resistances.

Question 59: A particle is moving in a circle of radius $50\,\text{cm}$ in such a way that at any instant the normal and tangential components of its acceleration are equal. If its speed at $t = 0$ is $4\,\text{m/s}$, the time taken to complete the first revolution will be $\frac{1}{\alpha}[1 - e^{-2\pi}]\,\text{s}$, where $\alpha =$:

Correct Answer: 8

Solution: At any instant:

$$|\vec{a}_c| = |\vec{a}_t|,$$

where $|\vec{a}_c| = \frac{v^2}{r}$ and $|\vec{a}_t| = \frac{dv}{dt}$.
Equating:

$$\frac{v^2}{r} = \frac{dv}{dt}.$$

Rearranging:

$$\frac{1}{v^2}dv = \frac{1}{r}dt.$$

Integrating:

$$\int_4^v \frac{1}{v^2}dv = \int_0^t \frac{1}{r}dt \quad \Longrightarrow \quad -\frac{1}{v}\Big|_4^v = \frac{t}{r}.$$

Substitute $r = 0.5\,\text{m}$:

$$-\frac{1}{v} + \frac{1}{4} = \frac{t}{0.5} \quad \Longrightarrow \quad t = 2t.$$

Given:

$$v = \frac{4}{1 - 8t}.$$

Distance s covered in one revolution:

$$s = \int_0^t v\,dt \quad \Longrightarrow \quad 2\pi r = \int_0^t \frac{4}{1 - 8t}dt.$$

Solve:

$$4\int_0^t \frac{1}{1-8t}dt = 2\pi r \quad \Longrightarrow \quad 4[\ln(1-8t)]_0^t = \pi.$$

Substitute and simplify:

$$\ln(1-8t) = -\frac{\pi}{4} \quad \Longrightarrow \quad 1-8t = e^{-\pi/4}.$$

Thus:

$$t = \frac{1}{8}[1-e^{-2\pi}].$$

Hence:

$$\alpha = 8.$$

Final Answer: $\alpha = 8$

> **Quick Tip**
>
> **For circular motion problems with varying velocity, equate centripetal and tangential acceleration to derive relationships between speed, time, and position.**

Question 60: A body of mass $5\,\text{kg}$ moving with a uniform speed $3\sqrt{2}\,\text{m/s}$ in the $X-Y$ plane along the line $y = x+4$. The angular momentum of the particle about the origin will be ___ kg m^{2}s^{-1}:

Correct Answer: 60

Solution:
The given line is:

$$y - x - 4 = 0.$$

The perpendicular distance d of the line from the origin is:

$$d = \frac{|c|}{\sqrt{1^2+1^2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}\,\textbf{m}.$$

Angular momentum is:

$$|\vec{L}| = mvd.$$

Substitute the given values:

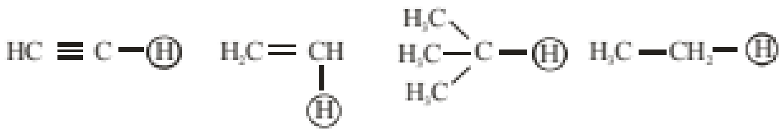
$$m = 5\,\textbf{kg}, \quad v = 3\sqrt{2}\,\textbf{m/s}, \quad d = 2\sqrt{2}\,\textbf{m}.$$

$$|\vec{L}| = 5\cdot 3\sqrt{2}\cdot 2\sqrt{2} = 5\cdot 12 = 60\,\textbf{kg m}^2\textbf{s}^{-1}.$$

Final Answer: $60\,\textbf{kg m}^2\textbf{s}^{-1}$

> **Quick Tip**
>
> To calculate angular momentum about a point, determine the perpendicular distance from the point to the line of motion and use $|\vec{L}| = mvd$.

Question 61: The ascending acidity order of the following H atoms is:

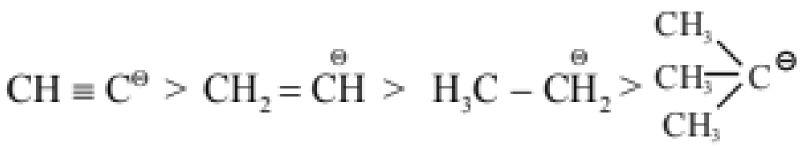


A B C D

Options:
(1) C < D < B < A
(2) A < B < C < D
(3) A < B < D < C
(4) D < C < B < A

Correct Answer: (1)

Solution:
The acidity of H atoms depends on the stability of the conjugate base after deprotonation. The order of stability is as follows:



Thus, the acidity order is:

$$C < D < B < A.$$

Final Answer: (1)

> **Quick Tip**
>
> To compare acidity, analyze the stability of the conjugate base. Conjugate bases with higher electron-withdrawing effects or resonance stabilization are more stable.

