Paper:	B.E_B.Tech
Set Name:	Item25
Exam Date:	29 July 2022
Exam Shift:	1
Langauge:	English

Topic:	Mathematics-Section A
Item No:	1
Question ID:	**100201**
Question Type:	MCQ
Question:	Let R be a relation from the set {1, 2, 3, . . . , 60} to itself such that $R = \{(a, b) : b = pq$, where $p, q \geq 3$ are prime numbers$\}$. Then, the number of elements in R is :
A:	600
B:	660
C:	540
D:	720

Topic:	Mathematics-Section A
Item No:	2
Question ID:	**100202**
Question Type:	MCQ
Question:	If $z = 2 + 3i$, then $z^5 + (\overline{z})^5$ is equal to :
A:	244
B:	224
C:	245
D:	265

Topic:	Mathematics-Section A
Item No:	3
Question ID:	**100203**
Question Type:	MCQ

Question:	Let A and B be two 3×3 non-zero real matrices such that AB is a zero matrix. Then
A:	the system of linear equations $AX=0$ has a unique solution
B:	the system of linear equations $AX=0$ has infinitely many solutions
C:	B is an invertible matrix
D:	adj(A) is an invertible matrix

Topic:	Mathematics-Section A
Item No:	4
Question ID:	**100204**
Question Type:	MCQ
Question:	If $\frac{1}{(20-a)(40-a)}+\frac{1}{(40-a)(60-a)}+\ldots+\frac{1}{(180-a)(200-a)}=\frac{1}{256}$, then the maximum value of a is :
A:	198
B:	202
C:	212
D:	218

Topic:	Mathematics-Section A
Item No:	5
Question ID:	**100205**
Question Type:	MCQ
Question:	If $\lim\limits_{x\to0}\frac{\alpha e^{x}+\beta e^{-x}+\gamma\sin x}{x\sin^{2}x}=\frac{2}{3}$, where $\alpha, \beta, \gamma \in \mathbf{R}$, then which of the following is **NOT** correct ?
A:	$\alpha^2+\beta^2+\gamma^2=6$
B:	$\alpha\beta+\beta\gamma+\gamma\alpha+1=0$
C:	$\alpha\beta^2+\beta\gamma^2+\gamma\alpha^2+3=0$

D:	$\alpha^2-\beta^2+\gamma^2=4$

Topic:	Mathematics-Section A
Item No:	6
Question ID:	**100206**
Question Type:	MCQ
Question:	The integral $\int_0^{\frac{\pi}{2}} \frac{1}{3+2\sin x+\cos x}\,dx$ is equal to :
A:	$\tan^{-1}(2)$
B:	$\tan^{-1}(2)-\frac{\pi}{4}$
C:	$\frac{1}{2}\tan^{-1}(2)-\frac{\pi}{8}$
D:	$\frac{1}{2}$

Topic:	Mathematics-Section A
Item No:	7
Question ID:	**100207**
Question Type:	MCQ
Question:	Let the solution curve $y=y(x)$ of the differential equation $\left(1+e^{2x}\right)\left(\frac{dy}{dx}+y\right)=1$ pass through the point $\left(0,\frac{\pi}{2}\right)$. Then, $\lim\limits_{x\to\infty} e^x y(x)$ is equal to :
A:	$\frac{\pi}{4}$
B:	$\frac{3\pi}{4}$

C:	$\frac{\pi}{2}$
D:	$\frac{3\pi}{2}$

Topic:	Mathematics-Section A
Item No:	8
Question ID:	**100208**
Question Type:	MCQ
Question:	Let a line L pass through the point of intersection of the lines $bx+10y-8=0$ and $2x-3y=0$, $b \in \mathbf{R}-\left\{\frac{4}{3}\right\}$. If the line L also passes through the point (1, 1) and touches the circle $17(x^2+y^2)=16$, then the eccentricity of the ellipse $\frac{x^2}{5}+\frac{y^2}{b^2}=1$ is :
A:	$\frac{2}{\sqrt{5}}$
B:	$\sqrt{\frac{3}{5}}$
C:	$\frac{1}{\sqrt{5}}$
D:	$\sqrt{\frac{2}{5}}$

Topic:	Mathematics-Section A
Item No:	9
Question ID:	**100209**
Question Type:	MCQ

Question:	If the foot of the perpendicular from the point $A(-1,4,3)$ on the plane $P: 2x+my+nz=4$, is $\left(-2, \frac{7}{2}, \frac{3}{2}\right)$, then the distance of the point A from the plane P, measured parallel to a line with direction ratios $3, -1, -4$, is equal to :
A:	1
B:	$\sqrt{26}$
C:	$2\sqrt{2}$
D:	$\sqrt{14}$