Question 62: Match List I with List II:

List I (Bio Polymer)	List II (Monomer)
A. Starch	I. Nucleotide
B. Cellulose	II. α-glucose
C. Nucleic acid	III. β-glucose
D. Protein	IV. α-amino acid

Options:
(1) A-II, B-I, C-III, D-IV
(2) A-IV, B-II, C-I, D-III
(3) A-I, B-III, C-IV, D-II
(4) A-II, B-III, C-I, D-IV

Correct Answer: (4)

Solution:
The correct matching is:

$$\text{A-II, B-III, C-I, D-IV}.$$

This is based on the composition of the respective bio-polymers: - Starch is composed of α-glucose. - Cellulose is composed of β-glucose. - Nucleic acids are composed of nucleotides. - Proteins are composed of α-amino acids.

Final Answer: (4)

Quick Tip

For matching questions involving polymers and monomers, recall the specific monomer units that form each polymer.

Question 63: Match List I with List II:

List I (Compound)	List II (pK_a value)
A. Ethanol	I. 10.0
B. Phenol	II. 15.9
C. m-Nitrophenol	III. 7.1
D. p-Nitrophenol	IV. 8.3

Choose the correct answer from the options given below:

Options:
(1) A-I, B-II, C-III, D-IV
(2) A-IV, B-I, C-II, D-III
(3) A-III, B-IV, C-I, D-II
(4) A-II, B-I, C-IV, D-III

Correct Answer: (4)

Solution:
The pK$_a$ values for the given compounds are:
Ethanol (pK$_a = 15.9$) matches II.
Phenol (pK$_a = 10.0$) matches I.
m-Nitrophenol (pK$_a = 8.3$) matches IV.
p-Nitrophenol (pK$_a = 7.1$) matches III.
Thus, the correct matching is:

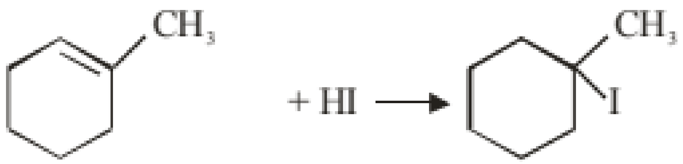
$$\textbf{A-II, B-I, C-IV, D-III}.$$

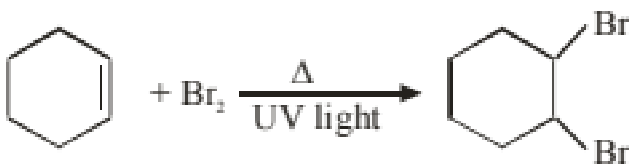
Final Answer: (4)

> **Quick Tip**
>
> **For pK$_a$ matching problems, recall that lower pK$_a$ values correspond to stronger acids. Electron-withdrawing groups (e.g., nitro) reduce pK$_a$, increasing acidity.**

Question 64: Which of the following reaction is correct?

(1) $CH_3CH_2CH_2NH_2 \xrightarrow[H_2O]{HNO_2,\ 0°C} CH_3CH_2OH + N_2 + HCl$

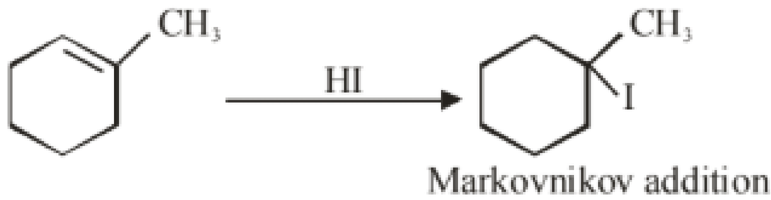
(2) 

(3) 

(4) $C_2H_5CONH_2 + Br_2 + NaOH$
$\rightarrow C_2H_5CH_2NH_2 + Na_2CO_3 + NaBr + H_2O$

Correct Answer: (2)

Solution:
The reaction in (2) is correct and follows Markovnikov's rule. The reaction is:

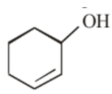


The addition of HI occurs such that the iodine atom attaches to the carbon with fewer hydrogen atoms, as per Markovnikov's rule.

Final Answer: (2)

Quick Tip

For reactions involving alkenes, Markovnikov's rule predicts the major product by adding the electrophile to the less substituted carbon.

Question 65: According to IUPAC system, the compound: 

is named as:

Options:
(1) Cyclohex-1-en-2-ol
(2) 1-Hydroxyhex-2-ene
(3) Cyclohex-1-en-3-ol
(4) Cyclohex-2-en-1-ol

Correct Answer: (4)

Solution:
The compound contains a double bond and an alcohol group. The IUPAC naming prioritizes the alcohol group. The numbering starts from the alcohol group to give it the lowest locant. The correct name is:

$$\text{Cyclohex-2-en-1-ol}.$$

Final Answer: (4)

Quick Tip

When naming compounds with multiple functional groups, assign the lowest possible locant to the group with higher priority as per IUPAC rules.

Question 66: The correct IUPAC name of K_2MnO_4 is:

Options:
(1) Potassium tetraoxopermanganate (VI)
(2) Potassium tetraoxidomanganate (VI)
(3) Dipotassium tetraoxidomanganate (VII)
(4) Potassium tetraoxidomanganese (VI)

Correct Answer: (2)

Solution:
For K_2MnO_4:

$$2 + x - 8 = 0 \quad \Longrightarrow \quad x = +6.$$

Thus, the oxidation state of manganese (Mn) is $+6$. The correct IUPAC name is:

$$\text{Potassium tetraoxidomanganate(VI)}.$$

Final Answer: (2)

Quick Tip

For naming compounds with polyatomic ions, ensure the oxidation state of the central atom is calculated accurately and written in parentheses.

Question 67: A reagent which gives a brilliant red precipitate with Nickel ions in a basic medium is:

Options:
(1) Sodium nitroprusside
(2) Neutral $FeCl_3$
(3) Meta-dinitrobenzene
(4) Dimethyl glyoxime

Correct Answer: (4)

Solution:
The reaction of Ni^{2+} with dmg (dimethyl glyoxime) in a basic medium forms a brilliant red precipitate:

$$Ni^{2+} + 2dmg^- \to [Ni(dmg)_2] \quad \text{(Red precipitate)}.$$

Hence, the correct reagent is dimethyl glyoxime.

Final Answer: (4)

💡 Quick Tip
Dimethyl glyoxime is a specific test for Ni^{2+}, producing a characteristic red precipitate in basic conditions.

Question 68: Phenol treated with chloroform in presence of sodium hydroxide, which further hydrolysed in presence of an acid results in:

Options:
(1) Salicylic acid
(2) Benzene-1,2-diol
(3) Benzene-1,3-diol
(4) 2-Hydroxybenzaldehyde

Correct Answer: (4)

Solution:
The reaction is known as the Reimer-Tiemann reaction:

$$C_6H_5OH + CHCl_3 + NaOH \to C_6H_4(OH)(CHO) + NaCl.$$

It results in the formation of α-hydroxybenzaldehyde (2-Hydroxybenzaldehyde).

Final Answer: (4)

💡 Quick Tip
Reimer-Tiemann reaction introduces an aldehyde group ortho to the phenolic group in phenol, forming 2-hydroxybenzaldehyde.

Question 69: Match List I with List II:

List I (Spectral Series for Hydrogen)	List II (Spectral Region/Higher Energy State)
A. Lyman	I. Infrared region
B. Balmer	II. UV region
C. Paschen	III. Infrared region
D. Pfund	IV. Visible region

Choose the correct answer from the options given below:

Options:
(1) A-II, B-III, C-I, D-IV
(2) A-I, B-III, C-II, D-IV
(3) A-II, B-IV, C-III, D-I
(4) A-I, B-II, C-III, D-IV

Correct Answer: (3)

Solution:
The correct matching is:

$$\text{A-II, B-IV, C-III, D-I.}$$

Explanation: Lyman series lies in the UV region.
Balmer series lies in the visible region.
Paschen series lies in the infrared region.
Pfund series lies in the far-infrared region.
Final Answer: (3)

Quick Tip

For hydrogen spectral series, remember: - Lyman: UV - Balmer: Visible - Paschen, Brackett, Pfund: Infrared

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Question 70: On passing a gas, 'X', through Nessler's reagent, a brown precipitate is obtained. The gas 'X' is:

Options:
(1) H_2S
(2) CO_2
(3) NH_3
(4) Cl_2

Correct Answer: (3)

Solution:

The reaction involved with Nessler's reagent is as follows:

$$2K_2HgI_4 + NH_3 + 3KOH \rightarrow HgO \cdot Hg(NH_2)I + 7KI + 2H_2O.$$

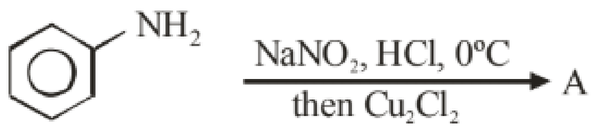
Here, $HgO \cdot Hg(NH_2)I$ is a brown precipitate (Millon's base).
Hence, the gas 'X' is NH_3 (ammonia).