Topic:	Mathematics-Section A
Item No:	10
Question ID:	**100210**
Question Type:	MCQ
Question:	Let $\vec{a}=3\hat{i}+\hat{j}$ and $\vec{b}=\hat{i}+2\hat{j}+\hat{k}$. Let $\vec{c}$ be a vector satisfying $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=\vec{b}+\lambda\vec{c}$. If $\vec{b}$ and $\vec{c}$ are non-parallel, then the value of λ is :
A:	-5
B:	5
C:	1
D:	-1

Topic:	Mathematics-Section A
Item No:	11
Question ID:	**100211**
Question Type:	MCQ
Question:	The angle of elevation of the top of a tower from a point A due north of it is α and from a point B at a distance of 9 units due west of A is $\cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$. If the distance of the point B from the tower is 15 units, then $\cot\alpha$ is equal to :

A:	$\frac{6}{5}$
B:	$\frac{9}{5}$
C:	$\frac{4}{3}$
D:	$\frac{7}{3}$

Topic:	Mathematics-Section A
Item No:	12
Question ID:	**100212**
Question Type:	MCQ
Question:	The statement $(p \wedge q) \Rightarrow (p \wedge r)$ is equivalent to :
A:	$q \Rightarrow (p \wedge r)$
B:	$p \Rightarrow (p \wedge r)$
C:	$(p \wedge r) \Rightarrow (p \wedge q)$
D:	$(p \wedge q) \Rightarrow r$

Topic:	Mathematics-Section A
Item No:	13
Question ID:	**100213**
Question Type:	MCQ
Question:	Let the circumcentre of a triangle with vertices A(a, 3), B(b, 5) and C(a, b), $ab > 0$ be P(1, 1). If the line AP intersects the line BC at the point $Q(k_1, k_2)$, then $k_1 + k_2$ is equal to :
A:	2
B:	$\frac{4}{7}$

C:	$\frac{2}{7}$
D:	4

Topic:	Mathematics-Section A
Item No:	14
Question ID:	**100214**
Question Type:	MCQ
Question:	Let $\hat{a}$ and $\hat{b}$ be two unit vectors such that the angle between them is $\frac{\pi}{4}$. If θ is the angle between the vectors $\left(\hat{a}+\hat{b}\right)$ and $\left(\hat{a}+2\hat{b}+2\left(\hat{a}\times\hat{b}\right)\right)$, then the value of $164\cos^2\theta$ is equal to :
A:	$90+27\sqrt{2}$
B:	$45+18\sqrt{2}$
C:	$90+3\sqrt{2}$
D:	$54+90\sqrt{2}$

Topic:	Mathematics-Section A
Item No:	15
Question ID:	**100215**
Question Type:	MCQ
Question:	If $f(\alpha)=\int_1^{\alpha}\frac{\log_{10}t}{1+t}\,dt,\ \alpha>0$, then $f(e^3)+f(e^{-3})$ is equal to :
A:	9
B:	$\frac{9}{2}$
C:	$\frac{9}{\log_e(10)}$

D:	$\frac{9}{2\ \log_e(10)}$

Topic:	Mathematics-Section A
Item No:	16
Question ID:	**100216**
Question Type:	MCQ
Question:	The area of the region $\left\{(x, y): \lvert x-1\rvert \le y \le \sqrt{5-x^2}\right\}$ is equal to :
A:	$\frac{5}{2}\sin^{-1}\left(\frac{3}{5}\right)-\frac{1}{2}$
B:	$\frac{5\pi}{4}-\frac{3}{2}$
C:	$\frac{3\pi}{4}+\frac{3}{2}$
D:	$\frac{5\pi}{4}-\frac{1}{2}$

Topic:	Mathematics-Section A
Item No:	17
Question ID:	**100217**
Question Type:	MCQ
Question:	Let the focal chord of the parabola P : $y^2=4x$ along the line L : $y=mx+c$, m > 0 meet the parabola at the points M and N. Let the line L be a tangent to the hyperbola H : $x^2-y^2=4$. If O is the vertex of P and F is the focus of H on the positive x-axis, then the area of the quadrilateral OMFN is :
A:	$2\sqrt{6}$
B:	$2\sqrt{14}$
C:	$4\sqrt{6}$

D:	$4\sqrt{14}$

Topic:	Mathematics-Section A
Item No:	18
Question ID:	**100218**
Question Type:	MCQ
Question:	The number of points, where the function $f: \mathbf{R} \to \mathbf{R}$, $f(x) = \lvert x-1\rvert\cos\lvert x-2\rvert \sin\lvert x-1\rvert + (x-3)\lvert x^2-5x+4\rvert$, is **NOT** differentiable, is :
A:	1
B:	2
C:	3
D:	4

Topic:	Mathematics-Section A
Item No:	19
Question ID:	**100219**
Question Type:	MCQ
Question:	Let S = {1, 2, 3, ..., 2022}. Then the probability, that a randomly chosen number n from the set S such that HCF (n, 2022) = 1, is :
A:	$\frac{128}{1011}$
B:	$\frac{166}{1011}$
C:	$\frac{127}{337}$
D:	$\frac{112}{337}$