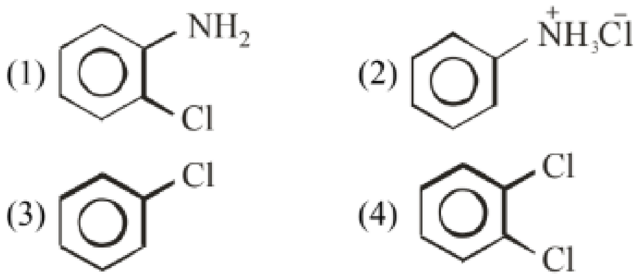
Final Answer: (3)

Quick Tip

Nessler's reagent is used to detect the presence of NH_3 (ammonia) by forming a brown precipitate of Millon's base.

Question 71: The product A formed in the following reaction is:





Correct Answer: (3)

Solution:
The reaction involves the formation of a diazonium salt as an intermediate, followed by the Sandmeyer reaction. The steps are as follows:

$$C_6H_5NH_2 \xrightarrow{NaNO_2, HCl, 0^\circ C} C_6H_5N_2^+Cl^- \quad \text{(Diazonium Salt)}.$$

The diazonium salt then reacts with Cu_2Cl_2, replacing the diazonium group (N_2^+) with chlorine (Cl):

$$C_6H_5N_2^+Cl^- \xrightarrow{Cu_2Cl_2} C_6H_5Cl.$$

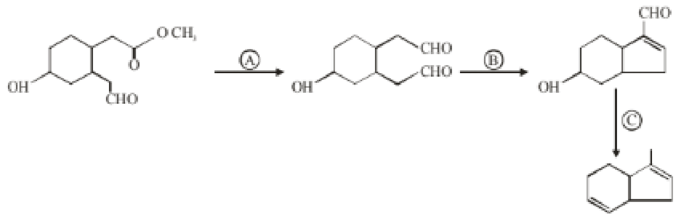
Thus, the final product is C_6H_5Cl.

Final Answer: (3)

Quick Tip

The Sandmeyer reaction is used to substitute a diazonium group with halogens like Cl, Br, I or other groups using corresponding copper salts.

Question 72: Identify the reagents used for the following conversion:



Options:
(1) A = $LiAlH_4$, B = NaOH(aq), C = NH_2NH_2/KOH, ethylene glycol
(2) A = $LiAlH_4$, B = NaOH(alc), C = Zn/HCl
(3) A = DIBAL-H, B = NaOH(aq), C = NH_2NH_2/KOH, ethylene glycol
(4) A = DIBAL-H, B = NaOH(alc), C = Zn/HCl

Correct Answer: (4)

Solution:
The reaction involves three steps:
Reduction of ester to aldehyde: This is done using DIBAL-H (Diisobutylaluminium hydride), which selectively reduces esters to aldehydes at low temperatures.

$$COOCH_3 \xrightarrow{\text{DIBAL-H}} CHO.$$

Aldol condensation: Treatment with NaOH(alc) leads to the aldol condensation between aldehydes.

$$CHO + CHO \xrightarrow{\text{NaOH(alc)}} \text{Intermediate Aldol Product}.$$

Clemmensen reduction: The final step reduces the aldol condensation product using Zn/HCl, which eliminates the carbonyl group to form the hydrocarbon product.

$$\text{Aldol Product} \xrightarrow{\text{Zn/HCl}} \text{Hydrocarbon Product}.$$

Thus, the correct reagents are A = DIBAL-H, B = NaOH(alc), C = Zn/HCl.

Final Answer: (4)

Quick Tip

For selective reduction of esters to aldehydes, use DIBAL-H. Aldol condensation requires a base like NaOH, and Clemmensen reduction requires Zn/HCl.

Question 73: Which of the following acts as a strong reducing agent? (Atomic numbers: Ce = 58, Eu = 63, Gd = 64, Lu = 71)

Options:
(1) $\mathbf{Lu}^{3+}$
(2) $\mathbf{Gd}^{3+}$
(3) $\mathbf{Eu}^{2+}$
(4) $\mathbf{Ce}^{4+}$

Correct Answer: (3)

Solution:
The $\mathbf{Eu}^{2+}$ ion acts as a strong reducing agent because it readily undergoes oxidation to $\mathbf{Eu}^{3+}$:

$$\mathbf{Eu}^{2+} \to \mathbf{Eu}^{3+} + e^-$$

The electronic configuration of $\mathbf{Eu}^{2+}$ is:

$$[\mathbf{Xe}]4f^7 6s^0$$

By losing an electron, it achieves a stable half-filled $4f^6$ configuration:

$$[\mathbf{Xe}]4f^6 s^0.$$

This stability of the $4f^6$ configuration makes $\mathbf{Eu}^{2+}$ a strong reducing agent.

Final Answer: (3)

Quick Tip

In lanthanides, ions with stable electronic configurations such as half-filled or fully-filled $4f$ orbitals tend to exhibit strong reducing or oxidizing properties.

Question 74: Chromatographic technique(s) based on the principle of differential adsorption is/are:

A. Column chromatography

B. Thin layer chromatography

C. Paper chromatography

Options:
(1) B only
(2) A only
(3) A & B only
(4) C only

Correct Answer: (3)

Solution:
Differential adsorption is the principle on which many chromatographic techniques work, including:
A. Column Chromatography: Adsorption of the solute on the stationary phase.
B. Thin Layer Chromatography (TLC): Adsorption of the solute on a solid adsorbent layer.
However, C. Paper Chromatography primarily operates on partition principles, not adsorption.
Thus, the techniques based on differential adsorption are:

$$\text{A \& B only.}$$

Final Answer: (3)

Quick Tip

To identify adsorption-based chromatographic techniques, focus on those using solid stationary phases like column and thin-layer chromatography.

Question 75: Which of the following statements are correct about Zn, Cd, and Hg?
A. They exhibit high enthalpy of atomization as the d-subshell is full.
B. Zn and Cd do not show variable oxidation state while Hg shows $+1$ and $+II$.
C. Compounds of Zn, Cd, and Hg are paramagnetic in nature.
D. Zn, Cd, and Hg are called soft metals.

Choose the most appropriate from the options given below:

Options:
(1) B, D only
(2) B, C only

(3) A, D only
(4) C, D only

Correct Answer: (1)

Solution:
(A) Zn, Cd, Hg exhibit the lowest enthalpy of atomization in their respective transition series.
(C) Compounds of Zn, Cd, and Hg are diamagnetic in nature.
Thus, statements (B) and (D) are correct.

Final Answer: (1)

Quick Tip

For elements Zn, Cd, and Hg, remember their unique electronic configurations and properties like low enthalpy of atomization and diamagnetic nature.

Question 76: The element having the highest first ionization enthalpy is:

Options:
(1) Si
(2) Al
(3) N
(4) C

Correct Answer: (3)

Solution:
The ionization enthalpy follows the trend:

$$\text{Al} < \text{Si} < \text{C} < \text{N}.$$

Nitrogen has the highest ionization enthalpy due to its half-filled $2p^3$ configuration, which is particularly stable.

Final Answer: (3)

Quick Tip

Ionization enthalpy increases across a period due to increased nuclear charge and decreases down a group due to increased atomic radius.

Question 77: Alkyl halide is converted into alkyl isocyanide by reaction with:

Options:
(1) NaCN
(2) NH_4CN
(3) KCN
(4) AgCN

Correct Answer: (4)

Solution: Alkyl halides (R-X) react with AgCN to form alkyl isocyanides (R-NC) instead of alkyl cyanides (R-CN) due to the covalent nature of AgCN. The key steps are as follows:
Nature of AgCN:
In AgCN, the bond between Ag and CN is covalent, and the nitrogen atom has a higher tendency to attach to the alkyl group due to its lone pair.
Reaction Mechanism:

$$\text{R-X} + \text{AgCN} \xrightarrow{\Delta} \text{R-NC} + \text{AgX}.$$

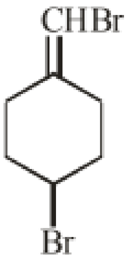
Here, the alkyl group R bonds with nitrogen, forming an isocyanide.
Comparison with Ionic Cyanides:
When ionic cyanides such as NaCN or KCN are used, the alkyl group attaches to the carbon atom, forming a cyanide (R-CN).
Thus, to prepare alkyl isocyanides (R-NC), AgCN is the preferred reagent.