Topic:	Mathematics-Section A
Item No:	20
Question ID:	**100220**

Question Type:	MCQ
Question:	Let $f(x)=3^{(x^2-2)^3+4}$, $x \in \mathbf{R}$. Then which of the following statements are true ? P : $x=0$ is a point of local minima of f Q : $x=\sqrt{2}$ is a point of inflection of f R : f' is increasing for $x>\sqrt{2}$
A:	Only P and Q
B:	Only P and R
C:	Only Q and R
D:	All P, Q and R

Topic:	Mathematics-Section B
Item No:	21
Question ID:	**100221**
Question Type:	Numeric Answer
Question:	Let $S=\{\theta \in (0, 2\pi) : 7\cos^2\theta-3\sin^2\theta-2\cos^2 2\theta=2\}$. Then, the sum of roots of all the equations $x^2-2(\tan^2\theta+\cot^2\theta)x+6\sin^2\theta=0$, $\theta \in S$, is _____.

Topic:	Mathematics-Section B
Item No:	22
Question ID:	**100222**
Question Type:	Numeric Answer
Question:	Let the mean and the variance of 20 observations $x_1, x_2, \ldots, x_{20}$ be 15 and 9, respectively. For $\alpha \in \mathbf{R}$, if the mean of $(x_1+\alpha)^2, (x_2+\alpha)^2, \ldots, (x_{20}+\alpha)^2$ is 178, then the square of the maximum value of α is equal to _____.

Topic:	Mathematics-Section B
Item No:	23
Question ID:	**100223**
Question Type:	Numeric Answer

Question:	Let a line with direction ratios a, $-4a$, -7 be perpendicular to the lines with direction ratios 3, -1, 2b and b, a, -2. If the point of intersection of the line $\frac{x+1}{a^2+b^2} = \frac{y-2}{a^2-b^2} = \frac{z}{1}$ and the plane $x-y+z=0$ is (α, β, γ), then $\alpha+\beta+\gamma$ is equal to _____.

Topic:	Mathematics-Section B
Item No:	24
Question ID:	**100224**
Question Type:	Numeric Answer
Question:	Let $a_1, a_2, a_3, \ldots$ be an A.P. If $\sum_{r=1}^{\infty} \frac{a_r}{2^r} = 4$, then $4a_2$ is equal to _____.

Topic:	Mathematics-Section B
Item No:	25
Question ID:	**100225**
Question Type:	Numeric Answer
Question:	Let the ratio of the fifth term from the beginning to the fifth term from the end in the binomial expansion of $\left(\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}}\right)^n$, in the increasing powers of $\frac{1}{\sqrt[4]{3}}$ be $\sqrt[4]{6} : 1$. If the sixth term from the beginning is $\frac{\alpha}{\sqrt[4]{3}}$, then α is equal to _____.

Topic:	Mathematics-Section B
Item No:	26
Question ID:	**100226**
Question Type:	Numeric Answer
Question:	The number of matrices of order 3×3, whose entries are either 0 or 1 and the sum of all the entries is a prime number, is _____.

Topic:	Mathematics-Section B
Item No:	27
Question ID:	**100227**

Question Type:	Numeric Answer
Question:	Let p and $p+2$ be prime numbers and let $\Delta = \begin{vmatrix} p! & (p+1)! & (p+2)! \\ (p+1)! & (p+2)! & (p+3)! \\ (p+2)! & (p+3)! & (p+4)! \end{vmatrix}$ Then the sum of the maximum values of α and β, such that p^{α} and $(p+2)^{\beta}$ divide Δ, is _____.

Topic:	Mathematics-Section B
Item No:	28
Question ID:	**100228**
Question Type:	Numeric Answer
Question:	If $\frac{1}{2\times3\times4}+\frac{1}{3\times4\times5}+\frac{1}{4\times5\times6}+\ldots+\frac{1}{100\times101\times102}=\frac{k}{101}$, then 34 k is equal to _____.

Topic:	Mathematics-Section B
Item No:	29
Question ID:	**100229**
Question Type:	Numeric Answer
Question:	Let $S=\{4, 6, 9\}$ and $T=\{9, 10, 11, ..., 1000\}$. If $A=\{a_1+a_2+...+a_k : k \in \mathbf{N}, a_1, a_2, a_3, ..., a_k \in S\}$, then the sum of all the elements in the set $T-A$ is equal to _____.

Topic:	Mathematics-Section B
Item No:	30
Question ID:	**100230**
Question Type:	Numeric Answer
Question:	Let the mirror image of a circle $c_1 : x^2+y^2-2x-6y+\alpha=0$ in line $y=x+1$ be $c_2 : 5x^2+5y^2+10gx+10fy+38=0$. If r is the radius of circle c_2, then $\alpha+6r^2$ is equal to _____.