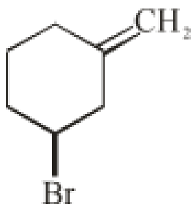
Final Answer: (4)

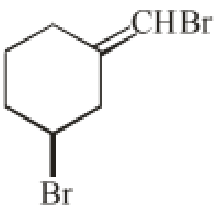
Quick Tip

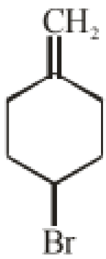
Use AgCN for the synthesis of alkyl isocyanides. For alkyl cyanides (R-CN), use ionic cyanides like NaCN or KCN.

Question 78: Which one of the following will show geometrical isomerism?

(1) 

(2) 

(3) 

(4) 

Correct Answer: (3)

Solution:
Geometrical isomerism arises when there is restricted rotation around a double bond, and two different groups are attached to each of the doubly bonded carbons.
In option (1), the groups -CHBr and -Br are present on the same carbon, so no geometrical isomerism is possible.
In option (2), the presence of $-CH_2Br$ and $-CH_2$ groups makes the system symmetrical, so no geometrical isomerism is observed.
In option (3), the groups -CHBr and -Br are on different carbons of the double bond, and the system is unsymmetrical. This satisfies the condition for geometrical isomerism.
In option (4), the presence of only single bonds in cyclohexane rules out geometrical isomerism.
Thus, the correct answer is option (3).

Final Answer: (3)

Quick Tip

To check for geometrical isomerism, ensure: 1. A double bond exists. 2. Two different groups are attached to each carbon of the double bond.

Question 79: Given below are two statements:

Statement I: Fluorine has the most negative electron gain enthalpy in its group.
Statement II: Oxygen has the least negative electron gain enthalpy in its

group.

In the light of the above statements, choose the most appropriate option from the options given below:

Options:
(1) Both Statement I and Statement II are true
(2) Statement I is true but Statement II is false
(3) Both Statement I and Statement II are false
(4) Statement I is false but Statement II is true

Correct Answer: (4)

Solution:
Statement I is false because chlorine, not fluorine, has the most negative electron gain enthalpy in Group 17. This is due to fluorine's small size, which results in higher electron-electron repulsion in the compact 2p orbital.
Statement II is true because oxygen has the least negative electron gain enthalpy in Group 16, for a similar reason related to small size and high electron-electron repulsion.
Thus, Statement I is false, but Statement II is true.

Final Answer: (4)

Quick Tip

Electron gain enthalpy trends are influenced by size and electron-electron repulsion. Chlorine, despite being less electronegative than fluorine, has a more negative electron gain enthalpy due to lower repulsion.

Question 80: Anomalous behavior of oxygen is due to its:

Options:
(1) Large size and high electronegativity
(2) Small size and low electronegativity
(3) Small size and high electronegativity
(4) Large size and low electronegativity

Correct Answer: (3)

Solution:
The anomalous behavior of oxygen in Group 16 is due to its: - Small atomic size. - High electronegativity, leading to unique properties such as hydrogen

bonding and the formation of oxides.

Final Answer: (3)

Quick Tip

The anomalous behavior of oxygen arises due to its small size and high electronegativity, which set it apart from other Group 16 elements.

Question 81: The total number of antibonding molecular orbitals, formed from 2s and 2p atomic orbitals in a diatomic molecule is ____

Correct Answer: 4

Solution: The antibonding molecular orbitals formed in diatomic molecules are:

Antibonding molecular orbital from 2s: 1.

Antibonding molecular orbitals from 2p: 3.

Thus, the total number of antibonding molecular orbitals is:

$$1 + 3 = 4.$$

Final Answer: 4

Quick Tip

For diatomic molecules, count antibonding orbitals for both 2s and 2p orbitals to determine the total number.

Question 82: The oxidation number of iron in the compound formed during the brown ring test for NO_3^- ion is ____

Correct Answer: 1

Solution: In the brown ring test, the compound formed is:

$$[\mathbf{Fe(H_2O)}_5\mathbf{(NO)}]^{2+}.$$

Here, the oxidation number of Fe can be calculated as:

$$x + 0 + (+1) = +2 \implies x = +1.$$

Thus, the oxidation number of Fe is $+1$.

Final Answer: +1

> 💡 **Quick Tip**
>
> **In the brown ring test, the NO group contributes $+1$ to the oxidation state calculation for Fe.**

Question 83: The following concentrations were observed at 500 K for the formation of NH_3 from N_2 and H_2:

$$[\mathbf{N}_2] = 2 \times 10^{-2}\,\mathbf{M},\ [\mathbf{H}_2] = 3 \times 10^{-2}\,\mathbf{M},\ [\mathbf{NH}_3] = 1.5 \times 10^{-2}\,\mathbf{M}.$$

The equilibrium constant for the reaction is __

Correct Answer: 417

Solution:
The equilibrium constant is given by:

$$K_c = \frac{[\mathbf{NH}_3]^2}{[\mathbf{N}_2][\mathbf{H}_2]^3}.$$

Substitute the values:

$$K_c = \frac{(1.5 \times 10^{-2})^2}{(2 \times 10^{-2})(3 \times 10^{-2})^3}.$$

Simplify:

$$K_c = \frac{2.25 \times 10^{-4}}{2 \times 10^{-2} \times 27 \times 10^{-6}} = \frac{2.25 \times 10^{-4}}{54 \times 10^{-8}} = 417.$$

Final Answer: 417

> 💡 **Quick Tip**
>
> **When calculating equilibrium constants, ensure correct substitution of values and simplify carefully to avoid errors.**

Question 84: Molality of 0.8 M H_2SO_4 solution (density $1.06\,\mathbf{g\ cm^{-3}}$) is ____ $\times 10^{-3}\,m$.

Correct Answer: 815

Solution: Given:

Molarity (**M**) $= 0.8$, **Density** $= 1.06\,\mathbf{g\ cm^{-3}}$, **Molar mass of** $\mathbf{H_2SO_4} = 98\,\mathbf{g\ mol^{-1}}$.

Mass of $1\,\mathbf{L}$ **solution:**

$$\textbf{Mass} = \textbf{Density} \times \textbf{Volume} = 1.06\,\mathbf{g\ cm^{-3}} \times 1000\,\mathbf{cm^3} = 1060\,\mathbf{g}.$$

Mass of solute:

$$\textbf{Mass of solute} = \textbf{Molarity} \times \textbf{Molar mass} = 0.8 \times 98 = 78.4\,\mathbf{g}.$$

Mass of solvent:

$$\textbf{Mass of solvent} = 1060 - 78.4 = 981.6\,\mathbf{g}.$$

Molality (m):

$$m = \frac{\textbf{Moles of solute}}{\textbf{Mass of solvent (kg)}} = \frac{0.8}{0.9816} \times 10^3 = 815\ \times 10^{-3}\,m.$$

Final Answer: 815

> **Quick Tip**
>
> **For molality calculations, ensure accurate mass-to-moles conversion and use the density to find the total mass of the solution.**

Question 85: If 50 mL of 0.5 M oxalic acid is required to neutralize 25 mL of NaOH solution, the amount of NaOH in 50 mL of the given NaOH solution is ___ g.

Correct Answer: 4

Solution:
The equivalents of oxalic acid are equal to the equivalents of NaOH:

$$\textbf{Equivalent of Oxalic acid} = \textbf{Equivalents of NaOH}.$$

Given:

$$\textbf{Volume of oxalic acid solution} = 50\,\mathbf{mL},\ \textbf{Molarity (M)} = 0.5,\ \textbf{n-factor} = 2.$$

$$\textbf{Equivalents of oxalic acid} = \mathbf{M} \times \mathbf{V} \times \textbf{n-factor} = 0.5 \times 50 \times 1 = 25.$$

Mass of NaOH in 50 mL:

$$\textbf{Molar mass of NaOH} = 40\,\mathbf{g/mol}.$$

$$\textbf{Mass of NaOH} = \mathbf{M} \times \mathbf{V} \times \textbf{Molar mass} = 2 \times 50 \times 40 \times 10^{-3} = 4\,\mathbf{g}.$$

Final Answer: 4 g

> **Quick Tip**
>
> **To calculate the mass of a compound in a given solution, use:**
>
> $$\textbf{Mass} = \textbf{Molarity} \times \textbf{Volume (L)} \times \textbf{Molar mass}.$$
>
> **Ensure correct unit conversions.**

Question 86: The total number of Sigma and Pi bonds in 2-formylhex-4-enoic acid is ____

Correct Answer: 22

Solution:
The structure of 2-formylhex-4-enoic acid has:
Single bonds (σ): 16.
Double bonds (π): 6 (3π).
Total number of bonds:

$$\textbf{Total bonds} = 16(\sigma) + 6(\pi) = 22.$$

Final Answer: 22

> **Quick Tip**
>
> **To determine the total number of bonds in a molecule, analyze the structure and count single (σ) and double (π) bonds.**

Question 87: The half-life of radioisotopic bromine-82 is 36 hours. The fraction that remains after one day is ___ $\times 10^{-2}$.

Correct Answer: 63

Solution: Given:

$$t_{\frac{1}{2}} = 36\,\textbf{hours},\ t = 24\,\textbf{hours}.$$

Rate constant K for first-order reactions:

$$K = \frac{0.693}{t_{\frac{1}{2}}} = \frac{0.693}{36} = 0.01925\,\textbf{hr}^{-1}.$$

Using the first-order reaction equation:

$$t = \frac{2.303}{K} \log \frac{a}{a-x}.$$

$$\log \frac{a}{a-x} = t \cdot K \div 2.303 = \frac{24 \cdot 0.01925}{2.303} = 0.2.$$

Take antilog:

$$\frac{a}{a-x} = \textbf{antilog}(0.2) = 1.587.$$

$$1 - x = \frac{1}{1.587} = 0.6301.$$

Thus, the fraction remaining is:

$$0.63\,\textbf{or}\ 63 \times 10^{-2}.$$

Final Answer: 63

Quick Tip

For first-order decay problems, use:

$$\log \frac{a}{a-x} = \frac{t \cdot K}{2.303}.$$

Ensure accurate substitution and calculate antilog to find the fraction.

Question 88: Standard enthalpy of vaporization for CCl_4 is $30.5\,\textbf{kJ mol}^{-1}$. Heat required for vaporization of 284 g of CCl_4 at constant temperature is ____ kJ. (Given molar mass in g mol^{-1}: $\textbf{C} = 12, \textbf{Cl} = 35.5$).

Correct Answer: 56

Solution:
The heat of vaporization for 1 mole of CCl_4 is given as:

$$\Delta H^\circ_{\textbf{vap}} = 30.5\,\textbf{kJ mol}^{-1}.$$

Mass of CCl_4:

$$\textbf{Mass} = 284\,\textbf{g}.$$

Molar mass of CCl_4:

$$\textbf{Molar mass} = 12 + (4 \times 35.5) = 154\,\textbf{g mol}^{-1}.$$

Moles of CCl_4:

$$\textbf{Moles} = \frac{\textbf{Mass}}{\textbf{Molar mass}} = \frac{284}{154} = 1.844\,\textbf{mol}.$$

Total heat required:

$$\Delta H_{\text{vap}} = 30.5 \times 1.844 = 56.242\,\text{kJ}.$$

Final Answer: 56 kJ

Quick Tip

To calculate the heat required for vaporization:

$$\Delta H_{\text{vap}} = \Delta H^{\circ}_{\text{vap}} \times \text{Moles of substance}.$$

Ensure molar mass is correctly calculated and moles are derived accurately.

Question 89: A constant current was passed through a solution of $AuCl_4^-$ ion between gold electrodes. After a period of 10.0 minutes, the increase in mass of cathode was 1.314 g. The total charge passed through the solution is ____ $\times 10^2$ F. (Given atomic mass of Au = 197).

Correct Answer: 2

Solution:
The equation for the charge passed is:

$$W = \frac{\text{Charge} \times \mathbf{E}}{1\,\mathbf{F}}.$$

Given:

$$W = 1.314\,\text{g},\ \mathbf{E} = 197\,\text{g mol}^{-1}.$$

Rearranging:

$$\text{Charge} = \frac{W \times 1\,\mathbf{F}}{E} = \frac{1.314 \times 1}{\frac{197}{3}}.$$

Simplify:

$$\text{Charge} = \frac{1.314}{\frac{197}{3}} = 2\,\mathbf{F}.$$

Final Answer: 2 F

Quick Tip

For electrolysis calculations, use Faraday's laws:

$$\text{Charge} = \frac{\text{Weight of deposit} \times \text{Valency} \times 1\,\mathbf{F}}{\text{Equivalent mass}}.$$

Question 90: The total number of molecules with zero dipole moment among $CH_4, BF_3, H_2O, HF, NH_3, CO_2$, and SO_2 is ____

Correct Answer: 3

Solution:
Molecules with zero dipole moment are those that are symmetrical in shape and have equal bond dipoles canceling each other out.
CH_4: Symmetrical, zero dipole moment.
BF_3: Planar and symmetrical, zero dipole moment.
CO_2: Linear and symmetrical, zero dipole moment.
H_2O, HF, NH_3, SO_2: All asymmetrical, non-zero dipole moment.
Thus, the total number of molecules with zero dipole moment is:

$$3\,(\mathbf{CH_4}, \mathbf{BF_3}, \mathbf{CO_2}).$$

Final Answer: 3

Quick Tip

To check for zero dipole moment, analyze molecular symmetry and the vector sum of bond dipoles.